Searching for collectivity and testing the limits of hydrodynamics: results from the 2016 d+Au beam energy scan

Ron Belmont
University of Colorado Boulder

Brookhaven National Lab
Upton, NY
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PHENIX

University of Colorado Boulder

Outline

- Introduction
- Physics motivation for small systems
- Toolkit
  - Experimental setup
  - AMPT simulations
- Results
  - Two-particle correlations ($\Delta\phi$, ridge, etc)
  - Event plane results
  - Multiparticle correlations
- Summary and outlook

Утро в сосновом лесу

Утро в сосновом лесу

A very brief history of recent heavy ion physics

- 1980s and 1990s—AGS and SPS... QGP at SPS!
- Early 2000s—QGP at RHIC! No QGP at SPS. d+Au as control.
- Mid-late 2000s—Detailed, quantitative studies of strongly coupled QGP. d+Au as control.
- 2010—Ridge in high multiplicity p+p (LHC)! Probably CGC!
- Early 2010s—QGP in p+Pb!
- Early 2010s—QGP in d+Au!
- Mid 2010s and now-ish—QGP in high multiplicity p+p? QGP in mid-multiplicity p+p?? QGP in d+Au even at low energies???

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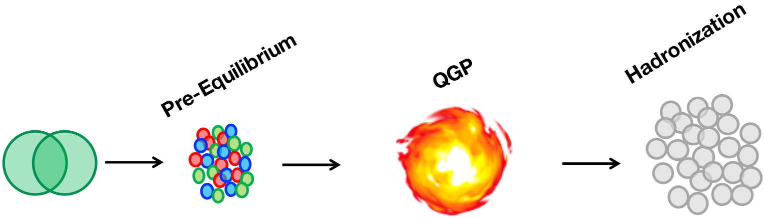
- Early 2010s—QGP in d+Au!
- Mid 2010s and now-ish—QGP in high multiplicity p+p? QGP in mid-multiplicity p+p?? QGP in d+Au even at low energies???

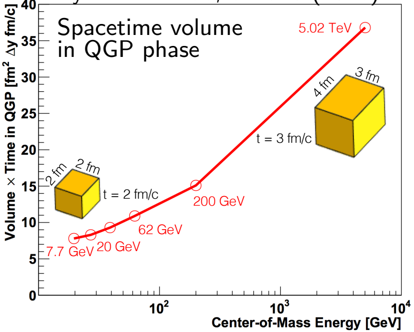
"Twenty years ago, the challenge in heavy ion physics was to find the QGP. Now, the challenge is to not find it." —Jürgen Schukraft, QM17

Testing hydro by controlling system size



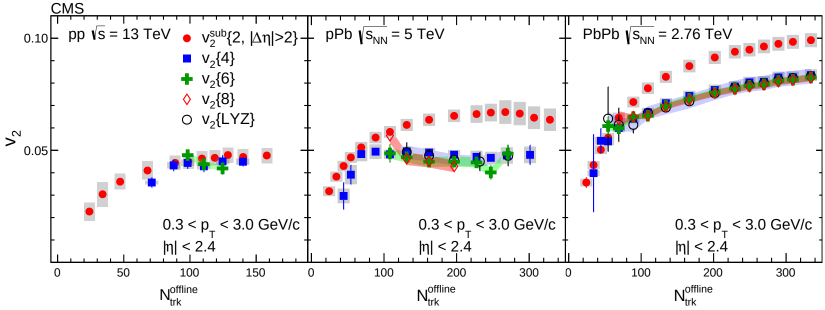
- Standard picture for A+A: QGP in hydro evolution
- What about small systems? And lower energies?
- Use collisions species and energy to control system size, test limits of hydro applicability

J.D. Orjuela Koop et al
Phys. Rev. C 93, 044910 (2016)



Multiparticle correlations in small systems

CMS, Phys. Lett. B 765 (2017) 193-220



- Multiparticle correlations: a strong case for collectivity in small systems
- Gaussian fluctuations:

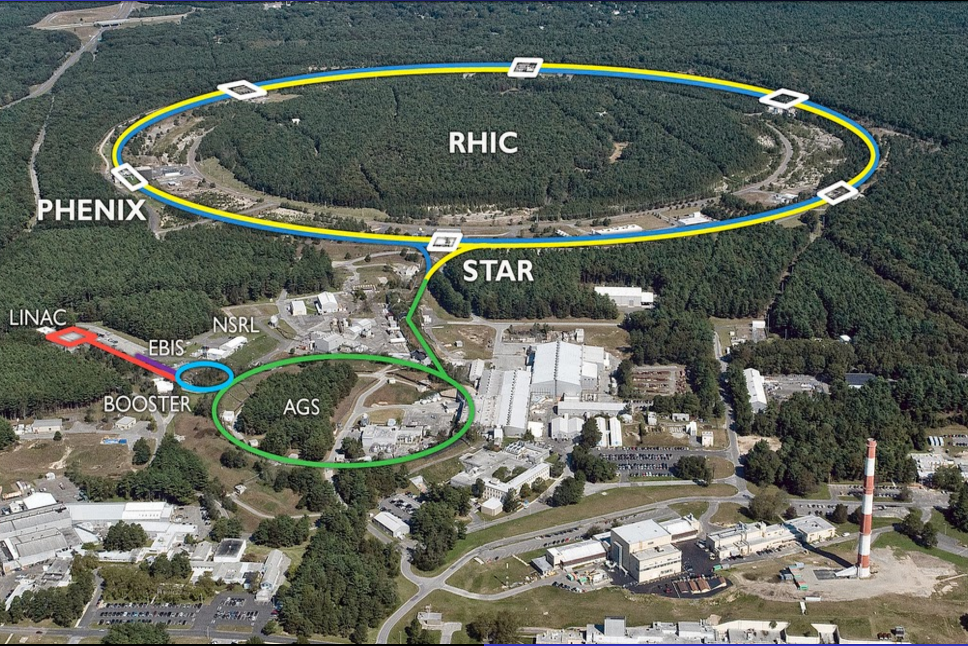
$$v_2\{2\} = \sqrt{v_2^2 + \sigma^2 + \delta} \qquad \delta \text{ non-flow, } \sigma \text{ variance}$$

$$v_2\{2, |\Delta\eta| > 2\} = \sqrt{v_2^2 + \sigma^2} \qquad \text{eta gap removes some non-flow}$$

$$v_2\{4\} \approx v_2\{6\} \approx v_2\{8\} \approx \sqrt{v_2^2 - \sigma^2} \qquad \text{higher orders remove non-flow}$$

- Can multiparticle correlations be measured in small systems at RHIC?

The Relativistic Heavy Ion Collider



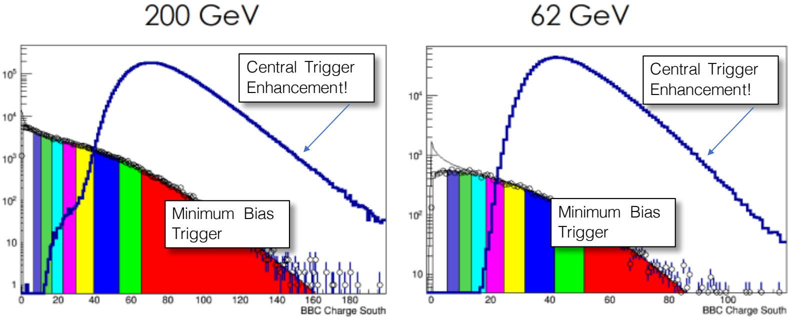
The Relativistic Heavy Ion Collider

- RHIC is the only polarized proton collider in the world
- RHIC is one of two heavy ion colliders, the other being the LHC
- RHIC is a dedicated ion collider and is designed to collide many different species of ions at many different energies—vastly more flexible than the LHC

Collision Species	Collision Energies (GeV)
p↑+p↑	510, 500, 200, 62.4
p+Al	200
p+Au	200
d+Au	200, 62.4, 39, 19.6
^{3}He+Au	200
Cu+Cu	200, 62.4, 22.5
Cu+Au	200
Au+Au	200, 130, 62.4, 56, 39, 27, 19.6, 15, 11.5, 7.7, 5, ...
U+U	193

And lots more to come!

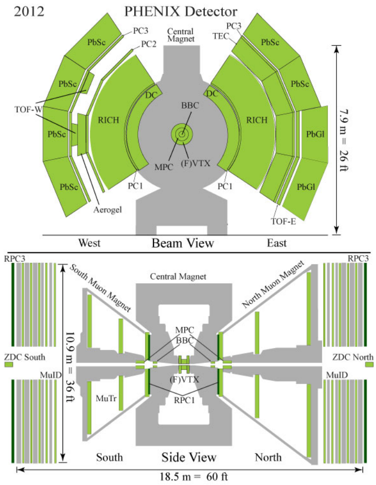
2016 d+Au beam energy scan in PHENIX



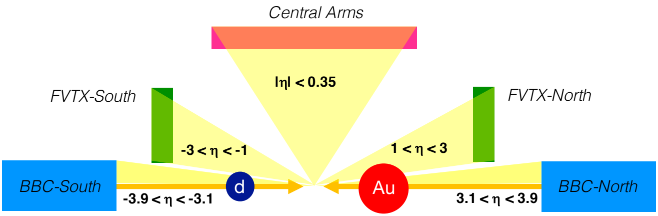
d+Au collision energy	total events analyzed	central events analyzed
200 GeV	636 million	585 million
62.4 GeV	131 million	76 million
39 GeV	137 million	49 million
19.6 GeV	15 million	3 million

PHENIX

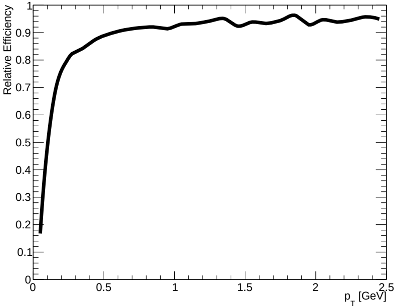
- Central arms (CNT) cover $|\eta| < 0.35$, tracking, momentum determination, PID, etc.
- Forward vertex detector (FVTX) covers $1 < |\eta| < 3$, tracking only (no momentum vector information)
- Beam beam counters (BBC): centrality and vertex determination
- BBC and FVTX used for triggering
- BBC south and FVTX south used for event plane determination for Run16 flow analysis (south = backward)
- CNT, FVTXS, BBCS used for event plane resolution



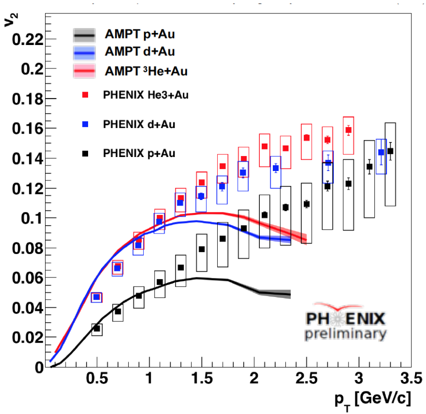
The PHENIX forward vertex detector



- FVTX: forward vertex detector —silicon strip technology
- Very precise vertex/DCA determination
- No momentum determination, p_T dependent efficiency — measured v_2 roughly 18% higher than true



A Multi-Phase Transport model



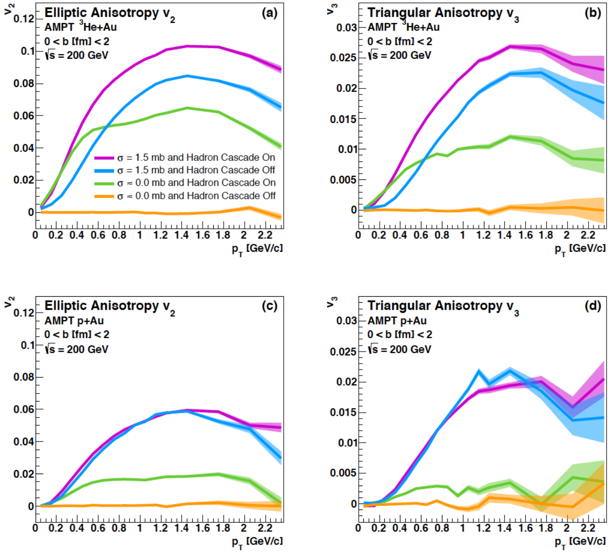
AMPT basic features

Initial conditions	MC Glauber
Particle production	String melting
Pre-equilibrium	None
Expansion	Parton scattering (tunable)
Hadronization	Spatial coalescence
Final stage	Hadron cascade (tunable)

- AMPT has significant success in describing flow-like signatures (for low p_T and p_T-integrated)
- AMPT produces final state particles over the full available phasespace —possible to perform exact same analysis on data and model

AMPT with no scattering

J.D. Orjuela Koop et al Phys. Rev. C 92, 054903 (2015)

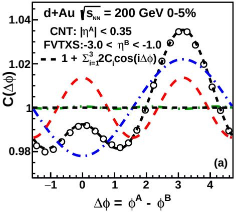


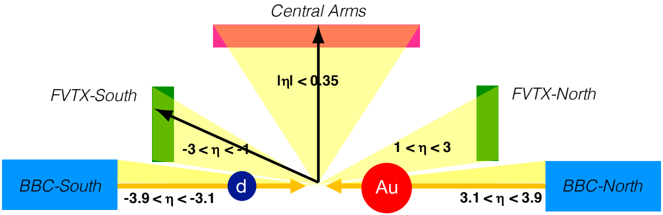
- Turn off scattering in AMPT—remove all correlations with initial geometry $\sigma_{parton} = 0$ and $\sigma_{hadron} = 0$
- Participant plane v_2 goes to zero
- Other sources of correlation remain—non-flow

Intermission

Results

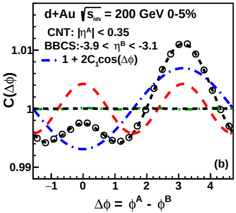
Two particle correlations

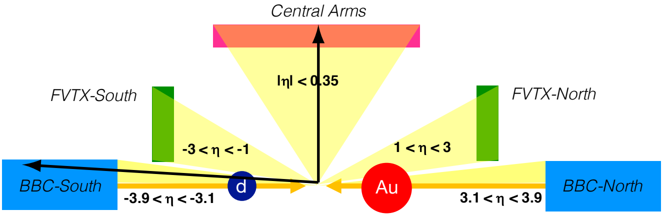




- $0.65 < |\Delta\eta| < 3.35$

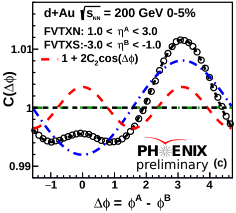
Two particle correlations

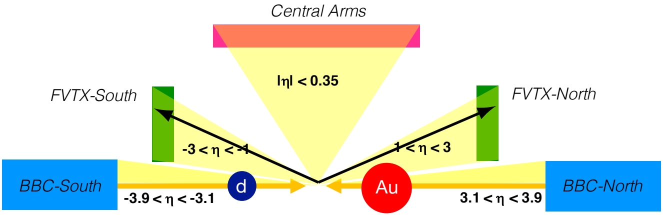




- $2.75 < |\Delta\eta| < 4.25$

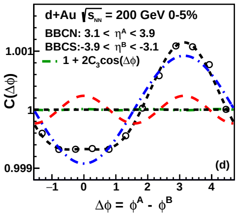
Two particle correlations

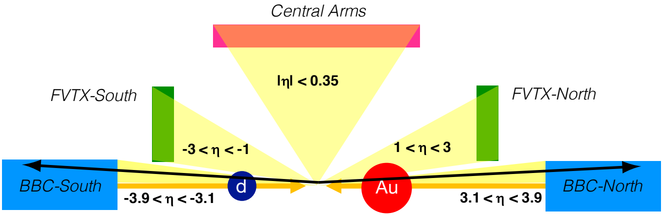




- $2.0 < |\Delta\eta| < 6.0$

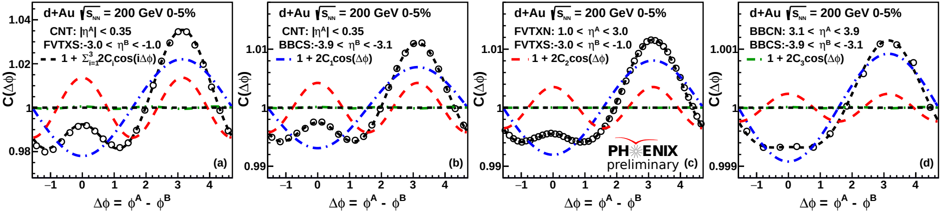
Two particle correlations

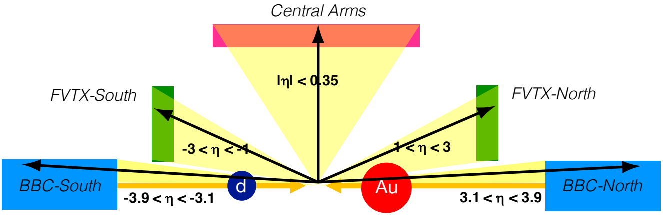




- $6.2 < |\Delta\eta| < 7.8$

Two particle correlations





- $0.65 < |\Delta\eta| < 7.8$
- Wide range of pseudorapidity separation has potential to provide significant leverage to disentangle various flow and non-flow effects
- Ridge observed for $|\Delta\eta| > 6.2$—long range indeed! (means early times)

Intermission

Event plane results

Experimental method and details

Definition of Q-vectors

$$Q_{n,x} = \sum_{i=1}^{M} \cos n\phi_i = \Re Q_n, \quad Q_{n,y} = \sum_{i=1}^{M} \sin n\phi_i = \Im Q_n$$

Calculation of event plane

$$n\psi_n = \arctan \frac{Q_{n,y}}{Q_{n,x}}$$

Calculation of harmonic coefficients v_n

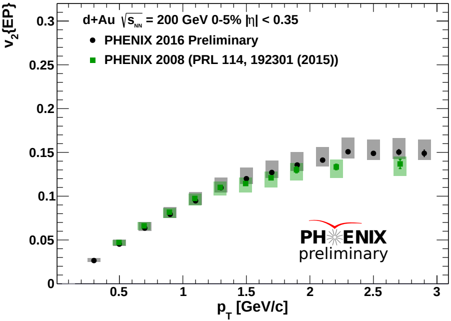
$$v_n = \langle \cos(n(\phi - \psi_n)) \rangle / R(\psi_n)$$

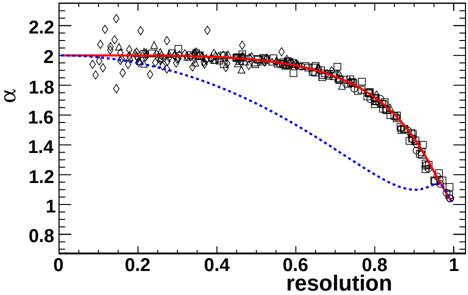
Determination of event plane resolutions

$$R(\psi_n^A) = \sqrt{\frac{\langle \cos(n(\psi_n^A - \psi_n^B)) \rangle \langle \cos(n(\psi_n^A - \psi_n^C)) \rangle}{\langle \cos(n(\psi_n^B - \psi_n^C)) \rangle}}$$

(CNT, FVTXS, BBCS used for event plane resolution)

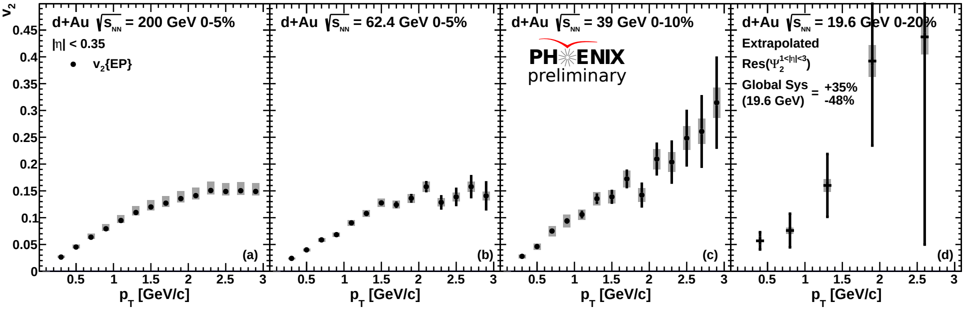
J.Y. Ollitrault et al Phys Rec C 80, 014904 (2009)





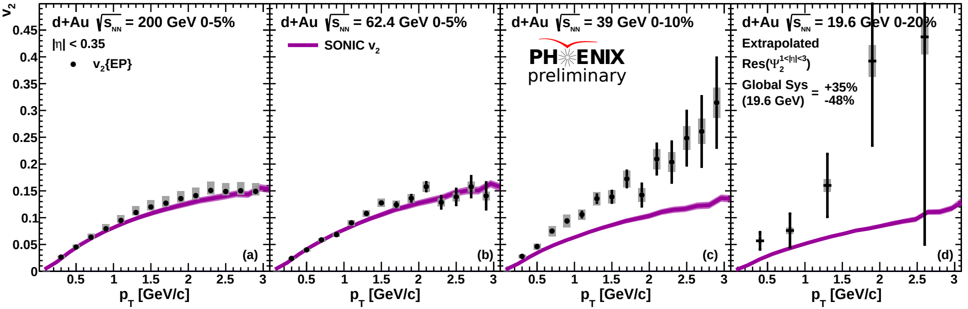
- Important to remember that $v_n\{\text{EP}\}$ is an estimator of $\langle v_n^\alpha \rangle^{1/\alpha}$
- High multiplicity → high resolution → $\alpha = 1$
- Low multiplicity → low resolution → $\alpha = 2$
- For all RHIC small systems results, $\alpha = 2$
  —same dependence on fluctuations as two-particle methods

v_2 vs p_T, comparisons to theory

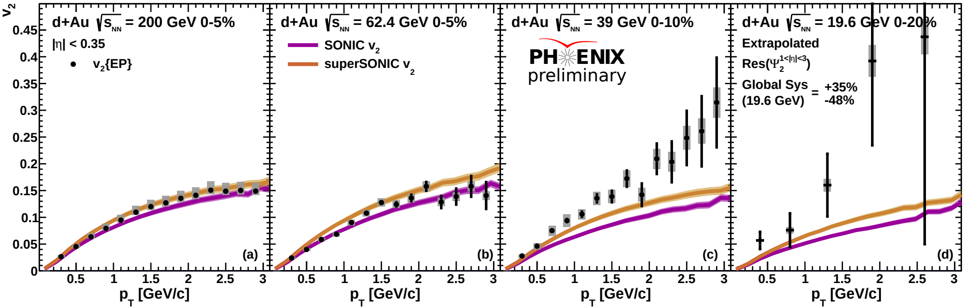


- Event plane v_2 vs p_T measured for all energies

v_2 vs p_T, comparisons to theory



- Event plane v_2 vs p_T measured for all energies
- Hydro theory agrees with higher energies very well, far underpredicts lower energies—lots of non-flow at lower energies (you probably expected that?)



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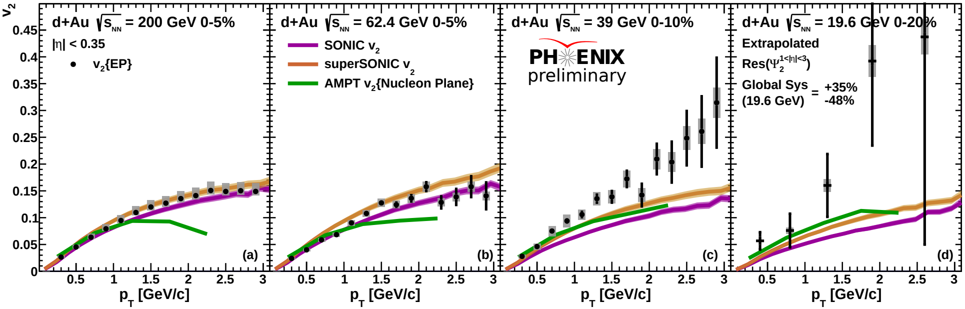

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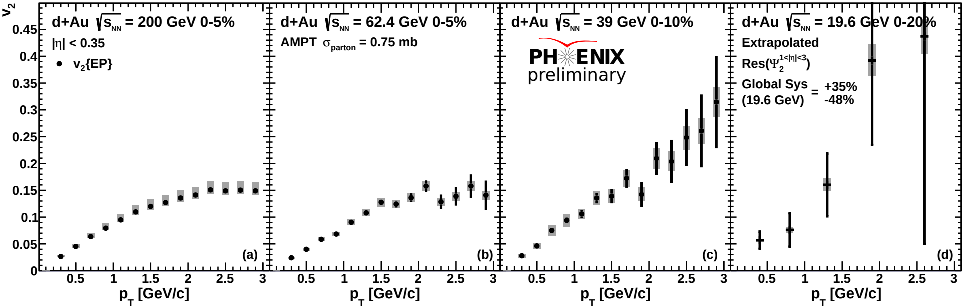
- Hydro theory with pre-flow very similar (pre-flow must be there...)
- AMPT nucleon plane similar to hydro at low p_T

v_2 vs p_T, comparisons to AMPT



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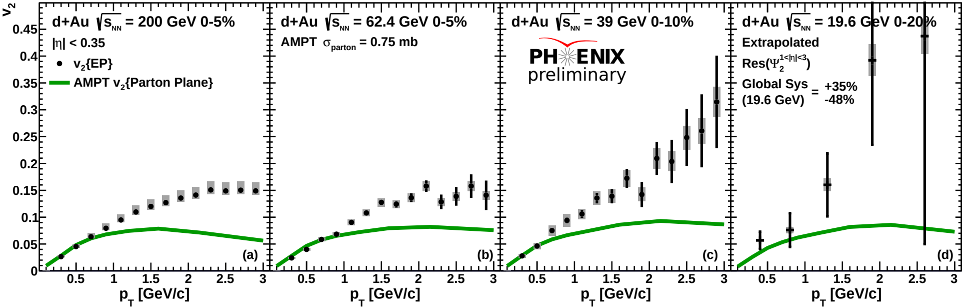
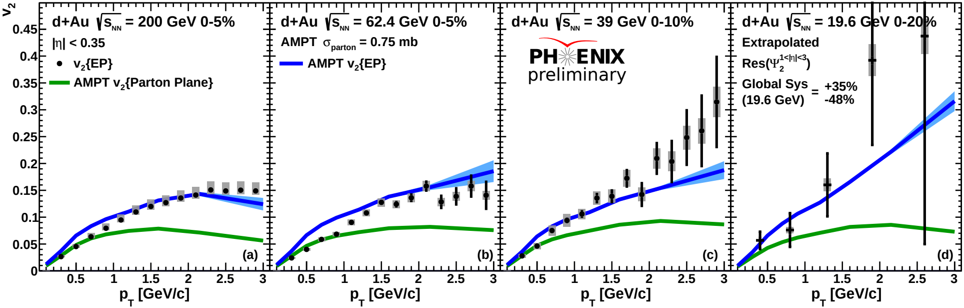
v_2 vs p_T, comparisons to AMPT



- Event plane v_2 vs p_T measured for all energies
- AMPT flow only shows good agreement at low p_T

v_2 vs p_T, comparisons to AMPT



- Event plane v_2 vs p_T measured for all energies
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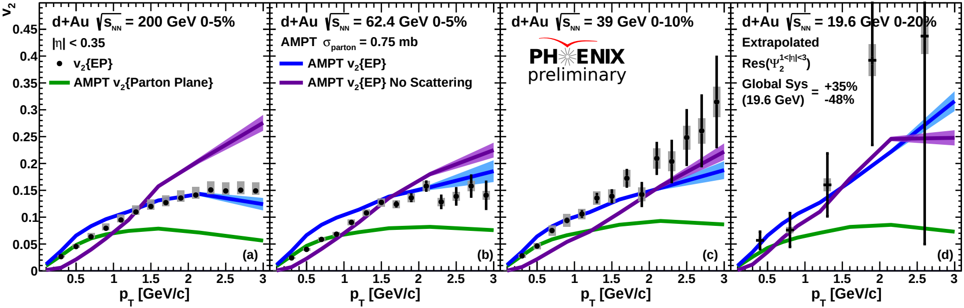
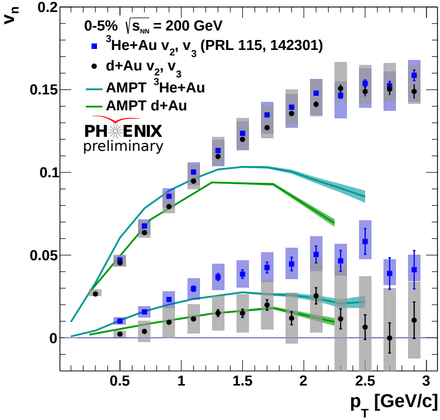
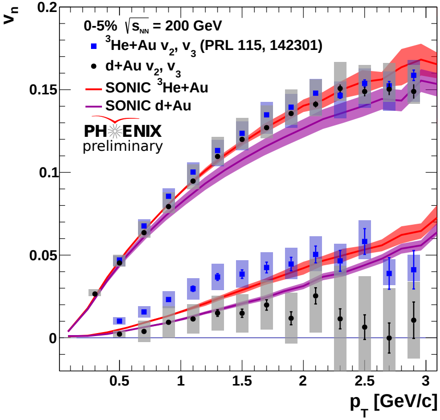
- AMPT flow+non-flow shows reasonable agreement for all p_T

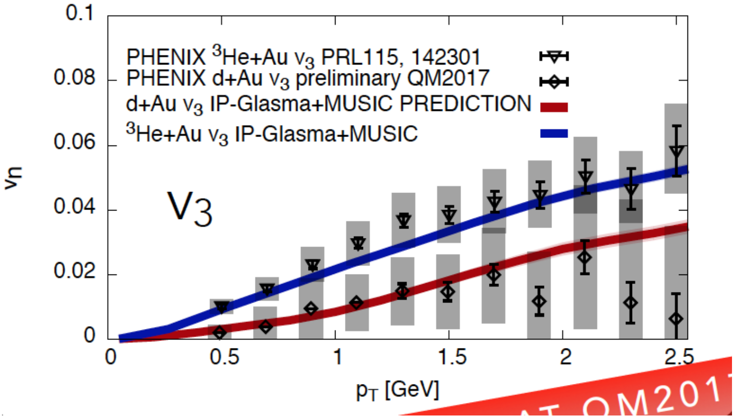


- Event plane v_2 vs p_T measured for all energies
- AMPT flow only shows good agreement at low p_T
- AMPT flow+non-flow shows reasonable agreement for all p_T
- AMPT non-flow only far under-predicts for low p_T, too high for high p_T





- v_3 is non-zero and lower in d+Au compared to ^{3}He+Au
- Excellent further confirmation that geometry engineering works



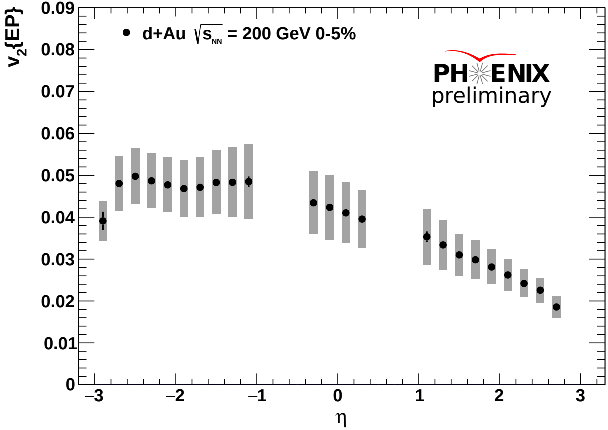
- v_3 is non-zero and lower in d+Au compared to ^{3}He+Au
- Excellent further confirmation that geometry engineering works
- New hydro prediction from Björn shows excellent agreement with data

Intermission

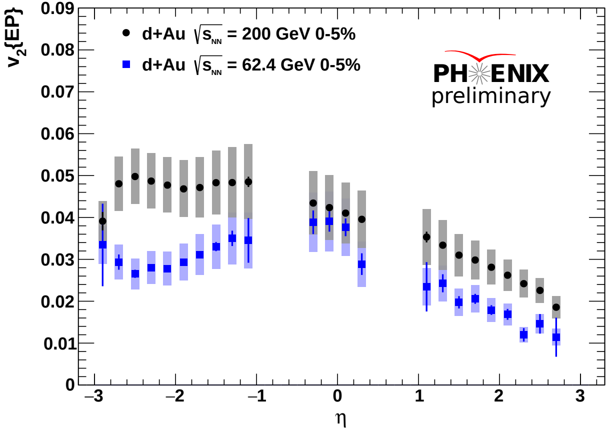
- Ridge observed even for $|\Delta\eta| > 6.2$
- Positive v_2 vs p_T observed at all energies
- Hydro theory describes 200 GeV and 62.4 GeV with flow only while 39 and 19.6 GeV must have some significant non-flow
- AMPT suggests flow dominates at low p_T, mix of flow+non-flow at mid and high p_T
- v_3 in d/^{3}He+Au confirm geometry engineering, excellent agreement with hydro (Björn)

Intermission

v_2 vs η



- BBC south ($-3.9 < \eta < -3.1$) used to estimate the event plane
- 200 GeV shows strong forward/backward asymmetry



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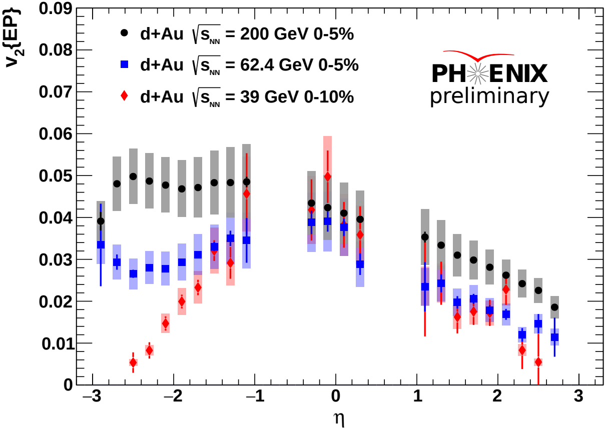
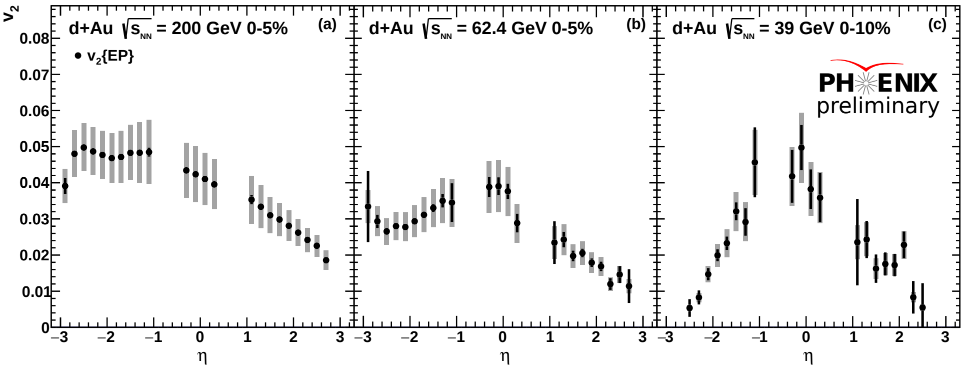


- BBC south ($-3.9 < \eta < -3.1$) used to estimate the event plane
- 200 GeV shows strong forward/backward asymmetry
- 62.4 GeV shows similar weaker forward/backward asymmetry
- 39 GeV seems to have very little forward/backward asymmetry...

v_2 vs η, comparison to theory



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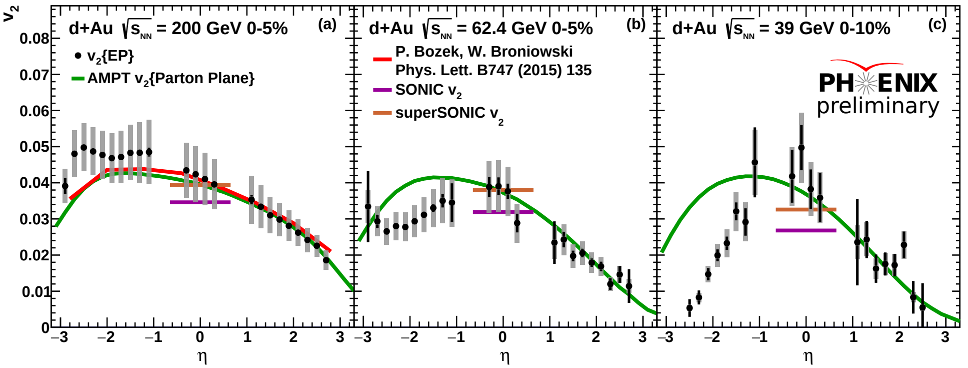
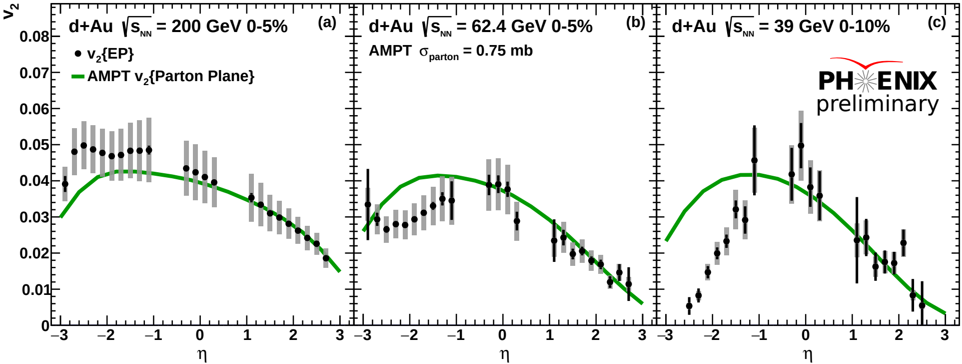
v_2 vs η, comparison to theory



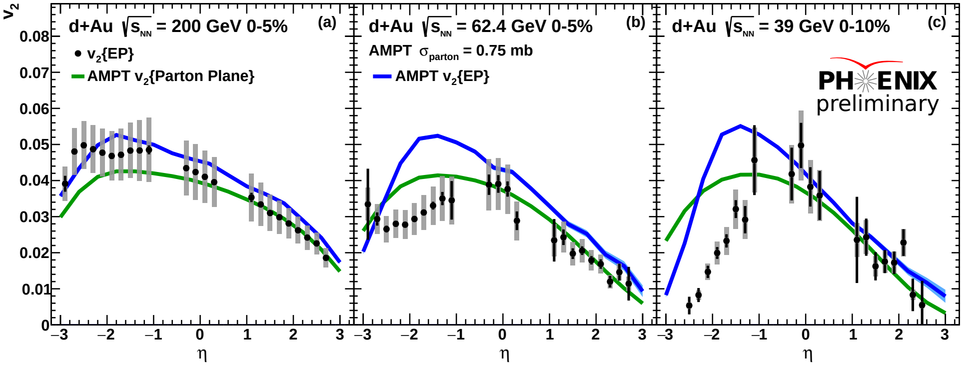
- BBC south ($-3.9 < \eta < -3.1$) used to estimate the event plane
- Hydro theory describes 200 GeV very well
- AMPT flow only completely consistent with hydro theory at 200 GeV
- AMPT flow only describes lower energies very well at mid and forward, but has large forward/backward asymmetry at all energies... what gives?

v_2 vs η, comparison with AMPT



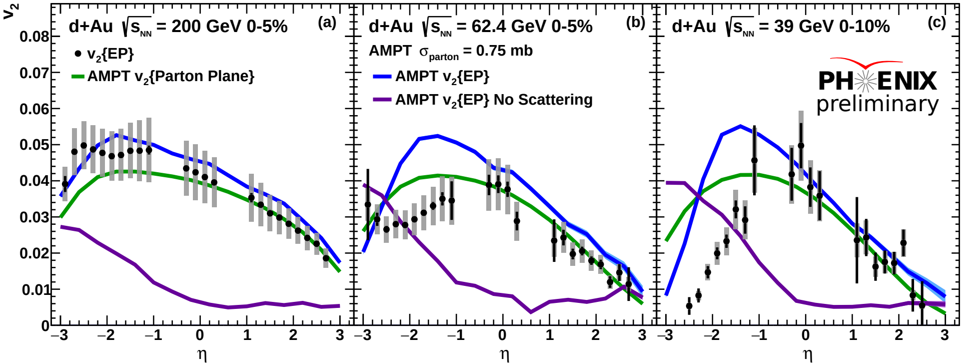
- AMPT flow only agrees with mid and forward rapidity very well, but shows higher v_2 at backward for lower energies

v_2 vs η, comparison with AMPT



- AMPT flow only agrees with mid and forward rapidity very well, but shows higher v_2 at backward for lower energies
- AMPT flow+non-flow is very similar at mid and forward
- AMPT flow+non-flow shows striking anti-correlation at backward rapidity

v_2 vs η, comparison with AMPT



- AMPT flow only agrees with mid and forward rapidity very well, but shows higher v_2 at backward for lower energies
- AMPT flow+non-flow is very similar at mid and forward
- AMPT flow+non-flow shows striking anti-correlation at backward rapidity
- AMPT non-flow only shows nothing at mid and forward, large v_2 at backward rapidity near the detector

Intermission

- More hydro theory calculations for η dependence would be very helpful
- The data shows large forward/backward asymmetry that decreases with energy, but is that what's really happening?
- AMPT flow only shows forward/backward asymmetry at all energies
- AMPT flow+non-flow shows strong anticorrelation between flow and non-flow at backward rapidity that brings v_2 backward down significantly

Intermission

Multiparticle correlations

Experimental method and details

Definition of Q-vectors is the same as before

Two particle correlator:

$$\begin{aligned} \langle 2 \rangle &= \langle \cos(n(\phi_1 - \phi_2)) \rangle \quad (= v_n^2) \\ &= \frac{Q_n Q_n^* - M}{M(M-1)} \end{aligned}$$

Four particle correlator:

$$\begin{aligned} \langle 4 \rangle &= \langle \cos(n(\phi_1 + \phi_2 - \phi_3 - \phi_4)) \rangle \quad (= v_n^4) \\ &= \frac{|Q_n|^4 + |Q_{2n}|^2 - 2\mathfrak{Re}\left[Q_{2n} Q_n^* Q_n^*\right]}{M(M-1)(M-2)(M-3)} - 2\,\frac{2(M-2)\,|Q_n|^2 - M(M-3)}{M(M-1)(M-2)(M-3)}. \end{aligned}$$

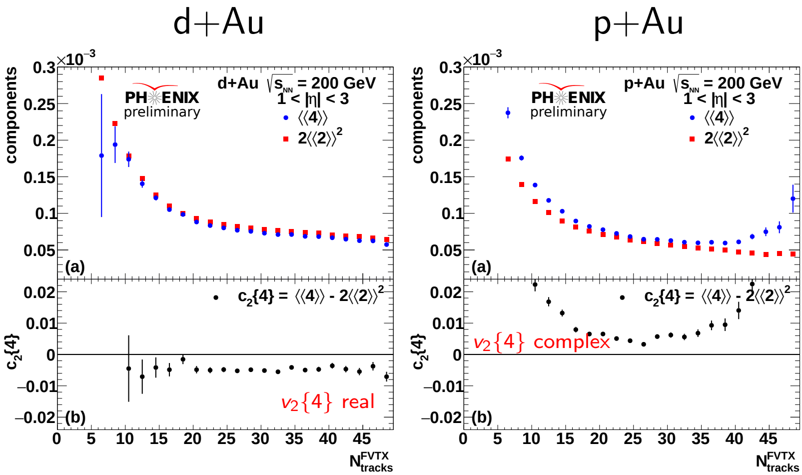
Calculation of cumulants and harmonic coefficients

$$c_n\{2\} = \langle\langle 2 \rangle\rangle \qquad = \langle v_n^2 \rangle \qquad v_n\{2\} = \sqrt{c_n\{2\}}$$

$$c_n\{4\} = \langle\langle 4 \rangle\rangle - 2\langle\langle 2 \rangle\rangle^2 \qquad = \langle v_n^4 \rangle - 2\langle v_n^2 \rangle^2 \qquad v_n\{4\} = \sqrt[4]{-c_n\{4\}}$$

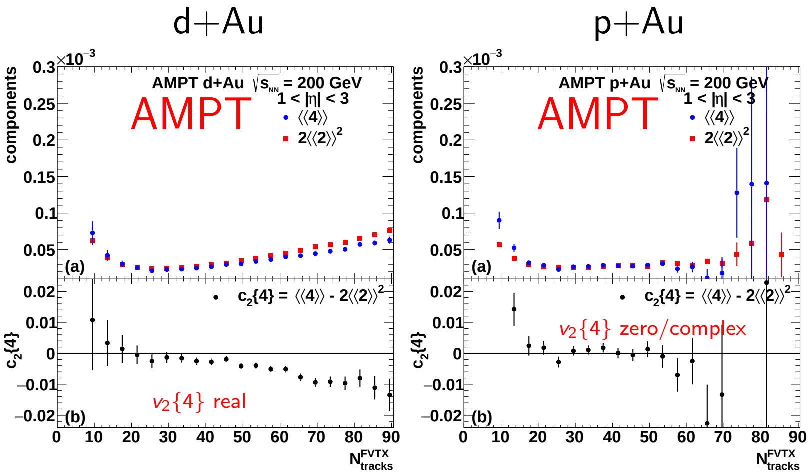
(FVTXN and FVTXS used for multiparticle calculations)

Components and cumulants in p+Au and d+Au at 200 GeV

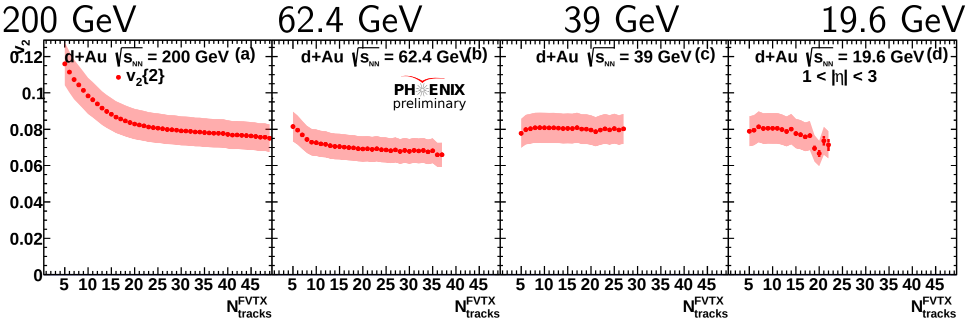


- Real $v_2\{4\}$ in d+Au, complex $v_2\{4\}$ in p+Au
- Fluctuations could dominate in the p+Au ($v_2\{4\} \approx \sqrt{v_2^2 - \sigma^2}$)

Components and cumulants in p+Au and d+Au in AMPT

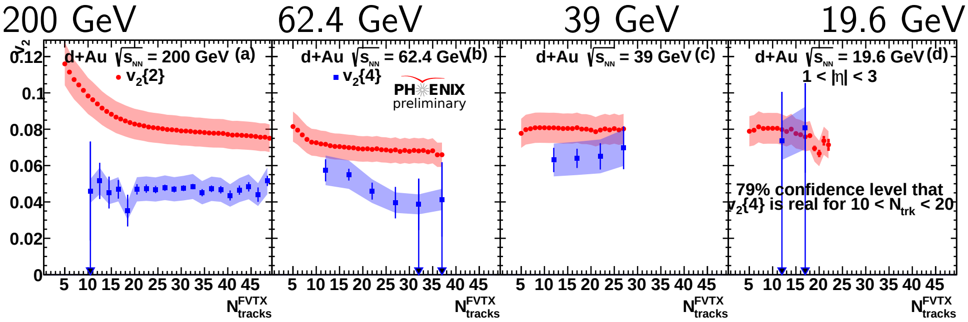


- AMPT similar to data—real $v_2\{4\}$ in d+Au, complex $v_2\{4\}$ in p+Au
- Fluctuations could dominate in the p+Au ($v_2\{4\} \approx \sqrt{v_2^2 - \sigma^2}$)



- $v_2\{2\}$ relatively constant with N_{tracks}^{FVTX} and collision energy

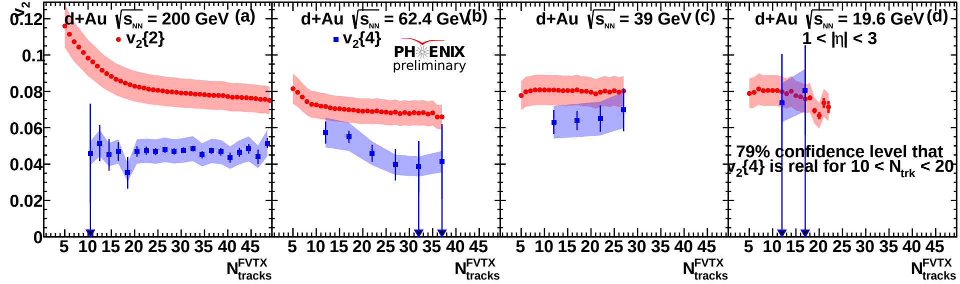
$v_2\{2\}$ and $v_2\{4\}$ in the d+Au beam energy scan



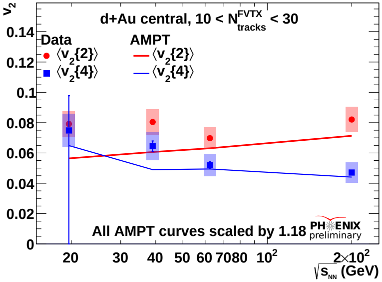
- $v_2\{2\}$ relatively constant with N_{tracks}^{FVTX} and collision energy
- Observation of real $v_2\{4\}$ in d+Au at all energies!
- Strong evidence for collectivity

$v_2\{2\}$ and $v_2\{4\}$ in the d+Au beam energy scan

200 GeV 62.4 GeV 39 GeV 19.6 GeV



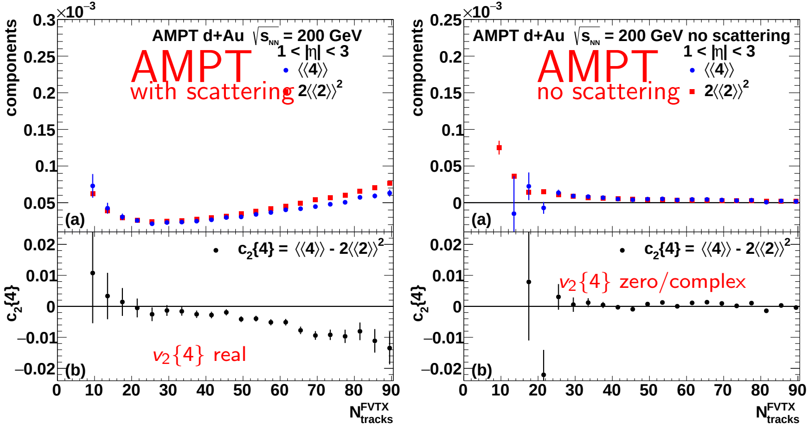
- Select $10 < N^{FVTX}_{tracks} < 30$, integrate
- Trend of $v_2\{2\}$ and $v_2\{4\}$ merging as $\sqrt{s_{NN}}$ is lowered
- AMPT sees the same trend



The story so far:

- Real $v_2\{4\}$ in d+Au collisions at all energies!
- Complex $v_2\{4\}$ in p+Au at 200 GeV, maybe fluctuations dominate? Good reason to believe collectivity/flow in p+Au as well
- Is real-valued $v_2\{4\}$ really a good measure for collectivity?
- Good news: we can turn the knobs in AMPT to see if we can draw a clear connection between $v_2\{4\}$ and initial geometry in d+Au

AMPT with no scattering



- Turn off scattering in AMPT—remove all correlations with initial geometry
- Components show different trend but are still non-zero
- But $v_2\{4\}$ goes from real to ∼zero—connection between real $v_2\{4\}$ and geometry in d+Au

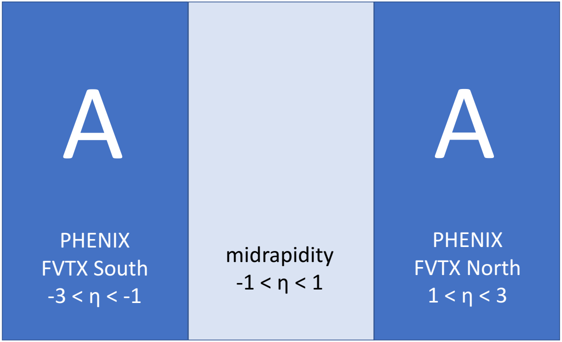
Intermission

The story so far:

- Real $v_2\{4\}$ in d+Au collisions at all energies!
- Clear connection between real $v_2\{4\}$ and initial geometry—strong evidence for collective behavior

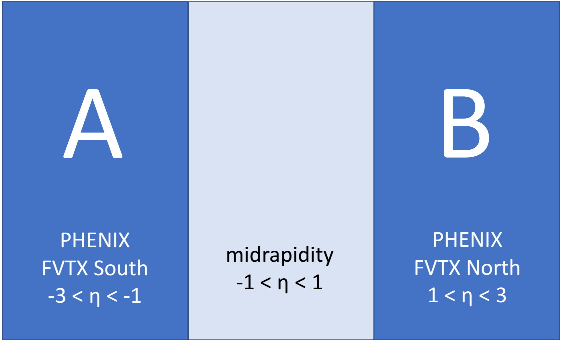
What about the non-flow?

- We've shown $v_2\{2\}$ but potentially significant non-flow
- We assume $v_2\{4\}$ removes all the non-flow, but are we sure?
- Try to apply an eta gap on the 2-particle ($v_2\{2, |\Delta\eta| > 2\}$) to get a better handle on non-flow



- $v_2\{2\}$ and $v_2\{4\}$—use tracks anywhere in the FVTX

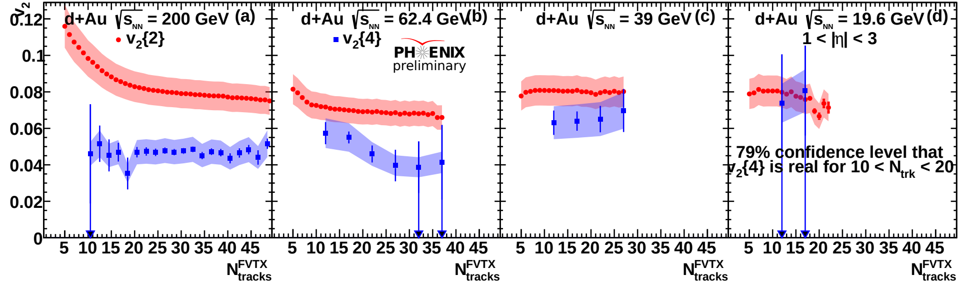
How to apply an eta gap in the FVTX?



- $v_2\{2\}$ and $v_2\{4\}$—use tracks anywhere in the FVTX
- $v_2\{2, |\Delta\eta| > 2\}$—require one track in south (backward rapidity) and one in north (forward)

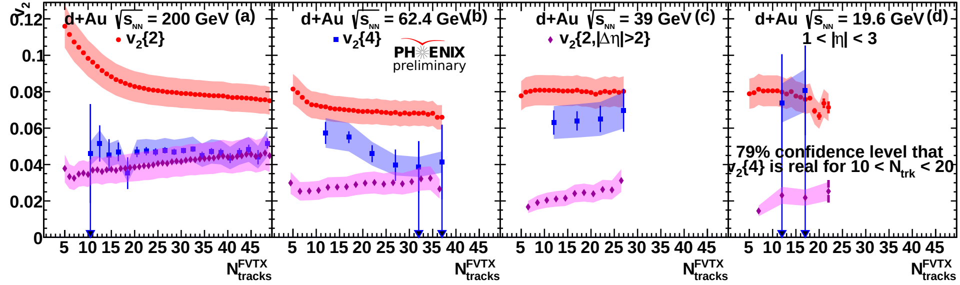
Can we apply an eta gap to get a better handle on the non-flow?

200 GeV | 62.4 GeV | 39 GeV | 19.6 GeV



- $v_2\{2\}$ and $v_2\{4\}$ vs N^{FVTX}_{tracks}, all tracks anywhere in FVTX

200 GeV 62.4 GeV 39 GeV 19.6 GeV



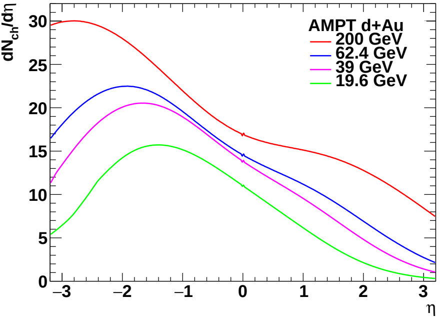
- $v_2\{2\}$ and $v_2\{4\}$ vs N^{FVTX}_{tracks}, all tracks anywhere in FVTX
- $v_2\{2,|\Delta\eta|>2\}$ vs N^{FVTX}_{tracks}, one track backward, the other forward

$$v_2\{2,|\Delta\eta|>2\} = \sqrt{v_2^2+\sigma^2} \qquad v_2\{2\} = \sqrt{v_2^2+\sigma^2+\delta}$$

$$v_2\{4\} \approx \sqrt{v_2^2-\sigma^2}$$

- Hard to understand this result based on fluctuations
- The eta gap reduces the non-flow, but what else does it do?

What can AMPT tell us about asymmetric collisions?



- Asymmetric collision systems have:
  —asymmetric $dN_{ch}/d\eta$
  —asymmetric v_2 vs η
- The FVTX combined is weighted by $dN_{ch}/d\eta$ towards backward rapidity, where v_2 is also higher—the effect is more pronounced at lower energies
- The FVTX two subevent is equally weighted between forward and back: $\sqrt{v_2^B v_2^F}$

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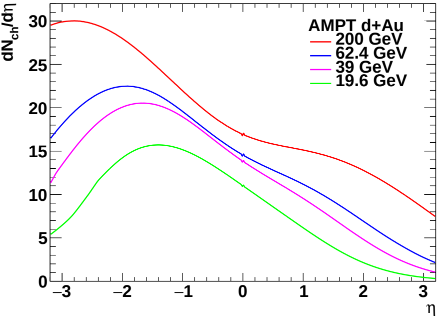

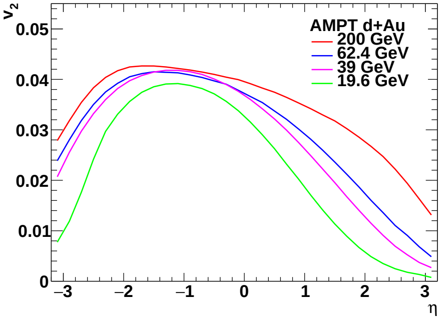

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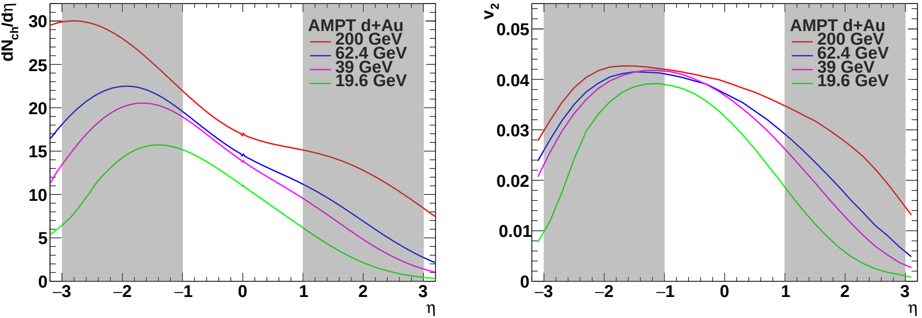

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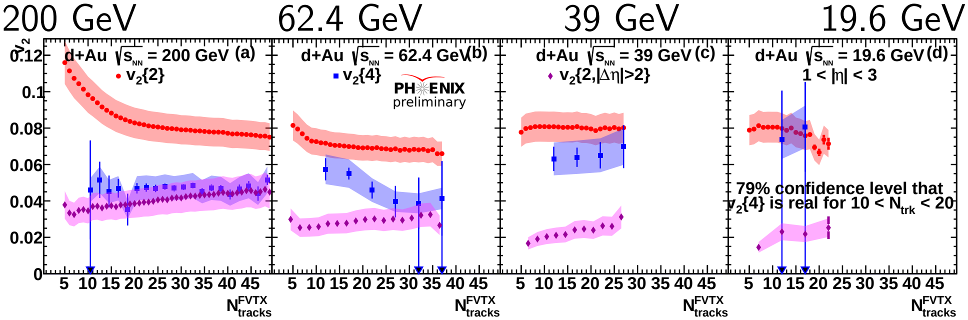
Understanding $v_2\{2\}$, $v_2\{4\}$, and $v_2\{2, |\Delta\eta| > 2\}$



- $v_2\{2\}$ and $v_2\{4\}$ vs N^{FVTX}_{tracks}—weighted average of v_2^B and v_2^F
- $v_2\{2, |\Delta\eta| > 2\}$ vs N^{FVTX}_{tracks}—fixed, equal weighting $\sqrt{v_2^B v_2^F}$
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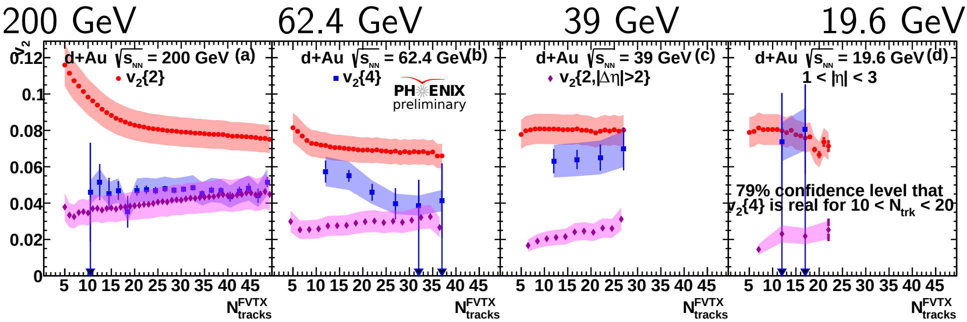
Understanding $v_2\{2\}$, $v_2\{4\}$, and $v_2\{2, |\Delta\eta| > 2\}$



- $v_2\{2\}$ and $v_2\{4\}$ vs N_{tracks}^{FVTX}—weighted average of v_2^B and v_2^F
- $v_2\{2, |\Delta\eta| > 2\}$ vs N_{tracks}^{FVTX}—fixed, equal weighting $\sqrt{v_2^B v_2^F}$
- $dN_{ch}/d\eta$ and v_2 vs η alone may explain these results
- There may be additional effects like event plane decorrelation, e.g. $v_2\{2, |\Delta\eta| > 2\} = \sqrt{v_2^B v_2^F \cos(2(\psi_2^B - \psi_2^F))}$

The cumulant story so far...

- Observation of real-valued $v_2\{4\}$ in d+Au collisions at 200, 62.4, 39, and 19.6 GeV from the 2016 d+Au beam energy scan
  - Connection between real-valued $v_2\{4\}$ and initial geometry established
  - Strong evidence for collectivity
- $v_2\{4\}$ observed to be everywhere complex in p+Au collisions at 200 GeV
  - Could be dominance of fluctuations ($\sigma/v_2 > 1$)
  - Good reason to believe collectivity exists in p+Au (many PHENIX measurements on the topic)
  - Contrast with p+Pb at 5.02 TeV—what happens in between RHIC and LHC energies?
- The trend of $v_2\{2\}$, $v_2\{2, |\Delta\eta| > 2\}$, and $v_2\{4\}$ with collision energy is consistent with expectations based on $dN_{ch}/d\eta$ and v_2 vs η
  - AMPT shows similar trends
  - Additional effects may be at play—event plane decorrelation?
- Additional measurements are possible and potentially very valuable
  - We plan to explore $v_2\{6\}$ in d+Au
  - It may be possible to look at $v_3\{4\}$ in ^{3}He+Au

Intermission

Clearly, "fluctuations" are doing a lot of work for us. What do we mean, and how well do we understand them?

Cumulants: a closer inspection

- We always say $v_2\{4\} \approx v_2\{6\} \approx v_2\{8\} \approx \sqrt{v_2^2 - \sigma^2}$

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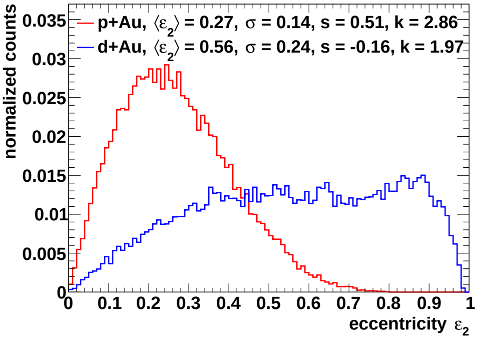
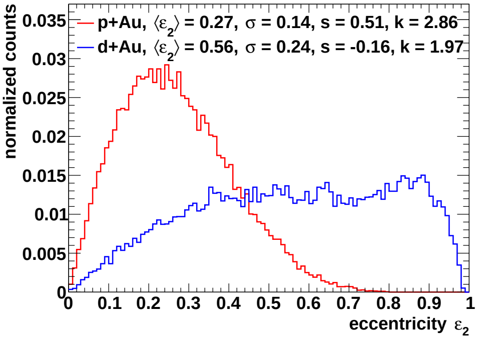
    - Gaussian fluctuations
    - Small relative variance, $\sigma/v_n \ll 1$
- Are these assumptions valid? Let's have a look...

Eccentricity distributions and cumulants



	p+Au	d+Au
$\langle \varepsilon_2\{2\} \rangle$	0.303	0.610
$\langle \varepsilon_2 \rangle$	0.270	0.560
$\langle \varepsilon_2\{4\} \rangle$ Approx.	0.232	0.505
$\langle \varepsilon_2\{4\} \rangle$ Exact	0.166	0.508

- Eccentricity cumulants: $\varepsilon_2\{2\} = (\langle \varepsilon_2^2 \rangle)^{1/2}$, $\varepsilon_2\{4\} = (-(\langle \varepsilon_2^4 \rangle - 2\langle \varepsilon_2^2 \rangle^2))^{1/4}$
- We don't have the v_n distribution but in the hydro limit $v_n \propto \varepsilon_n$



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- We don't have the v_n distribution but in the hydro limit $v_n \propto \varepsilon_n$
- Gaussian? No. Small relative variance? No.

Back to basics (a brief excursions)

The (raw) moments of a probability distribution function $f(x)$:

$$\mu_n = \langle x^n \rangle \equiv \int_{-\infty}^{+\infty} x^n f(x) dx$$

The moment generating function:

$$M_x(t) \equiv \langle e^{tx} \rangle = \int_{-\infty}^{+\infty} e^{tx} f(x) dx = \int_{-\infty}^{+\infty} \sum_{n=0}^{\infty} \frac{t^n}{n!} x^n f(x) dx = \sum_{n=0}^{\infty} \mu_n \frac{t^n}{n!}$$

Moments from the generating function:

$$\mu_n = \left. \frac{d^n M_x(t)}{dt^n} \right|_{t=0}$$

Key point: the moment generating function uniquely describe $f(x)$

Back to basics (a brief excursions)

Can also uniquely describe $f(x)$ with the cumulant generating function:

$$K_x(t) \equiv \ln M_x(t) = \sum_{n=0}^{\infty} \kappa_n \frac{t^n}{n!}$$

Cumulants from the generating function:

$$\kappa_n = \left.\frac{d^n K_x(t)}{dt^n}\right|_{t=0}$$

Since $K_x(t) = \ln M_x(t)$, $M_x(t) = \exp(K_x(t))$, so

$$\mu_n = \left.\frac{d^n \exp(K_x(t))}{dt^n}\right|_{t=0}, \quad \kappa_n = \left.\frac{d^n \ln M_x(t)}{dt^n}\right|_{t=0}$$

End result: (details left as an exercise for the interested reader)

$$\begin{aligned} \mu_n &= \sum_{k=1}^{n} B_{n,k}(\kappa_1, \ldots, \kappa_{n-k+1}) \quad (= B_n(\kappa_1, \ldots, \kappa_{n-k+1})) \\ \kappa_n &= \sum_{k=1}^{n} (-1)^{k-1}(k-1)! B_{n,k}(\mu_1, \ldots, \mu_{n-k+1}), \end{aligned}$$

Back to basics (a brief excursions)

Evaluating the Bell polynomials gives

$$\begin{aligned}
\langle x \rangle &= \kappa_1 \\
\langle x^2 \rangle &= \kappa_2 + \kappa_1^2 \\
\langle x^3 \rangle &= \kappa_3 + 3\kappa_1\kappa_2 + \kappa_1^3 \\
\langle x^4 \rangle &= \kappa_4 + 4\kappa_1\kappa_3 + 3\kappa_2^2 + 6\kappa_1^2\kappa_2 + \kappa_1^4
\end{aligned}$$

One can tell by inspection (or derive explicitly) that κ_1 is the mean, κ_2 is the variance, etc.

Back to basics (a brief excursions)

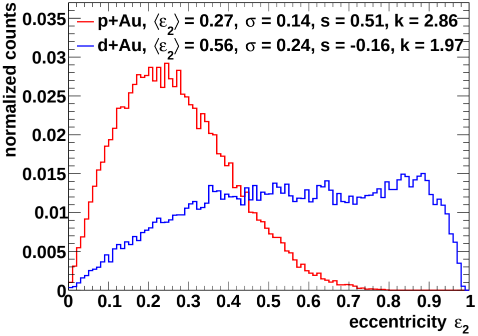
Subbing in $x = v_n$, $\kappa_2 = \sigma^2$, we find

$$\begin{aligned} \Big(\langle v_n^4\rangle &= v_n^4 + 6v_n^2\sigma^2 + 3\sigma^4 + 4v_n\kappa_3 + \kappa_4\Big) \\ -\Big(2\langle v_n^2\rangle^2 &= 2v_n^4 + 4v_n^2\sigma^2 + 2\sigma^4\Big) \\ &\rightarrow \\ \langle v_n^4\rangle - 2\langle v_n^2\rangle^2 &= -v_n^4 + 2v_n^2\sigma^2 + \sigma^4 + 4v_n\kappa_3 + \kappa_4 \end{aligned}$$

Skewness s: $\kappa_3 = s\sigma^3$
Kurtosis k: $\kappa_4 = (k-3)\sigma^4$

$$\begin{aligned} v_n\{2\} &= (v_n^2 + \sigma^2)^{1/2} \\ v_n\{4\} &= (v_n^4 - 2v_n^2\sigma^2 - 4v_n s\sigma^3 - (k-2)\sigma^4)^{1/4} \end{aligned}$$

So the correct form is actually much more complicated than we tend to think...



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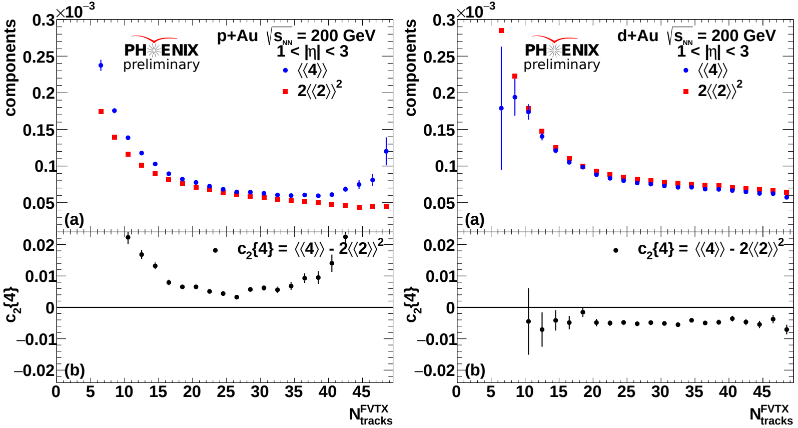
$$\varepsilon_2\{4\} = (\varepsilon_2^4 - 2\varepsilon_2^2\sigma^2 - 4\varepsilon_2 s\sigma^3 - (k-2)\sigma^4)^{1/4}$$

- the variance brings $\varepsilon_2\{4\}$ down
- positive skew brings $\varepsilon_2\{4\}$ further down, negative skew brings it back up
- kurtosis > 2 brings $\varepsilon_2\{4\}$ further down, kurtosis < 2 brings it back up —recall Gaussian has kurtosis $= 3$

Eccentricity distributions and cumulants



$$v_2\{4\} = \left(v_2^4 - 2v_2^2\sigma^2 - 4v_2 s\sigma^3 - (k-2)\sigma^4\right)^{1/4}$$

- Eccentricity fluctuations alone go a long way towards explaining this
- Additional fluctuations in the (imperfect) translation of ε_2 to v_2?

Final thoughts

- Ridge observed for $|\Delta\eta| > 6.2$—long range means early times
- Positive v_2 vs p_T observed from 200 GeV all the way down to 19.6 GeV
  - 200 and 62.4 GeV can be described either with flow only or with flow+non-flow
  - 39 and 19.6 GeV require significant non-flow in any scenario
- Positive v_2 vs η observed from 200 GeV down to 39 GeV
  - 200 GeV can be described as flow only for almost all η
  - Lower energies can be described as flow only for mid and forward rapidity
  - Lower energies show possible flow/non-flow anti-correlation at backward rapidity—this can obscure what is likely a strong forward/backward asymmetry at all energies
- Real valued $v_2\{4\}$ observed from 200 GeV all the way down to 19.6 GeV
  - Multiparticle correlations generally held as best evidence for collectivity
  - There is still the risk of some non-flow contribution to $v_2\{4\}$
  - It's very important to understand the details of the fluctuations