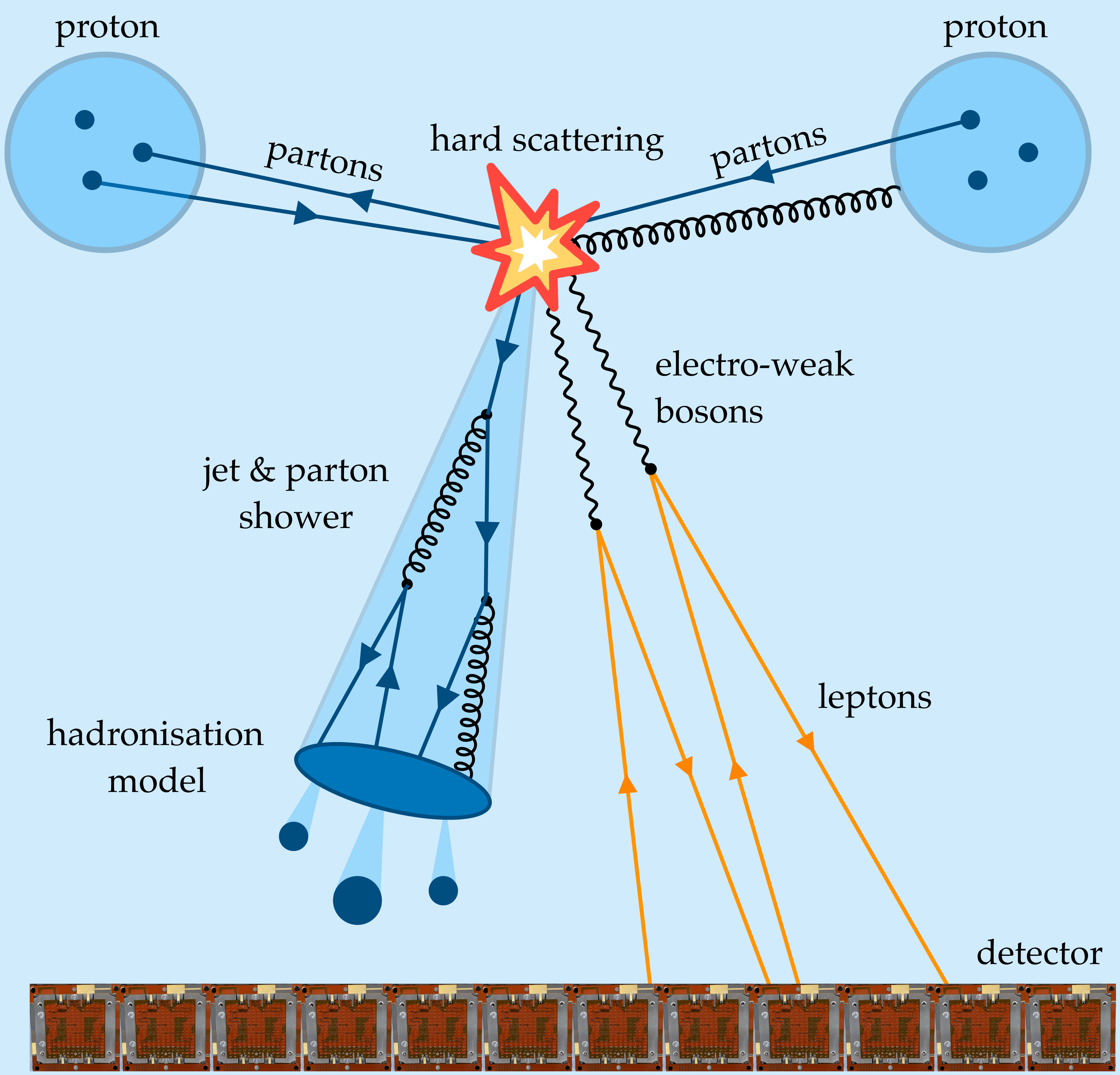
The KLN theorem meets collinear factorisation



Zeno Capatti

Based on work in collaboration with Lucien Huber and Michael Ruf

Hadronic cross-sections

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The problem of computing hadronic cross-sections is mapped to partonic scattering

$$\sigma(PP \to Z + X) = \sum_{\mathfrak{q}_1, \mathfrak{q}_2 \in \{q, \bar{q}, g\}} \int \frac{\mathrm{d}x_1}{x_1} \int \frac{\mathrm{d}x_2}{x_2} f^{(0)}_{\mathfrak{q}_1}(x_1) f^{(0)}_{\mathfrak{q}_2}(x_2) \sigma_{\mathrm{p}}(\mathfrak{q}_1 \mathfrak{q}_2 \to Z + X)$$

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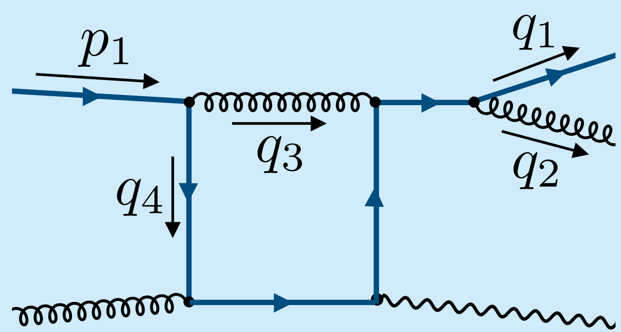

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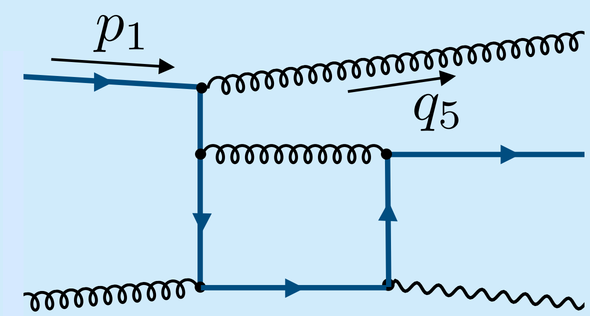

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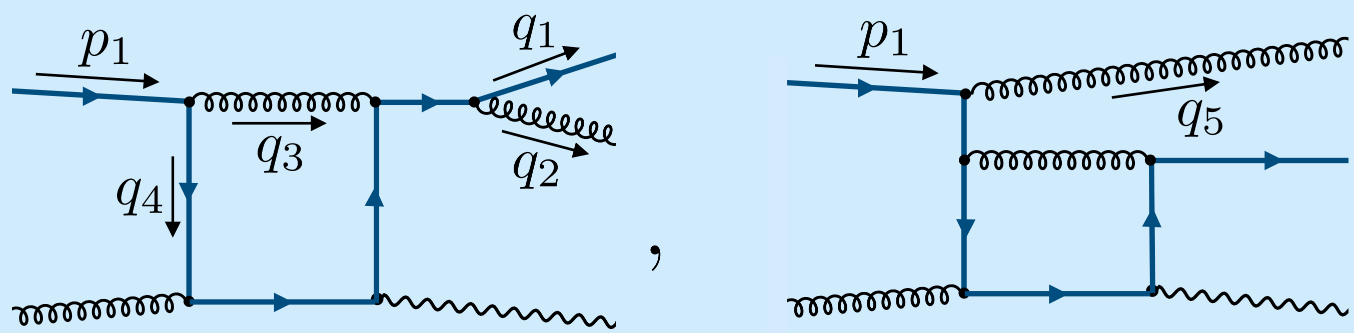

If the singularity involves an initial-state parton, then call it initial-state

$q_1//q_2$	**Final-State**
$q_5//p_1$	**Initial-State**
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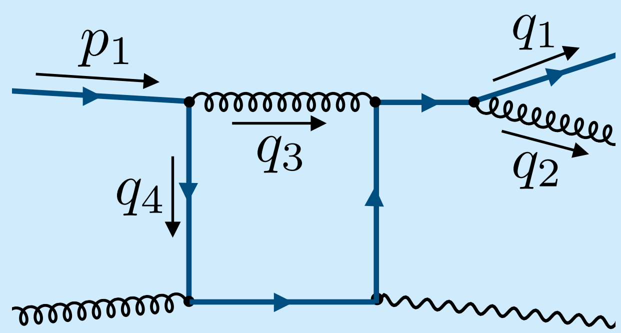

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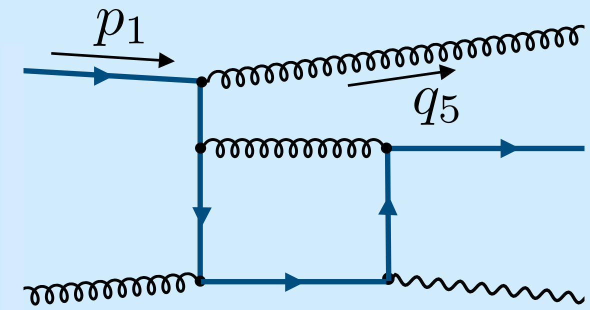

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Final-state singularities cancel by the *Kinoshita-Lee-Nauenberg theorem*.

$$\underbrace{\sigma_{\mathrm{p}}(\mathfrak{q}_1\mathfrak{q}_2 \to Z + X)}_{\text{Does not have FS singularities.}} = \sum_{n=0}^{\infty} \int \mathrm{d}\Pi_n \underbrace{\sigma_{\mathrm{p}}(\mathfrak{q}_1\mathfrak{q}_2 \to Z + \mathfrak{q}'_1 ... \mathfrak{q}'_n)}_{\text{Has FS singularities.}} \mathbb{O}_n$$

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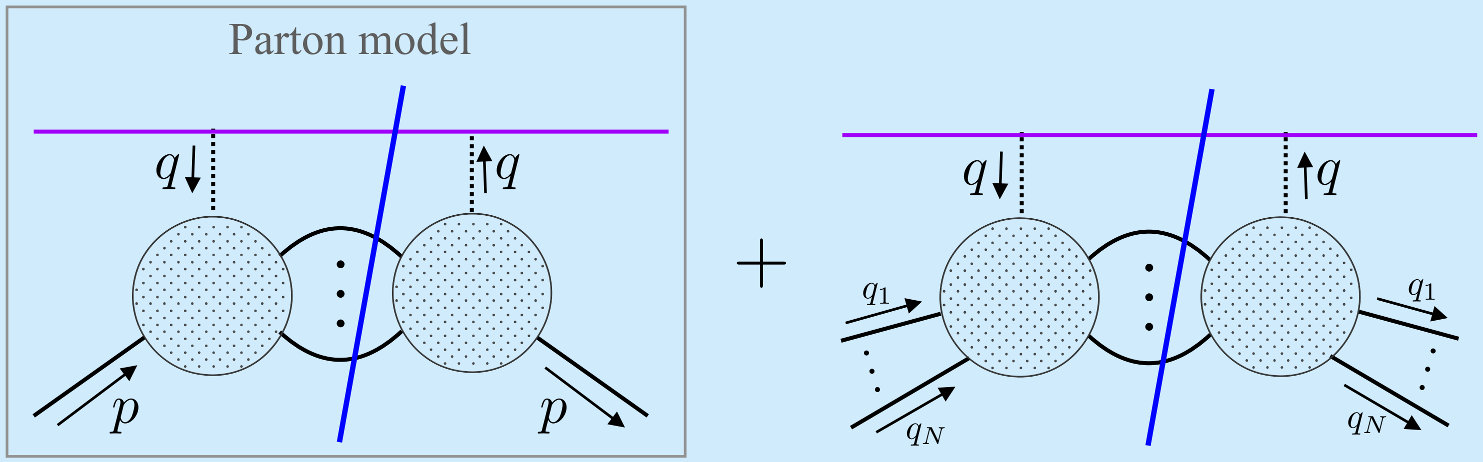

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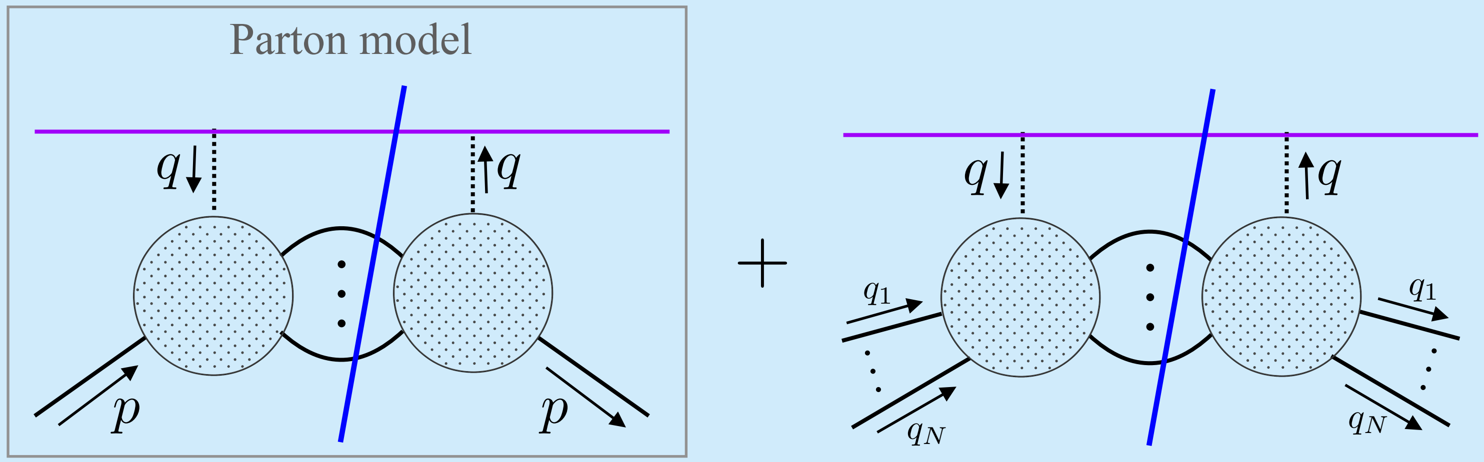

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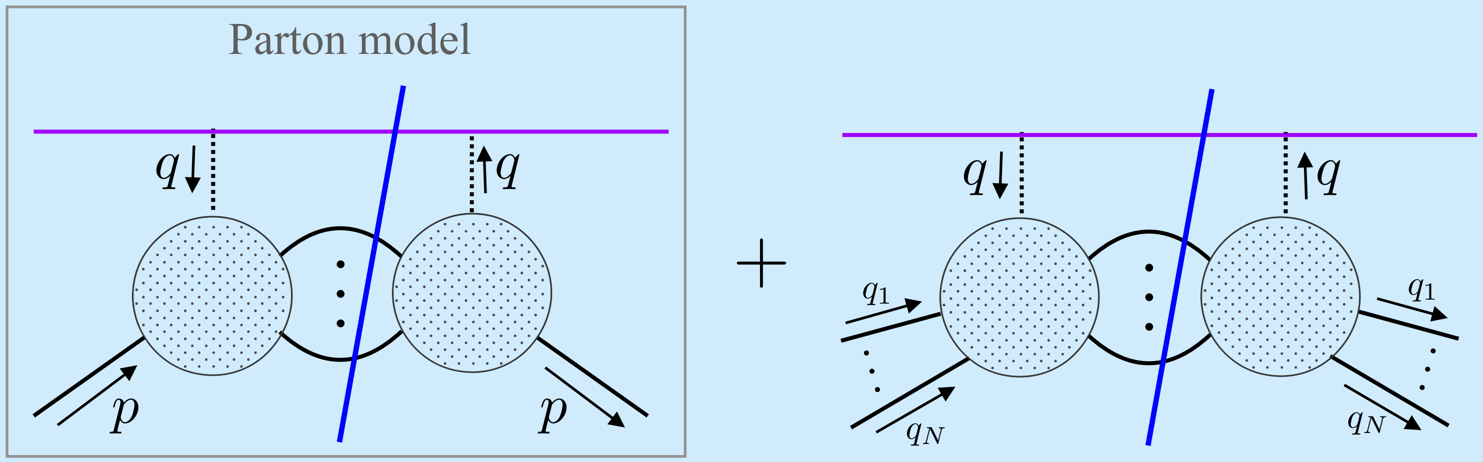

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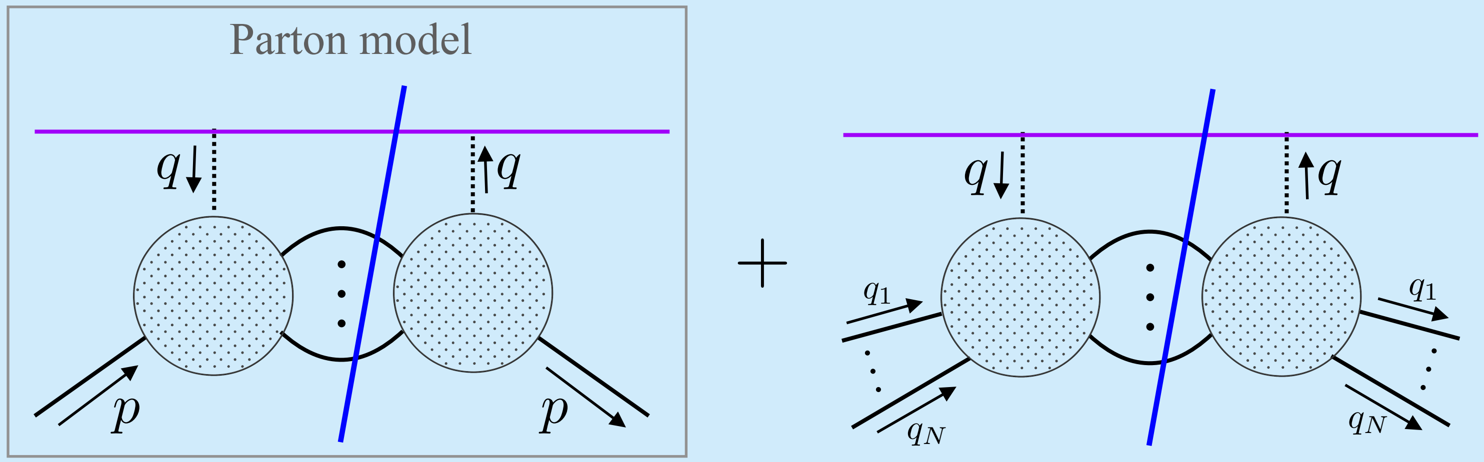

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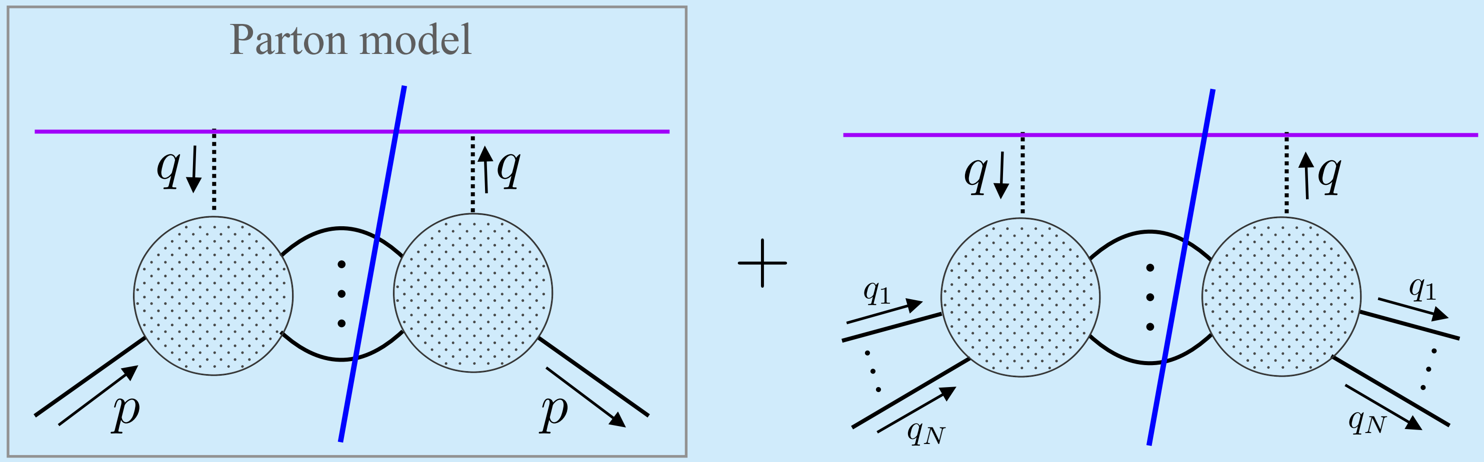

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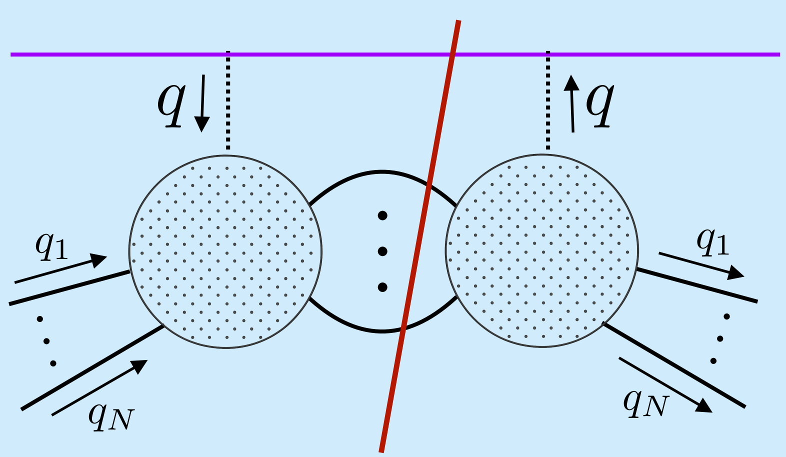
Following Sterman and Weinberg, we cluster them into jets (one per hadron)



$$\begin{cases} q_1 + \ldots + q_n = p \\ p^2 < \Lambda^2 \\ \mathfrak{q}_1 + \ldots + \mathfrak{q}_n = \mathfrak{q} \end{cases}$$

Satisfies that, as $\Lambda^2 \to 0$, $q_i = x_i p + \mathcal{O}(\Lambda^2)$

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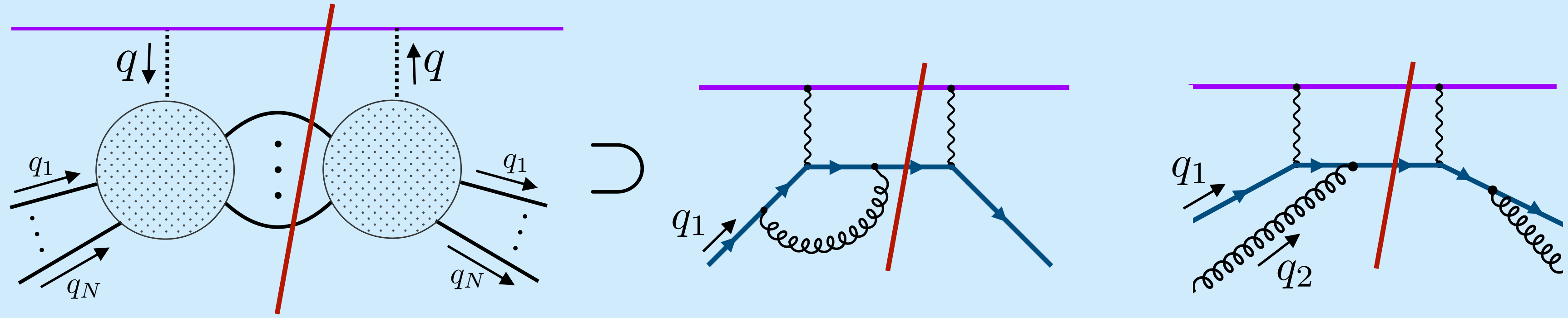

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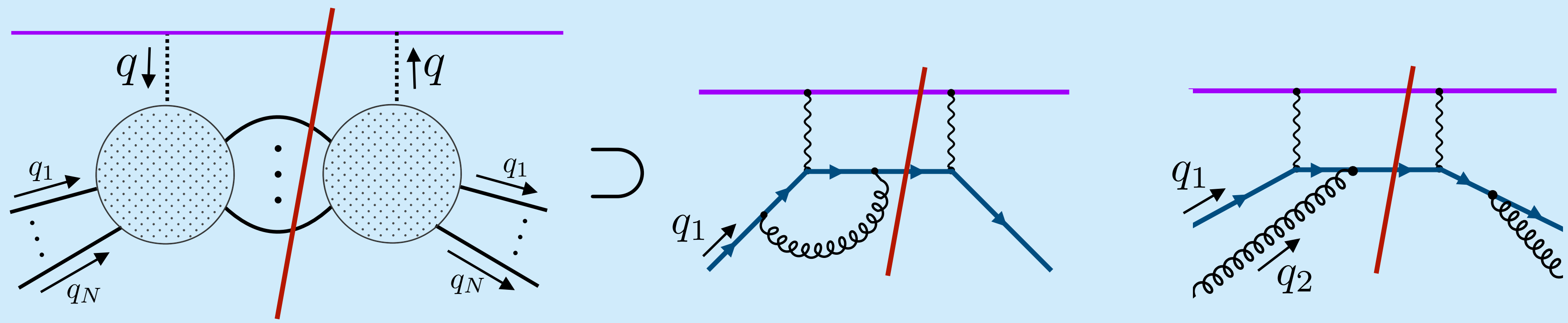

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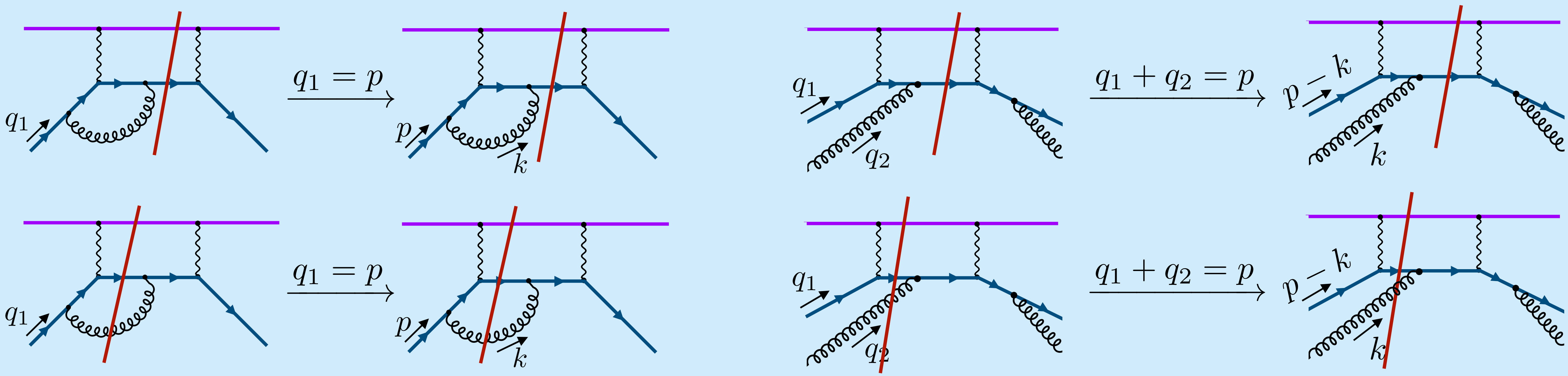
q
q_1
q_N
q_1
q_N
$\supset$
q_1
q_1
q_2



q_1
$q_1 = p$
p
k
q_1
q_2
$q_1 + q_2 = p$
$p - k$
k
q_1
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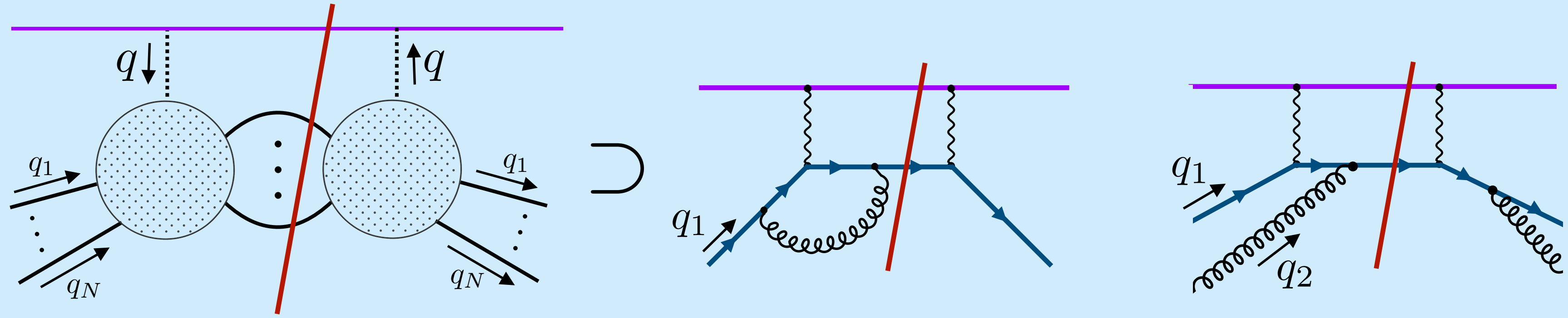
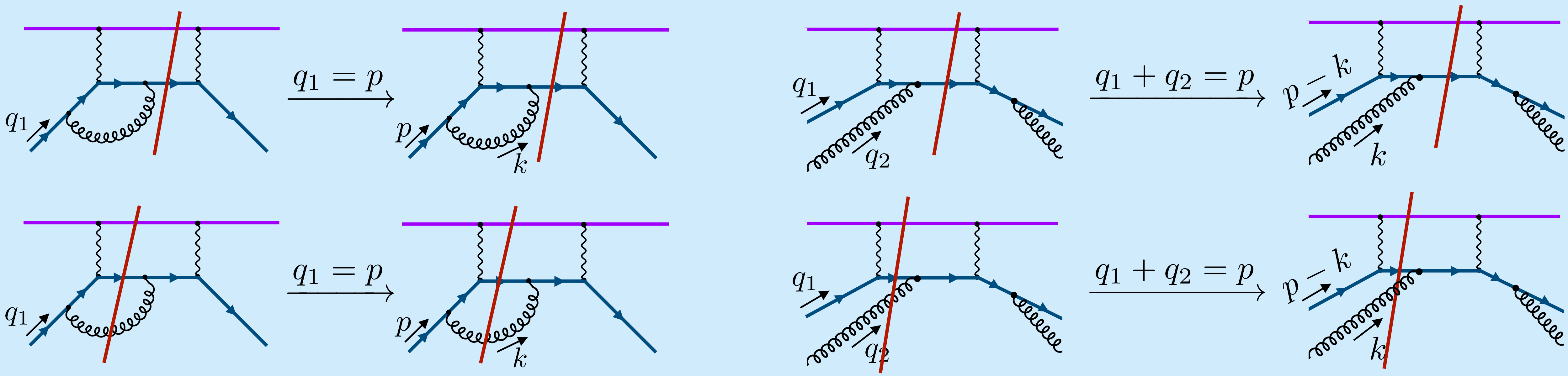
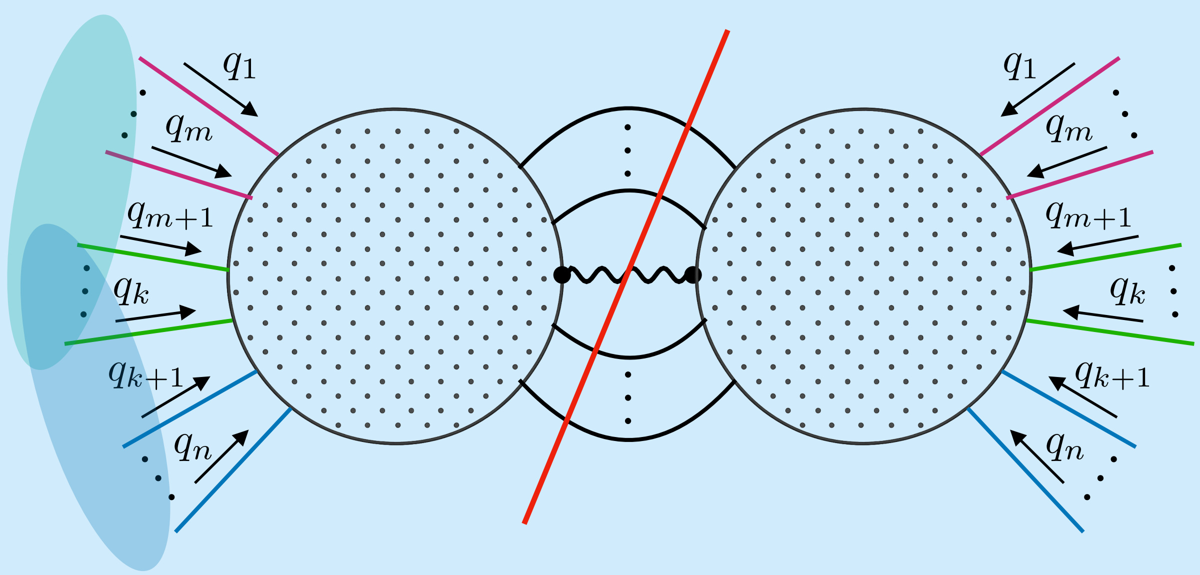
p
k
q_1
q_2
$q_1 + q_2 = p$
$p - k$
k

For example:





For Drell-Yan we need two “initial-state” jets; we sum over overlapping partitions:



$$
\begin{cases}
S_1 = \{1, \ldots, k\}, \quad S_2 = \{m+1, \ldots, n\} \\
q_1 + \ldots + q_m + q_{m+1} + \ldots + q_k = p_1 \\
q_{m+1} + \ldots + q_k + q_{m+1} + \ldots + q_n = p_2 \\
p_1^2 < \Lambda_1^2, \quad p_2^2 < \Lambda_2^2
\end{cases}
$$

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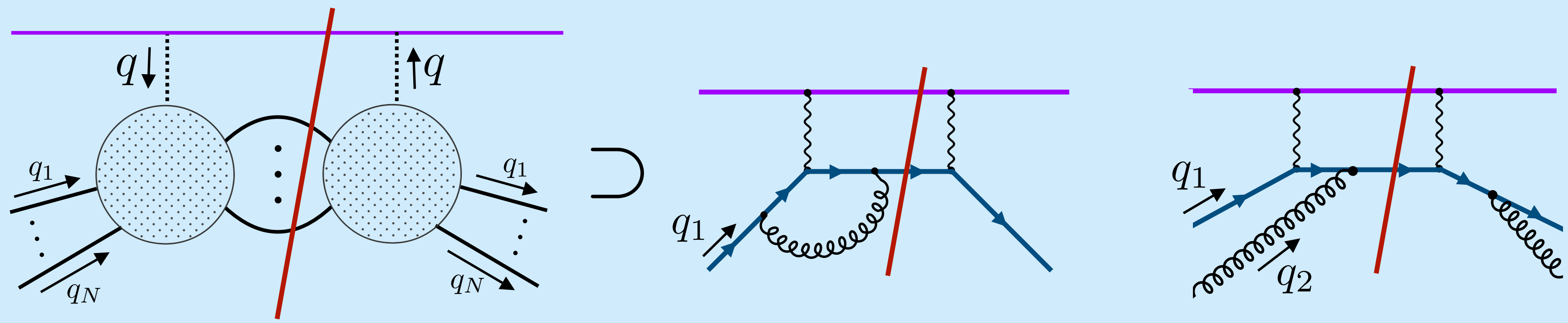

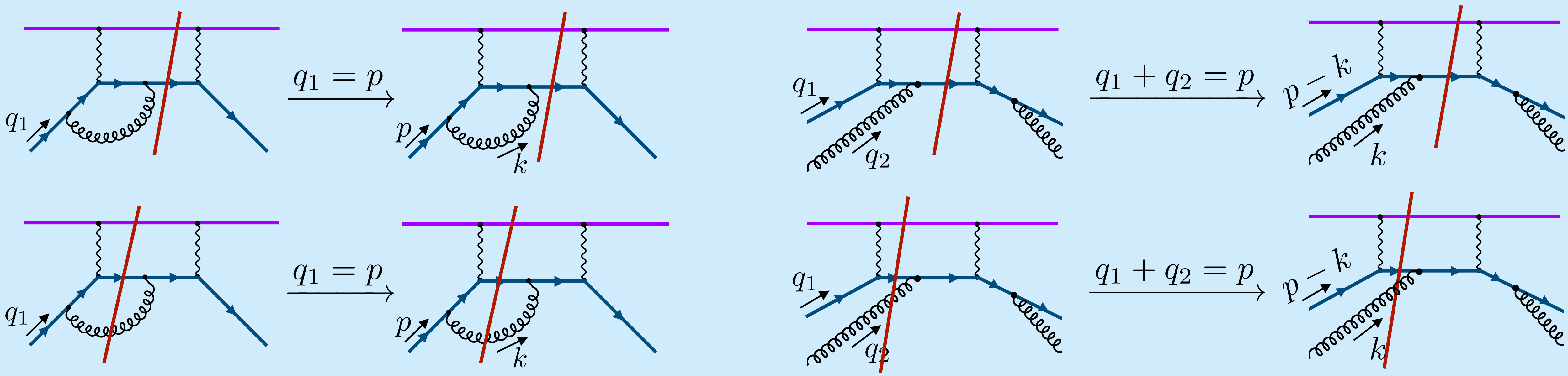

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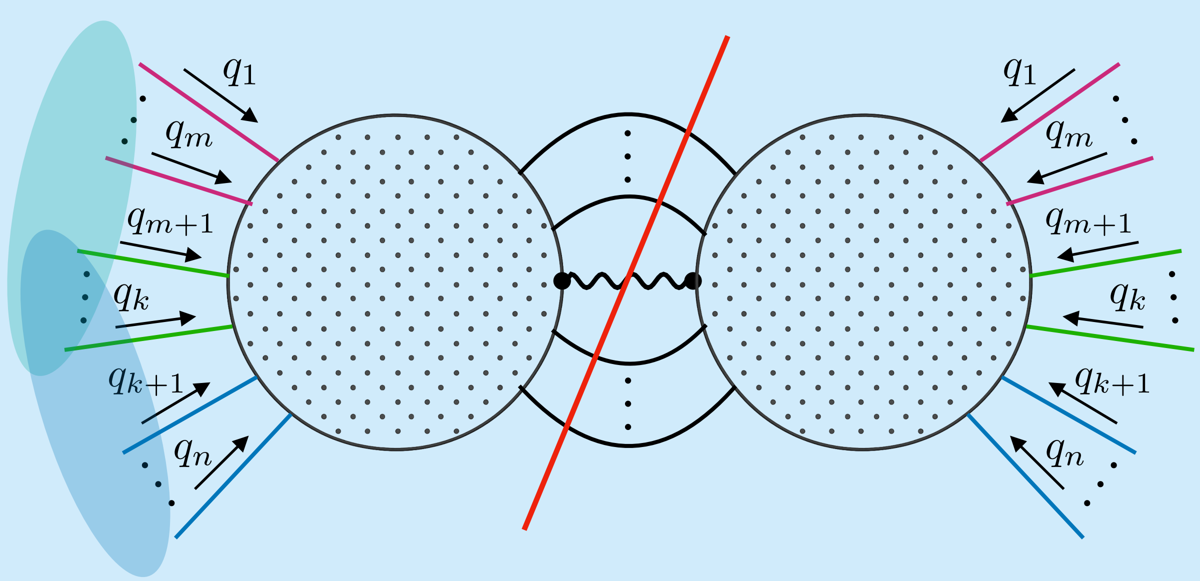


$$\begin{cases} S_1 = \{1, \ldots, k\}, \quad S_2 = \{m+1, \ldots, n\} \\ q_1 + \ldots + q_m + q_{m+1} + \ldots + q_k = p_1 \\ q_{m+1} + \ldots + q_k + q_{m+1} + \ldots + q_n = p_2 \\ p_1^2 < \Lambda_1^2, \quad p_2^2 < \Lambda_2^2 \end{cases}$$

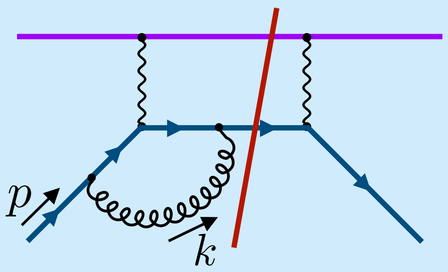
For small virtualities:

$$q_i = x_i p_1 + \mathcal{O}(\Lambda_1^2) \qquad q_j = x_j p_2 + \mathcal{O}(\Lambda_2^2) \qquad q_l = 0 + \mathcal{O}(\Lambda_1^2, \Lambda_2^2)$$

Then we can write the corresponding integrals

Then we can write the corresponding integrals

Then we can write the corresponding integrals

<table>
<tr>
<td></td>
<td>$\delta^+(p^2)\delta^+((p+q)^2)\int \frac{\mathrm{d}^d k}{(2\pi)^d} \frac{1}{k^2(p-k)^2(p+q-k)^2}$</td>
<td>$\delta^+(p^2)\delta^+((p+q)^2)T(p,q)$</td>
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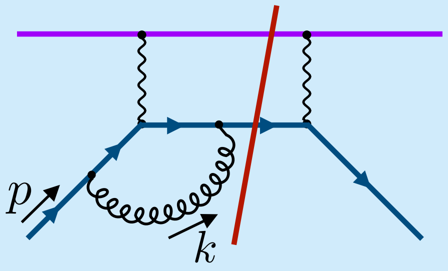
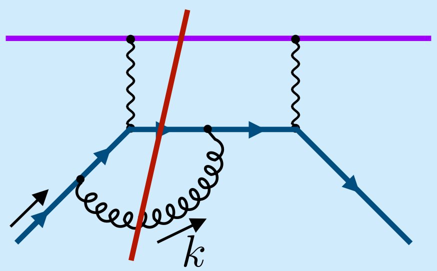
	$\delta^+(p^2)\delta^+((p+q)^2)\int \frac{\mathrm{d}^d k}{(2\pi)^d}\frac{1}{k^2(p-k)^2(p+q-k)^2}$	$\delta^+(p^2)\delta^+((p+q)^2)T(p,q)$
	$\delta^+(p^2)\delta^+((p+q)^2)\int \frac{\mathrm{d}^d k}{(2\pi)^d}\frac{\delta^-(k)\delta^+(p+q-k)}{(p-k)^2}$	$\delta^+(p^2)\frac{\mathrm{disc}_{(p+q)^2}T(p,q)}{(p+q)^2}$

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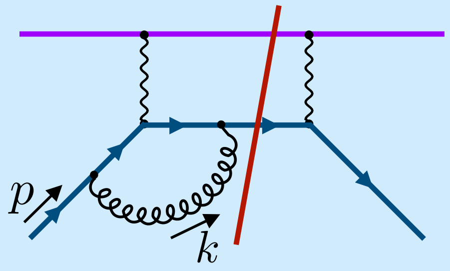
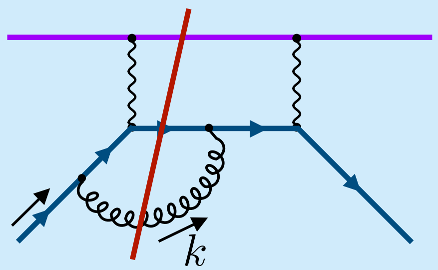
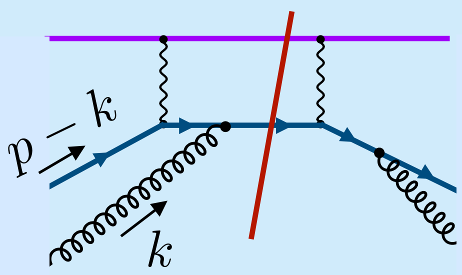
	$\delta^+(p^2)\delta^+((p+q)^2)\int \frac{\mathrm{d}^d k}{(2\pi)^d}\frac{1}{k^2(p-k)^2(p+q-k)^2}$	$\delta^+(p^2)\delta^+((p+q)^2)T(p,q)$
	$\delta^+(p^2)\delta^+((p+q)^2)\int \frac{\mathrm{d}^d k}{(2\pi)^d}\frac{\delta^-(k)\delta^+(p+q-k)}{(p-k)^2}$	$\delta^+(p^2)\frac{\mathrm{disc}_{(p+q)^2}T(p,q)}{(p+q)^2}$
	$\frac{\delta^+((p+q)^2)}{p^2}\int \frac{\mathrm{d}^d k}{(2\pi)^d}\frac{\delta^-(k)\delta^+(p-k)}{(p+q-k)^2}$	$\delta^+((p+q)^2)\frac{\mathrm{disc}_{p^2}T(p,q)}{p^2}$

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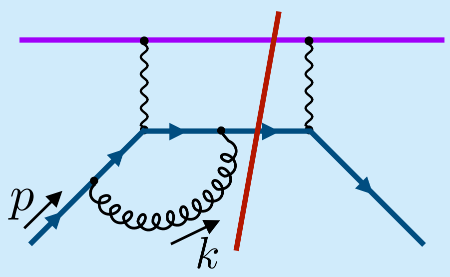
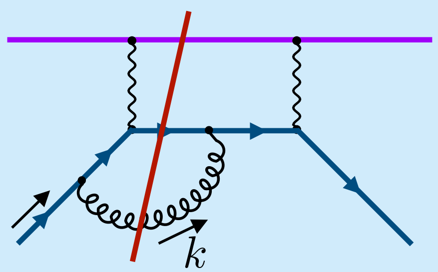
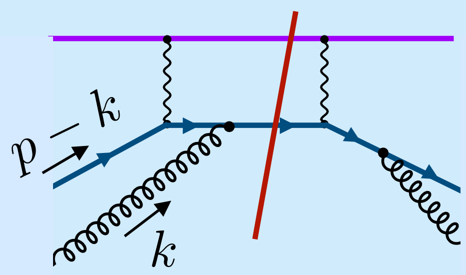
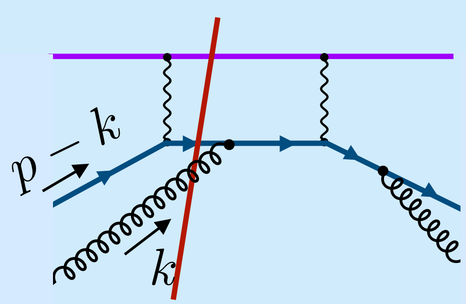
	$\delta^+(p^2)\delta^+((p+q)^2)\int \frac{\mathrm{d}^d k}{(2\pi)^d}\frac{1}{k^2(p-k)^2(p+q-k)^2}$	$\delta^+(p^2)\delta^+((p+q)^2)T(p,q)$
	$\delta^+(p^2)\delta^+((p+q)^2)\int \frac{\mathrm{d}^d k}{(2\pi)^d}\frac{\delta^-(k)\delta^+(p+q-k)}{(p-k)^2}$	$\delta^+(p^2)\frac{\mathrm{disc}_{(p+q)^2}T(p,q)}{(p+q)^2}$
	$\frac{\delta^+((p+q)^2)}{p^2}\int \frac{\mathrm{d}^d k}{(2\pi)^d}\frac{\delta^-(k)\delta^+(p-k)}{(p+q-k)^2}$	$\delta^+((p+q)^2)\frac{\mathrm{disc}_{p^2}T(p,q)}{p^2}$
	$\frac{1}{p^2(p+q)^2}\int \frac{\mathrm{d}^d k}{(2\pi)^d}\delta^-(k)\delta^+(p-k)\delta^+(p+q-k)$	$\frac{\mathrm{disc}_{p^2}\mathrm{disc}_{(p+q)^2}T(p,q)}{p^2(p+q)^2}$

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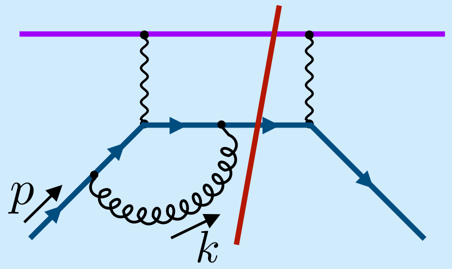
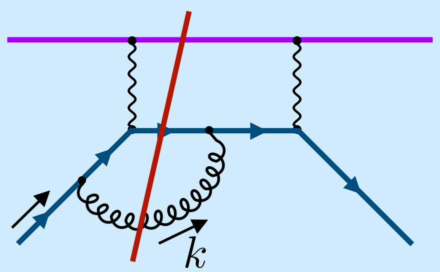
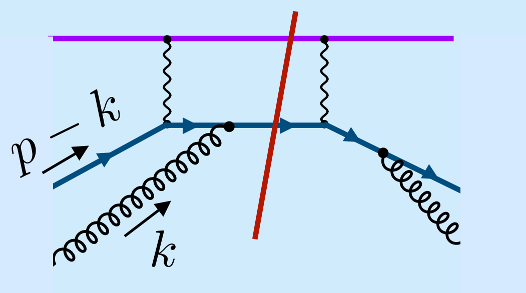
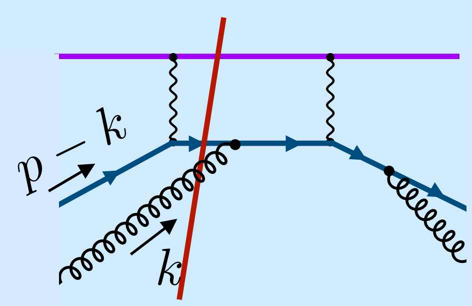
	$\delta^+(p^2)\delta^+((p+q)^2)\int \frac{\mathrm{d}^d k}{(2\pi)^d}\frac{1}{k^2(p-k)^2(p+q-k)^2}$	$\delta^+(p^2)\delta^+((p+q)^2)T(p,q)$
	$\delta^+(p^2)\delta^+((p+q)^2)\int \frac{\mathrm{d}^d k}{(2\pi)^d}\frac{\delta^-(k)\delta^+(p+q-k)}{(p-k)^2}$	$\delta^+(p^2)\frac{\mathrm{disc}_{(p+q)^2}T(p,q)}{(p+q)^2}$
	$\frac{\delta^+((p+q)^2)}{p^2}\int \frac{\mathrm{d}^d k}{(2\pi)^d}\frac{\delta^-(k)\delta^+(p-k)}{(p+q-k)^2}$	$\delta^+((p+q)^2)\frac{\mathrm{disc}_{p^2}T(p,q)}{p^2}$
	$\frac{1}{p^2(p+q)^2}\int \frac{\mathrm{d}^d k}{(2\pi)^d}\delta^-(k)\delta^+(p-k)\delta^+(p+q-k)$	$\frac{\mathrm{disc}_{p^2}\mathrm{disc}_{(p+q)^2}T(p,q)}{p^2(p+q)^2}$

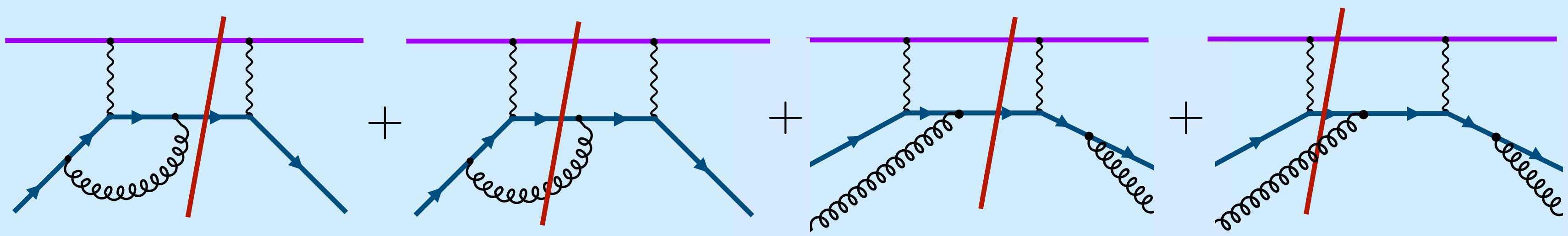
“Easy” integrals! Computed by Reverse Unitarity! That’s why “simplest clustering”.

C. Anastasiou, K. Melnikov, F. Petriello (2003)

C. Anastasiou, C. Duhr, F. Dulat, F. Herzog, B. Mistlberger (2015)

Each integral is badly divergent…

Turns out that we can write these contributions very compactly



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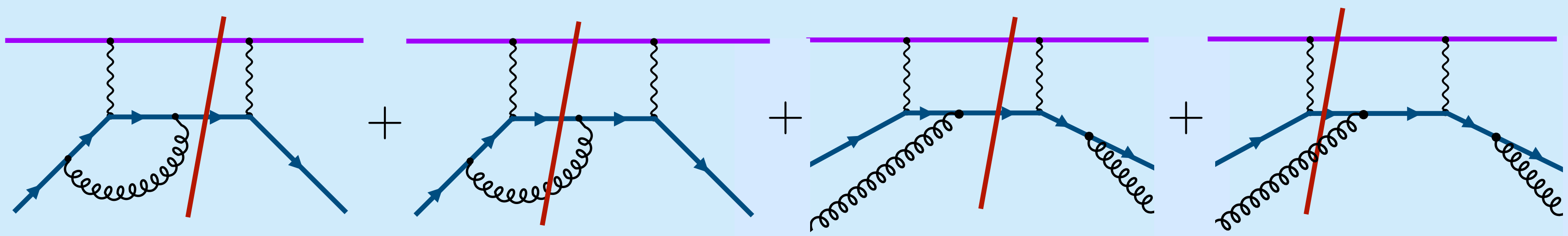

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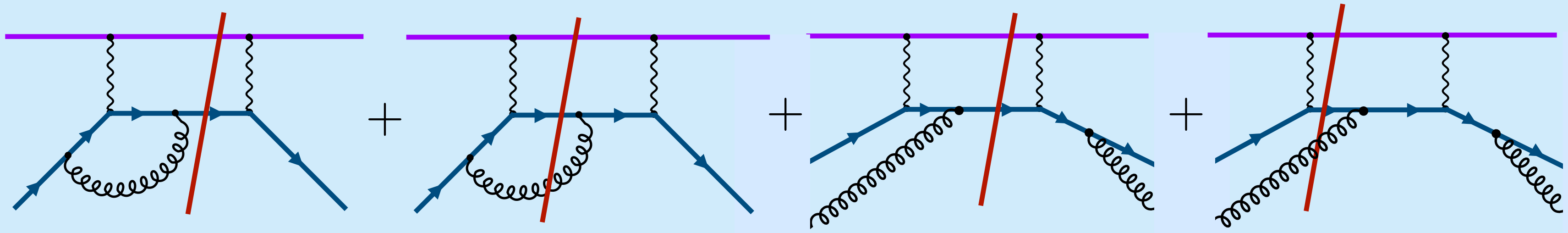

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This provides a generalised optical theorem!

Interference diagrams $\xleftarrow{\mathrm{disc}_{(p+q)^2}}$ Forward-scattering diagrams $\xleftarrow{\mathrm{disc}_{p^2}}$ …?

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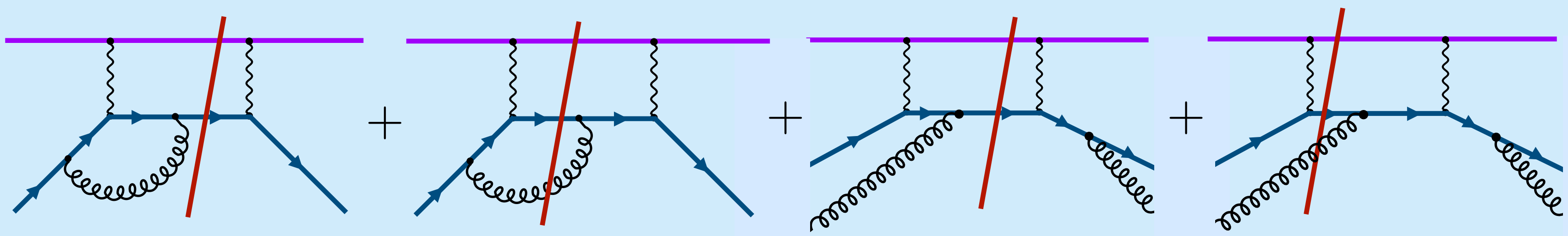

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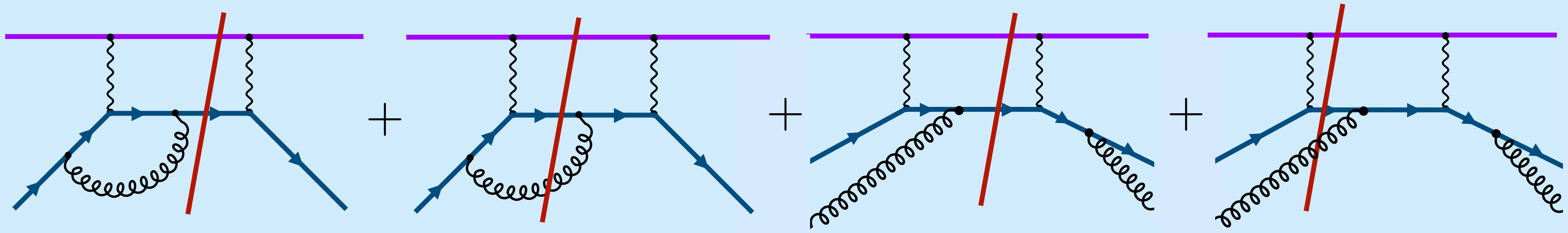

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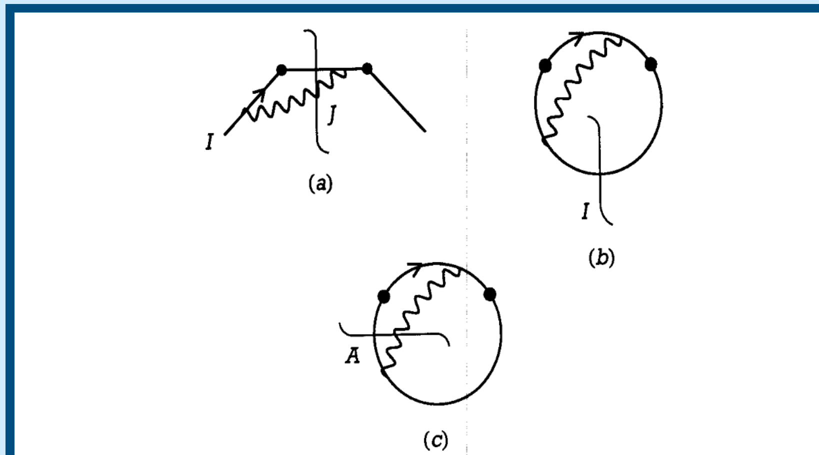
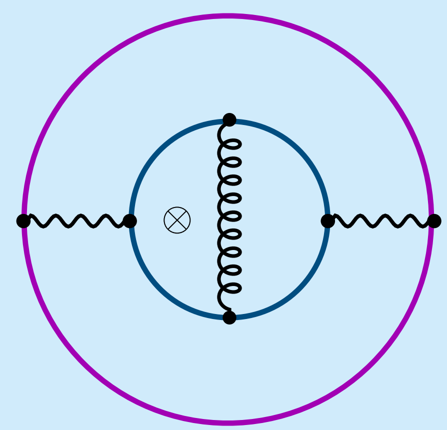


Fig. 13.16 (*a*) Deeply inelastic scattering diagram (leptonic part omitted); (*b*) circular diagram found from (*a*); (*c*) related circular diagram.

G. Sterman
“An Introduction to Quantum field theory” (1993)

Embedding of vacuum diagrams in punctured space ($\otimes$)



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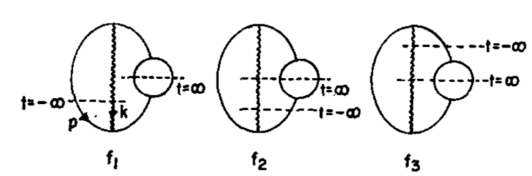
Fig. 15. Additional double cut diagrams which are introduced to take account of the degeneracy of the initial state.

T. Kinoshita
“Mass singularities of Feynman amplitudes” (1962)

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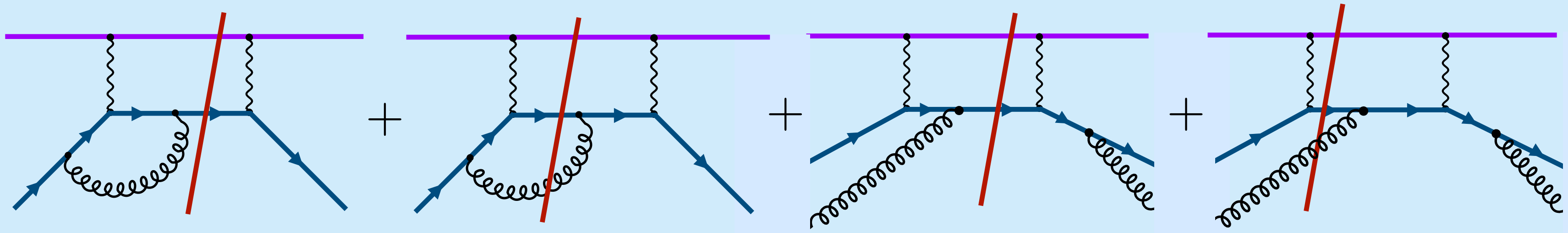

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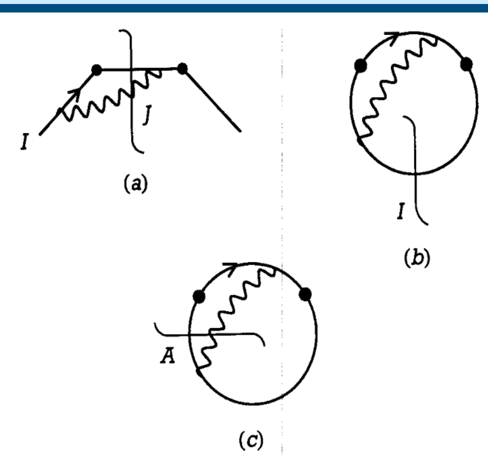

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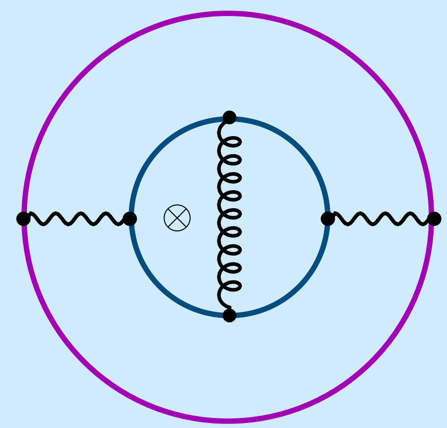

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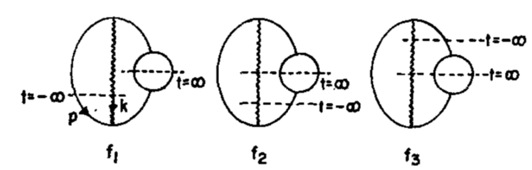

FIG. 15. Additional double cut diagrams which are introduced to take account of the degeneracy of the initial state.

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The embedding allows us to organise the zoo of topologies we get into IR-finite classes!

The cross-section is obtained by integrating over the invariant mass of the cluster

$$\sigma_{\triangle} = \int_0^{\Lambda^2} \frac{\mathrm{d}p^2}{8\pi^2} \mathrm{disc}_{p^2} \mathrm{disc}_{(p+q)^2} \frac{T(p, q, -p-q)}{p^2 q^2}$$

To see that it is IR-finite we can work at the level of cuts using standard KLN proofs…

Capatti, Hirschi, Pelloni, Ruijl (2020)

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$$\sigma_{\triangle} = \underbrace{a_0 \log(\Lambda^2) + a_1}_{\text{Leading Power} \sim \text{Parton Model}} + \overbrace{a_2 \Lambda^2 \log(\Lambda^2) + \ldots}^{\text{Power corrections} \sim \text{MPI+…}}$$

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Leading Power ~ Parton Model

To obtain the leading power result we can use the expansion-by-regions method:

$$\frac{T(p^2,q^2,(p+q)^2)}{p^2q^2} = \frac{T_h(q^2,(p+q)^2)}{p^2(p+q)^2} + \frac{T_c(q^2,(p+q)^2)}{(p^2)^{1+\epsilon}(p+q)^2} + \frac{T_{\bar{c}}(q^2,(p+q)^2)}{p^2((p+q)^2)^{1+\epsilon}} + \frac{T_s(q^2,(p+q)^2)}{(p^2)^{1+\epsilon}((p+q)^2)^{1+\epsilon}} + \mathcal{O}((p^2)^{-1})$$

$$\sigma_{\triangle} = \Big[\delta((p+q)^2)T_h((p+q)^2,q^2) + \sin(\pi\epsilon)((p+q)^2)^{-1-\epsilon}T_c((p+q)^2,q^2)\Big]$$

$$+(\Lambda^2)^{-\epsilon}\Big[\delta((p+q)^2)\sin(\pi\epsilon)T_{\bar{c}}((p+q)^2,q^2) + \sin(\pi\epsilon)^2((p+q)^2)^{-1-\epsilon}T_s((p+q)^2,q^2)\Big] + \mathcal{O}(\Lambda^2\log(\Lambda^2))$$

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We observe that there is a close relation between regions and interference diagrams…

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Capatti, Hirschi, Pelloni, Ruijl (2020)

However another proof technique arises if we now add the virtuality expansion:

$$\sigma_\triangle = \underbrace{a_0 \log(\Lambda^2) + a_1}_{\text{Leading Power} \sim \text{Parton Model}} + \overbrace{a_2 \Lambda^2 \log(\Lambda^2) + \ldots}^{\text{Power corrections} \sim \text{MPI}+\ldots}$$

To obtain the leading power result we can use the expansion-by-regions method:

$$\frac{T(p^2,q^2,(p+q)^2)}{p^2q^2} = \frac{T_h(q^2,(p+q)^2)}{p^2(p+q)^2} + \frac{T_c(q^2,(p+q)^2)}{(p^2)^{1+\epsilon}(p+q)^2} + \frac{T_{\bar{c}}(q^2,(p+q)^2)}{p^2((p+q)^2)^{1+\epsilon}} + \frac{T_s(q^2,(p+q)^2)}{(p^2)^{1+\epsilon}((p+q)^2)^{1+\epsilon}} + \mathcal{O}((p^2)^{-1})$$

$$\sigma_\triangle = \Big[\delta((p+q)^2)T_h((p+q)^2,q^2) + \sin(\pi\epsilon)((p+q)^2)^{-1-\epsilon}T_c((p+q)^2,q^2)\Big]$$
$$+(\Lambda^2)^{-\epsilon}\Big[\delta((p+q)^2)\sin(\pi\epsilon)T_{\bar{c}}((p+q)^2,q^2) + \sin(\pi\epsilon)^2((p+q)^2)^{-1-\epsilon}T_s((p+q)^2,q^2)\Big] + \mathcal{O}(\Lambda^2\log(\Lambda^2))$$

We observe that there is a close relation between regions and interference diagrams…

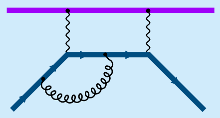
The KLN theorem is here a deformed version of the pole cancellation mechanism between regions!

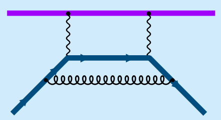
Results: NLO DIS

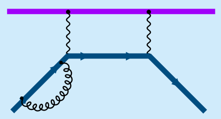
Results: NLO DIS

In summary:

$$F_{i,q} = \int_0^{\Lambda^2} \mathrm{d}p^2 P^{(i)}_{\mu\nu} \mathrm{disc}_{p^2} \mathrm{disc}_{(p+q)^2} [I^{\mu\nu}_{\triangle} + I^{\mu\nu}_{\square} + I^{\mu\nu}_{\circ}] \begin{cases} I^{\mu\nu}_{\triangle} = \dfrac{1}{p^2(p+q)^2} \displaystyle\int \dfrac{\mathrm{d}^d k}{(2\pi)^d} \dfrac{\mathcal{N}^{\mu\nu}_{\triangle}(k,p,q)}{k^2(p-k)^2(p+q-k)^2} \\ I^{\mu\nu}_{\square} = \dfrac{1}{p^2} \displaystyle\int \dfrac{\mathrm{d}^d k}{(2\pi)^d} \dfrac{\mathcal{N}^{\mu\nu}_{\square}(k,p,q)}{k^2[(p-k)^2]^2(p+q-k)^2} \\ I^{\mu\nu}_{\circ} = \dfrac{1}{(p^2)^2} \displaystyle\int \dfrac{\mathrm{d}^d k}{(2\pi)^d} \dfrac{\mathcal{N}^{\mu\nu}_{\circ}(k,p,q)}{k^2(p-k)^2(p+q)^2} \end{cases}$$







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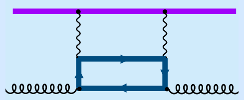

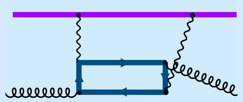

Results: NLO DIS

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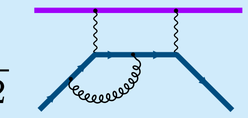
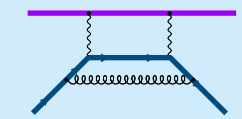
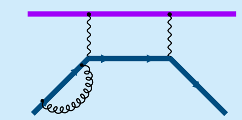

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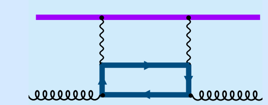
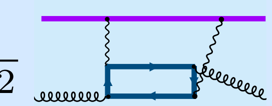

Computing the leading behaviour in Λ^2

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$$F_{Lq} = e_q^2 \frac{\alpha_s}{2\pi} x \left[2C_F x\right]$$

$$\left(+\mathcal{O}(\Lambda^2/Q^2)\right)$$

Results: NLO DIS

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$$F_{i,q} = \int_0^{\Lambda^2} \mathrm{d}p^2 P_{\mu\nu}^{(i)} \mathrm{disc}_{p^2} \mathrm{disc}_{(p+q)^2} [I_\triangle^{\mu\nu} + I_\square^{\mu\nu} + I_\circ^{\mu\nu}]$$

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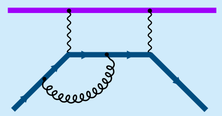

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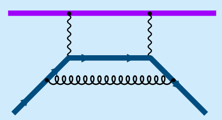

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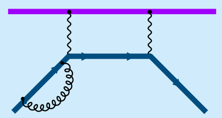

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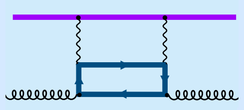

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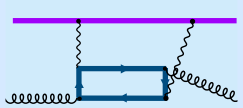

Computing the leading behaviour in Λ^2

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Everything is IR finite!

Results: NLO DIS

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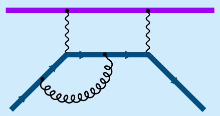
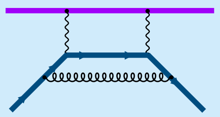
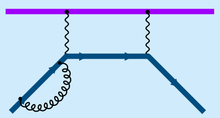

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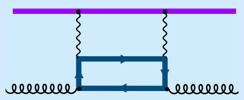
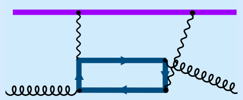

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Everything is IR finite!

Longitudinal structure functions and scale evolution are the same as in the parton model

Results: NLO DIS

In summary:

$$F_{i,q} = \int_0^{\Lambda^2} \mathrm{d}p^2 P^{(i)}_{\mu\nu} \mathrm{disc}_{p^2} \mathrm{disc}_{(p+q)^2} [I^{\mu\nu}_{\triangle} + I^{\mu\nu}_{\square} + I^{\mu\nu}_{\circ}]$$

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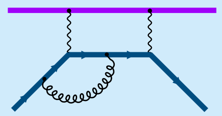

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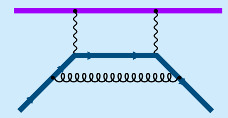

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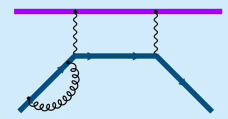

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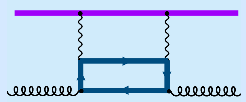

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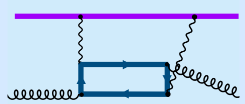

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Longitudinal structure functions and scale evolution are the same as in the parton model

Thus, there exists a scheme change mapping this result to the MSbar parton model result

Blowing-up PDF counter-terms

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Everything works as expected! Too nice to be a coincidence… this can happen only if

$$\sigma_{\mathrm{extra}} = \alpha_s \left(\frac{c_1}{\epsilon} + d_1 \right) \sigma(\mathbb{O}) + \mathcal{O}(\alpha_s)$$

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The extra diagrams "factorise exactly", i.e. they have no remainder. We can verify this:

Locally finite two-loop amplitudes for off-shell multi-photon production in electron-positron annihilation #7

Charalampos Anastasiou (Zurich, ETH), Rayan Haindl (Zurich, ETH), George Sterman (YITP, Stony Brook), Zhou Yang (Zurich, ETH), Mao Zeng (Zurich, ETH) (Aug 27, 2020)

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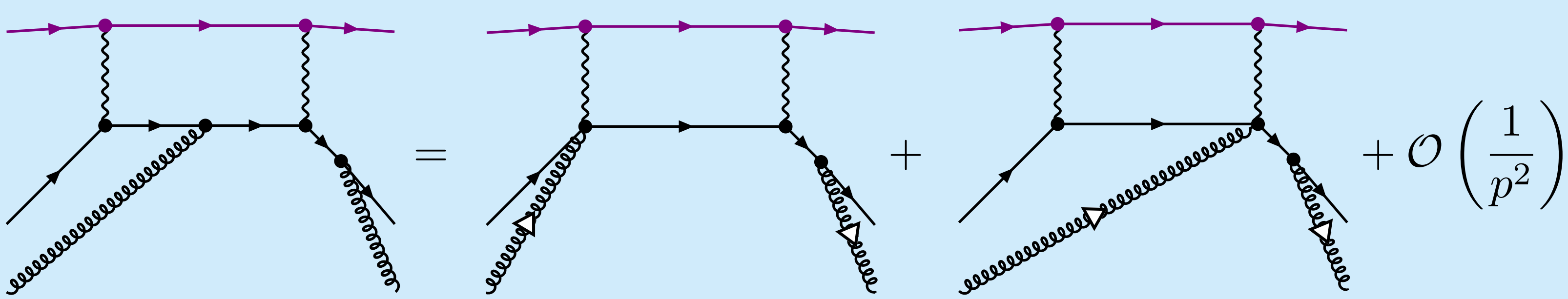

$$= \frac{\eta^\mu k^\nu}{k^2[(k+\eta)^2 - \eta^2]}$$

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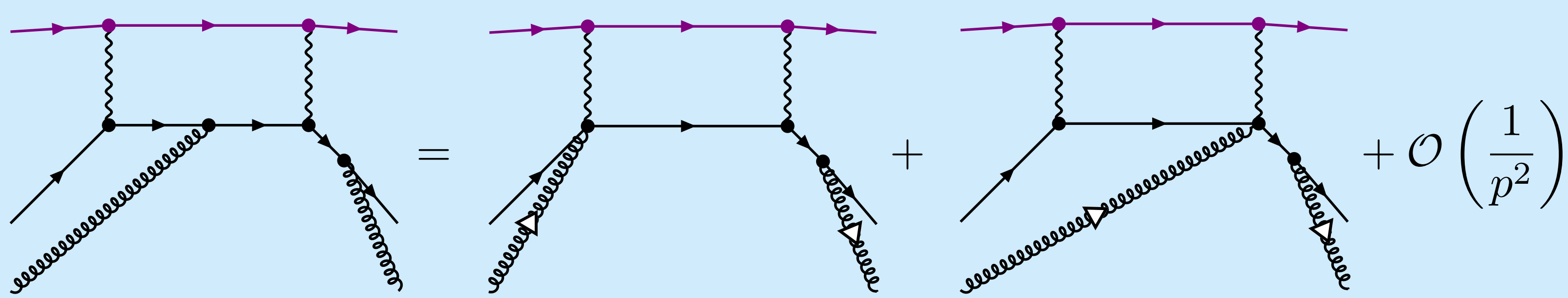
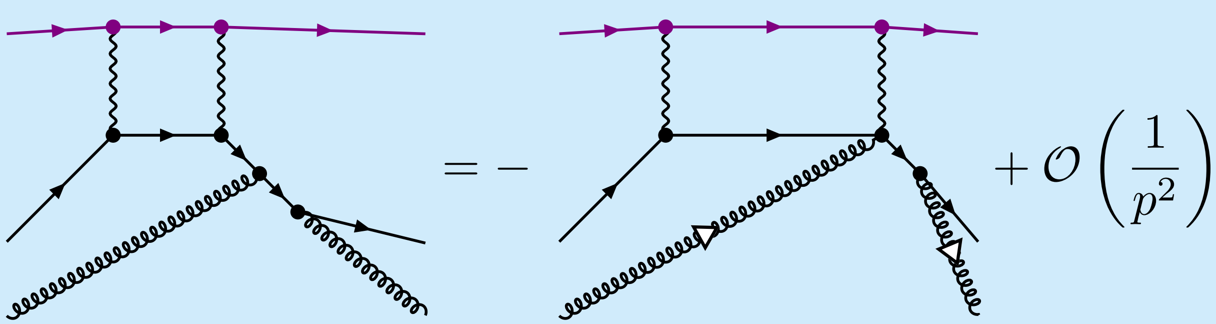
Charalampos Anastasiou (Zurich, ETH), Rayan Haindl (Zurich, ETH), George Sterman (YITP, Stony Brook), Zhou Yang (Zurich, ETH), Mao Zeng (Zurich, ETH) (Aug 27, 2020)



$$= \frac{\eta^\mu k^\nu}{k^2[(k+\eta)^2 - \eta^2]}$$



Blowing-up PDF counter-terms

Everything works as expected! Too nice to be a coincidence… this can happen only if

$$\sigma_{\text{extra}} = \alpha_s \left(\frac{c_1}{\epsilon} + d_1 \right) \sigma(\mathbb{O}) + \mathcal{O}(\alpha_s)$$

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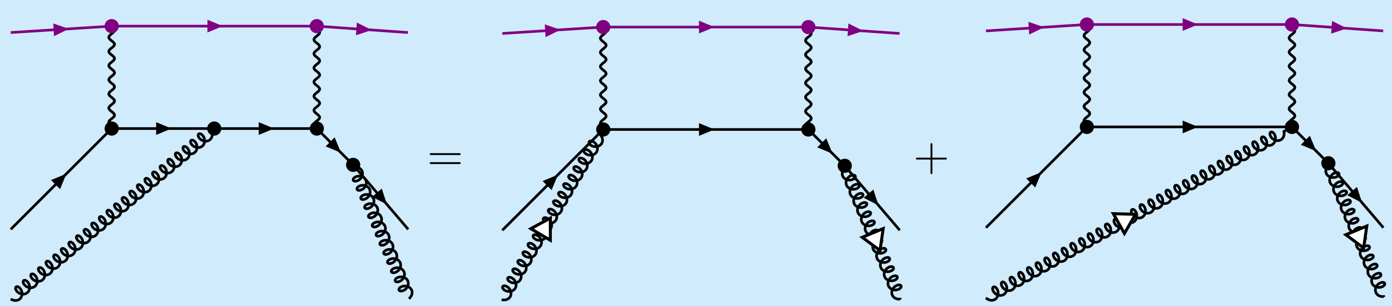

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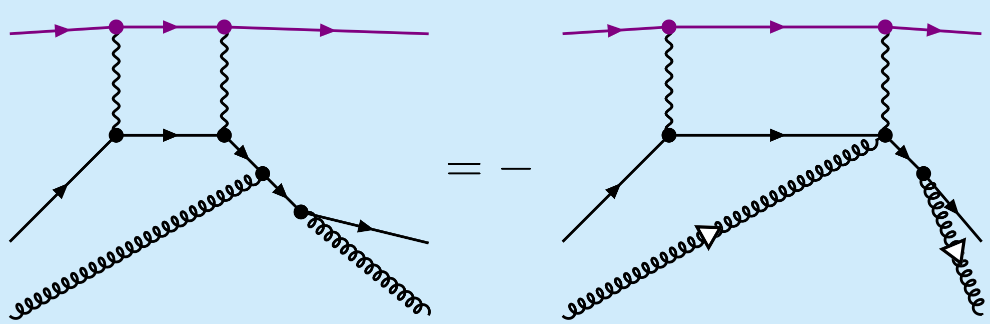

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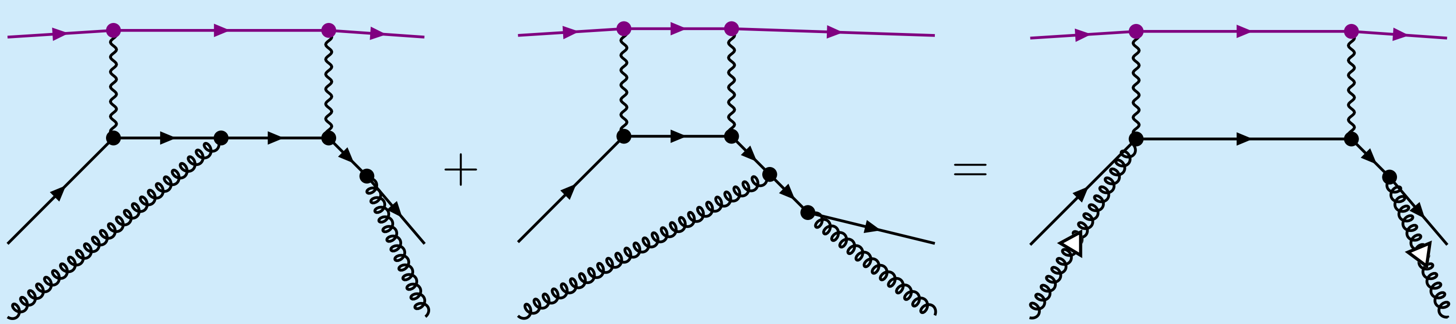

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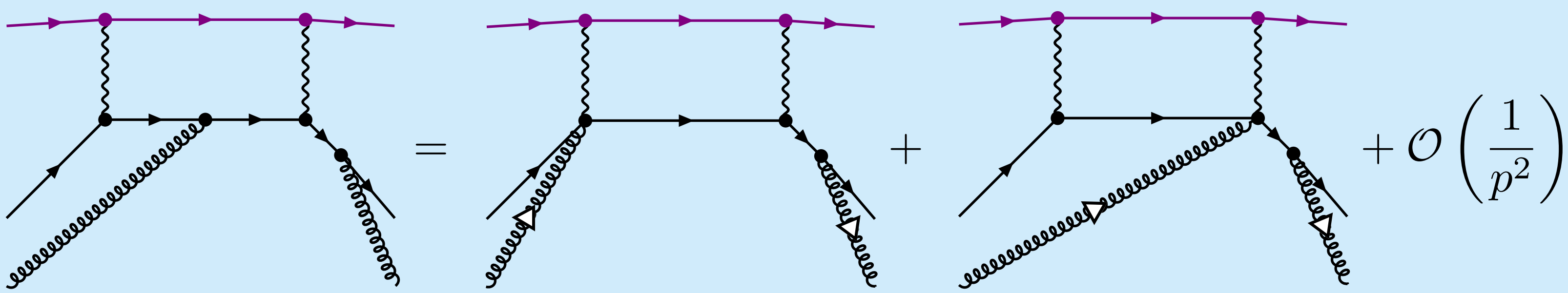

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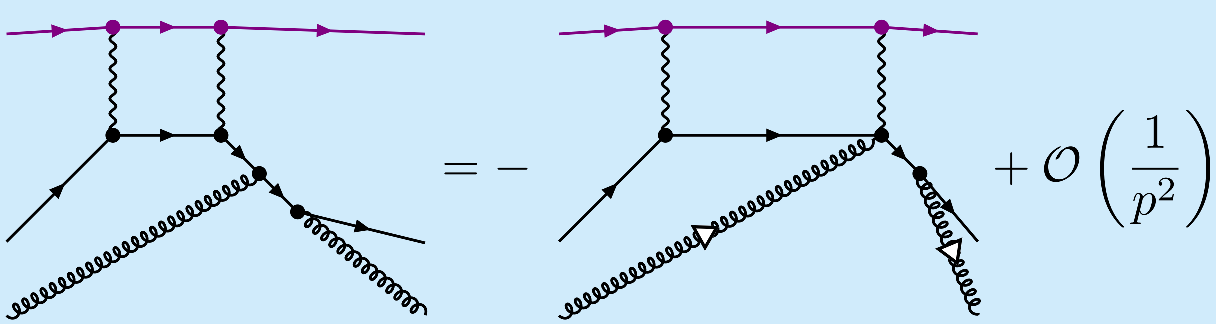
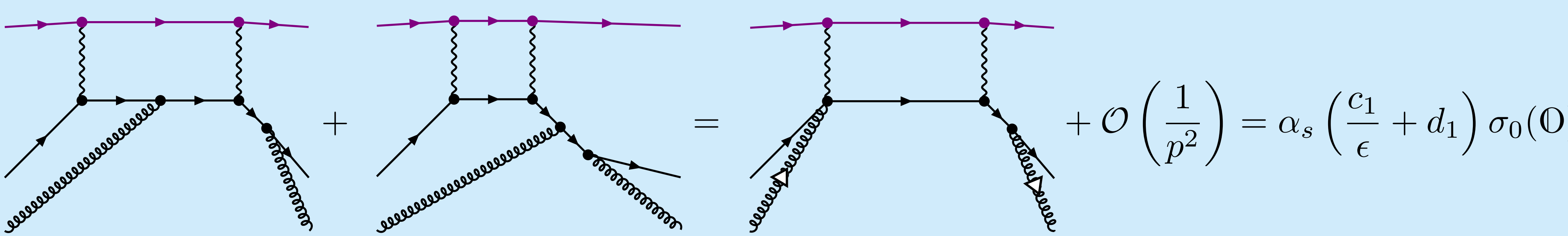


The extra diagrams, at leading virtuality, provide "blown-up PDF counter-terms", i.e. local representations of the PDF counter-terms in a certain scheme



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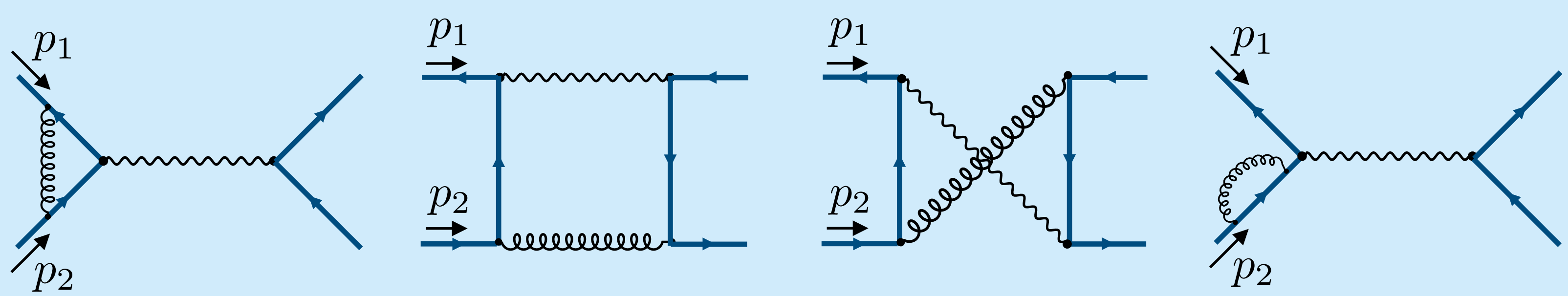

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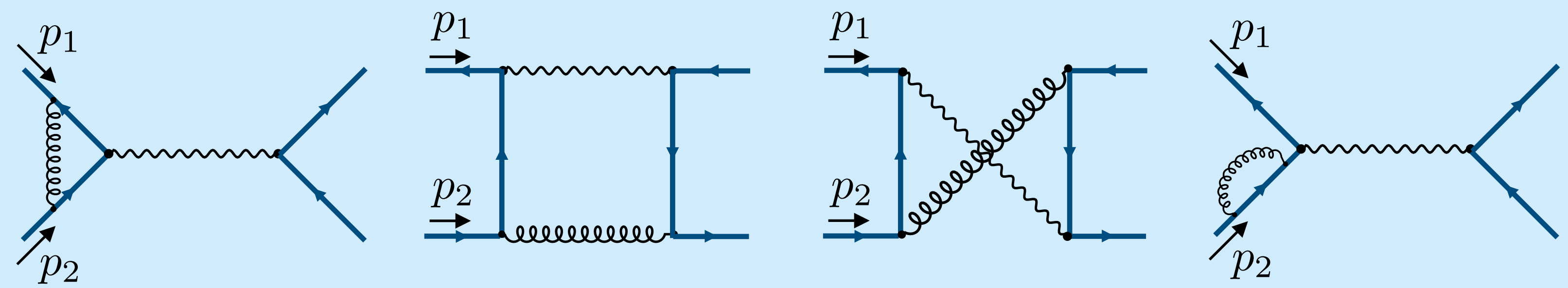

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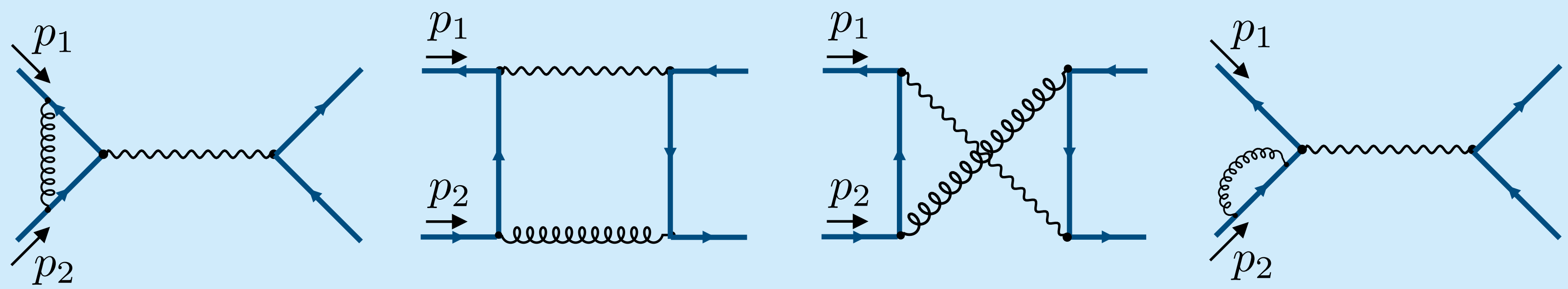

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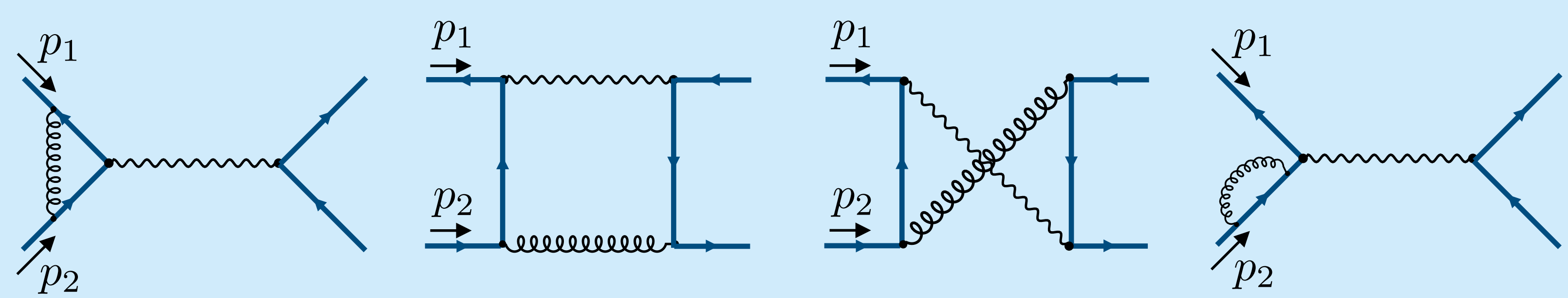

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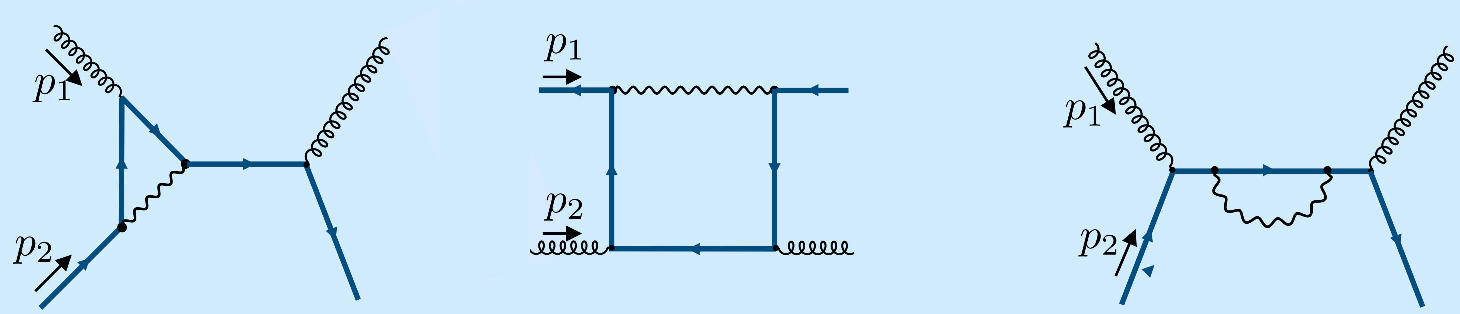

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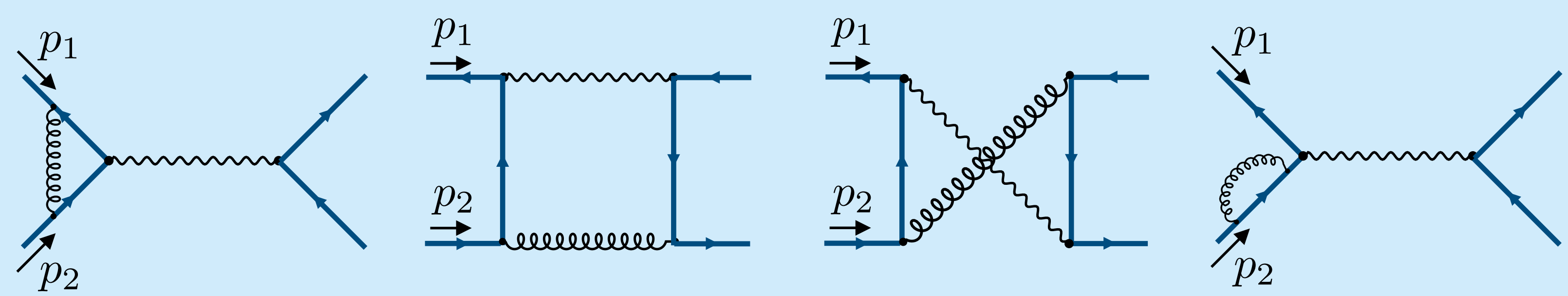

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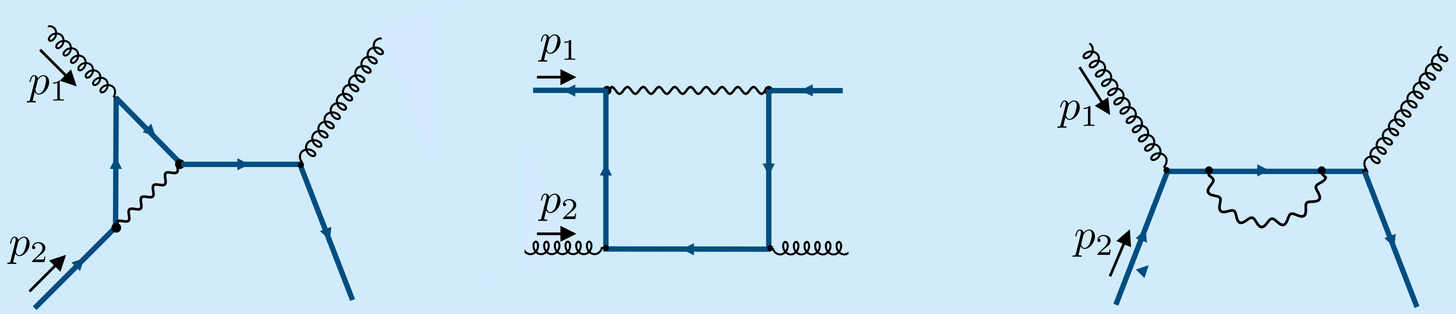

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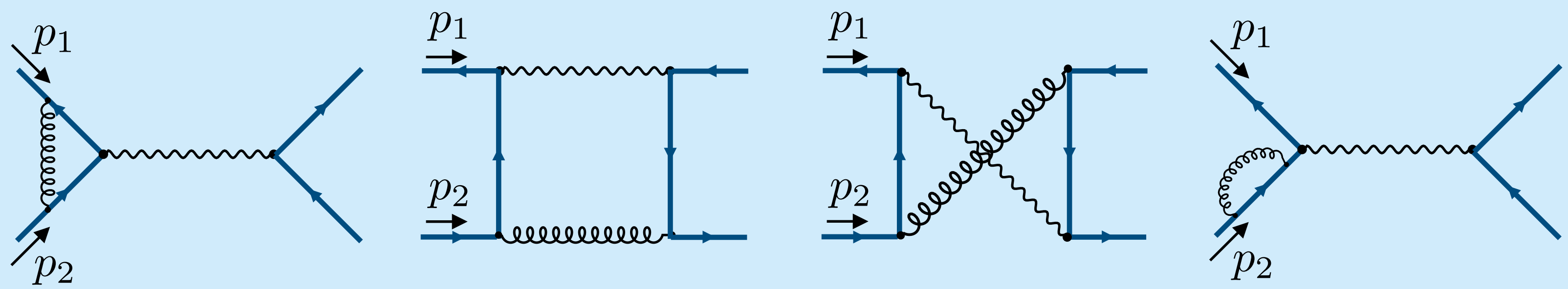

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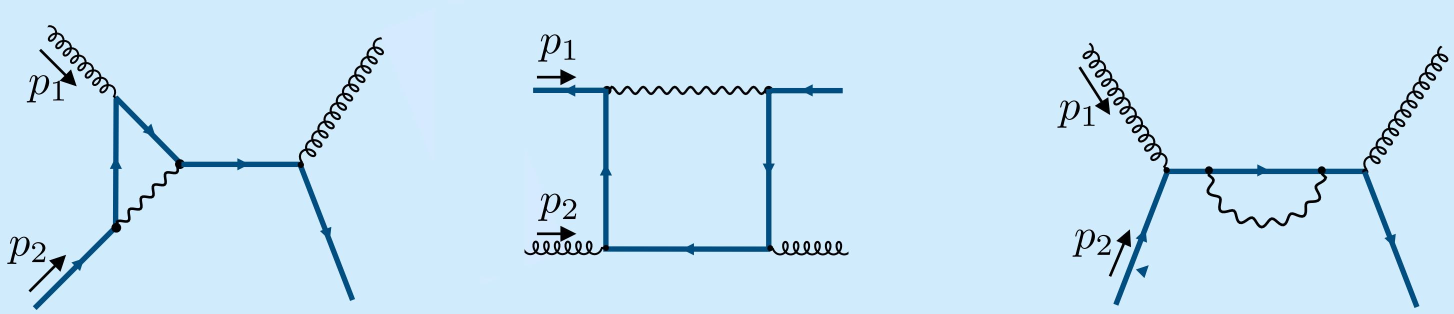

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Using this now known quantity, we **verify that**

$$\frac{\mathrm{d}\sigma_{qg}}{\mathrm{d}Q^2}(x_1,x_2) = D(\xi_1)\otimes_{\xi_1}\frac{\mathrm{d}\sigma_{qg}^{\overline{\mathrm{MS}}}}{\mathrm{d}Q^2}(x_1/\xi_1,x_2/\xi_2)\otimes_{\xi_2}D(\xi_2)$$

The same holds for quark-anti-quark contributions.

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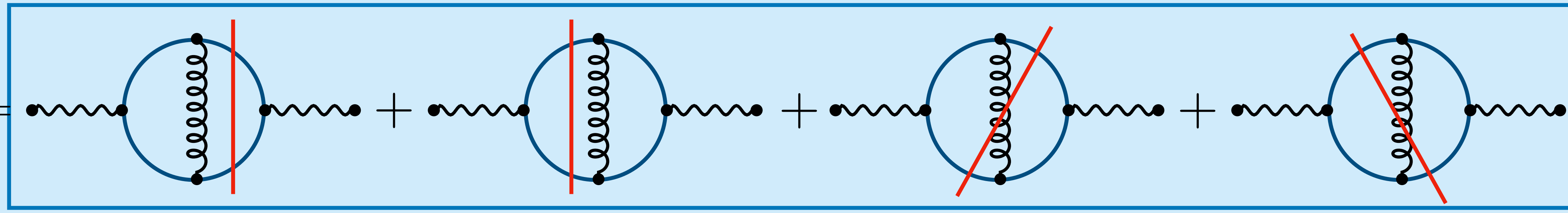

IR-finite

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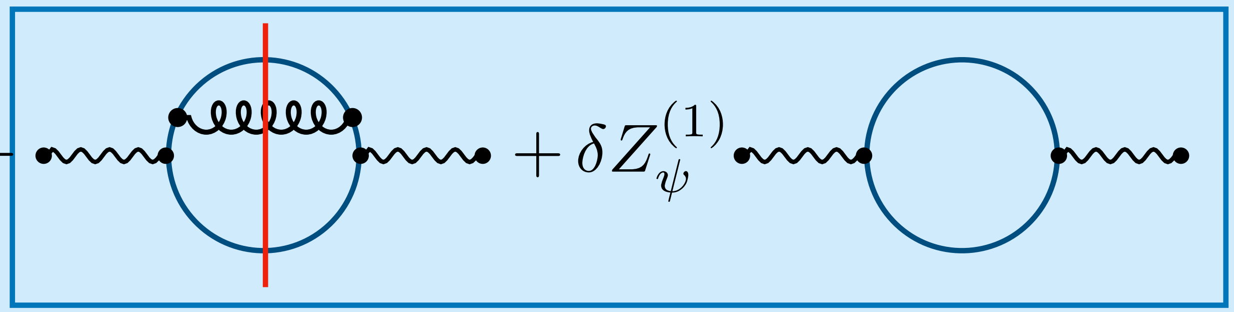

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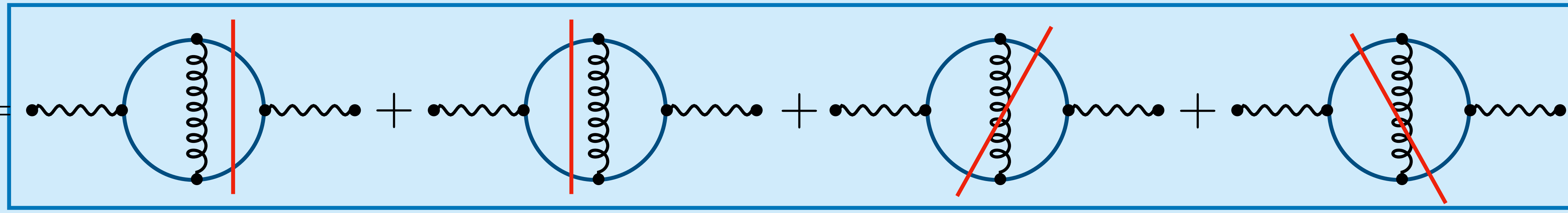

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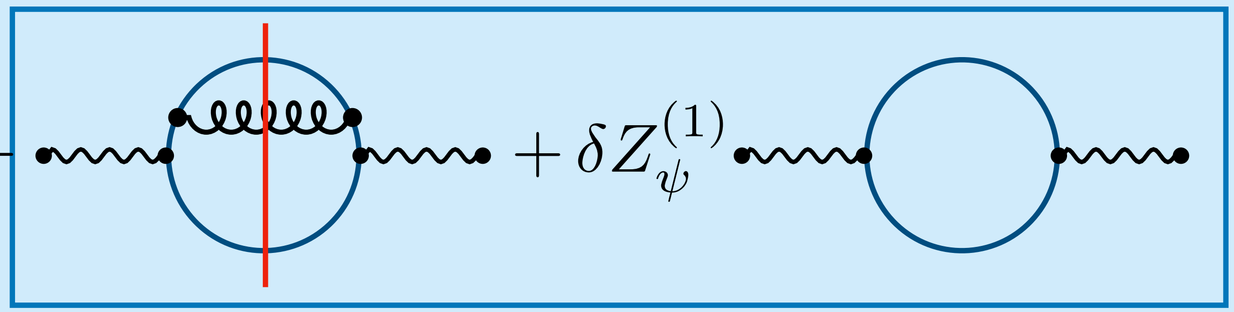

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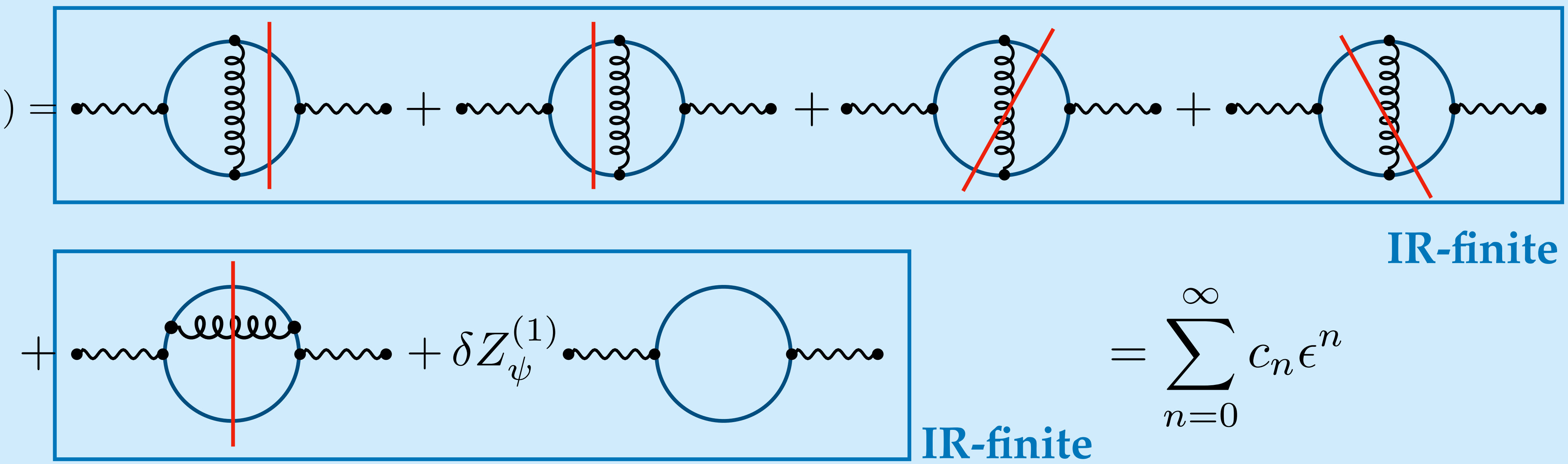

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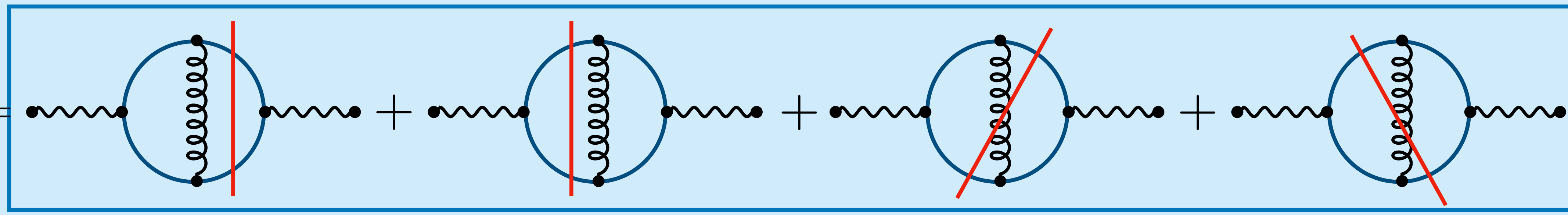

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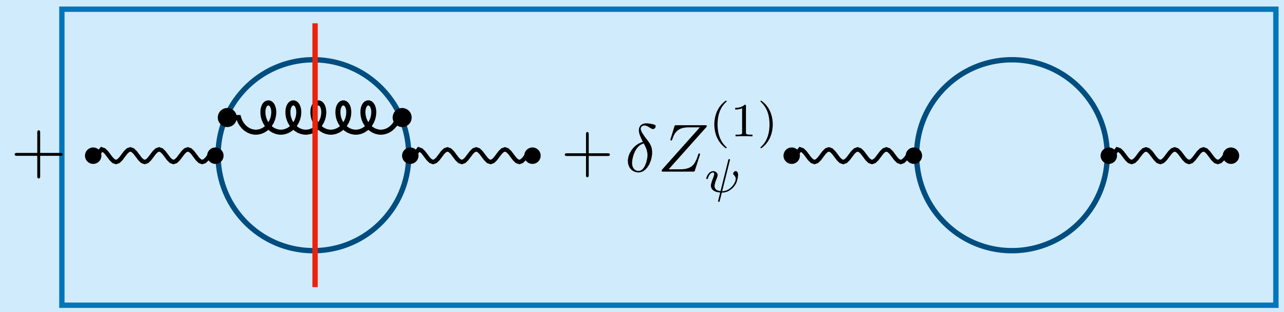

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This means we can obtain the integral by direct Monte Carlo integration!

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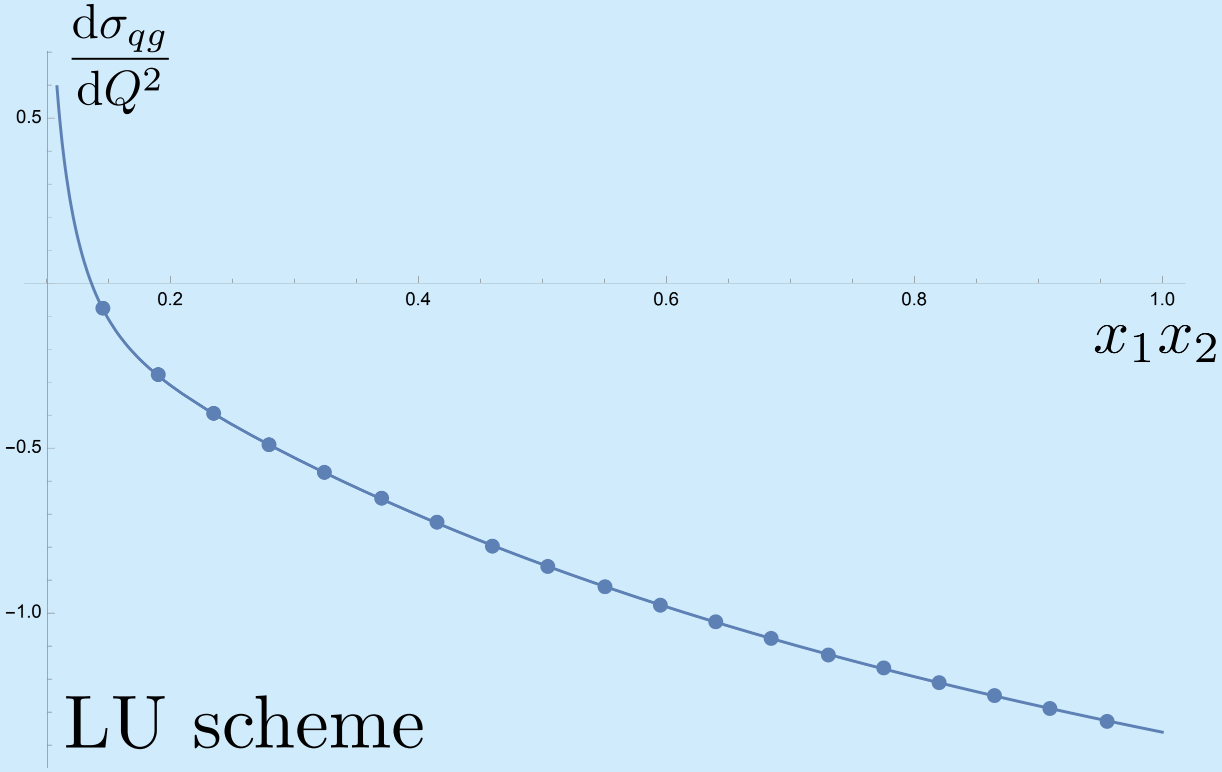
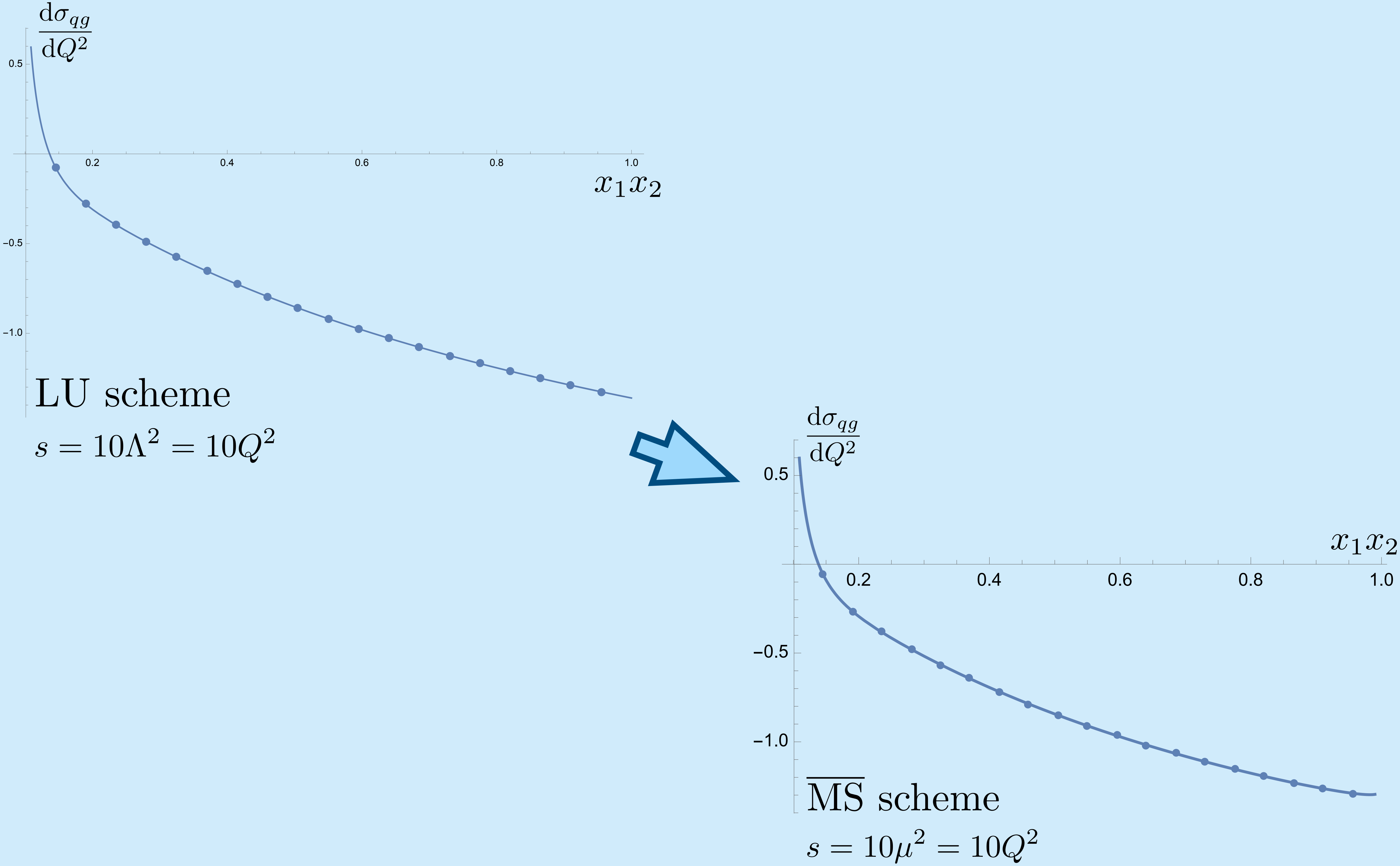


$s = 10\Lambda^2 = 10Q^2$

Local Unitarity for Hadronic Cross-sections

We can apply this technology to also exploit the pole cancellation mechanism that we constructed for Drell-Yan in the initial state:



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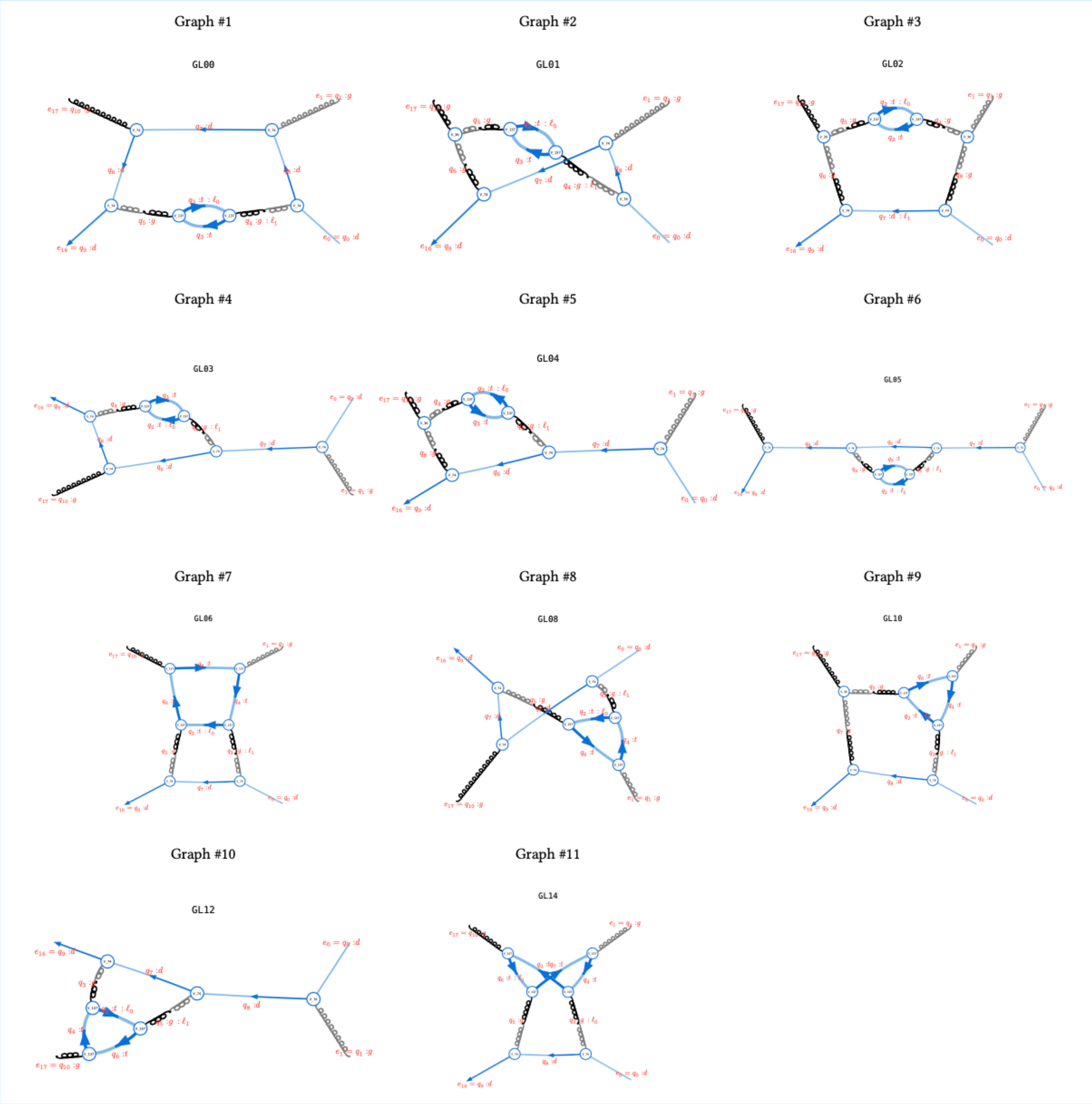

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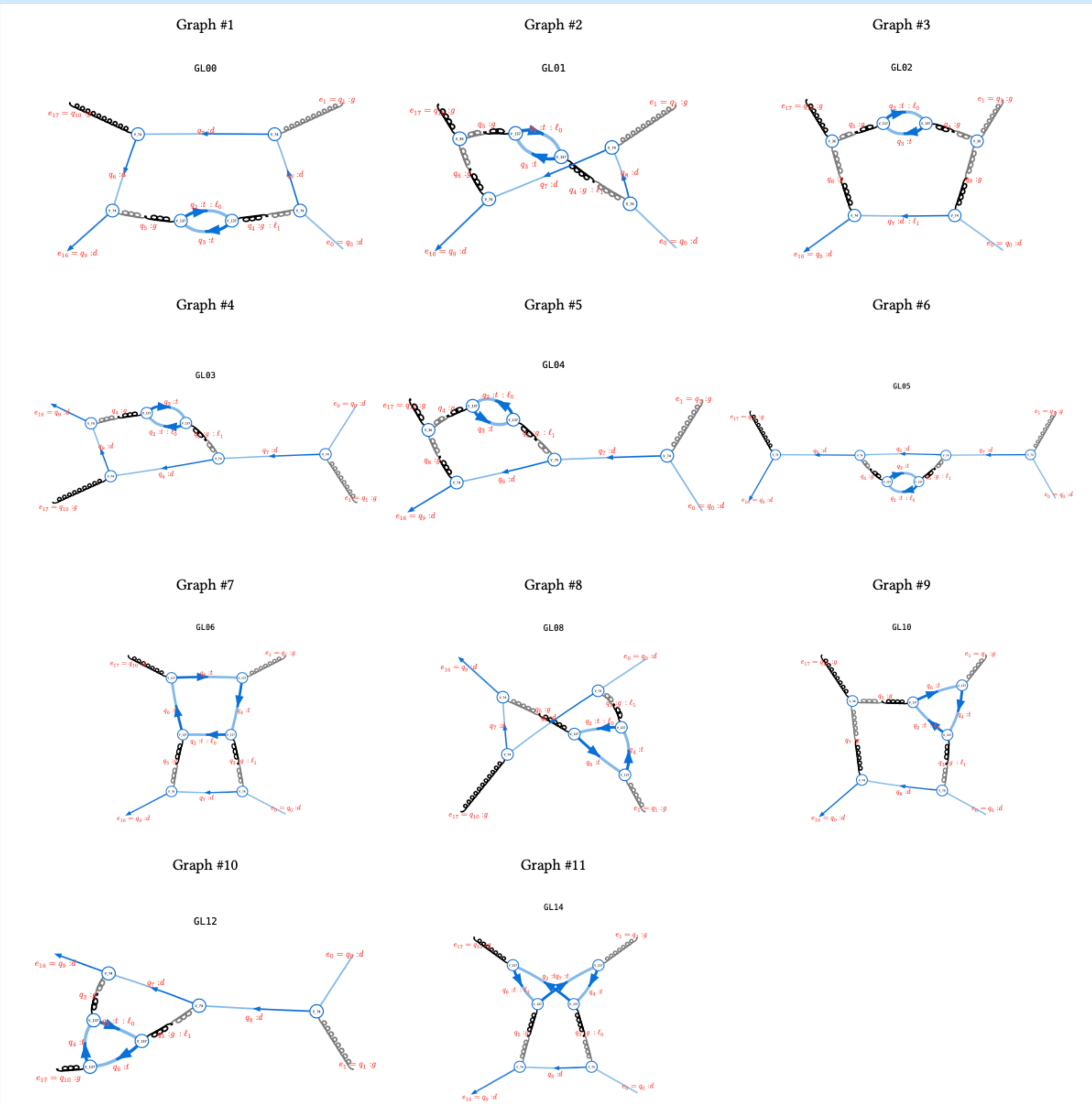

$$\Lambda_1 = \Lambda_2 = m_Z$$

$\sqrt{s}$[GeV]	$\sigma_{qg}[\text{GeV}^{-2}]$	$\Delta_{\text{bench}}/\sigma_{\text{bench}}$
600	$1.505(10) \cdot 10^{-9}$	0.0099
1000	$6.787(52) \cdot 10^{-9}$	0.0088
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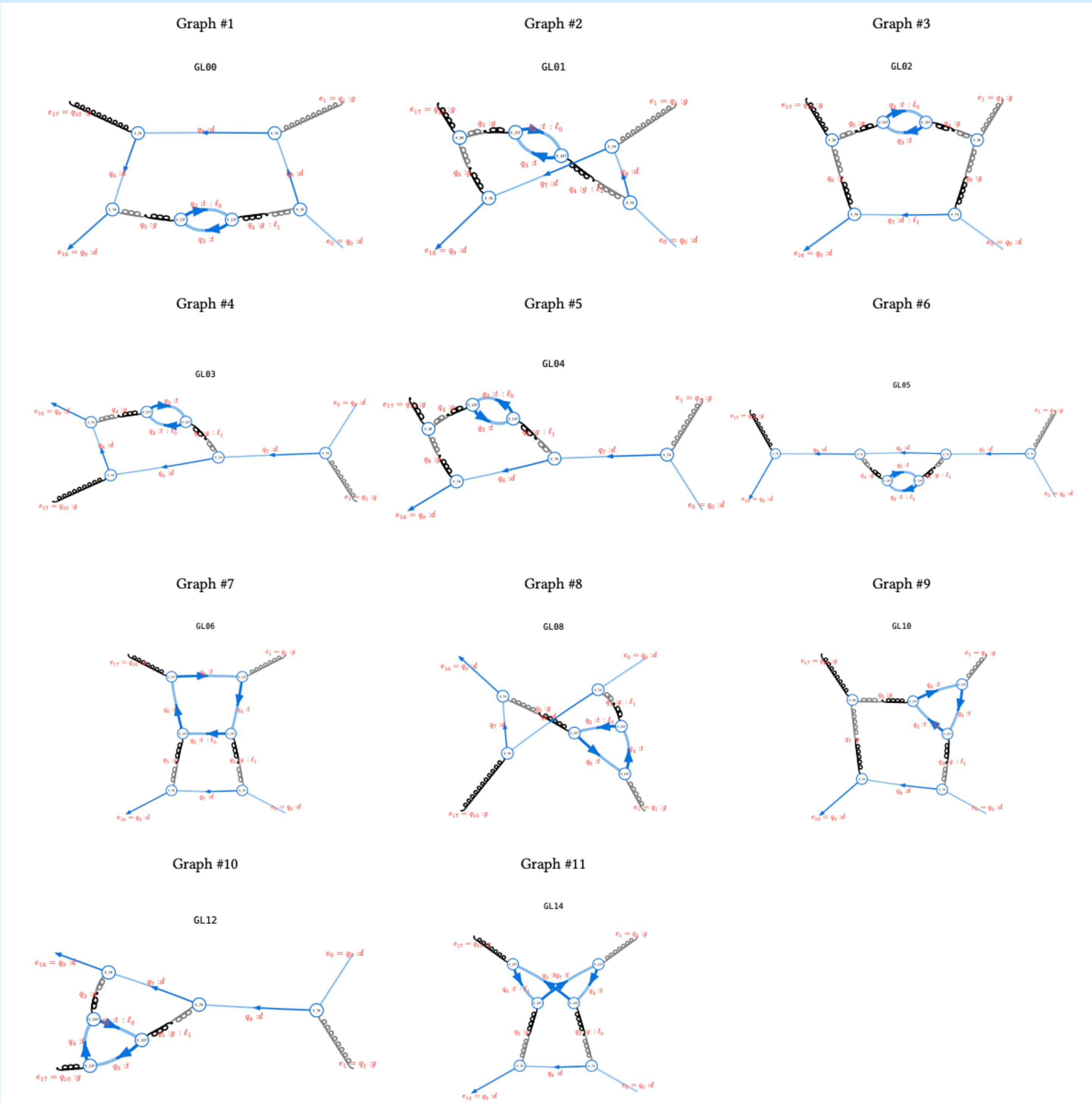

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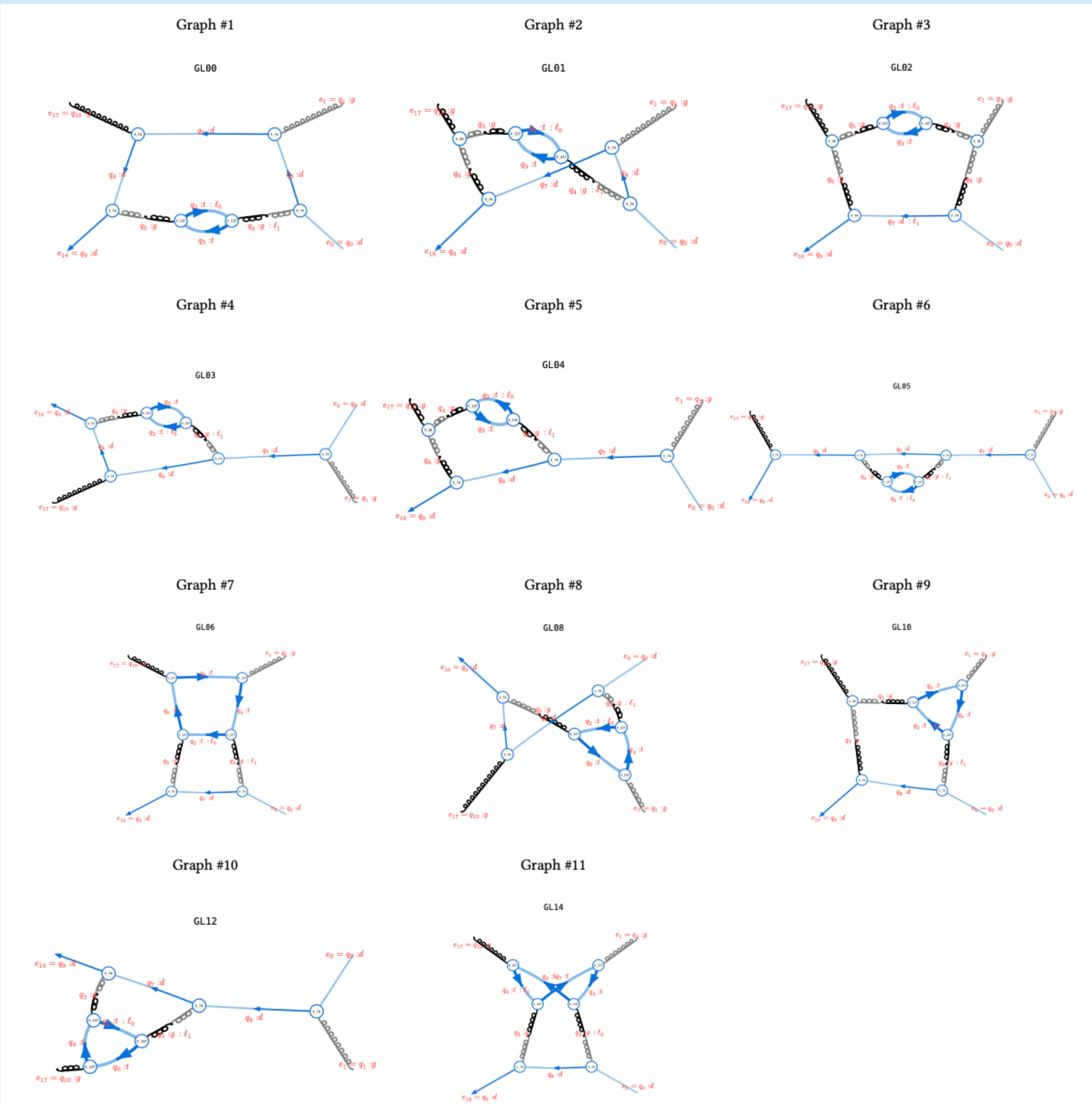

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Need to determine all remaining "splitting counter-terms":

$D_{gq}, \quad D_{qq}$ **DIS/DY**

$D_{qg}, \quad D_{gg}$ **Dedicated "AP" computation**

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- ★ Construct a MC method that integrates any NLO cross-section without the need for dimensional regularisation in either phase-space nor loops (barred DIS)
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Investigate power-corrections in Λ^2

★ Ideally one would start comparing with DIS, which has a well-defined OPE, and some work has been done on determining twist four operators, especially in the so-called handbag model

Jaffe, Soldate (1982)   Ellis, Furmanski, Petronzio (1983)   Qiu (1990)