The Reasonable and Unreasonable Effectiveness of Hydrodynamics in Exotic Quantum Matter

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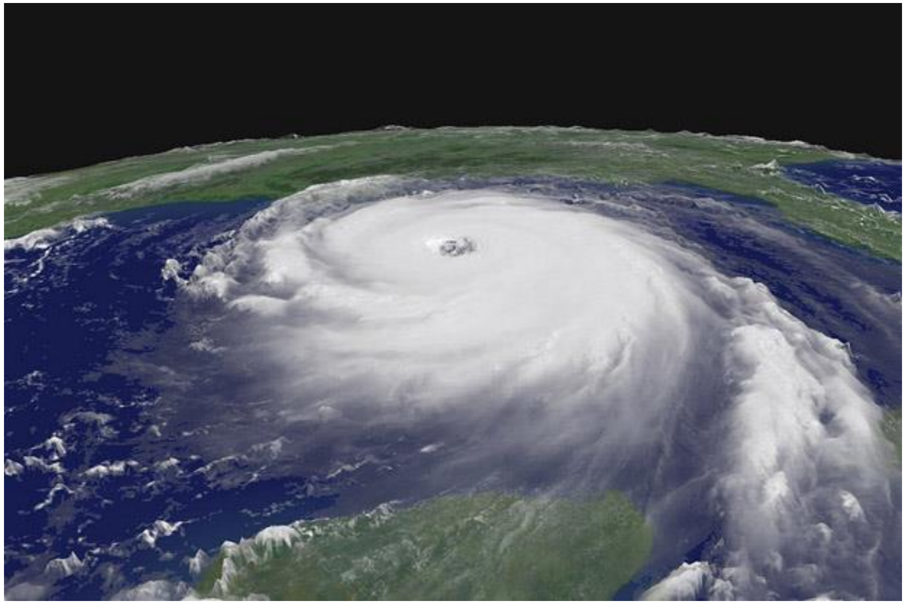
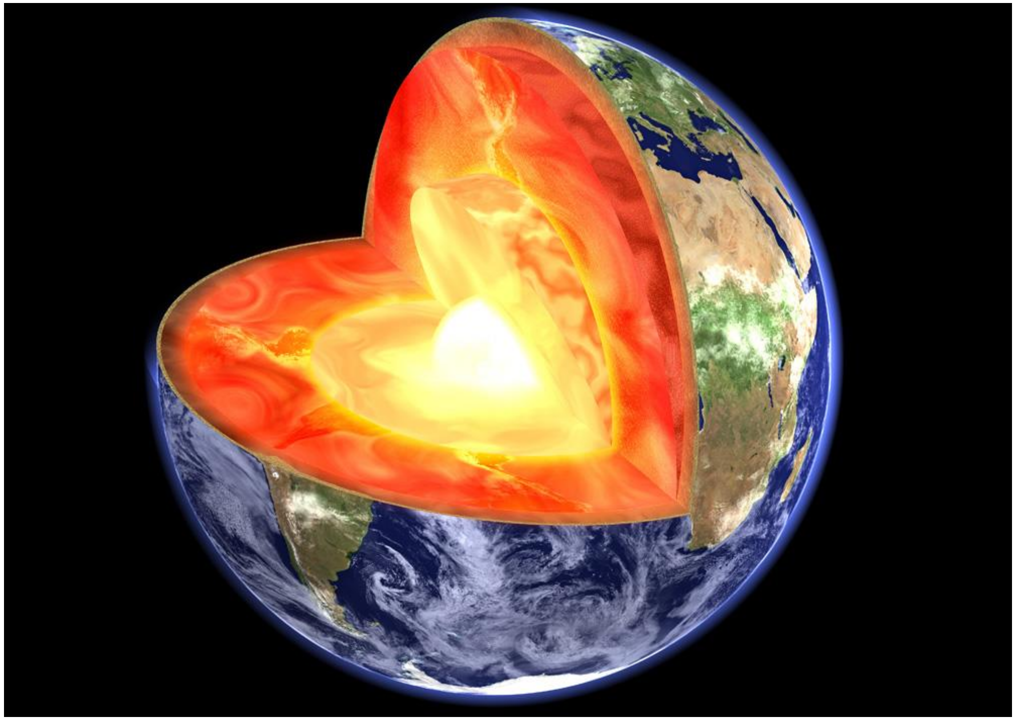
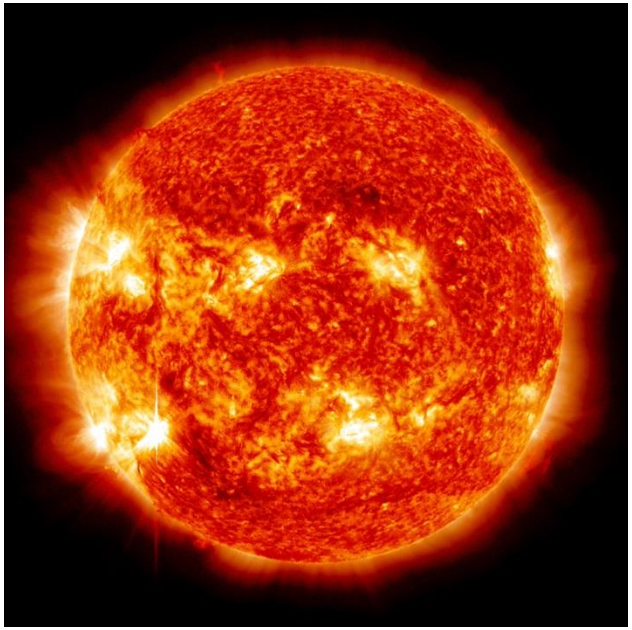
Colloquium, Brookhaven National Laboratory

Sep. 23rd , 2025

Fluid phenomena are ubiquitous in nature:




Hydrodynamics

Long history, dating back to Archimedes (~200 BC), Da Vinci, Newton, Euler, Bernoulli, Navier, Stokes,……

Fluid approximation: a continuum of fluid elements each of which is considered to be a macroscopic object in local equilibrium:

$$\rho(t,\vec{x}), T(t,\vec{x}), v^i(t,\vec{x})$$

(Eulerian)

Express energy, momentum in terms of these variables (constitutive relations)

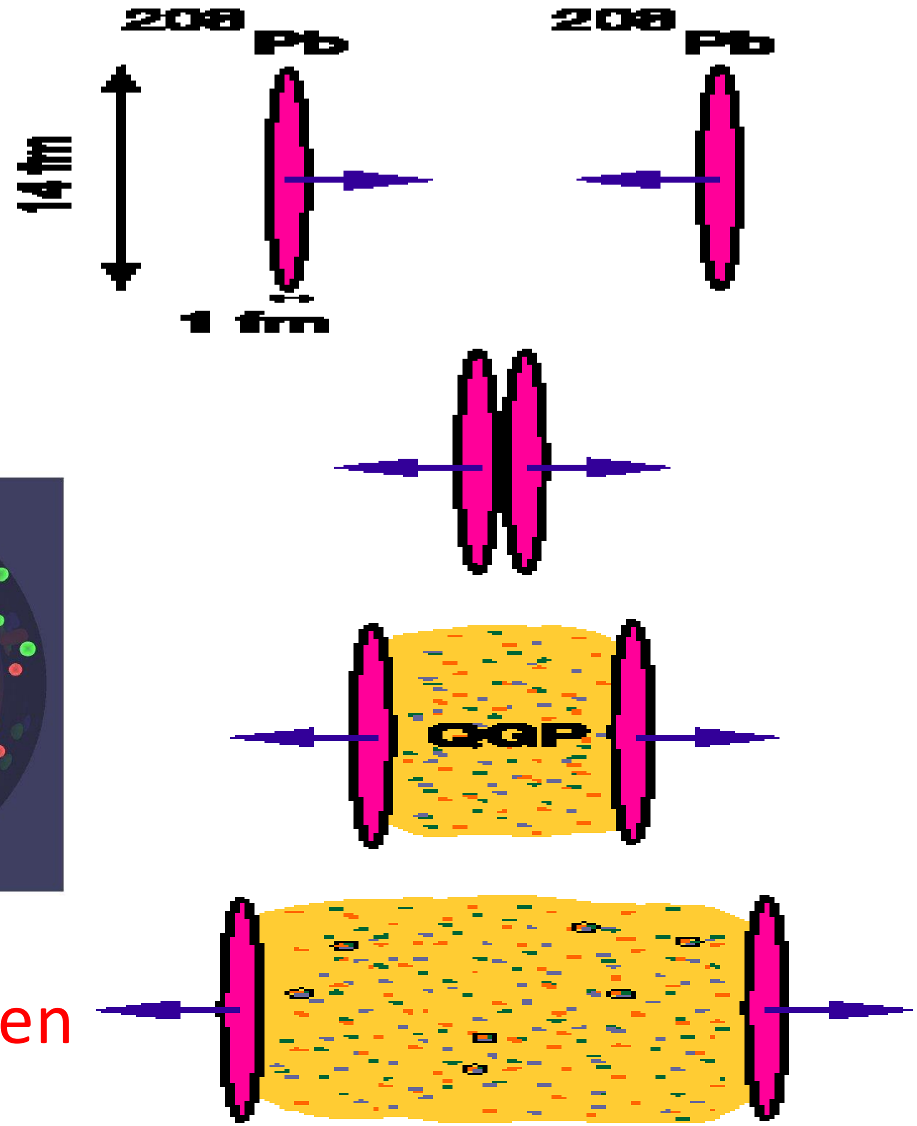
Equations: Energy + momentum conservation, continuity equation

Hydrodynamics has also made unexpected entries in 21st century physics.

I will quickly describe three examples.

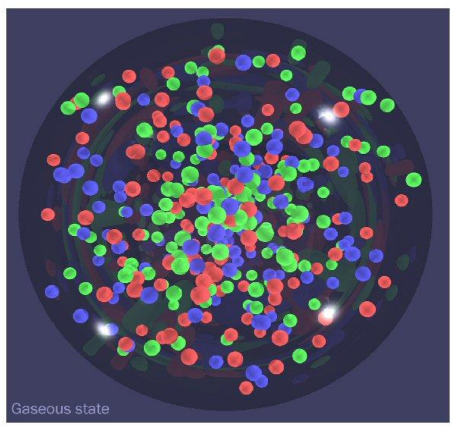
Quark-Gluon Plasma from heavy ion collisions

At room temperature, quarks and gluons are always confined inside hadrons.



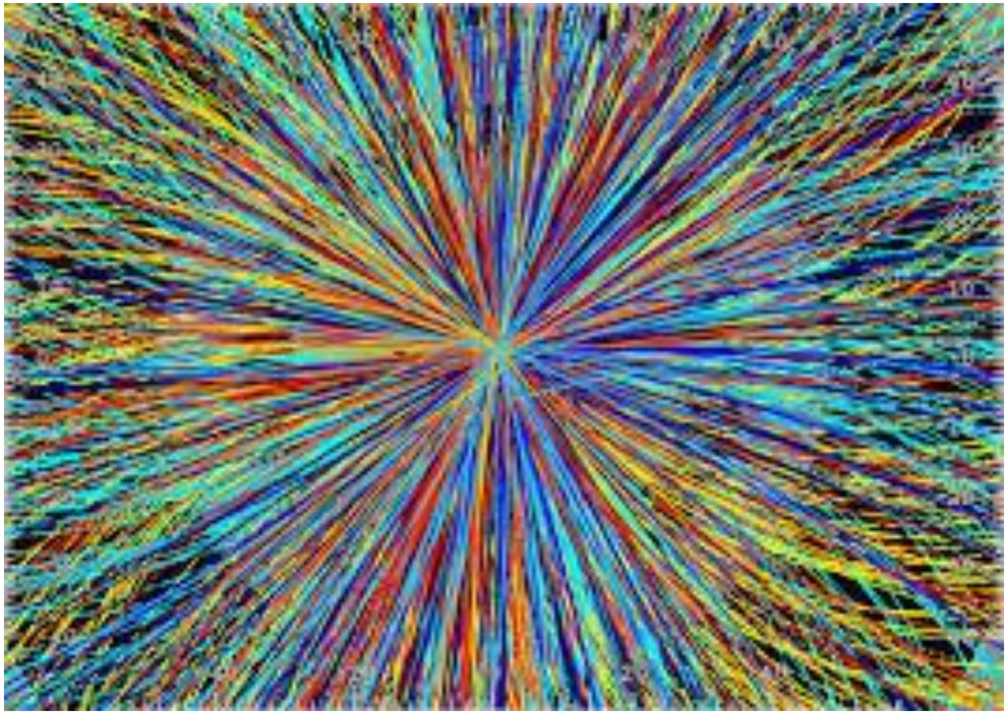
Hadrons melt at sufficient high temperatures

→ Quark-gluon plasma (QGP)



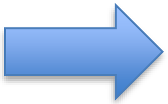
First produced at Brookhaven in 2005!

QGP created:
Size: 10^{-14} m
Lifetime: 10^{-23} sec
Temperature: ~ 10^{12} K

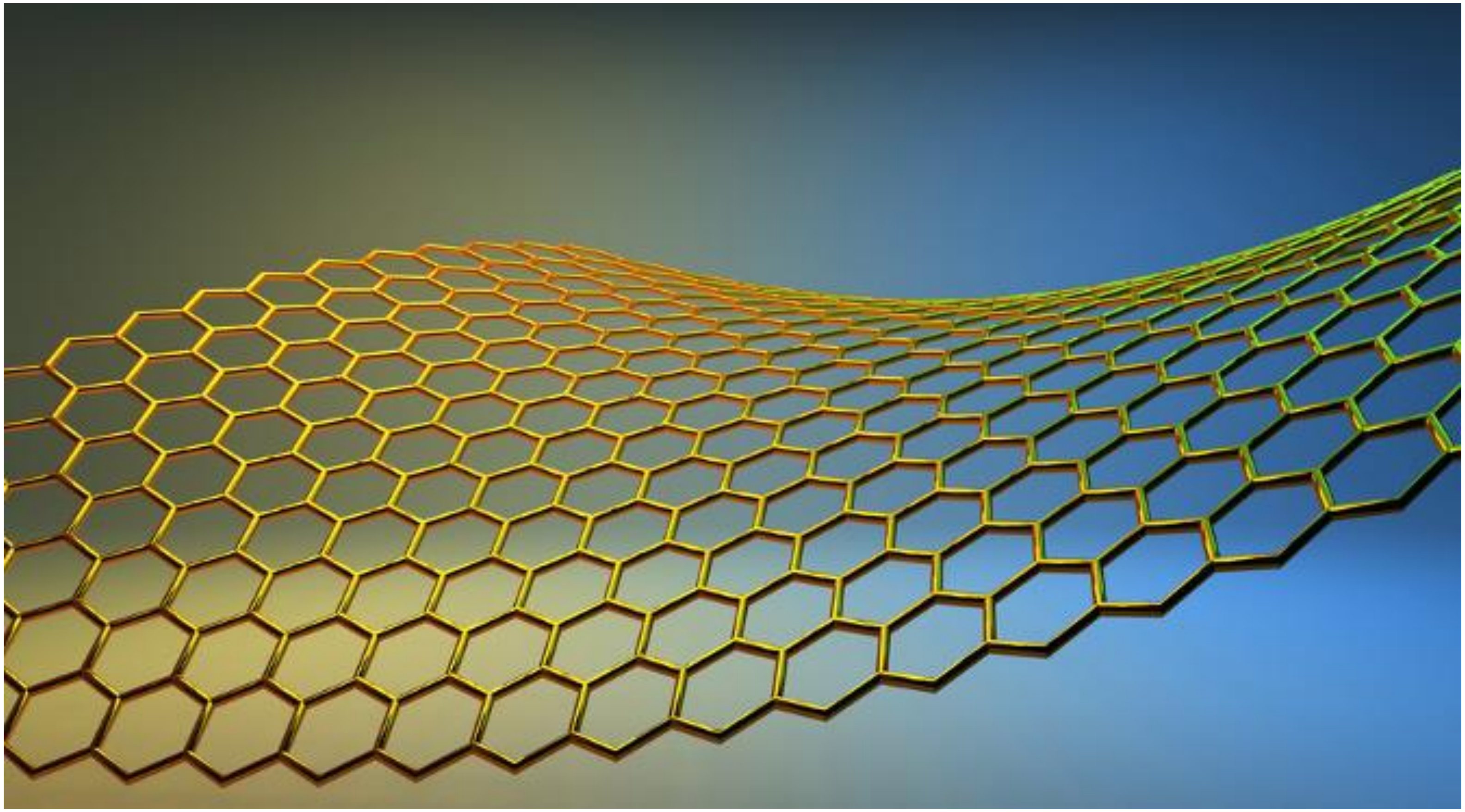


The correlations of detected particles can be well explained: if

evolution of QGP after its creation follows hydrodynamics

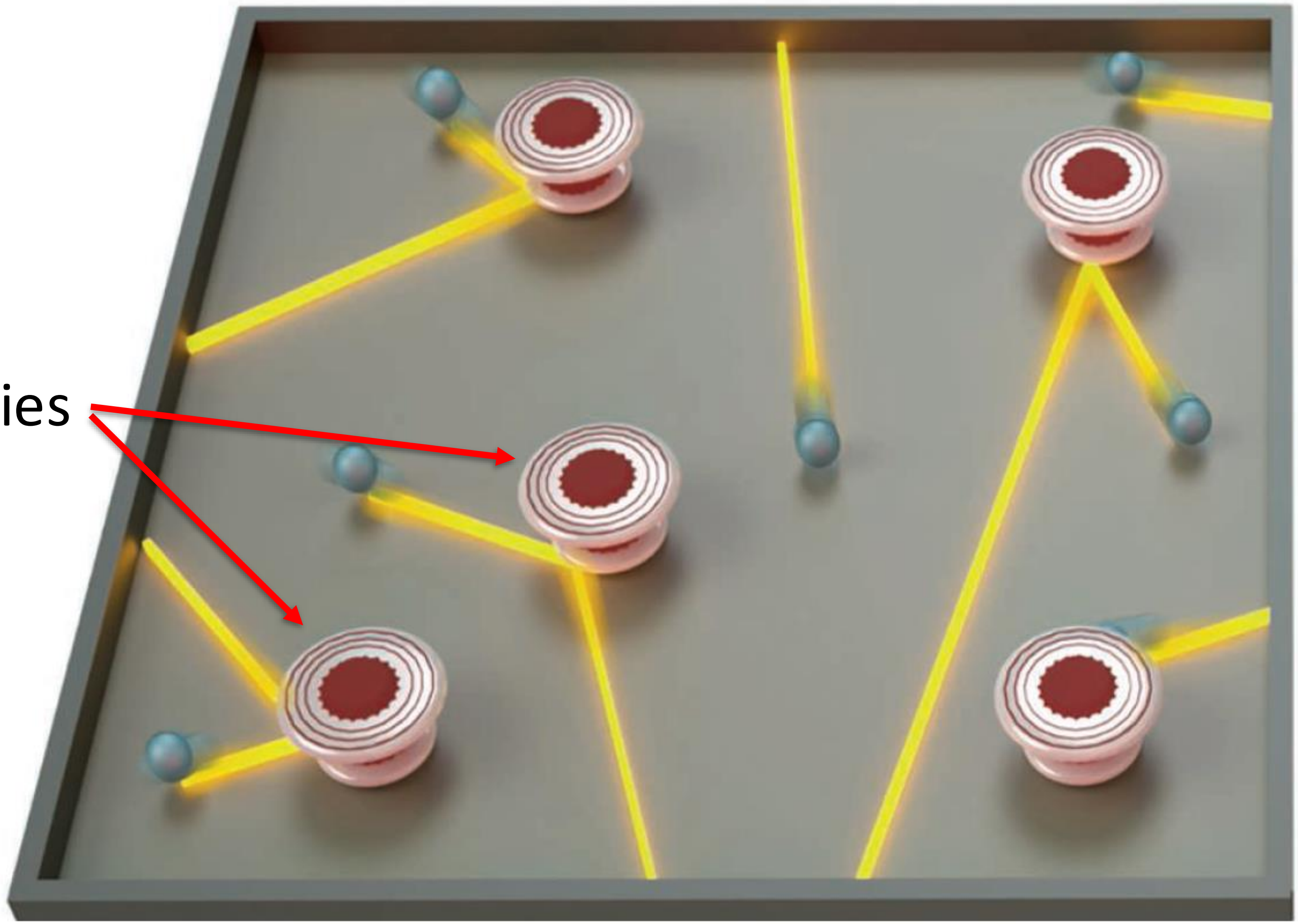
 The QGP behaves like a fluid

Graphene



A very good conductor and can be made very pure.

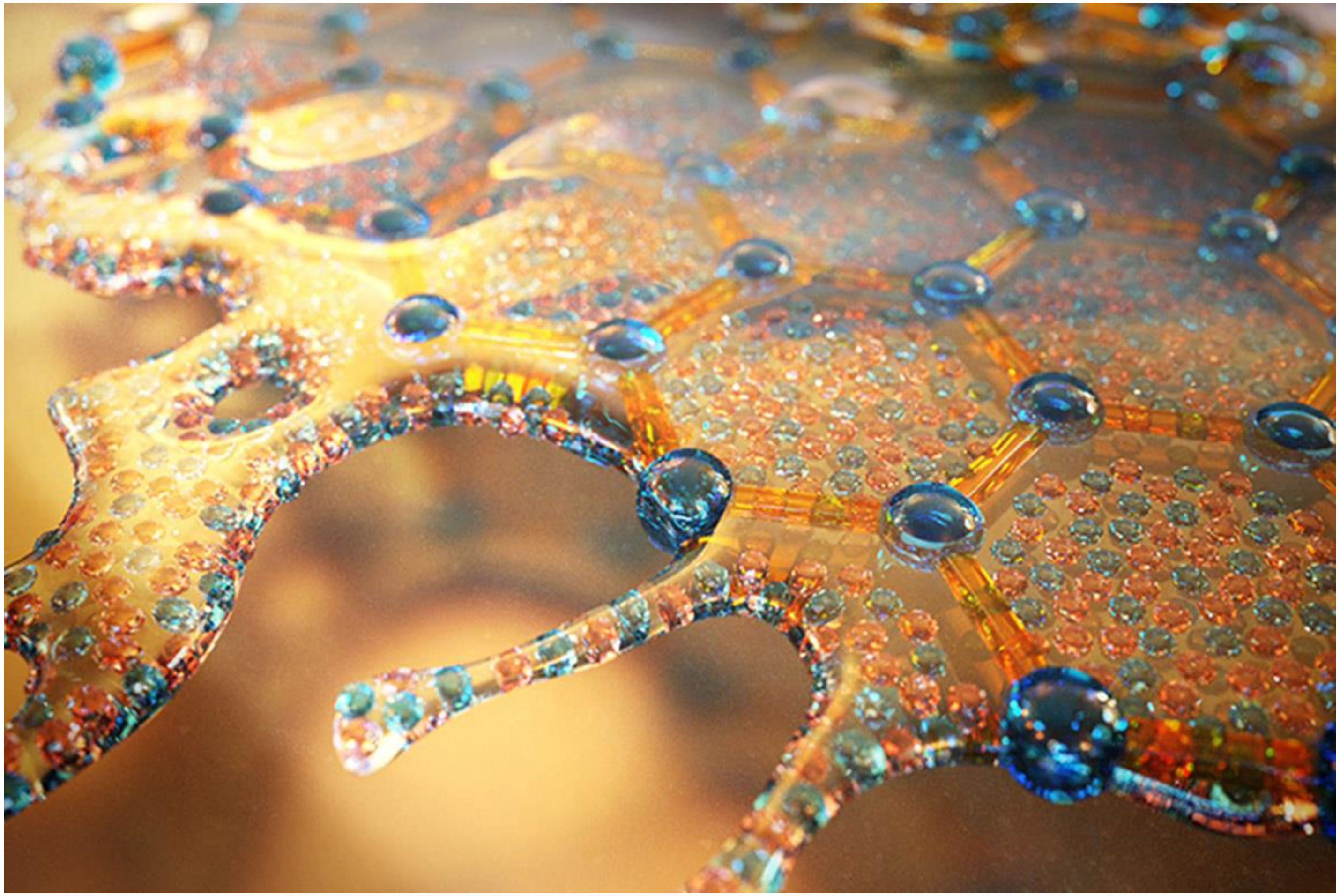
Electrons in a metal



Taken from: J. Zaanen Science 351 (2016)

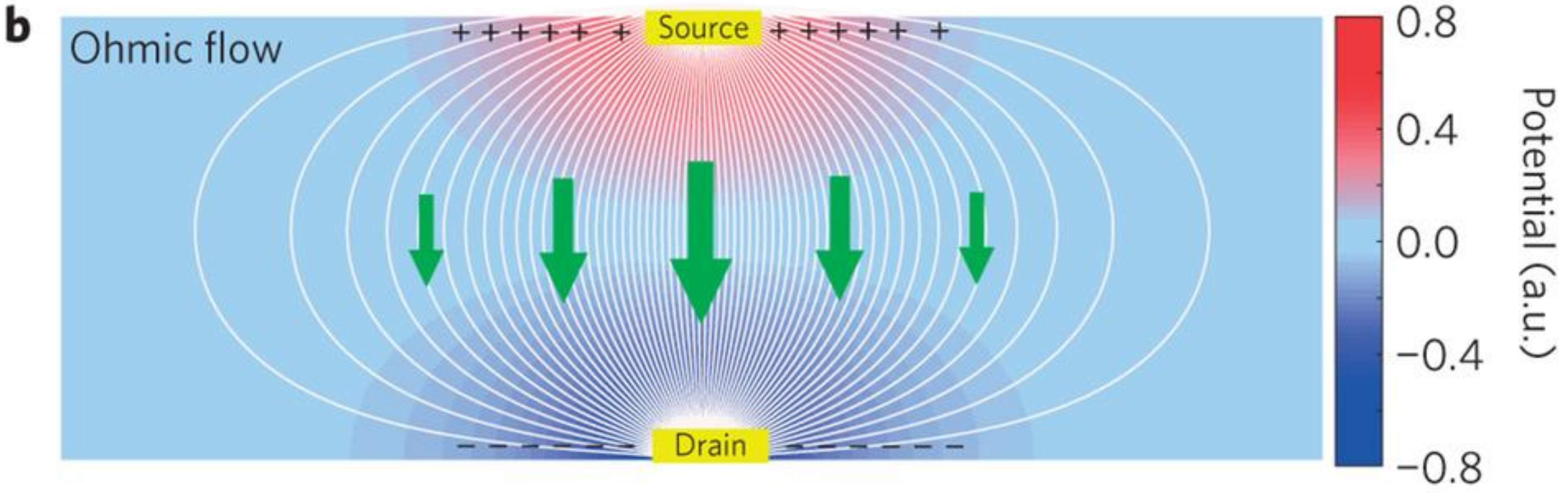
Electrons in Graphene

Graphene can made very pure and one can assume impurities do not exists.



J. Trinastic, GotScience Magazine, 2016

From Levitov and Falkovich, Nature Physics, Feb. 2016



Science **315** March 2016

REPORTS

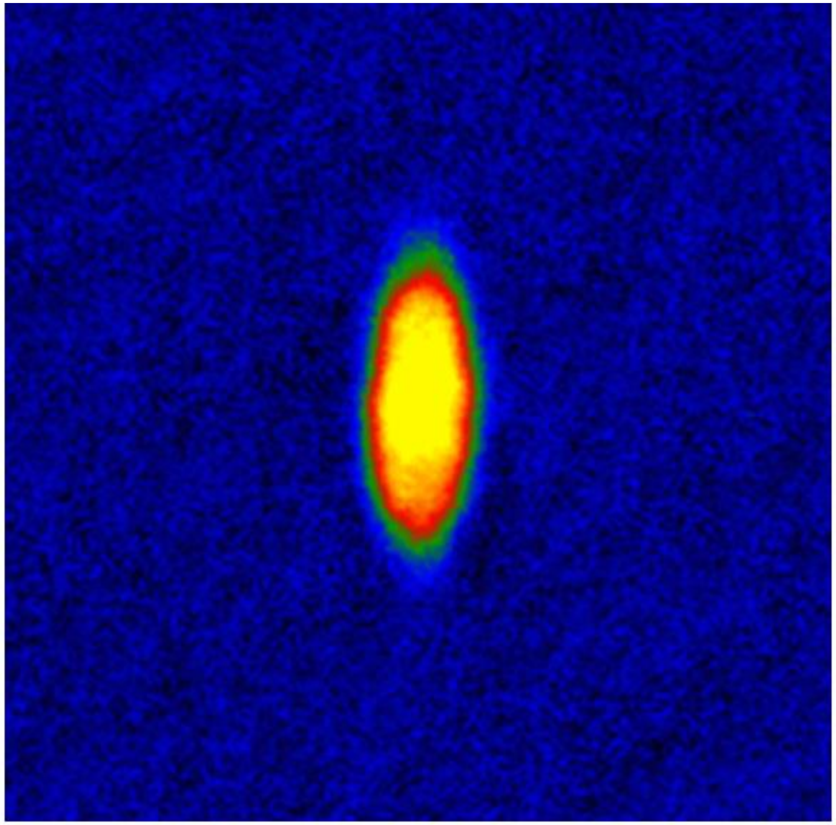
ELECTRON TRANSPORT

Negative local resistance caused by viscous electron backflow in graphene

D. A. Bandurin,[1] I. Torre,[2] R. Krishna Kumar,[1,3] M. Ben Shalom,[1,4] A. Tomadin,[5] A. Principi,[6] G. H. Auton,[4] E. Khestanova,[1,4] K. S. Novoselov,[4] I. V. Grigorieva,[1] L. A. Ponomarenko,[1,3] A. K. Geim,[1*] M. Polini[7*]

Graphene hosts a unique electron system in which electron-phonon scattering is extremely weak but electron-electron collisions are sufficiently frequent to provide local equilibrium above the temperature of liquid nitrogen. Under these conditions, electrons can behave as a viscous liquid and exhibit hydrodynamic phenomena similar to classical liquids. Here we report strong evidence for this transport regime. We found that doped graphene exhibits an anomalous (negative) voltage drop near current-injection contacts, which is attributed to the formation of submicrometer-size whirlpools in the electron flow. The viscosity of graphene's electron liquid is found to be ~0.1 square meters per second, an order of magnitude higher than that of honey, in agreement with many-body theory. Our work demonstrates the possibility of studying electron hydrodynamics using high-quality graphene.

Ultracold Fermi gases



Courtesy of John Thomas's group

T: 10^{-9} K

A confined cigar-shaped cloud of fermionic ^{6}Li atoms, strongly interacting

O’Hara et al
***Science*, 298**, (2002)

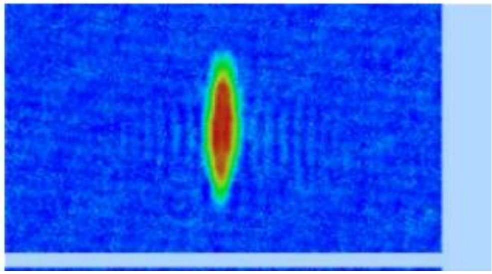
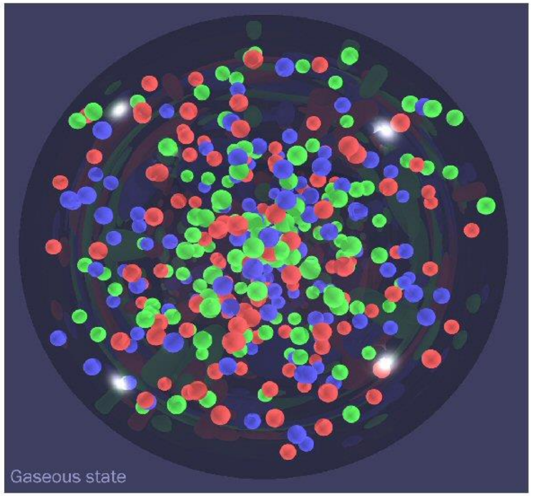


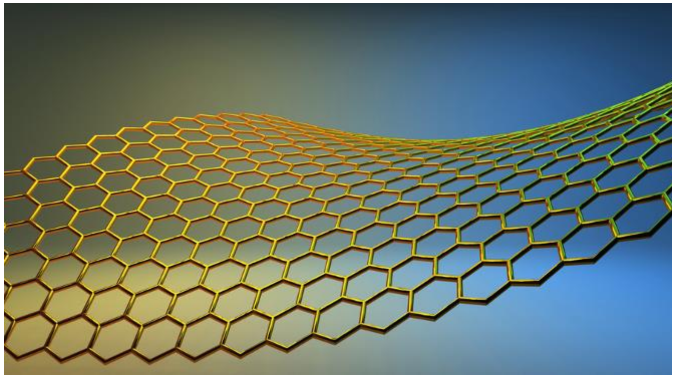
Exhibit collective flows governed by hydrodynamics, indicating a viscous fluid.

Why is hydrodynamics so effective in describing these exotic quantum matter?



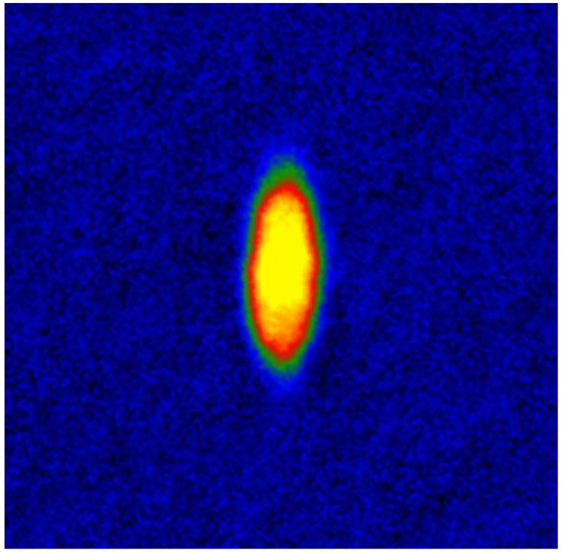
Strong interactions

$\sim 10^{12} K$



Coulomb interactions

$\sim 300 K$



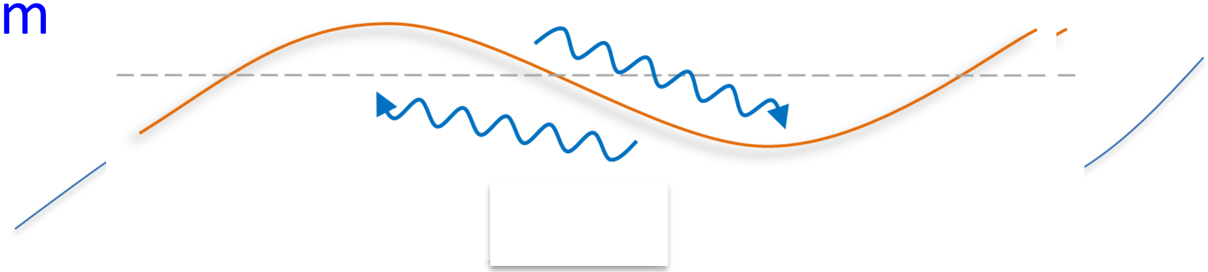
Atomic interactions at unitarity limit

$\sim 10^{-9} K$

There is in fact a simple reason behind it.

Universality of hydrodynamics

Consider a long wavelength disturbance of a system in thermal equilibrium



non-conserved quantities: relax locally, $\tau_{\rm relax} \sim \tau_{\rm mfp}$

conserved quantities: cannot relax locally, only via transports

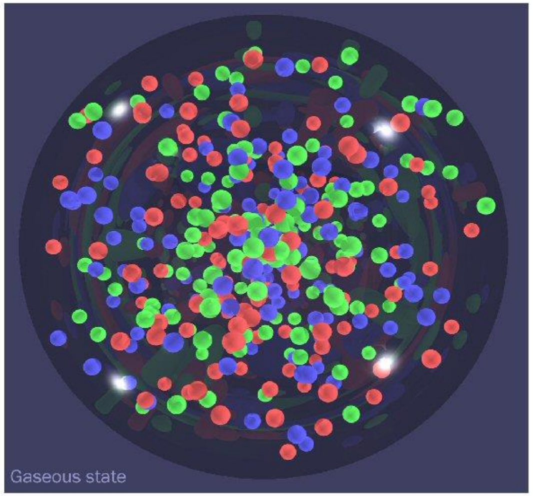
$$\lambda \to \infty, \quad \Rightarrow \quad \tau_{\rm relax} \to \infty$$

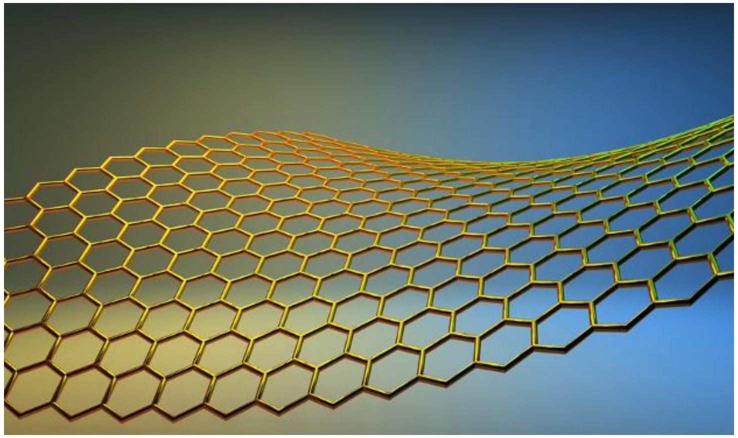
If we are interested in physics at scales: $L \gg \ell_{\rm mfp}, \quad t \gg \tau_{\rm mfp}$

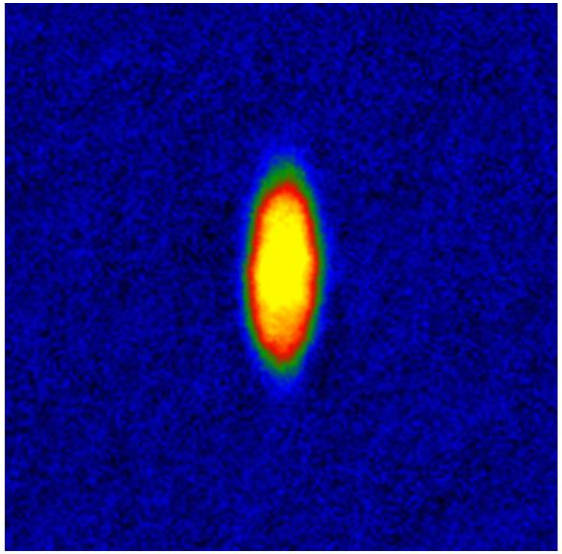
Only dynamics of conserved quantities are relevant, all other details are washed out by interactions !

Hydrodynamics is the theory of conserved quantities.

Key: $L \gg \ell_{\mathrm{mfp}}, \quad t \gg \tau_{\mathrm{mfp}}$







Their mean free paths have to be sufficiently short.

They must be strongly interacting!

There are many other physical applications of hydrodynamics:

Particle physics, cosmology, astrophysics, black holes, holography .....

Hydrodynamics provides a universal theory for non-equilibrium dynamics of quantum many-body systems at sufficiently long distances and times!

Traditional formulation: conservation equations with constitutive relations (require phenomenological inputs).

- As equations of motion, cannot capture fluctuations

  (There exist phenomenological fixes for simplest cases, but not applicable to general or far-from-equilibrium situations.)

- There are many situations, phenomenological considerations not enough to fix equations of motion

Does it have an action formulation from first principle?

Searching for an action principle for dissipative hydrodynamics has been a long standing problem, dating back at least to the ideal fluid action of G. Herglotz in 1911.

The last decade has seen a renewed interest:

Dubovsky, Gregoire, Nicolis and Rattazzi hep-th/0512260

Dubovsky, Hui, Nicolis and Son, arXiv:1107.0731

Grozdanov and Polonyi, arXiv:1305.3670

Kovtun, Moore and Romatschke, arXiv:1405.3967

Harder, Kovtun, and Ritz, arXiv:1502.03076

Haehl, Loganayagam and Rangamani, arXiv:1502.00636, 1511.07809

…….

Challenges

1. Dissipation

Standard lore: Dissipative systems don't have an action formulation

$$m\ddot{x} + \nu\dot{x} = 0$$

2. Dynamical variables

Standard variables: $\rho(t,\vec{x}), T(t,\vec{x}) \quad v^i(t,\vec{x})$ Unsuitable!

3. Symmetries

What symmetries define a fluid?

Paolo Glorioso   Michael Crossley

A few years ago, my students and I were able to develop a complete formulation of hydrodynamics from first principles (i.e. based on symmetries and action principle, no need to have phenomenological inputs).

arXiv: 1511.03646, 1612.07705, 1701.07817, 1701.07445

A review: 1805.09331   Paolo Glorioso, HL

Used techniques and insights from quantum field theories, gravity, and string theories.

Effective field theory (EFT)

Full path integral of a quantum many-body system

Identify ϕ : Low energy degrees of freedom

Integrate out the rest

$$\int D\phi \, e^{iS_{eff}[\phi]}$$

$S_{eff}[\phi]$: low energy effective action

Direct computation: rarely possible

Identify symmetries and constraints of $S_{eff}[\phi]$

Write down the most general theory consistent with the symmetries

The EFT paradigm has been extremely powerful in many disciplines of physics:

Nuclear, particle, condensed matter, ……

Essentially all applications have been to equilibrium systems.

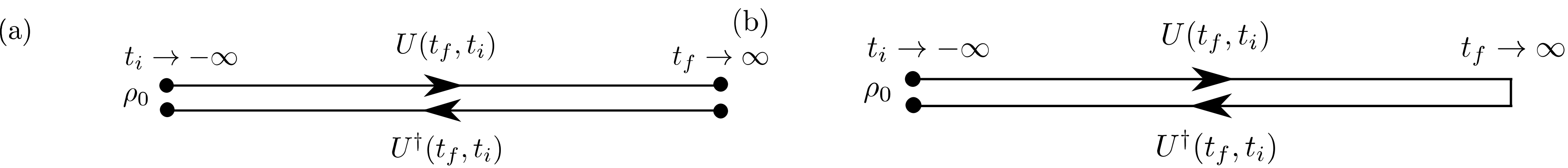
We developed a framework for formulating EFTs for non-equilibrium systems

→ action principle for hydrodynamics

Dissipations

This issue is naturally resolved by quantum mechanics.

interested in dynamics of a non-equilibrium state.



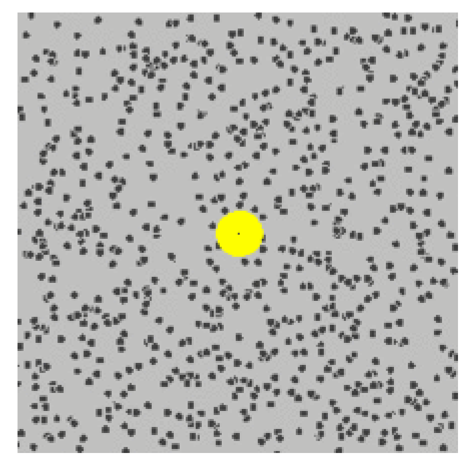
$$\rho(t_f) = U(t_f, t_i)\rho_0 U^\dagger(t_f, t_i) \qquad \mathrm{Tr}\left(\rho_0 \cdots\right)$$

Closed time path (CTP) or Schwinger-Keldysh contour

Key: develop effective field theories for systems on a closed time path (double d.o.f.)

Example: Brownian motion

Quantum → Classical (action principle for Langevin equation)



Dynamical variables

Key: identify universal variables associated with energy-momentum conservation.

Trick: put the system in a curved spacetime: because of energy-momentum conservation, the system should be diffeomorphism invariant

That is, invariant under any coordinate transformations

Promote spacetime coordinates into dynamical variables

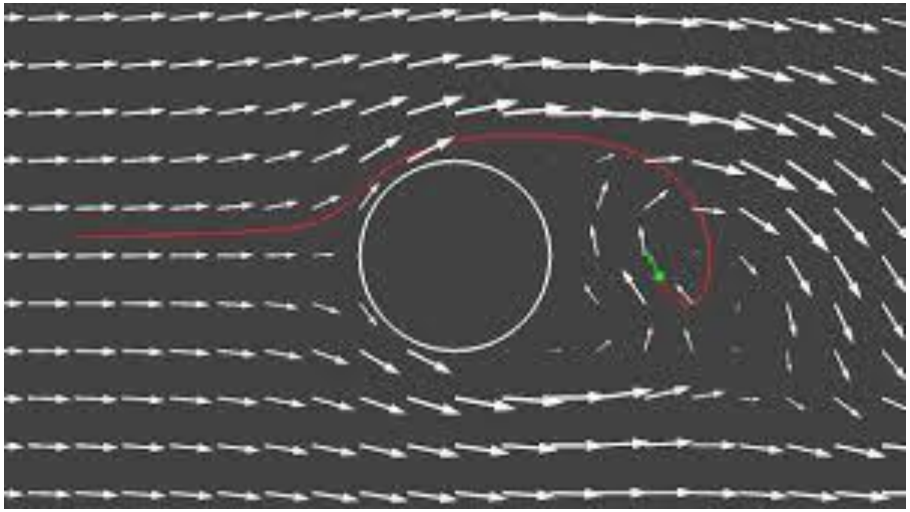
$$x^\mu \to X^\mu(\sigma^a)$$

⇒ Equations of X^μ equivalent to energy-momentum conservation.

Need a new auxiliary spacetime with coordinates σ^a

Dynamical variables: $X_1^\mu(\sigma^i, \sigma^0), \quad X_2^\mu(\sigma^i, \sigma^0)$

This is just a generalization of the Lagrange description!



σ^i: label fluid elements

$x^i(t, \sigma^i),$

σ^i label individual fluid elements, σ^0 internal time

Symmetries

1. Symmetries defining a fluid:

$$\sigma^i \to \sigma'^i(\sigma^i), \quad \sigma^0 \to \sigma^0$$

$$\sigma^0 \to \sigma'^0 = f(\sigma^0, \sigma^i), \quad \sigma^i \to \sigma^i$$

2. Constraints from quantum unitarity (survive in the classical limit)

3. A Z_2 symmetry: dynamical KMS symmetry, which imposes micro-time-reversibility and local equilibrium

⇨ A "statistical" field theory which fully recovers the standard hydrodynamic as equations of motion, but also treats statistical and quantum hydrodynamic fluctuations systematically.

Emergent entropy as a Noether charge

Glorioso, HL, 2017

Combination of unitarity constraints and dynamical KMS symmetry leads to a remarkable consequence:

One can construct a local current s^μ, the "charge" of which never decreases.

$$\Delta S \equiv \int_{t=t_f} d^{d-1}x \, s^0 - \int_{t=t_i} d^{d-1}x \, s^0 = \mathcal{R} \geq 0$$

A new derivation of the second law of thermodynamics

$\mathcal{R}$ can be found explicitly using the action

Universal expression for entropy production.

This framework is very general:

- Can be applied to situations where phenomenological considerations are inadequate

Magnetohydrodynamics with strong magnetic field

fracton hydrodynamics, systems with discrete rotation symmetries

Floquet systems, certain lattice systems, systems with anomalies

Biological systems …….

Quantum chaos and scrambling

- can be generalized to other continuous media

solids, liquid crystals, quasicrystals,
systems undergoing chemical reactions, …….

EFT for dynamical EM field in a general medium

Consider electromagnetic field strongly couped to microscopic matter at some finite temperature

- Insulator: dielectrics
- Conducting medium: magnetohydrodynamics (MHD)

Magnetohydrodynamics (MHD)

MHD: a universal theory for charged fluids in the presence of dynamical EM field.

Dynamical variables: hydro variables for conserved quantities and magnetic field $\vec{B}$

Difficulty: lack of a general principle to write down constitutive relations at strong field.

By using non-equilibrium EFT we have developed a systematic formulation of MHD and found new terms not known before.

Grozdanov, Leutheusser, HL, Vardhan
Landry, HL

New results on MHD

For illustration, focus on E and B fields, neglecting fluid motions.

Textbook version:

$$\mathbf{E} = c\mathbf{j}, \quad \mathbf{j} = \nabla \times \mathbf{B}$$

Goldreich-Reisenegger (1992)

$$\mathbf{E} = c_\eta \, \mathbf{j} + c_a \, (\mathbf{B} \cdot \mathbf{j}) \, \mathbf{B} + c_H \, \mathbf{j} \times \mathbf{B},$$

Our new results:

$$\mathbf{E} = c_\eta \, \mathbf{j}_B + c_a \, (\mathbf{B} \cdot \mathbf{j}) \, \mathbf{B} + c_H \, \mathbf{j}_B \times \mathbf{B}$$

$$\mathbf{j}_B \equiv \mathbf{j} - \nabla \ln \mu \times \mathbf{B}$$

μ : magnetic permeability.

Lead to new magnetic diffusion behavior

The above formulation was for non-chiral matter

Now consider including chiral matter with ABJ anomaly:

$$\partial_\mu J_5^\mu = \frac{c}{4}\epsilon^{\mu\nu\alpha\beta}F_{\mu\nu}F_{\alpha\beta}$$

One of breakthroughs of hydrodynamics in last two decades was the realization microscopic t' Hooft anomaly can have macroscopic effects on hydrodynamics and transports.

t' Hooft anomaly: global symmetry, source not dynamical

ABJ anomaly: source dynamical

Its possible effects on hydro been an outstanding open question

Chiral anomalous MHD

Landry, HL
arXiv:2212.09757

φ, φ_a : hydro variables for the chiral current

$$J_5^i = (a_{50} - 2cA_0)B_i - \kappa_{ij}\partial_j\mu_5 - \lambda_{ij}^-\partial_t A_j + \epsilon_{ijk}\partial_j(mB_k)$$

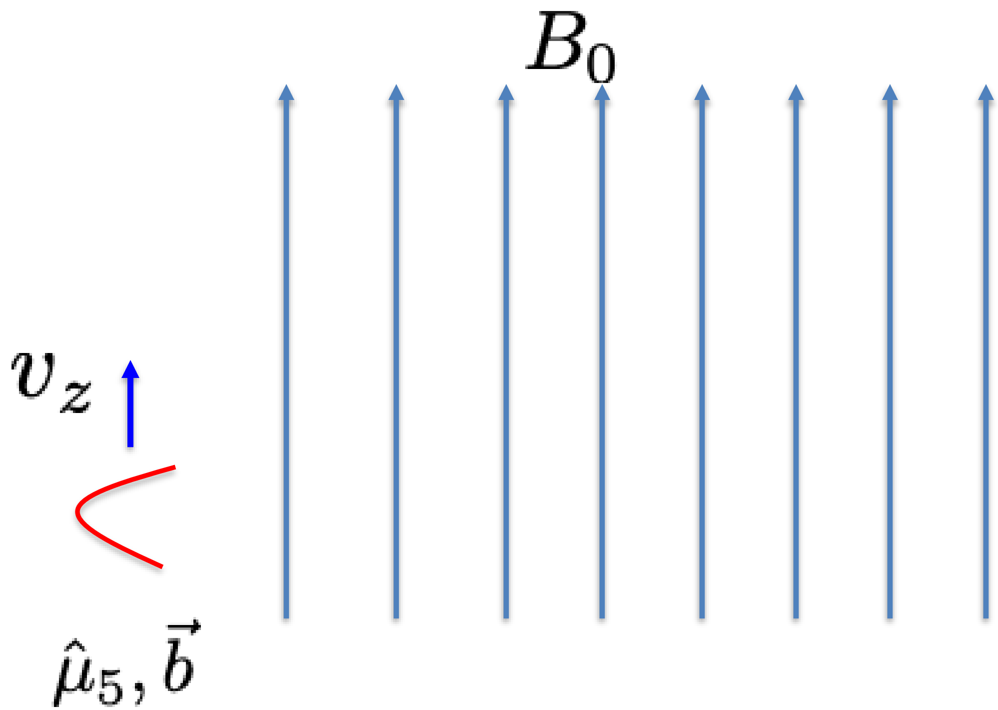
$$\partial_t A_i = -r_{ij}(\epsilon_{jkl}\partial_k H_l + 2c\hat{\mu}_5 B_j + \lambda_{kj}^+\partial_k\mu_5)$$

$$n_5 = \frac{\partial F}{\partial \mu_5}, \quad H_i = -\frac{\partial F}{\partial B_i} \qquad F(A_0, \mu_5, B^2)$$: Equilibrium free energy

$$\hat{\mu}_5 \equiv \mu_5 - \frac{a_{00}}{2c}$$

A prediction: chiral wave

Equilibrium $\quad \hat{\mu}_5 = 0, \quad B_z = B_0$



$$v_z \equiv \frac{4cB_0 r_{\|} \lambda_{\|}}{\chi_5}$$

Chiral instability

Equilibrium $\hat{\mu}_5 = \text{const}, \quad B_i = 0$

Helical unstable mode:

$$B_x = \mathcal{B}(t)\cos kz, \quad B_y = \mathcal{B}(t)\sin kz, \quad \mathcal{B}(t) = B_0 e^{2cr\hat{\mu}_5 kt - D_B k^2 t}$$

first pointed out by Akamatsu and Yamamoto (2013)

We presented argument that the system will evolve to

$$\hat{\mu}_5 = 0, \quad B_i = \text{const}$$

New way of generating magnetic field in astrophysics or cosmology?

Summary

- Hydrodynamics plays important role in characterizing various exotic quantum matter.

a universal definition of strongly coupled quantum liquids

- We now have a first principle formulation of hydrodynamics which incorporates statistical and quantum fluctuations.

- Many applications

Thank you!