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Factorization and Resummation for PDFs at threshold

Based on WIP with M.Beneke, R.Szafron

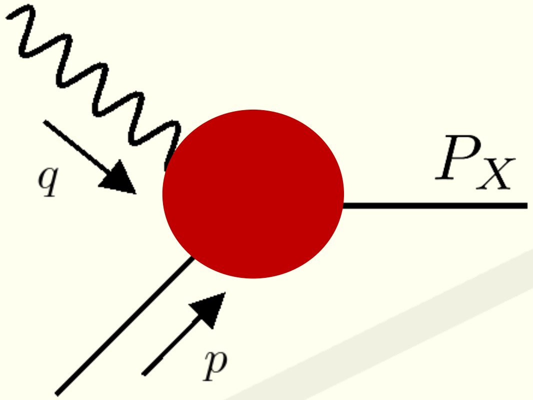
LoopFest XXIV
BNL New Amsterdam

"Toy environment": DIS at threshold

- To study the PDF behaviour in the endpoint region, look at DIS for x→1
- Collinear parton p is struck by hard momentum q
- Kinematics:

$$p + q = P_X \qquad x = \frac{Q^2}{2p \cdot q} \qquad Q^2 = -q^2 \qquad p^2 = \Lambda^2_{\mathrm{QCD}}$$

$$P_X^2 = Q^2 \frac{1-x}{x}$$



[Korchemsky, Marchesini, hep-ph/9210281]
[Moch, Vermaseren, Vogt, hep-ph/0506288]
[Becher, Neubert, hep-ph/0605050]

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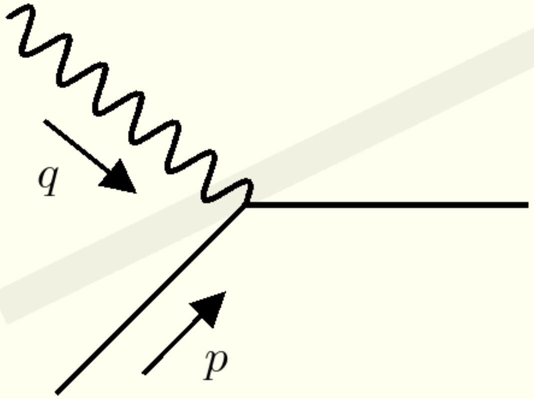

$$W^{\mu\nu} = -g_\perp^{\mu\nu} |C|^2 \int_x^1 \mathrm{d}\xi \, J\left(Q^2 \frac{\xi - x}{x}\right) f_{q/N}(\xi) + \mathcal{O}(1-x)$$

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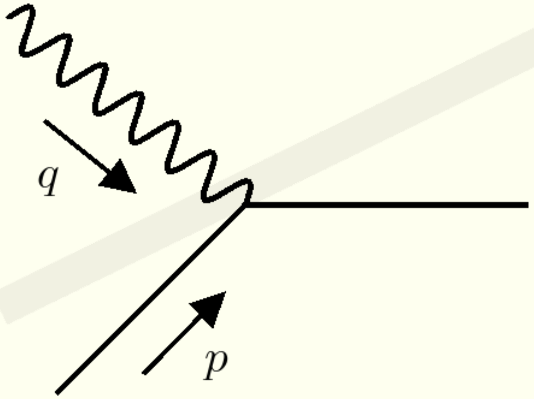

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Hard coefficient — PDF

Collinear factorisation

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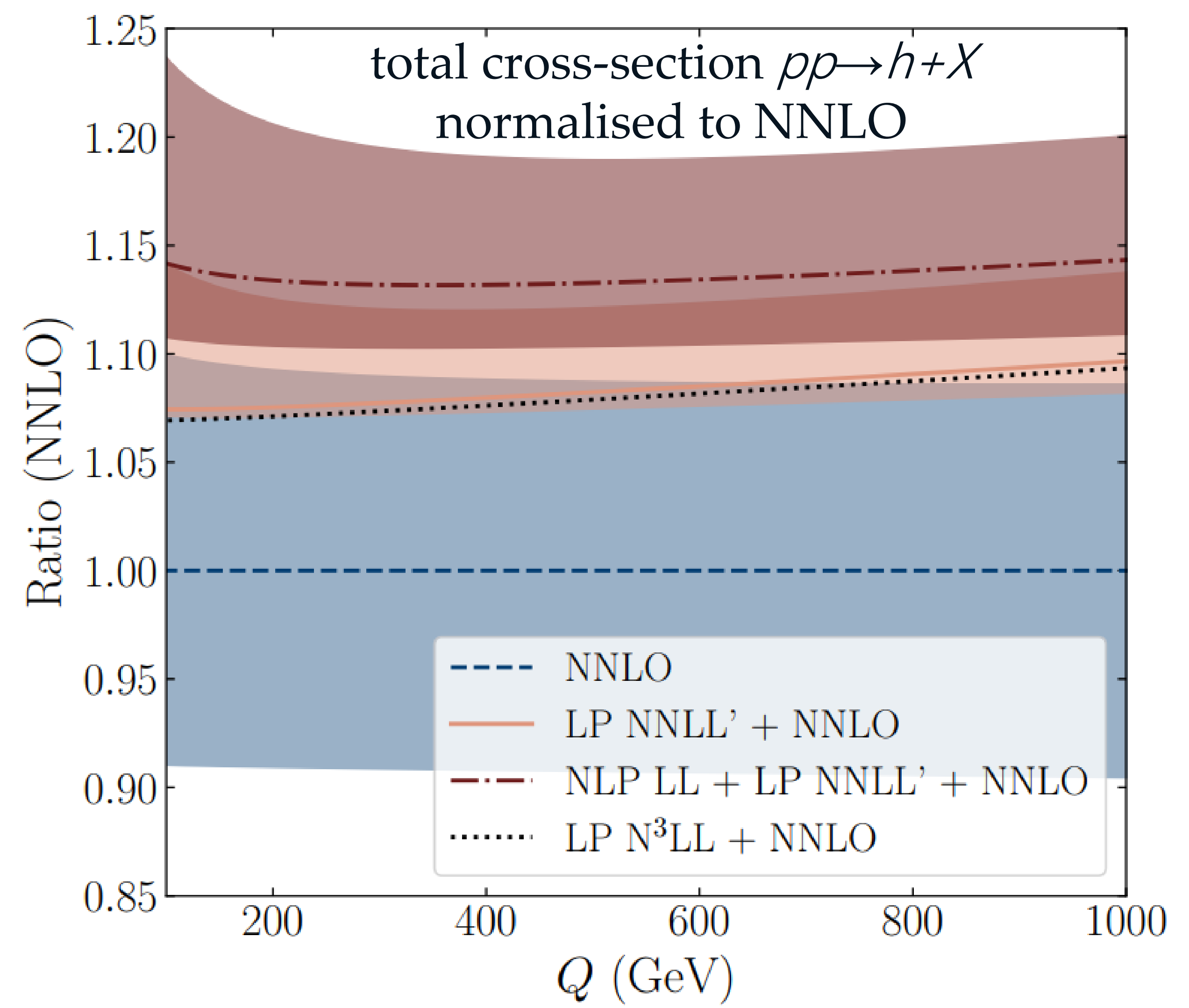
Power corrections

[Korchemsky, Marchesini, hep-ph/9210281]
[Moch, Vermaseren, Vogt, hep-ph/0506288]
[Becher, Neubert, hep-ph/0605050]

Why care about (perturbative) power corrections?

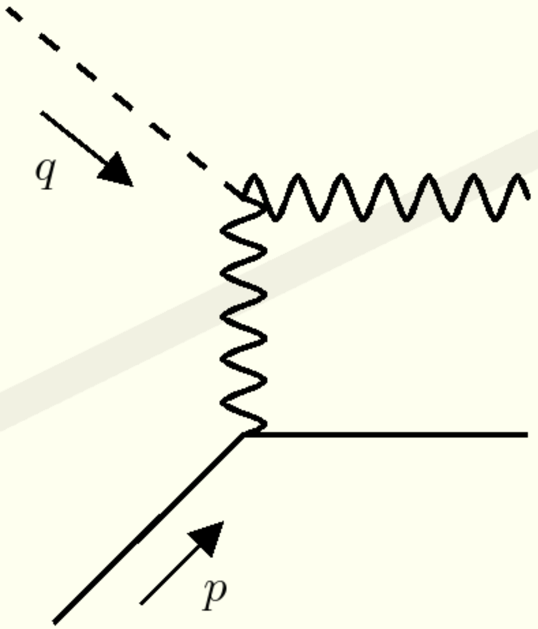
- LP is often well understood to high perturbative and logarithmic accuracy
- Eventually, observables become sensitive to power corrections!
- With current and future collider sensitivities, now is the time to think about power corrections!



[van Beekveld, Laenen, Damsté, Vernazza, 2101.07270]

NLP DIS at threshold

- The off-diagonal channel contribution is a pure NLP process → No LP contamination like kinematic power corrections



$$W_\phi = C^{\text{NLP}} \otimes f^{\text{LP}} + C^{\text{LP}} \otimes f^{\text{NLP}}$$

- Use series of EFTs to derive factorisation theorem: QCD → SCET-I → SCET-II

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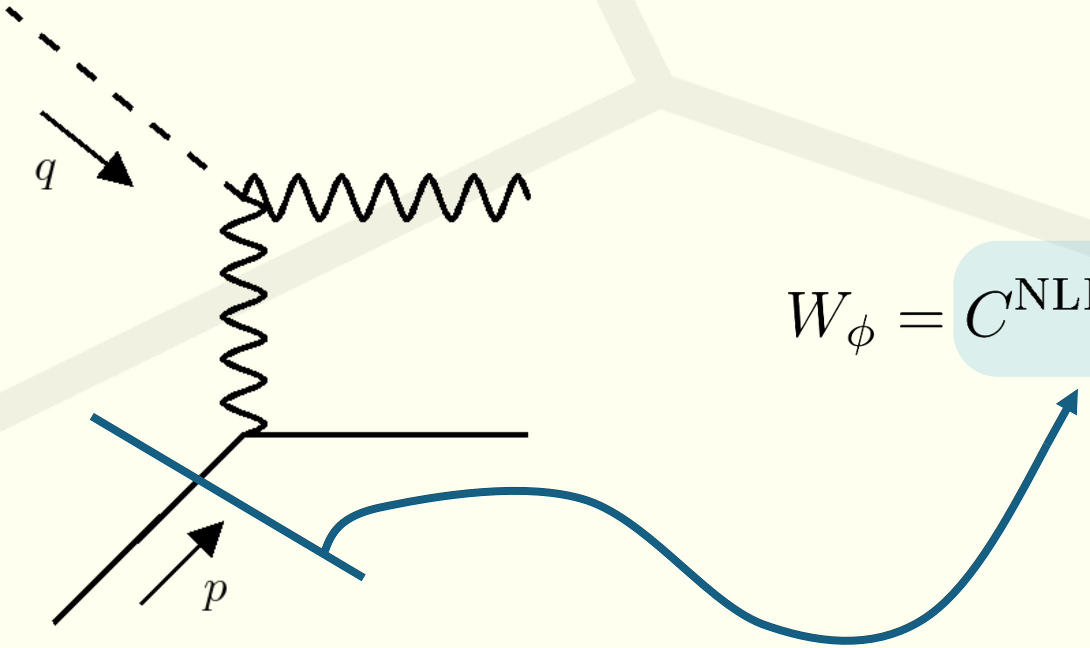

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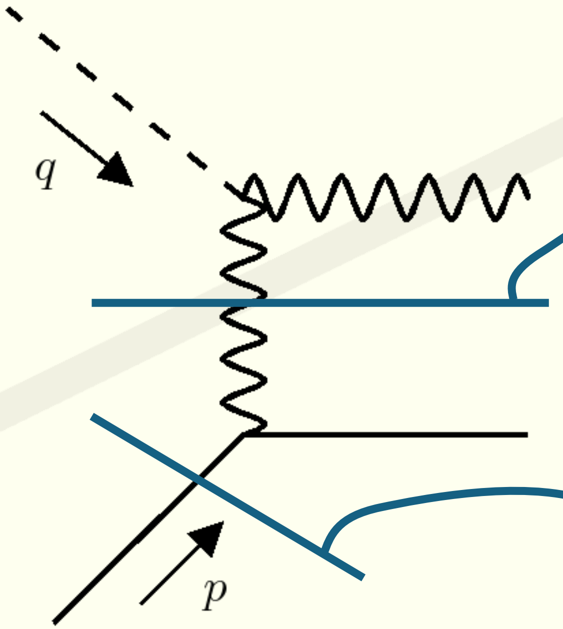

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- Use series of EFTs to derive factorisation theorem:
  QCD → SCET-I → SCET-II

Derivation of the factorisation theorem

- Procedure: match full QED/QCD onto SCET currents

$$W = \int \mathrm{dPS}_X \delta^4(p+q-p_X)\langle e(p)F^2(0)|X\rangle\langle X|F^2(0)|e(p)\rangle$$

$$F^2(0) = J^{\mathrm{A0}}(0) + J^{\mathrm{B1}}(0)$$ ← SCET currents

$$J^{\mathrm{A0}} = C^{\mathrm{A0}} \otimes G_{\overline{\mathrm{hc}}} \otimes G_{\mathrm{hc}} \qquad J^{\mathrm{B1}} = C^{\mathrm{B1}} \otimes G_{\overline{\mathrm{hc}}} \otimes \bar{\psi}_{\overline{\mathrm{hc}}} \otimes \psi_c$$

$$\mathcal{L}_{\mathrm{sc}} = \bar{\psi}_c \otimes G_{\mathrm{hc}} \otimes q_{sc}$$

hard (virtual loops)
anti-hard-collinear (final state jet)
hard-collinear (intermediate matching)
collinear (PDF)
soft-collinear (final state radiation)

Derivation of the factorisation theorem

- Full factorisation theorem:

$$W = C^{B1*}(r')C^{B1}(r) \overset{r,r'}{\otimes} \mathcal{J}_{\overline{hc}}^{(qg)}(\xi, r, r') \overset{\xi}{\otimes} f_{q/N}(\xi) + |C^{A0}|^2 \mathcal{J}_{\overline{hc}}^{(g)}(\xi) \overset{\xi}{\otimes} f_{g/N}^{\text{thr,NLP}}(\xi)$$

$$f_{g/N}^{\text{thr,NLP}}(\xi) = D^{B1*}(\omega')D^{B1}(\omega) \overset{\omega,\omega'}{\otimes} S_{sc,\text{sub}}^{\text{NLP}}(\xi', \omega, \omega') \overset{\xi'}{\otimes} f_{q/N}^{\text{thr,LP}}(1 + \xi - \xi') .$$

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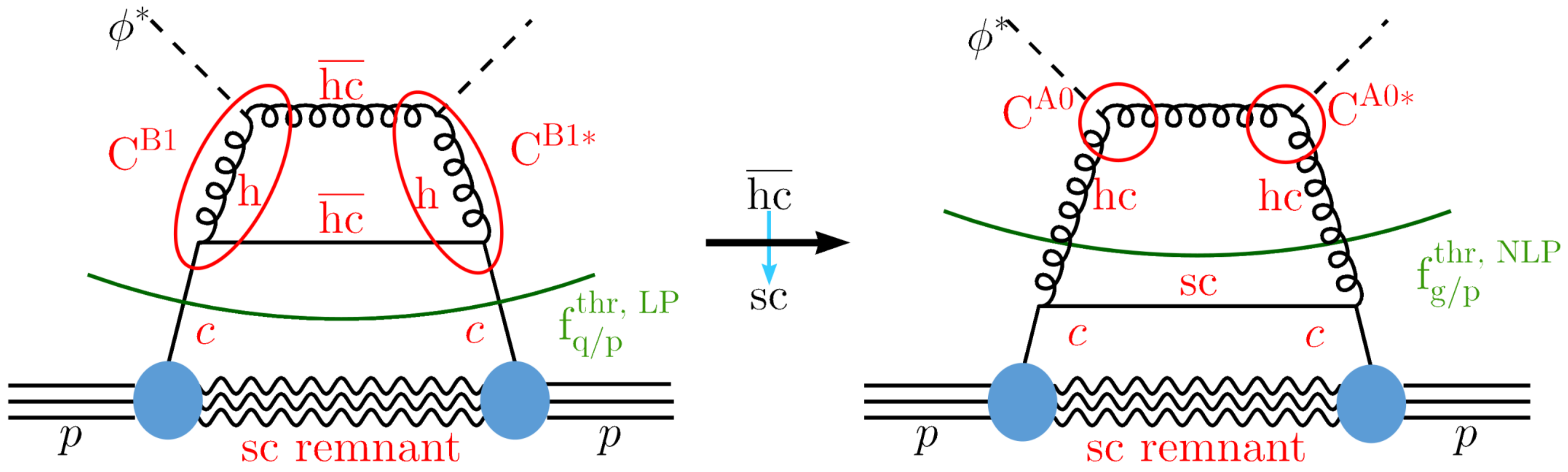


Endpoint divergences...

$$W = C^{B1*}(r')C^{B1}(r) \overset{r,r'}{\otimes} \mathcal{J}^{(qg)}_{\overline{hc}}(\xi, r, r') \overset{\xi}{\otimes} f_{q/N}(\xi)$$
$$+ |C^{A0}|^2 \, \mathcal{J}^{(g)}_{\overline{hc}}(\xi) \overset{\xi}{\otimes} f^{\text{thr,NLP}}_{g/N}(\xi)$$

$$f^{\text{thr,NLP}}_{g/N}(\xi) = D^{B1*}(\omega')D^{B1}(\omega) \overset{\omega,\omega'}{\otimes} S^{\text{NLP}}_{sc,\text{sub}}(\xi', \omega, \omega') \overset{\xi'}{\otimes} f^{\text{thr,LP}}_{q/N}(1 + \xi - \xi') \,.$$

- Convolutions are divergent in the endpoint limits, i.e. $r^{(\prime)} \to 0, \; \omega^{(\prime)} \to \infty$
- Stems from regions of phase space where the initial assumptions to derive factorisation are violated.

… and their cancellations

- Full hadronic tensor is divergence free → The endpoint divergences must cancel in the end!
- Two possible solutions: 1) Stay in d dimensions 2) use refactorization theorems
- First method is possible, because endpoint divergences can be regularized with dimensional regulator
- Resums poles of component functions to all orders, which eventually cancel out *after* the convolutions in r, ω have been performed, agrees with literature

$$f_{g/q}^{\text{thr,NLP}} = \frac{1}{2NL_N}\frac{C_F}{C_F - C_A}\mathcal{B}_0\left(\frac{\alpha_s}{\pi}(C_F - C_A)L_N^2\right) f_{q/q}^{\text{thr,LP}}$$

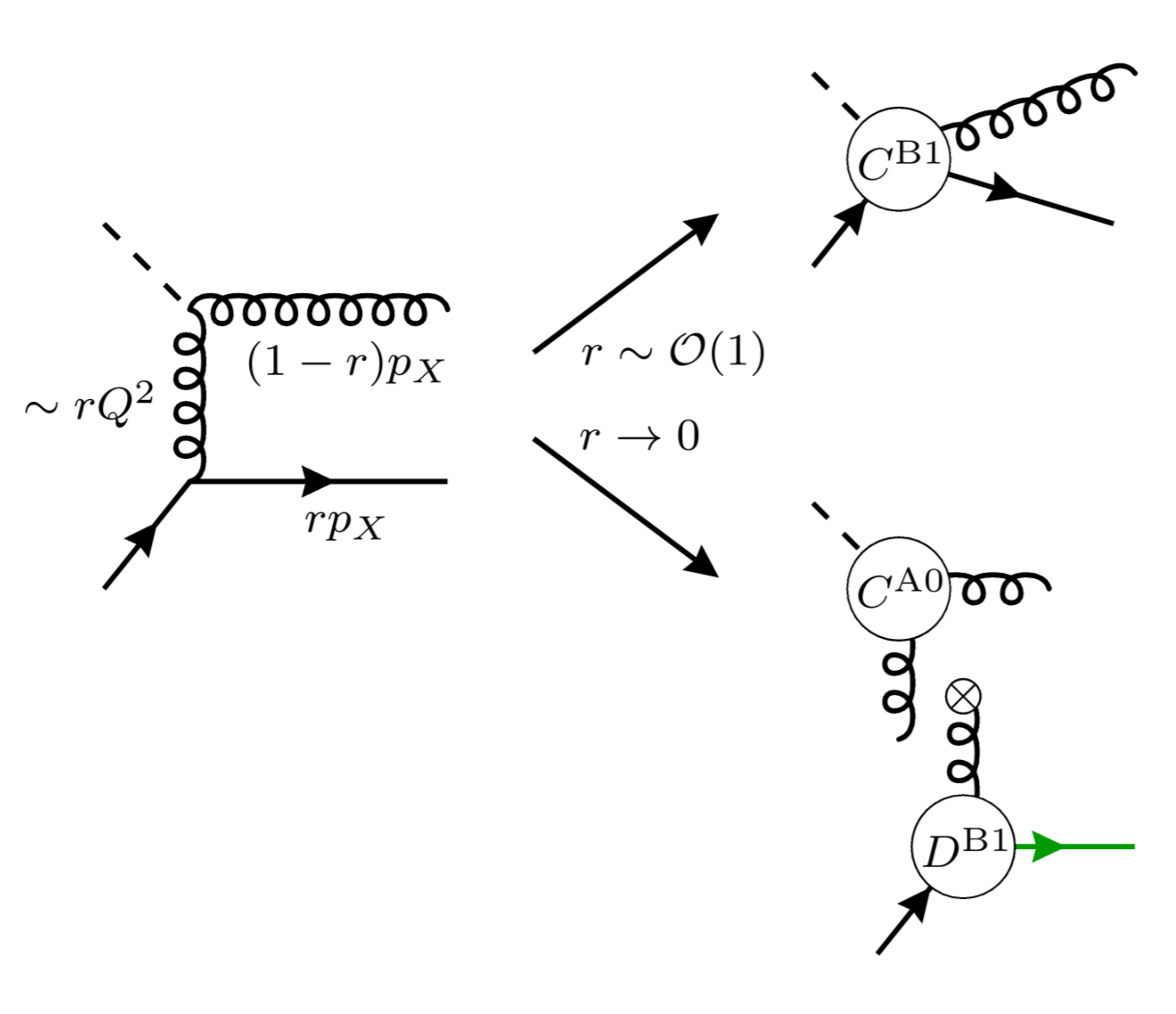
$L_N = \ln N$ After Mellin transform, x → 1 ↔ N → ∞

… and their cancellations

- Limitations of this approach:
  - Divergences cancel only after convolutions are performed
  - d dimensional resummation is not straight-forward, no intuitive 4-dimensional interpretation
- Refactorisation-based subtractions rearrange the factorisation formula s.t. the divergences are cured on the integrand level!

[Liu, Neubert, 1912.08818]
[Beneke, Garny, Jaskiewicz, Szafron, Vernazza, Wang 2008.04943]
[Liu, Mecaj, Neubert, Wang, 2009.06779]
[Beneke, Garny, Jaskiewicz, Strohm, Szafron, Vernazza, 2205.04479]
[Bell, Böer, Feldmann, 2205.06021]
[Liu, Neubert, MS, Wang, 2212.10447]
[Cornella, König, Neubert, 2212.14430]
[Hurth, Szafron, 2301.01739]
[Cornella, Ferré, König, Neubert, 2601.14361]

Refactorisation theorems



- When r ~ 1, then the factorisation follows as previously, but in region r → 0 an additional factorisation is due instead:

$$\left[\!\left[C^{\mathrm{B1}}(Q^2, r) \right]\!\right] = -2 C_g^{\mathrm{A0}}(Q^2)\, \frac{D_g^{\mathrm{B1}}(rQ^2)}{r}$$

- Similarly:

$$\frac{1}{Q} \left[\!\left[\mathcal{J}_{hc}^{\mathrm{NLP}}(k^2, r, r') \right]\!\right] \overset{\Omega}{\otimes} S^{\mathrm{LP}}_{\mathrm{sc,\ sub}}(\Omega)$$

$$= \mathcal{J}_{hc}^{\mathrm{LP}}(k^2) \overset{\Omega}{\otimes} S^{\mathrm{NLP}}_{\mathrm{sc,\ sub}}(\Omega, rQ, r'Q)$$

Refactorisation theorems

- This introduces a new scale Λ where the refactorisation-based subtractions are implemented, that can be chosen freely $0 < \Lambda < Q$

$$W = C^{B1*}(r')C^{B1}(r) \overset{r,r' \geq \Lambda/Q}{\otimes} \mathcal{J}^{(qg)}_{\overline{hc}}(\xi, r, r') \overset{\xi}{\otimes} f_{q/N}(\xi)$$

$$+ |C^{A0}|^2 \, \mathcal{J}^{(g)}_{\overline{hc}}(\xi) \overset{\xi}{\otimes} f^{\mathrm{NLP},®}_{g/N}(\xi)$$

$$f^{\mathrm{NLP},®}_{g/N}(\xi) = D^{B1*}(\omega')D^{B1}(\omega) \overset{\omega,\omega' \leq \Lambda}{\otimes} S^{\mathrm{NLP}}_{sc,\mathrm{sub}}(\xi', \omega, \omega') \overset{\xi'}{\otimes} f^{\mathrm{thr,LP}}_{q/N}(1 + \xi - \xi') \, .$$

- This effectively is a *new* PDF definition!

Advantages of the new PDF scheme

$$f_{g/N}^{\mathrm{NLP},®}(\xi) = D^{B1*}(\omega')D^{B1}(\omega) \overset{\omega,\omega'\leq\Lambda}{\otimes} S_{sc,\mathrm{sub}}^{\mathrm{NLP}}(\xi',\omega,\omega') \overset{\xi'}{\otimes} f_{q/N}^{\mathrm{thr,LP}}(1+\xi-\xi')\,.$$

- Regularisation of endpoint divergences and UV renormalisation are disentangled and commute
  - Factorisation formula depends only on renormalised component functions
  - Can use standard RG evolution to resum large logarithms

$$f_{g/N}^{\mathrm{NLP},®}(\Lambda) = \frac{1}{2NL_N}\frac{C_F}{C_F-C_A}\exp\left[\frac{\alpha_s}{\pi}(C_F-C_A)\left(L_N L_\Lambda + \frac{L_N^2}{2}\right)\right] f_{q/N}^{\mathrm{thr,LP}}$$

$L_\Lambda = \ln \mu^2/(Q\Lambda)$

- Choice of Λ allows us to shuffle large logs (at LL) to either of the A0 or B1 terms
- Possibility to choose renormalisation scale $\mu^2 = Q\Lambda/\sqrt{N}$ s.t. there are no log corrections in the NLP PDF

Scheme conversion

- Define scheme conversion factor

$$\Delta f_g^{\text{NLP}} \equiv f_{g/q}^{\text{NLP},®}(\Lambda,\mu) - f_{g/q}^{\text{thr,NLP}}(\mu)$$
$$\equiv \Delta Z_{gq}^{\text{NLP}}(\Lambda) f_{q/q}^{\text{thr,LP}}$$

- ΔZ is perturbatively calculable

$$\Delta Z_{gq}^{\text{NLP}}(\Lambda) = \left[\int_\Lambda^\infty \frac{d\omega}{\omega}\int_\Lambda^\infty \frac{d\omega'}{\omega'} - Z_{gg}^{\text{LP}}(Z_{qq}^{\text{LP}})^{(-1)} D_{g,\text{bare}}^{\text{B1}}(-n_+p\omega) D_{g,\text{bare}}^{\text{B1}*}(-n_+p\omega') S_{\text{sc,sub,bare}}^{\text{NLP}}(\omega,\omega')\right]_{\text{fin}}$$
$$= \frac{1}{2NL_N}\frac{C_F}{C_F - C_A}\left\{\exp\left[\frac{\alpha_s}{\pi}(C_F - C_A)\left(L_N L_\Lambda + \frac{L_N^2}{2}\right)\right]\right.$$
$$\left. - \mathcal{B}_0\left(\frac{\alpha_s}{\pi}(C_F - C_A)L_N^2\right)\right\}$$

- This computation from first principles serves as a strong cross-check.

Conclusions & Outlook

- Derived factorisation formula for off-diagonal channel DIS in the threshold limit $x \to 1$ at NLP
- NLP gluon PDF factorises into perturbative coefficient x LP quark PDF
- Endpoint divergences are cured with
  - d dimensional treatment
  - Refactorisations → new (in our opinion advantageous) PDF definition

○ Resummation of large logs at higher accuracies

Thank you!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Back up

Soft functions & soft subtractions

- In full theorem, collinear modes are matched to frozen-collinear modes before collecting them in the PDF when lowering virtuality to the soft scale
- Pole parts of soft functions mix UV & IR divergences
- Soft and frozen-collinear functions are not separately well-defined (explicit dependence on IR regulator)
- Soft subtraction solves this problem $S^{\mathrm{NLP}}_{\mathrm{sc,sub}}(n_-z,\omega,\omega') = \dfrac{S^{\mathrm{NLP}}_{sc}(n_-z,\omega,\omega')}{S^{\mathrm{LP}}_{sc}(n_-z)}$