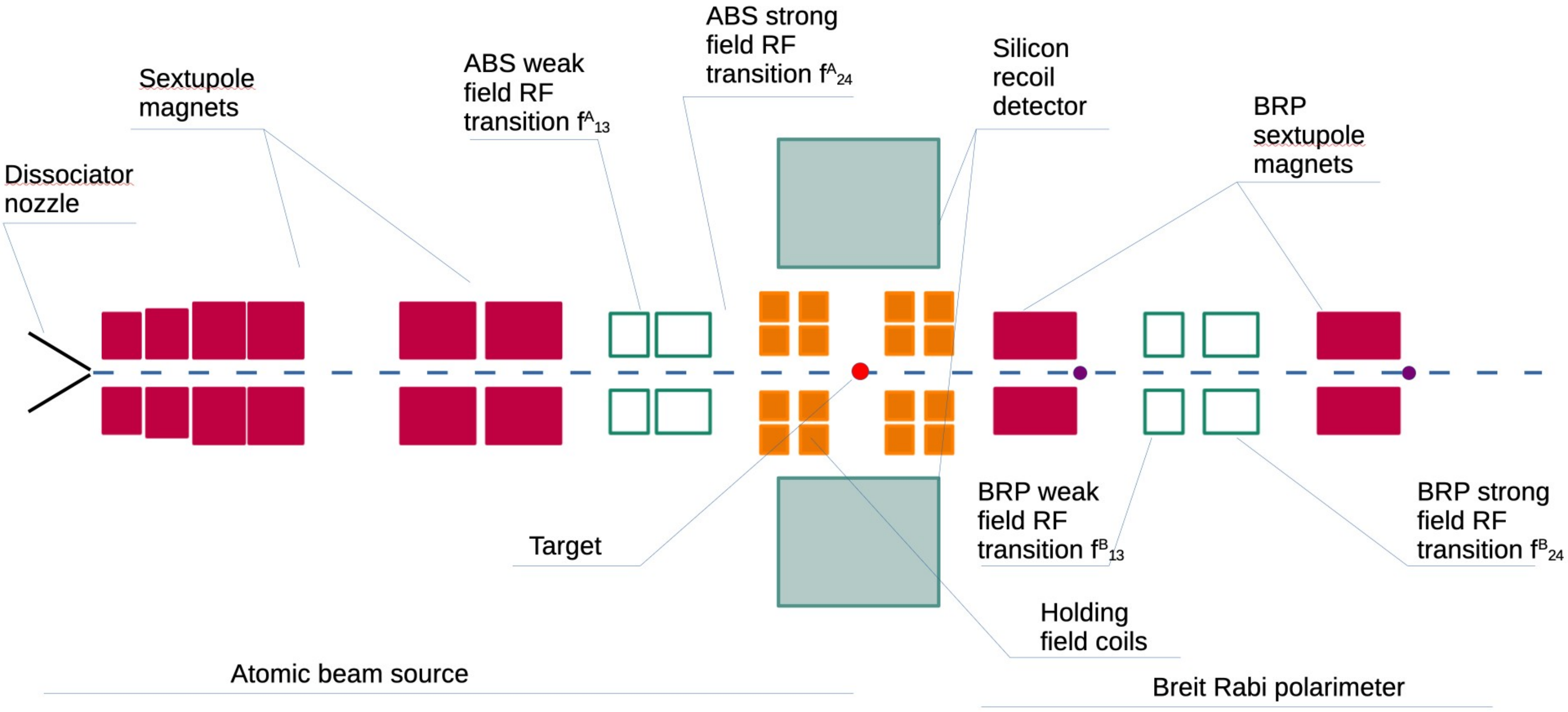
Nuclear polarization measurement of the hydrogen jet target using the Breit-Rabi polarimeter

Vera Shmakova,
Frank Rathmann,
Zhengqiao Zheng

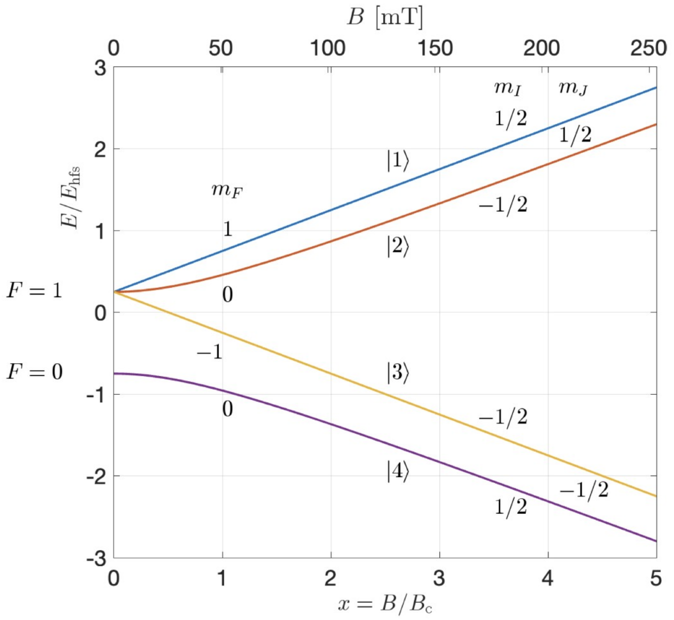
H-JET schematic view



Nuclear polarization determination using data of BRP

- Ultimate goal of atomic beam system – produce a hydrogen jet with controlled polarization in interaction region
- Nuclear polarization depends on hyperfine state composition and magnetic holding field
- In the magnetic holding field B_0 at IP, each hyperfine state has a specific polarization:

$$Q_1(B_0) = \text{nuclear polarization of state } |1\rangle$$
$$Q_2(B_0) = \text{nuclear polarization of state } |2\rangle$$
$$Q_3(B_0) = \text{nuclear polarization of state } |3\rangle$$
$$Q_4(B_0) = \text{nuclear polarization of state } |4\rangle$$



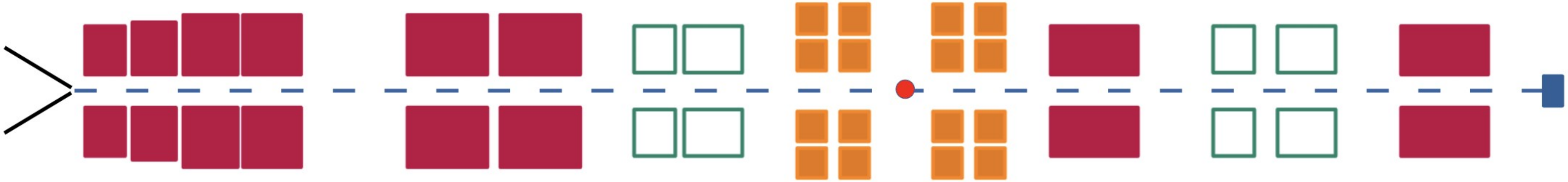
- Overall nuclear polarization of H-Jet:

$$Q = \frac{\sum_{i=1}^{4} n_i \cdot Q_i(B_0)}{\sum_{i=1}^{4} n_i}$$

Nuclear polarization determination

- Initially equally populated states exiting the nozzle with n_i, i=1...4.
- The populations pass the ABS sextupole focusing system, transmitted efficiency σ_i^A, focusing states 1,2 and defocusing states 3,4.

$$\begin{pmatrix} n \\ n \\ n \\ n \end{pmatrix} \xrightarrow{\text{transmission A}} \begin{pmatrix} n\,\sigma_1^A \\ n\,\sigma_2^A \\ n\,\sigma_3^A \\ n\,\sigma_4^A \end{pmatrix}$$



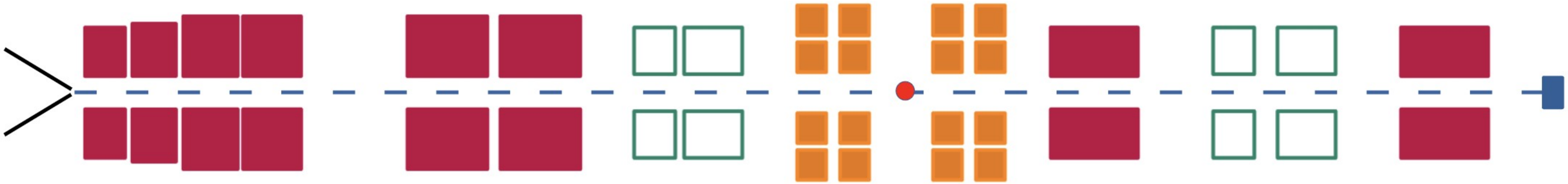
Nuclear polarization determination

- Selective RF transition units in ABS operate with efficiencies ε_{ij}^{A}, creating desired hyperfine state combinations

$$\begin{pmatrix} n\,\sigma_1^A \\ n\,\sigma_2^A \\ n\,\sigma_3^A \\ n\,\sigma_4^A \end{pmatrix} \xrightarrow{\text{transition } f_{24}^{A}} \begin{pmatrix} n\,\sigma_1^A \\ n\,(\sigma_2^A\,\varepsilon_{24}^A + \sigma_4^A(1-\varepsilon_{24}^A)) \\ n\,\sigma_3^A \\ n\,(\sigma_4^A\,\varepsilon_{24}^A + \sigma_2^A(1-\varepsilon_{24}^A)) \end{pmatrix} \qquad \begin{pmatrix} n\,\sigma_1^A \\ n\,\sigma_2^A \\ n\,\sigma_3^A \\ n\,\sigma_4^A \end{pmatrix} \xrightarrow{\text{transition } f_{13}^{A}} \begin{pmatrix} n\,(\sigma_1^A\,\varepsilon_{13}^A + \sigma_3^A(1-\varepsilon_{13}^A)) \\ n\,\sigma_2^A \\ n\,(\sigma_3^A\,\varepsilon_{13}^A + \sigma_1^A(1-\varepsilon_{13}^A)) \\ n\,\sigma_4^A \end{pmatrix}$$

$$Q_{+1}^{\mathrm{A}} = \frac{\sigma_1^A \cdot Q_1(B_0) + (\sigma_2^A\varepsilon_{24}^A + \sigma_4^A(1-\varepsilon_{24})) \cdot Q_2(B_0) + \sigma_3^A \cdot Q_3(B_0) + (\sigma_2^A(1-\varepsilon_{24}^A) + \sigma_4^A\varepsilon_{24}) \cdot Q_4(B_0)}{\sigma_1^A + \sigma_2^A + \sigma_3^A + \sigma_4^A}$$

$$Q_{-1}^{\mathrm{A}} = \frac{(\sigma_1^A\varepsilon_{13}^A + \sigma_3^A(1-\varepsilon_{13}^A)) \cdot Q_1(B_0) + \sigma_2^A \cdot Q_2(B_0) + (\sigma_1^A(1-\varepsilon_{13}^A) + \sigma_3^A\varepsilon_{13}^A) \cdot Q_3(B_0) + \sigma_4^A \cdot Q_4(B_0)}{\sigma_1^A + \sigma_2^A + \sigma_3^A + \sigma_4^A}$$



Signal in Breit Rabi polarimeter

- Both transition efficiencies ε_{ij}^{A} and sextupole transmissions σ_i^{A} need to be known to determine nuclear polarization at the target.
- Way to extract this information from signal measured in BRP of 16 combinations of states.
- 4 possible combinations at the target:

$$\begin{pmatrix} n \\ n \\ n \\ n \end{pmatrix} \xrightarrow{\text{transitions in ABS off}} \begin{pmatrix} n\,\sigma_1^A \\ n\,\sigma_2^A \\ n\,\sigma_3^A \\ n\,\sigma_4^A \end{pmatrix}$$

$$\begin{pmatrix} n\,\sigma_1^A \\ n\,\sigma_2^A \\ n\,\sigma_3^A \\ n\,\sigma_4^A \end{pmatrix} \xrightarrow{\text{transition } f_{13}^{A}} \begin{pmatrix} n\,(\sigma_1^A\,\varepsilon_{13}^A + \sigma_3^A(1-\varepsilon_{13}^A)) \\ n\,\sigma_2^A \\ n\,(\sigma_3^A\,\varepsilon_{13}^A + \sigma_1^A(1-\varepsilon_{13}^A)) \\ n\,\sigma_4^A \end{pmatrix}$$

$$\begin{pmatrix} n\,\sigma_1^A \\ n\,\sigma_2^A \\ n\,\sigma_3^A \\ n\,\sigma_4^A \end{pmatrix} \xrightarrow{\text{transition } f_{24}^{A}} \begin{pmatrix} n\,\sigma_1^A \\ n\,(\sigma_2^A\,\varepsilon_{24}^A + \sigma_4^A(1-\varepsilon_{24}^A)) \\ n\,\sigma_3^A \\ n\,(\sigma_4^A\,\varepsilon_{24}^A + \sigma_2^A(1-\varepsilon_{24}^A)) \end{pmatrix}$$

$$\begin{pmatrix} n\,\sigma_1^A \\ n\,\sigma_2^A \\ n\,\sigma_3^A \\ n\,\sigma_4^A \end{pmatrix} \xrightarrow{\text{transition } f_{24}^{A}, f_{13}^{A}} \begin{pmatrix} n\,(\sigma_1^A\,\varepsilon_{13}^A + \sigma_3^A(1-\varepsilon_{13}^A)) \\ n\,(\sigma_2^A\,\varepsilon_{24}^A + \sigma_4^A(1-\varepsilon_{24}^A)) \\ n\,(\sigma_3^A\,\varepsilon_{13}^A + \sigma_1^A(1-\varepsilon_{13}^A)) \\ n\,(\sigma_4^A\,\varepsilon_{24}^A + \sigma_2^A(1-\varepsilon_{24}^A)) \end{pmatrix}$$

- Each of these combinations passing through BRP weak and straong transitions units split into 4 more combinations.

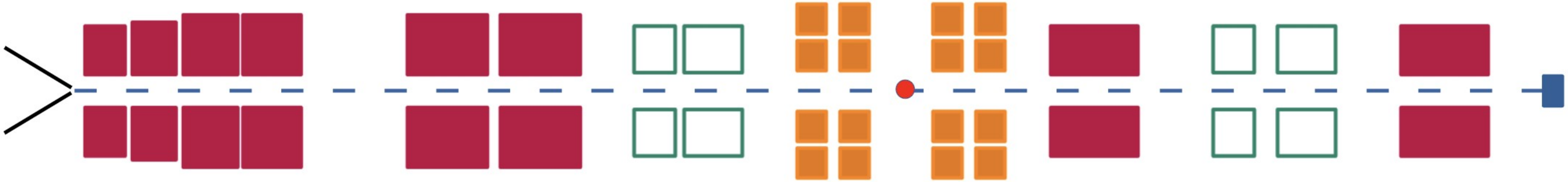
Signal in Breit Rabi polarimeter

$$\begin{pmatrix} n\,\sigma_1^A \\ n\,\sigma_2^A \\ n\,\sigma_3^A \\ n\,\sigma_4^A \end{pmatrix} \xrightarrow{\text{transition in BRP off}} \begin{pmatrix} n\,\sigma_1^A\,\sigma_1^{B1}\,\sigma_1^{B2} \\ n\,\sigma_2^A\,\sigma_2^{B1}\,\sigma_2^{B2} \\ n\,\sigma_3^A\,\sigma_3^{B1}\,\sigma_3^{B2} \\ n\,\sigma_4^A\,\sigma_4^{B1}\,\sigma_4^{B2} \end{pmatrix}$$

$$\begin{pmatrix} n\,\sigma_1^A \\ n\,\sigma_2^A \\ n\,\sigma_3^A \\ n\,\sigma_4^A \end{pmatrix} \xrightarrow{\text{transition } f_{24}^{\mathrm{B}}} \begin{pmatrix} n\,\sigma_1^A\,\sigma_1^{B1}\,\sigma_1^{B2} \\ n\,(\sigma_2^A\sigma_2^{B1}\varepsilon_{24}^B + \sigma_4^A\sigma_4^{B1}(1-\varepsilon_{24}^B))\sigma_2^{B2} \\ n\,\sigma_3^A\,\sigma_3^{B1}\,\sigma_3^{B2} \\ n\,(\sigma_4^A\sigma_4^{B1}\varepsilon_{24}^B + \sigma_2^A\sigma_2^{B1}(1-\varepsilon_{24}^B))\sigma_4^{B2} \end{pmatrix}$$

$$\begin{pmatrix} n\,\sigma_1^A \\ n\,\sigma_2^A \\ n\,\sigma_3^A \\ n\,\sigma_4^A \end{pmatrix} \xrightarrow{\text{transition } f_{13}^{\mathrm{B}}} \begin{pmatrix} n\,(\sigma_1^A\sigma_1^{B1}\varepsilon_{13}^B + \sigma_3^A\sigma_3^{B1}(1-\varepsilon_{13}^B))\sigma_1^{B2} \\ n\,\sigma_2^A\,\sigma_2^{B1}\,\sigma_2^{B2} \\ n\,(\sigma_3^A\sigma_3^{B1}\varepsilon_{13}^B + \sigma_1^A\sigma_1^{B1}(1-\varepsilon_{13}^B))\sigma_3^{B2} \\ n\,\sigma_4^A\,\sigma_4^{B1}\,\sigma_4^{B2} \end{pmatrix}$$

$$\begin{pmatrix} n\,\sigma_1^A \\ n\,\sigma_2^A \\ n\,\sigma_3^A \\ n\,\sigma_4^A \end{pmatrix} \xrightarrow{\text{transition } f_{13}^{\mathrm{B}}, f_{24}^{\mathrm{B}}} \begin{pmatrix} n\,(\sigma_1^A\sigma_1^{B1}\varepsilon_{13}^B + \sigma_3^A\sigma_3^{B1}(1-\varepsilon_{13}^B))\sigma_1^{B2} \\ n\,(\sigma_2^A\sigma_2^{B1}\varepsilon_{24}^B + \sigma_4^A\sigma_4^{B1}(1-\varepsilon_{24}))\sigma_2^{B2} \\ n\,(\sigma_3^A\sigma_3^{B1}\varepsilon_{13}^B + \sigma_1^A\sigma_1^{B1}(1-\varepsilon_{13}^B))\sigma_3^{B2} \\ n\,(\sigma_4^A\sigma_4^{B1}\varepsilon_{24}^B + \sigma_2^A\sigma_2^{B1}(1-\varepsilon_{24}^B))\sigma_4^{B2} \end{pmatrix}$$



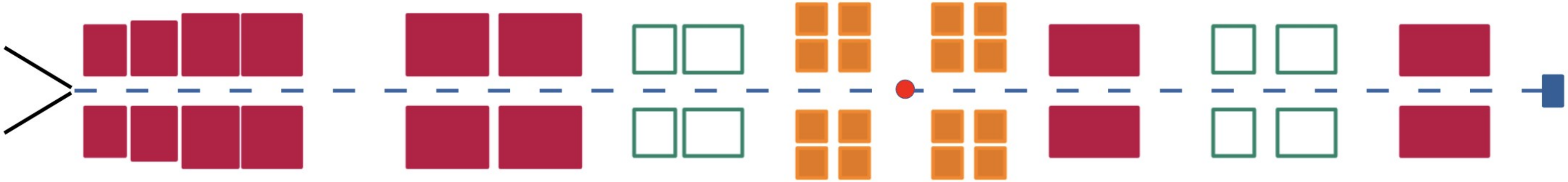
Signal in Breit Rabi polarimeter

- Combinations after ABS strong transition field transitions:

$$\begin{pmatrix} n\,\sigma_1^A \\ n\,(\sigma_2^A\,\varepsilon_{24}^A + \sigma_4^A(1-\varepsilon_{24}^A)) \\ n\,\sigma_3^A \\ n\,(\sigma_4^A\,\varepsilon_{24}^A + \sigma_2^A(1-\varepsilon_{24}^A)) \end{pmatrix} \xrightarrow{\text{transition in BRP off}} \begin{pmatrix} n\,\sigma_1^A\,\sigma_1^{B1}\,\sigma_1^{B2} \\ n\,(\sigma_2^A\,\varepsilon_{24}^A + \sigma_4^A(1-\varepsilon_{24}^A))\sigma_2^{B1}\,\sigma_2^{B2} \\ n\,\sigma_3^A\,\sigma_3^{B1}\,\sigma_3^{B2} \\ n\,(\sigma_4^A\,\varepsilon_{24}^A + \sigma_2^A(1-\varepsilon_{24}^A))\sigma_4^{B1}\,\sigma_4^{B2} \end{pmatrix}$$

$$\begin{pmatrix} n\,\sigma_1^A \\ n\,(\sigma_2^A\,\varepsilon_{24}^A + \sigma_4^A(1-\varepsilon_{24}^A)) \\ n\,\sigma_3^A \\ n\,(\sigma_4^A\,\varepsilon_{24}^A + \sigma_2^A(1-\varepsilon_{24}^A)) \end{pmatrix} \xrightarrow{\text{transition } f_{24}^{\mathrm{B}}}$$

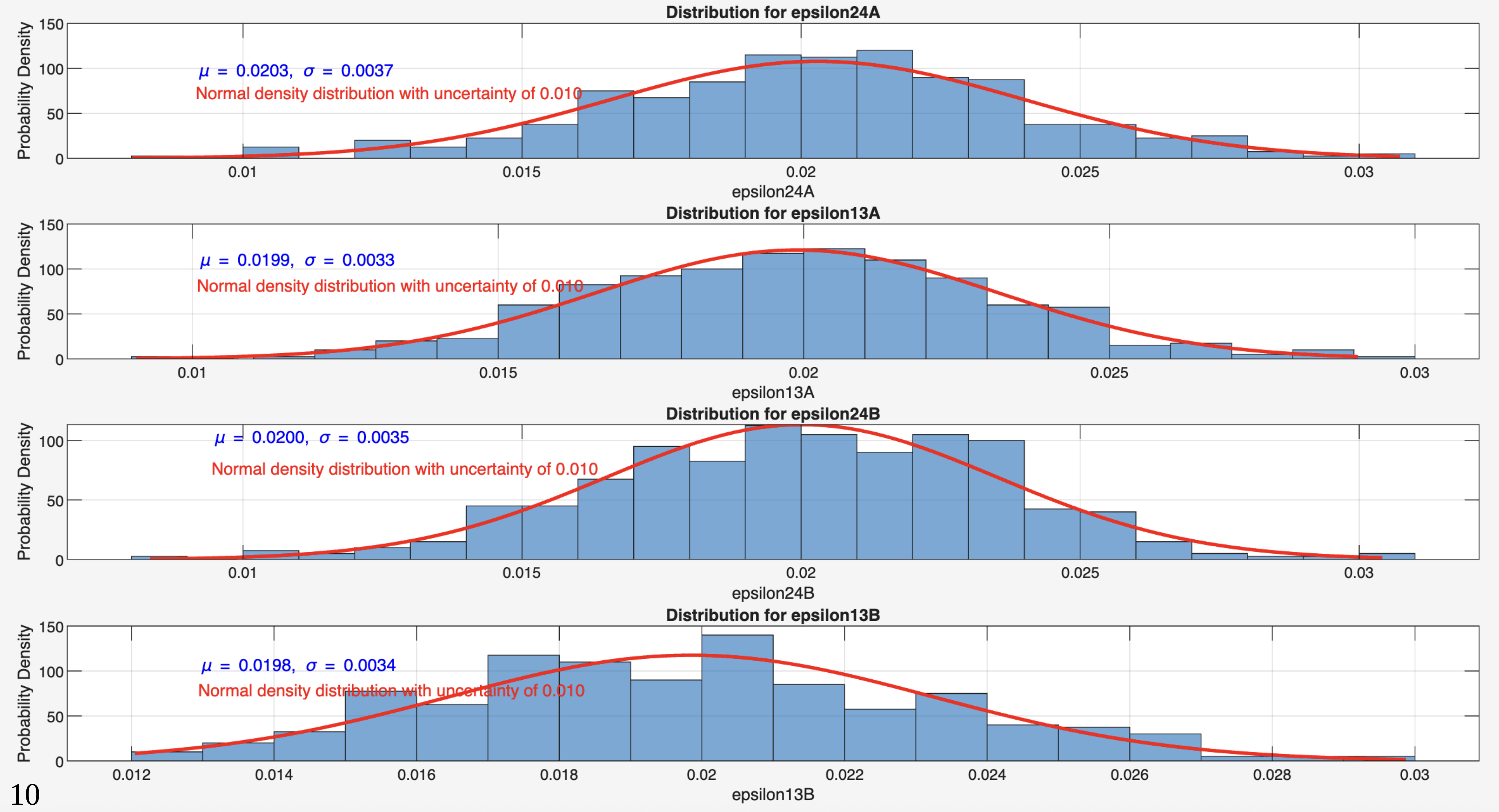
$$\xrightarrow{\text{transition } f_{24}^{\mathrm{B}}} \begin{pmatrix} n\,\sigma_1^A\,\sigma_1^{B1}\,\sigma_1^{B2} \\ n\,((\sigma_2^A\,\varepsilon_{24}^A + \sigma_4^A(1-\varepsilon_{24}^A))\sigma_2^{B1}\varepsilon_{24}^B + (\sigma_4^A\,\varepsilon_{24}^A + \sigma_2^A(1-\varepsilon_{24}^A))\sigma_4^{B1}(1-\varepsilon_{24}^B))\sigma_2^{B2} \\ n\,\sigma_3^A\,\sigma_3^{B1}\,\sigma_3^{B2} \\ n\,((\sigma_4^A\,\varepsilon_{24}^A + \sigma_2^A(1-\varepsilon_{24}^A))\sigma_4^{B1}\varepsilon_{24}^B + (\sigma_2^A\,\varepsilon_{24}^A + \sigma_4^A(1-\varepsilon_{24}^A))\sigma_2^{B1}(1-\varepsilon_{24}^B))\sigma_4^{B2} \end{pmatrix}$$

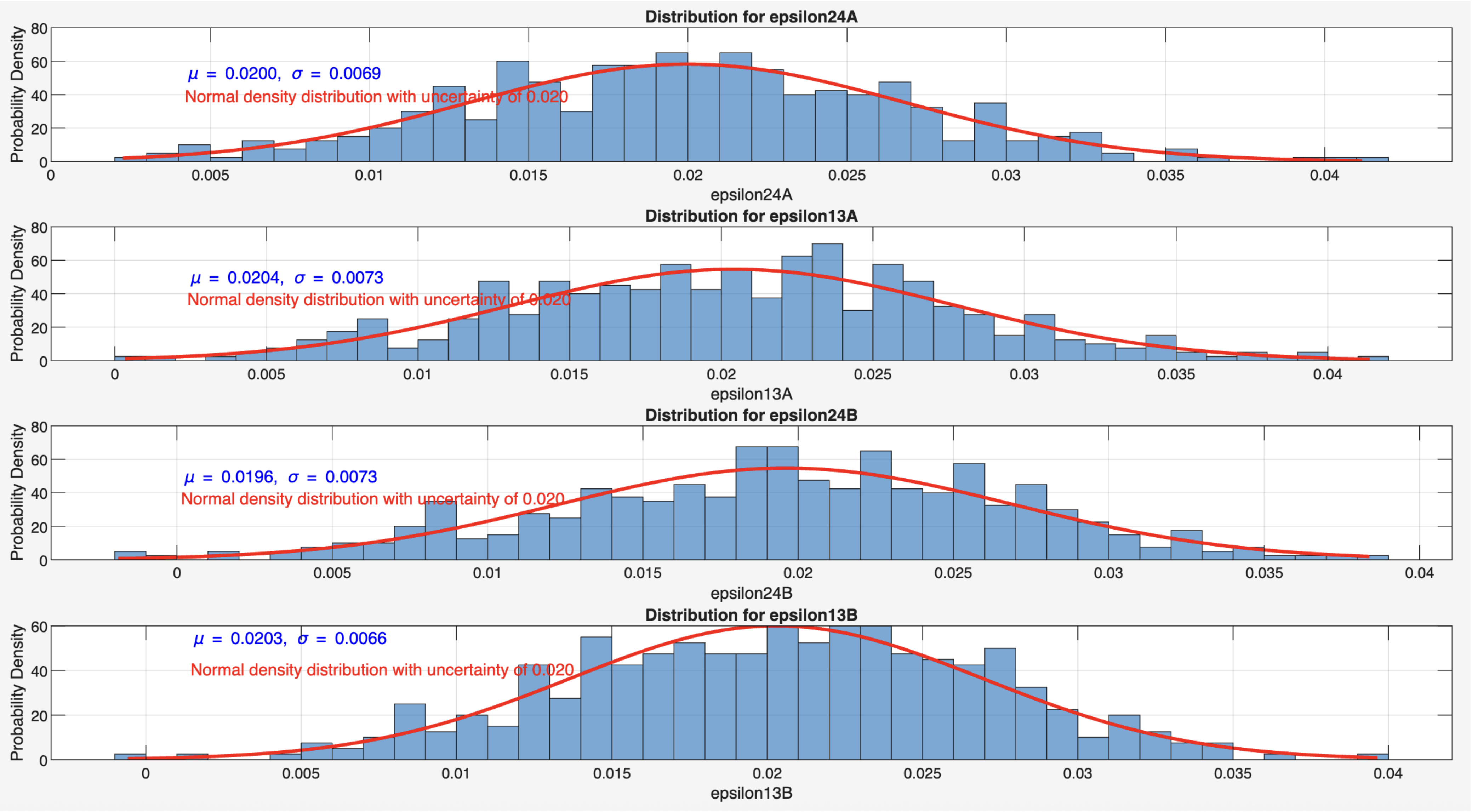


Solution of system of 16 overdetermined equations

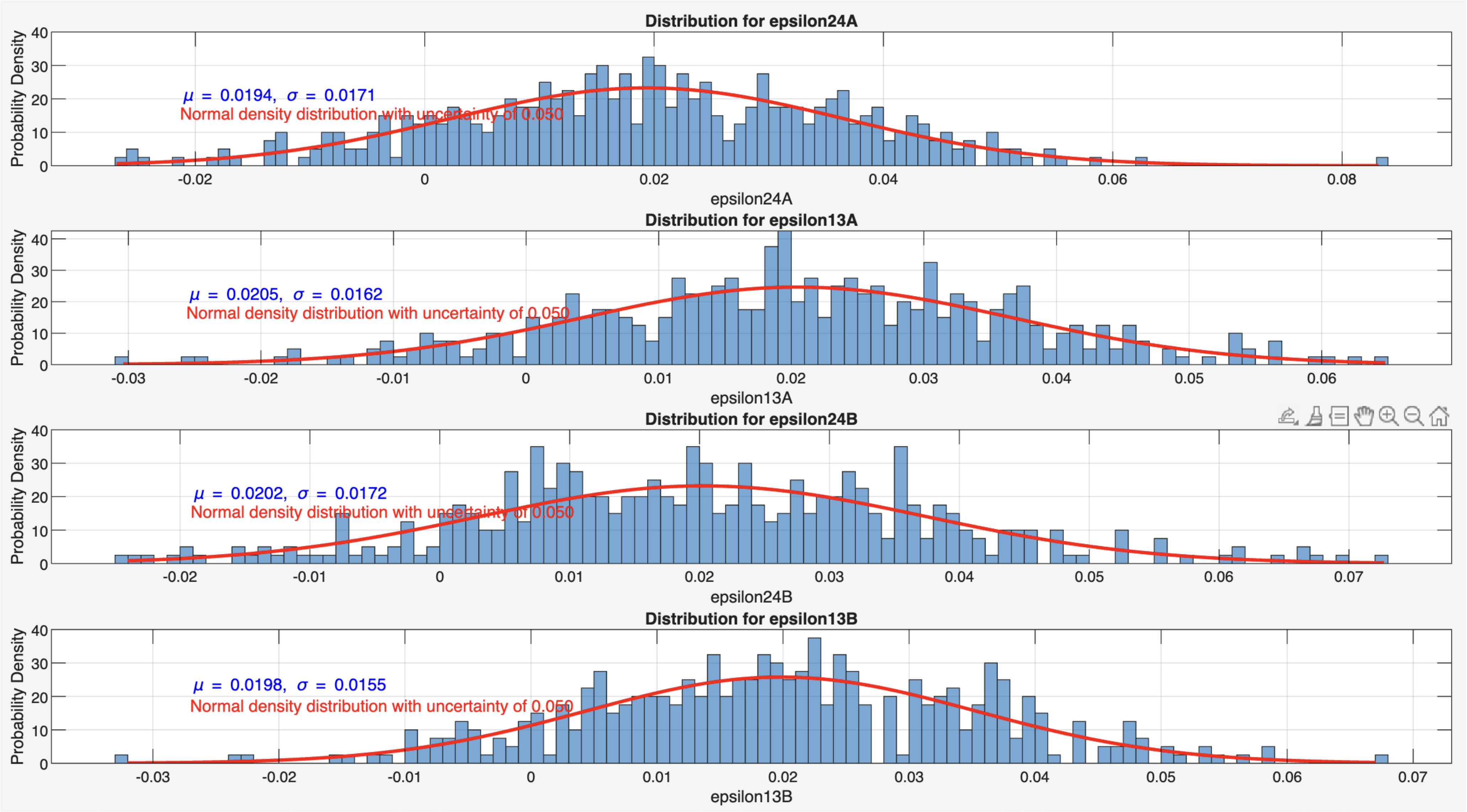
- 16 equations
- For now assume σ_i^A, σ_i^{B1}, σ_i^{B2} and ε_{ij}^A , ε_{ij}^B = 0.02
- Randomize 16 signals with normal distribution with standard deviation of 0.10, 0.05, 0.02, 0.01
- Minimize sum of squared residuals

sigma1A	0.950
sigma2A	0.940
sigma3A	0.010
sigma4A	0.015
sigma1B1	0.930
sigma2B1	0.920
sigma3B1	0.009
sigma4B1	0.012
sigma1B2	0.900
sigma2B2	0.880
sigma3B2	0.011
sigma4B2	0.016


Distribution for epsilon24A
μ = 0.0203, σ = 0.0037
Normal density distribution with uncertainty of 0.010
Probability Density
epsilon24A
Distribution for epsilon13A
μ = 0.0199, σ = 0.0033
Normal density distribution with uncertainty of 0.010
Probability Density
epsilon13A
Distribution for epsilon24B
μ = 0.0200, σ = 0.0035
Normal density distribution with uncertainty of 0.010
Probability Density
epsilon24B
Distribution for epsilon13B
μ = 0.0198, σ = 0.0034
Normal density distribution with uncertainty of 0.010
Probability Density
epsilon13B



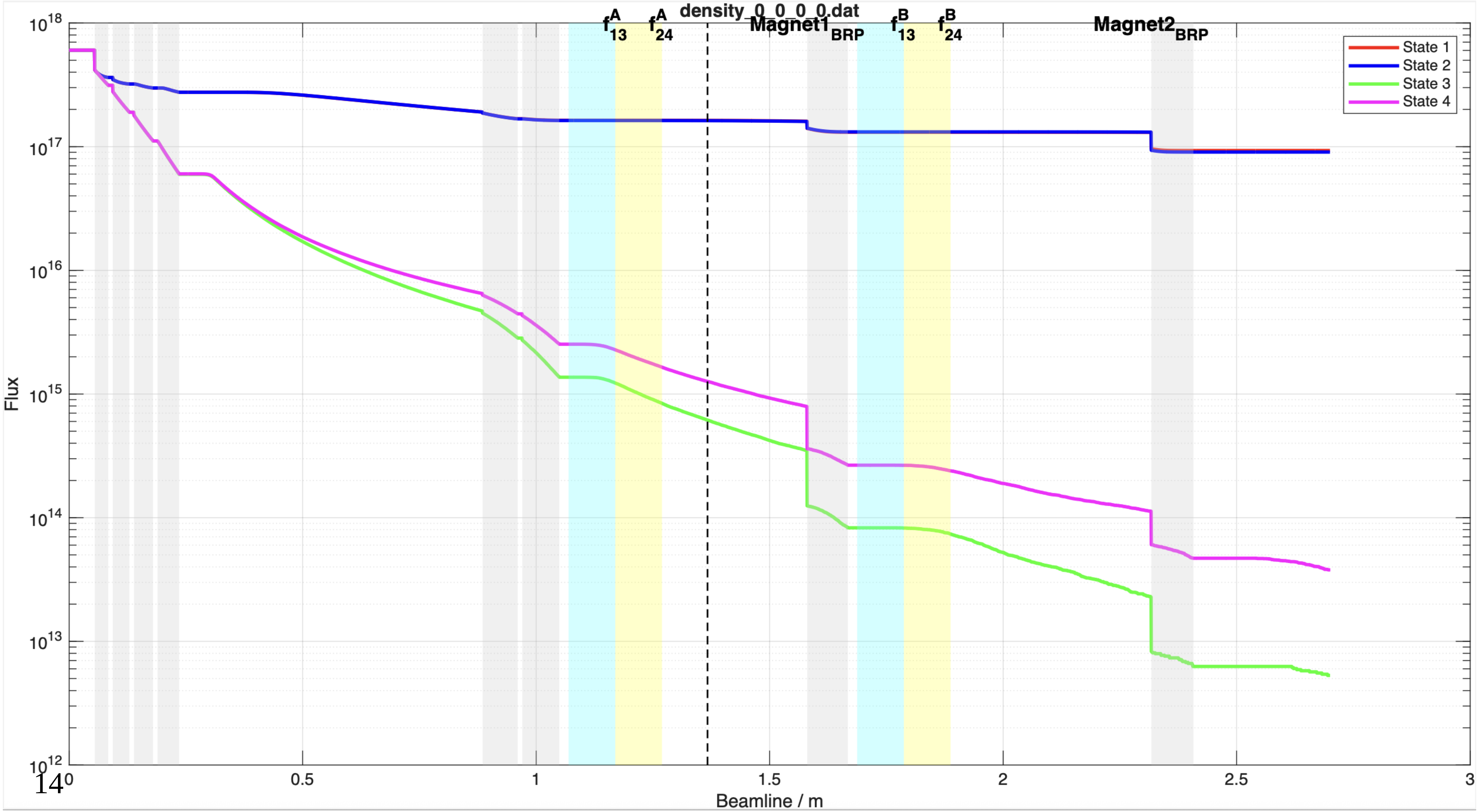
Distribution for epsilon24A
μ = 0.0200, σ = 0.0069
Normal density distribution with uncertainty of 0.020
Probability Density
epsilon24A
Distribution for epsilon13A
μ = 0.0204, σ = 0.0073
Normal density distribution with uncertainty of 0.020
Probability Density
epsilon13A
Distribution for epsilon24B
μ = 0.0196, σ = 0.0073
Normal density distribution with uncertainty of 0.020
Probability Density
epsilon24B
Distribution for epsilon13B
μ = 0.0203, σ = 0.0066
Normal density distribution with uncertainty of 0.020
Probability Density
epsilon13B


Distribution for epsilon24A
Probability Density
μ = 0.0194, σ = 0.0171
Normal density distribution with uncertainty of 0.050
epsilon24A
Distribution for epsilon13A
μ = 0.0205, σ = 0.0162
Normal density distribution with uncertainty of 0.050
epsilon13A
Distribution for epsilon24B
μ = 0.0202, σ = 0.0172
Normal density distribution with uncertainty of 0.050
epsilon24B
Distribution for epsilon13B
μ = 0.0198, σ = 0.0155
Normal density distribution with uncertainty of 0.050
epsilon13B

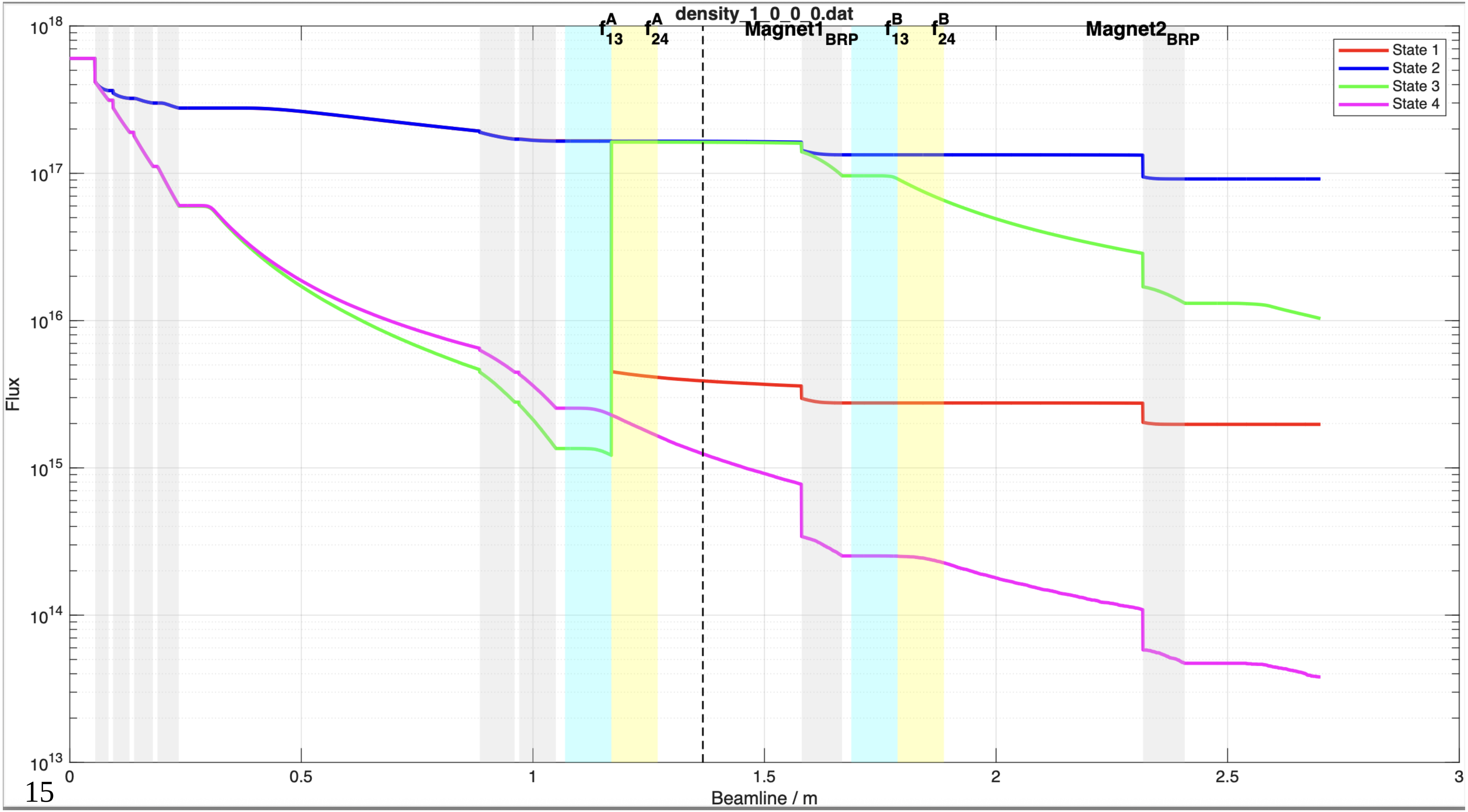
Comparison to simulated data

- Simulations by Zhengqiao
- σ_i^A, σ_i^{B1}, σ_i^{B2} are taken from simulated data
- Calculated signals assume known efficiencies ε_{ij}^A , $\varepsilon_{ij}^A = 0.02$
- Calculated signals are always smaller than simulated (?)

					ABS		BRP	
	Signals calculated	Signals from simulation		Calculated/Simulated	1-3	2-4	1-3	2-4
n1	0.0762	0.0762	*2.409853e+18	1.000	0	0	0	0
n2	0.0479	0.0563		0.851	0	0	0	1
n3	0.0418	0.0566		0.739	0	0	1	0
n4	0.0135	0.0357		0.378	0	0	1	1
n5	0.0413	0.0442		0.934	0	1	0	0
n6	0.0454	0.0458		0.991	0	1	0	1
n7	0.0161	0.0234		0.688	0	1	1	0
n8	0.0110	0.0259		0.425	0	1	1	1
n9	0.0389	0.0431		0.903	1	0	0	0
n10	0.0106	0.0222		0.477	1	0	0	1
n11	0.0407	0.0445		0.915	1	0	1	0
n12	0.0124	0.0244		0.508	1	0	1	1
n13	0.0040	0.0102		0.392	1	1	0	0
n14	0.0081	0.0122		0.664	1	1	0	1
n15	0.0059	0.0120		0.492	1	1	1	0
n16	0.0100	0.0143		0.699	1	1	1	1



density_0_0_0_0.dat
f^A_{13}
f^A_{24}
$Magnet1_{BRP}$
f^B_{13}
f^B_{24}
$Magnet2_{BRP}$
State 1
State 2
State 3
State 4
Flux
10^{18}
10^{17}
10^{16}
10^{15}
10^{14}
10^{13}
10^{12}
0
0.5
1
1.5
2
2.5
3
Beamline / m


density_1_0_0_0.dat
f_{13}^{A}
f_{24}^{A}
$\mathrm{Magnet1}_{BRP}$
f_{13}^{B}
f_{24}^{B}
$\mathrm{Magnet2}_{BRP}$
State 1
State 2
State 3
State 4
Flux
10^{18}
10^{17}
10^{16}
10^{15}
10^{14}
10^{13}
0
0.5
1
1.5
2
2.5
3
Beamline / m

Particle density and flux

- $\dot{N}$ $\frac{atoms}{s}$
- Area m^2
- Flux $\Gamma = \frac{\dot{N}}{A}$ $\frac{atoms}{m^2 s}$
- Depending on area included results would change.
- Density depends on velocity distribution $n = \frac{\Gamma}{\langle v_z \rangle}$ $\frac{atoms}{m^3}$
- Beam size need to be considered
- Could explain discrepancy between calculations and simulations