

Integrated Subtraction Terms and Finite Remainders for Arbitrary Massless QCD Processes With Jets at Colliders Using the Nested Soft- Collinear Subtraction Scheme

Davide Maria Tagliabue

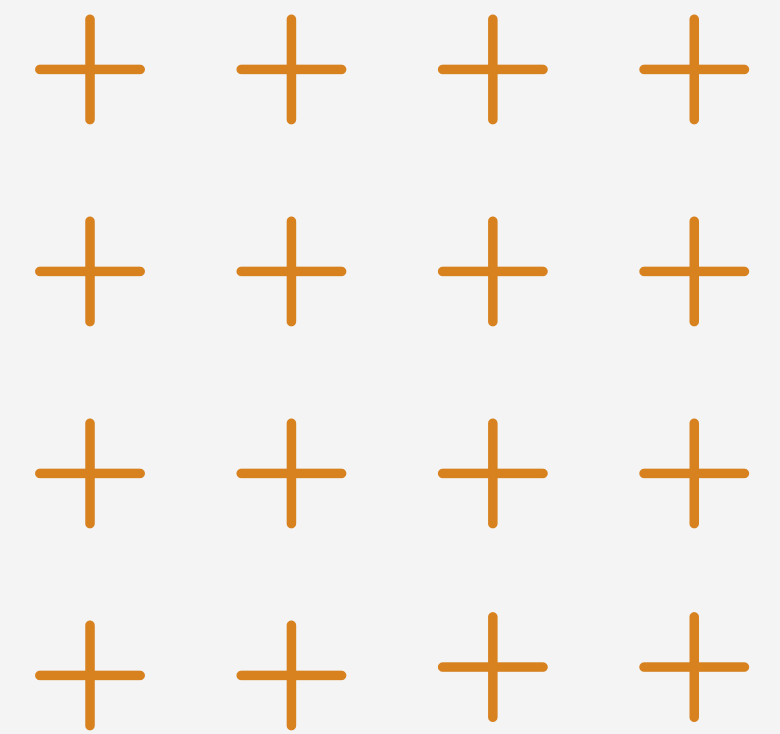
Based on the following Refs.

[Devoto, Melnikov, Röntsch, Signorile-Signorile, DMT, 2310.17598]

[Devoto, Melnikov, Röntsch, Signorile-Signorile, DMT, Tresoldi, 2503.15251]

[Devoto, Melnikov, Röntsch, Signorile-Signorile, DMT, Tresoldi, 2509.08594]

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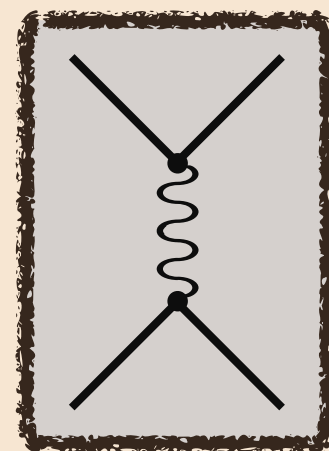
The Factorization Theorem

PDFs |

Hadronic cross section |
$$d\sigma_H = \sum_{ab} \int_0^1 dx_a dx_b f_a(x_a) f_b(x_b) \times d\hat{\sigma}_{ab}(x_a, x_b, \alpha_s(\mu))$$

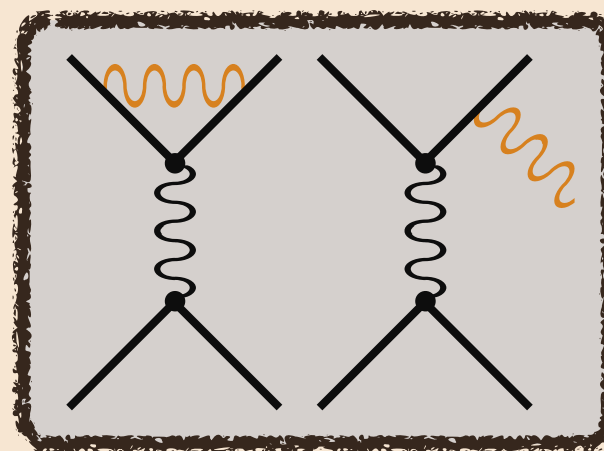
Partonic cross section

$$d\hat{\sigma}_{ab} = d\hat{\sigma}_{ab}^{\text{LO}} + d\hat{\sigma}_{ab}^{\text{NLO}} + d\hat{\sigma}_{ab}^{\text{NNLO}}$$



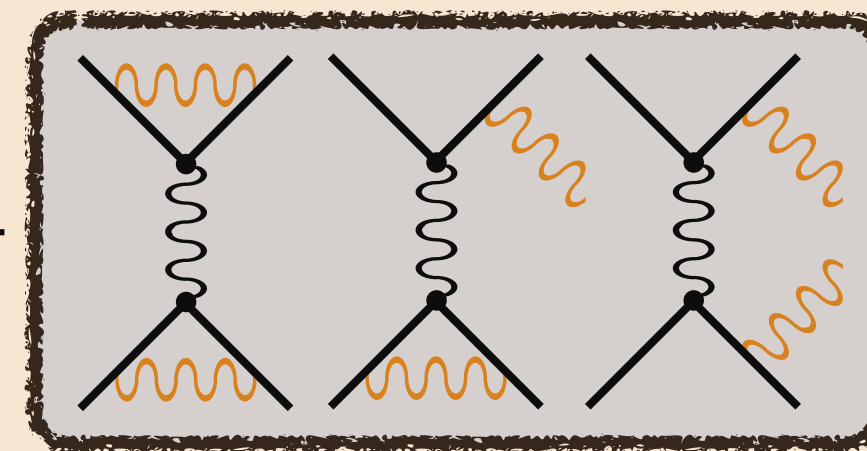
LO

+



NLO

+



NNLO

Slicing and Subtraction ←

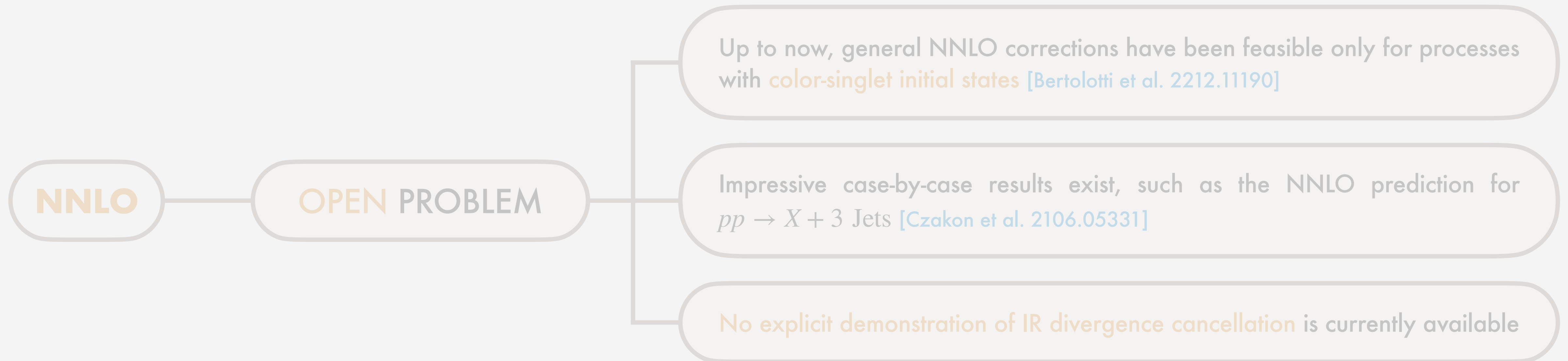
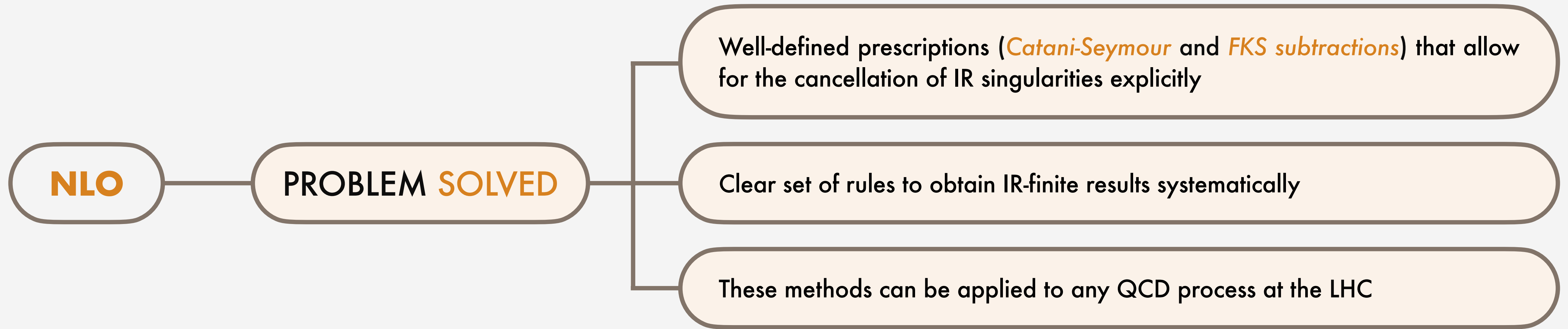
At NLO

- CATANI - SEYMOUR SUBTRACTION [[hep-ph/9605323](#)]
- FRIXIONE - KUNSZT - SIGNER SUBTRACTION (FKS) [[hep-ph/9512328](#)]

At NNLO

- ANTENNA SUBTRACTION [[hep-ph/0505111](#), [1301.4693](#)]
- COLORFUL SUBTRACTION [[1501.07226](#)]
- GEOMETRIC SUBTRACTION [[1804.07949](#)]
- LOCAL ANALYTIC SECTOR SUBTRACTION [[1806.09570](#), [1809.05444](#)]
- NESTED SOFT-COLLINEAR SUBTRACTION (NSC) [[1702.01352](#)] ←
- N - JETTINESS [[1505.04794](#), [1504.02131](#), [1504.02540](#)]
- PROJECTION TO BORN [[1506.02660](#)]
- q_T - SUBTRACTION [[hep-ph/0703012](#)]
- SECTOR-IMPROVED RESIDUE SUBTRACTION [[1005.0274](#), [1101.0642](#)]

THE PROBLEM



NLO

PROBLEM SOLVED

Well-defined prescriptions (*Catani-Seymour* and *FKS subtractions*) that allow for the cancellation of IR singularities explicitly

Clear set of rules to obtain IR-finite results systematically

These methods can be applied to any QCD process at the LHC

NNLO

OPEN PROBLEM

Up to now, general NNLO corrections have been feasible only for processes with *color-singlet initial states* [Bertolotti et al. 2212.11190]

Impressive case-by-case results exist, such as the NNLO prediction for $pp \rightarrow X + 3 \text{ Jets}$ [Czakon et al. 2106.05331]

No explicit demonstration of IR divergence cancellation is currently available

NLO

PROBLEM SOLVED

Well-defined prescriptions (*Catani-Seymour* and *FKS subtractions*) that allow for the cancellation of IR singularities explicitly

Clear set of rules to obtain IR-finite results systematically

These methods can be applied to any QCD process at the LHC

NNLO

WHY OPEN ?

Overlapping singularities: at NNLO, one encounters simultaneous soft and collinear divergences that overlap

Phase-space partitioning: to disentangle these singularities, we must **partition** and **sector** the phase space

Loss of transparency: **sectoring obscures** the **physical interpretation** of the cancellation mechanisms

Overlapping Singularities and Need for Sectoring

$$\langle \Delta^{(mn)} F_{LM}^{ab}[\dots | \mathfrak{m}, \mathfrak{n}] \rangle = \underbrace{\langle S_{mn} F_{LM}^{ab}[\dots | \mathfrak{m}, \mathfrak{n}] \rangle}_{\text{Double-Soft Counterterm}} + \underbrace{\langle \bar{S}_{mn} S_n \Delta^{(mn)} F_{LM}^{ab}[\dots | \mathfrak{m}, \mathfrak{n}] \rangle}_{\text{Single-Soft Counterterm}} + \underbrace{\langle \bar{S}_{mn} \bar{S}_n \Delta^{(mn)} F_{LM}^{ab}[\dots | \mathfrak{m}, \mathfrak{n}] \rangle}_{\text{Soft-Regulated Term}}$$

Overlapping Singularities and Need for Sectoring

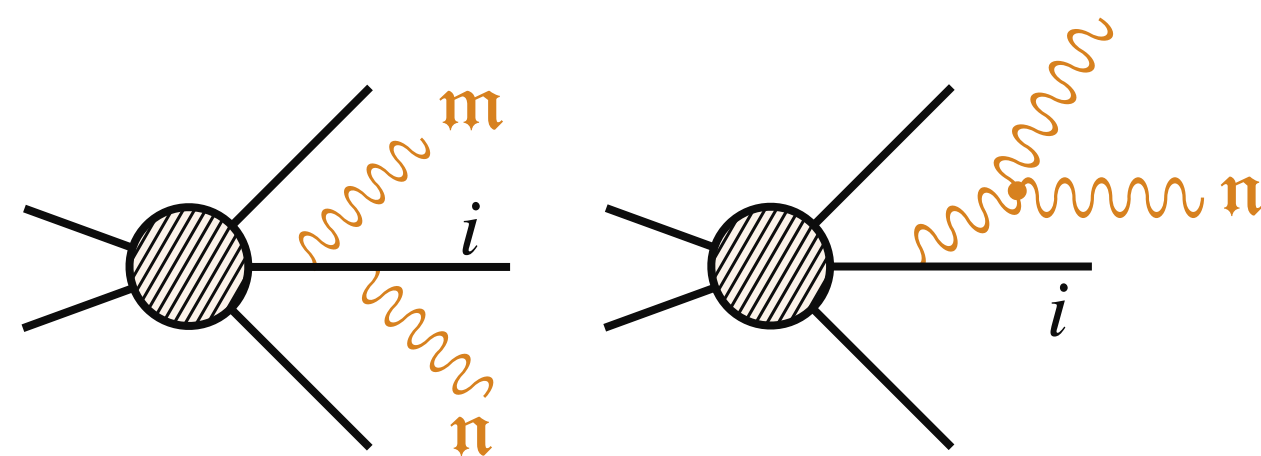
$$\langle \Delta^{(mn)} F_{LM}^{ab}[\dots | \mathfrak{m}, \mathfrak{n}] \rangle = \underbrace{\langle S_{mn} F_{LM}^{ab}[\dots | \mathfrak{m}, \mathfrak{n}] \rangle}_{\text{Double-Soft Counterterm}} + \underbrace{\langle \bar{S}_{mn} S_n \Delta^{(mn)} F_{LM}^{ab}[\dots | \mathfrak{m}, \mathfrak{n}] \rangle}_{\text{Single-Soft Counterterm}} + \underbrace{\langle \bar{S}_{mn} \bar{S}_n \Delta^{(mn)} F_{LM}^{ab}[\dots | \mathfrak{m}, \mathfrak{n}] \rangle}_{\text{Soft-Regulated Term}}$$

Problem of Overlapping Singularities

$$1 = \sum_{i,j} \omega^{mi,nj}$$

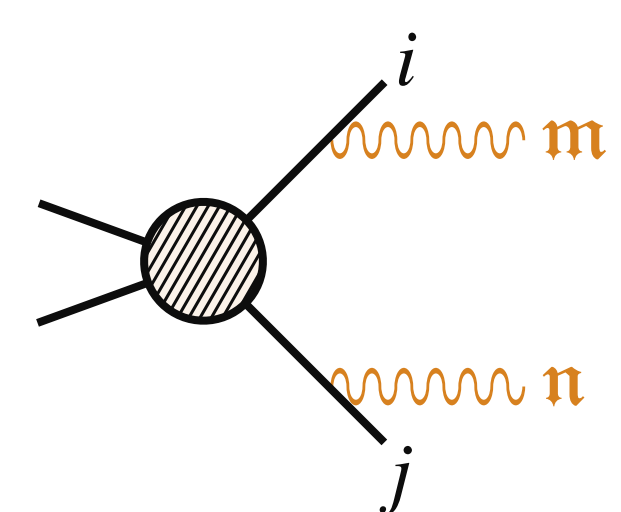
Triple-Collinear Sector

$$C_{im} - C_{in} - C_{mn} - C_{mn,i}$$



Double-Collinear Sector

$$C_{im} - C_{jn}$$



Overlapping Singularities and Need for Sectoring

$$\langle \Delta^{(mn)} F_{LM}^{ab}[\dots | \mathbf{m}, \mathbf{n}] \rangle = \underbrace{\langle S_{mn} F_{LM}^{ab}[\dots | \mathbf{m}, \mathbf{n}] \rangle}_{\text{Double-Soft Counterterm}} + \underbrace{\langle \bar{S}_{mn} S_n \Delta^{(mn)} F_{LM}^{ab}[\dots | \mathbf{m}, \mathbf{n}] \rangle}_{\text{Single-Soft Counterterm}} + \underbrace{\langle \bar{S}_{mn} \bar{S}_n \Delta^{(mn)} F_{LM}^{ab}[\dots | \mathbf{m}, \mathbf{n}] \rangle}_{\text{Soft-Regulated Term}}$$

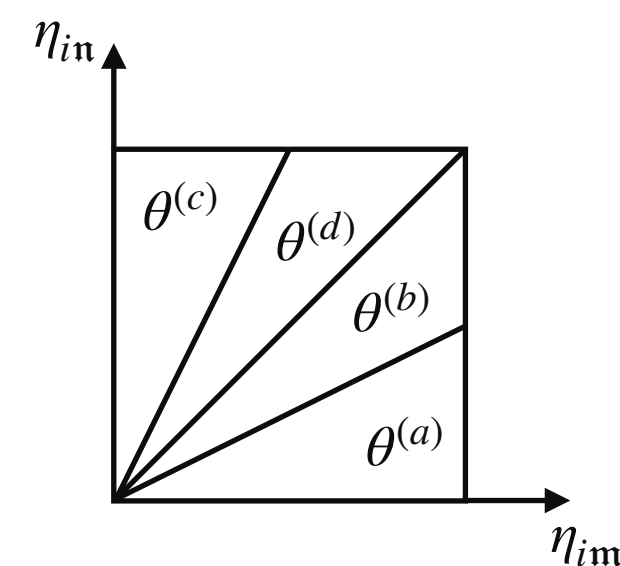
In **principle**, the original formula of the Nested can be applied to any process at the LHC.

In **practice**, identifying structures that can be combined with the VV and RV contributions becomes nearly impossible, rendering the calculation heavily process-dependent.

Sector Decomposition

[Czakon '10]

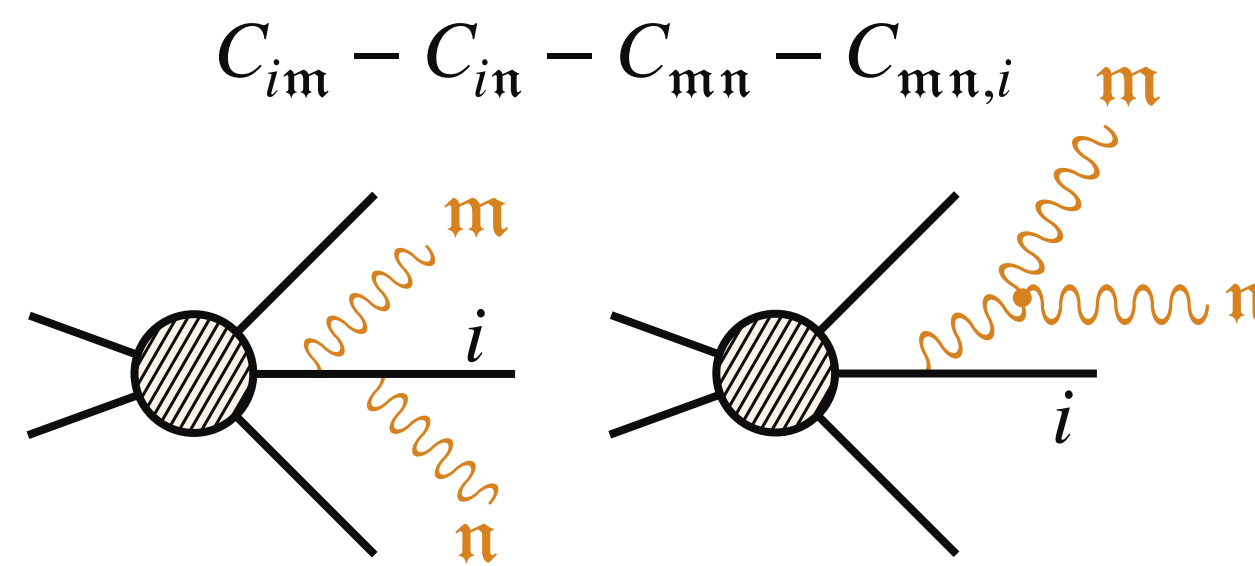
Angular Ordering



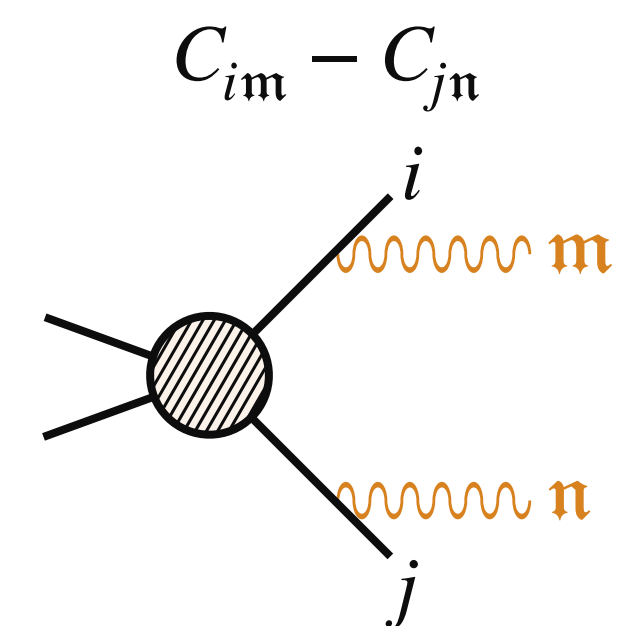
Problem of Overlapping Singularities

$$1 = \sum_{i,j} \omega^{mi,nj}$$

Triple-Collinear Sector



Double-Collinear Sector



THE SOLUTION

The Simple and Complete Subtraction Formula Of the Nested Soft-Collinear Scheme

$$2s_{ab} d\hat{\sigma}_{ab}^{\text{NNLO}} = 2s_{ab} \left[d\hat{\sigma}_{ab}^{\text{SU}} + d\hat{\sigma}_{ab}^{\text{DU}} + d\hat{\sigma}_{ab}^{\text{FR}} \right]$$

$$d\hat{\sigma}_{ab}^{\text{SU, sb, a}} = [\alpha_s] \sum_x \sum_m \sum_n \left\{ \langle \mathcal{O}_{\text{NLO}}^{(a,m)} \omega_{a||n}^{ma,na} \Delta^{(m)} \log\left(\frac{\eta_{am}}{2}\right) \hat{P}_{xa}^{(0)} \otimes F_{\text{LM}}^{xb}[\mathcal{B}_{N+1,n}(\mathbf{m})] \rangle \right. \\ \left. + \langle \mathcal{O}_{\text{NLO}}^{(m)} \Delta^{(m)} \mathcal{P}_{xa}^{\text{NLO}} \otimes F_{\text{LM}}^{xb}[\mathcal{B}_{N+1,n}(\mathbf{m})] \rangle \right\},$$

$$d\hat{\sigma}_{ab}^{\text{SU, sb, b}} = [\alpha_s] \sum_x \sum_m \sum_n \left\{ \langle \mathcal{O}_{\text{NLO}}^{(b,m)} \omega_{b||n}^{mb,nb} \Delta^{(m)} \log\left(\frac{\eta_{bm}}{2}\right) F_{\text{LM}}^{ax}[\mathcal{B}_{N+1,n}(\mathbf{m})] \otimes F_{\text{LM}}^{yb}[\mathcal{B}_{N+1,n}(\mathbf{m})] \rangle \right. \\ \left. + \langle \mathcal{O}_{\text{NLO}}^{(m)} \Delta^{(m)} F_{\text{LM}}^{ax}[\mathcal{B}_{N+1,n}(\mathbf{m})] \otimes \mathcal{P}_{xb}^{\text{NLO}} \rangle \right\}.$$

$$d\hat{\sigma}_{ab}^{\text{SU, el}} = \sum_m \sum_n \left\{ [\alpha_s] \langle \mathcal{O}_{\text{NLO}}^{(m)} \Delta^{(m)} [\delta_{mg} I_{\text{T}}^{(0)}(E_m) + \bar{\delta}_{mg} I_{\text{T}}^{(0)}(E_{\text{max}})] F_{\text{LM}}^{ab}[\mathcal{B}_{N+1,n}(\mathbf{m})] \rangle \right. \\ - \sum_{i \in \mathcal{H}} [\alpha_s] \langle \mathcal{O}_{\text{NLO}}^{(i,m)} \omega_{i||n}^{mi,ni} \log\left(\frac{\eta_{im}}{2}\right) \Delta^{(m)} [\gamma_i^{\perp, r} I_{\text{T}}^{(0)}(E_m) + \bar{\delta}_{mg} (\gamma_i + 2T_i^2 L_i)] F_{\text{LM}}^{ab}[\mathcal{B}_{N+1,n}(\mathbf{m})] \rangle \\ - \sum_{i \in \mathcal{H}} [\alpha_s] \langle \mathcal{O}_{\text{NLO}}^{(i,m)} \omega_{m||n}^{mi,ni} \log\left(\frac{\eta_{im}}{4(1-\eta_{im})}\right) \Delta^{(m)} [\gamma_m^{\perp, r} I_{\text{T}}^{(0)}(E_m) + 2T_q^2 L_m \bar{\delta}_{mg}] F_{\text{LM}}^{ab}[\mathcal{B}_{N+1,n}(\mathbf{m})] \rangle \\ \left. + \langle \mathcal{O}_{\text{NLO}}^{(m)} \Delta^{(m)} F_{\text{RV,fin}}^{ab}[\mathcal{B}_{N+1,n}(\mathbf{m})] \rangle + \sum_n \sum_{i \in \mathcal{H}} \frac{[\alpha_s]}{2} \langle \mathcal{O}_{\text{NLO}}^{(i,m)} \omega_{m||n}^{mi,ni} \Delta^{(m)} [\gamma_g^{\perp, r} F_{\text{LM}}^{ab}[\mathcal{B}_{N+1,n}(\mathbf{m}_g)]] \right. \\ \left. + \gamma_g^{\perp} (r_i^\mu r_i^\nu + g^{\mu\nu}) F_{\text{LM},\mu\nu}^{ab}[\mathcal{B}_{N+1,n}(\mathbf{m}_g)] \right\}.$$

$$d\hat{\sigma}_{ab}^{\text{FR}} = \langle \bar{S}_m \bar{S}_n \Omega_1 \Delta^{(mn)} \Theta_{mn} F_{\text{LM}}^{ab, \text{DS}}[\mathbf{m}, \mathbf{n}] \rangle + \langle \bar{S}_{mn} \bar{S}_n \Omega_1 \Delta^{(mn)} F_{\text{LM}}^{ab, \text{DS}}[\mathbf{m}, \mathbf{n}] \rangle$$

$$d\hat{\sigma}_{ab}^{\text{DU, db}} = \sum_{x,y} \sum_n [\alpha_s]^2 \langle \mathcal{P}_{xa}^{\text{NLO}} \otimes F_{\text{LM}}^{xy}[\mathcal{B}_{N,n}] \otimes \mathcal{P}_{yb}^{\text{NLO}} \rangle$$

$$d\hat{\sigma}_{ab}^{\text{DU, sb, a}} = \sum_x \sum_n \left\{ [\alpha_s] \langle \mathcal{P}_{xa}^{\text{NLO}} \otimes \mathcal{F}^{xb}[\mathcal{B}_{N,n}] \rangle + [\alpha_s]^2 \langle \mathcal{P}_{xa}^{\text{NNLO}} \otimes F_{\text{LM}}^{xb}[\mathcal{B}_{N,n}] \rangle \right\} \\ + \sum_{n'} [\alpha_s]^2 \langle \mathcal{P}_{aa}^{\mathcal{W}} \otimes [\mathcal{W}_a^{a||n, \text{fin}} \cdot F_{\text{LM}}^{ab}[\mathcal{B}_{N,n'}]] \rangle$$

$$d\hat{\sigma}_{ab}^{\text{DU, sb, b}} = \sum_x \sum_n \left\{ [\alpha_s] \langle \mathcal{F}^{ax}[\mathcal{B}_{N,n}] \otimes \mathcal{P}_{xb}^{\text{NLO}} \rangle + [\alpha_s]^2 \langle F_{\text{LM}}^{ax}[\mathcal{B}_{N,n}] \otimes \mathcal{P}_{xb}^{\text{NNLO}} \rangle \right\} \\ + \sum_{n'} [\alpha_s]^2 \langle [\mathcal{W}_b^{b||n, \text{fin}} \cdot F_{\text{LM}}^{ab}[\mathcal{B}_{N,n'}]] \otimes \mathcal{P}_{bb}^{\mathcal{W}} \rangle$$

$$d\hat{\sigma}_{ab}^{\text{DU, el}} = \sum_n \left\{ [\alpha_s]^2 \langle [I_{\text{cc}}^{\text{fin}} + I_{\text{ss}}^{\text{fin}} + I_{\text{tri}}^{\text{fin}} + I_{\text{unc}}^{\text{fin}}] \cdot F_{\text{LM}}^{ab}[\mathcal{B}_{N,n}] \rangle \right. \\ \left. + [\alpha_s]^2 \sum_{i \in \mathcal{H}} \langle [\theta_{\mathcal{H}_i} \gamma_{z, f_i \rightarrow f_i g}^{\mathcal{W}}(L_i) \mathcal{W}_i^{i||n, \text{fin}} + \delta^{(0)} \mathcal{W}_i^{m||n, \text{fin}} + \delta^{\perp, (0)} \mathcal{W}_i^{(i)}] \cdot F_{\text{LM}}^{ab}[\mathcal{B}_{N,n}] \rangle \right. \\ \left. + [\alpha_s] \langle I_{\text{T}}^{(0)} \cdot F_{\text{LV,fin}}^{ab}[\mathcal{B}_{N,n}] \rangle + \langle F_{\text{LV}^2, \text{fin}}^{ab}[\mathcal{B}_{N,n}] \rangle + \langle F_{\text{VV,fin}}^{ab}[\mathcal{B}_{N,n}] \rangle \right\}$$

SIMPLE FORMULA

The **Simple** and Complete Subtraction Formula Of the Nested Soft-Collinear Scheme

$$2s_{ab} d\hat{\sigma}_{ab}^{\text{NNLO}} = 2s_{ab} \left[d\hat{\sigma}_{ab}^{\text{SU}} + d\hat{\sigma}_{ab}^{\text{DU}} + d\hat{\sigma}_{ab}^{\text{FR}} \right]$$

$$d\hat{\sigma}_{ab}^{\text{SU, sb, a}} = [\alpha_s] \sum_x \sum_{\mathbf{m}} \sum_n \left\{ \langle \mathcal{O}_{\text{NLO}}^{(a, \mathbf{m})} \omega_{a||n}^{\mathbf{m}a, \mathbf{n}a} \Delta^{(\mathbf{m})} \log\left(\frac{\eta_{a\mathbf{m}}}{2}\right) \hat{P}_{xa}^{(0)} \otimes F_{\text{LM}}^{xb}[\mathcal{B}_{N+1, n}(\mathbf{m})] \rangle \right. \\ \left. + \langle \mathcal{O}_{\text{NLO}}^{(\mathbf{m})} \Delta^{(\mathbf{m})} \mathcal{P}_{xa}^{\text{NLO}} \otimes F_{\text{LM}}^{xb}[\mathcal{B}_{N+1, n}(\mathbf{m})] \rangle \right\},$$

$$d\hat{\sigma}_{ab}^{\text{SU, sb, b}} = [\alpha_s] \sum_x \sum_{\mathbf{m}} \sum_n \left\{ \langle \mathcal{O}_{\text{NLO}}^{(b, \mathbf{m})} \omega_{b||n}^{\mathbf{m}b, \mathbf{n}b} \Delta^{(\mathbf{m})} \log\left(\frac{\eta_{b\mathbf{m}}}{2}\right) F_{\text{LM}}^{ax}[\mathcal{B}_{N+1, n}(\mathbf{m})] \otimes \hat{P}_{xb}^{(0)} \rangle \right. \\ \left. + \langle \mathcal{O}_{\text{NLO}}^{(\mathbf{m})} \Delta^{(\mathbf{m})} F_{\text{LM}}^{ax}[\mathcal{B}_{N+1, n}(\mathbf{m})] \otimes \mathcal{P}_{xb}^{\text{NLO}} \rangle \right\}.$$

$$d\hat{\sigma}_{ab}^{\text{SU, el}} = \sum_{\mathbf{m}} \sum_n \left\{ [\alpha_s] \langle \mathcal{O}_{\text{NLO}}^{(\mathbf{m})} \Delta^{(\mathbf{m})} [\delta_{\mathbf{m}g} I_{\text{T}}^{(0)}(E_{\mathbf{m}}) + \bar{\delta}_{\mathbf{m}g} I_{\text{T}}^{(0)}(E_{\text{max}})] \cdot F_{\text{LM}}^{ab}[\mathcal{B}_{N+1, n}(\mathbf{m})] \rangle \right. \\ - \sum_{i \in \mathcal{H}} [\alpha_s] \langle \mathcal{O}_{\text{NLO}}^{(i, \mathbf{m})} \omega_{i||n}^{\mathbf{m}i, \mathbf{n}i} \log\left(\frac{\eta_{i\mathbf{m}}}{2}\right) \Delta^{(\mathbf{m})} [\delta_{\mathbf{m}g} (\gamma_i + 2\mathbf{T}_i^2 L_i(E_{\mathbf{m}})) + \bar{\delta}_{\mathbf{m}g} (\gamma_i + 2\mathbf{T}_i^2 L_i)] F_{\text{LM}}^{ab}[\mathcal{B}_{N+1, n}(\mathbf{m})] \rangle \\ - \sum_{i \in \mathcal{H}} [\alpha_s] \langle \mathcal{O}_{\text{NLO}}^{(i, \mathbf{m})} \omega_{\mathbf{m}||n}^{\mathbf{m}i, \mathbf{n}i} \log\left(\frac{\eta_{i\mathbf{m}}}{4(1-\eta_{i\mathbf{m}})}\right) \Delta^{(\mathbf{m})} [\gamma_{\mathbf{m}} + 2\mathbf{T}_q^2 L_{\mathbf{m}} \bar{\delta}_{\mathbf{m}g}] F_{\text{LM}}^{ab}[\mathcal{B}_{N+1, n}(\mathbf{m})] \rangle \\ \left. + \langle \mathcal{O}_{\text{NLO}}^{(\mathbf{m})} \Delta^{(\mathbf{m})} F_{\text{RV, fin}}^{ab}[\mathcal{B}_{N+1, n}(\mathbf{m})] \rangle \right\} + \sum_n \sum_{i \in \mathcal{H}} \frac{[\alpha_s]}{2} \langle \mathcal{O}_{\text{NLO}}^{(i, \mathbf{m})} \omega_{\mathbf{m}||n}^{\mathbf{m}i, \mathbf{n}i} \Delta^{(\mathbf{m})} [\gamma_g^{\perp, r} F_{\text{LM}}^{ab}[\mathcal{B}_{N+1, n}(\mathbf{m}_g)]] \\ + \gamma_g^{\perp} (r_i^\mu r_i^\nu + g^{\mu\nu}) F_{\text{LM, } \mu\nu}^{ab}[\mathcal{B}_{N+1, n}(\mathbf{m}_g)] \rangle.$$

$$d\hat{\sigma}_{ab}^{\text{FR}} = \langle \bar{S}_{\mathbf{m}n} \bar{S}_n \Omega_1 \Delta^{(\mathbf{m}n)} \Theta_{\mathbf{m}n} F_{\text{LM}}^{ab, \text{DS}}[\mathbf{m}, \mathbf{n}] \rangle + \langle \bar{S}_{\mathbf{m}n} \bar{S}_n \Omega_1 \Delta^{(\mathbf{m}n)} F_{\text{LM}}^{ab, \text{DS}}[\mathbf{m}, \mathbf{n}] \rangle$$

$$d\hat{\sigma}_{ab}^{\text{DU, db}} = \sum_{x, y} \sum_n [\alpha_s]^2 \langle \mathcal{P}_{xa}^{\text{NLO}} \otimes F_{\text{LM}}^{xy}[\mathcal{B}_{N, n}] \otimes \mathcal{P}_{yb}^{\text{NLO}} \rangle$$

$$d\hat{\sigma}_{ab}^{\text{DU, sb, a}} = \sum_x \sum_n \left[[\alpha_s] \langle \mathcal{P}_{xa}^{\text{NLO}} \otimes \mathcal{F}^{xb}[\mathcal{B}_{N, n}] \rangle + [\alpha_s]^2 \langle \mathcal{P}_{xa}^{\text{NNLO}} \otimes F_{\text{LM}}^{xb}[\mathcal{B}_{N, n}] \rangle \right] \\ + \sum_{n'} [\alpha_s]^2 \langle \mathcal{P}_{aa}^{\mathcal{W}} \otimes [\mathcal{W}_a^{a||n, \text{fin}} \cdot F_{\text{LM}}^{ab}[\mathcal{B}_{N, n'}]] \rangle$$

$$d\hat{\sigma}_{ab}^{\text{DU, sb, b}} = \sum_x \sum_n \left[[\alpha_s] \langle \mathcal{F}^{ax}[\mathcal{B}_{N, n}] \otimes \mathcal{P}_{xb}^{\text{NLO}} \rangle + [\alpha_s]^2 \langle F_{\text{LM}}^{ax}[\mathcal{B}_{N, n}] \otimes \mathcal{P}_{xb}^{\text{NNLO}} \rangle \right] \\ + \sum_{n'} [\alpha_s]^2 \langle [\mathcal{W}_b^{b||n, \text{fin}} \cdot F_{\text{LM}}^{ab}[\mathcal{B}_{N, n'}]] \otimes \mathcal{P}_{bb}^{\mathcal{W}} \rangle$$

$$d\hat{\sigma}_{ab}^{\text{DU, el}} = \sum_n \left\{ [\alpha_s]^2 \langle [I_{\text{cc}}^{\text{fin}} + I_{\text{ss}}^{\text{fin}} + I_{\text{tri}}^{\text{fin}} + I_{\text{unc}}^{\text{fin}}] \cdot F_{\text{LM}}^{ab}[\mathcal{B}_{N, n}] \rangle \right. \\ + [\alpha_s]^2 \sum_{i \in \mathcal{H}} \langle [\theta_{\mathcal{H}_i} \gamma_{z, f_i \rightarrow f_i g}^{\mathcal{W}}(L_i) \mathcal{W}_i^{i||n, \text{fin}} + \delta^{(0)} \mathcal{W}_i^{\mathbf{m}||n, \text{fin}} + \delta^{\perp, (0)} \mathcal{W}_i^{(i)}] \cdot F_{\text{LM}}^{ab}[\mathcal{B}_{N, n}] \rangle \\ \left. + [\alpha_s] \langle I_{\text{T}}^{(0)} \cdot F_{\text{LV, fin}}^{ab}[\mathcal{B}_{N, n}] \rangle + \langle F_{\text{LV}^2, \text{fin}}^{ab}[\mathcal{B}_{N, n}] \rangle + \langle F_{\text{VV, fin}}^{ab}[\mathcal{B}_{N, n}] \rangle \right\}$$

The **Simple** and Complete Subtraction Formula Of the Nested Soft-Collinear Scheme

$$2s_{ab} d\hat{\sigma}_{ab}^{\text{NNLO}} = 2s_{ab} \left[d\hat{\sigma}_{ab}^{\text{SU}} + d\hat{\sigma}_{ab}^{\text{DU}} + d\hat{\sigma}_{ab}^{\text{FR}} \right]$$

The only INPUTS are the LO configurations

$$d\hat{\sigma}_{ab}^{\text{DU,db}} = \sum_{x,y} \sum_n [\alpha_s]^2 \langle \mathcal{P}_{xa}^{\text{NLO}} \otimes F_{\text{LM}}^{xy}[\mathcal{B}_{N,n}] \otimes \mathcal{P}_{yb}^{\text{NLO}} \rangle$$

$$d\hat{\sigma}_{ab}^{\text{DU,sb,a}} = \sum_x \sum_n \left[[\alpha_s] \langle \mathcal{P}_{xa}^{\text{NLO}} \otimes \mathcal{F}^{xb}[\mathcal{B}_{N,n}] \rangle + [\alpha_s]^2 \langle \mathcal{P}_{xa}^{\text{NNLO}} \otimes F_{\text{LM}}^{xb}[\mathcal{B}_{N,n}] \rangle \right] \\ + \sum_{n'} [\alpha_s]^2 \langle \mathcal{P}_{aa}^{\mathcal{W}} \otimes [\mathcal{W}_a^{a||n,\text{fin}} \cdot F_{\text{LM}}^{ab}[\mathcal{B}_{N,n'}]] \rangle$$

$$d\hat{\sigma}_{ab}^{\text{DU,sb,b}} = \sum_x \sum_n \left[[\alpha_s] \langle \mathcal{F}^{ax}[\mathcal{B}_{N,n}] \otimes \mathcal{P}_{xb}^{\text{NLO}} \rangle + [\alpha_s]^2 \langle F_{\text{LM}}^{ax}[\mathcal{B}_{N,n}] \otimes \mathcal{P}_{xb}^{\text{NNLO}} \rangle \right] \\ + \sum_{n'} [\alpha_s]^2 \langle [\mathcal{W}_b^{b||n,\text{fin}} \cdot F_{\text{LM}}^{ab}[\mathcal{B}_{N,n'}]] \otimes \mathcal{P}_{bb}^{\mathcal{W}} \rangle$$

$$d\hat{\sigma}_{ab}^{\text{DU,el}} = \sum_n \left\{ [\alpha_s]^2 \langle [I_{\text{cc}}^{\text{fin}} + I_{\text{ss}}^{\text{fin}} + I_{\text{tri}}^{\text{fin}} + I_{\text{unc}}^{\text{fin}}] \cdot F_{\text{LM}}^{ab}[\mathcal{B}_{N,n}] \rangle \right. \\ + [\alpha_s]^2 \sum_{i \in \mathcal{H}} \left\langle \left[\theta_{\mathcal{H}_f} \gamma_{z,f_i \rightarrow f_i g}^{\mathcal{W}}(L_i) \mathcal{W}_i^{i||n,\text{fin}} + \delta^{(0)} \mathcal{W}_i^{\text{m}||n,\text{fin}} + \delta^{\perp,(0)} \mathcal{W}_f^{(i)} \right] \cdot F_{\text{LM}}^{ab}[\mathcal{B}_{N,n}] \right\rangle \\ \left. + [\alpha_s] \langle I_{\text{T}}^{(0)} \cdot F_{\text{LV,fin}}^{ab}[\mathcal{B}_{N,n}] \rangle + \langle F_{\text{LV}^2,\text{fin}}^{ab}[\mathcal{B}_{N,n}] \rangle + \langle F_{\text{VV,fin}}^{ab}[\mathcal{B}_{N,n}] \rangle \right\}$$

The **Simple** and Complete Subtraction Formula Of the Nested Soft-Collinear Scheme

$$2s_{ab} d\hat{\sigma}_{ab}^{\text{NNLO}} = 2s_{ab} \left[d\hat{\sigma}_{ab}^{\text{SU}} + d\hat{\sigma}_{ab}^{\text{DU}} + d\hat{\sigma}_{ab}^{\text{FR}} \right]$$

The only INPUTS are the LO configurations

The formula sums over all possible LO configurations

$$\begin{aligned}
 d\hat{\sigma}_{ab}^{\text{DU,db}} &= \sum_{x,y} \sum_n [\alpha_s]^2 \langle \mathcal{P}_{xa}^{\text{NLO}} \otimes F_{\text{LM}}^{xy}[\mathcal{B}_{N,n}] \otimes \mathcal{P}_{yb}^{\text{NLO}} \rangle \\
 d\hat{\sigma}_{ab}^{\text{DU,sb,a}} &= \sum_x \sum_n \left[[\alpha_s] \langle \mathcal{P}_{xa}^{\text{NLO}} \otimes \mathcal{F}^{xb}[\mathcal{B}_{N,n}] \rangle + [\alpha_s]^2 \langle \mathcal{P}_{xa}^{\text{NNLO}} \otimes F_{\text{LM}}^{xb}[\mathcal{B}_{N,n}] \rangle \right] \\
 &\quad + \sum_{n'} [\alpha_s]^2 \langle \mathcal{P}_{aa}^{\mathcal{W}} \otimes [\mathcal{W}_a^{a||n,\text{fin}} \cdot F_{\text{LM}}^{ab}[\mathcal{B}_{N,n'}]] \rangle \\
 d\hat{\sigma}_{ab}^{\text{DU,sb,b}} &= \sum_x \sum_n \left[[\alpha_s] \langle \mathcal{F}^{ax}[\mathcal{B}_{N,n}] \otimes \mathcal{P}_{xb}^{\text{NLO}} \rangle + [\alpha_s]^2 \langle F_{\text{LM}}^{ax}[\mathcal{B}_{N,n}] \otimes \mathcal{P}_{xb}^{\text{NNLO}} \rangle \right] \\
 &\quad + \sum_{n'} [\alpha_s]^2 \langle [\mathcal{W}_b^{b||n,\text{fin}} \cdot F_{\text{LM}}^{ab}[\mathcal{B}_{N,n'}]] \otimes \mathcal{P}_{bb}^{\mathcal{W}} \rangle \\
 d\hat{\sigma}_{ab}^{\text{DU,el}} &= \sum_n \left\{ [\alpha_s]^2 \langle [I_{\text{cc}}^{\text{fin}} + I_{\text{ss}}^{\text{fin}} + I_{\text{tri}}^{\text{fin}} + I_{\text{unc}}^{\text{fin}}] \cdot F_{\text{LM}}^{ab}[\mathcal{B}_{N,n}] \rangle \right. \\
 &\quad + [\alpha_s]^2 \sum_{i \in \mathcal{H}} \left\langle \left[\theta_{\mathcal{H}_i} \gamma_{z,f_i \rightarrow f_i g}^{\mathcal{W}}(L_i) \mathcal{W}_i^{i||n,\text{fin}} + \delta^{(0)} \mathcal{W}_i^{m||n,\text{fin}} + \delta^{\perp,(0)} \mathcal{W}_i^{(i)} \right] \cdot F_{\text{LM}}^{ab}[\mathcal{B}_{N,n}] \right\rangle \\
 &\quad \left. + [\alpha_s] \langle I_{\text{T}}^{(0)} \cdot F_{\text{LV,fin}}^{ab}[\mathcal{B}_{N,n}] \rangle + \langle F_{\text{LV}^2,\text{fin}}^{ab}[\mathcal{B}_{N,n}] \rangle + \langle F_{\text{VV,fin}}^{ab}[\mathcal{B}_{N,n}] \rangle \right\}
 \end{aligned}$$

The **Simple** and Complete Subtraction Formula Of the Nested Soft-Collinear Scheme

Modular Structure

- = Boosted Contributions
- = Elastic Contributions

$$2s_{ab} d\hat{\sigma}_{ab}^{\text{NNLO}} = 2s_{ab} \left[d\hat{\sigma}_{ab}^{\text{SU}} + d\hat{\sigma}_{ab}^{\text{DU}} + d\hat{\sigma}_{ab}^{\text{FR}} \right]$$

$$d\hat{\sigma}_{ab}^{\text{SU, sb, a}} = [\alpha_s] \sum_x \sum_m \sum_n \left\{ \langle \mathcal{O}_{\text{NLO}}^{(a,m)} \omega_{a||n}^{ma,na} \Delta^{(m)} \log\left(\frac{\eta_{am}}{2}\right) \hat{P}_{xa}^{(0)} \otimes F_{\text{LM}}^{xb}[\mathcal{B}_{N+1,n}(\mathbf{m})] \rangle \right. \\ \left. + \langle \mathcal{O}_{\text{NLO}}^{(m)} \Delta^{(m)} \mathcal{P}_{xa}^{\text{NLO}} \otimes F_{\text{LM}}^{xb}[\mathcal{B}_{N+1,n}(\mathbf{m})] \rangle \right\}$$

$$d\hat{\sigma}_{ab}^{\text{SU, sb, b}} = [\alpha_s] \sum_x \sum_m \sum_n \left\{ \langle \mathcal{O}_{\text{NLO}}^{(b,m)} \omega_{b||n}^{mb,nb} \Delta^{(m)} \log\left(\frac{\eta_{bm}}{2}\right) F_{\text{LM}}^{ax}[\mathcal{B}_{N+1,n}(\mathbf{m})] \otimes \hat{P}_{xb}^{(0)} \rangle \right. \\ \left. + \langle \mathcal{O}_{\text{NLO}}^{(m)} \Delta^{(m)} F_{\text{LM}}^{ax}[\mathcal{B}_{N+1,n}(\mathbf{m})] \otimes \mathcal{P}_{xb}^{\text{NLO}} \rangle \right\}$$

$$d\hat{\sigma}_{ab}^{\text{SU, el}} = \sum_m \sum_n \left\{ [\alpha_s] \langle \mathcal{O}_{\text{NLO}}^{(m)} \Delta^{(m)} [\delta_{\text{mg}} I_{\text{T}}^{(0)}(E_m) + \bar{\delta}_{\text{mg}} I_{\text{T}}^{(0)}(E_{\text{max}})] \cdot F_{\text{LM}}^{ab}[\mathcal{B}_{N+1,n}(\mathbf{m})] \rangle \right. \\ - \sum_{i \in \mathcal{H}} [\alpha_s] \langle \mathcal{O}_{\text{NLO}}^{(i,m)} \omega_{i||n}^{mi,ni} \log\left(\frac{\eta_{im}}{2}\right) \Delta^{(m)} [\delta_{\text{mg}} (\gamma_i + 2\mathbf{T}_i^2 L_i(E_m)) + \bar{\delta}_{\text{mg}} (\gamma_i + 2\mathbf{T}_i^2 L_i)] F_{\text{LM}}^{ab}[\mathcal{B}_{N+1,n}(\mathbf{m})] \rangle \\ - \sum_{i \in \mathcal{H}} [\alpha_s] \langle \mathcal{O}_{\text{NLO}}^{(i,m)} \omega_{m||n}^{mi,ni} \log\left(\frac{\eta_{im}}{4(1-\eta_{im})}\right) \Delta^{(m)} [\gamma_m + 2\mathbf{T}_q^2 L_m \bar{\delta}_{\text{mg}}] F_{\text{LM}}^{ab}[\mathcal{B}_{N+1,n}(\mathbf{m})] \rangle \\ \left. + \langle \mathcal{O}_{\text{NLO}}^{(m)} \Delta^{(m)} F_{\text{RV,fin}}^{ab}[\mathcal{B}_{N+1,n}(\mathbf{m})] \rangle \right\} + \sum_n \sum_{i \in \mathcal{H}} \frac{[\alpha_s]}{2} \langle \mathcal{O}_{\text{NLO}}^{(i,m)} \omega_{m||n}^{mi,ni} \Delta^{(m)} [\gamma_g^{\perp,r} F_{\text{LM}}^{ab}[\mathcal{B}_{N+1,n}(\mathbf{m}_g)] \\ + \gamma_g^{\perp} (r_i^\mu r_i^\nu + g^{\mu\nu}) F_{\text{LM},\mu\nu}^{ab}[\mathcal{B}_{N+1,n}(\mathbf{m}_g)] \rangle$$

$$d\hat{\sigma}_{ab}^{\text{FR}} = \langle \bar{S}_{\text{mn}} \bar{S}_n \Omega_1 \Delta^{(\text{mn})} \Theta_{\text{mn}} F_{\text{LM}}^{ab, \text{DS}}[\mathbf{m}, \mathbf{n}] \rangle + \langle \bar{S}_{\text{mn}} \bar{S}_n \Omega_1 \Delta^{(\text{mn})} F_{\text{LM}}^{ab, \text{DS}}[\mathbf{m}, \mathbf{n}] \rangle$$

$$d\hat{\sigma}_{ab}^{\text{DU, db}} = \sum_{x,y} \sum_n [\alpha_s]^2 \langle \mathcal{P}_{xa}^{\text{NLO}} \otimes F_{\text{LM}}^{xy}[\mathcal{B}_{N,n}] \otimes \mathcal{P}_{yb}^{\text{NLO}} \rangle$$

$$d\hat{\sigma}_{ab}^{\text{DU, sb, a}} = \sum_x \sum_n \left[[\alpha_s] \langle \mathcal{P}_{xa}^{\text{NLO}} \otimes \mathcal{F}^{xb}[\mathcal{B}_{N,n}] \rangle + [\alpha_s]^2 \langle \mathcal{P}_{xa}^{\text{NNLO}} \otimes F_{\text{LM}}^{xb}[\mathcal{B}_{N,n}] \rangle \right] \\ + \sum_{n'} [\alpha_s]^2 \langle \mathcal{P}_{aa}^{\mathcal{W}} \otimes [\mathcal{W}_a^{a||n, \text{fin}} \cdot F_{\text{LM}}^{ab}[\mathcal{B}_{N,n'}]] \rangle$$

$$d\hat{\sigma}_{ab}^{\text{DU, sb, b}} = \sum_x \sum_n \left[[\alpha_s] \langle \mathcal{F}^{ax}[\mathcal{B}_{N,n}] \otimes \mathcal{P}_{xb}^{\text{NLO}} \rangle + [\alpha_s]^2 \langle F_{\text{LM}}^{ax}[\mathcal{B}_{N,n}] \otimes \mathcal{P}_{xb}^{\text{NNLO}} \rangle \right] \\ + \sum_{n'} [\alpha_s]^2 \langle [\mathcal{W}_b^{b||n, \text{fin}} \cdot F_{\text{LM}}^{ab}[\mathcal{B}_{N,n'}]] \otimes \mathcal{P}_{bb}^{\mathcal{W}} \rangle$$

$$d\hat{\sigma}_{ab}^{\text{DU, el}} = \sum_n \left\{ [\alpha_s]^2 \langle [I_{\text{cc}}^{\text{fin}} + I_{\text{ss}}^{\text{fin}} + I_{\text{tri}}^{\text{fin}} + I_{\text{unc}}^{\text{fin}}] \cdot F_{\text{LM}}^{ab}[\mathcal{B}_{N,n}] \rangle \right. \\ \left. + [\alpha_s]^2 \sum_{i \in \mathcal{H}} \langle [\theta_{\mathcal{H}_i} \gamma_{z, f_i \rightarrow f_i g}^{\mathcal{W}}(L_i) \mathcal{W}_i^{i||n, \text{fin}} + \delta^{(0)} \mathcal{W}_i^{m||n, \text{fin}} + \delta^{\perp, (0)} \mathcal{W}_r^{(i)}] \cdot F_{\text{LM}}^{ab}[\mathcal{B}_{N,n}] \rangle \right. \\ \left. + [\alpha_s] \langle I_{\text{T}}^{(0)} \cdot F_{\text{LV,fin}}^{ab}[\mathcal{B}_{N,n}] \rangle + \langle F_{\text{LV}^2, \text{fin}}^{ab}[\mathcal{B}_{N,n}] \rangle + \langle F_{\text{VV,fin}}^{ab}[\mathcal{B}_{N,n}] \rangle \right\}$$

The **Simple** and Complete Subtraction Formula Of the Nested Soft-Collinear Scheme

It Works Also for
Processes With a **Color-Singlet** Initial State

$$2s_{ab} d\hat{\sigma}_{ab}^{\text{NNLO}} = 2s_{ab} \left[d\hat{\sigma}_{ab}^{\text{SU}} + d\hat{\sigma}_{ab}^{\text{DU}} + d\hat{\sigma}_{ab}^{\text{FR}} \right]$$

● = Elastic Contributions

$$d\hat{\sigma}_{ab}^{\text{SU, sb, a}} = [\alpha_s] \sum_x \sum_m \sum_n \left\{ \langle \mathcal{O}_{\text{NLO}}^{(a,m)} \omega_{a||n}^{ma,na} \Delta^{(m)} \log\left(\frac{\eta_{am}}{2}\right) \hat{P}_{xa}^{(0)} \otimes F_{\text{LM}}^{xb}[\mathcal{B}_{N+1,n}(\mathbf{m})] \rangle \right. \\ \left. + \langle \mathcal{O}_{\text{NLO}}^{(m)} \Delta^{(m)} \mathcal{P}_{xa}^{\text{NLO}} \otimes F_{\text{LM}}^{xb}[\mathcal{B}_{N+1,n}(\mathbf{m})] \rangle \right\}$$

$$d\hat{\sigma}_{ab}^{\text{SU, sb, b}} = [\alpha_s] \sum_x \sum_m \sum_n \left\{ \langle \mathcal{O}_{\text{NLO}}^{(b,m)} \omega_{b||n}^{mb,nb} \Delta^{(m)} \log\left(\frac{\eta_{bm}}{2}\right) F_{\text{LM}}^{ax}[\mathcal{B}_{N+1,n}(\mathbf{m})] \otimes \hat{P}_{xb}^{(0)} \rangle \right. \\ \left. + \langle \mathcal{O}_{\text{NLO}}^{(m)} \Delta^{(m)} F_{\text{LM}}^{ax}[\mathcal{B}_{N+1,n}(\mathbf{m})] \otimes \mathcal{P}_{xb}^{\text{NLO}} \rangle \right\}$$

$$d\hat{\sigma}_{ab}^{\text{SU, el}} = \sum_m \sum_n \left\{ [\alpha_s] \langle \mathcal{O}_{\text{NLO}}^{(m)} \Delta^{(m)} [\delta_{\text{mg}} I_{\text{T}}^{(0)}(E_{\text{m}}) + \bar{\delta}_{\text{mg}} I_{\text{T}}^{(0)}(E_{\text{max}})] \cdot F_{\text{LM}}^{ab}[\mathcal{B}_{N+1,n}(\mathbf{m})] \rangle \right. \\ - \sum_{i \in \mathcal{H}} [\alpha_s] \langle \mathcal{O}_{\text{NLO}}^{(i,m)} \omega_{i||n}^{mi,ni} \log\left(\frac{\eta_{im}}{2}\right) \Delta^{(m)} [\delta_{\text{mg}} (\gamma_i + 2\mathbf{T}_i^2 L_i(E_{\text{m}})) + \bar{\delta}_{\text{mg}} (\gamma_i + 2\mathbf{T}_i^2 L_i)] F_{\text{LM}}^{ab}[\mathcal{B}_{N+1,n}(\mathbf{m})] \rangle \\ - \sum_{i \in \mathcal{H}} [\alpha_s] \langle \mathcal{O}_{\text{NLO}}^{(i,m)} \omega_{m||n}^{mi,ni} \log\left(\frac{\eta_{im}}{4(1-\eta_{im})}\right) \Delta^{(m)} [\gamma_m + 2\mathbf{T}_q^2 L_m \bar{\delta}_{\text{mg}}] F_{\text{LM}}^{ab}[\mathcal{B}_{N+1,n}(\mathbf{m})] \rangle \\ + \langle \mathcal{O}_{\text{NLO}}^{(m)} \Delta^{(m)} F_{\text{RV,fin}}^{ab}[\mathcal{B}_{N+1,n}(\mathbf{m})] \rangle \left. + \sum_n \sum_{i \in \mathcal{H}} \frac{[\alpha_s]}{2} \langle \mathcal{O}_{\text{NLO}}^{(i,m)} \omega_{m||n}^{mi,ni} \Delta^{(m)} [\gamma_g^{\perp,r} F_{\text{LM}}^{ab}[\mathcal{B}_{N+1,n}(\mathbf{m}_g)]] \right. \\ \left. + \gamma_g^{\perp} (r_i^\mu r_i^\nu + g^{\mu\nu}) F_{\text{LM},\mu\nu}^{ab}[\mathcal{B}_{N+1,n}(\mathbf{m}_g)] \right\}$$

$$d\hat{\sigma}_{ab}^{\text{FR}} = \langle \bar{S}_{\text{mn}} \bar{S}_n \Omega_1 \Delta^{(\text{mn})} \Theta_{\text{mn}} F_{\text{LM}}^{ab, \text{DS}}[\mathbf{m}, \mathbf{n}] \rangle + \langle \bar{S}_{\text{mn}} \bar{S}_n \Omega_1 \Delta^{(\text{mn})} F_{\text{LM}}^{ab, \text{DS}}[\mathbf{m}, \mathbf{n}] \rangle$$

$$d\hat{\sigma}_{ab}^{\text{DU, db}} = \sum_{x,y} \sum_n [\alpha_s]^2 \langle \mathcal{P}_{xa}^{\text{NLO}} \otimes F_{\text{LM}}^{xy}[\mathcal{B}_{N,n}] \otimes \mathcal{P}_{yb}^{\text{NLO}} \rangle$$

$$d\hat{\sigma}_{ab}^{\text{DU, sb, a}} = \sum_x \sum_n \left[[\alpha_s] \langle \mathcal{P}_{xa}^{\text{NLO}} \otimes F_{\text{LM}}^{xb}[\mathcal{B}_{N,n}] \rangle + [\alpha_s]^2 \langle \mathcal{P}_{xa}^{\text{NNLO}} \otimes F_{\text{LM}}^{xb}[\mathcal{B}_{N,n}] \rangle \right] \\ + \sum_{n'} [\alpha_s]^2 \langle \mathcal{P}_{aa}^{\mathcal{W}} \otimes [\mathcal{W}_a^{a||n, \text{fin}} \cdot F_{\text{LM}}^{ab}[\mathcal{B}_{N,n'}]] \rangle$$

$$d\hat{\sigma}_{ab}^{\text{DU, sb, b}} = \sum_x \sum_n \left[[\alpha_s] \langle F_{\text{LM}}^{ax}[\mathcal{B}_{N,n}] \otimes \mathcal{P}_{xb}^{\text{NLO}} \rangle + [\alpha_s]^2 \langle F_{\text{LM}}^{ax}[\mathcal{B}_{N,n}] \otimes \mathcal{P}_{xb}^{\text{NNLO}} \rangle \right] \\ + \sum_{n'} [\alpha_s]^2 \langle [\mathcal{W}_b^{b||n, \text{fin}} \cdot F_{\text{LM}}^{ab}[\mathcal{B}_{N,n'}]] \otimes \mathcal{P}_{bb}^{\mathcal{W}} \rangle$$

$$d\hat{\sigma}_{ab}^{\text{DU, el}} = \sum_n \left\{ [\alpha_s]^2 \langle [I_{\text{cc}}^{\text{fin}} + I_{\text{ss}}^{\text{fin}} + I_{\text{tri}}^{\text{fin}} + I_{\text{unc}}^{\text{fin}}] \cdot F_{\text{LM}}^{ab}[\mathcal{B}_{N,n}] \rangle \right. \\ + [\alpha_s]^2 \sum_{i \in \mathcal{H}} \left\langle \left[\theta_{\mathcal{H}_i} \gamma_{z, f_i \rightarrow f_i g}(L_i) \mathcal{W}_i^{i||n, \text{fin}} + \delta^{(0)} \mathcal{W}_i^{m||n, \text{fin}} + \delta^{\perp, (0)} \mathcal{W}_i^{(i)} \right] \cdot F_{\text{LM}}^{ab}[\mathcal{B}_{N,n}] \right\rangle \\ \left. + [\alpha_s] \langle I_{\text{T}}^{(0)} \cdot F_{\text{LV,fin}}^{ab}[\mathcal{B}_{N,n}] \rangle + \langle F_{\text{LV}^2, \text{fin}}^{ab}[\mathcal{B}_{N,n}] \rangle + \langle F_{\text{VV,fin}}^{ab}[\mathcal{B}_{N,n}] \rangle \right\}$$

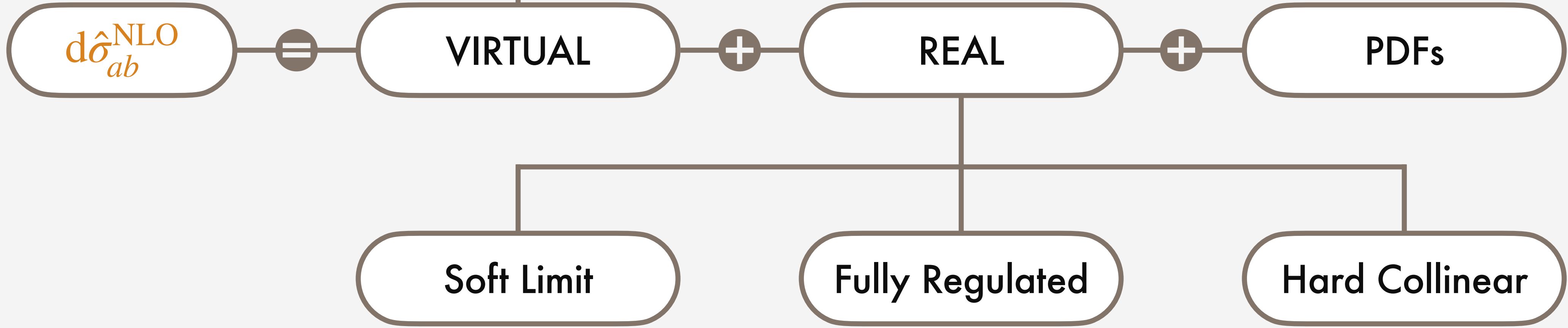
CONCLUSIONS

- a) The problem of subtracting IR singularities at NLO was solved more than 20 years ago. Nowadays, the computation of NLO QCD corrections for generic processes is well understood and fully automated.
- b) At NNLO, however, a fully general framework comparable to the NLO case is still lacking.
- c) The presence of **overlapping singularities** typically requires the use of **partitioning** and **sectoring** procedures, which in general lead to a loss of transparency.
- d) Through the *Nested Soft-Collinear* subtraction scheme, we have addressed this issue by deriving a general NNLO subtraction formula for processes with massless partons of the type $pp \rightarrow X + N \text{ Jets}$.
- e) In this formula, the number N of jets is a **free** computation parameter, and the **only required inputs** are the **LO configurations**.
- f) The **formula is modular**, as it is constructed leg by leg, and is therefore suited for **automation**.
- g) The remaining step is the numerical implementation of this general formula in numerical code.
- h) Prototype versions of such codes already exist for specific processes, but not yet for a fully general process such as $pp \rightarrow X + N \text{ Jets}$. This work is currently in progress.

BACK UP SLIDES

NLO

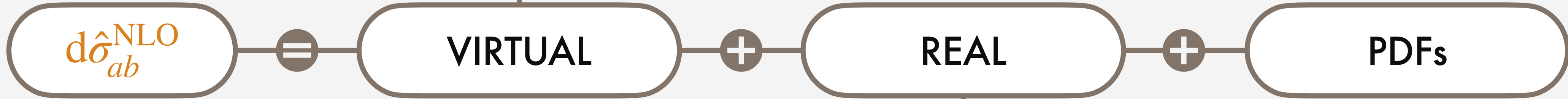
$$I_V(\epsilon) = \bar{I}_1(\epsilon) + \bar{I}_1^\dagger(\epsilon)$$
$$\bar{I}_1(\epsilon) = \frac{1}{2} \sum_{i \neq j}^N \left(\frac{1}{\epsilon^2} + \frac{\gamma_i}{\epsilon T_i^2} \right) \left(\frac{\mu^2 e^{i\pi\lambda_{ij}}}{s_{ij}} \right)^\epsilon (\mathbf{T}_i \cdot \mathbf{T}_j)$$



NLO

$$I_V(\epsilon) = \bar{I}_1(\epsilon) + \bar{I}_1^\dagger(\epsilon)$$

$$\bar{I}_1(\epsilon) = \frac{1}{2} \sum_{i \neq j}^N \left(\frac{1}{\epsilon^2} + \frac{\gamma_i}{\epsilon T_i^2} \right) \left(\frac{\mu^2 e^{i\pi\lambda_{ij}}}{s_{ij}} \right)^\epsilon (\mathbf{T}_i \cdot \mathbf{T}_j)$$



Soft Limit

Fully Regulated

Hard Collinear

$$I_S(\epsilon) = - \frac{(2E_{\text{max}}/\mu)^{-2\epsilon}}{\epsilon^2} \sum_{i \neq j}^N \eta_{ij}^{-\epsilon} K_{ij} (\mathbf{T}_i \cdot \mathbf{T}_j)$$

$$I_C(\epsilon) = \sum_i^N \frac{\Gamma_{i,f_i}}{\epsilon}$$

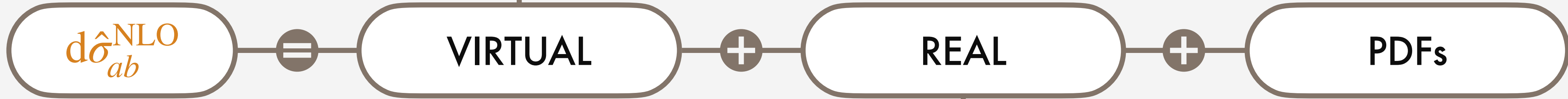
$$\Gamma_{i,f_i} = [\text{irrelevant pref.}] \times \left[T_i^2 \frac{1 - e^{-2\epsilon L_i}}{\epsilon} + \gamma_i \right]$$

$$\Gamma_{i,f_i} = [\text{irrelevant pref.}] \times \gamma_{z,g \rightarrow gg}^{22}(\epsilon, L_i)$$

NLO

$$I_V(\epsilon) = \bar{I}_1(\epsilon) + \bar{I}_1^\dagger(\epsilon)$$

$$\bar{I}_1(\epsilon) = \frac{1}{2} \sum_{i \neq j}^N \left(\frac{1}{\epsilon^2} + \frac{\gamma_i}{\epsilon T_i^2} \right) \left(\frac{\mu^2 e^{i\pi\lambda_{ij}}}{s_{ij}} \right)^\epsilon (T_i \cdot T_j)$$



Soft Limit

Fully Regulated

Hard Collinear

$$I_S(\epsilon) = - \frac{(2E_{\text{max}}/\mu)^{-2\epsilon}}{\epsilon^2} \sum_{i \neq j}^N \eta_{ij}^{-\epsilon} K_{ij} (T_i \cdot T_j)$$

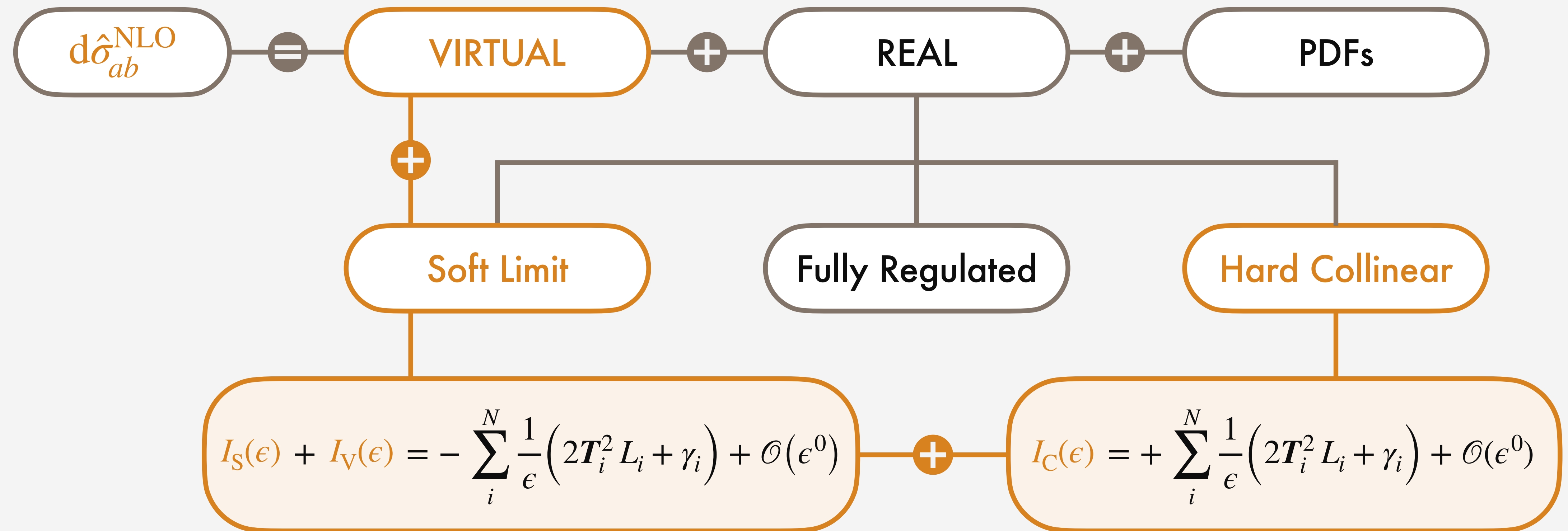
$$I_C(\epsilon) = \sum_i^N \frac{\Gamma_{i,f_i}}{\epsilon}$$

$$\Gamma_{i,f_i} = [\text{irrelevant pref.}] \times \left[T_i^2 \frac{1 - e^{-2\epsilon L_i}}{\epsilon} + \gamma_i \right]$$

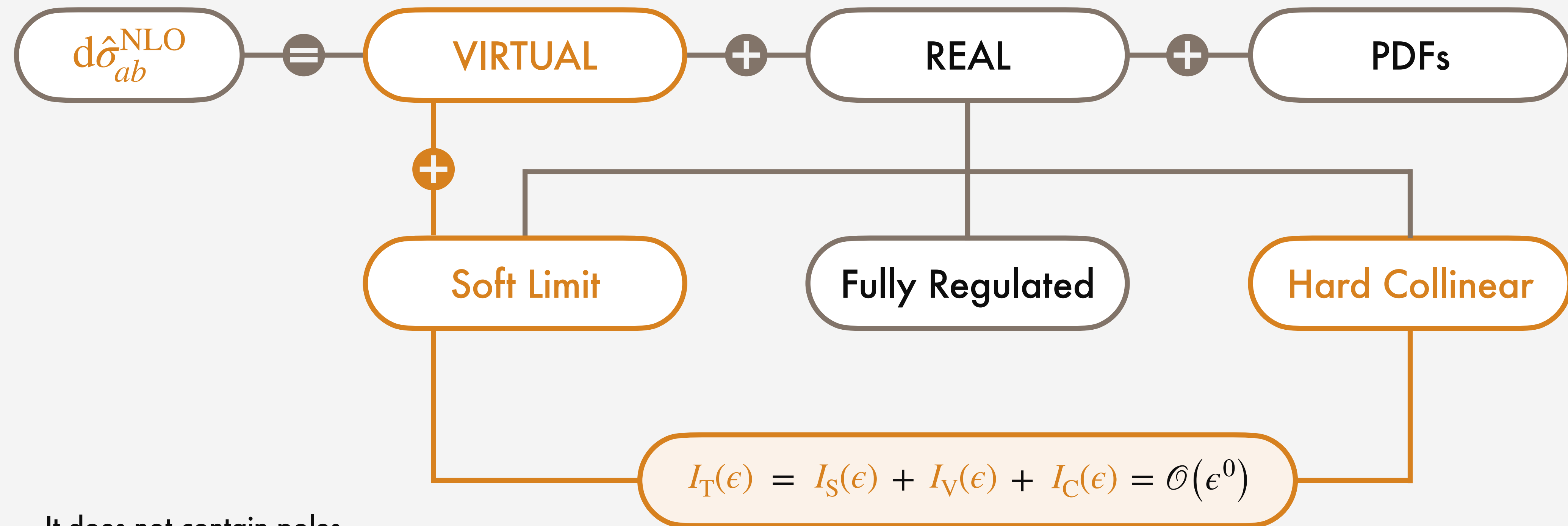
$$\Gamma_{i,f_i} = [\text{irrelevant pref.}] \times \gamma_{z,g \rightarrow gg}^{22}(\epsilon, L_i)$$

● = Color Correlations $\implies T_i^2 = C_i$ and $(T_i \cdot T_j) = T_i^a T_j^a$

NLO



NLO

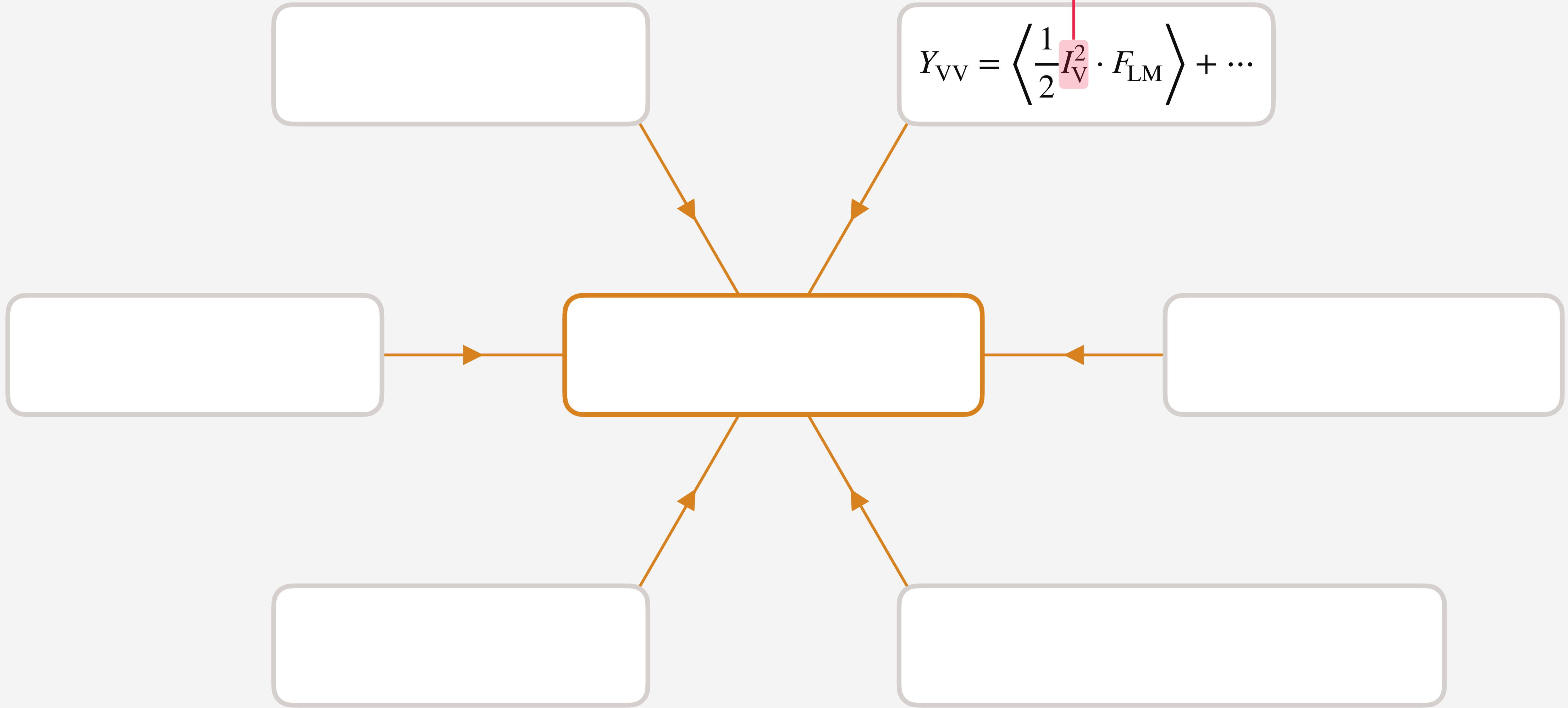


- It does not contain poles
- General procedure for **color - correlations**
- **Trivially dependent on** the number of hard partons

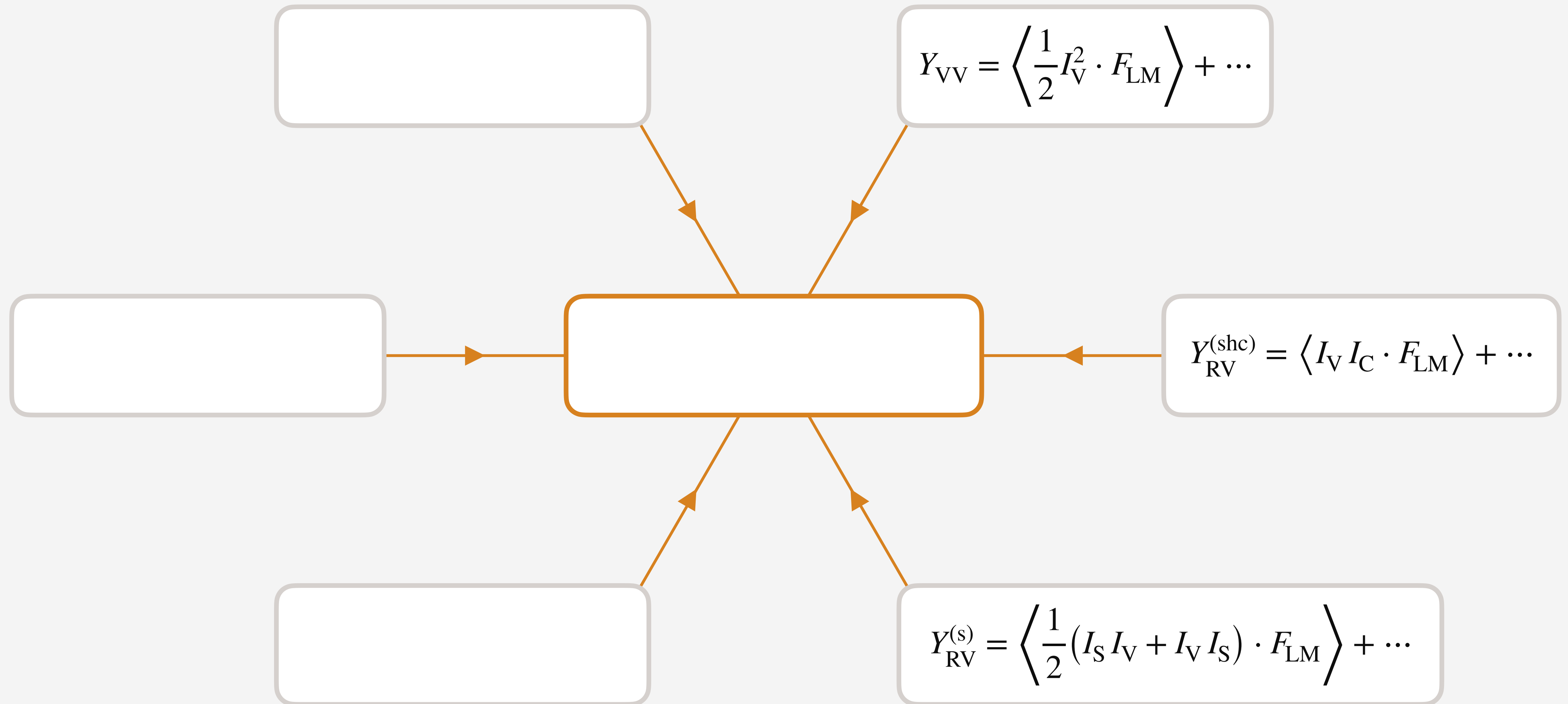
NNLO

Quartic Color Correlations
 $(\mathbf{T}_i \cdot \mathbf{T}_j)(\mathbf{T}_k \cdot \mathbf{T}_l)$

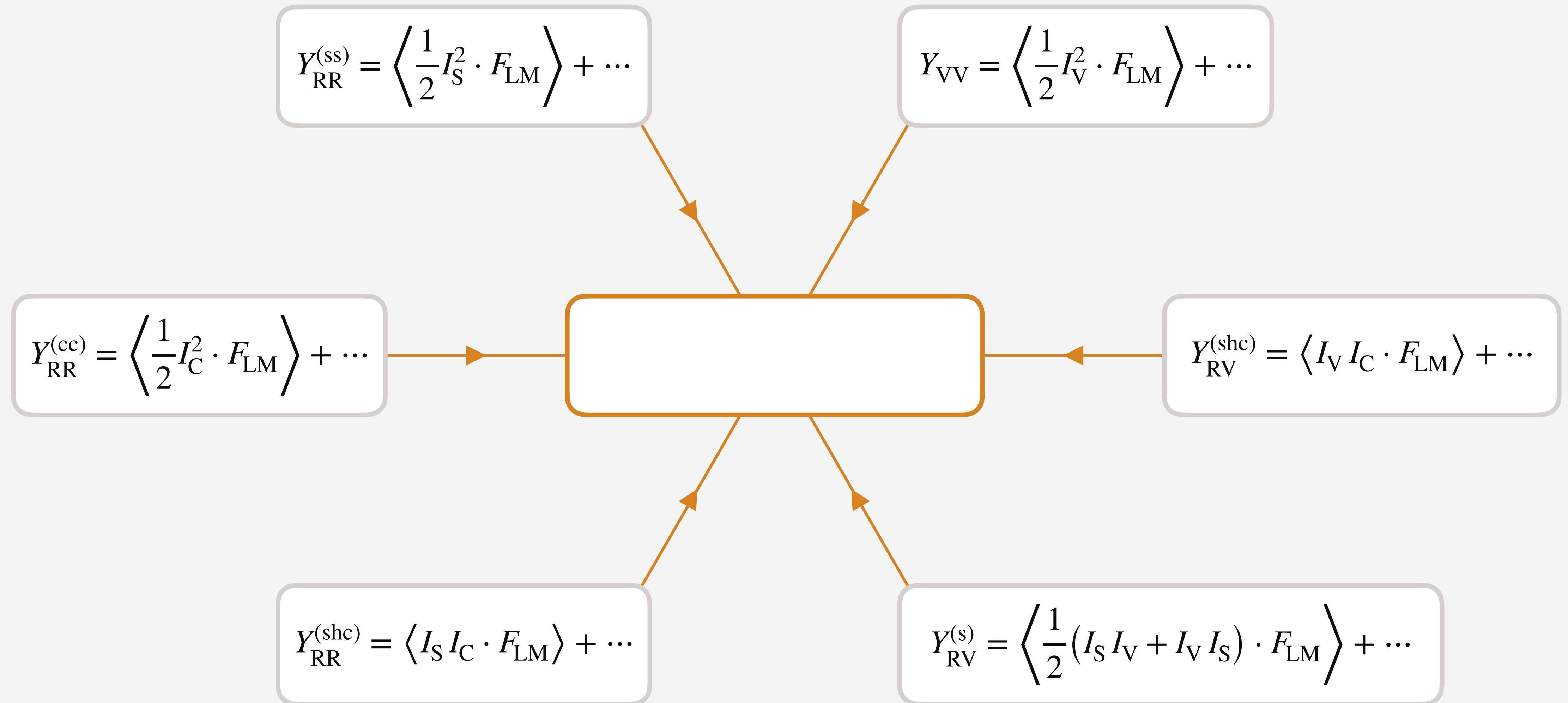
$$Y_{VV} = \left\langle \frac{1}{2} I_V^2 \cdot F_{LM} \right\rangle + \dots$$



NNLO



NNLO



NNLO

$$Y_{\text{RR}}^{(\text{ss})} = \left\langle \frac{1}{2} I_{\text{S}}^2 \cdot F_{\text{LM}} \right\rangle + \dots$$

$$Y_{\text{VV}} = \left\langle \frac{1}{2} I_{\text{V}}^2 \cdot F_{\text{LM}} \right\rangle + \dots$$

$$Y_{\text{RR}}^{(\text{cc})} = \left\langle \frac{1}{2} I_{\text{C}}^2 \cdot F_{\text{LM}} \right\rangle + \dots$$

$$d\hat{\sigma}_{ab}^{\text{NNLO}} \sim \left\langle \frac{1}{2} I_{\text{T}}^2 \cdot F_{\text{LM}} \right\rangle + \dots$$

$$Y_{\text{RV}}^{(\text{shc})} = \left\langle I_{\text{V}} I_{\text{C}} \cdot F_{\text{LM}} \right\rangle + \dots$$

$$Y_{\text{RR}}^{(\text{shc})} = \left\langle I_{\text{S}} I_{\text{C}} \cdot F_{\text{LM}} \right\rangle + \dots$$

$$Y_{\text{RV}}^{(\text{s})} = \left\langle \frac{1}{2} (I_{\text{S}} I_{\text{V}} + I_{\text{V}} I_{\text{S}}) \cdot F_{\text{LM}} \right\rangle + \dots$$