Parity Violating Electron Scattering at EW Bosonic NNLO

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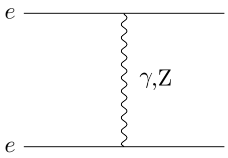
Parity-Violating Electron (Møller) Scattering

Parity violating asymmetry in polarized electron scattering

$$A_{LR} = \frac{d\sigma_L - d\sigma_R}{d\sigma_L + d\sigma_R} = \frac{G_F Q^2}{\sqrt{2}\pi\alpha} \frac{1-y}{1+y^4+(1-y)^4} \left(1 - 4\sin^2\theta_W + \delta Q_W^e\right)$$



where $y = Q^2/s = -t/s$.

In particular, $m_Z^2 \gg Q^2$ ($\sim 10^{-3}$ GeV^2) $\gg m_e^2$:

- Measurements Of Lepton-Lepton EW Reaction (MOLLER) at JLab; construction underway, first results expected by mid-2027

  MOLLER Collaboration '14, Demiroglu '25

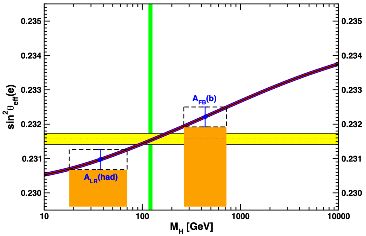
- Two-loop EW corrections can be computed analytically

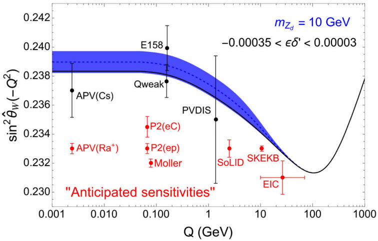
Weak Mixing Angle for SM and BSM

Best Z-pole $\sin^2\theta_W$ measurements in $\sim 3\sigma$ tension with one another, as illustrated by their Higgs mass predictions — Erler arXiv:1710.06503

Test of BSM scenarios, e.g. dark Z boson
Davoudiasl et al. PRD '23, see also Cadeddu et al. PRD '21

Bounds for $4e$ contact interaction (Z', $H^{\pm\pm}$, compositeness, ...) up to $\mathcal{O}(10\ \text{TeV})$
MOLLER Collaboration '14, Demiroglu '25

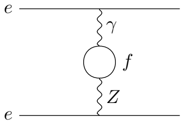




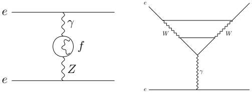
PVES and Precision Calculation

$$A_{LR} = \frac{G_F Q^2}{\sqrt{2}\pi\alpha} \frac{1-y}{1+y^4+(1-y)^4} \left(1 - 4\sin^2\theta_W + \delta Q_W^e\right)$$

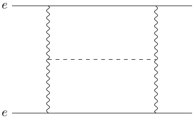
NLO: $\sim 40 \pm 3\%$ of tree level, $\delta Q^e_{W,\mathrm{ferm}} \sim 10 \cdot \delta Q^e_{W,\mathrm{bos}}$
Czarnecki & Marciano PRD '96



Fermionic NNLO: $\sim 1.3 \pm 0.01\%$ of tree level
roughly MOLLER sensitivity
Du, Freitas, Patel, and Ramsey-Musolf PRL '21



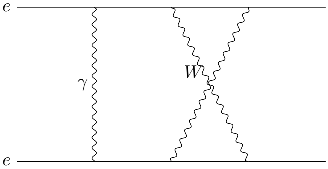
This work: NNLO without fermion loops
approximate $\sin^2\theta_W = 1/2$;
include NNLL $\sim \log^2(q^2/m_Z^2)$.

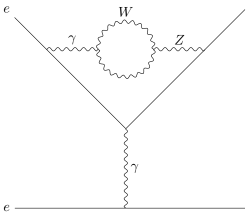


Method of Regions

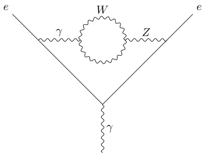
1. Split loop momenta into regions according to the scales of the problem
2. Expand the integrands with corresponding scaling
3. Extend the bound of integration to $[0, \infty)$ and evaluate the integrals
4. Add contributions from all regions

Beneke & Smirnov Nucl. Phys. '98, Smirnov '01, Pak & Smirnov '10,...





Method of Regions



$$\sim \frac{1}{(q_1^2 - m_e^2)((q_1 - p_a)^2 - m_\gamma^2)((q_1 - p_a)^2 - m_Z^2)}$$
$$\times \frac{1}{((q_1 - p_a)^2 - m_e^2)(q_2^2 - m_W^2)((q_2 - q_1 - p_a)^2 - m_W^2)}$$

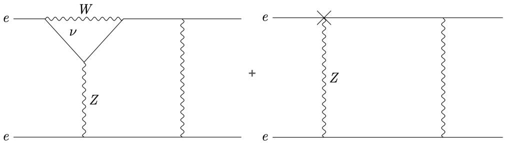
where $\{p_a, p_b\} \ll m_Z$ are (comb. of) external momenta

This in principle gives us many regions:

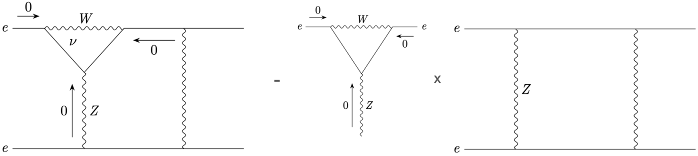
- $\{q_1, q_2\} \sim m_Z \gg \{p_a, p_b\}$
- ~~$q_1 \sim m_Z \gg q_2 \sim \{p_a, p_b\}$~~
- $q_2 \sim m_Z \gg q_1 \sim \{p_a, p_b\}$
- ~~$\{q_1, q_2\} \sim \{p_a, p_b\}$~~ $\ll m_Z \ldots$

γ_5 with "Momentum Subtraction"

Must treat γ_5 in some self-energy and vertex subloop in a box carefully, e.g.,

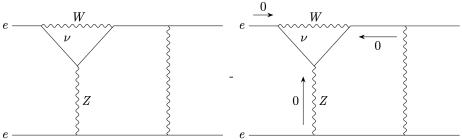


The triangle subloop in the two-loop diagram is divergent;
naive application of dim. reg. result in ambiguities.
$\Rightarrow$ Subtract 0 s.t. the subset is finite, then compute in 4D:

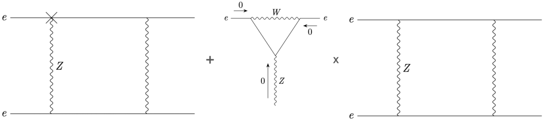


γ_5 with "Momentum Subtraction"

First calculate



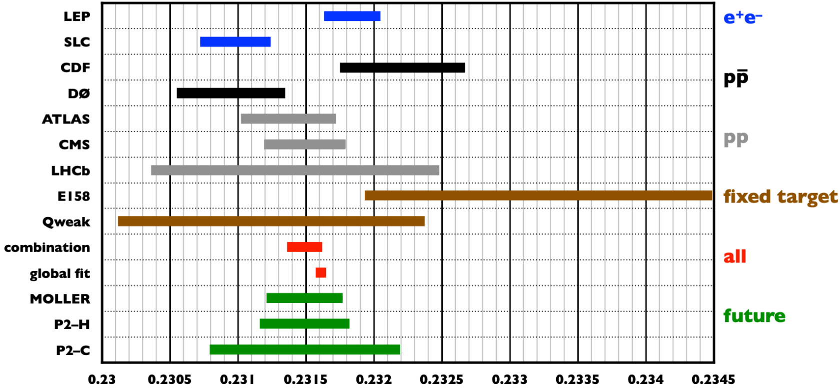
which is now finite, i.e. no need for dim. reg. Then calculate



which is also finite. The sum of the two lines gives the desired result!

Also applicable to diagrams with IR singularities, regulated by fictitious photon mass that cancels in the final result!

Status of $\sin^2\theta_W$



See Erler '24 and refs therein

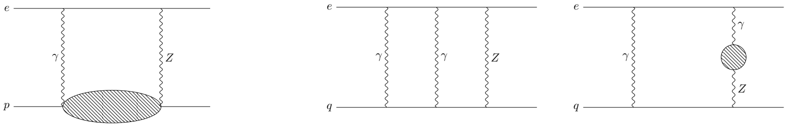
Towards NNLO for Mainz P2

Weak charge measurements for proton and C-12 at $Q^2 \sim 4.5 \times 10^{-3}$ GeV^2;
Becker et al. EPJA '18

Under construction (follow progress on website!), results anticipated $\sim$ 2030;

One-loop corrections studied; Erler et al. PRD '03

Framework for EW two-loop calculation laid out. Needs to be computed!
Erler et al. JHEP '22, Cadeddu et al. PRC '24



See Gorchtein & Horowitz PRL '08 for the discussion of AZ boxes.

Summary

- Low-energy weak charge measurements are complementary to Z-pole studies and are important for precision tests of SM and constraining BSM;
- Electroweak bosonic two-loop corrections for PVES can be computed analytically with method of regions. Results for MOLLER coming soon!
- These corrections could be more significant for electron-proton and electron-C-12 scattering at Mainz P2, to be computed!