A perturbative framework to probe IR sensitivity in non-Abelian gauge theories

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- In an era of ultraprecise measurements, **non-perturbative corrections** need to be addressed

$$d\sigma_x = \sum_{ij} \int dx_i\, dx_j\, f_i(x_i)\, f_j(x_j)\, d\sigma^{\text{part}}_{ij\to x} \times \left[1 + \mathcal{O}(\lambda^p)\right], \quad \text{with} \quad \lambda \equiv \frac{\Lambda_{\text{QCD}}}{Q}$$

- They are **power corrections**, which can be probed by studying **IR sensitivity**

- Given $\lambda \ll 1$, power corrections will matter only if

  a) the observable is measured in an ultraprecise way, *b)* $\lambda^p = \lambda^1$: **LINEAR** IR sensitivity

- Linear corrections are currently a source of debate in the interpretation of some observables…

  …and are expected to be a recurrent issue as the expertimental precision keeps increasing

- Studying IR sensitivity from first principles is difficult. One usually resorts either to phenomenological approaches, or approaches based on perturbation theory

- In the context of the latter, the renormalon model was often used [Beneke, 9807443]
  - One considers a class of diagrams that give factorial divergence at high orders
  - The sensitivity to linear corrections can be probed by computing perturbative QCD corrections with massive gluons
  - Yet, such class of diagrams is only known whenever there are no non-Abelian interactions

How to probe IR sensitivity for general processes involving non-Abelian interactions ?

- In particular, could linear terms *cancel* in suitable observables $\sigma[O] = \int d\Phi \, \frac{d\sigma}{d\Phi} \, O(\Phi)$?

  This would be a generalization of the KLN theorem, which concerns the logarithmic accuracy

  [Kinoshita, 1963]
  [Lee, Nauenberg, 1964]

  In more detail,

  - **Logarithmic** accuracy:

    $$\sigma[O]\big|_{\log\lambda} = \int d\Phi \left[\frac{d\sigma}{d\Phi}\Big|_{\log\lambda} O(\Phi)\big|_{\lambda^0} + \frac{d\sigma}{d\Phi}\Big|_{\lambda^0} O(\Phi)\big|_{\log\lambda} \right]$$

    At this accuracy, an infra-red safe observable obeys $\int d\Phi \, \frac{d\sigma}{d\Phi}\Big|_{\log\lambda} O(\Phi)\big|_{\lambda^0} = 0$

  - **Linear accuracy:**

    $$\sigma[O]\big|_{\lambda^1} = \int d\Phi \left[\frac{d\sigma}{d\Phi}\Big|_{\lambda^1} O(\Phi)\big|_{\lambda^0} + \frac{d\sigma}{d\Phi}\Big|_{\lambda^0} O(\Phi)\big|_{\lambda^1} \right]$$

    What happens at this accuracy? Could it be that $\int d\Phi \, \frac{d\sigma}{d\Phi}\Big|_{\lambda^1} O(\Phi)\big|_{\lambda^0} = 0$?

- This argument was developed several years ago, but we lack a clear proof in the non-Abelian case

  [Akhoury, Zakharov, 9512433]

- We need a framework to probe IR sensitivity — and, in particular, linear IR sensitivity — in general processes governed by non-Abelian interactions

- In this talk I present one such framework
  - Inspired by the renormalon model, we promote the gluon mass to a parameter of a proper QFT
  - Such QFT is a non-Abelian gauge theory, where the gluon gets its mass via the Higgs mechanism
  - The gluon mass $m_{\rm g}$, assumed to be the smallest scale, plays the role of the IR scale
    - In particular, a linear dependence of an observable on $m_{\rm g}$ is a sign of linear power corrections
  - This framework provides a consistent laboratory to unambiguously probe, from a perturbative perspective, IR sensitivity of observables at colliders
- I start by discussing the theory — a renormalizable, non-Abelian gauge theory based on SU(2)
- Then, as a first step towards probing linear sensitivity of observables at colliders, I calculate two ingredients through 2-loops:

  i) the $\mathcal{O}(m_{\rm g})$ contributions to the relation between the pole and $\overline{\rm MS}$ masses of a heavy quark

  ii) the $\mathcal{O}(m_{\rm g})$ contributions to the relation between the on-shell subtraction and $\overline{\rm MS}$ field counterterms of the heavy quark

- The model is a renormalizable gauge theory, where SU(2) is spontaneously broken
- In analogy with QCD,
  - I refer to the gauge fields G as gluons
  - I refer to the massive fermion ψ_t as top quark
  - I consider n_f flavors of massless fermions ψ_k
- Defining $D_\mu = \partial_\mu + ig\hat{T}^a G^a_\mu$, the Lagrangian is

$$\mathcal{L} = -\frac{1}{4} G^a_{\mu\nu} G^{a,\mu\nu} + \overline{\psi}_\mathrm{t}(\mathrm{i}\not{D} - m_\mathrm{t})\psi_\mathrm{t} + \sum_{k=1}^{n_\mathrm{f}} \overline{\psi}_k \mathrm{i}\not{D}\psi_k + (D_\mu\Phi)^\dagger(D^\mu\Phi) + \mu^2\Phi^\dagger\Phi - \lambda(\Phi^\dagger\Phi)^2 + \mathcal{L}_\mathrm{GF} + \mathcal{L}_\mathrm{Ghost}$$

- With $\mu^2 > 0, \ \lambda > 0,$ the Higgs doublet Φ acquires a vacuum expectation value v ,

$$\Phi(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} -\varphi_2(x) - \mathrm{i}\varphi_1(x) \\ v + H(x) + \mathrm{i}\varphi_3(x) \end{pmatrix} \quad \text{with} \quad \varphi_{1,2,3} \quad \text{the would-be Goldstone bosons}$$

- The gluon and the Higgs boson acquire masses

$$m_\mathrm{g} = \frac{gv}{2}\,, \qquad m_\mathrm{H} = \sqrt{2\lambda}\,v\,, \quad \text{with} \quad v = \sqrt{\frac{\mu^2}{\lambda}}$$

m_g → probes IR sensitivity

- A proper quantization requires gauge-fixing and ghosts,

$$\mathcal{L}_{\rm GF} = -\frac{1}{2\xi} F_a F_a \,, \qquad F_a = \partial^\mu G^a_\mu + \xi m_{\rm g} \varphi_a, \quad a = 1,2,3$$

$$\mathcal{L}_{\rm Ghost} = -\,\bar{c}_a(\partial_\mu\partial^\mu + \xi m_{\rm g}^2)c_a - g\epsilon^{abc}\bar{c}_a\left[(\partial_\mu c_b)G^\mu_c + c_b(\partial_\mu G^\mu_c)\right] - \frac{g\xi m_{\rm g}}{2}\bar{c}_a c_a H - \frac{g\xi m_{\rm g}}{2}\epsilon^{abc}\bar{c}_a c_b \varphi_c$$

- To renormalize the model, we write

$$m^2_{\rm H(0)} = Z_{m^2_{\rm H}} m^2_{\rm H} \,, \quad m^2_{\rm g(0)} = Z_{m^2_{\rm g}} m^2_{\rm g} \,, \quad m_{\rm t(0)} = Z_{m_{\rm t}} m_{\rm t} \,, \quad g_{(0)} = \left[S_\epsilon \mu^{2\epsilon}\right]^{\frac{1}{2}} Z_{\rm g}\, g$$

$$\psi_{\rm t(0)} = Z^{1/2}_{\psi_{\rm t}} \psi_{\rm t} \,, \qquad \psi_{k(0)} = Z^{1/2}_{\psi_k} \psi_k \,, \qquad H_{(0)} = Z^{1/2}_{\rm H} H \,, \qquad G^a_{\mu(0)} = Z^{1/2}_{\rm G} G^a_\mu$$

where $Z_X = 1 + \delta Z_X = 1 + \sum_i \delta Z_X^{(i)}$ → i-th loop contribution to the counterterm δZ_X

- We renormalize all counterterms in on-shell subtraction, except $Z_{\rm g}$. The latter could be fixed in $\overline{\rm MS}$, but it is more useful to define a g that absorbs certain finite parts of loop diagrams

- So, $\delta Z_{\rm g}$ absorbs the finite contribution from top quark and Higgs boson contributions to δZ_G . This leads to

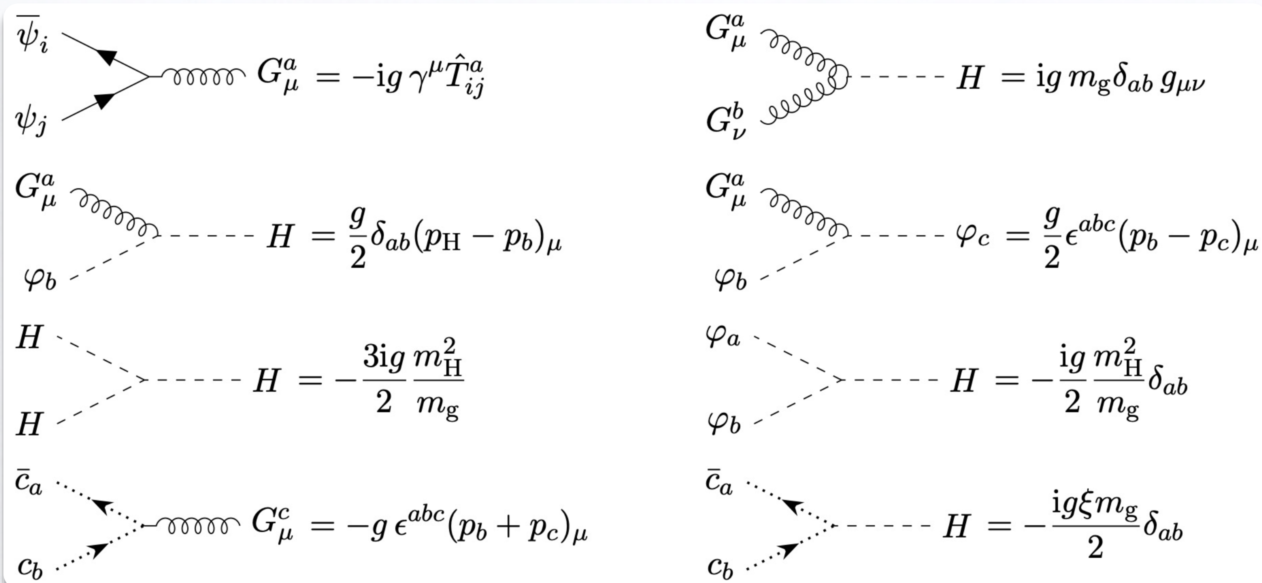
$$\mu \frac{{\rm d}\alpha_{\rm s}}{{\rm d}\mu} = -\frac{\beta_0}{2\pi}\alpha_{\rm s}^2 \,, \qquad \beta_0 = \frac{29}{8} C_{\rm A} - \frac{4}{3} T_{\rm R} n_{\rm f}$$

for a not too high $n_{\rm f}$, the theory is asymptotically free

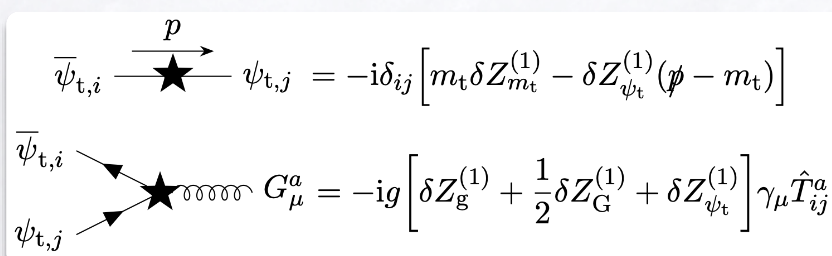
- I assume the mass hierarchy $m_{\rm H} \sim m_{\rm t} \gg m_{\rm g}$ and use renormalized perturbation theory.

  Examples of Feynman rules:

  - for the renormalized interactions



  - for the counterterm interactions



- We start with

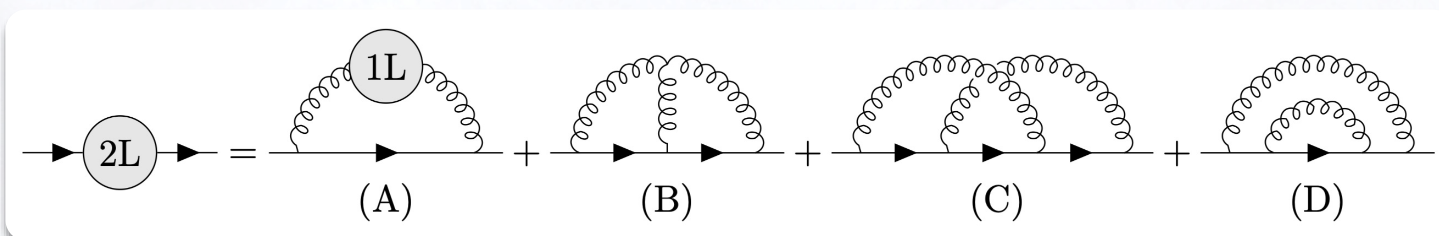
$$m_{t(0)} = Z_{m_t}^{OS} m_t = Z_{m_t}^{\overline{MS}}(\mu)\, m_t^{\overline{MS}}(\mu)\,, \qquad \psi_{t(0)} = \sqrt{Z_{\psi_t}^{OS}}\psi_t^{OS} = \sqrt{Z_{\psi_t}^{\overline{MS}}}\psi_t^{\overline{MS}}(\mu)$$

implying

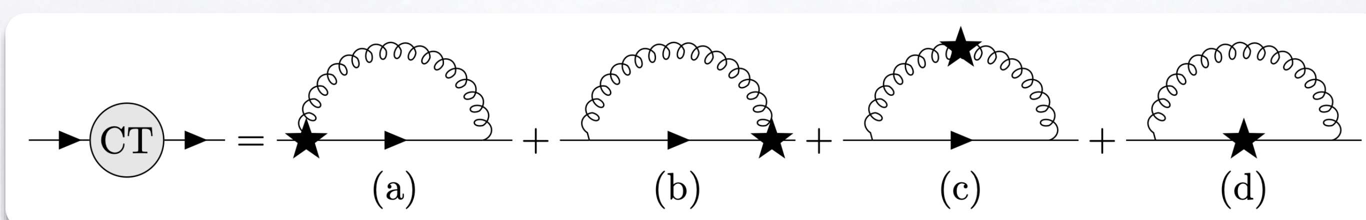
$$\frac{m_t^{\overline{MS}}(\mu)}{m_t} = \frac{Z_{m_t}^{OS}}{Z_{m_t}^{\overline{MS}}(\mu)}\,, \qquad \frac{\psi_t^{\overline{MS}}(\mu)}{\psi_t^{OS}} = \sqrt{\frac{Z_{\psi_t}^{OS}}{Z_{\psi_t}^{\overline{MS}}}}$$

I am only interested in the linear terms in m_g

- The non-trivial quantities to calculate are $\delta Z_{m_t}^{OS(2)}$ and $\delta Z_{\psi_t}^{OS(2)}$. They are obtained from



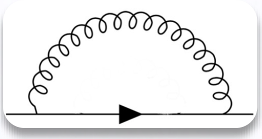
as well as from



- These diagrams depend on different mass scales $m_{\mathrm{H}} \sim m_{\mathrm{t}} \gg m_{\mathrm{g}}$ — difficult to evaluate exactly

- Yet, since our focus is on terms linear in m_{g}, we expand the integrands in powers of m_{g} before solving the integral. This is the **strategy of regions** [Beneke, Smirnov, 9711391]

- Example: the 1-loop top-quark self-energy



  - The integral is $I = \int \frac{\mathrm{d}^d k}{(2\pi)^d} \frac{\mu^{2\epsilon}}{(k^2 - m_{\mathrm{g}}^2)(k^2 + 2p\cdot k)}, \qquad p^2 = m_{\mathrm{t}}^2.$

  - By solving it exactly and only then expanding in powers of $\frac{m_{\mathrm{g}}}{m_{\mathrm{t}}}$,

$$I = \frac{\mathrm{i}\Gamma(1+\epsilon)}{(4\pi)^{d/2}} \left\{ \left[\frac{1}{\epsilon} + 2\log\frac{\mu}{m_{\mathrm{t}}} + 2 \right] - \pi \boxed{\frac{m_{\mathrm{g}}}{m_{\mathrm{t}}}} + \mathcal{O}\left(m_{\mathrm{g}}^2\right) \right\}$$

  - In the strategy of regions, $I = I^{(\mathrm{h})} + I^{(\mathrm{s})} + \mathcal{O}\left(m_{\mathrm{g}}^2\right)$, with

hard region ($k \sim m_{\mathrm{t}}$): $$I^{(\mathrm{h})} = \int \frac{\mathrm{d}^d k}{(2\pi)^d} \frac{\mu^{2\epsilon}}{k^2\,(k^2 - 2p\cdot k)} + \mathcal{O}\left(m_{\mathrm{g}}^2\right) = \frac{\mathrm{i}\Gamma(1+\epsilon)}{(4\pi)^{d/2}} \left[\frac{1}{\epsilon} + 2\log\frac{\mu}{m_{\mathrm{t}}} + 2 \right] + \mathcal{O}\left(m_{\mathrm{g}}^2\right),$$

soft region ($k \sim m_{\mathrm{g}}$): $$I^{(\mathrm{s})} = \int \frac{\mathrm{d}^d k}{(2\pi)^d} \frac{\mu^{2\epsilon}}{(k^2 - m_{\mathrm{g}}^2)\,(-2p\cdot k)} + \mathcal{O}\left(m_{\mathrm{g}}^2\right) = \frac{\mathrm{i}\Gamma(1+\epsilon)}{(4\pi)^{d/2}} \left(-\pi \boxed{\frac{m_{\mathrm{g}}}{m_{\mathrm{t}}}} \right) + \mathcal{O}\left(m_{\mathrm{g}}^2\right).$$

- In this one-loop example, the linear terms are entirely determined by the soft region
- This example extends to the 2-loop case in a natural way
- Here, four regions can be considered:

$$\text{hard-hard: } k_1 \sim m_{\rm t},\ k_2 \sim m_{\rm t}\,, \qquad \text{hard-soft: } k_1 \sim m_{\rm t},\ k_2 \sim m_{\rm g}\,,$$
$$\text{soft-hard: } k_1 \sim m_{\rm g},\ k_2 \sim m_{\rm t}\,, \qquad \text{soft-soft: } k_1 \sim m_{\rm g},\ k_2 \sim m_{\rm g}$$

such that only hard-soft, soft-hard and soft-soft can lead to the linear term in $m_{\rm g}$

- We followed a standard workflow for the calculation:
  - Generation of Feynman rules and diagrams
  - Manipulation of amplitudes
  - Expansion of scalar integrals by regions
  - Integration-by-parts identities

resorting to a variety of publicly available software → FeynMaster, FeynRules, Qgraf, FeynCalc, FeynHelpers, Package-X, LiteRed, FIRE

- We performed the calculation in two independent ways

- We find four master integrals for the linear term in the soft-soft region,

$$I_{\mathrm{A}} = \int \frac{\mathrm{d}^d k_1}{(2\pi)^d} \frac{\mathrm{d}^d k_2}{(2\pi)^d} \frac{1}{(k_1^2 - m_{\mathrm{g}}^2)} \frac{1}{(k_2^2 - m_{\mathrm{g}}^2)} \frac{1}{(2k_2 \cdot p)}$$

$$I_{\mathrm{B}} = \int \frac{\mathrm{d}^d k_1}{(2\pi)^d} \frac{\mathrm{d}^d k_2}{(2\pi)^d} \frac{1}{(k_1^2 - m_{\mathrm{g}}^2)} \frac{1}{[(k_1 - k_2)^2 - m_{\mathrm{g}}^2]} \frac{1}{[2k_2 \cdot p]}$$

$$I_{\mathrm{C}} = \int \frac{\mathrm{d}^d k_1}{(2\pi)^d} \frac{\mathrm{d}^d k_2}{(2\pi)^d} \frac{1}{(k_1^2 - m_{\mathrm{g}}^2)} \frac{1}{[(k_1 - k_2)^2 - m_{\mathrm{g}}^2]} \frac{1}{(k_2^2 - m_{\mathrm{g}}^2)} \frac{1}{(2k_2 \cdot p)}$$

$$I_{\mathrm{D}} = \int \frac{\mathrm{d}^d k_1}{(2\pi)^d} \frac{\mathrm{d}^d k_2}{(2\pi)^d} \frac{1}{k_1^2} \frac{1}{(k_1 - k_2)^2} \frac{1}{(k_2^2 - m_{\mathrm{g}}^2)} \frac{1}{(2k_2 \cdot p)}$$

- These can be solved with standard methods. Yet some contain a closed subloop of massive gluons. This subloop admits the dispersion representation:

$$\int \frac{\mathrm{d}^d k_1}{(2\pi)^d} \frac{1}{(k_1^2 - m_{\mathrm{g}}^2)} \frac{1}{[(k_1 + k_2)^2 - m_{\mathrm{g}}^2]} = \frac{\mathrm{i}}{(4\pi)^{d/2}} \frac{\Gamma(1-\epsilon)}{\Gamma(2-2\epsilon)} \int_{4m_{\mathrm{g}}^2}^{\infty} \mathrm{d}s \, \frac{s^{-\epsilon}(1 - 4m_{\mathrm{g}}^2/s)^{\frac{1}{2}-\epsilon}}{s - k_2^2 - \mathrm{i}0}$$

- Further details about the calculation can be found in the paper [DF et al, 2603.22072]

- We ultimately find that the relation between the pole and the $\overline{\text{MS}}$ masses is

$$m_\mathrm{t} = m_\mathrm{t}^{\overline{\mathrm{MS}}}(\mu)\bar{f}_1\left(\alpha_\mathrm{s},\mu,m_\mathrm{t}^{\overline{\mathrm{MS}}}(\mu),m_\mathrm{H}\right) - \alpha_\mathrm{s}(m_\mathrm{g})\, m_\mathrm{g}\frac{C_\mathrm{F}}{2}\left\{1+\frac{\alpha_\mathrm{s}(m_\mathrm{g})}{2\pi}\left[3C_\mathrm{F}+C_\mathrm{A}\left(\frac{21\sqrt{3}\pi}{32}-\frac{19}{48}\right)-\frac{4}{9}n_\mathrm{f}T_\mathrm{R}\right]\right\}$$

  - The function $\bar{f}_1$ is independent of the gluon mass and therefore is of no interest to us
  - The 2nd line is the linear term in m_g and is our main result
  - It confirms the expectation that the relation between the $\overline{\text{MS}}$ and pole masses of a heavy quark has a linear dependence on m_g
  - It supports the assertion that our toy model provides **a consistent framework** to probe the IR sensitivity of observables through the parameter m_g
  - The pole mass must be gauge independent. We checked this explicitly

- The relation between the field counterterm in on-shell subtraction and $\overline{\text{MS}}$ is

$$\sqrt{Z_{\psi_t}^{\text{OS}}} = \sqrt{Z_{\psi_t}^{\overline{\text{MS}}}}(\mu_0)\Bigg\{ f'\left(\alpha_s, \mu, m_t, m_H, \log m_g\right) + \frac{3}{8} C_F\, \alpha_s(m_g) \frac{m_g}{m_t} \times \\ \times \left[1 + \frac{\alpha_s(m_g)}{2\pi} \left(C_F - C_A \left(\frac{17}{16} - \frac{31\pi}{32\sqrt{3}} - \log\frac{m_t}{m_g} \right) - \frac{4}{9} n_f T_R \right) \right] \Bigg\}$$

$$\text{with} \quad \sqrt{Z_{\psi_t}^{\overline{\text{MS}}}}(\mu_0) = \sqrt{Z_{\psi_t}^{\overline{\text{MS}}}}(\mu) \left[1 - \frac{\alpha_s(\mu) C_F}{4\pi} \log\frac{\mu}{\mu_0} \right], \quad \mu_0 = \frac{m_t^3}{m_g^2}$$

  - This relation contains not only a linear, but also a logarithmic dependence on m_g
  - This relation is gauge dependent, which we checked explicitly. The result is in Feynman gauge

- With increasing experimental precision, we need to consider **non-perturbative corrections**
  - Assessing the sensitivity of perturbative calculations to IR physics is a possible strategy
- In the renormalon approach, this sensitivity can be exposed by introducing a small gluon mass
  - Although such approach is useful, it does not allow studying processes with external gluons
- I proposed a framework which promotes the gluon mass to a parameter of a consistent QFT
  - It is a renormalizable toy model based on an SU(2) gauge theory whose symmetry is spontaneously broken through the Higgs mechanism
  - Gluons acquire a small mass m_{g} in a theoretically consistent manner. A linear dependence on m_{g} probes a linear IR sensitivity of observables
  - **This framework provides a laboratory** for investigating whether IR-safe observables studied at colliders exhibit linear IR sensitivity
- As a first application, I calculated the linear terms in m_{g} in the relation between the pole and the masses $\overline{\mathrm{MS}}$ through 2 loops, providing a nontrivial check of the consistency of the framework
- Next step: heavy-quark pair production. This will provide a more direct connection to realistic processes relevant for precision measurements at hadron colliders