

Fast Cycling Permanent Magnet Fixed Field Alternating (FFA) Synchrotron (LDRD24-010)

Dejan Trbojevic

December 18, 2025

    @BrookhavenLab

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Dejan Trbojevic, Stephen J Brooks, George J Mahler, Nicholaos Tsoupas, Scott J Berg - Brookhaven National Laboratory, & Professor MD Samuel Ruy, Qian Xin – Radiation Oncology, Stony Brook University Hospital [SBUH]

Abstract: In the recent decades there have been new investigations in cancer radiation therapy with **FLASH effect where a very large radiation dose is delivered in a very short time, 40 Grays per second (40 Gy/s)**. It was found that the FLASH radiation **significantly improves healthy tissue recovery**. The FLASH therapy requirements of 40 Gy/s are to be compared to the dose of 0.01 Gy/s used in conventional radio therapy. The current SBUH permanent magnet proton therapy project is based on the previous successful permanent magnet **Cornell University Brookhaven National Laboratory Electron Test Accelerator (CBETA)** project. Electrons were accelerated in the Energy Recovery Linac (ERL) by passing four times through the similar permanent magnet racetrack and superconducting linac with energies of 42, 78, 112, and 150 MeV and obtained 99.98% efficiency. The proton therapy SBUH accelerator delivers protons with required energies in a 100 ms without changing the magnetic field. The fast-cycling permanent magnet synchrotron has cycles repeating at a frequency of $f_{REV} \sim 500\text{Hz}$. Variable magnetic field requirements for different energy settings are major limitations at all existing cancer hadron radiation therapy facilities. This proton therapy accelerator removes such limitations.

OUTLINES

To be presented at CAD

A. FLASH CANCER THERAPY

A.1. LIMITATIONS
OF EXISTING
FACILITIES

A.2. DESIGN OF
THE FLASH
THERAPY FACILITY

A.3. COMPARISON
WITH THE PRESENT
FACILITIES

A.4. FLASH
THERAPY FACILITY

DESIGN OF THE
NEW CONCEPT

MAGNETIC FIELD
EXPANSION

DYNAMICAL
APERTURE
STUDIES

BETATRON
FUNCTIONS

B. ACCELERATOR PHYSICS

EXPERIMENTAL
RESULTS

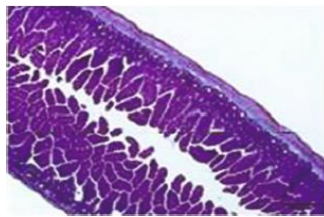
PERMANENT
MAGNETS
COMMISSIONING

ASSEMBLY SET-UP
NSRL-TANDEM
EXPERIMENTS

BEAM
MEASUREMENTS
RESULTS

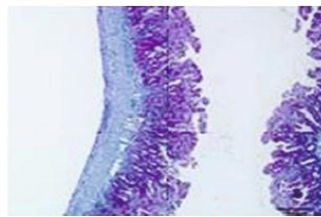
A. FLASH Radiation Therapy

It has been demonstrated, mostly through multiple experiments with animals and just a few human exposures that the **FLASH radiation can decrease the normal tissue injury while maintaining destruction of the tumor cells** as in conventional radiotherapy. **This proposal removes limitations of the existing proton therapy facilities.** It delivers protons with required energies in **a very short time** without varying the magnetic field. It uses the permanent magnets except in the spot scanning part. Permanent magnets proposal is based on **already achieved technology recently demonstrated in the Cornell-Brookhaven National Laboratory Electron Test Accelerator “CBETA” commissioning.**



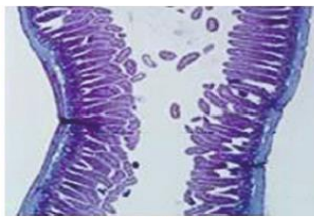
Normal tissue—0 Gy
0 Gy/s

Normal Tissue



Standard PT—18 Gy
0.71 Gy/s

Regular PT

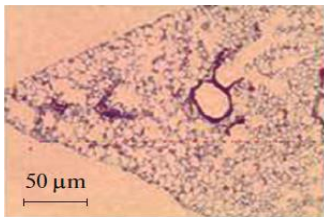


FLASH PT—18 Gy
63 Gy/s

FLASH

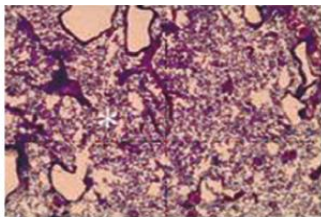
The proton FLASH therapy requirements are **40 Gy/s** with respect to **0.01 Gy/s** in **conventional RT.**

To obtain the dose of **40 Gy/s** in **100 ms** for a volume of **1000 ml (10x10x10 cm)** this becomes **4 Gray in 100 ms.**



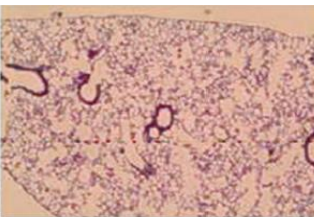
Normal tissue
0 Gy/s

Normal Tissue



Standard PT—17 Gy
0.03 Gy/s

Regular PT



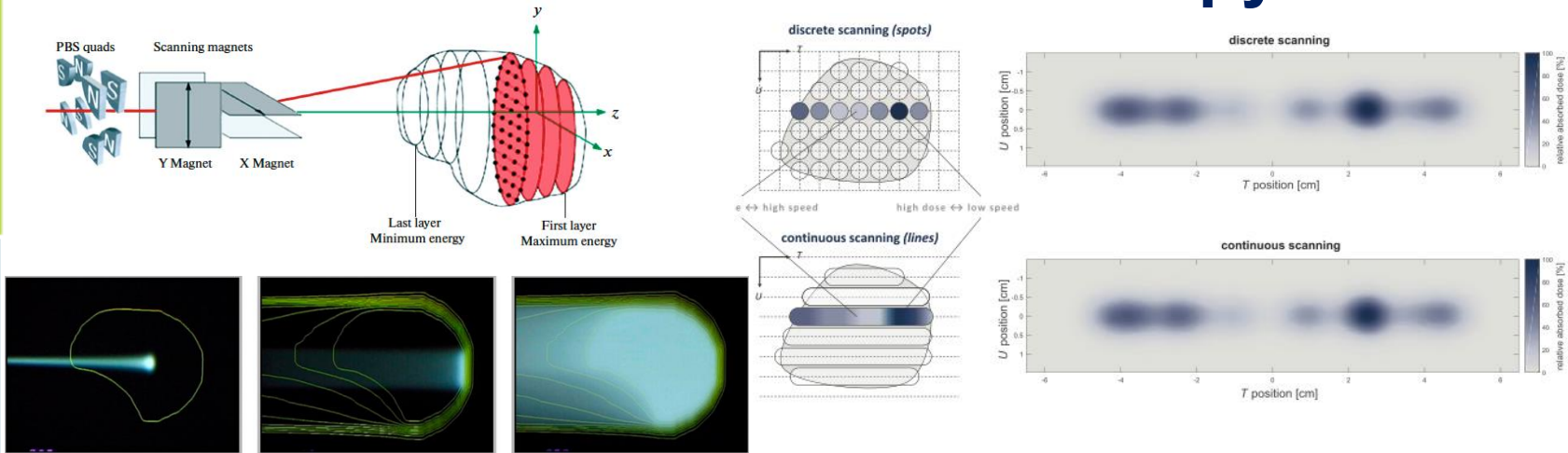
FLASH PT—17 Gy
60 Gy/s

FLASH

3.8×10^{11} protons or 60 nC are delivered in **100 ms** equivalent to **600 nA.**

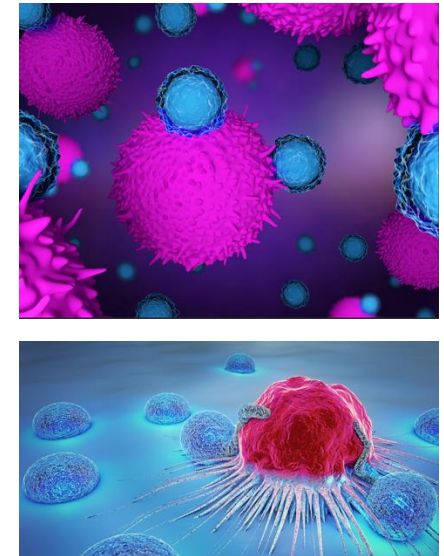
It is not yet clear why the high radiation in a flash spares the healthy tissue. It looks like **that radiochemical depletion of oxygen** occurs and induces the healthy tissue radio resistance.

FLASH Radiation Therapy



The scanning directs the pencil beam to the desired spot within the treatment volume. The spot-scanning is provided by steering magnets within the nozzle or upstream of the last bending magnet. The present systems require that every magnet within the entire beamline must adjust to match the beam energy for a given energy layer. This is avoided in our proposal. An important quote from the review shown in reference [5]: **...”In the future, rapid energy variation will demand beam delivery systems with large energy acceptance; rather than adjust the magnet settings for each energy layer, the beam dispersion is limited by a suitable beam-optical arrangement and sufficient aperture within the magnets and vacuum system provided to obtain a large energy acceptance.”**

Photographs of two cancerous tumors from 100 of different types

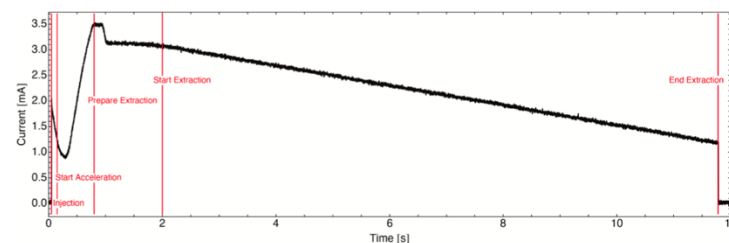
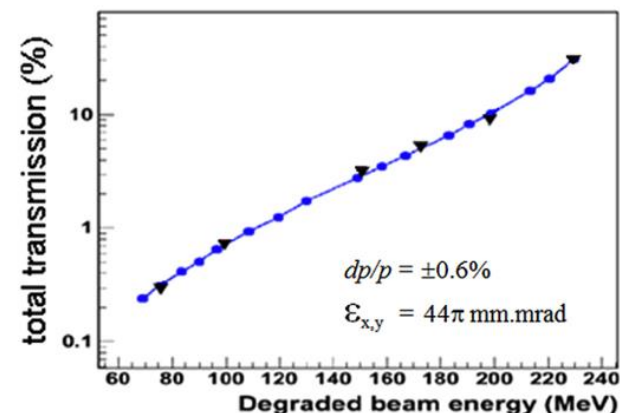


5. S. Jolly, H. Owen, M. Schippers, C. Welsch, “Technical challenges for FLASH proton therapy.” Phys. Med. 2020, 78, 71–82. <https://www.sciencedirect.com/science/article/pii/S1120179720301964>

A.1. Flash Therapy Limitations of the Present Facilities

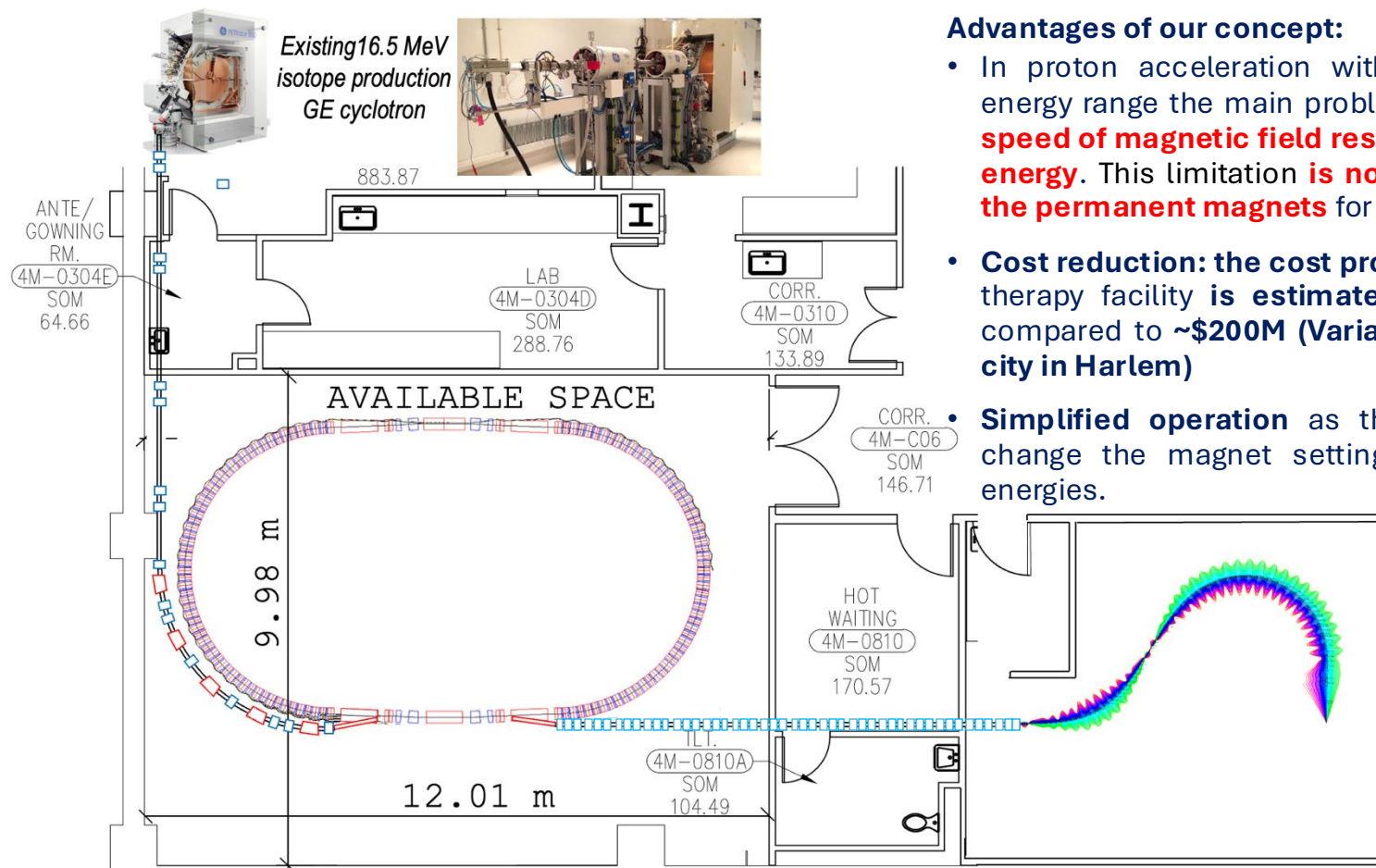
1. Major limitations at any existing cancer hadron radiation therapy system or facility are **the variable magnetic field requirements** for different energy settings. **It is hard to adjust the magnetic field within the short time (hysteresis) required for FLASH.**
2. CYCLOTRON based treatment centers **required energy degraders** as a single 230-250 MeV energy is extracted. **At low energies only 1% of the initial beam is delivered to the patient.** The beam size – emittance is always significantly enlarged. The synchrotrons are **presently cycling with low frequencies and the slow extraction** of the beam towards the patients is (Fig. 2) too long for FLASH.
3. For Synchro-cyclotrons (MEVION and IBA S2C2) the spot scanning has serious problems. It is unlikely that synchro-cyclotrons would be the eventual technology of choice for a pure spot-scanning FLASH proton therapy system.
4. At present, for AVO system the repetition rate of 200 Hz is simply too slow to enable delivery to anything but small volumes. The pulse repetition frequency of 200 Hz makes it possible to irradiate only 20 separate energy layers within a time window of 100 ms. An increase in this rate by **a factor of >500 would be necessary** [4] for spot-scanned FLASH delivery to a 1-liter volume.

Transmission due to Degradar



Beam extraction from synchrotrons requires much longer time with respect FLASH time

Proposal for the FLASH Radiation Cancer Therapy Facility at Stony Brook University Hospital: Fast-Cycling Permanent Magnet Synchrotron and Permanent Magnet Gantry



Advantages of our concept:

- In proton acceleration within the non-relativistic energy range the main problem is the limitation **the speed of magnetic field response to the change of energy**. This limitation **is now eliminated by using the permanent magnets** for the same energy range.
- **Cost reduction: the cost proposed FLASH radiation therapy facility is estimated to be ~\$40M** to be compared to ~\$200M (Varian Facility in New York city in Harlem)
- **Simplified operation** as there is not a need to change the magnet settings for different proton energies.

REDUCTION OF:

- **The power consumption** as the permanent magnets do not require electrical power.
- **The radiation shielding** as the beam losses in synchrotron if any are controlled.
- **magnet sizes and weights** (The gantry weight is significantly reduced).

Comparison of our proposal to the existing facilities

Accelerator Type	Isochronous Cyclotron		Synchrocyclotron		Synchrotron	Linear	FFA facility
Vendor	IBA	Varian	IBA	Mevion	Hitachi	AVO	BNL
System	C230	ProBeam	S2C2	S250	ProBeam	LIGHT	FFA
Maximum Energy (MeV)	230	250	250	250	250	250	250
Minimum Energy (MeV)	70	70	70	70	70	37.5	10 - 30
Peak Current	0.3 μ A	0.8 μ A	\sim 18 μ A	\sim 7 μ A	4.8 mA	\sim 40 μ A	1.2 mA
Max Ave. Current (nA)	300	800	\sim 130	\sim 32	4.8	32	530 - 897
Accel. Frequency (MHz)	106.1	72.8	87.6-63.2	133-90	1.3-10	3,000	378 MHz
Repetition Rate	CW	CW	1 kHz	500-750 Hz	CW	200 Hz	540-809 Hz
Treatment Pulse Length	>400 μ s	>400 μ s	7 μ s	6 μ s	0.5 - 5s	4 μ s	725 μ s
Bunch Length	\sim 2ns	\sim 2 ns	\sim 2 ns	\sim 2 ns	25-200 ns	0.5 ns	\sim 10–405 ns
Max Part. Per Bunch/Pulse	100,000	70,000	8 x 10 ⁸	4 x 10 ⁸	1.5 x 10 ¹¹	10 ¹⁰	1.8 x 10 ¹¹
Electric Central Field	1.7 T	2.4 T	5.75 T	9 T	1.7 T	25 MV/m	Max 1.8 T

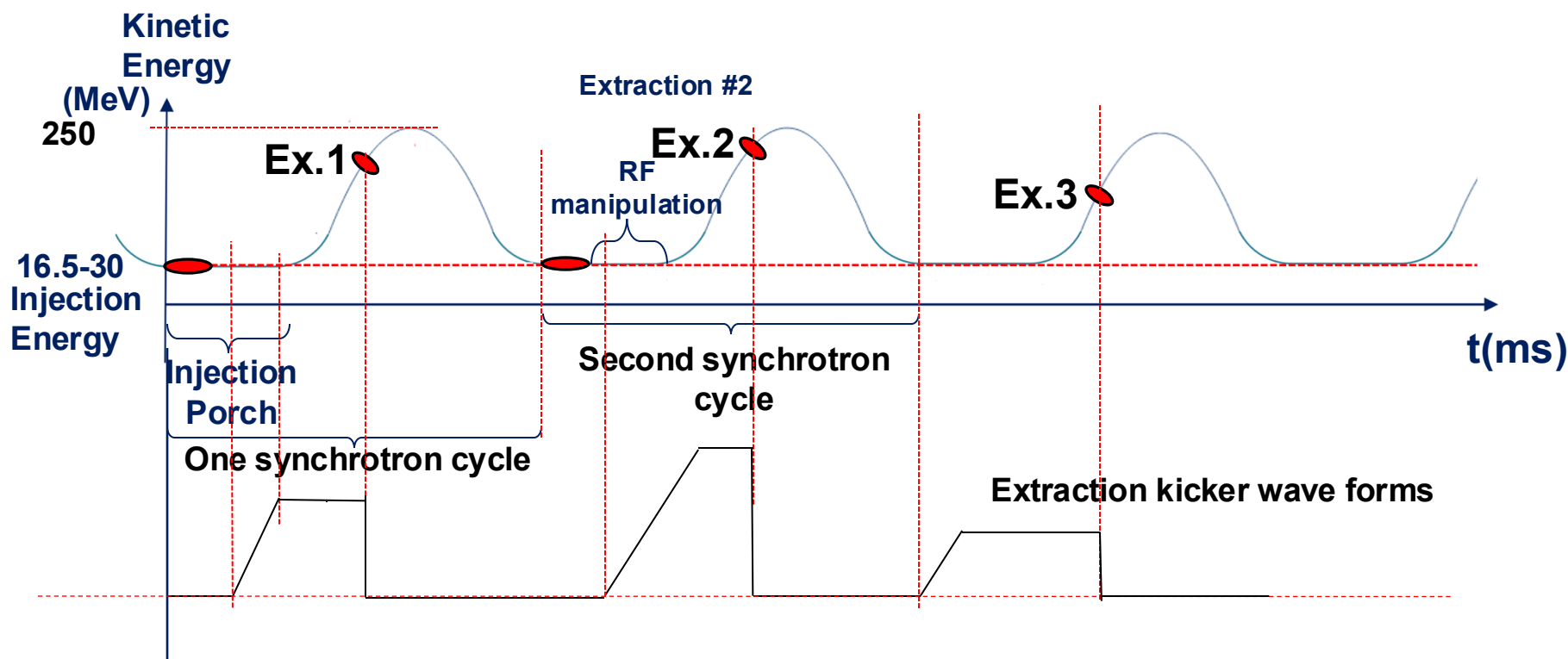
[1] S. Jolly, H. Owen, M. Schippers, C. Welsch, “Technical challenges for FLASH proton therapy.” Phys. Med. 2020, 78, 71–82. <https://pubmed.ncbi.nlm.nih.gov/32947086/>

A.4. Fast Cycling Acceleration with Ferrite cavities

By Stephen Brooks

	Choices for injector cyclotron				Units
	ACSI	GE	TR-19	IBA	
Energy of Injector Cyclotron	14	16.5	19	30	MeV
Cyclotron current	400	200	400	1200	μ A
Treatment charge	60	60	60	60	nC
Injection turns	4	4	4	2	#
RF acceleration per turn	29.70	29.70	29.70	44.55	kV
Calculation	Treatments Options				
Turn duration	585.74	540.6	504.8	405.1	ns
Charge per turn	0.234	0.108	0.151	0.486	nC
Charge per injection	0.937	0.432	0.606	0.972	nC
Machine cycles	65	139	100	62	#
Machine cycle rate	540	540	540	809.95	Hz
Outputs					
Treatment time	120	257	185	76.54	ms

A.4. Facility Cancer Treatment Cycles



$$0.8\text{ms} < t < 1.85\text{ms}$$

$$1.3\text{kHz} < f < 540\text{ Hz}$$

Time of bunch flight 405-550 ns
 At the injection porch time of 926 ms there are 1680 turns. 15 turns for 15 Cyclotron bunches and 1650 turns for RF manipulations

If the RF kicks are ± 30 kV, during the slip-stacking merge a difference in the time of flight for two 2 energies is 773ps. If the batches are 15m apart it would take 370 turns to get them aligned. Next step would be to merge into one bucket by snapping the new RF voltage on.

A.4. Time of flight dependence @ acceleration – Bunch Merging

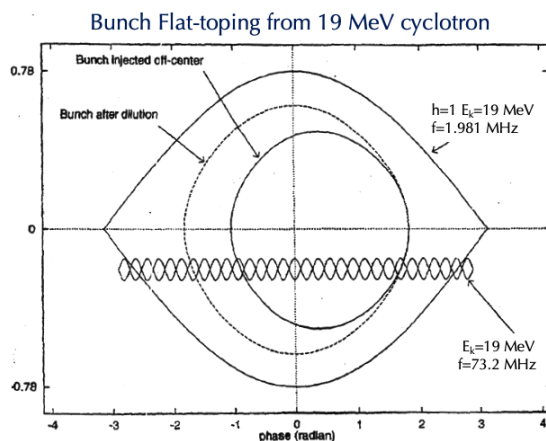
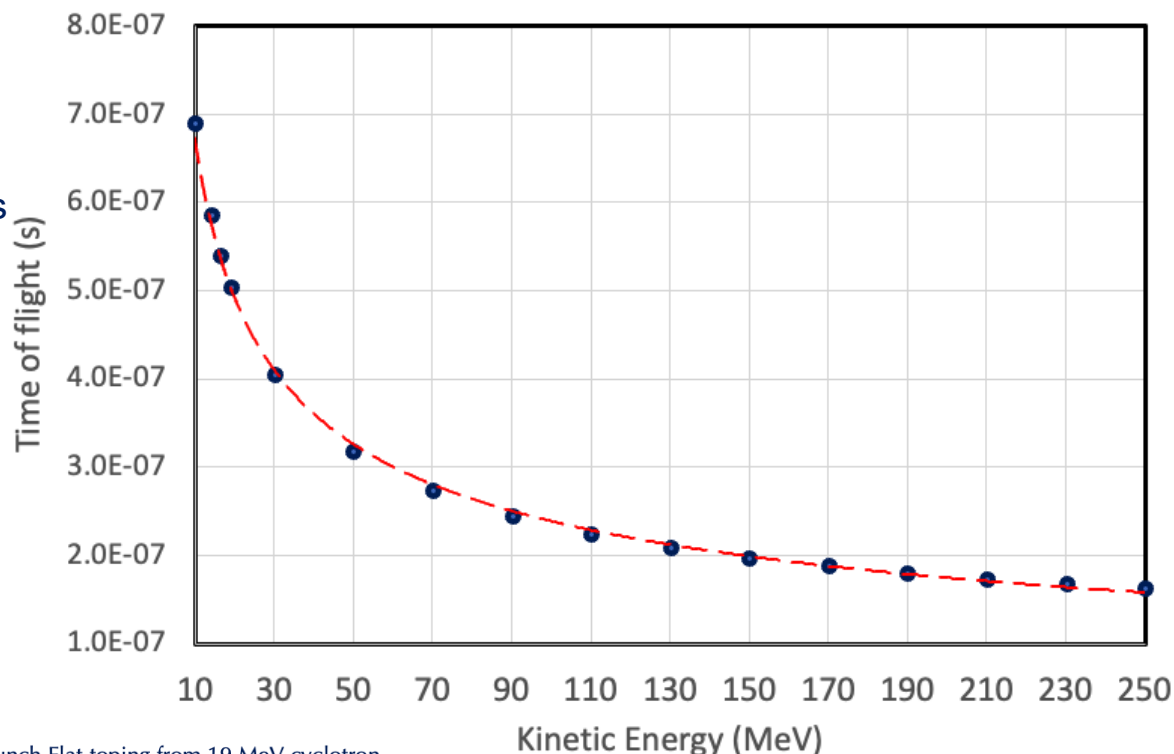
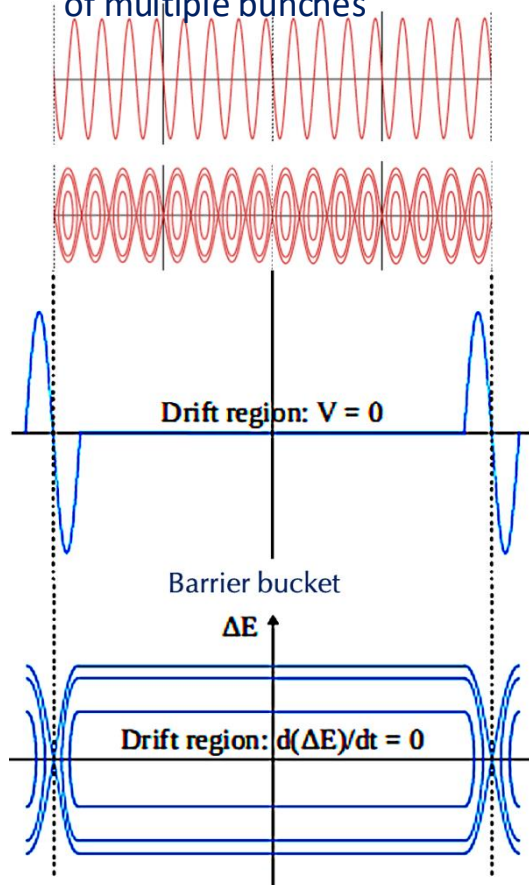
Radio Frequency (RF)-Acceleration:

Using the harmonic number $h=1$ the frequency must follow the time of flight:

$f=1.981$ MHz at 19 MeV

$f=6.132$ MHz at 250 MeV $t_{250}=163.08$ ns

Barrier Bucket merging
of multiple bunches



Time of flight for one turn at injection:

$\tau(10 \text{ MeV}) = 690.9 \text{ ns}$, $h=1 \rightarrow 1.447 \text{ MHz}$

$\tau(14 \text{ MeV}) = 585.7 \text{ ns}$, $h=1 \rightarrow 1.707 \text{ MHz}$

$\tau(16.5 \text{ MeV}) = 540.6 \text{ ns}$, $h=1 \rightarrow 1.850 \text{ MHz}$

$\tau(19 \text{ MeV}) = 504.8 \text{ ns}$, $h=1 \rightarrow 1.981 \text{ MHz}$

$\tau(30 \text{ MeV}) = 405.2 \text{ ns}$, $h=1 \rightarrow 2.468 \text{ MHz}$

Time of flight for one turn at maximum energy:

$\tau(250 \text{ MeV}) = 163.08 \text{ ns}$, $h=1 \rightarrow \text{MHz}$

**B.1. DESIGN OF THE
NEW CONCEPT**

**B.2. MAGNETIC FIELD
EXPANSION**

**B.3. BETATRON
FUNCTIONS**

**B.4. DYNAMICAL
APERTURE STUDIES**

B.5. SYNCHROTRON

ACCELERATOR PHYSICS

**To be shown
at CAD review**

**C.1. EXPERIMENTAL
RESULTS**

**C.2. PERMANENT MAGNETS
COMMISSIONING**

**C.3 ASSEMBLY SET-UP NSRL
TANDEM EXPERIMENTS**

**C.4. BEAM MEASUREMENTS
RESULTS**

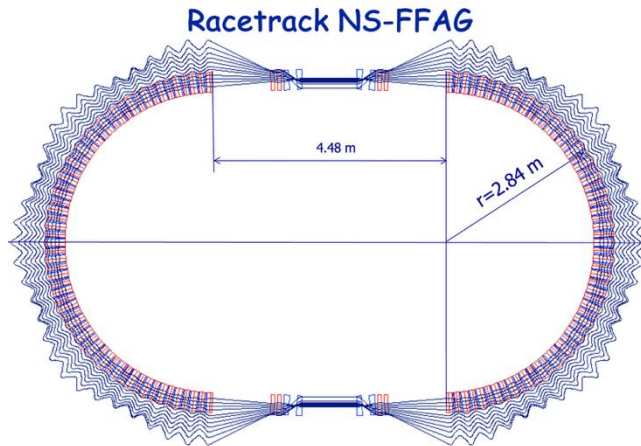
**C.5. ACCOMPLISHED TASKS
EXPERIMENT CONCLUSION
SUMMARY**

B.1. NOVEL ACCELERATOR PHYSICS CONCEPT

Background – FFA Gradient Accelerator

IPAC 2011 Contributed at
San Sebastian

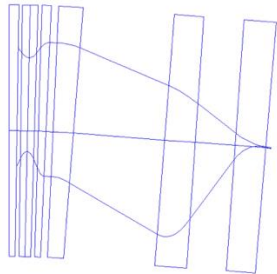
‘CBETA’ Project is a Proof of Principle



Dejan Trbojevic, September 21, 2011 IPAC 2011-San Sebastian, Spain

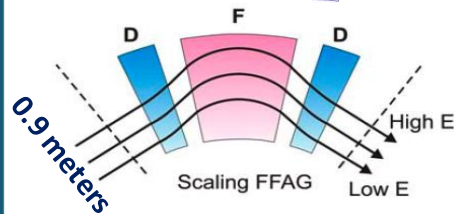
BROOKHAVEN
NATIONAL LABORATORY 28

Orbits of the maximum and minimum energy



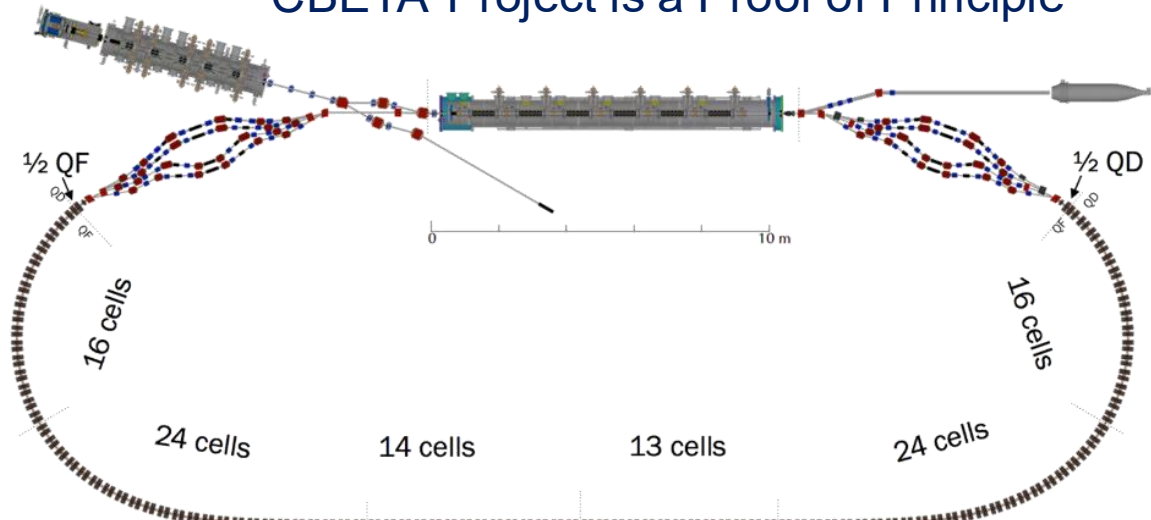
The proposal is based on the existing US patent D. Trbojevic, Title: “Non-scaling fixed field alternating gradient permanent magnet cancer therapy accelerator”, patent number: US 9661737 B2, Date of the patent: May 23, 2017.

<https://patentimages.storage.googleapis.com/42/5e/92/f7da1cf617d6e3/US9661737.pdf>



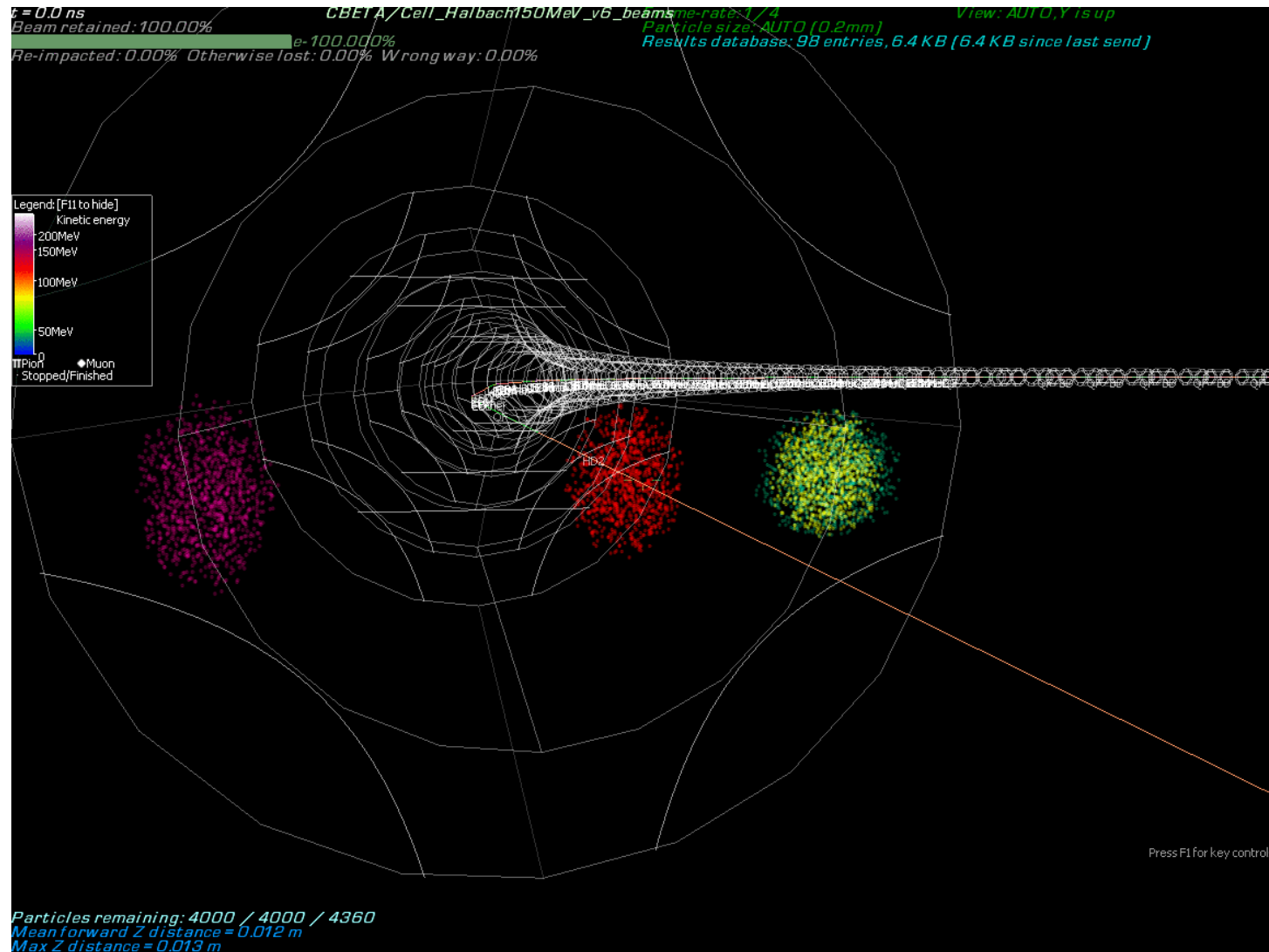
3 cm

$$B = B_0 + r G_0$$



B.1. Background: Particles with multiple energies are accelerated in the same Fixed Field Alternating (FFA) Gradient magnets.

Stephen Brooks' simulation of particles' motion in FFA with 42, 78, 114 and 150 MeV electrons



B.2. Magnetic Field Expansion

Multipole Field Components vs. Tunes

Energy 1:

$$B_{Fy1} = \sum_{k=0}^{12} \frac{b_{Fk1}}{k!} x^k = B_{Fo} + b_{F1,1}x + \frac{b_{F2,1}}{2} x^2 + \dots$$

$$B_{Dy1} = \sum_{k=0}^{12} \frac{b_{Dk1}}{k!} x^k = B_{Do} + b_{D1,1}x + \frac{b_{D2,1}}{2} x^2 + \dots$$

$$\delta v_{x,1} = \int \frac{ds}{2\pi\rho} \beta_{x,1} \sum_{n=0}^{12} (C_{n,1}^{x1,b1} b_{n,1} + C_{n,1}^{x1,a1} a_{n,1})$$

$$\delta v_{y,1} = \int \frac{ds}{2\pi\rho} \beta_{y,1} \sum_{n=0}^{12} (C_{n,1}^{y1,b1} b_{n,1} + C_{n,1}^{y1,a1} a_{n,1})$$

Energy 2:

$$B_{Fy2} = \sum_{k=0}^{12} \frac{b_{Fk2}}{k!} x^k = B_{Fo} + b_{F1,2}x + \frac{b_{F2,2}}{2} x^2 + \dots$$

$$B_{Dy2} = \sum_{k=0}^{12} \frac{b_{Dk2}}{k!} x^k = B_{Do} + b_{D1,2}x + \frac{b_{D2,2}}{2} x^2 + \dots$$

$$\delta v_{x,2} = \int \frac{ds}{2\pi\rho} \beta_{x,2} \sum_{n=0}^{12} (C_{n,1}^{x2,b2} b_{n,2} + C_{n,2}^{x2,a2} a_{n,2})$$

$$\delta v_{y,2} = \int \frac{ds}{2\pi\rho} \beta_{y,2} \sum_{n=0}^{12} (C_{n,2}^{y2,b2} b_{n,2} + C_{n,1}^{y2,a2} a_{n,2})$$

Energy 59:

$$B_{Fy59n} = \sum_{k=0}^{12} \frac{b_{Fk59}}{k!} x^k = B_{Fo} + b_{F1,59}x + \frac{b_{F2,59}}{2} x^2 + \dots$$

$$B_{Dy59n} = \sum_{k=0}^{12} \frac{b_{Dk59}}{k!} x^k = B_{Do} + b_{D1,59}x + \frac{b_{D2,59}}{2} x^2 + \dots$$

$$\delta v_{x,59} = \int \frac{ds}{2\pi\rho} \beta_{x,59} \sum_{n=0}^{12} (C_{n,59}^{x59,b59} b_{n,59} + C_{n,59}^{x59,a59} a_{n,59})$$

$$\delta v_{y,59} = \int \frac{ds}{2\pi\rho} \beta_{y,59} \sum_{n=0}^{12} (C_{n,59}^{y59,b59} b_{n,59} + C_{n,59}^{y59,a59} a_{n,59})$$

Energy 60:

$$B_{Fy59,12} = \sum_{k=0}^{12} \frac{b_{Fk60}}{k!} x^k = B_{Fo} + b_{F1,60}x + \frac{b_{F2,60}}{2} x^2 + \dots$$

$$\delta v_{x,60} = \int \frac{ds}{2\pi\rho} \beta_{x,60} \sum_{n=0}^{12} (C_{n,60}^{x60,b60} b_{n,60} + C_{n,60}^{x60,a60} a_{n,60})$$

B.2. Magnetic Field Expansion Least Square Minimization Towards the Right Multipoles

$$\Delta B_{yF,j} = B_{yF,j} - B_{yF,C} = \sum_{k=3}^{14} \frac{b_{kF,j}}{k!} x_{\beta}^k = \frac{b_{3F,j}}{3!} x_{\beta}^3 + \dots$$

$$\Delta B_{yD,j} = B_{yD,j} - B_{yD,C} = \sum_{k=3}^{14} \frac{b_{kD,j}}{k!} x_{\beta}^k = \frac{b_{3D,j}}{3!} x_{\beta}^3 + \dots$$

$$\delta \nu_{xj} \cong \frac{l_i \beta_{xi}}{2\pi B \rho} \sum_{k=0}^{14} b_k \sum_{m=1}^k (\delta \eta_i)^{k-m} \hat{x}_{\beta}^{m-1} \frac{k!}{m!(k-m)!} C_{m+1}$$

$$\delta \nu_{yj} \cong -\frac{l_i \beta_{yi}}{2\pi B \rho} \sum_{k=0}^{14} b_k \sum_{m=1}^k (\delta \eta_i)^{k-m} \hat{x}_{\beta}^{m-1} \frac{k!}{(k-m)!(m-1)!} C_{m-1}$$

$$C_m = \frac{(m-1)(m-3)\dots 1}{m(m-2)\dots 2}, \quad C_2 = \frac{1}{2}, C_4 = \frac{3}{8}, C_6 = \frac{5}{24}, C_8 = \frac{7}{48} \dots$$

Script for adjusting the magnetic multipoles

A new approach to the Non-Scaling Fixed Field Alternating Gradient Accelerators where additional multipoles in the magnets are added following the Taylor expansion of the magnetic field in the focusing and defocusing alternating gradient magnets. The goal is to obtain very large momentum or energy range with the fixed betatron tunes throughout the whole range between $-69\% < \Delta p/p < 64\%$ or for example for proton accelerator to cover the kinetic energy range between $10 \text{ MeV} < E_k < 250 \text{ MeV}$.

1. First step is, setting the reference energy. The reference energy I define as the circular orbit without betatron oscillations. The **closed orbit** is found by **variation of the gradients** using the dipole bending field \mathbf{B}_0 which defines the circular orbit.
2. At the energies higher or lower from the reference orbit the **closed orbit oscillates with higher or lower radii, respectively**. Additional multipoles are required to obtain the closed orbit with the same **betatron tunes established at the reference** orbit. They start with the **sextupoles**. As the energy is growing or reducing, with maximum orbit offsets getting larger, **to keep the same tunes additional multipoles like octupoles, decapoles and twelve poles are needed**.

B.2. Lattice Perturbation and Amplitude Dependent Tune spread – Distortion Functions

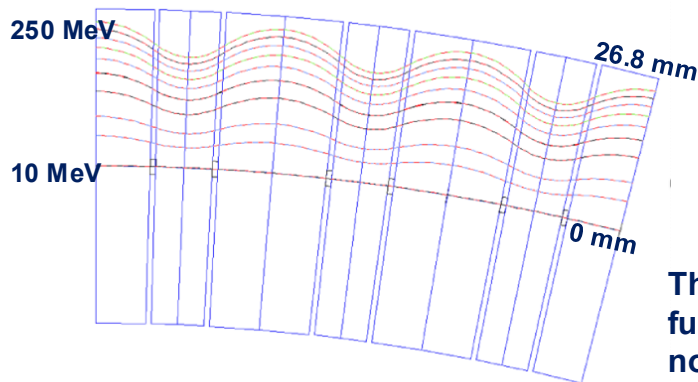
Hamiltonian describing the motion of a single particle beam from N. Merminga and K.Y. Ng Fermilab Note FN-403 1988:

$$H_1 = \frac{1}{2} [P_x^2 + K_x(s)X^2] + \frac{1}{2} [P_y^2 + K_y(s)Y^2] - \frac{B'_x}{B\rho} XY + \frac{B''_y}{6(B\rho)} (X^3 - 3XY^2) - \frac{B''_x}{6(B\rho)} (3X^2Y - Y^3) + \frac{B'''_y}{24(B\rho)} (X^4 - 6X^2Y^2 + Y^4) - \frac{B'''_x}{6(B\rho)} (X^3Y - XY^3), \quad (1.1)$$

Labels for the Hamiltonian terms:

- Skew quadrupole potential: $-\frac{B'_x}{B\rho} XY$
- Normal Sextupole potential: $\frac{B''_y}{6(B\rho)} (X^3 - 3XY^2)$
- Skew Sextupole potential: $-\frac{B''_x}{6(B\rho)} (3X^2Y - Y^3)$
- Normal Octupole Potential: $\frac{B'''_y}{24(B\rho)} (X^4 - 6X^2Y^2 + Y^4)$
- Skew Octupole Potential: $-\frac{B'''_x}{6(B\rho)} (X^3Y - XY^3)$

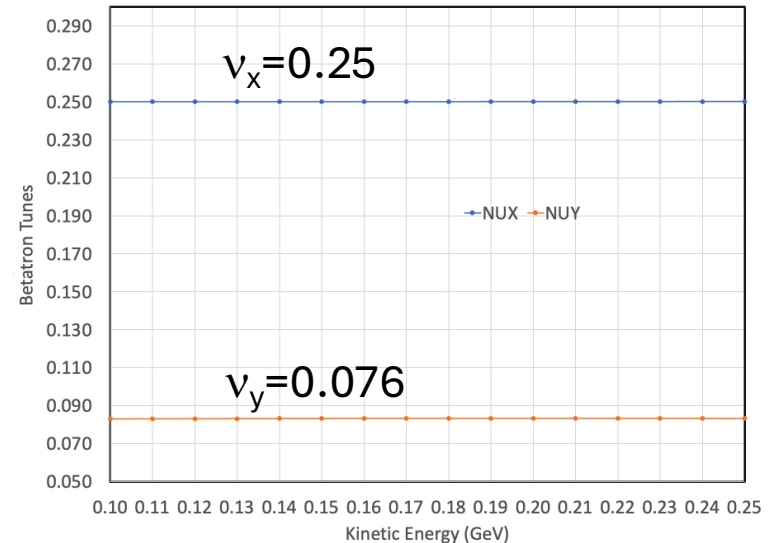
P_x and P_y are the canonical momenta to the horizontal and vertical displacements X and Y , $K_x(s)$ and $K_y(s)$ are proportional to the restoring forces due to the ring's curvature and the field gradients of the normal quadrupoles.



The first results with respect to the tune dependence and betatron functions were satisfied but the dynamical aperture at low energies did not show satisfying results. A conclusion was that the beam size with respect to the non-linear magnetic field dependence was too large.

First Solution

Betatron Tune Dependence on Proton Kinetic Energy



B.2. Magnetic Field Expansion

Taylor expansion by PTC and Bmad had disagreements with MUON1-by S. Brooks and ZGOUBI-by F. Meot due to the end fields presentation

Taylor Expansion:

$$B_y = \sum_{k=0}^{\infty} \frac{b_k}{k!} x^k = B_0 + b_1 x + \frac{b_2}{2} x^2 + \dots$$

$$\begin{aligned} b_{1F} = K_{BF} &= 38.054 \\ b_{1D} = K_{BD} &= -54.87 \\ b_{2F} = K_{SF} &= 1524.60 \\ b_{2D} = K_{BD} &= -3073.40 \\ b_{3F} = K_{OF} &= 23660 \\ b_{3D} = K_{OD} &= -30300 \\ b_{4F} = K_{FDEC} &= 277500 \\ b_{4D} = K_{DDEC} &= -1.085E6 \\ B_{5F} = K_{FDDC} &= 4.6E6 \\ B_{5D} = K_{DDC} &= -1.54E6 \end{aligned}$$

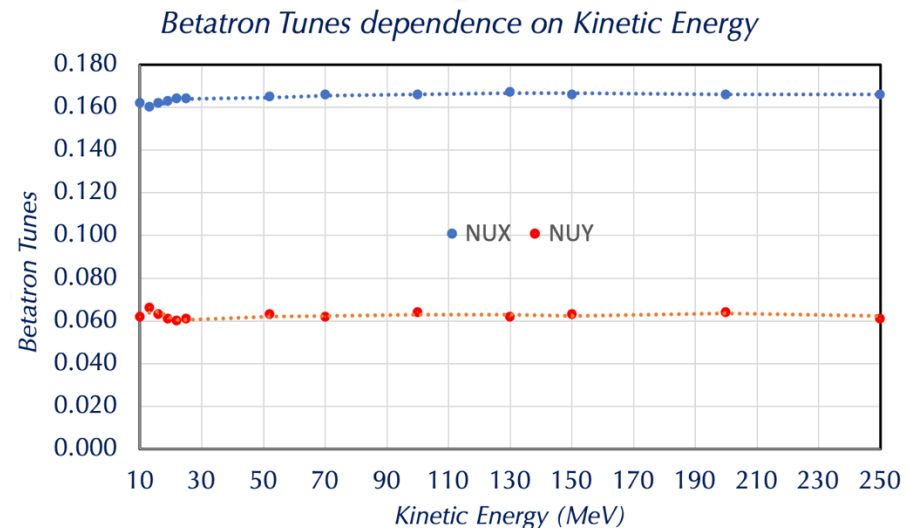
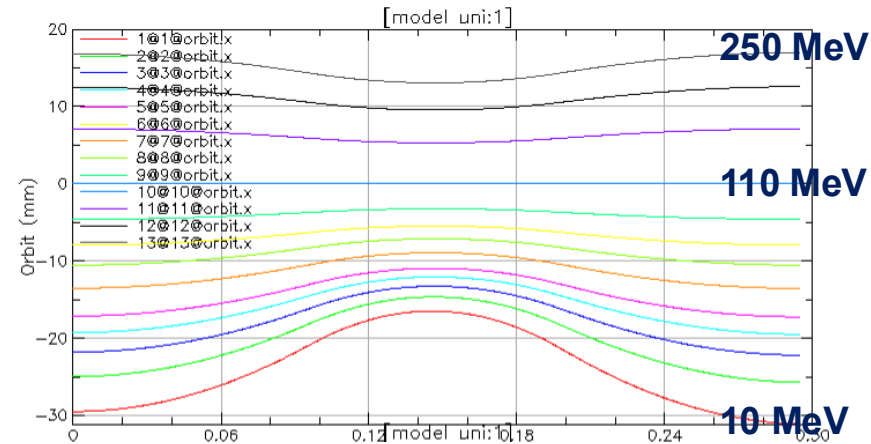
QUADRUPOLES

SEXTUPOLES

OCTUPOLES

DECAPOLES

TWELVE POLES



B.2. Transverse Magnetic Field Taylor expansion is replaced by Fourier series by S. Brooks

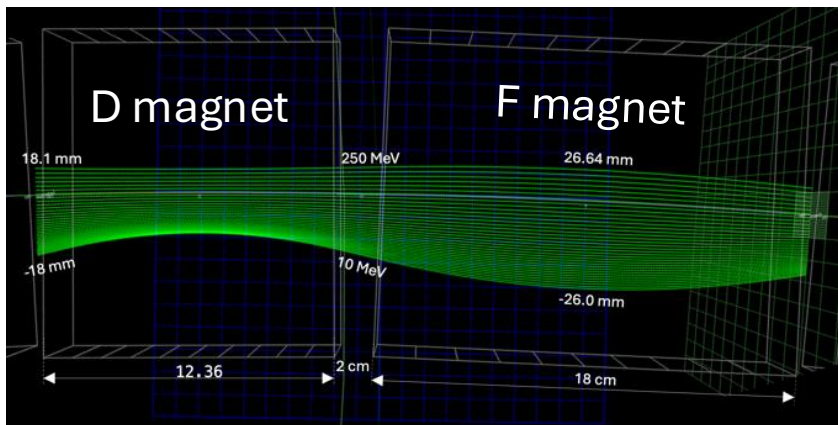
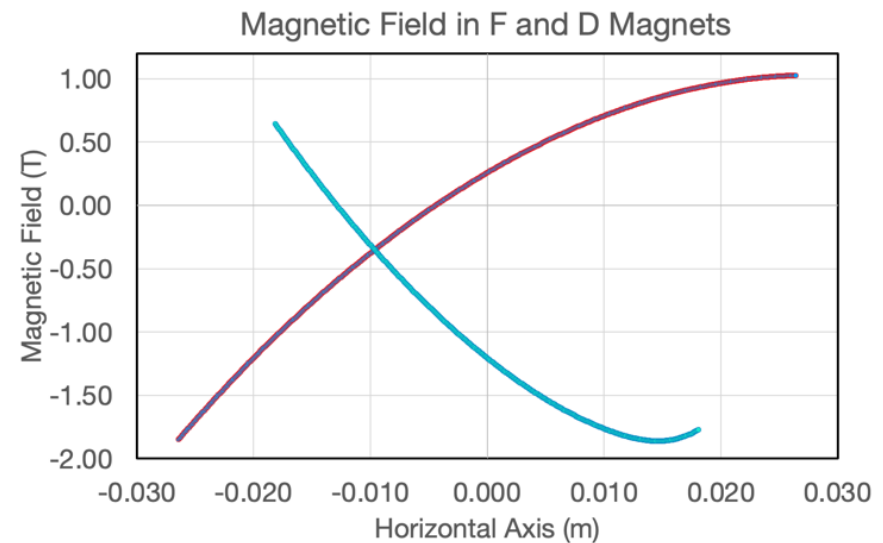
$$g(x) = c_0 + \sum_{n=1}^8 s_n \sin(nkx) + c_n \cos(nkx),$$

where k is a transverse scale factor and $c_0 \dots c_8$ are the coefficients defining the field (units Tesla). The values of these coefficients are provided in Table below.

The stable orbits in a wide energy range are obtained **by adjustments of the F and D sextupoles and additional multipoles up to eighteen poles.**

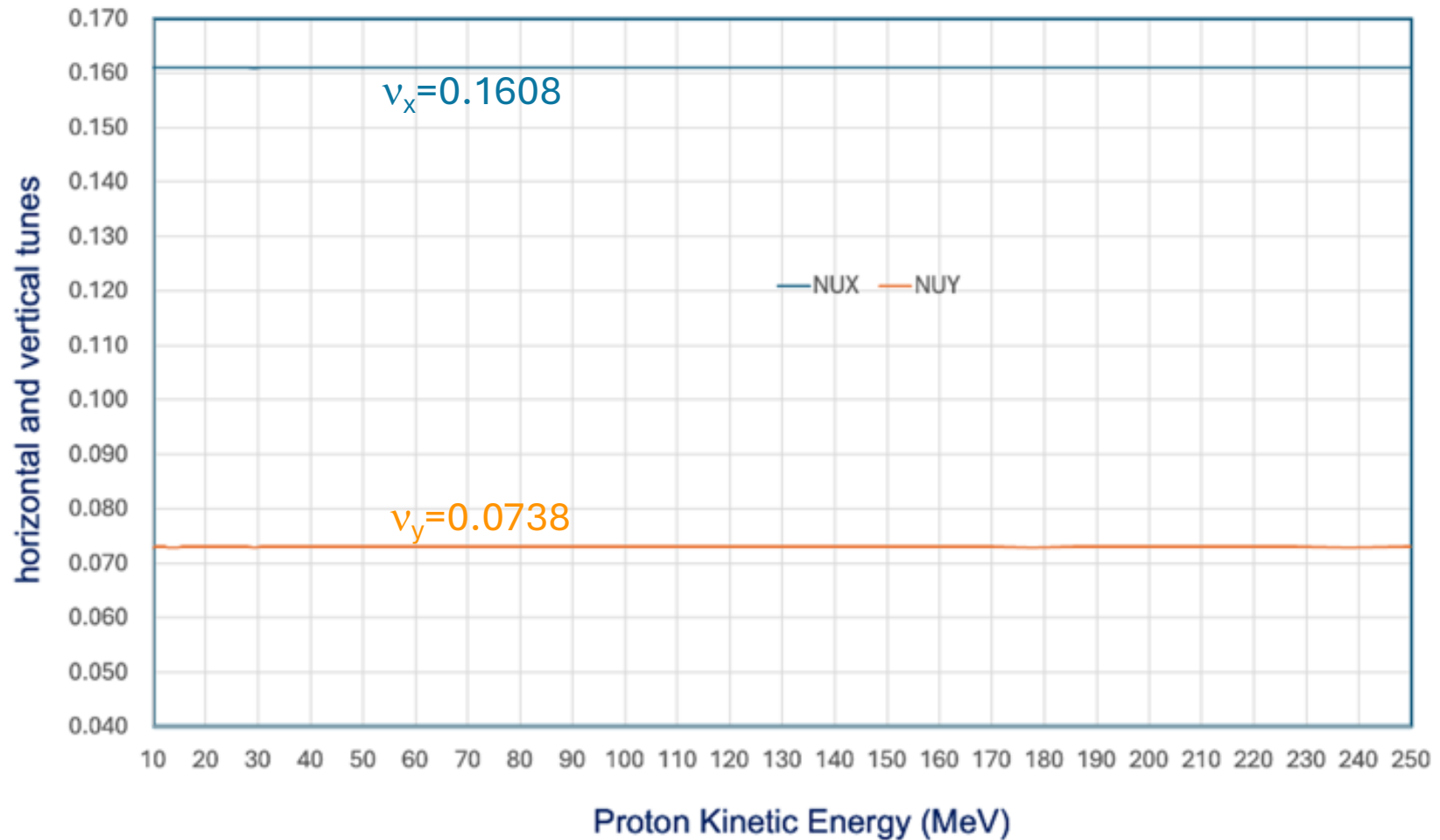
Fourier Coefficients for the Magnet Body Fields

n	Magnet F, k=60 (m ⁻¹)		Magnet D, k=60 (m ⁻¹)	
	c _n (T)	s _n (T)	c _n (T)	s _n (T)
0	-0.69803	-	0.98934	-
1	0.07561	1.73518	-1.91338	-2.46075
2	0.30509	-0.44226	0.39756	0.493241
3	-0.13953	0.08267	-0.13952	-0.13428
4	0.02768	-0.02081	0.01161	0.07066
5	-0.00196	0.01916	-0.01078	0.00724
6	0.00342	-0.01124	0.02076	-0.04809
7	-0.00265	0.002711	-0.01478	0.03069
8	0.00390	-6.57E-05	0.00390	-0.00686

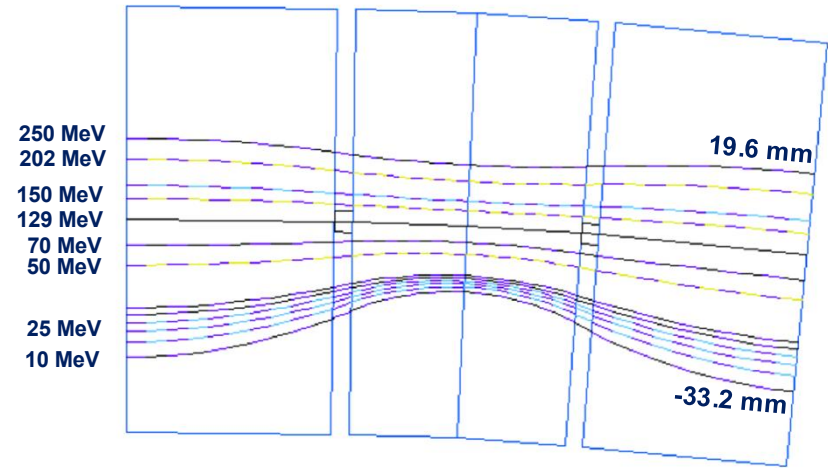
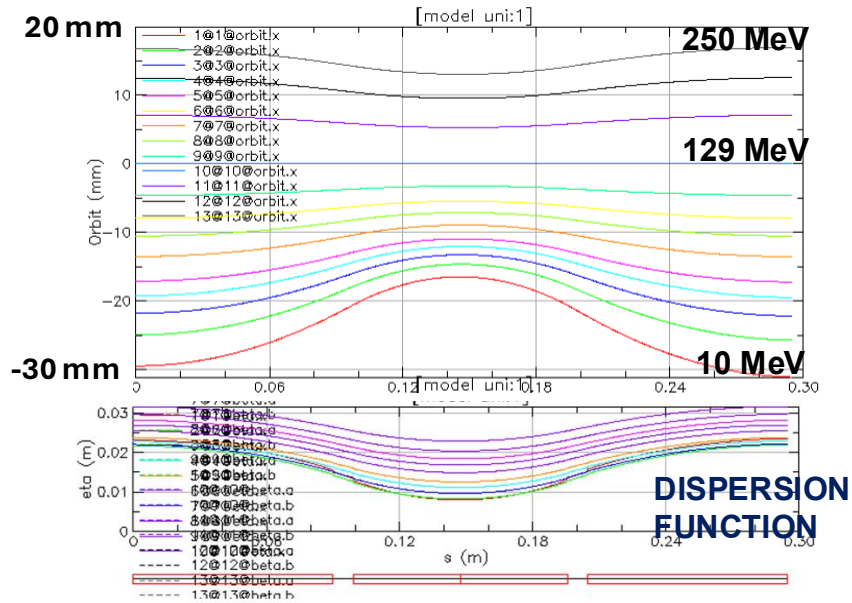


**Orbit dependence on energy
with rectangular non-linear D
and F permanent magnet layout**

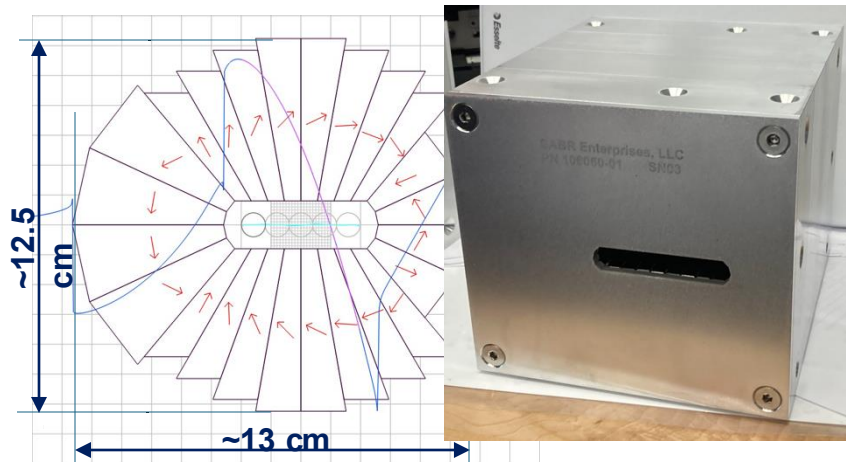
B.3. Fixed Betatron Tunes in Range $10 < E_k < 250$ MeV or in momentum space $-73\% < \Delta p/p < 43.4\%$



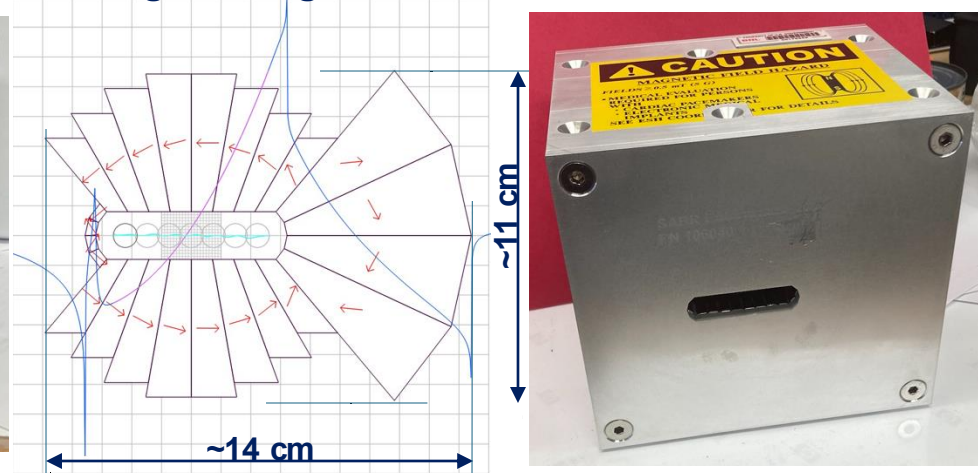
B.3. Orbit Dependence on Energy



F magnet length 18 cm

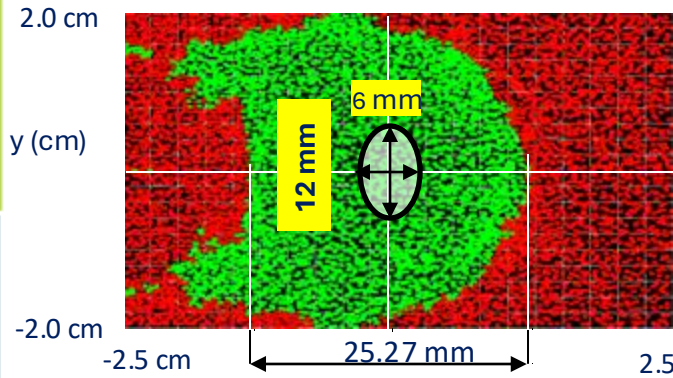


D magnet Length 9.652 cm

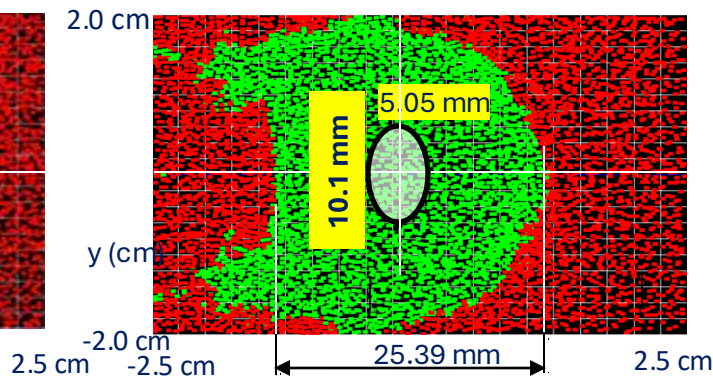


B.4. Dynamical Aperture Studies

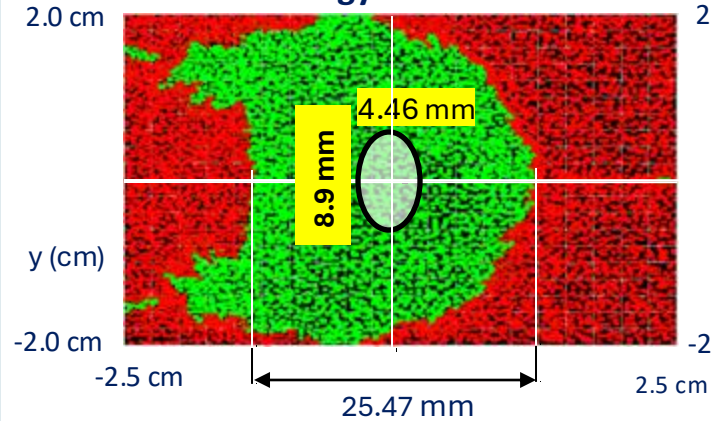
The kinetic energy $E_k=10\text{MeV}$



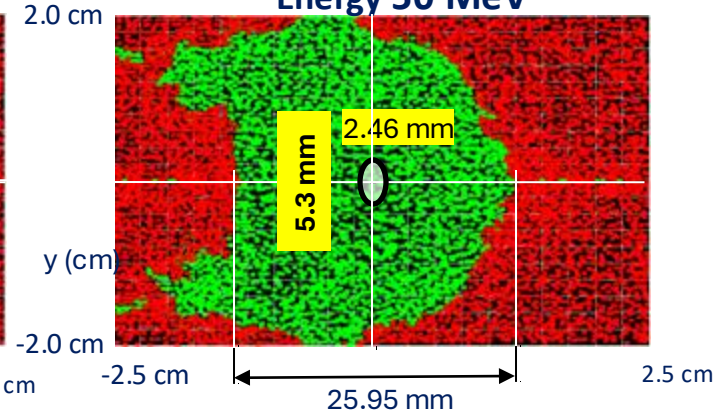
Energy 14 MeV



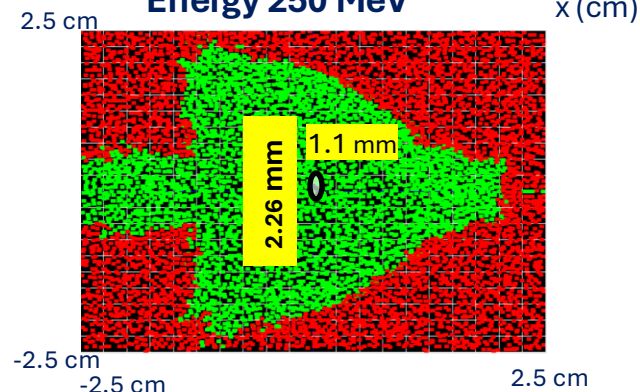
Energy 18 MeV



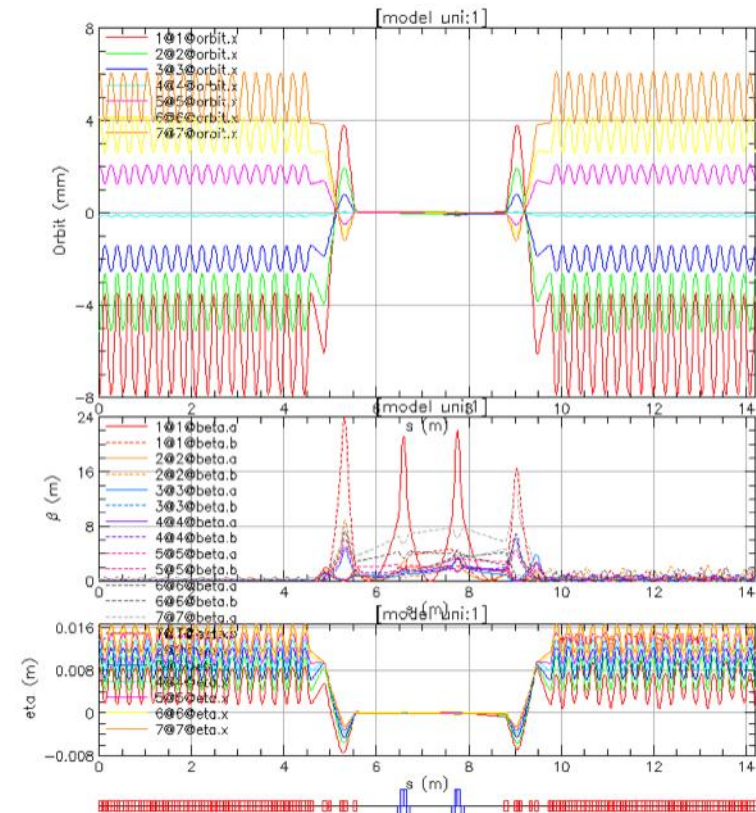
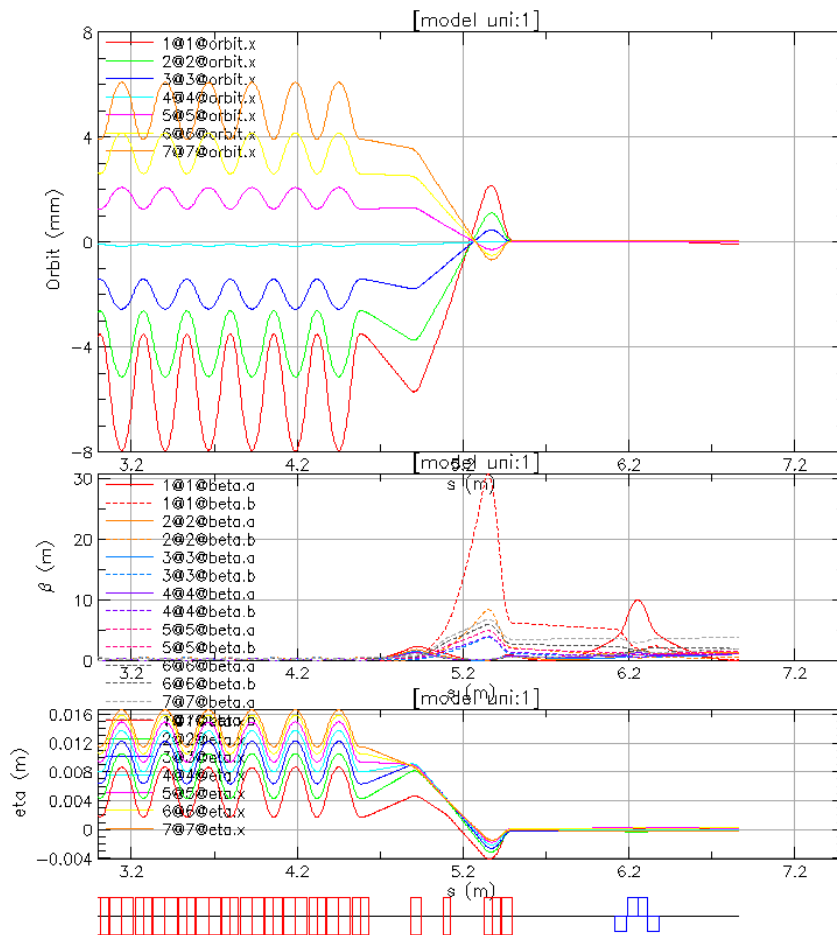
Energy 50 MeV



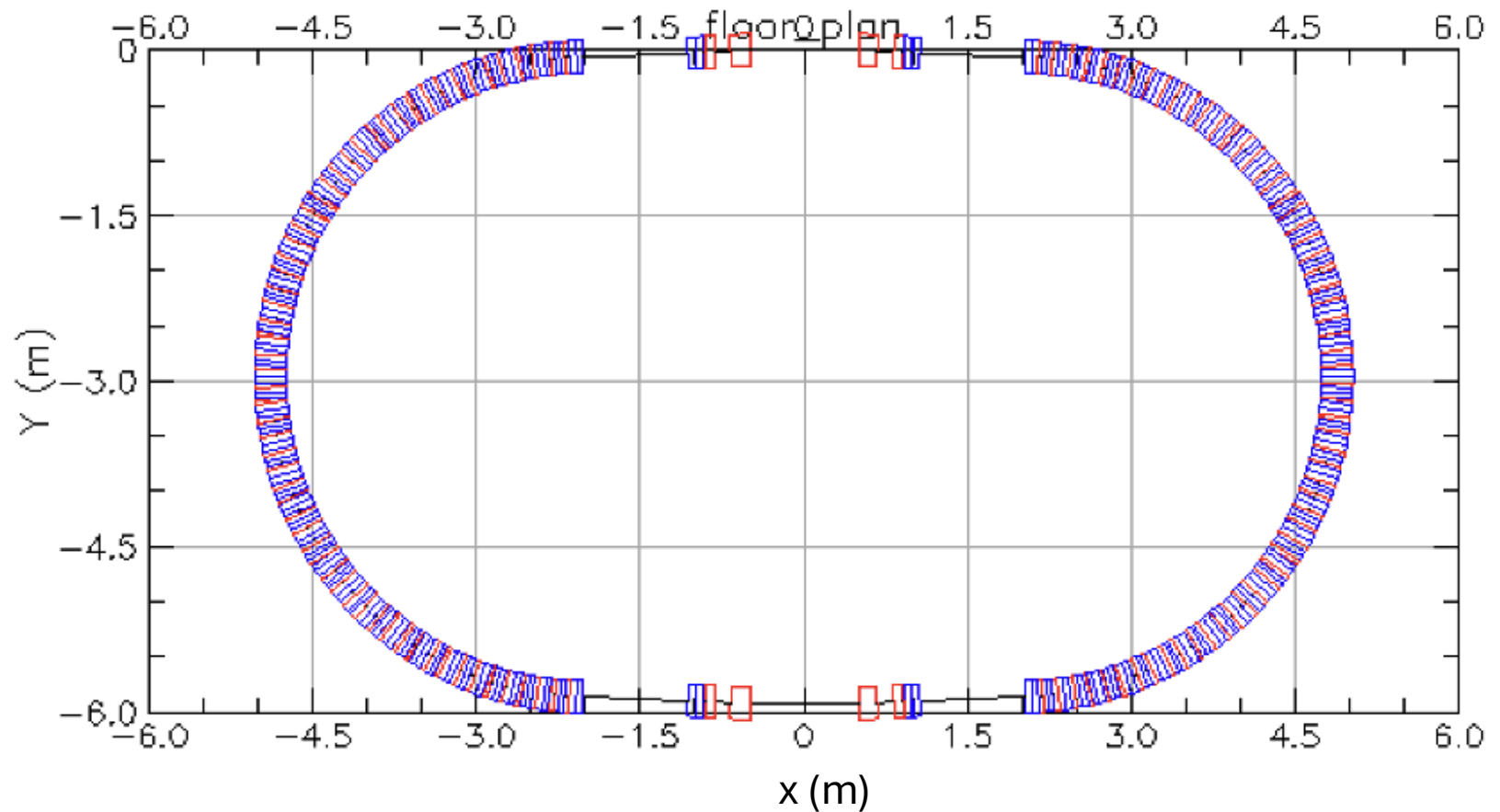
Energy 250 MeV



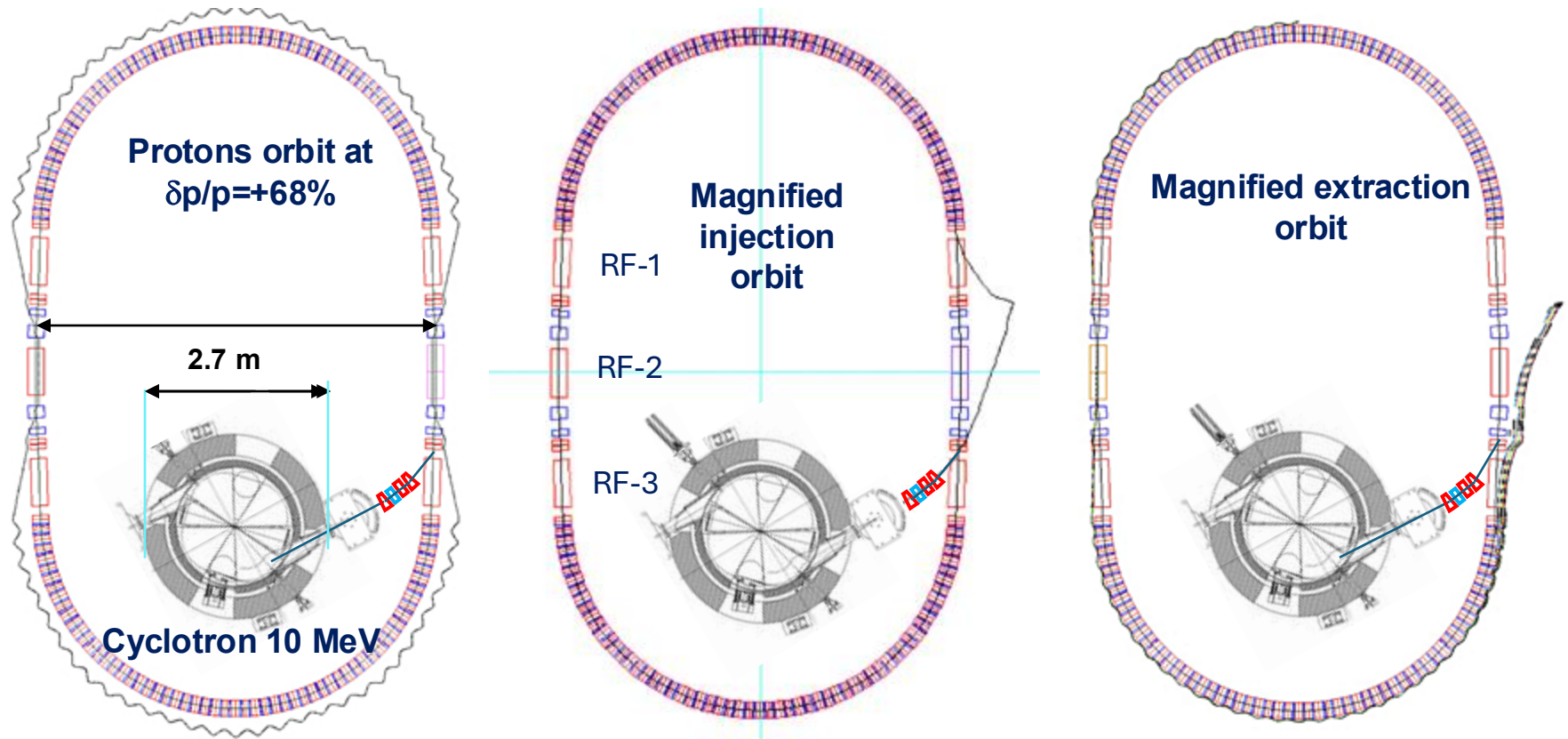
B.5. An example of Matching the Multiple Orbits from Arc to the Single Orbit in the Straight Section



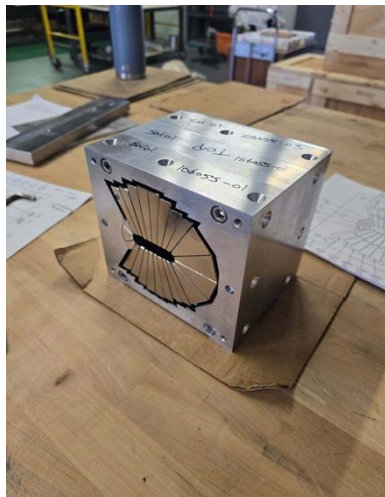
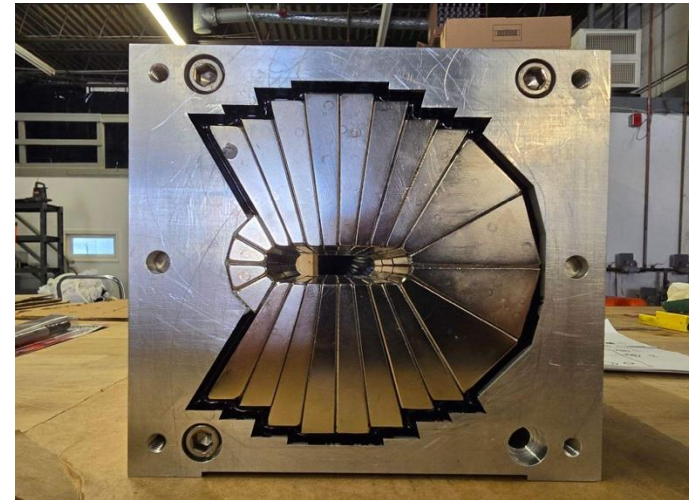
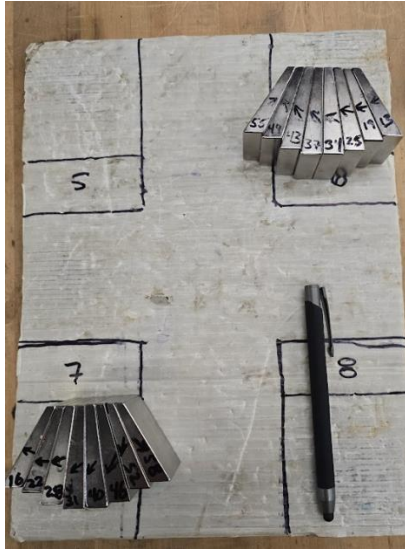
B.5. Permanent Magnet Synchrotron



B.5. Synchrotron: Injection, Acceleration, Extraction

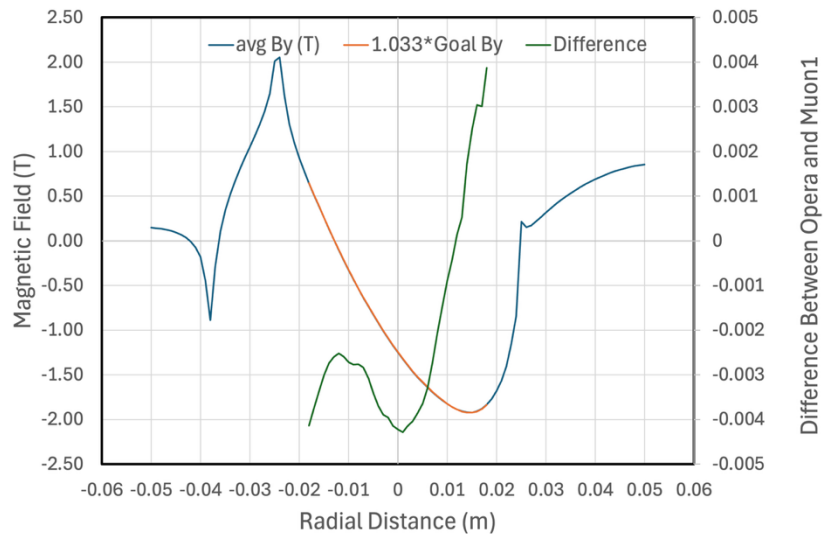


C.1. Permanent Magnet Production: SABR company (Massachusetts)

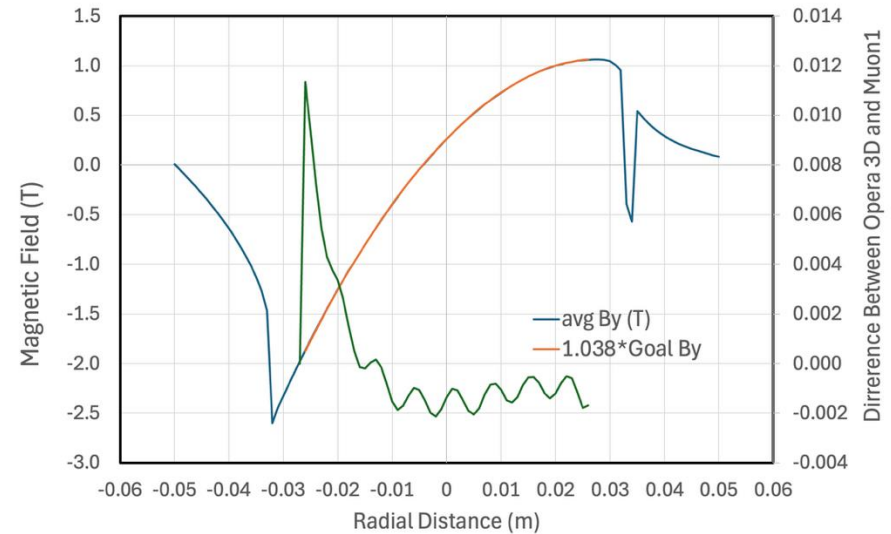


C.1. Permanent Magnet Design, Building & Measuring Magnetic Field

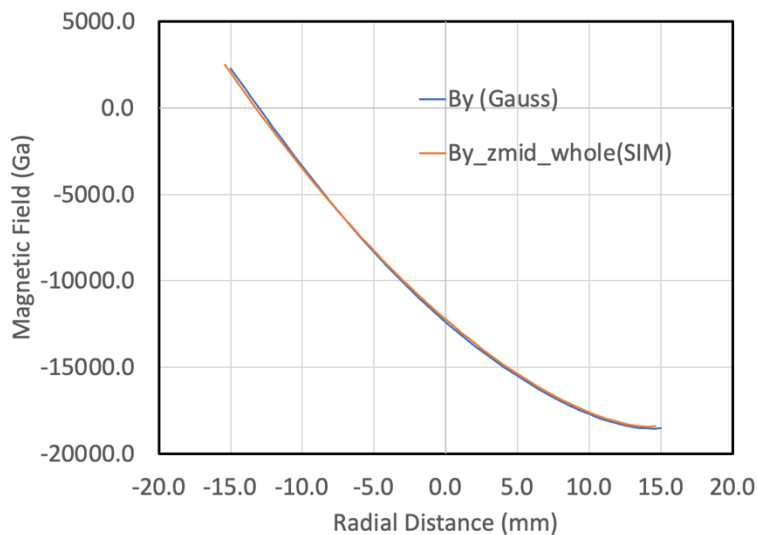
Comparison between OPERA 3D and Muon1 Calculation



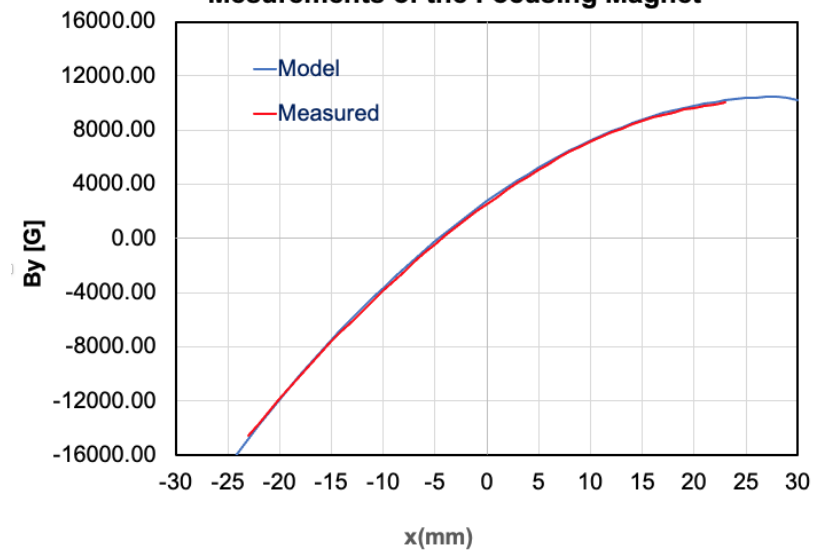
Comparison between Opera3D and Muon1 Predictions by Stephen Brooks



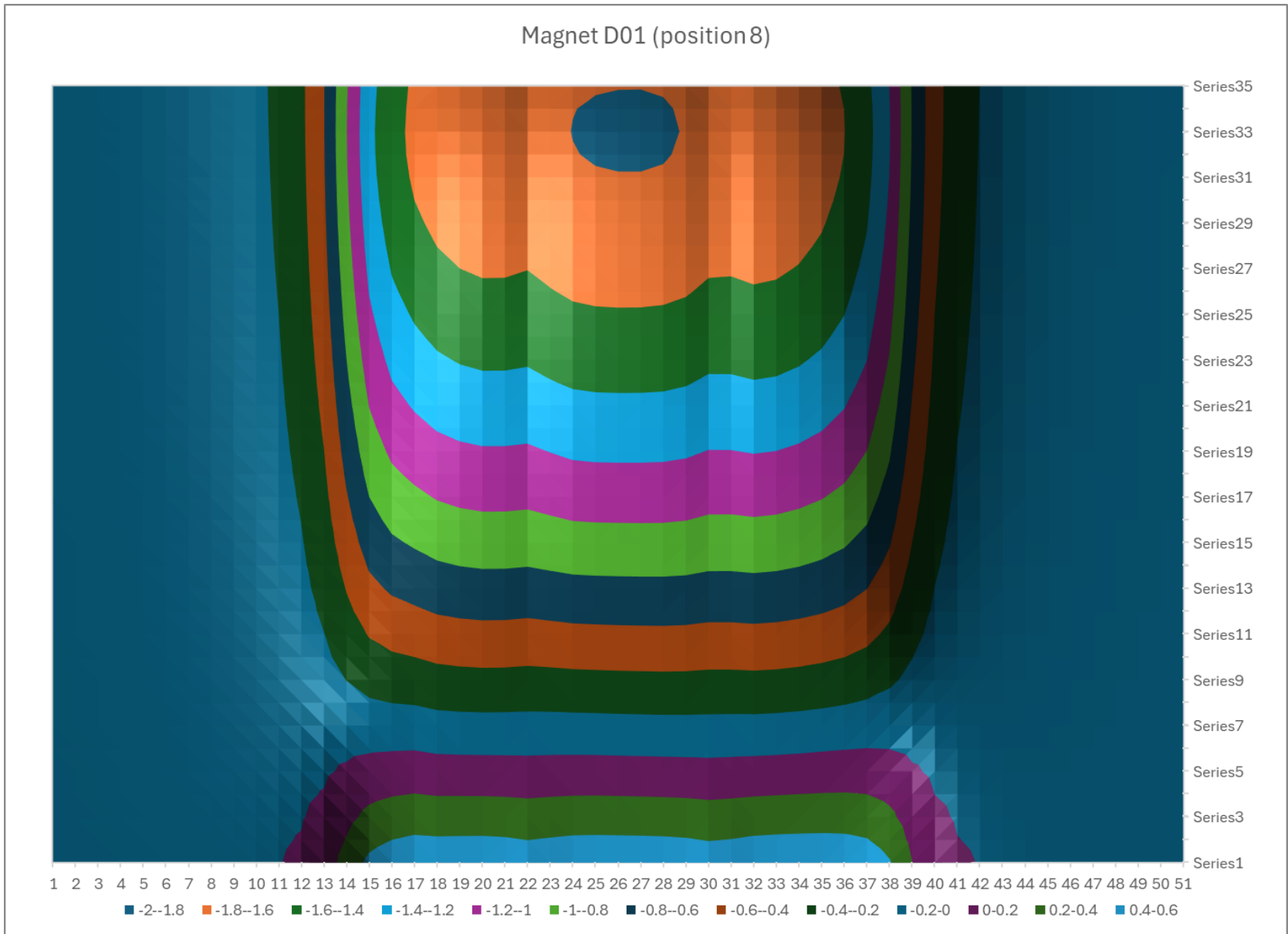
Comparison Between Measurements and Predictions



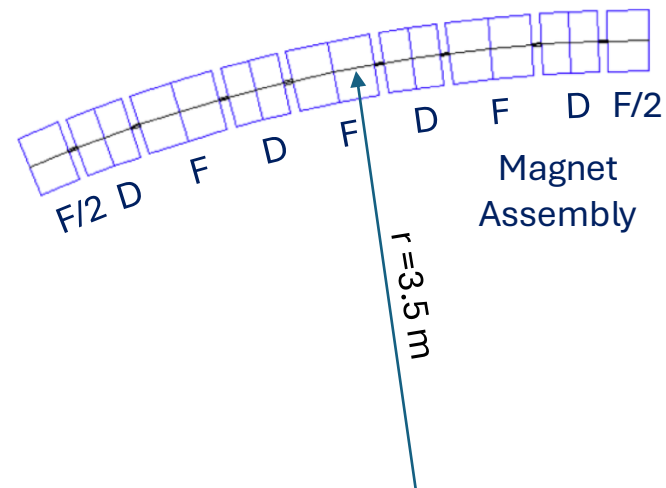
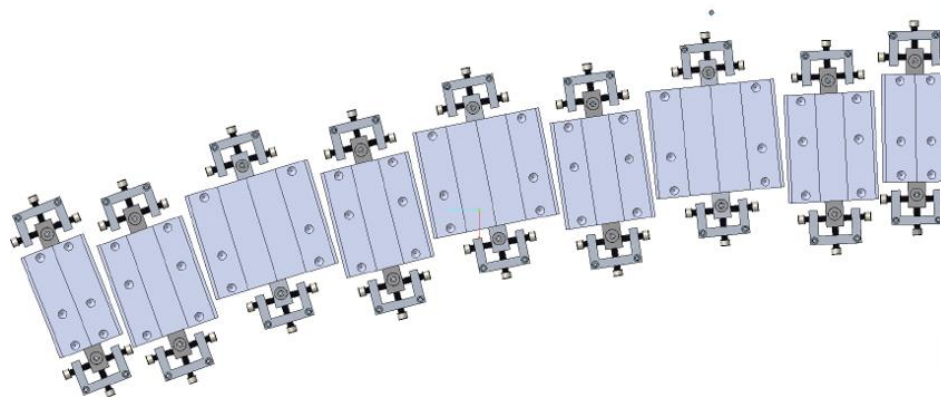
Measurements of the Focusing Magnet



C.1. Field-map of one magnet (Stephen Brooks)



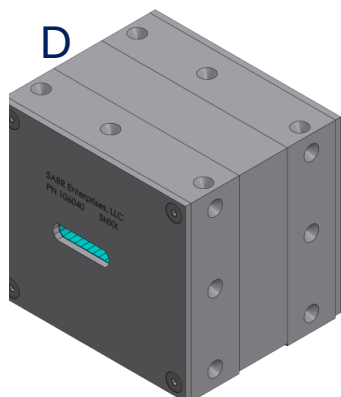
C.2. Arc Assembly with Nine Magnets



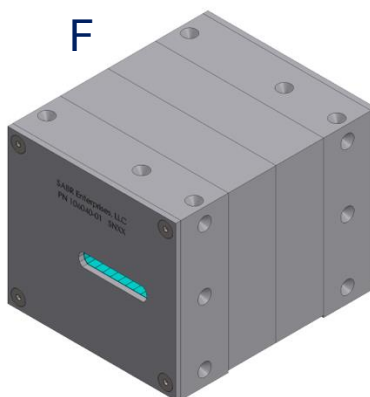
$\frac{1}{2} F$



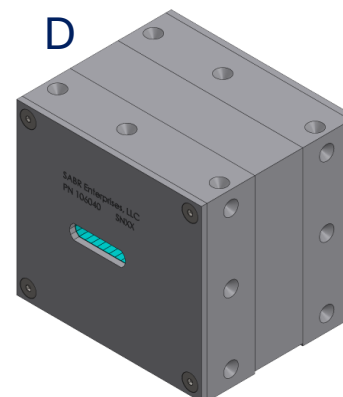
D



F



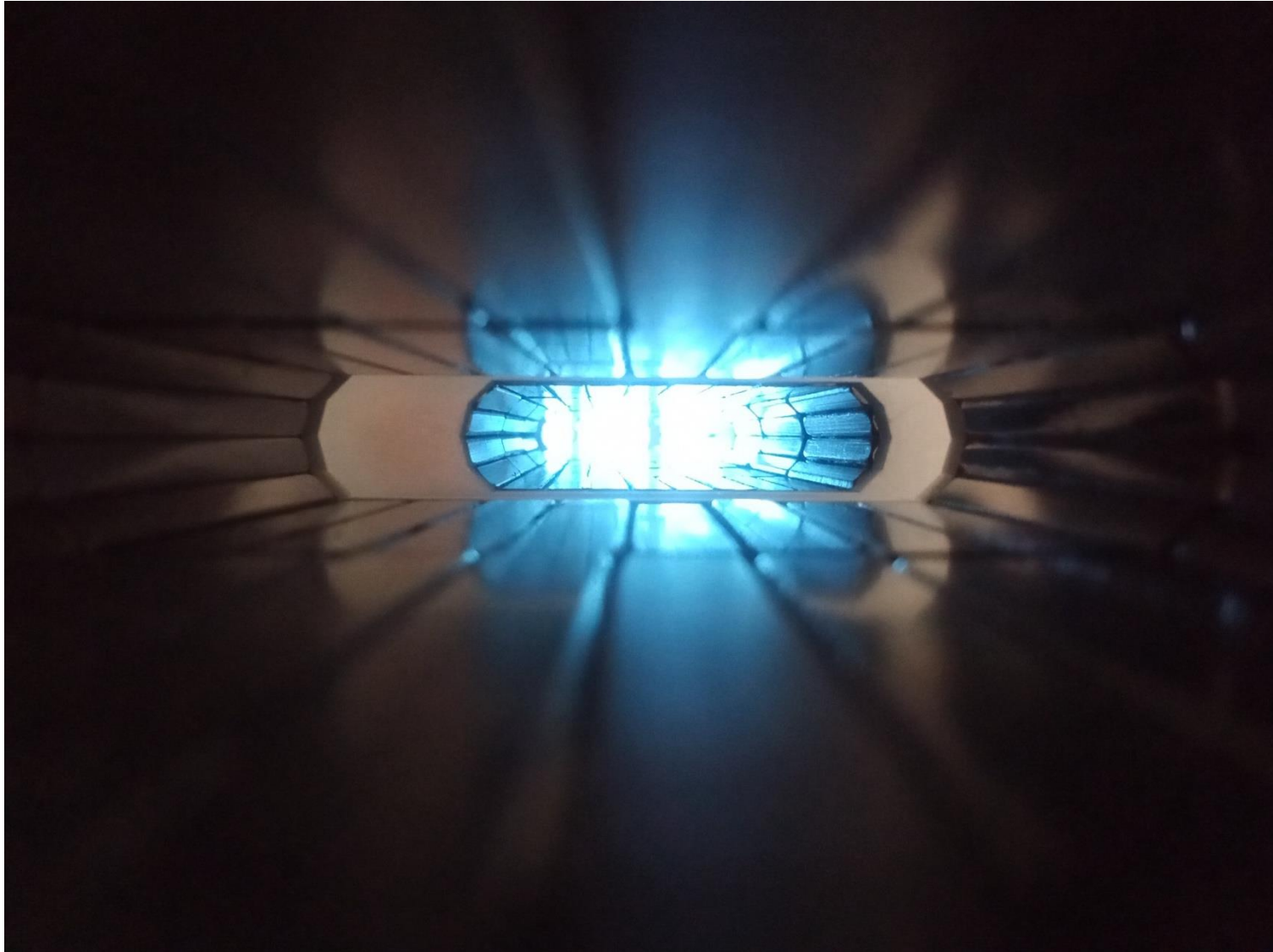
D



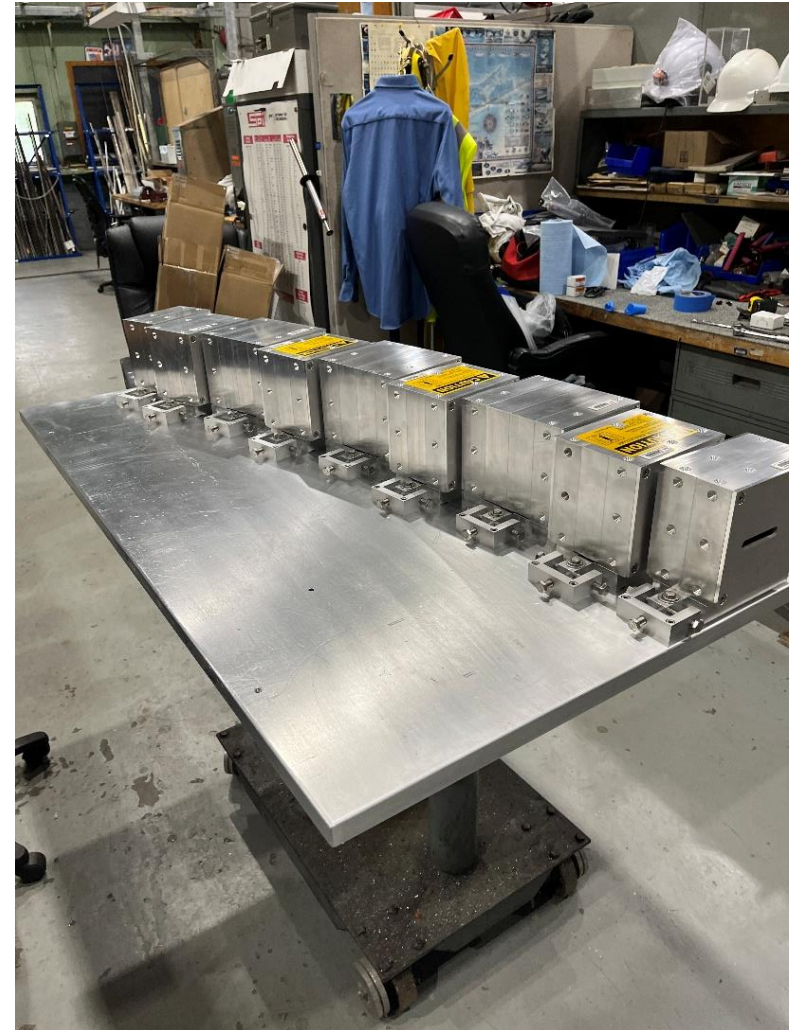
$\frac{1}{2} F$



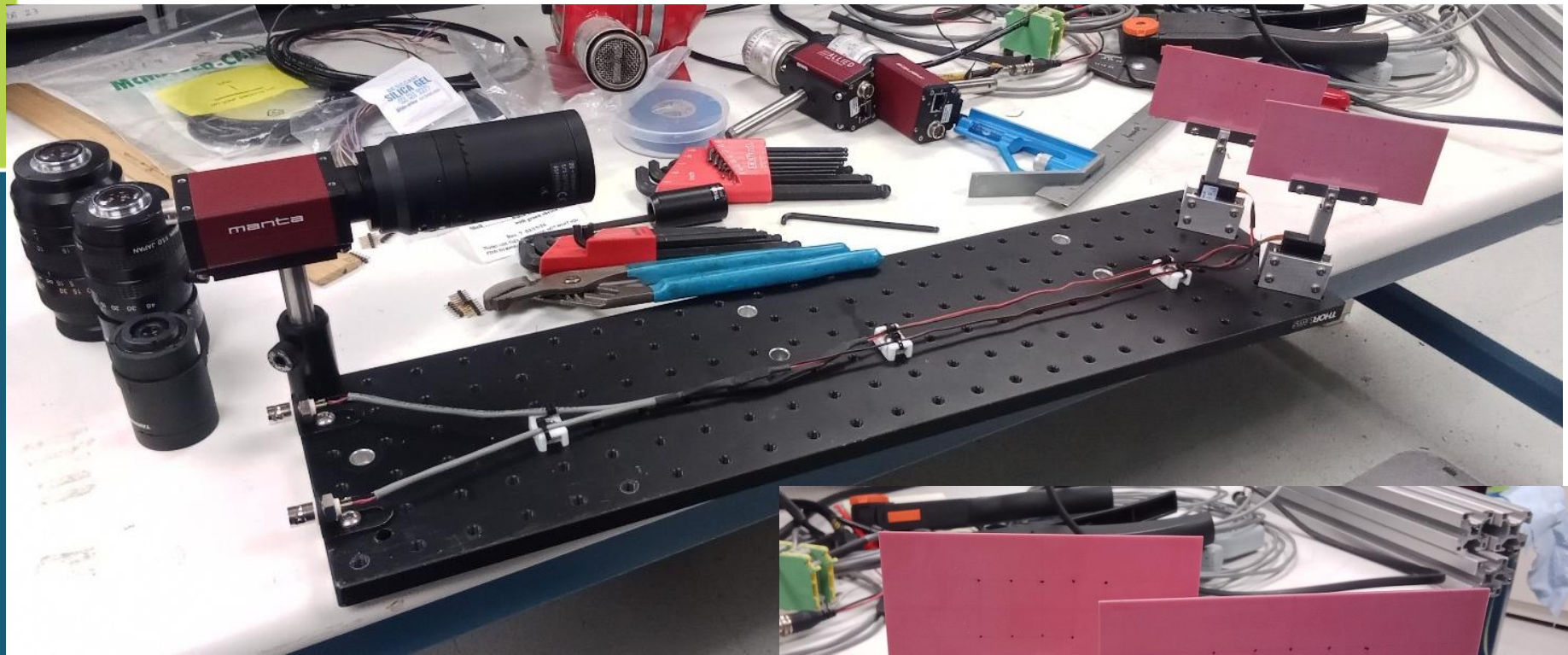
C.2. Magnet Apertures



C.3. Beamline Assembly

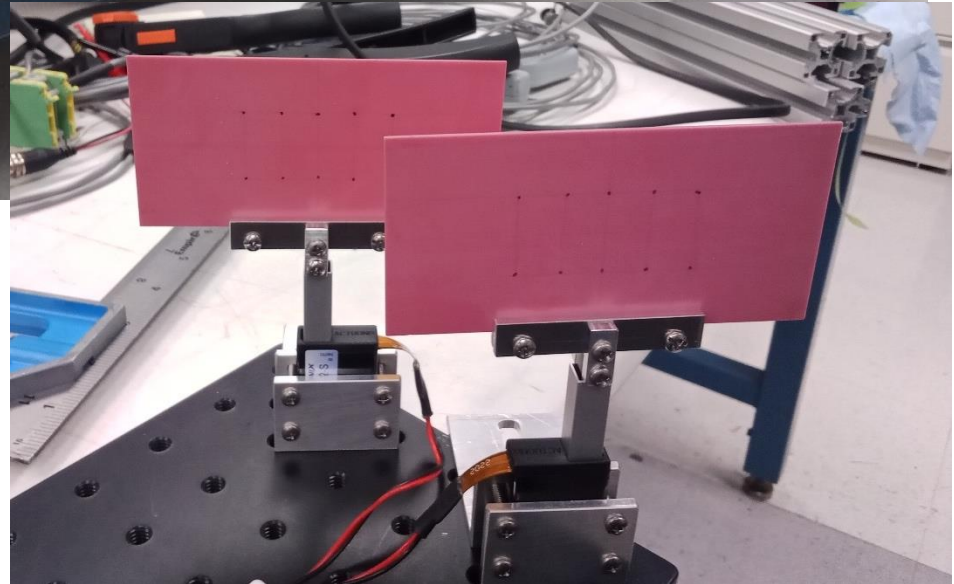


C.3. Camera and Beam Screen Plates

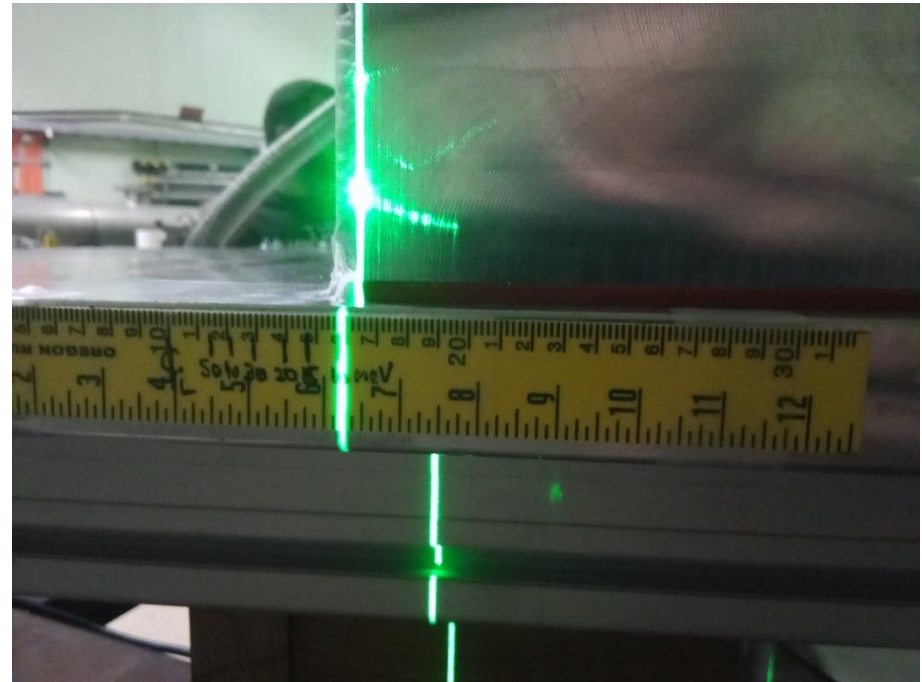


These were attached to both ends of the beamline.

4 cameras and 4 screens in total.

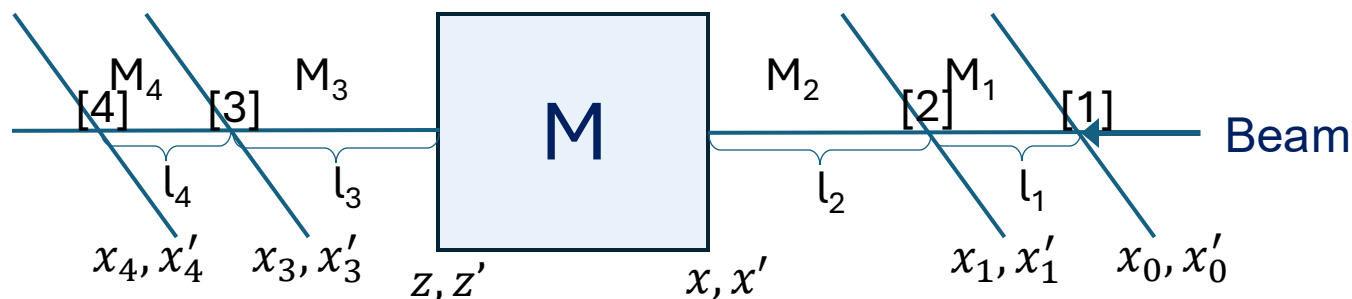


C.3. Installation (@Tandem)



Different beam energies marked on a ruler for alignment.

C.4. Beam Position Measurements at Few Positions



$$l_1=254\text{mm}, l_2=280.9\text{mm}, l_3=264\text{mm}, l_4=254\text{mm}$$

The values at different energies 'e' of z_e and z'_e and x_e and x'_e are determined from the beam flag positions $x_{4,e}$ $x_{3,e}$ and $x'_{4,e}$ $x'_{3,e}$ for the z_e and 1 and 2 for the $x_{1,e}$ $x_{2,e}$ and $x'_{1,e}$ $x'_{1,e}$.

$$\vec{z} = \begin{bmatrix} z \\ z' \end{bmatrix}$$

$$\vec{x} = \begin{bmatrix} x \\ x' \end{bmatrix}$$

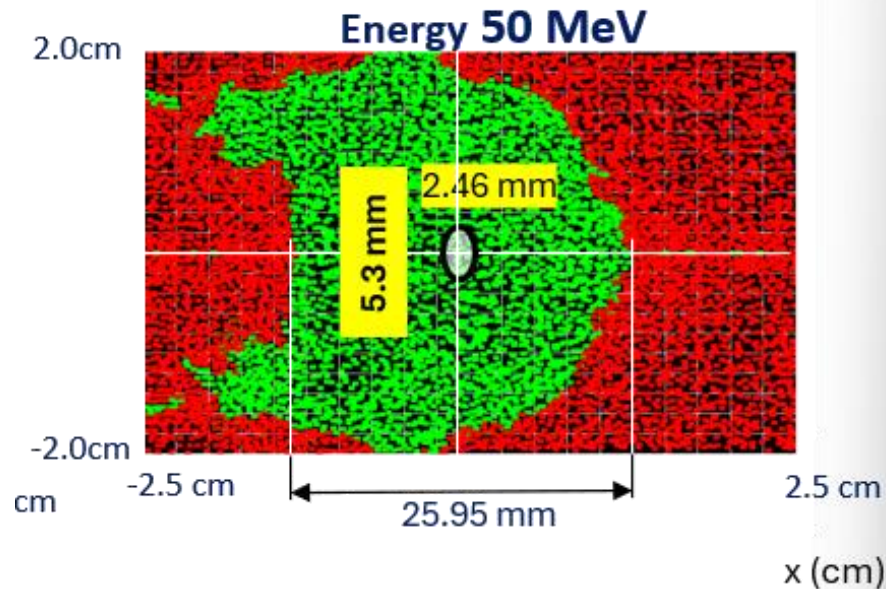
$$\vec{z}_t = M \vec{x} \quad M = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$\begin{bmatrix} z \\ z' \end{bmatrix}_t = \begin{bmatrix} x & x' & 0 & 0 \\ 0 & 0 & x & x' \end{bmatrix}_t \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}$$

$$\cos(\nu) = \frac{1}{2} \text{Trace } M$$

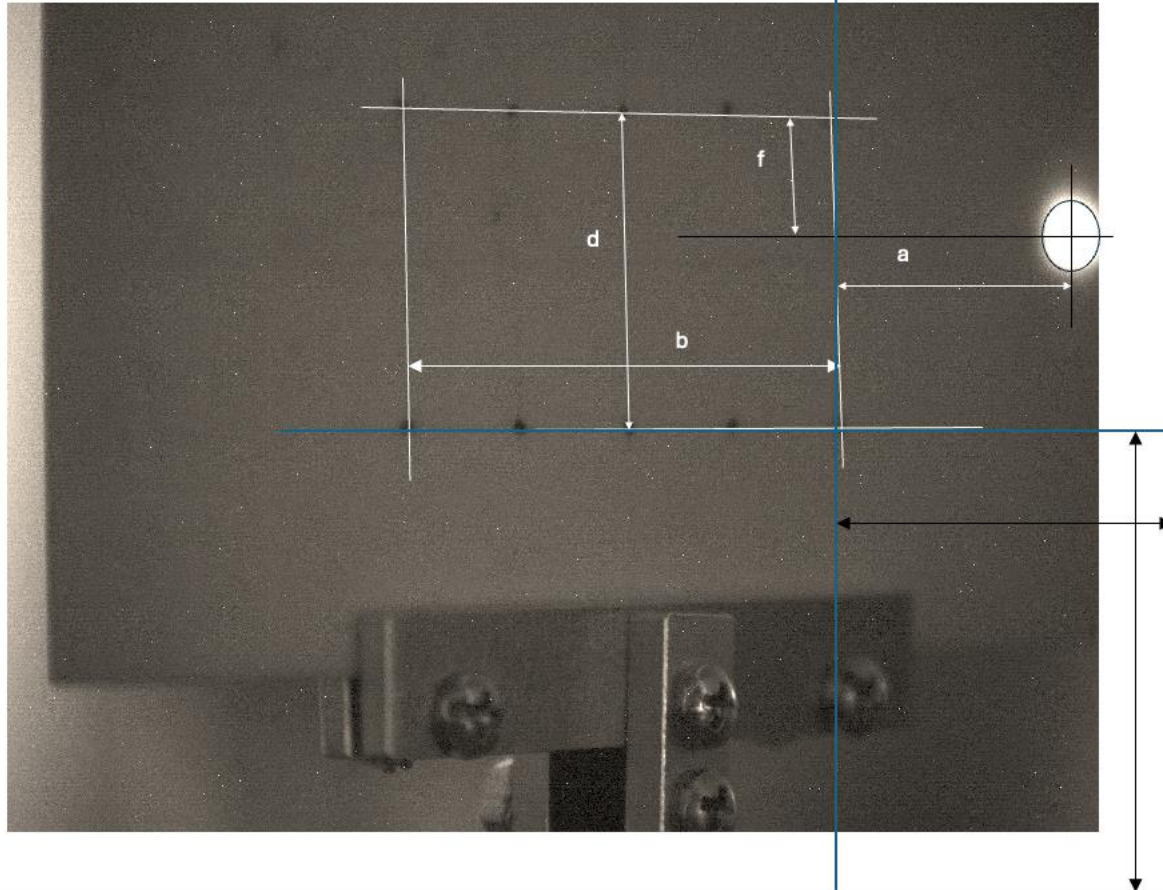
There were three beam runs: the first one at NSRL without the assembly checking the beam-air interaction and the second one with the assembly. The last run in October-25 was at Tandem. The more different beam input positions the better accuracy of the betatron function measurements would be.

C.4. At lower energies, the output beam gets very large due to the beam gas interaction and starts to assume a crescent shape as shown in dynamical aperture studies

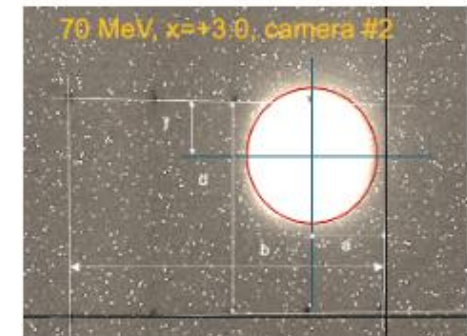
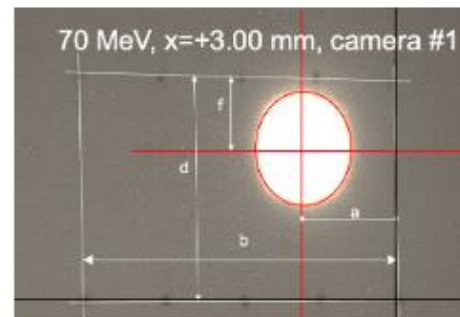
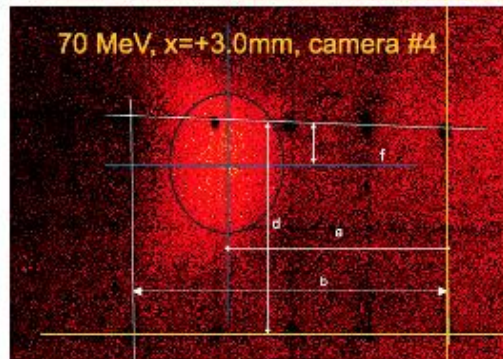
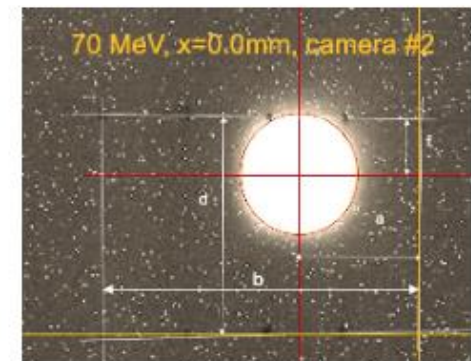
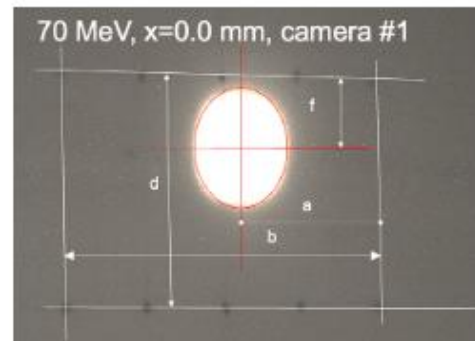
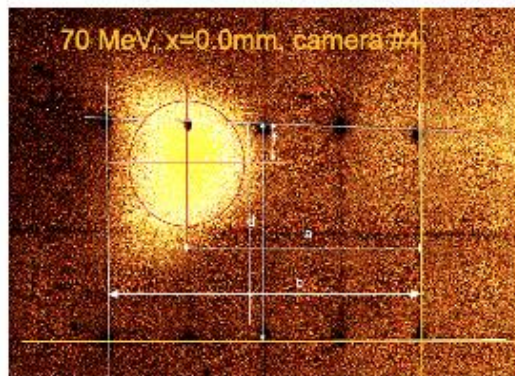
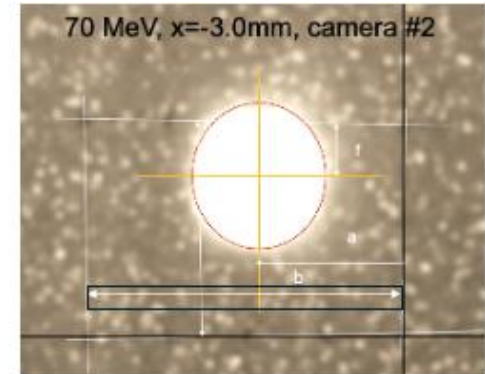
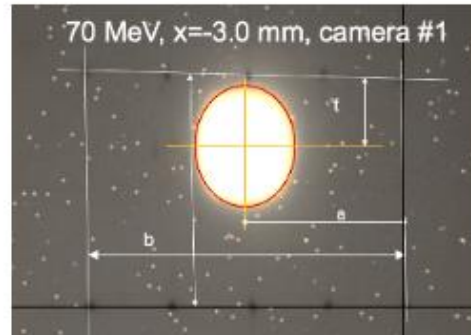
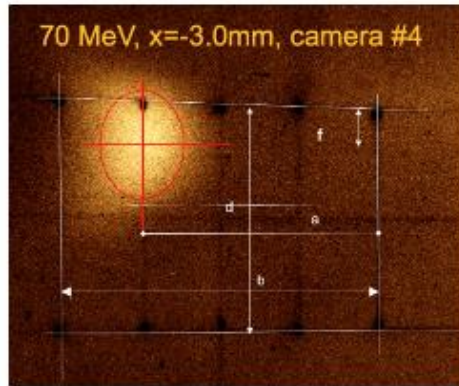


C.4. BEAM MEASUREMENTS

250 MeV, $x=+3.0$ mm, camera #1



C.4. Beam Position Measurements at Few Positions

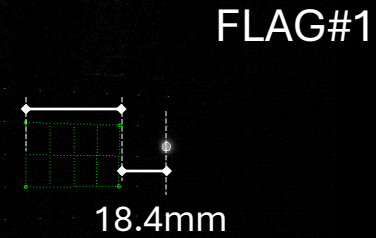
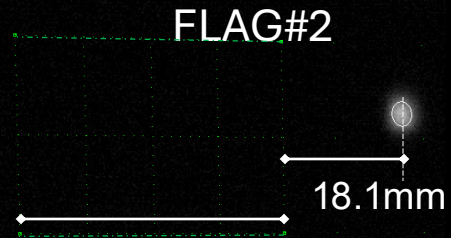


C.4. Calibration and 1s Ellipse Fit

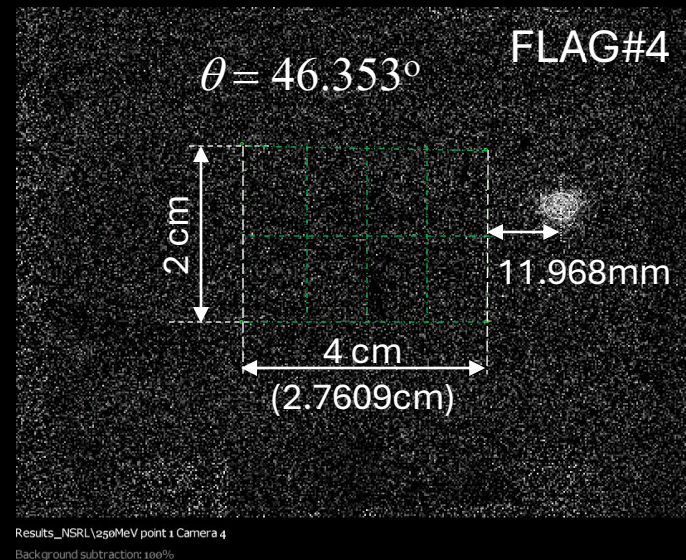
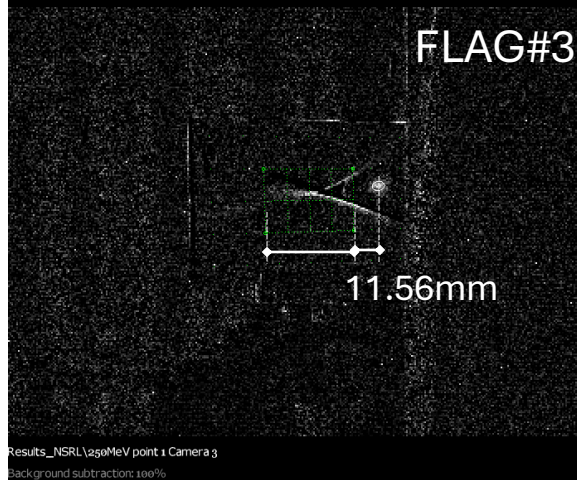


C.4. NASA Space Radiation Laboratory 250MeV protons

Beam in



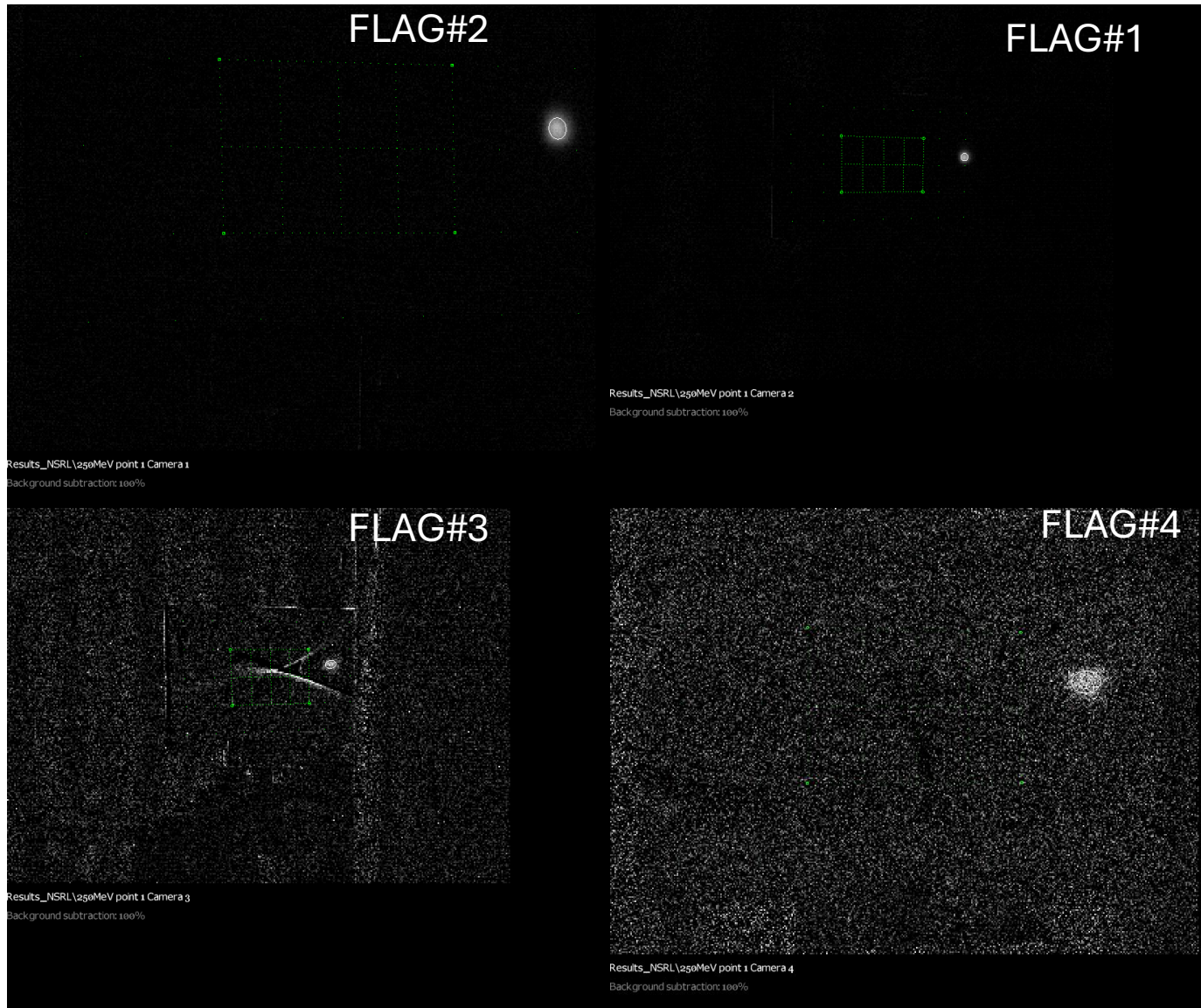
Beam out



C.4. NSRL 250MeV protons

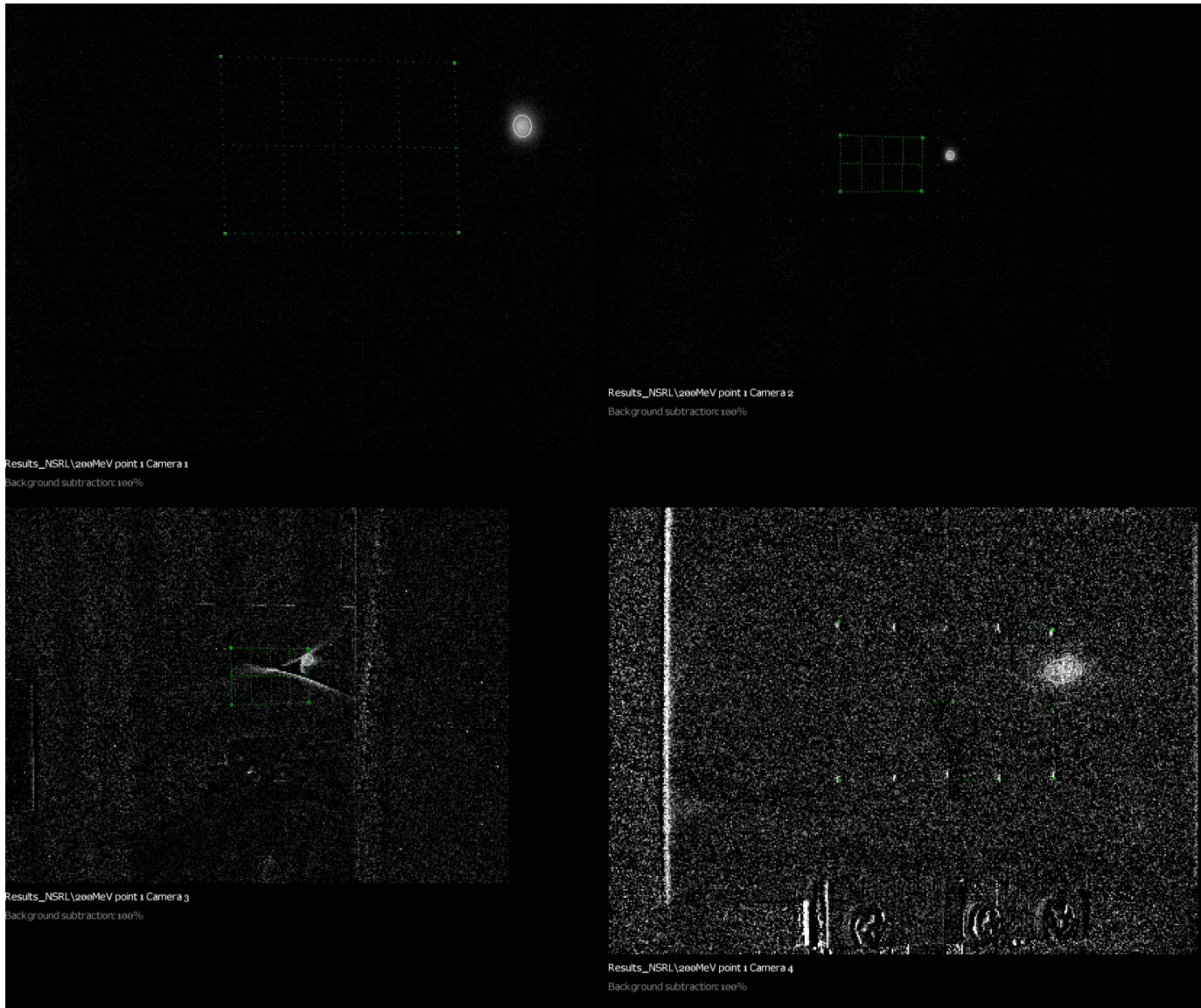
Beam in

Beam out



C.4. NSRL 200MeV protons

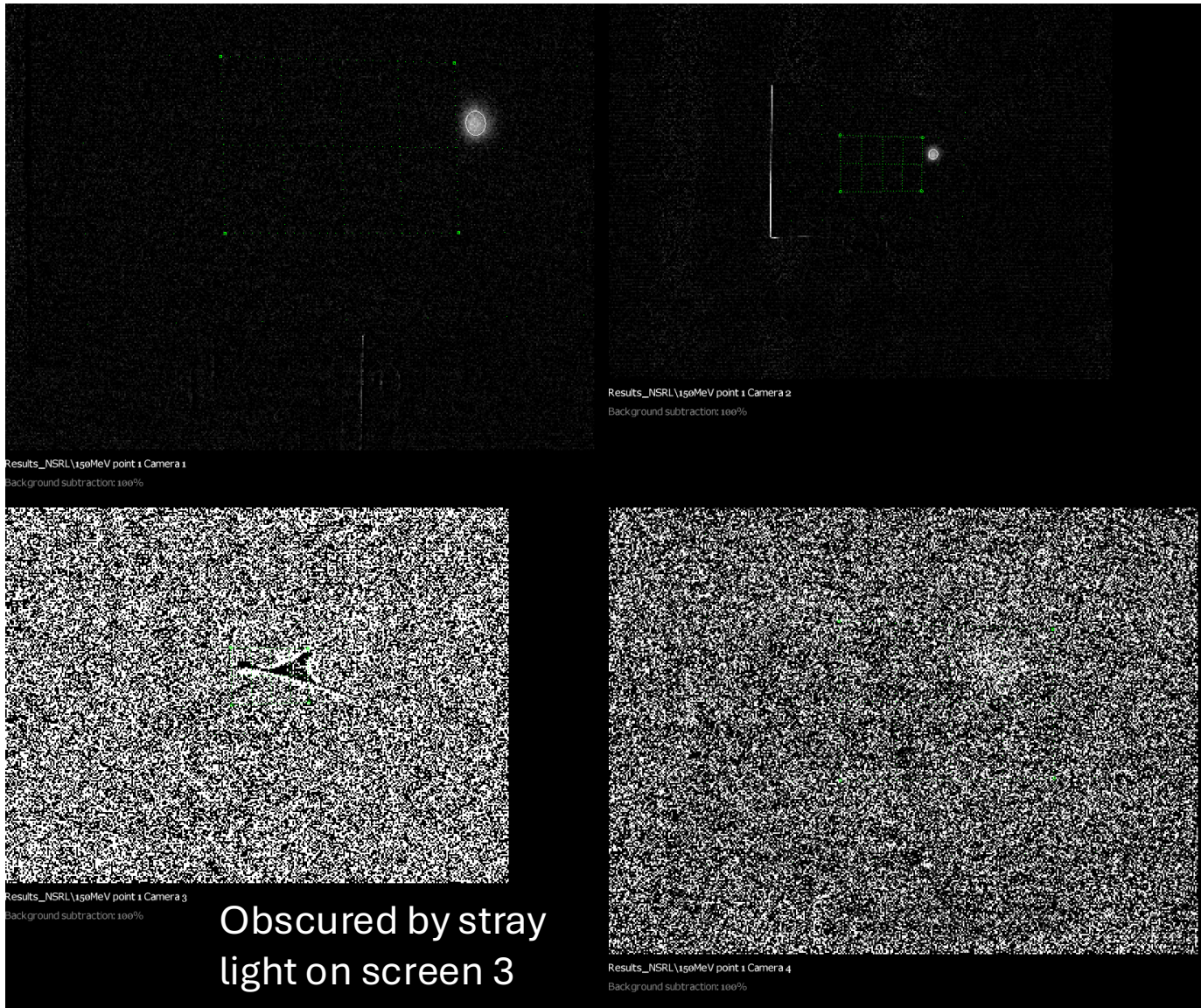
Beam in



Beam out

C.4. NSRL 150MeV Protons

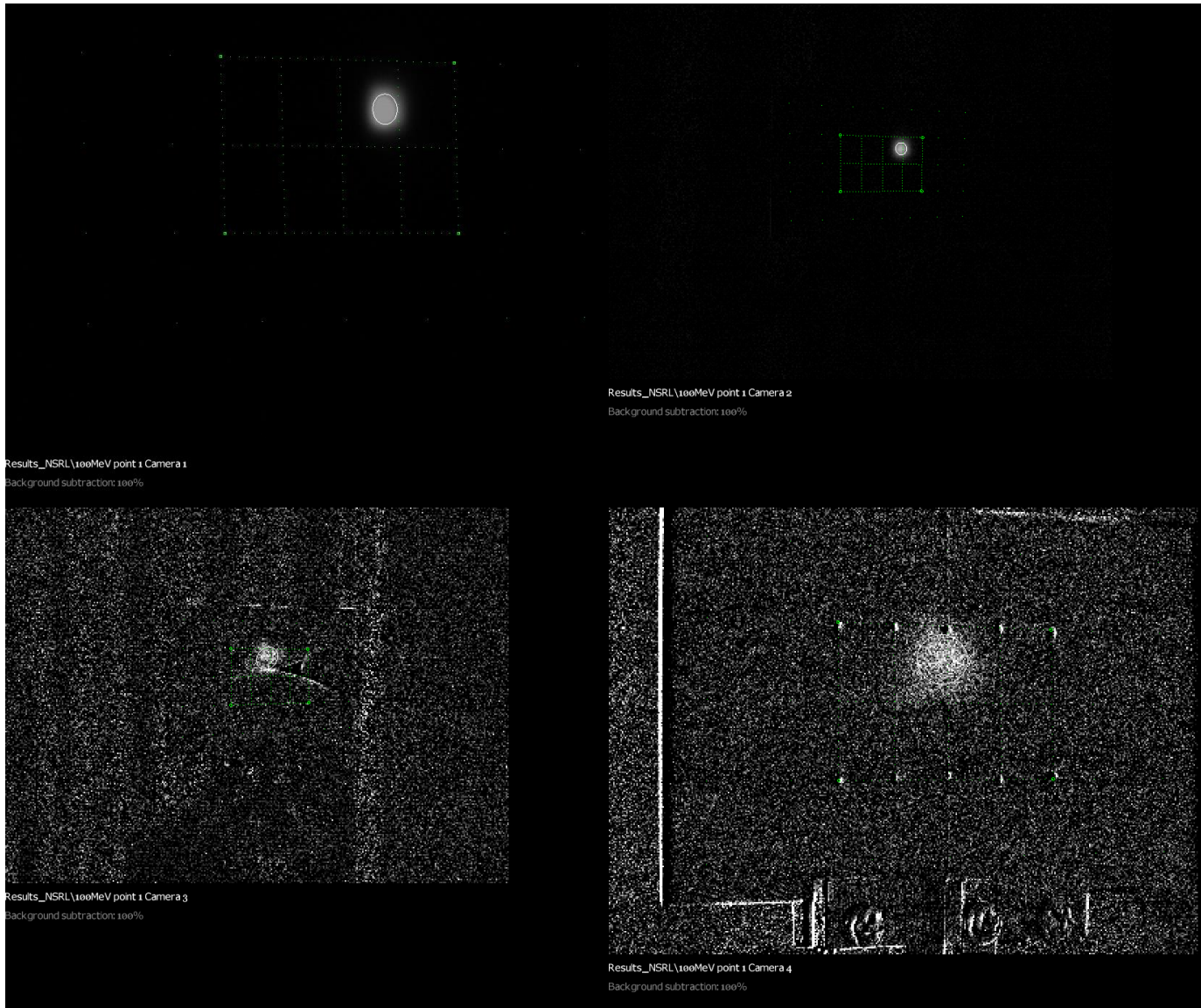
Beam in



Beam out

C.4.NSRL 100MeV protons

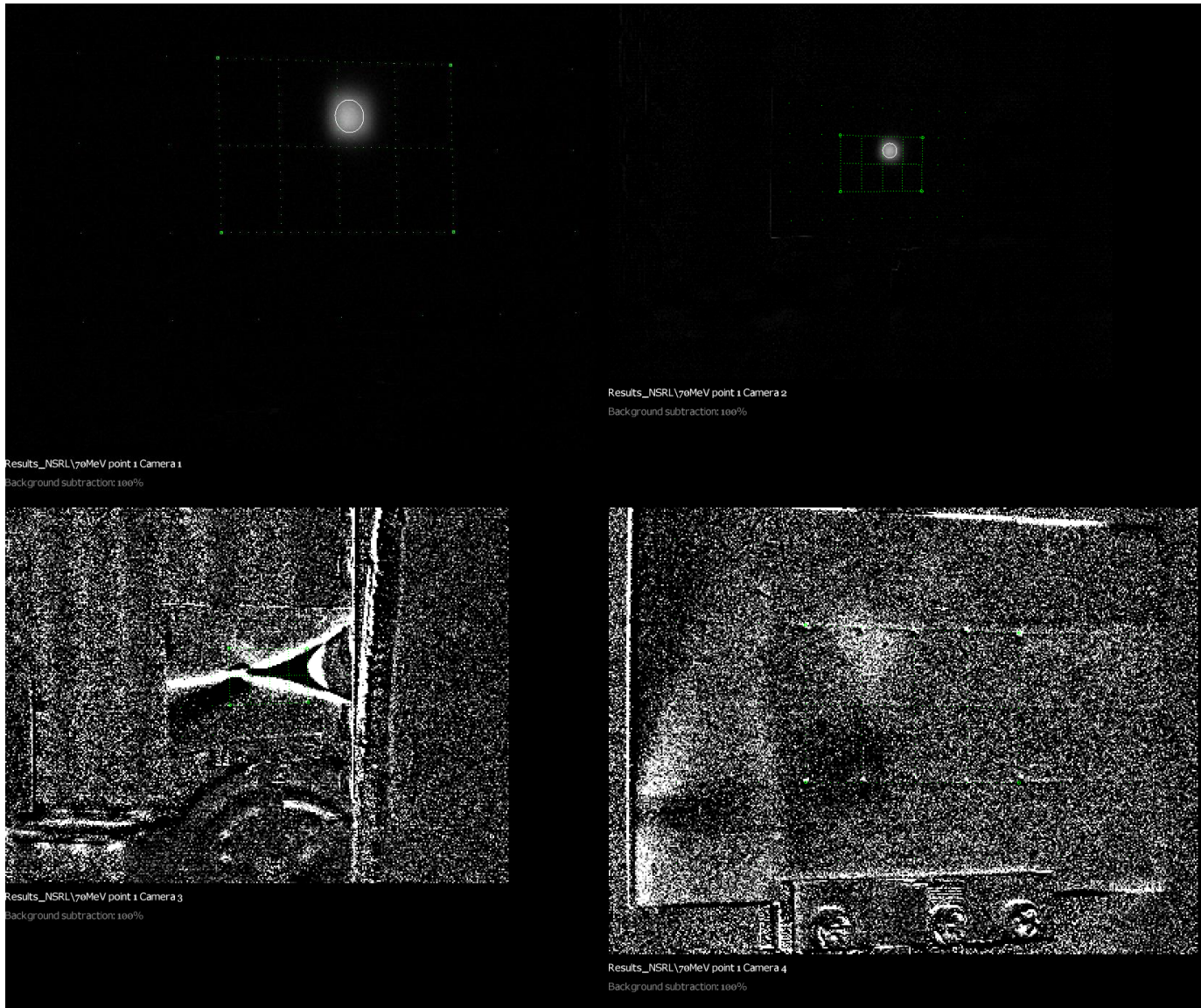
Beam in



Beam out

C.4. NSRL 70MeV protons

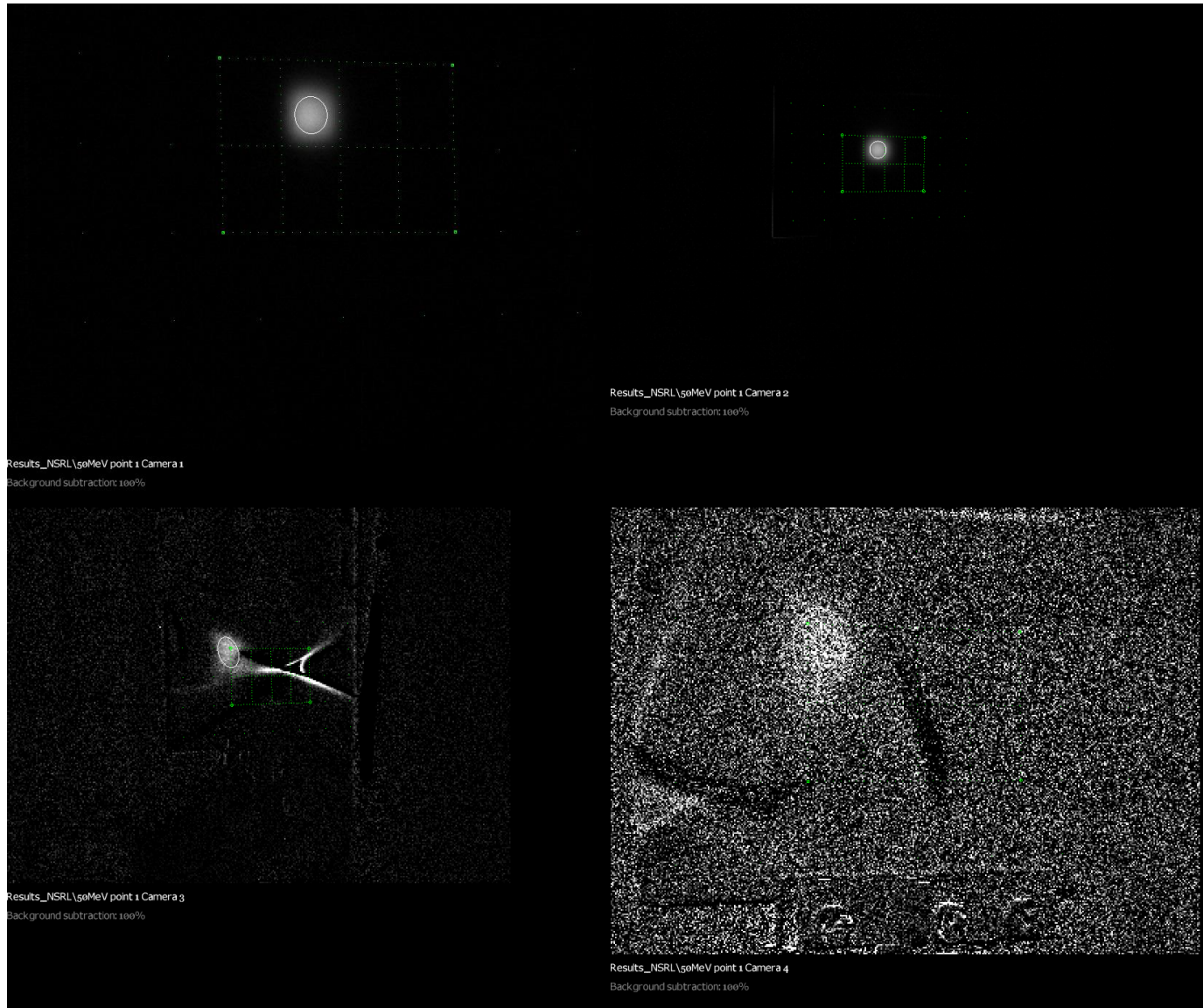
Beam in



Beam out

C.4. NSRL 50MeV protons

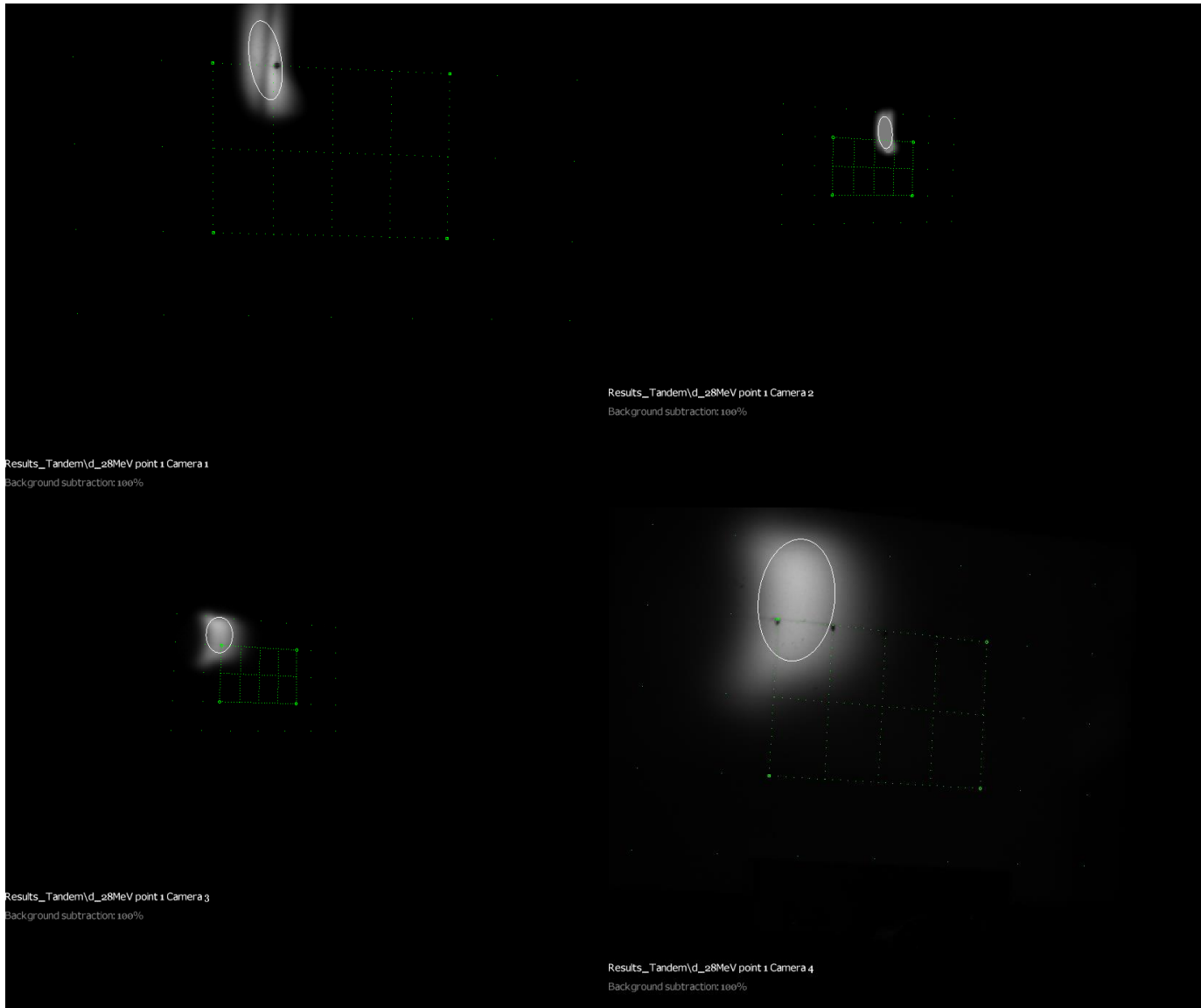
Beam in



Beam out

C.4. Tandem 28MeV d ~ 50MeV p

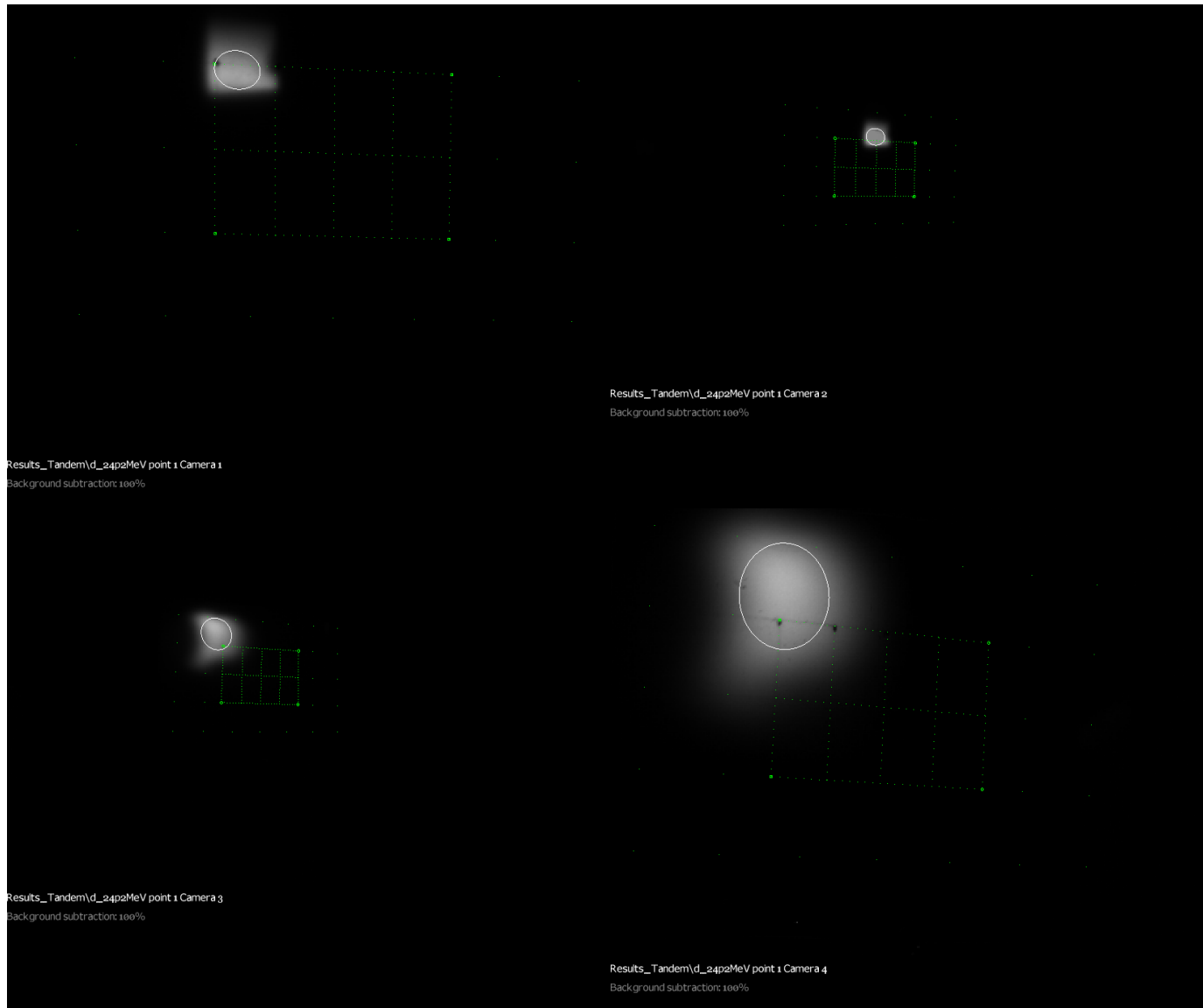
Beam in



Beam out

C.4. Tandem 24.2MeV d ~ 40MeV p

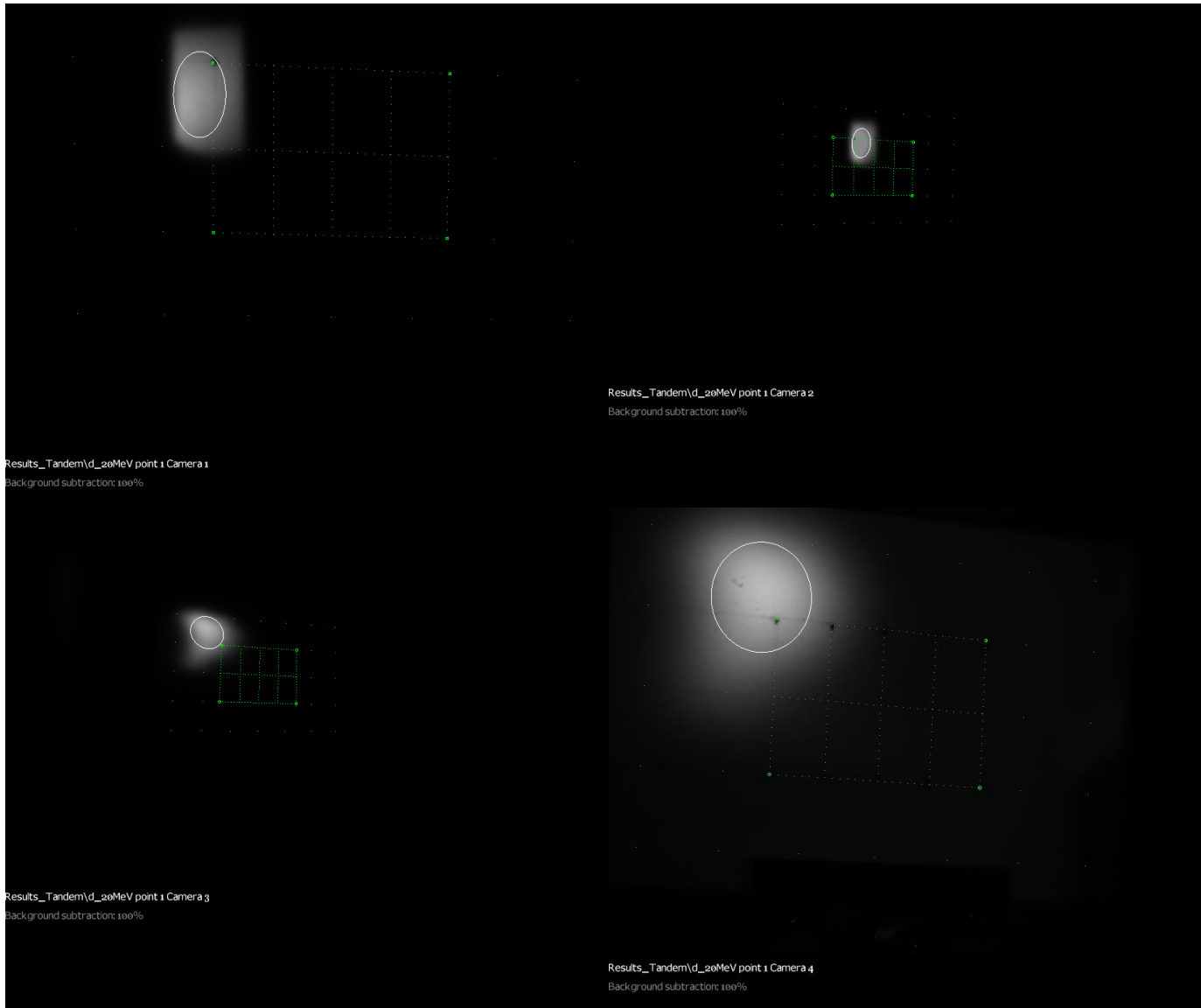
Beam in



Beam out

C.4. Tandem 20MeV d ~ 30MeV p

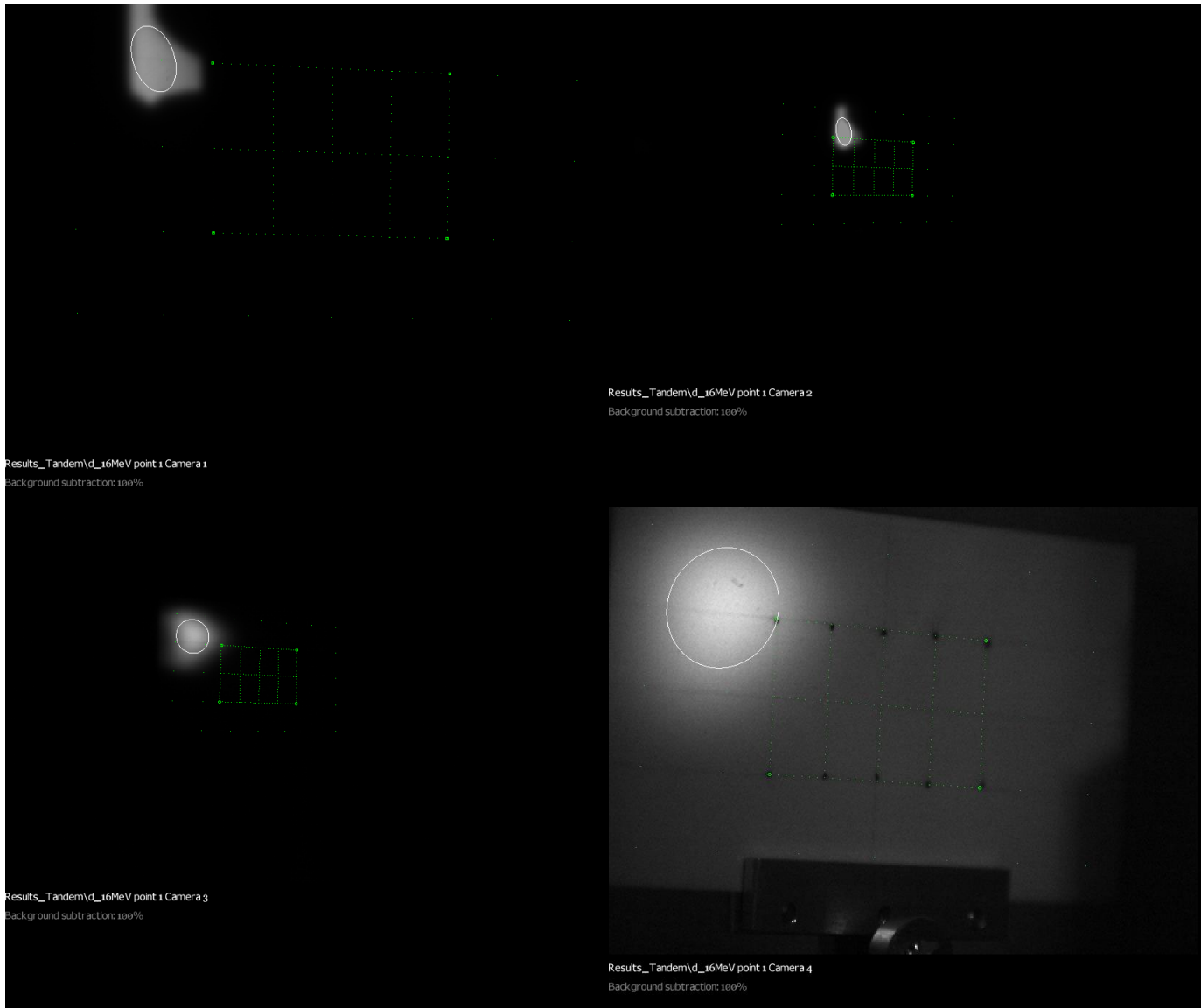
Beam in



Beam out

C.4. Tandem 16MeV d ~ 20MeV p

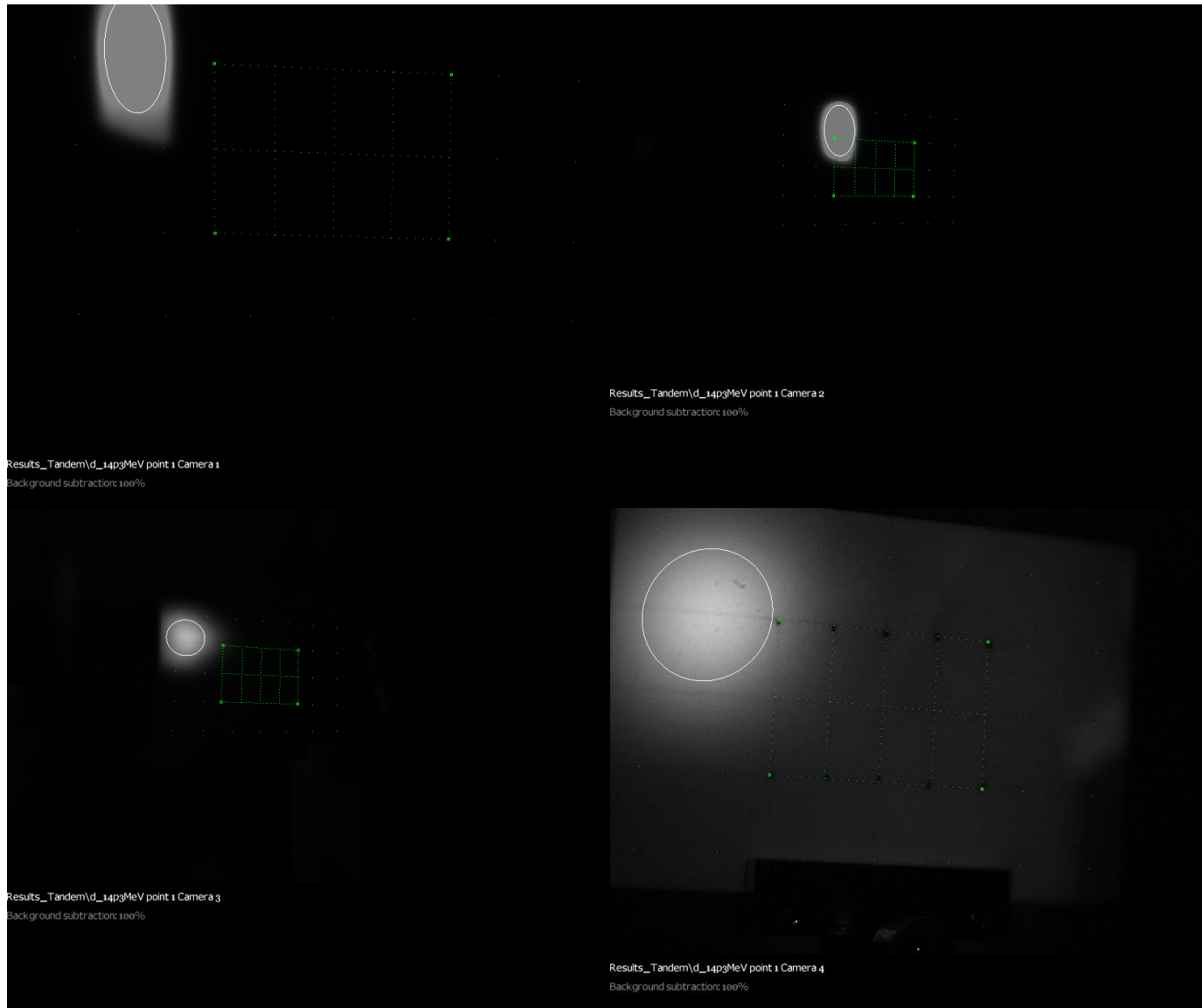
Beam in



Beam out

C.4. Tandem 14.3MeV d ~ 15MeV p

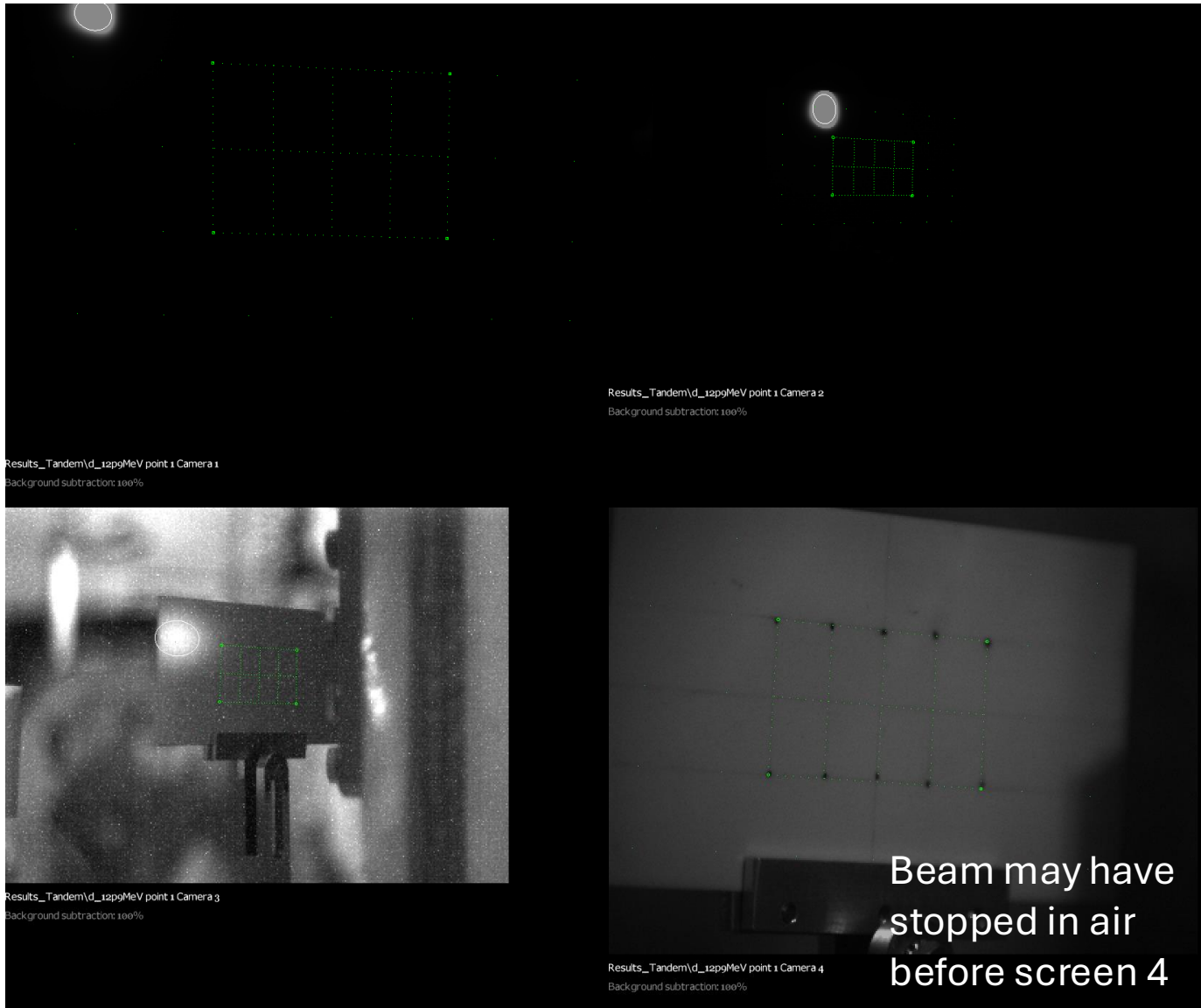
Beam in



Beam out

C.4. Tandem 12.9MeV d ~ 10MeV p

Beam
in



Beam
out

C.4. Tandem All Energies

Beam in



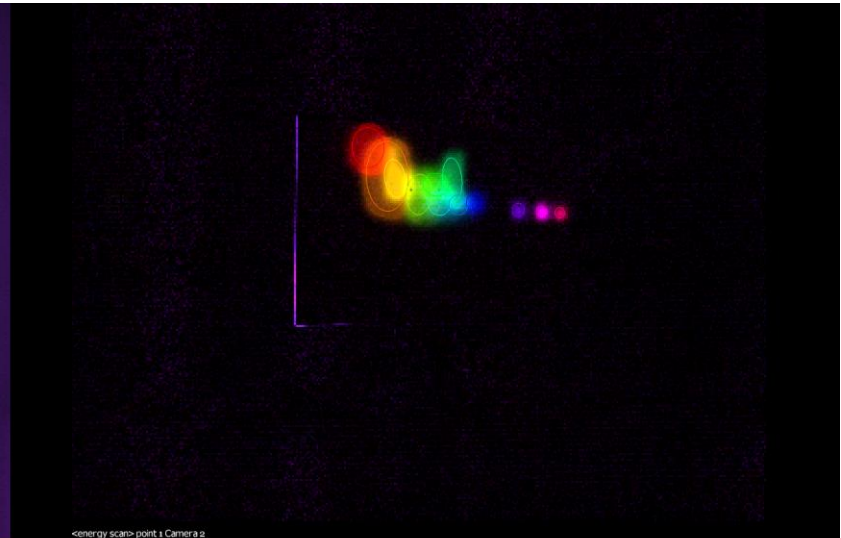
Beam out

All 12 Ellipses

Beam
in

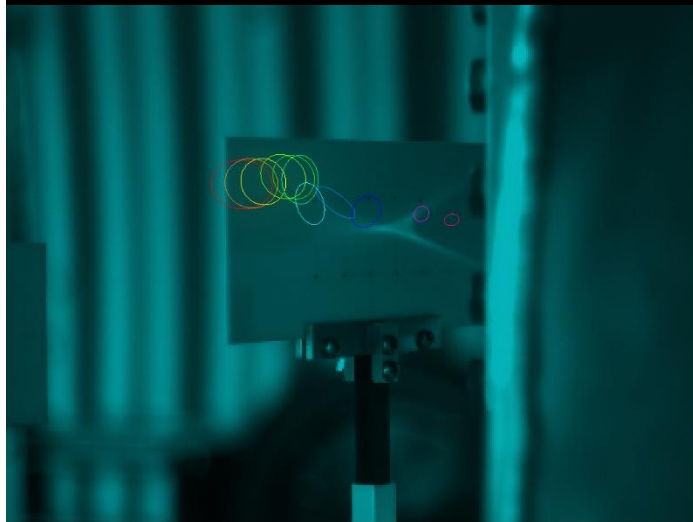


<energy scan> point 1 Camera 1

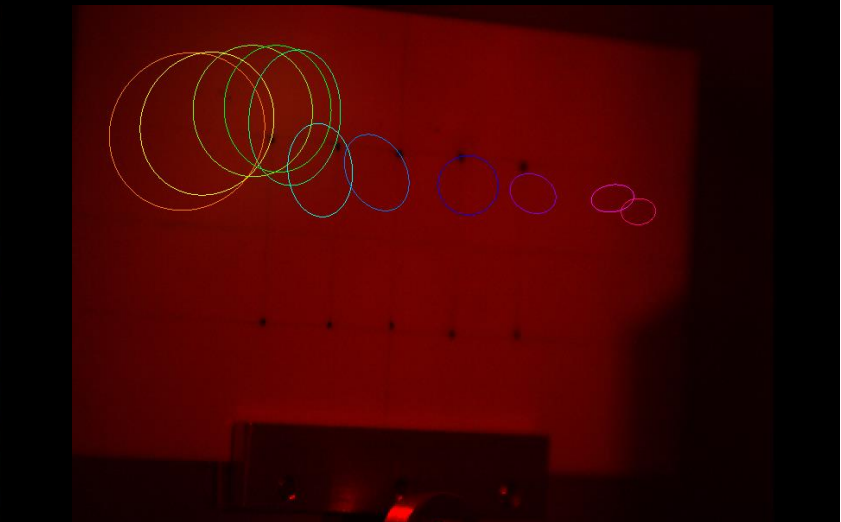


<energy scan> point 1 Camera 2

Beam
out



<energy scan> point 1 Camera 3

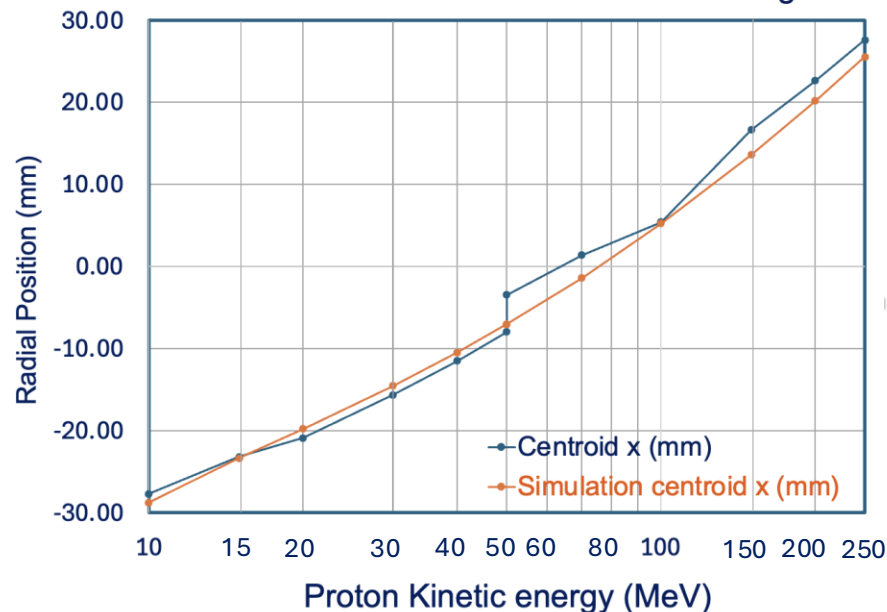


<energy scan> point 1 Camera 4

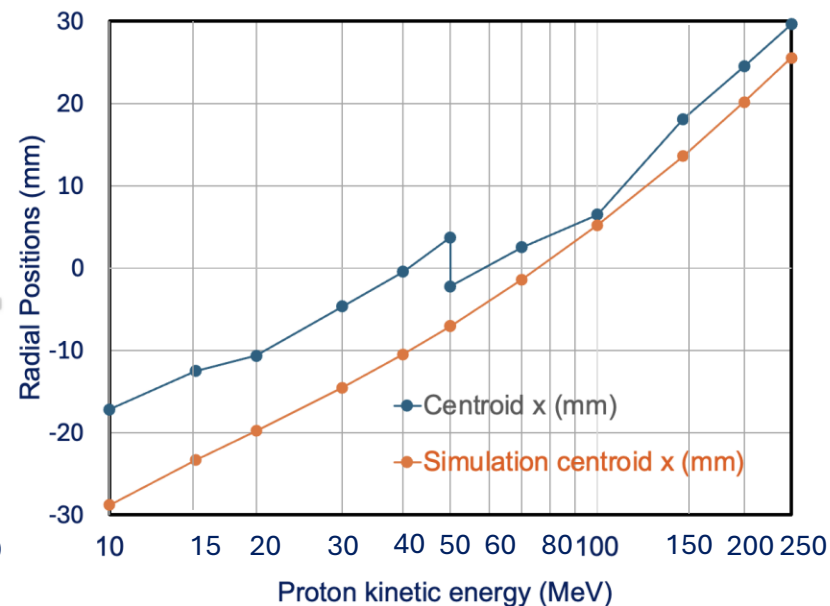
C.4. Centroid x Positions

Beam
in

Measured to Predicted Centroid Positions flag #1

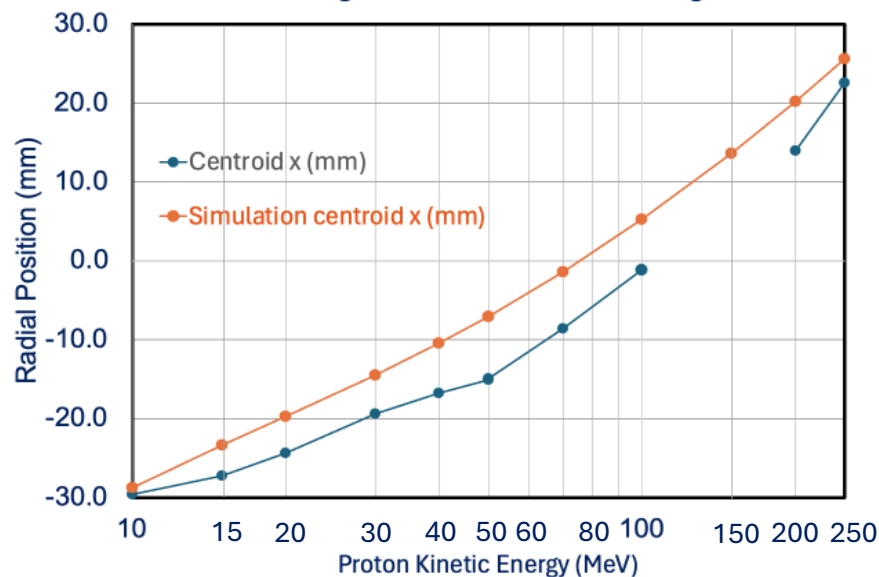


Masured and Predicted Centroid Positions Flag #2

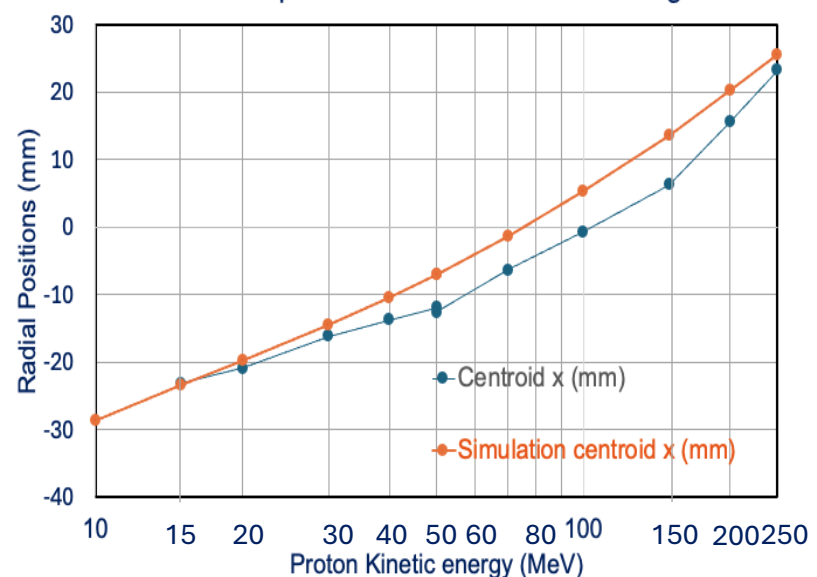


Beam
out

Beam Out Flag Centroid Positions Flag #3



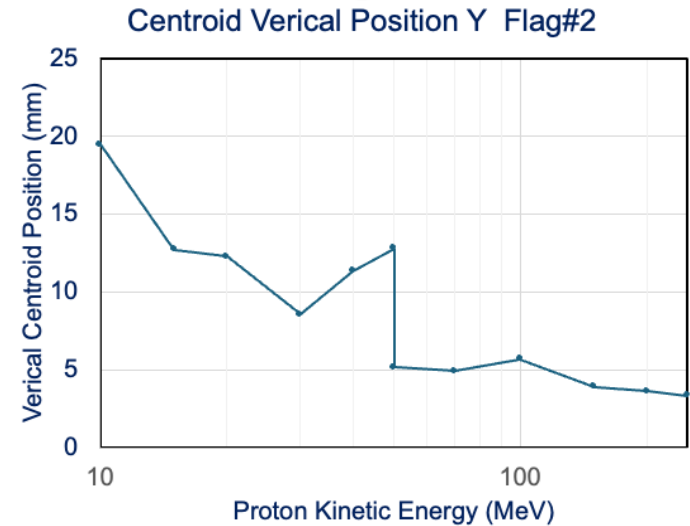
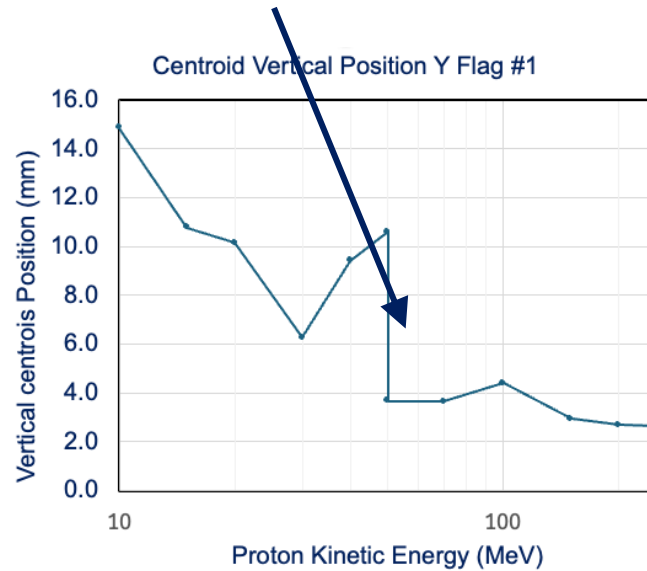
Measured and predicted Centroid Positions flag #4



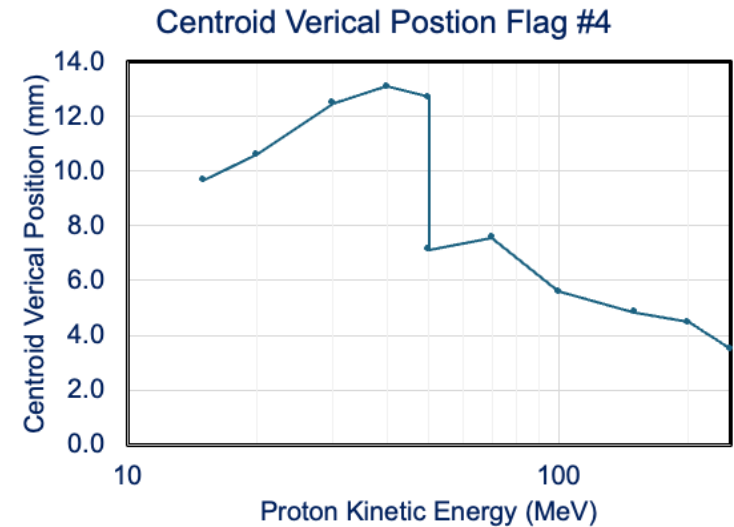
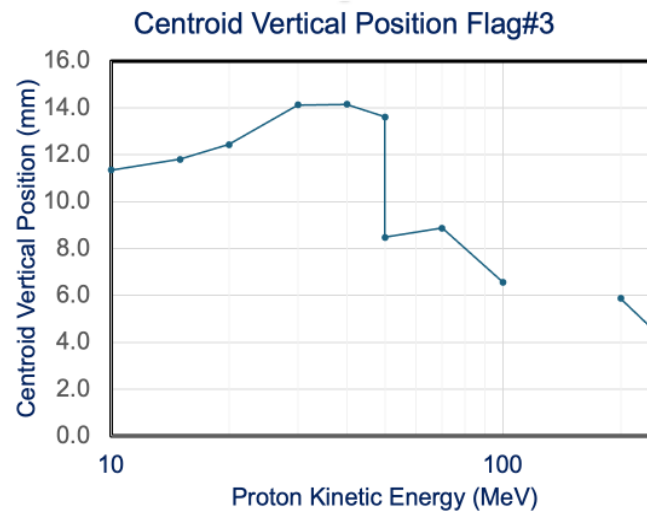
C.4. Centroid Y Positions

Known Y shift from reassembly of camera screens between NSRL and Tandem

Beam in



Beam out



C.4. Preliminary tune measurement

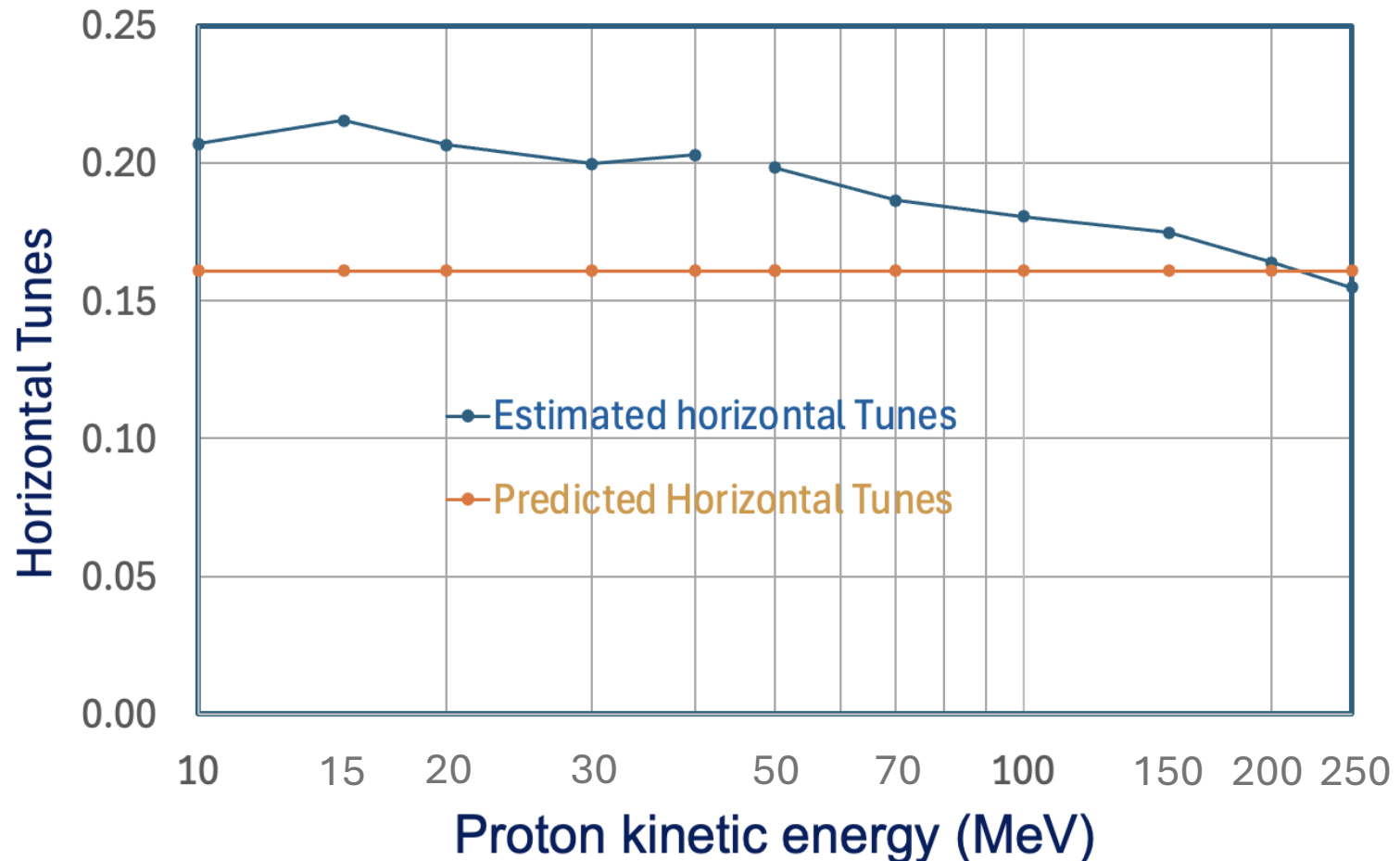
- $4\nu_{x,\text{cell}} = 0.644 \rightarrow \cos(\phi_x) = -0.618$
- Agrees at high energy

Cell tune ranges:

$$\nu_x = 0.1608 \pm 0.0004$$

$$\nu_y = 0.0738 \pm 0.0011$$

Initial Horizontal tune vs. energy analysis



C.5 ACCOMPLISHED TASKS

- The magnets were assembled at the aluminum plate, positions adjusted by the surveying team and covered by the aluminum chamber with Kevlar windows at the entrance and exit.
- The magnet assembly was transported to the NSRL laboratory and positioned for the first part of the experiment 50-250 MeV proton beam transport. The proton beam with energies between 50 MeV and 250 MeV was propagated through the assembly with radial horizontal positions adjusted for each energy. The proton beam positions were measured with two sequential flags in front and at the exit.
- The second part of the experiment was performed at Tandem BNL accelerator using deuterons to provide equivalent proton kinetic energies between 10 and 50 MeV.

C.5. EXPERIMENT CONCLUSION

- **Beams at all energies were transmitted through the 4-cell test girder (22.5° or $1/8$ arc)**
 - Protons at NSRL and deuterons at Tandem
- **10-250MeV is a factor of $5.3\times$ in momentum**
 - Very large for a non-scaling FFA, especially one with a flattened tune
 - 250MeV is highest ever energy in any NS-FFA line
- **Beam ought to propagate several cells around a ring at the start of commissioning**
 - (even without magnet shimming/correction here)

C.5. SUMMARY

- Further goals of the LDRD is to transfer the existing technology to the FLASH therapy cancer radiation facility at the Radiation Oncology Department of the Stony Brook University Hospital.
- The LDRD results will be used as a proof of principle for the concept of fast-cycling permanent magnet synchrotron for get funding to complete the synchrotron.
- This is a proof of principle for a novel type of accelerator where the magnetic field is fixed, and the acceleration rate depends only on the RF.
- The present collaboration includes the FLASH therapy experiments on mice at the BNL TANDEM accelerator with the 28.5MeV proton beam.
- When the whole proton therapy facility is built the FLASH radiation therapy could be explored. This is significant advantage to any other existing radiation therapy facility as due to fixed magnetic field the treatment could be done is 100ms.
- This accelerator opens the door for new proton drivers, Muon collider accelerator as the muon lifetime is very short, and for ADS.