Loops in Dense Fields: Perturbative QCD around classical color field

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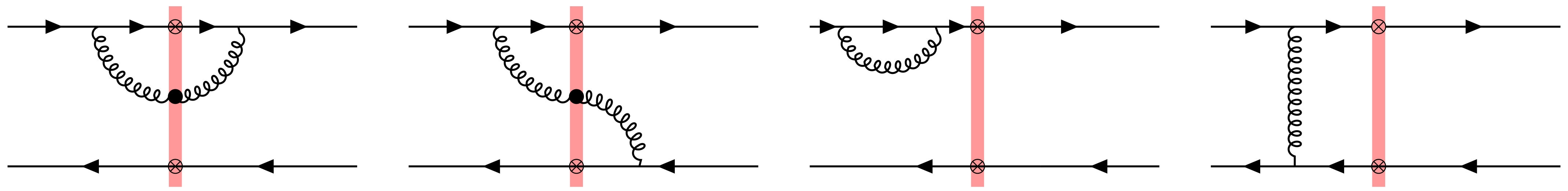
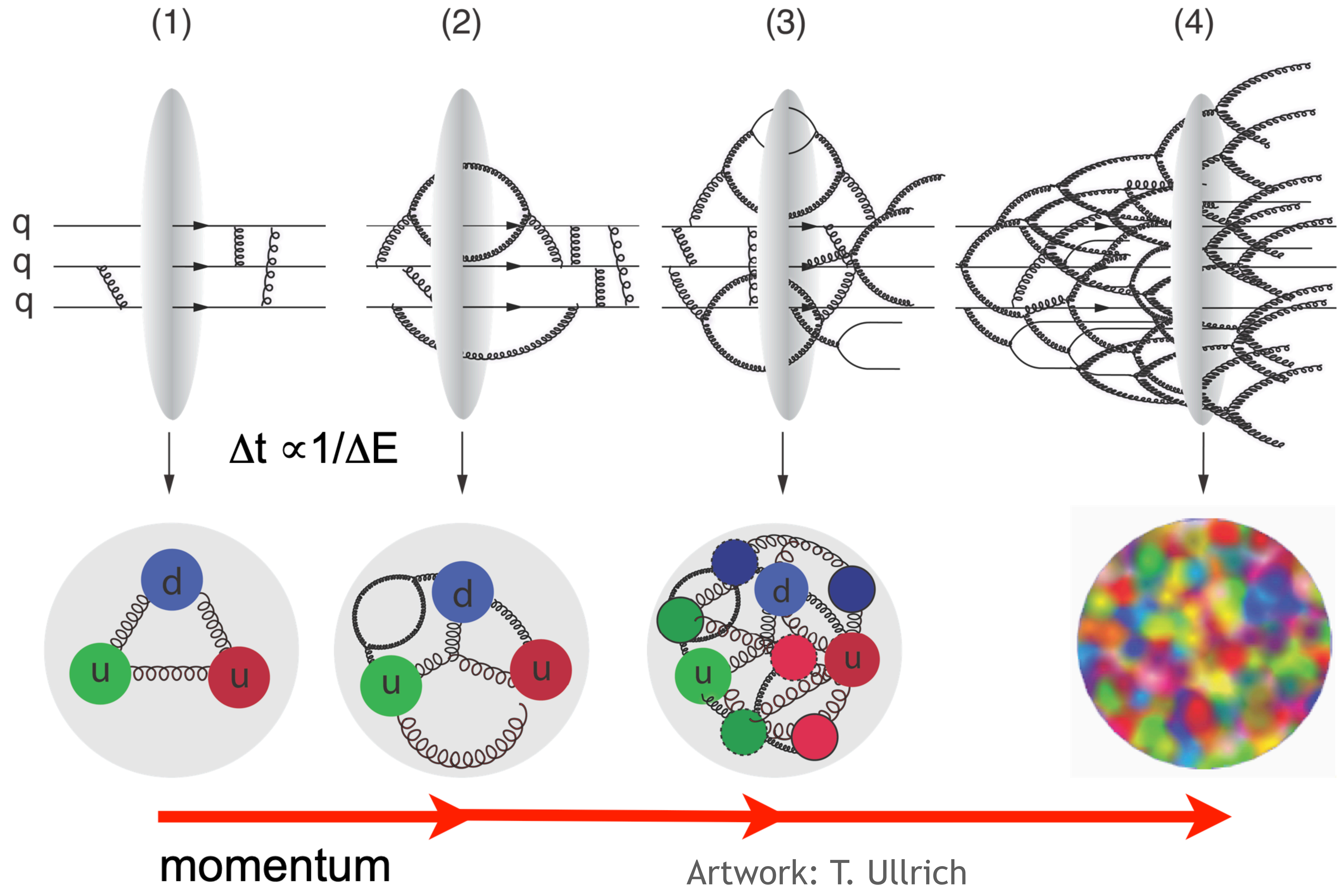


Loop Fest XXIV
May 28th, 2026

Anatomy of nuclear matter at high-energies



Artwork: T. Ullrich

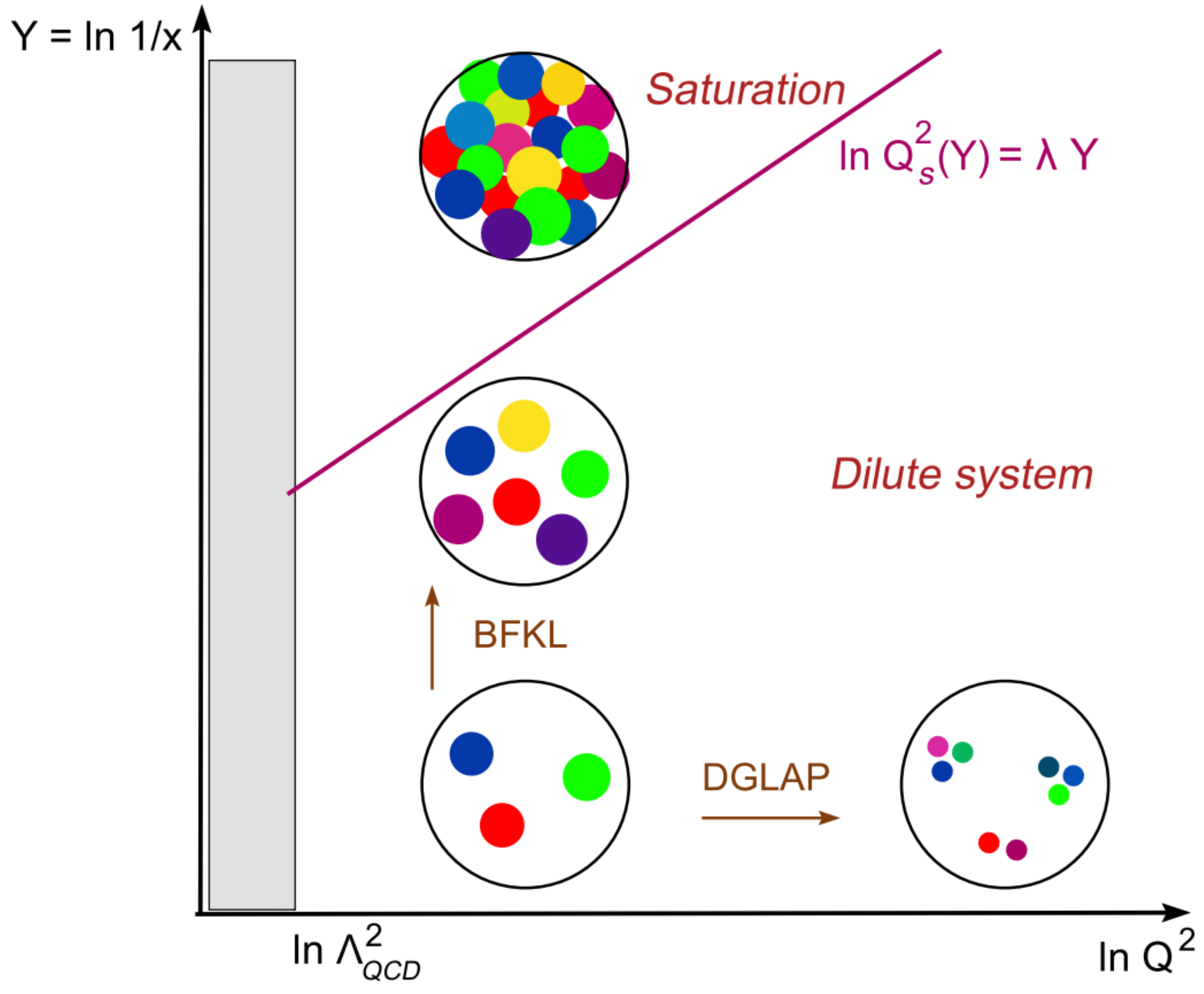


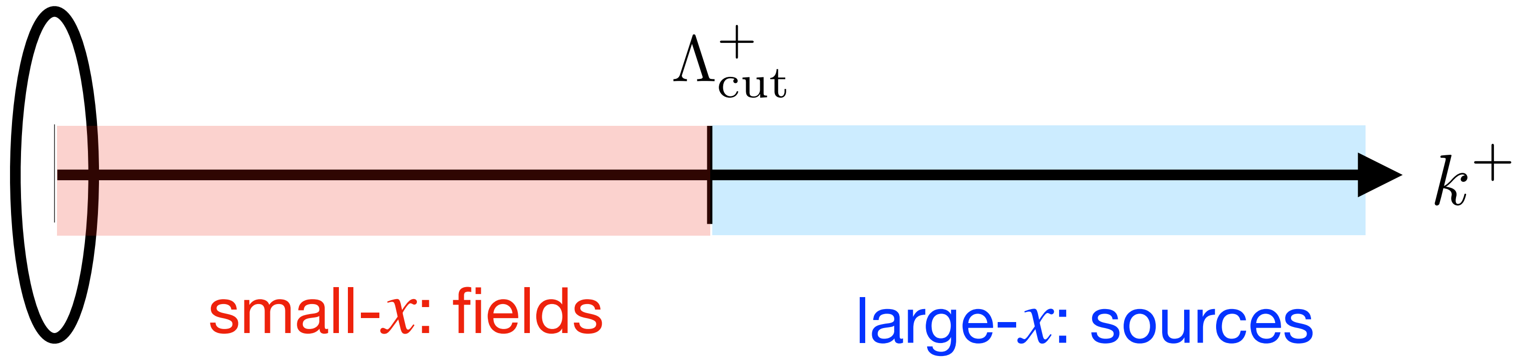
Figure from EIC White paper (2012)

Partonic picture superseded by **strong color fields**

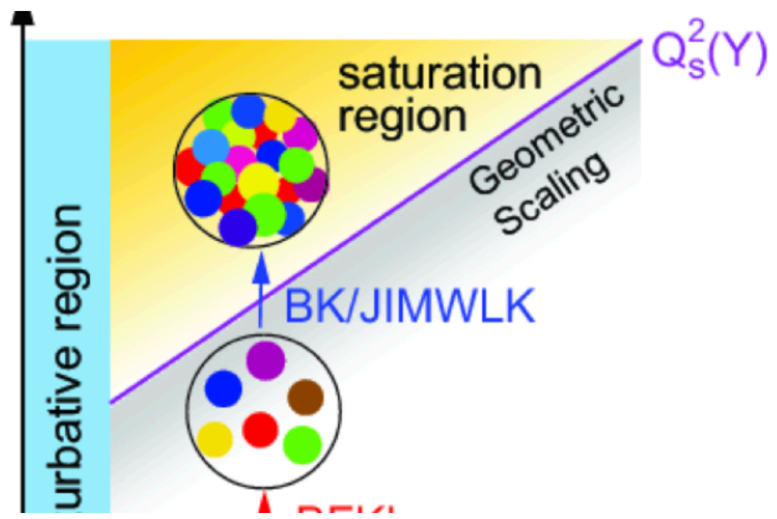
Universality: unified description of QCD at high-energies

The Color Glass Condensate in a nutshell

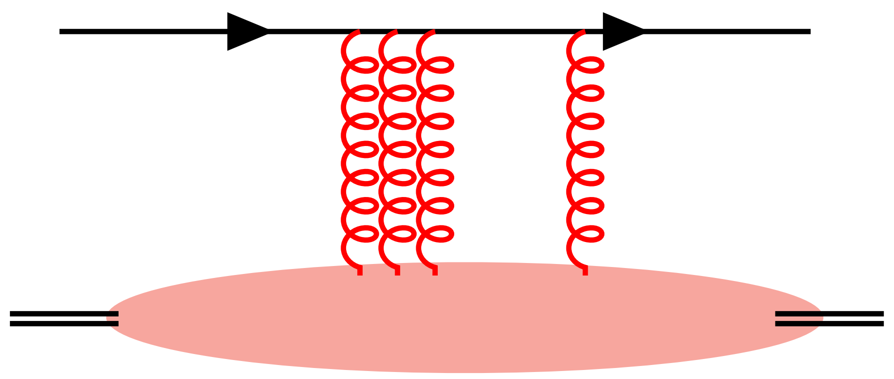
- Separation of degrees of freedom into sources and fields



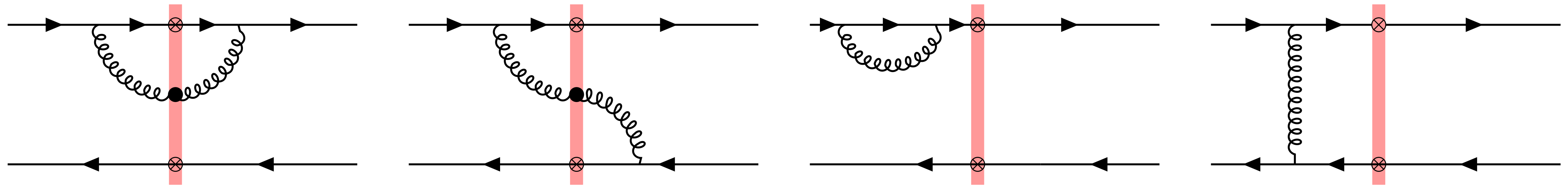
- CGC is an EFT of QCD providing a weak coupling approach for unitarization of cross-section



- Strong classical field $A \sim 1/g$ -> multiple scattering via light-like Wilson lines -> broadening (Glauber)



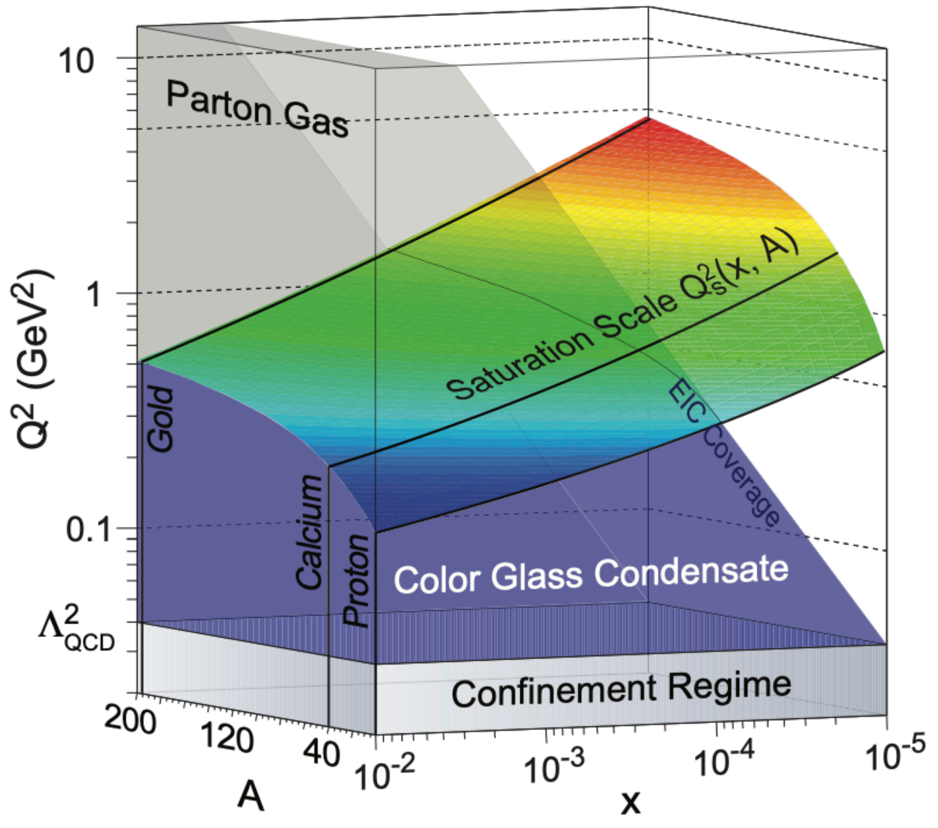
- Small-x radiation -> quantum (non-linear) evolution of Wilson line correlators-> suppression (Gribov)



- Emergence of an x-dependent and A-dependent momentum scale: $Q_s^2(x) \approx \Lambda_{QCD}^2 A^{1/3}(x_0/x)^\lambda$

- Saturation phenomena manifest in particle production of invariant mass with transverse momentum scale $\lesssim Q_s^2(x)$

- Potential to access at EIC or forward LHC



Power-counting in the CGC

Dilute-dilute: $Q_{sA}^2/k_{A\perp}^2 \ll 1$ and $Q_{sB}^2/k_{B\perp}^2 \ll 1$

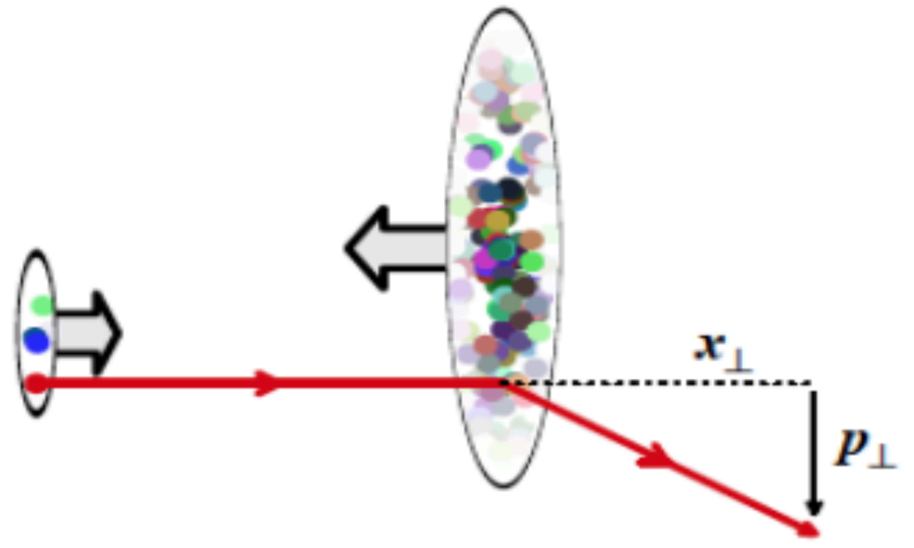
Match to pQCD computation of hard processes at small x

Hard production in hadron collisions

Dilute-dense: $Q_{sA}^2/k_{A\perp}^2 \ll 1$ and $Q_{sB}^2/k_{B\perp}^2 \sim 1$

Hybrid approach pQCD/CGC, advances at NLO and relation to TMD and GPDs

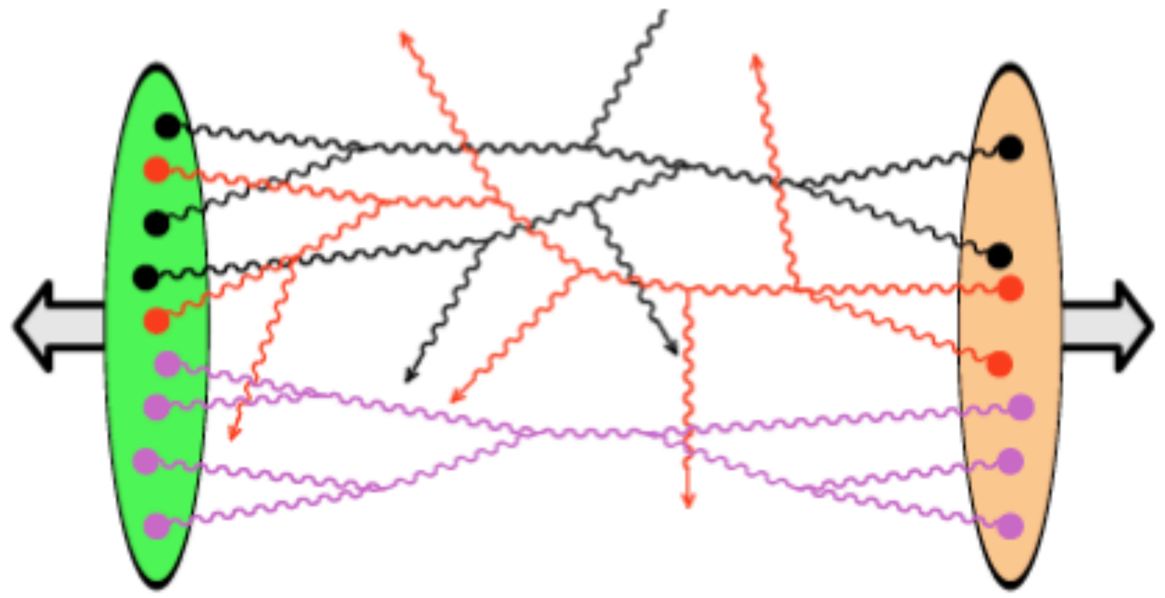
Semi-hard and forward particle production in proton-nucleus, electron-nucleus collisions



Dense-dense: $Q_{sA}^2/k_{A\perp}^2 \sim 1$ and $Q_{sB}^2/k_{B\perp}^2 \sim 1$

Solve classical YM equations numerically in 2+1 D (boost-invariant) /3+1 D

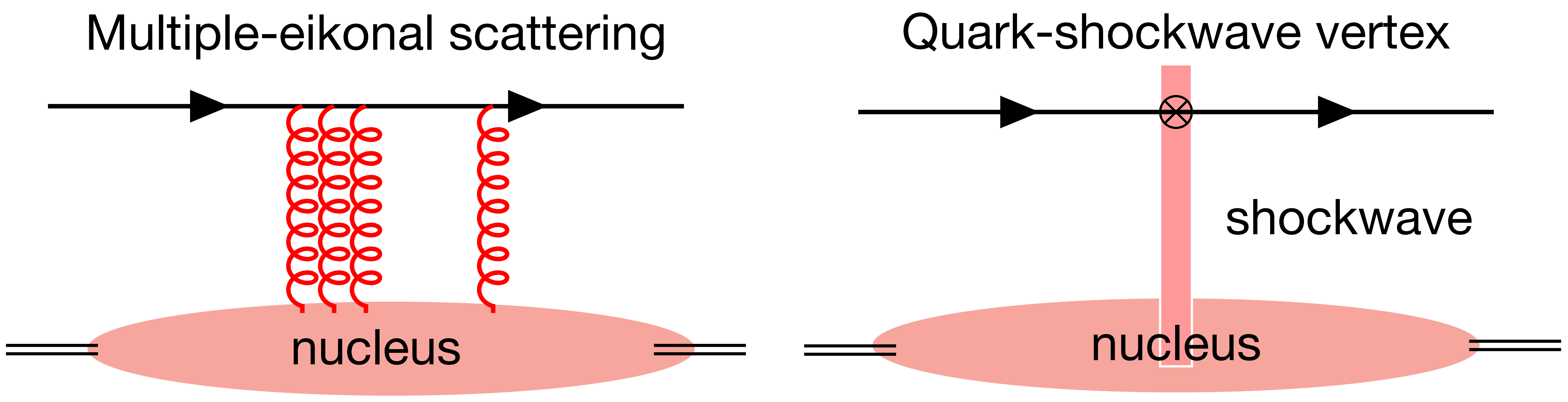
Semi-hard particle production in heavy-ion collisions



Regime is dictated by the colliding system, energy, centrality, rapidity, and transverse momentum of observed particles

Multiple scattering and quantum evolution

Shock-wave and Wilson lines



Effective vertex quark propagation

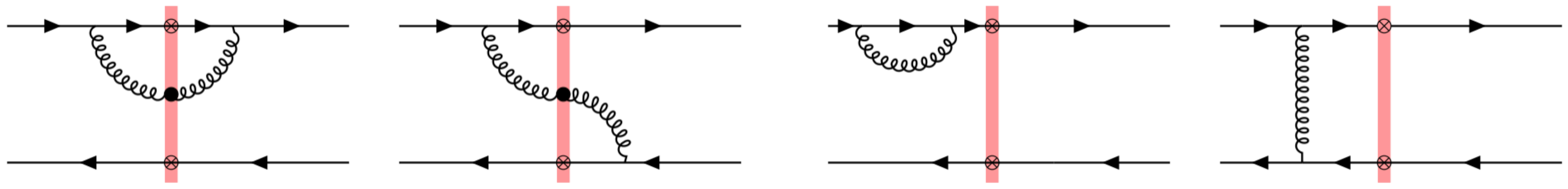
$$T^q_{ij}(l,l') = 2\pi\delta(n.(l-l'))\,(\gamma.n)\,\mathrm{sgn}(n.l) \times \int \mathrm{d}^2\boldsymbol{x}_\perp\, e^{-i(\boldsymbol{l}_\perp - \boldsymbol{l}'_\perp)\cdot\boldsymbol{x}_\perp}\, V_{ij}^{\mathrm{sgn}(n.l)}(\boldsymbol{x}_\perp)$$

Light-like Wilson line

$$V_{ij}(\boldsymbol{x}) = P\exp\left\{ig\int dx^- A^{+,a}_{cl}(\boldsymbol{x},x^-)t^a\right\}$$

Observables built from Wilson lines and their derivatives, convoluted with perturbative factor

Quantum evolution



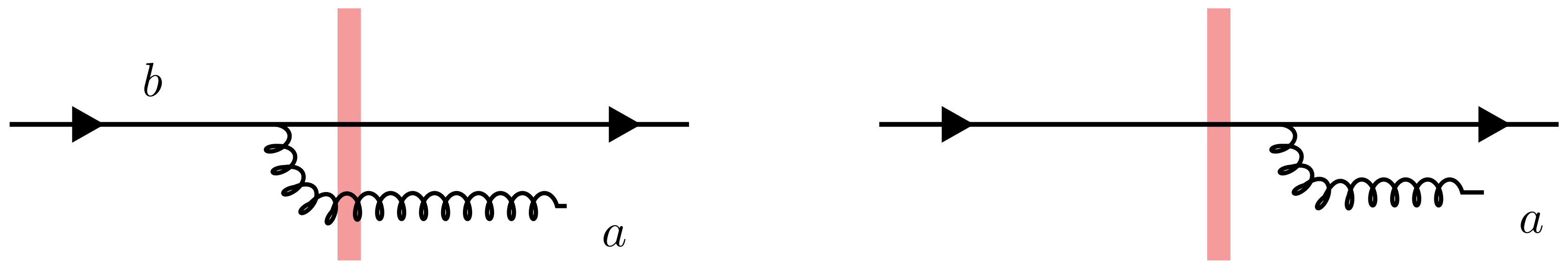
$$\frac{\partial S_Y(\boldsymbol{x}_1,\boldsymbol{x}_2)}{\partial Y} = \bar{\alpha}_s \int \frac{\mathrm{d}^2\boldsymbol{z}_\perp}{2\pi}\frac{(\boldsymbol{x}_1-\boldsymbol{x}_2)^2}{(\boldsymbol{x}_1-\boldsymbol{z})^2(\boldsymbol{z}-\boldsymbol{x}_2)^2}\left[S_Y(\boldsymbol{x}_1,\boldsymbol{z})\,S_Y(\boldsymbol{z},\boldsymbol{x}_2) - S_Y(\boldsymbol{x}_1,\boldsymbol{x}_2)\right]$$

Balitsky Kovchegov equations resums leading energy/rapidity logs $Y = \ln(1/x)$

$$S_Y(\boldsymbol{x}_\perp,\boldsymbol{y}_\perp) = \frac{1}{N_c}\left\langle \mathrm{Tr}\left[V(\boldsymbol{x}_\perp)V^\dagger(\boldsymbol{y}_\perp)\right]\right\rangle_Y$$

Quantum evolution of general operator

JIMWLK equation (leading log)



$$\boldsymbol{\Gamma}^a_m(\boldsymbol{z}) = \frac{ig}{2\pi}\frac{\boldsymbol{r}_m}{\boldsymbol{r}_m^2}\left(T^a_{L,m} - T^b_{R,m}\,U^{ab}(\boldsymbol{z})\right), \quad \boldsymbol{r}_m \equiv \boldsymbol{x}_m - \boldsymbol{z}$$

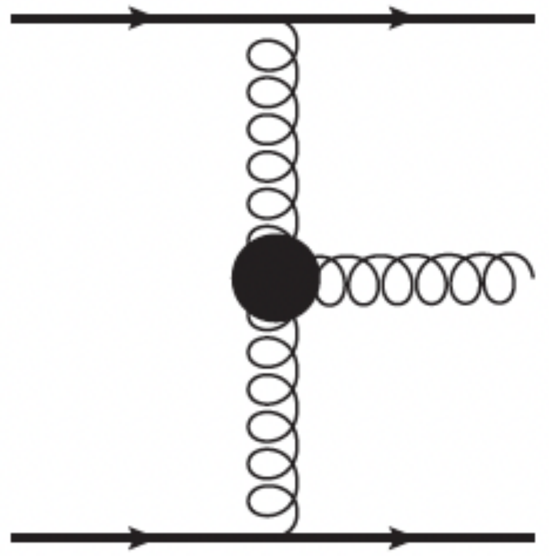
$$T^a_{L,m}\,V(\boldsymbol{x}_n) = \delta_{mn}t^a\,V(\boldsymbol{x}), \qquad T^a_{R,m}\,V(\boldsymbol{x}_n) = \delta_{mn}V(\boldsymbol{x})\,t^a$$

$$H_{\text{JIMWLK}} = \int\frac{\mathrm{d}^2\boldsymbol{z}}{2\pi}\sum_{m,n}\boldsymbol{\Gamma}^a_m(\boldsymbol{z})\cdot\boldsymbol{\Gamma}^a_n(\boldsymbol{z}) \qquad \frac{\partial\mathcal{O}_Y(\boldsymbol{x}_1,\dots,\boldsymbol{x}_N)}{\partial Y} = H_{\text{JIMWLK}}\,\mathcal{O}_Y(\boldsymbol{x}_1,\dots,\boldsymbol{x}_N)$$

The BK equation is the first equation in the JIMWLK hierarchy: two-point function

$$\frac{\partial S_Y(\boldsymbol{x}_1,\boldsymbol{x}_2)}{\partial Y} = \bar{\alpha}_s\int\frac{\mathrm{d}^2\boldsymbol{z}}{2\pi}\frac{(\boldsymbol{x}_1-\boldsymbol{x}_2)^2}{(\boldsymbol{x}_1-\boldsymbol{z})^2(\boldsymbol{z}-\boldsymbol{x}_2)^2}\left[S_Y(\boldsymbol{x}_1,\boldsymbol{z})\,S_Y(\boldsymbol{z},\boldsymbol{x}_2) - S_Y(\boldsymbol{x}_1,\boldsymbol{x}_2)\right]$$

Lipatov vertex



BFKL equation is the dilute $T \equiv 1 - S$, drop terms quadratic in T

$$\frac{\partial T_Y(\boldsymbol{x}_1,\boldsymbol{x}_2)}{\partial Y} = \bar{\alpha}_s\int\frac{\mathrm{d}^2\boldsymbol{z}}{2\pi}\frac{(\boldsymbol{x}_1-\boldsymbol{x}_2)^2}{(\boldsymbol{x}_1-\boldsymbol{z})^2(\boldsymbol{z}-\boldsymbol{x}_2)^2}\left[T_Y(\boldsymbol{x}_1,\boldsymbol{z}) + T_Y(\boldsymbol{z},\boldsymbol{x}_2) - T_Y(\boldsymbol{x}_1,\boldsymbol{x}_2)\right]$$

Lipatov vertex (coordinate space) squared

$$\boldsymbol{\Phi}(\boldsymbol{z};\boldsymbol{x}_1,\boldsymbol{x}_2) = \frac{\boldsymbol{r}_1}{\boldsymbol{r}_1^2} - \frac{\boldsymbol{r}_2}{\boldsymbol{r}_2^2}$$

Fixed order calculations

Shockwave breaks boost invariance ->
different regulator for transverse and longitudinal components

Effective vertices resum multiple scatterings ->
real and virtual loops also propagate in background

Two approaches: light-cone perturbation theory, or covariant approach (both in $A^- = 0$ projectile LC gauge)

both can be connected through spinor-helicity formalism, see e.g. Ayala, Hentschinski, Jalilian-Marian, Tejeda-Yeomans (2017) Caucal, Salazar, Venugopalan (2021)

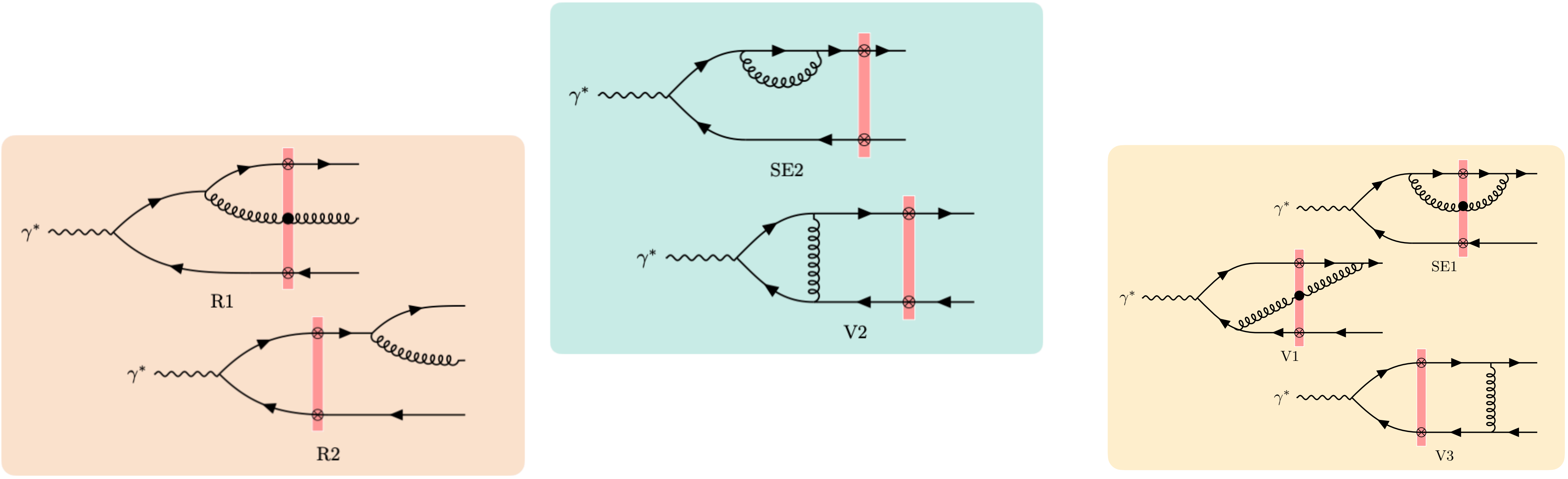
Impact factors at one-loop

Structure functions

light quarks	Balitsky, Chirilli (2011) Beuf (2017) Hänninen, Lappi, Paatelainen (2017)
massive quarks	Beuf, Lappi, Paatelainen (2021,2022)

Semi-inclusive processes in DIS

dijet+photon	Roy, Venugopalan (2019)
dijets	Caucal, Salazar, Venugopalan (2021)
back-to-back limit	Caucal, Salazar, Schenke, Stebel, Venugopalan (2024)
dijets (photo-production)	Taels, Altinoluk, Beuf, Marquet (2022)
dihadron	Bergabo, Jalilian-Marian (2022)
back-to-back limit	Caucal, Salazar (2024)
jet SIDIS	Caucal, Ferrand, Salazar (2024) Caucal, Iancu, Mueller, Yuan (2024)
hadron SIDIS	Bergabo, Jalilian-Marian (2022) Altinoluk, Marquet, Shi (2025)



Amplitude diagrams for dijet production in DIS. Caucal, Salazar, Venugopalan (2021)

Diffractive processes in DIS

Structure function	Beuf, Lappi, Mäntysaari, Paatelainen, Penttala (2024)
dijets and light vector meson	Boussarie, Grabovsky, Ivanov, Szymanowski, Wallon (2016)
vector meson	Mäntysaari, Penttala (2021, 2022)
single hadron	Fucilla, Grabovsky, Li, Szymanowski, Wallon (2023)

Semi-inclusive processes in pA

single hadron	Chirilli, Xiao, Yuan (2012), Stasto, Xiao, Zaslavsky (2013) …. Mäntysaari, Tawabutr (2023)
single jet	Liu, Xie, Kang, Liu (2022)
Drell-Yan	Taels (2023)

Missing in the literature:

-Single and double inclusive open/closed heavy flavor in DIS

-Two particle correlations in pA (photons, hadrons, jets)

-Isolated photon production in pA

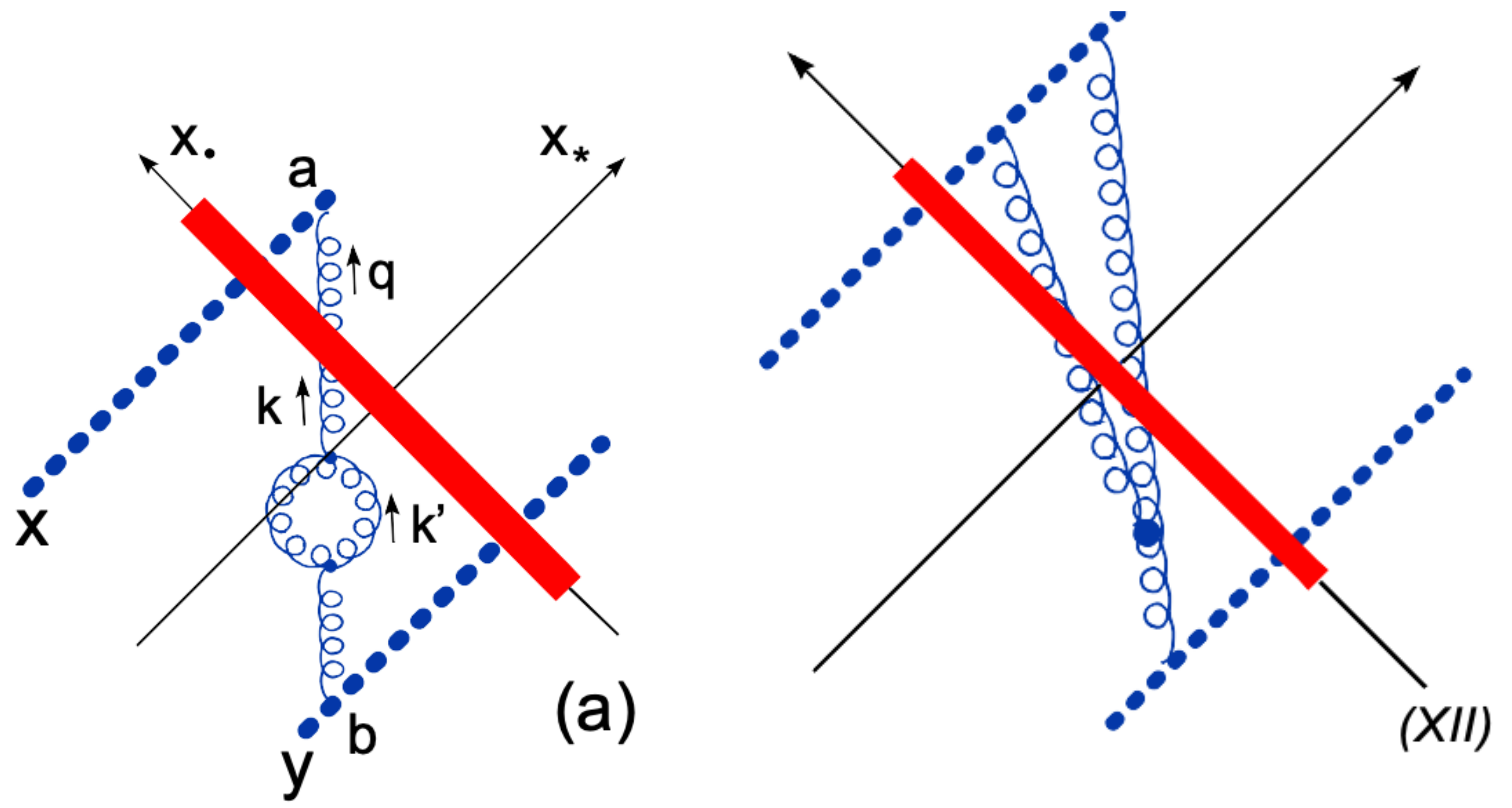
BK/JIMWLK

The evolution of the BK equation through the years

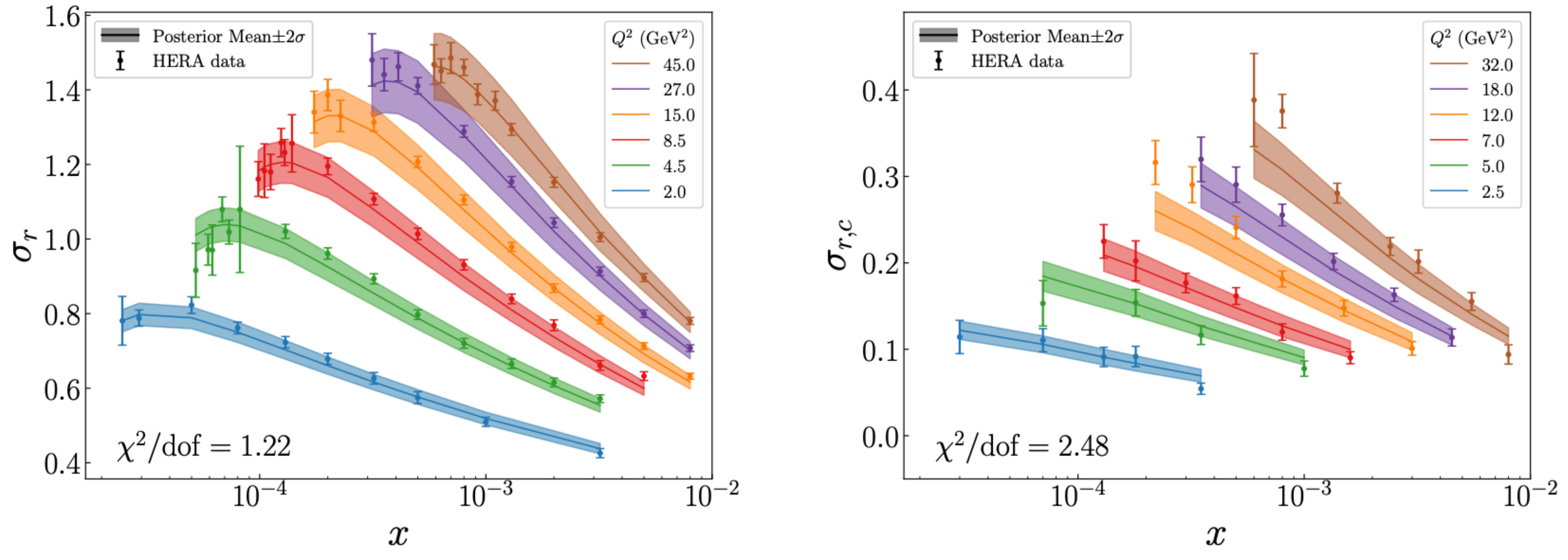
LL	**Balitsky (1996) Kovchegov (1999)**
running coupling	Kovchegov, Weigert (2007) Balitsky (2007)
NLL	**Balitsky, Chirilli (2008)**
NLL is unstable	Lappi, Mäntysaari (2015)
NLL with resummation	Ducloue, Iancu, Mueller, Soyez, Triantafyllopoulos (2015)
NLL with resummation is stable	Lappi, Mäntysaari (2016)
revisiting resummation	Boussarie, Caucal, Mehtar-Tani (2025)
NNLL	**Brunello, Caron-Huot, Crisanti, Giroux, Smith (2025)**

and the JIMWLK equation

LL	**Jalilian-Marian, Iancu, McLerran, Weigert, Leonidov, Kovner (1997-2001)**
running coupling	Lappi, H. Mäntysaari (2013)
NLL	**Balitsky, Chirilli (2013) Kovner, Lublinky, Mulian (2014)**
NLL with resummation	Hatta, Iancu (2016)
NLL with massive quarks	Dai, Lublinsky (2022)
running coupling revisited	Altinoluk, Beuf, Kovner, Lublinsky, Skokov (2023)



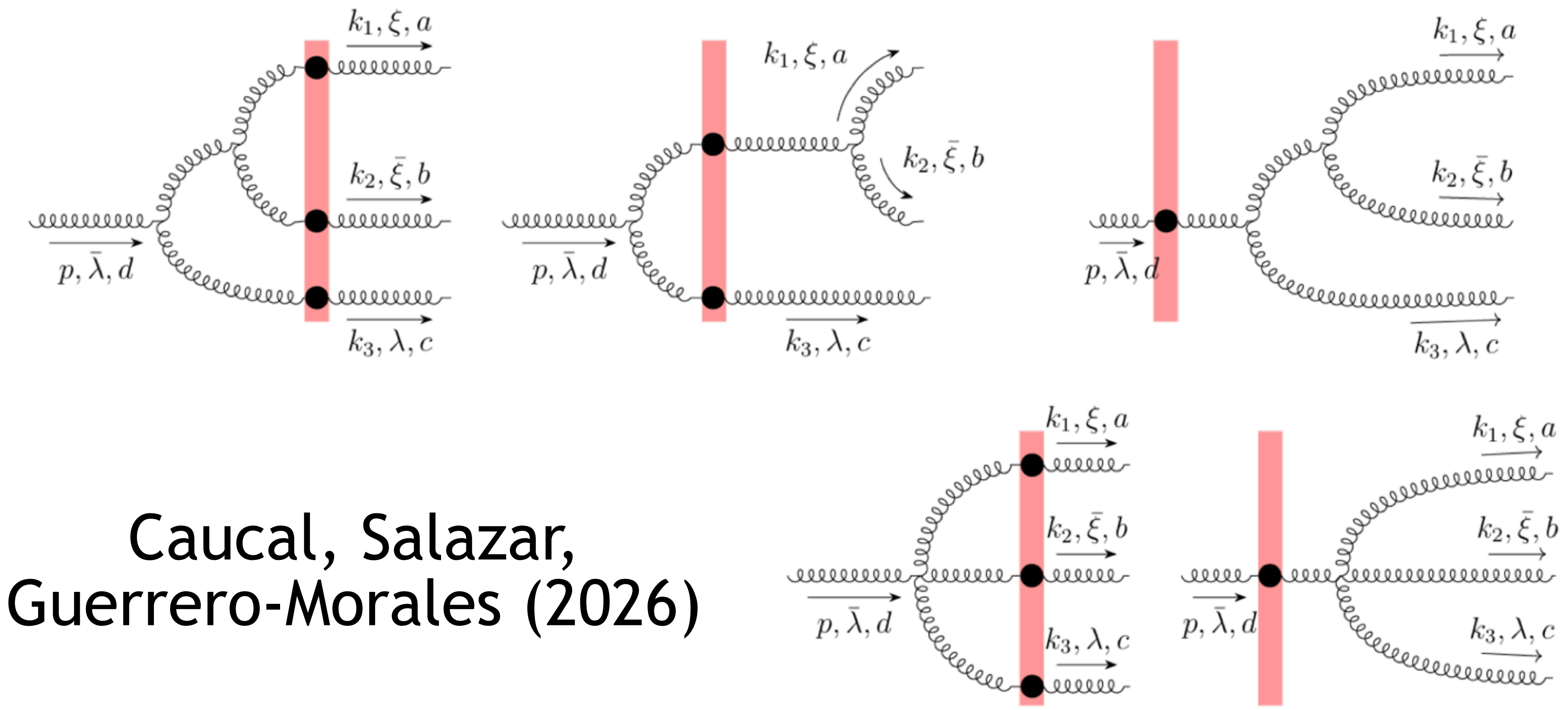
some diagrams for the NLL BK
Balitsky, Chirilli (2008)



Recent Bayesian fit of HERA data using
NLO impact factor + NLL BK
Casuga, Mäntysaari (2026)

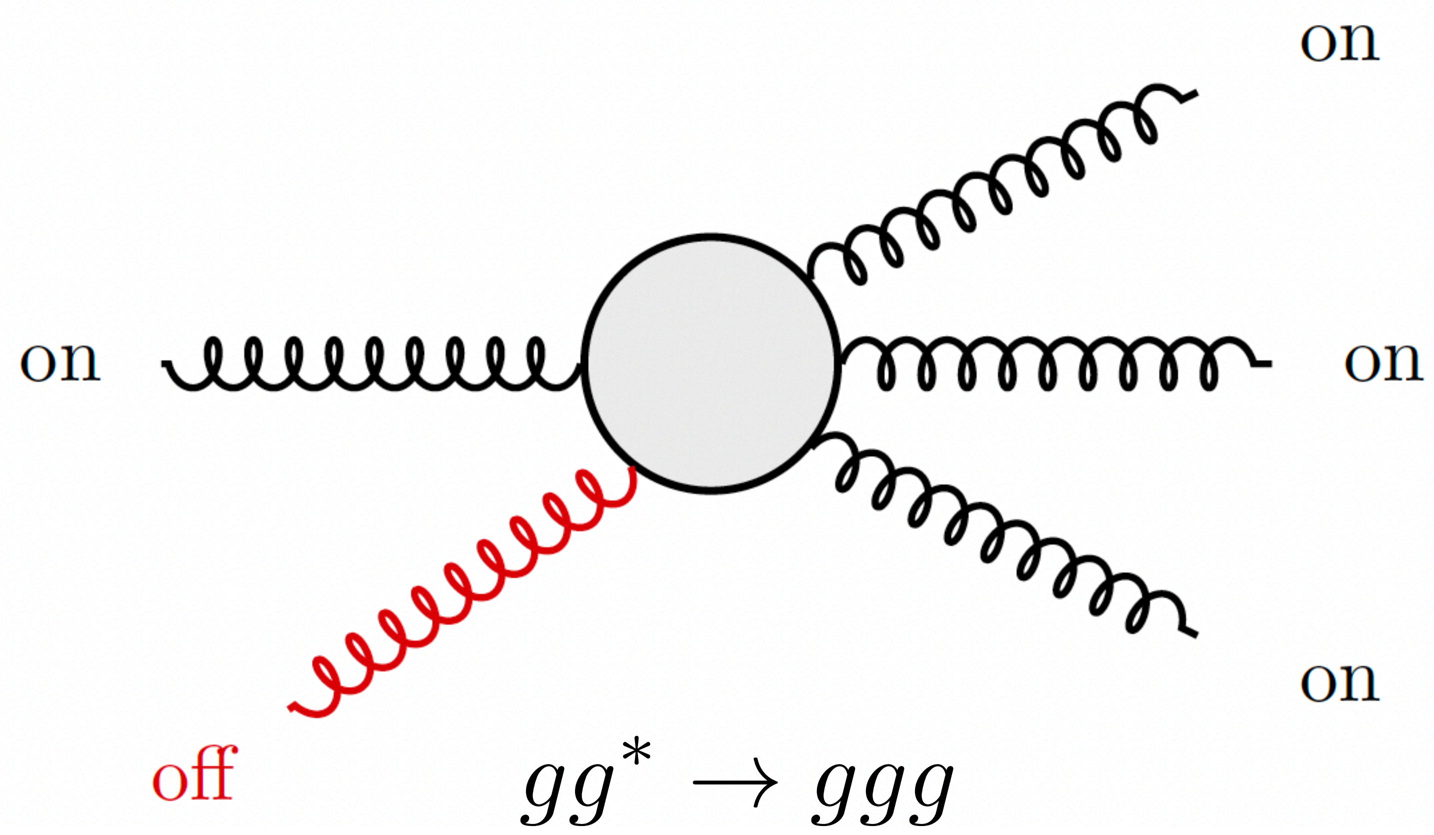
Dilute limit and kT factorization

- Expanding Wilson lines one should be able to recover multi-gluon amplitudes with one off-shell + m on-shell gluons. Amplitudes are important in kT factorization/BFKL

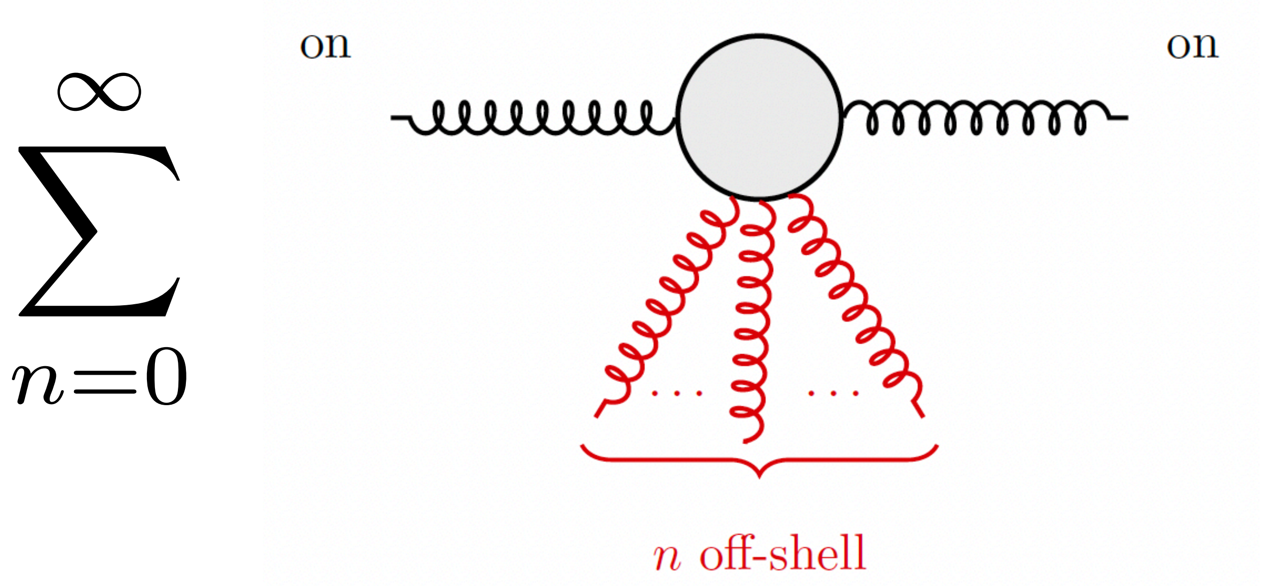


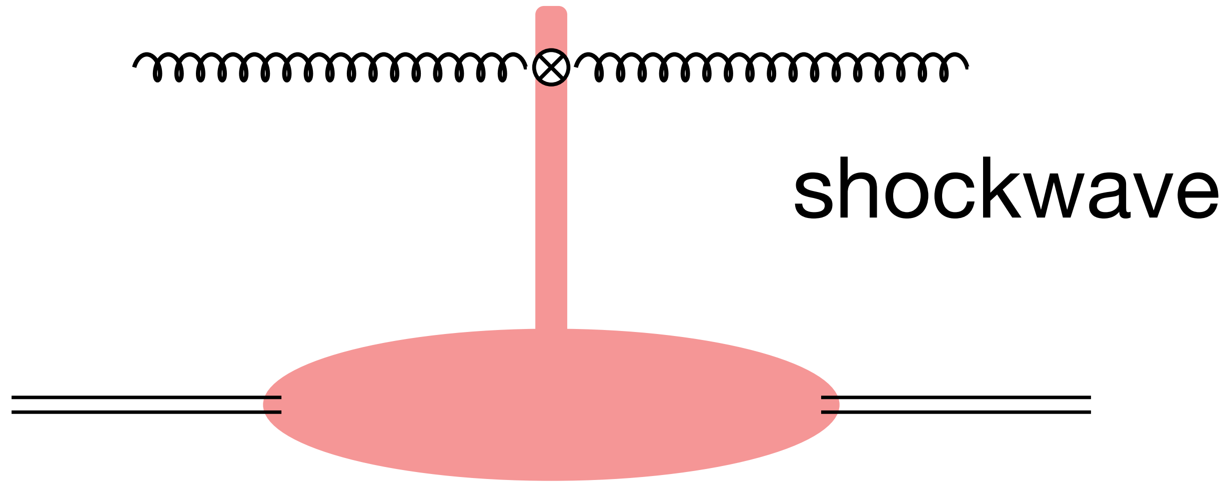
Caucal, Salazar, Guerrero-Morales (2026)

$$U^{ab}(\boldsymbol{x}_\perp) - \delta^{ab} = ig \int dx^- \, A^{+,c} \, (T^c)^{ab} + \mathcal{O}(A^2)$$



- Conversely, one use scattering amplitude techniques with n off-shell gluons to recover single gluon production in CGC in a gauge-invariant way using BCFW relations (Gelis 2026)





- Can these techniques be extended to "n" off-shell + "m" on-shell gluon production?
  Goal: generalized unitarity methods in the CGC, aka **"recycling trees for loops in dense fields"**

SCET + Glauber, and Lipatov effective action

There have been also recent progress in understanding low-x physics with SCET

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Effective field theory factorization for diffraction

Regular Article – Theoretical Physics | Open access | Published: 25 November 2025
Volume 2025, article number 157, (2025) Cite this article

Kyle Lee, Stella T. Schindler & Iain W. Stewart

LETTER | OPEN ACCESS

Power counting to saturation

Iain W. Stewart[1] and Varun Vaidya[2]

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Phys. Rev. D **110**, L011504 – **Published 23 July, 2024**

DOI: https://doi.org/10.1103/PhysRevD.110.L011504

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also from the Lipatov effective action and BFKL perspective

EDITORS' SUGGESTION | OPEN ACCESS

Color glass condensate formalism, Balitsky-JIMWLK evolution, and Lipatov's high energy effective action

Martin Hentschinski*

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Phys. Rev. D **97**, 114027 – **Published 26 June, 2018**

DOI: https://doi.org/10.1103/PhysRevD.97.114027

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Volume 85, article number 948, (2025) Cite this article

Ada Polizzi, Michael Fucilla & Alessandro Papa

Other topics

- Conformal BK equation in $\mathcal{N}=4$ SYM

High-energy amplitudes in $\mathcal{N}=4$ SYM in the next-to-leading order

Ian Balitsky [a,b,*], Giovanni A. Chirilli [c,d]

- Connection between CGC and TMD

PHYSICAL REVIEW D **109,** 034035 (2024)

Unified description of DGLAP, CSS, and BFKL evolution: TMD factorization bridging large and small x

Swagato Mukherjee [1,*] Vladimir V. Skokov [2,†] Andrey Tarasov [2,3,‡] and Shaswat Tiwari [2,§]

Gluon splitting at small x: a unified derivation for the JIMWLK, DGLAP and CSS equations

JHEP

Paul Caucal [a] Edmond Iancu [b] Farid Salazar [c,d,e] and Feng Yuan [f]

- BK and BMS equation correspondence

Relating e^+e^- annihilation to high energy scattering at weak and strong coupling

JHEP

Yoshitaka Hatta

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- Spin physics at low-x

PHYSICAL REVIEW D **99,** 054032 (2019)

Small-x helicity evolution: An operator treatment

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Theoretical Division, Los Alamos National Laboratory, Los Alamos, New Mexico 87545, USA

Summary and Outlook

- Dilute-dense version of the CGC = perturbation theory around classical gluon field

  Propagation of colored partons in the presence of classical gluon field, encoded in light-like Wilson lines

  Of great interest at the EIC program, and forward production at the LHC

- Status: many one-loop calculations are becoming available
  no complete (evolution + impact factor) two-loop computation available yet

  Shockwave breaks boost invariance -> different regulator for transverse and longitudinal components

  Effective vertices resum multiple scatterings -> real and virtual loops also propagate in background

- Can one import techniques from amplitude community to accelerate higher-order loop computations?

  Leverage on spinor-helicity, BCFW relations, generalized unitarity methods, automatization tools