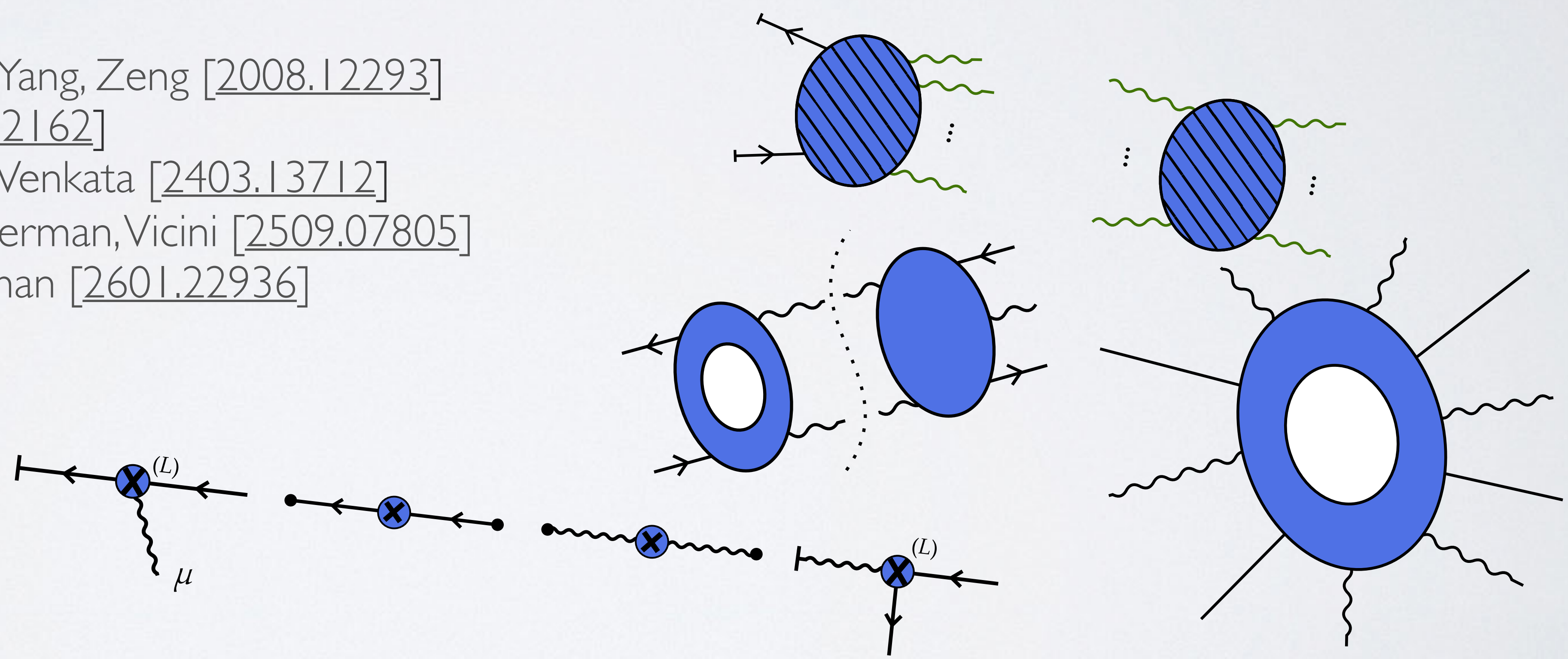
ON THE LOCAL IR STRUCTURE OF SCATTERING AMPLITUDES

To appear in: Anastasiou, Biello, Favorito, ***GG***, Sahoo, Sterman [26SO.ONISH]

Building on:

Anastasiou, Haindl, Sterman, Yang, Zeng [2008.12293]
Anastasiou, Sterman [2212.12162]
Anastasiou, Karlen, Sterman, Venkata [2403.13712]
Anastasiou, Karlen, Sahoo, Sterman, Vicini [2509.07805]
Anastasiou, Karlen, Ma, Sterman [2601.22936]



Giulio Gambuti - Loopfest XXIV - 05/28/2026

ETH zürich

Amplitude representations whose itegrand
factorises in all soft & collinear limits
for general two-loop QED processes.

A local Catani-like subtraction formula.

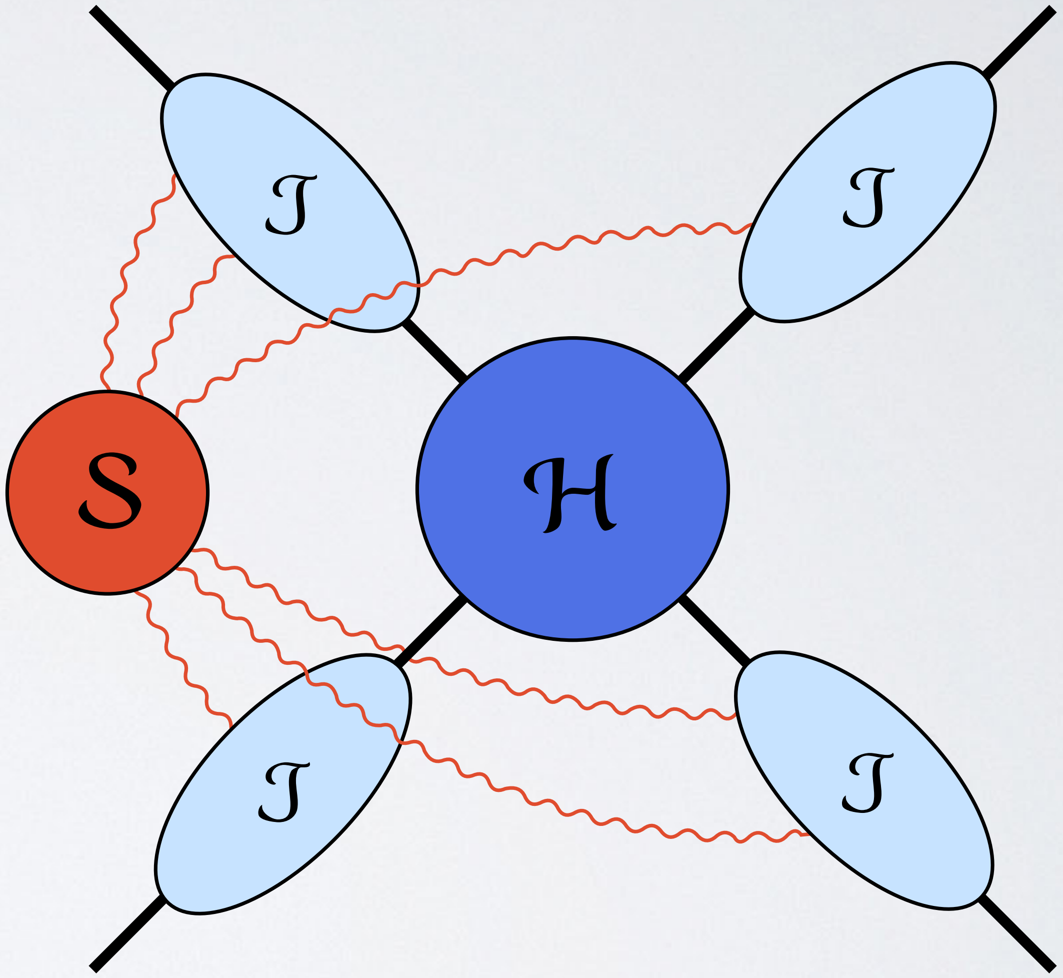
Both can be generalised.

AMPLITUDE *IR* FACTORISATION

$$A = J_1 J_2 \ldots J_n \, \mathcal{S} \cdot \mathcal{H}$$

Collins, Soper, Sterman ['88];
Catani: [9802439];
Kidonakis, Oderda, Sterman [9803241];
Sterman, Tejeda-Yeomans [0210130];
Gardi, Magnea: [0901.1091],[0908.3273];
Becher, Neubert: [0903.1126];
Feige, Schwartz: [1403.6472];

+ many more!!



IN SHORT

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1. local IR factorisation

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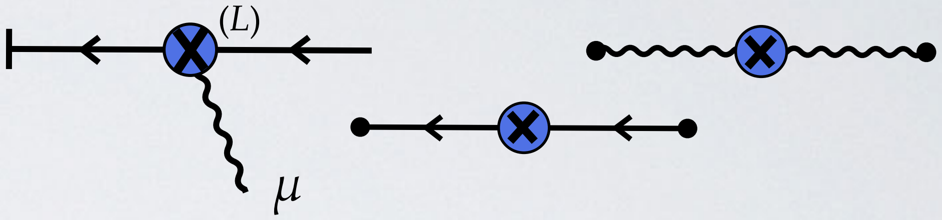
   → boundary terms



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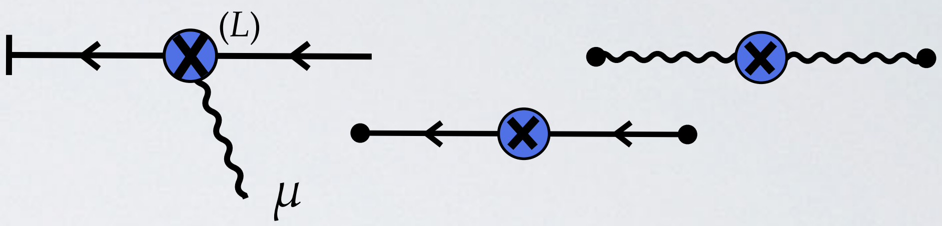

2. finite remainders

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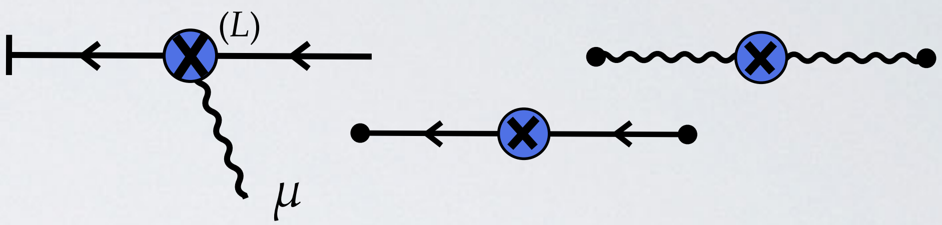

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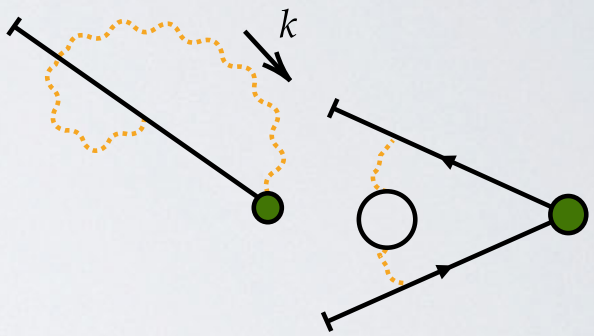
   → via simple+universal subtractions



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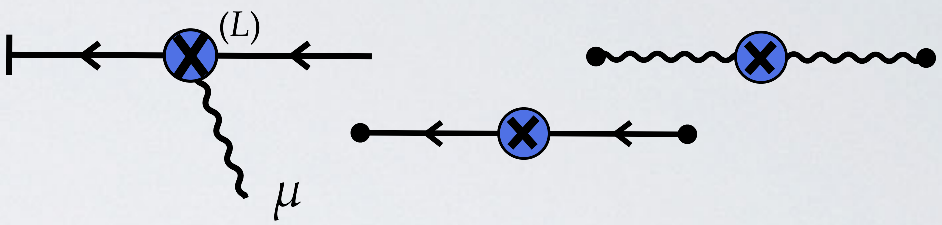

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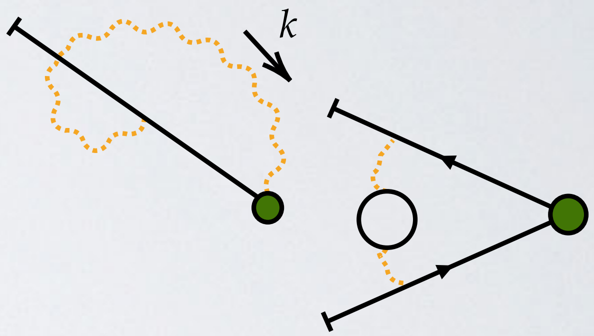
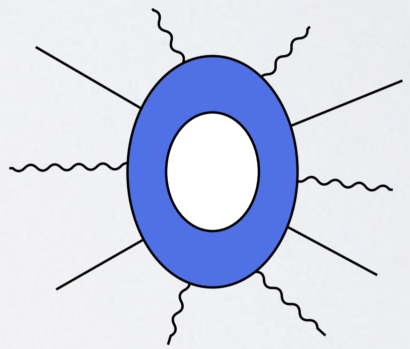


3. numerical integration



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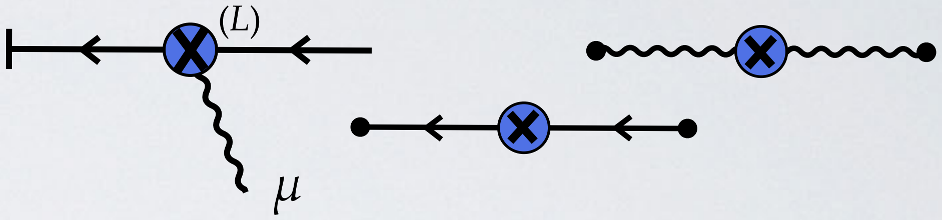

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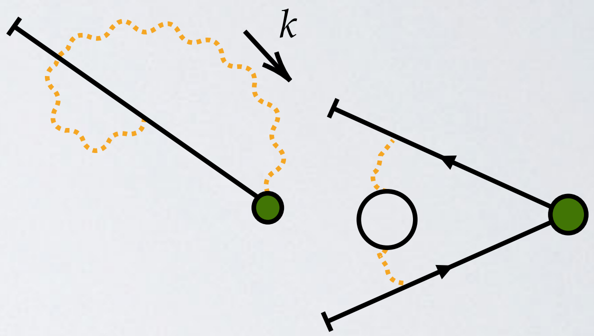

3. numerical integration

   → replace the many-scales problem
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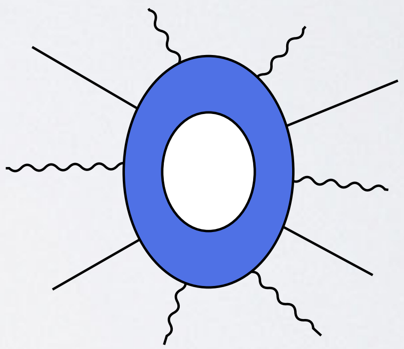

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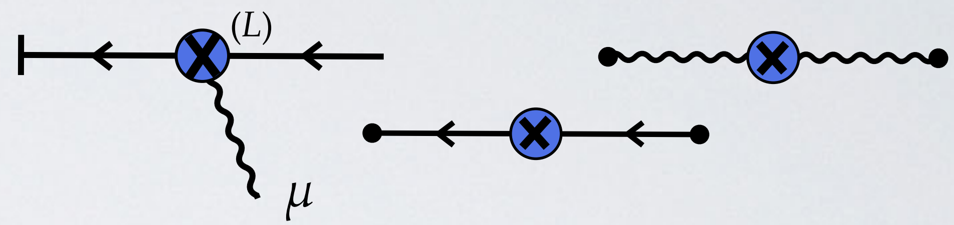

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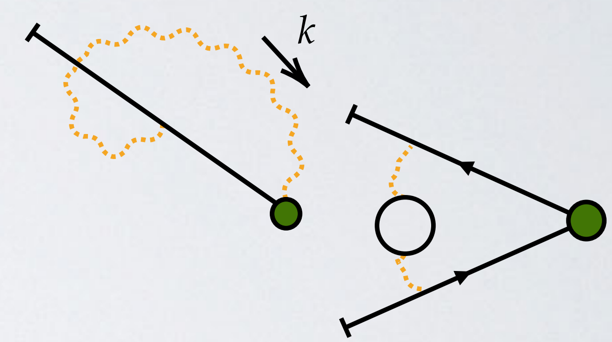

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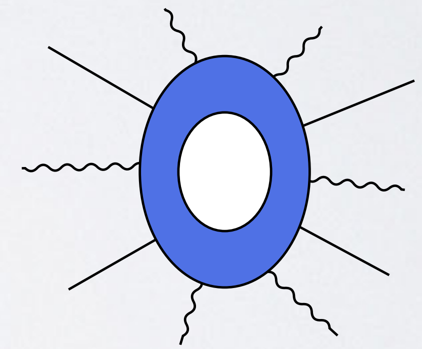

(2loop) QED $\Longrightarrow$

- Relevant for future e^+e^-colliders
- All kinematical complexity of QCD

THE ENGINE(S) OF LOCAL FACTORISATION

1. tree-level Ward identity

2. longitudinal polarisations

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$$(\not{r} + \not{q})^{-1} \not{q} \not{r}^{-1} = \not{r}^{-1} - (\not{r} + \not{q})^{-1}$$

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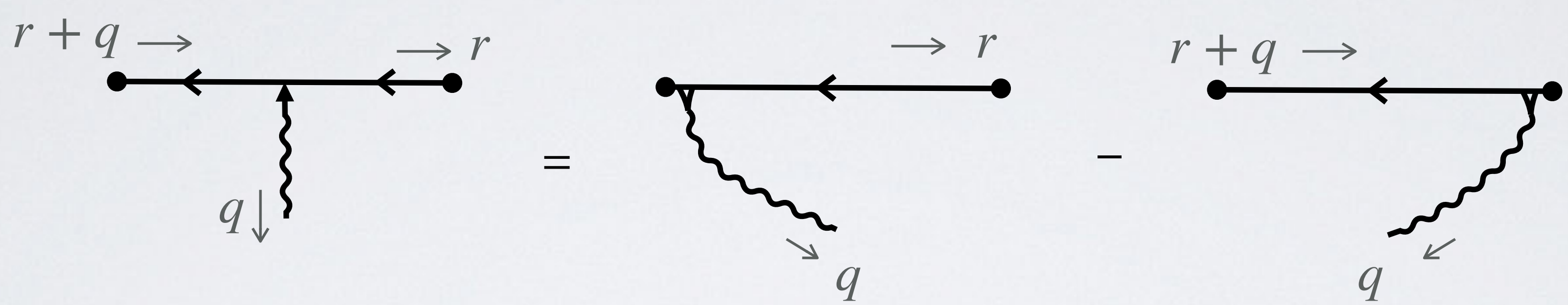

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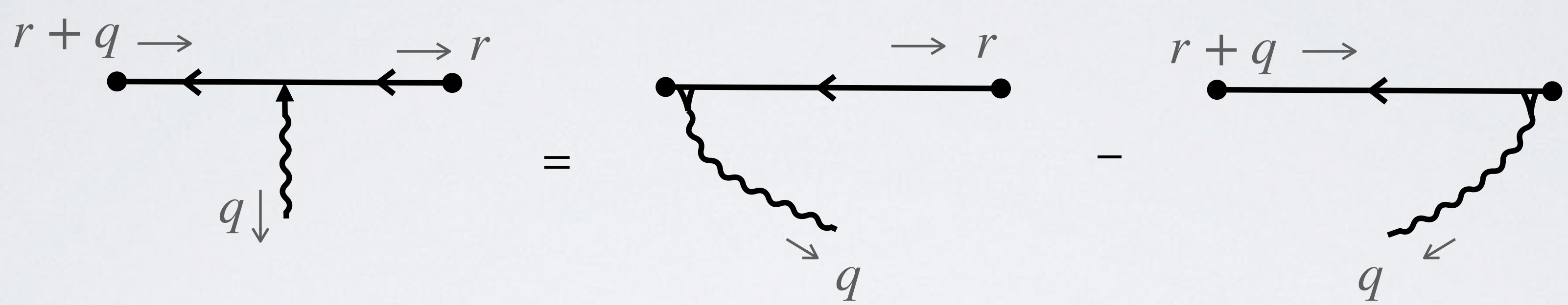

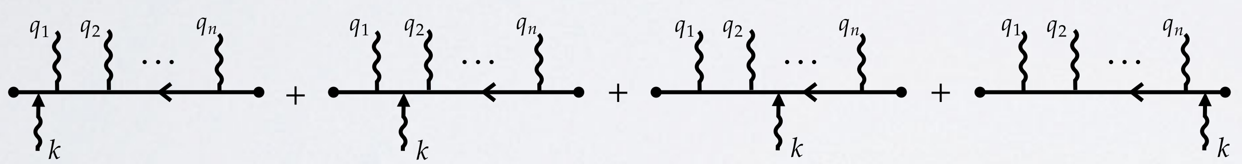

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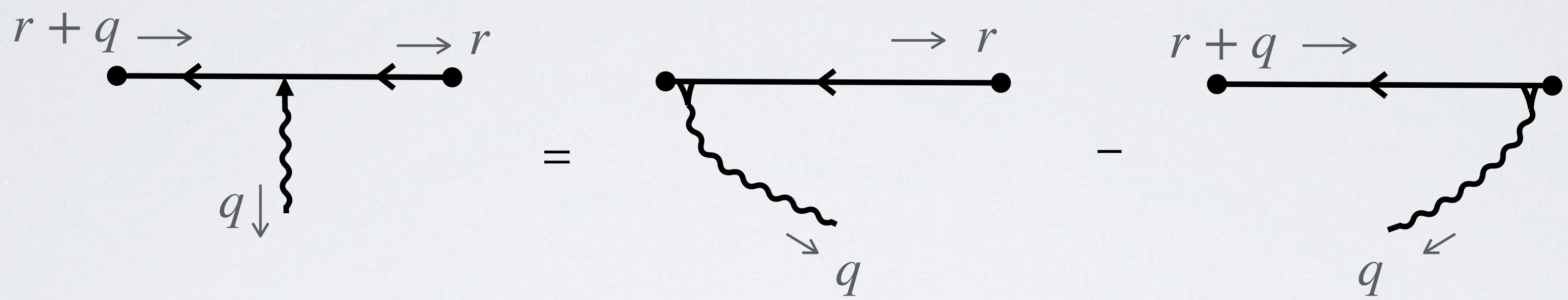

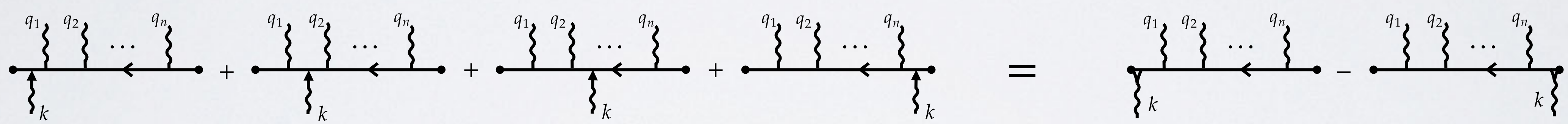

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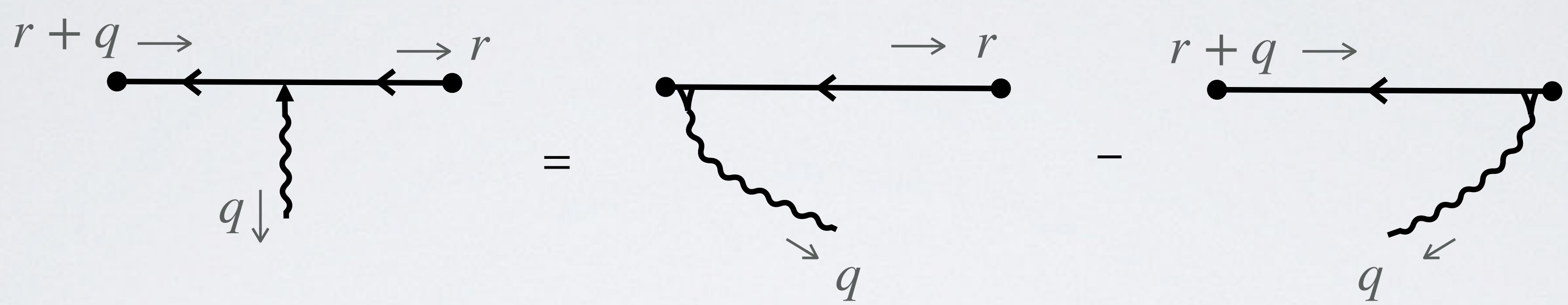
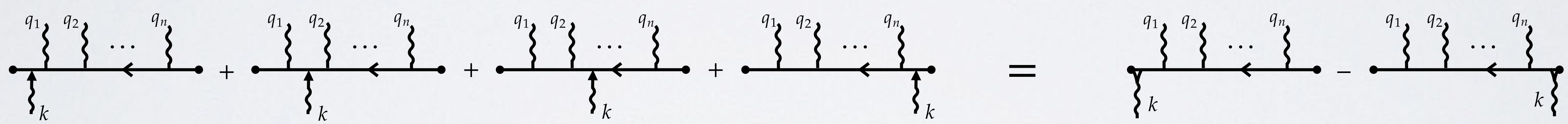
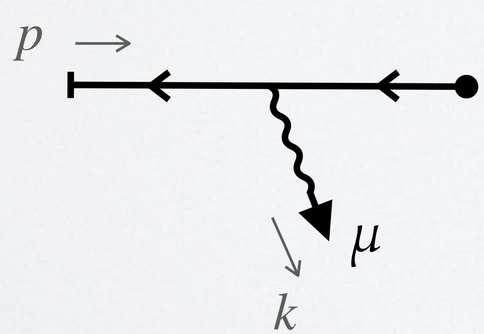




2. longitudinal polarisations

$$k^\mu \rightarrow \alpha \, p^\mu$$



$$\bar{u}(p)(-ie\gamma^\mu)\frac{-i(\not p - \not k)}{(p-k)^2} \propto k^\mu \, \bar{u}(p)$$

A COLLINEAR FACTORISATION EXAMPLE

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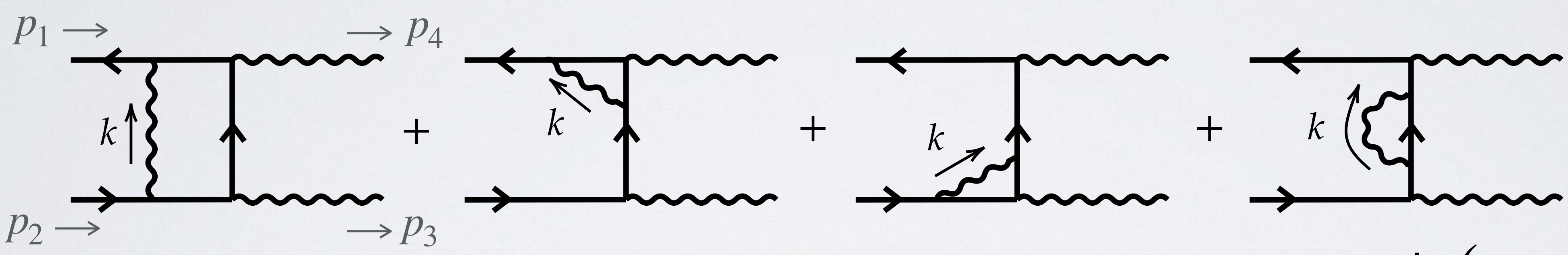
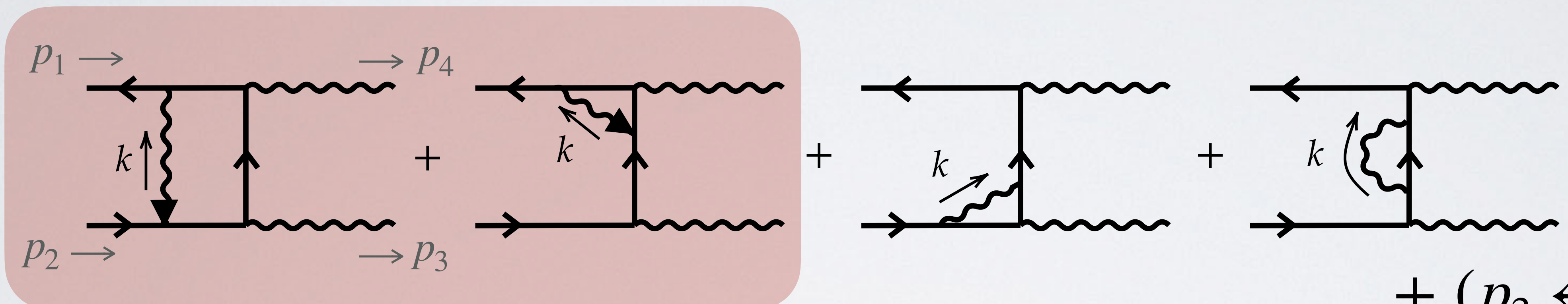


$+ (p_3 \leftrightarrow p_4)$

A COLLINEAR FACTORISATION EXAMPLE

$k^\mu \to \alpha\, p_1^\mu$



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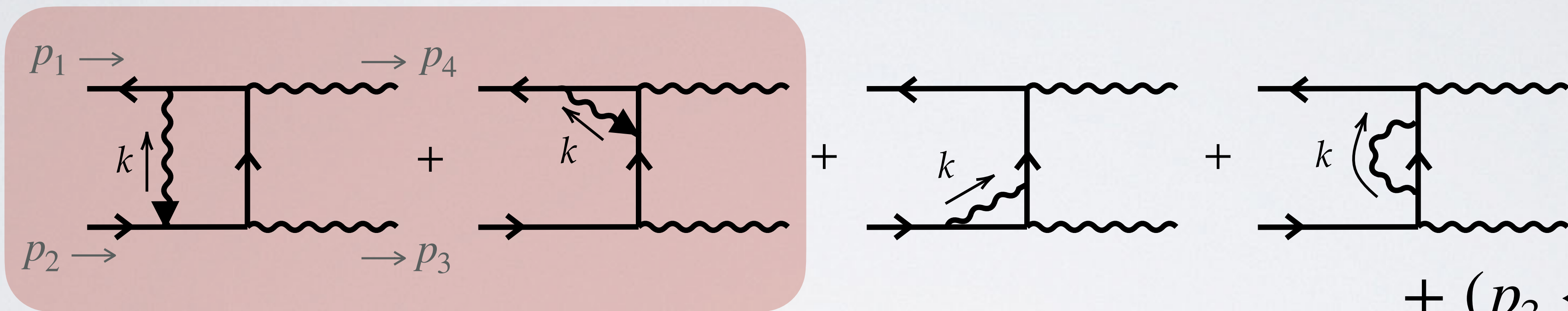

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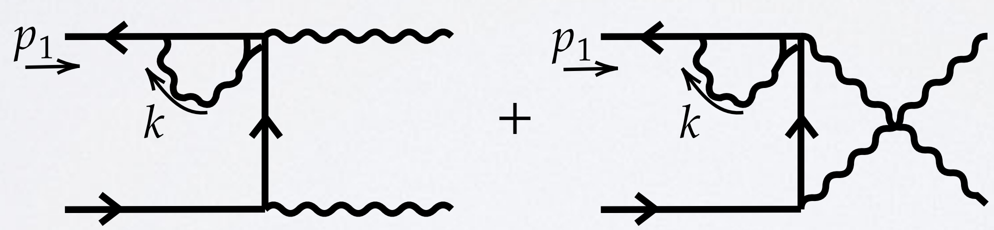
$$\simeq$$



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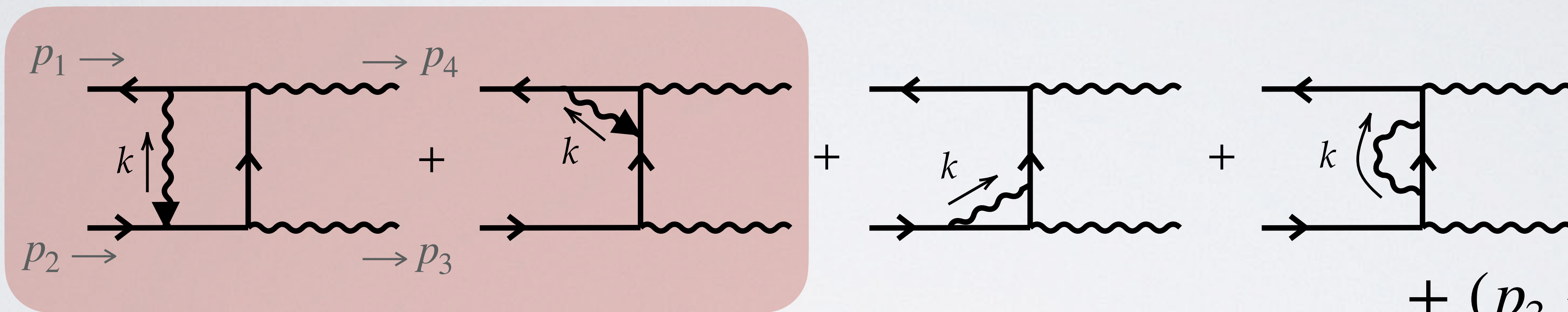

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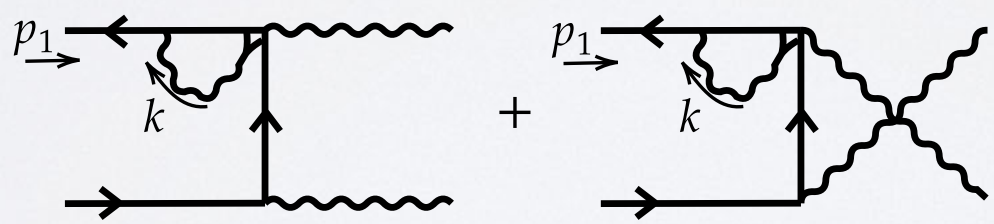

$$= \; B(p_1, k) \times A^{(0)}_{ee\to\gamma\gamma}$$

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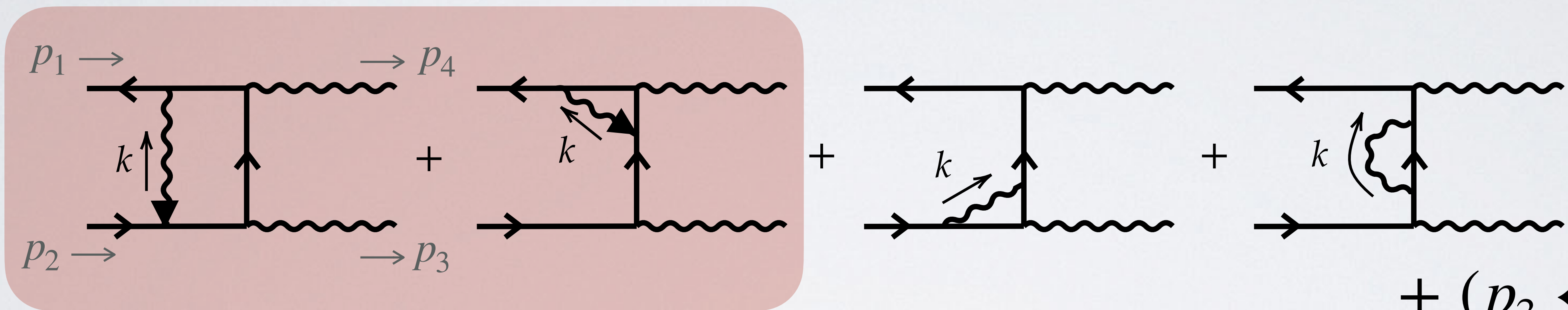

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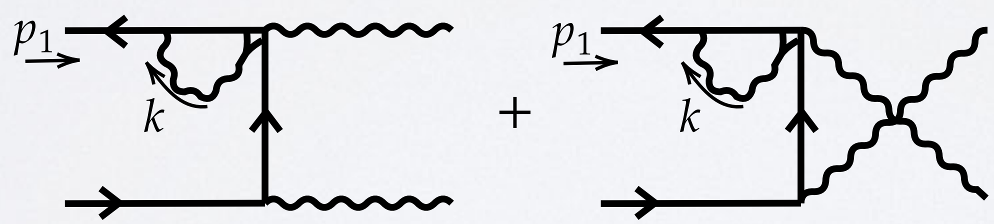

$= \; B(p_1, k) \; \times \; A^{(0)}_{ee\to\gamma\gamma}$

works* for all one-loop amplitudes

*provided a proper loop-momentum routing is imposed

OUR WISH LIST

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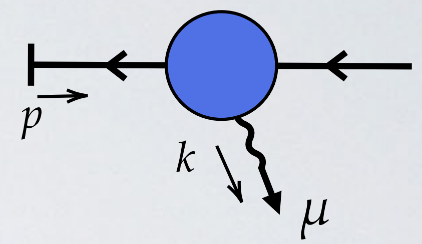
- collinear photons are longitudinally polarised



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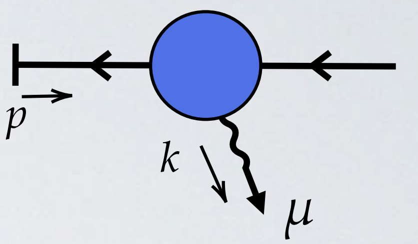
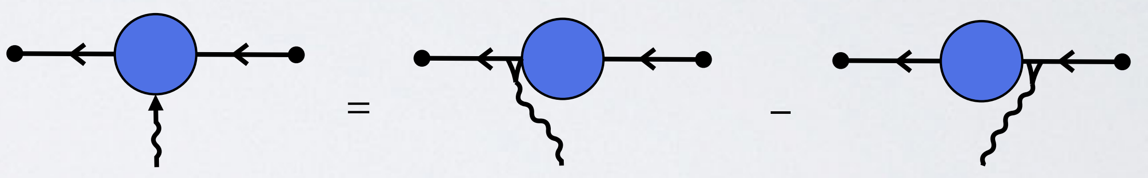


- Ward identities are locally preserved



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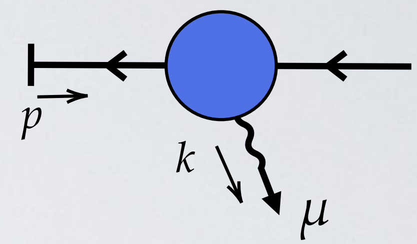

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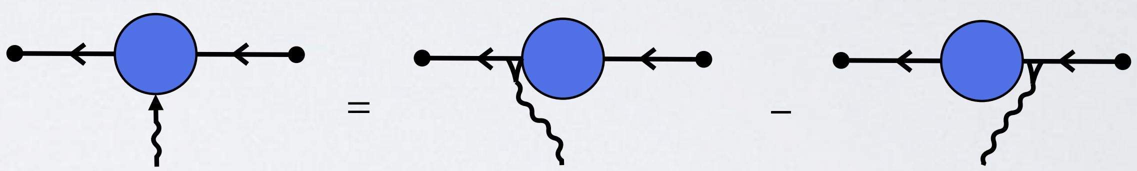
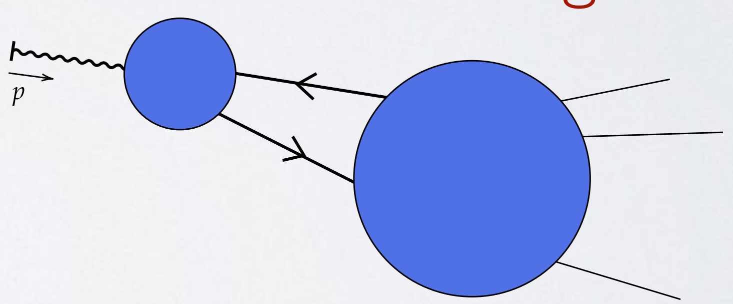


- spurious collinear divergences are absent

transient singularities



Anastasiou, Karlen, Sahoo, Sterman, Vicini: [2509.07805]

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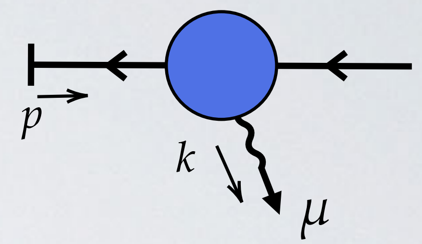

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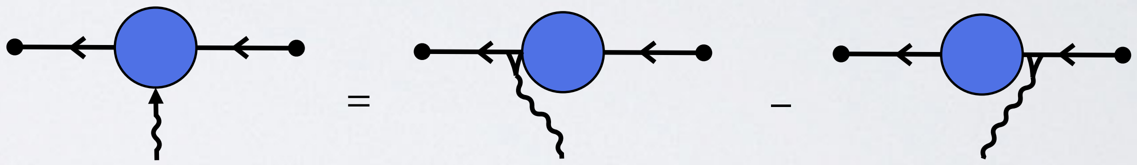

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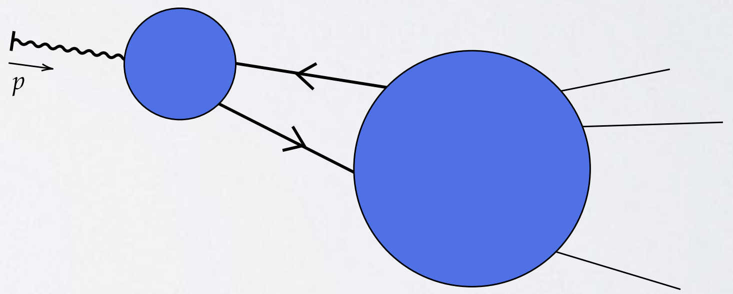
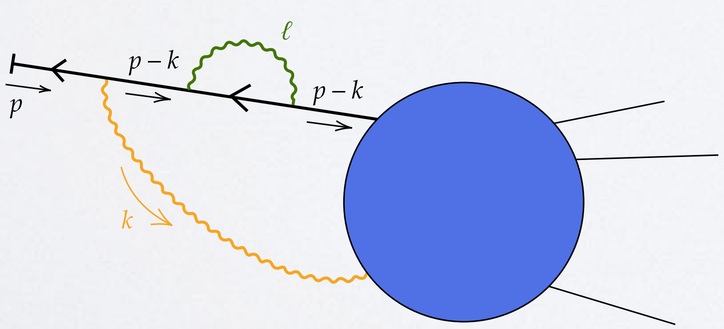


- all singularities are logarithmic



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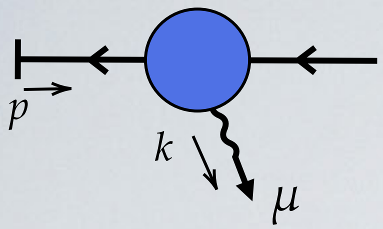
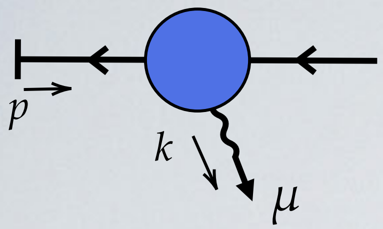


LOOP POLARISATIONS



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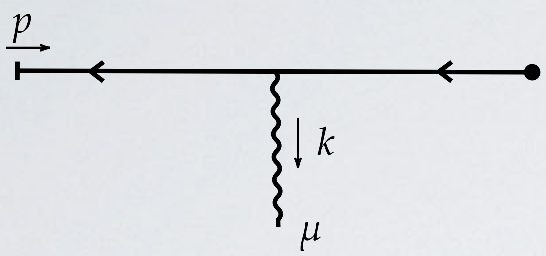

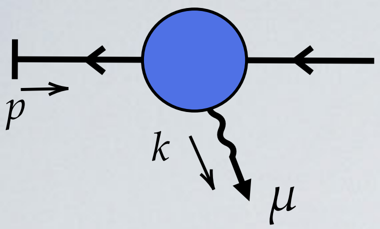

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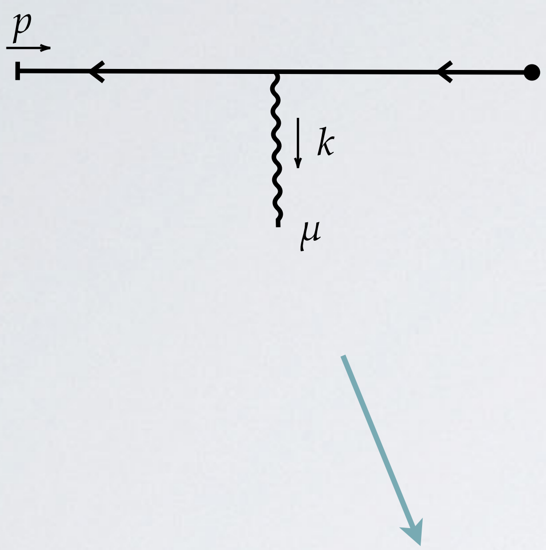
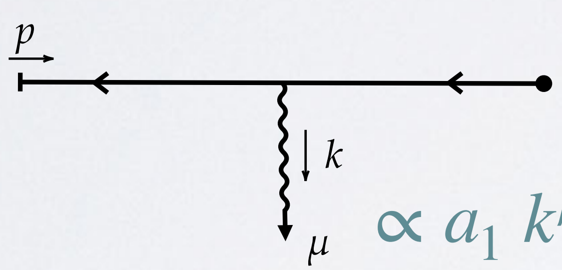
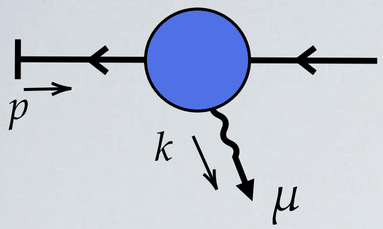




$\propto a_1 \, k^\mu$



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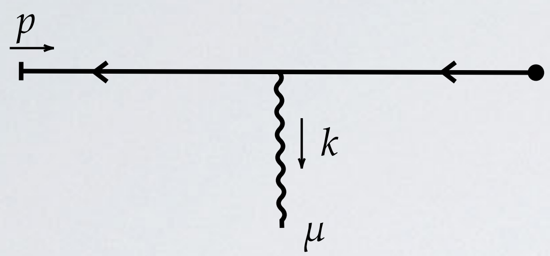

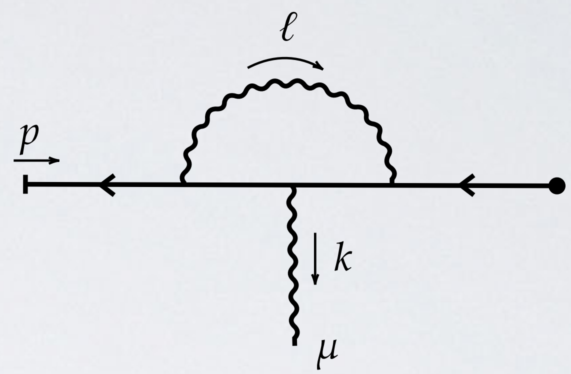

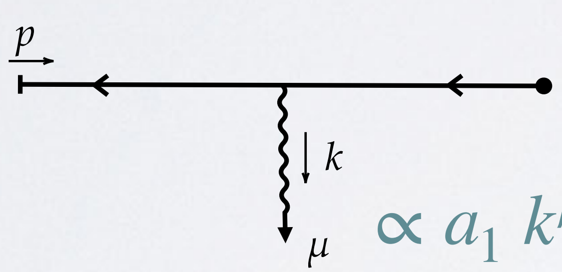

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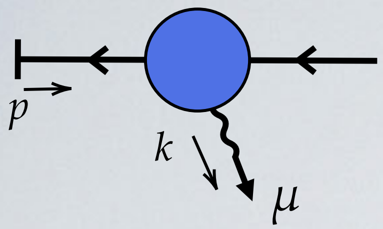

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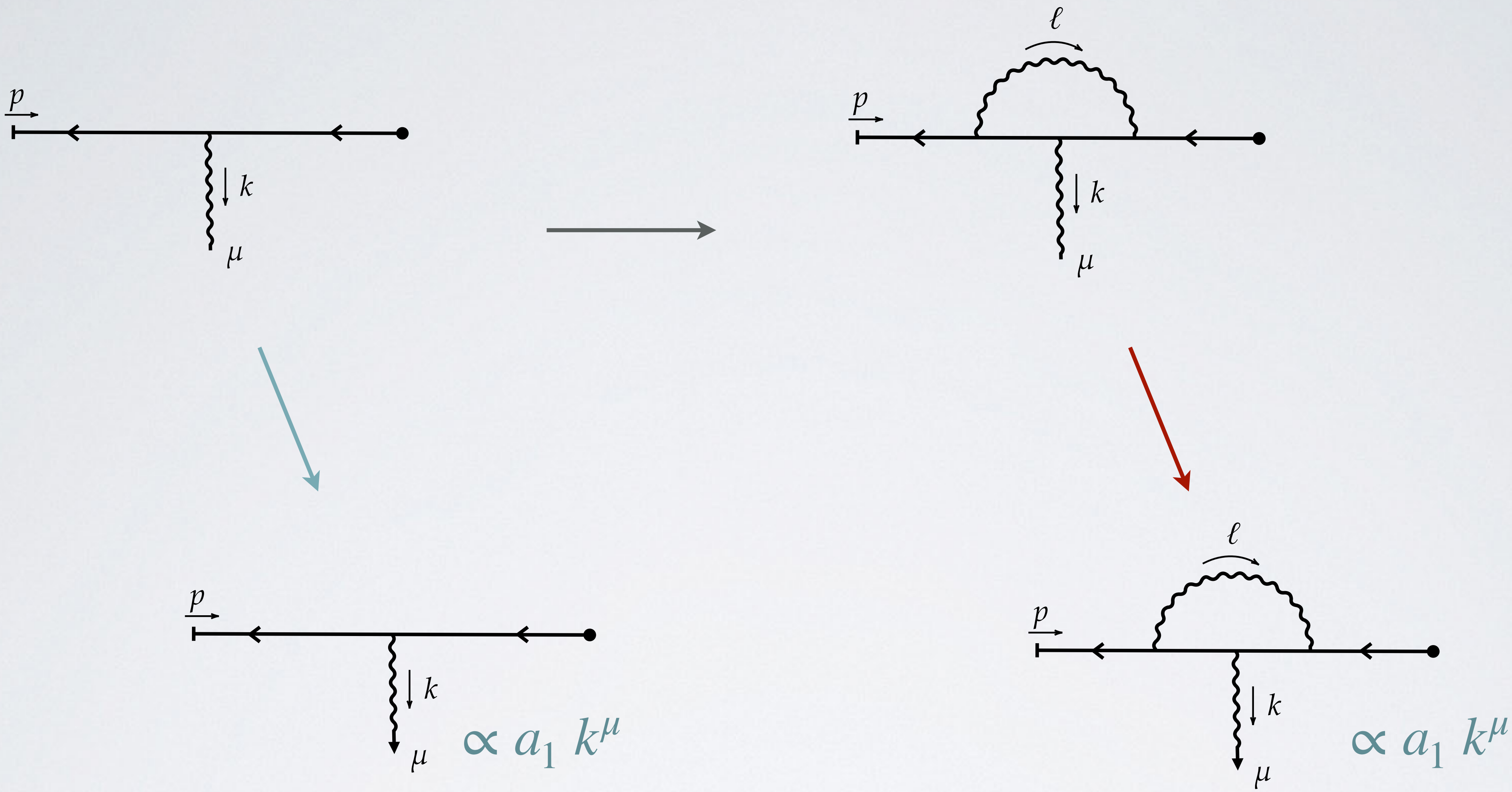

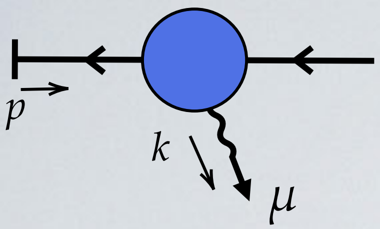

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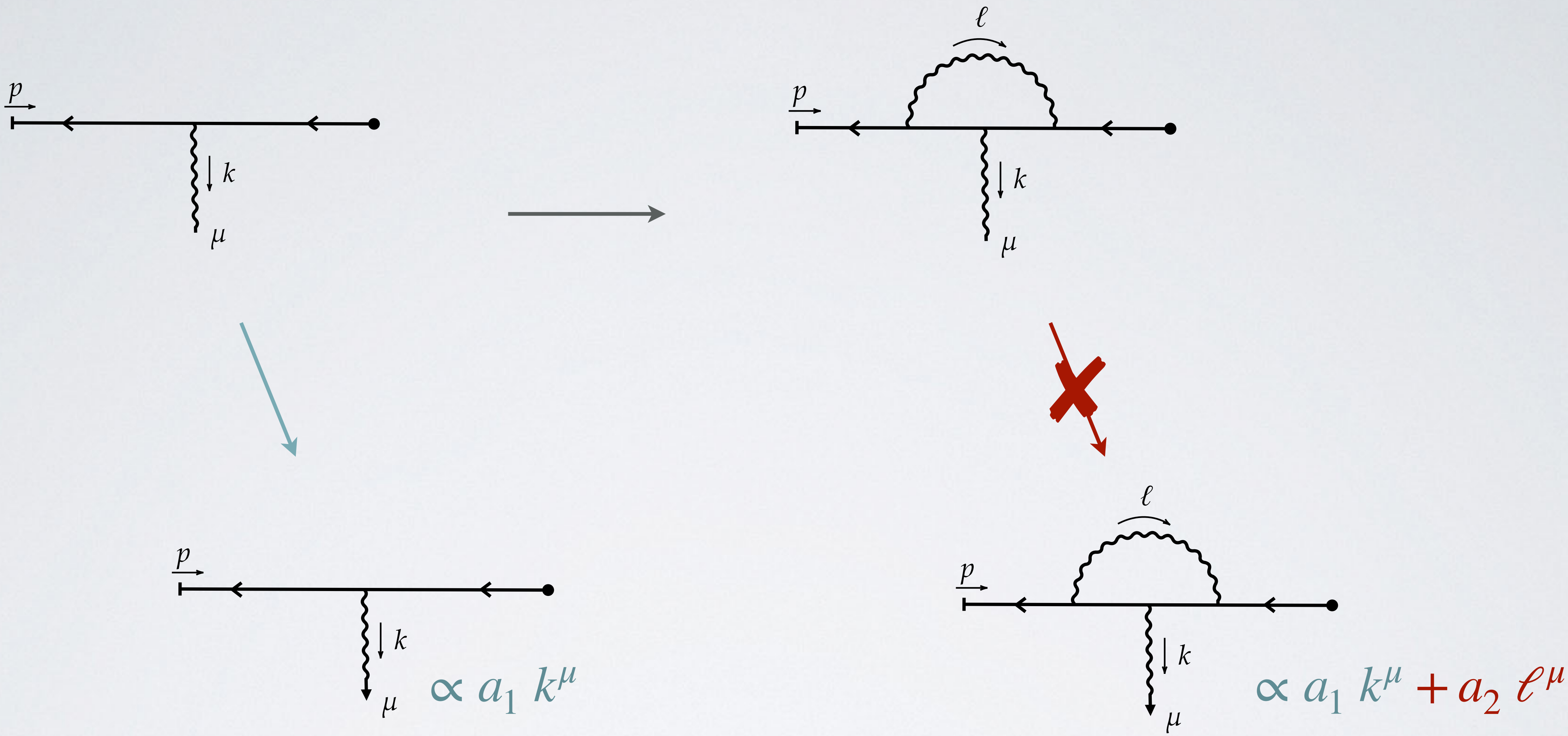

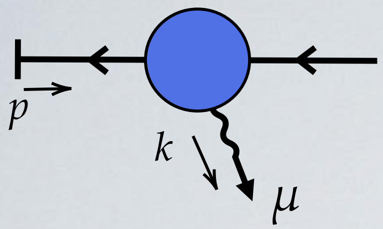

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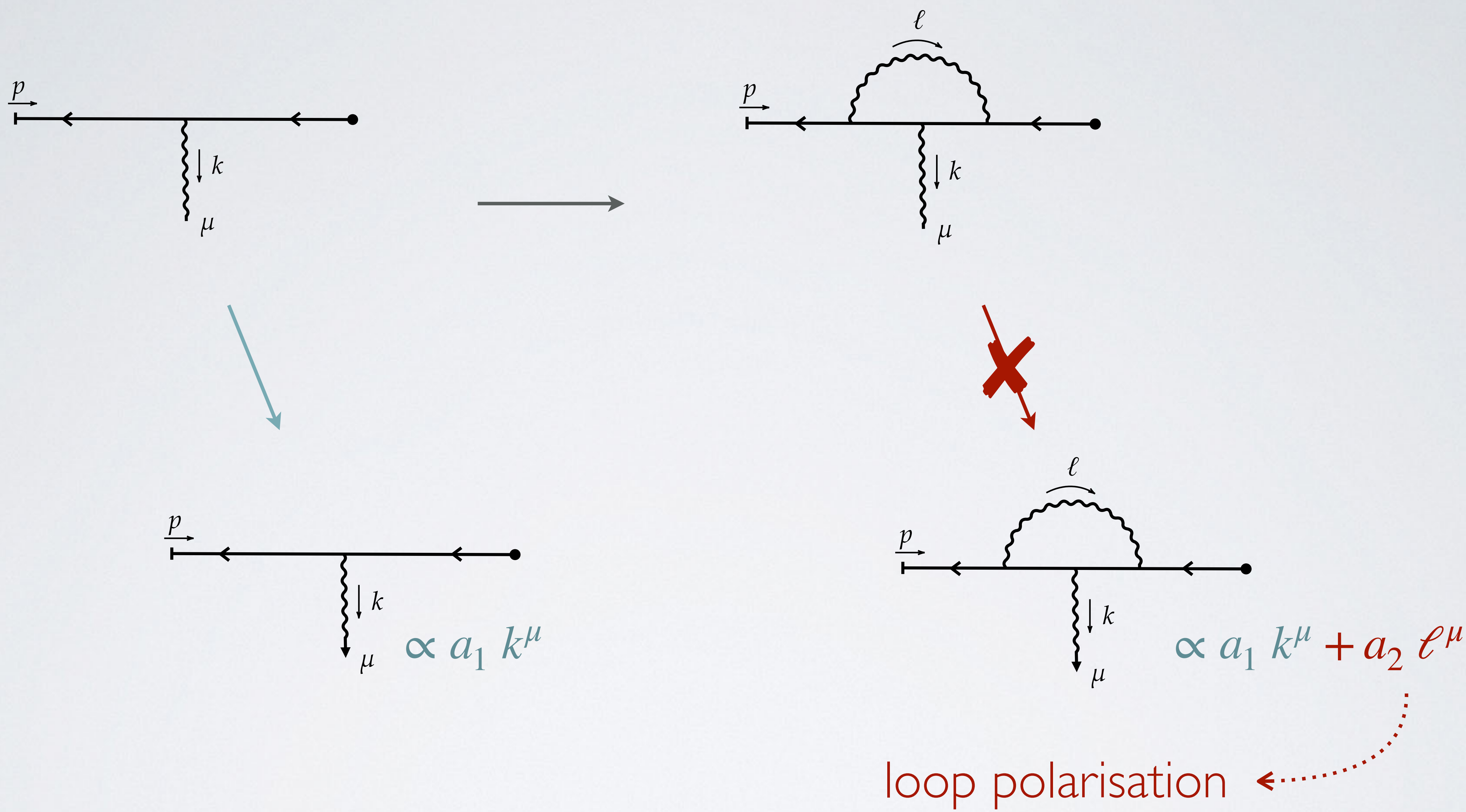

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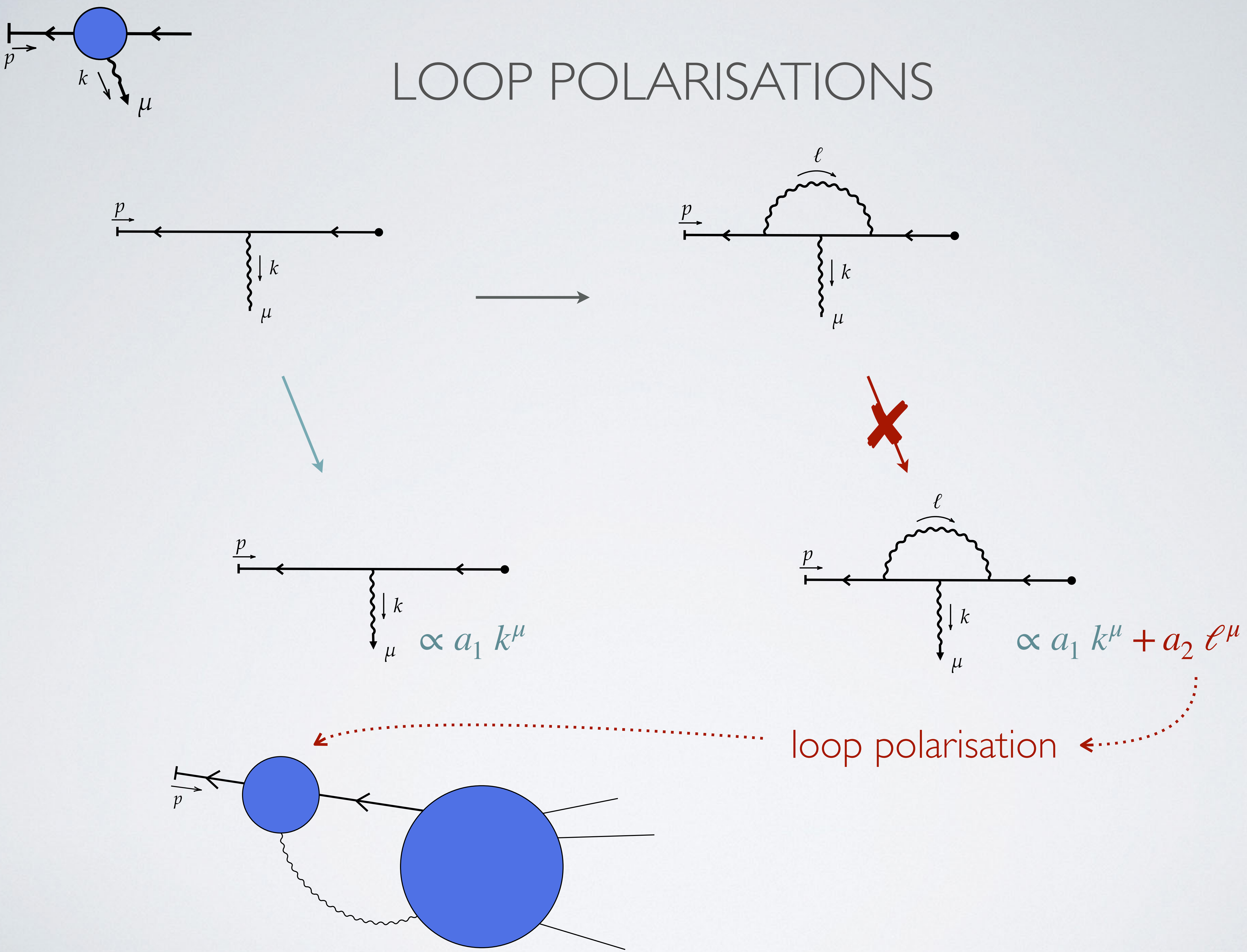
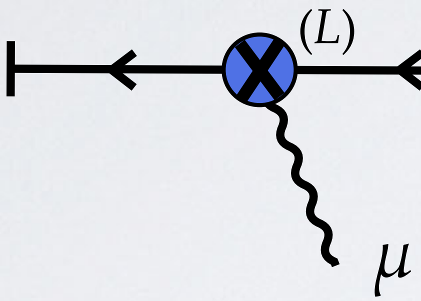
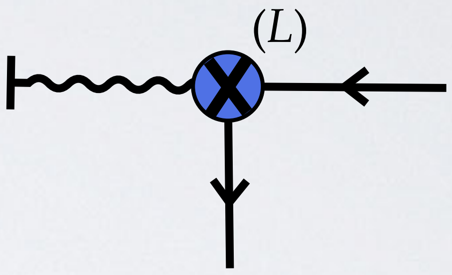
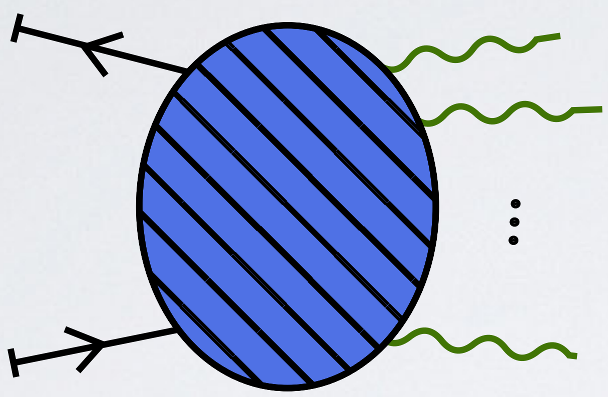


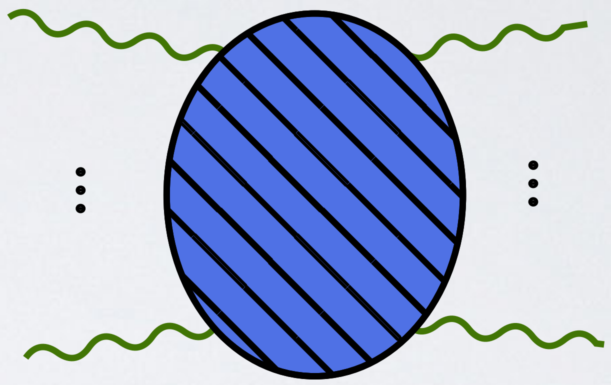
FEYNMAN-RULE COUNTERTERMS



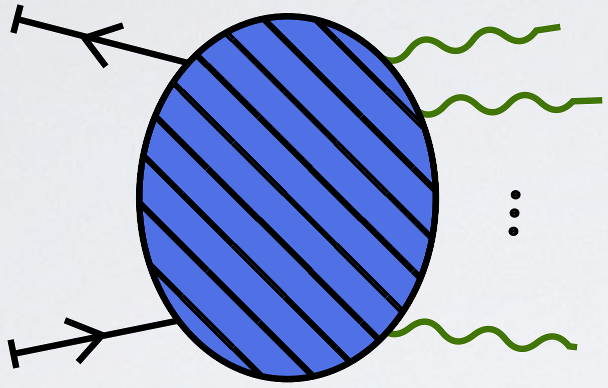


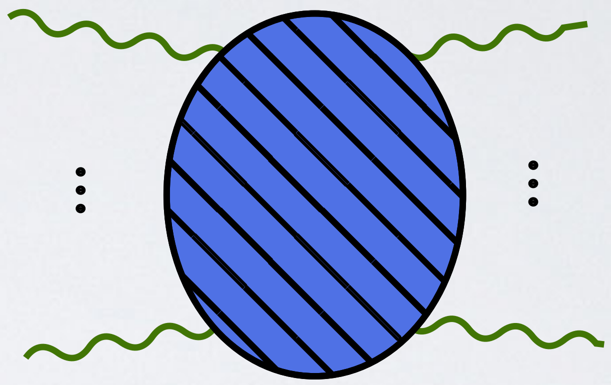
BUILDING-BLOCK AMPLITUDES

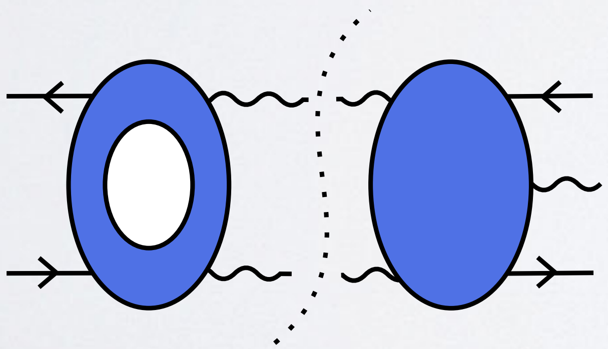


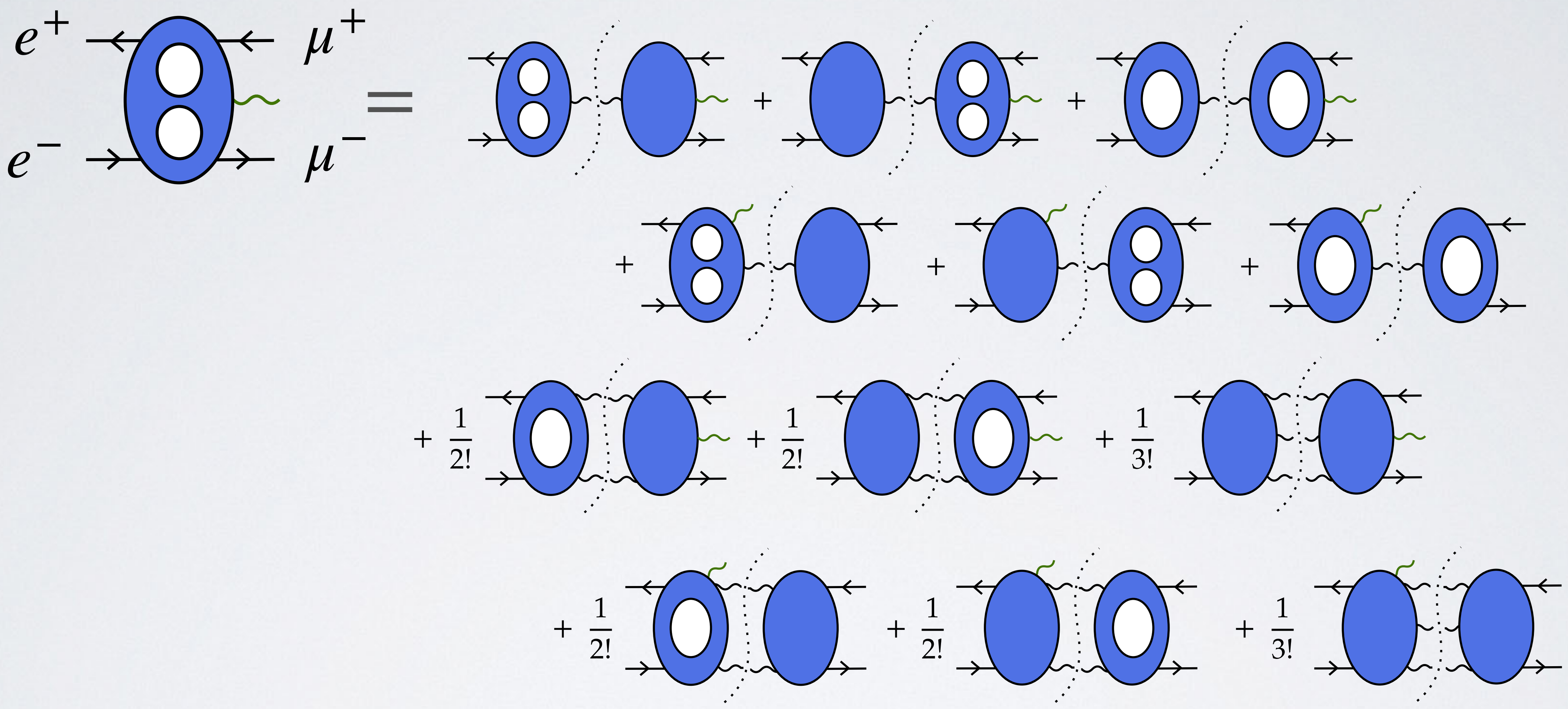


BUILDING-BLOCK AMPLITUDES



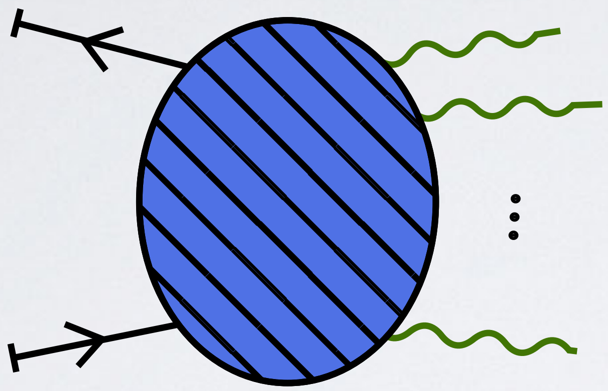


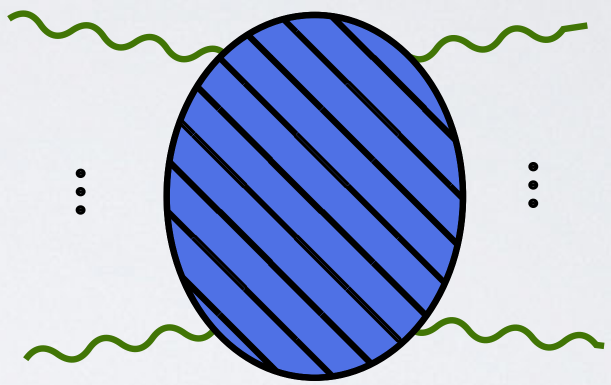


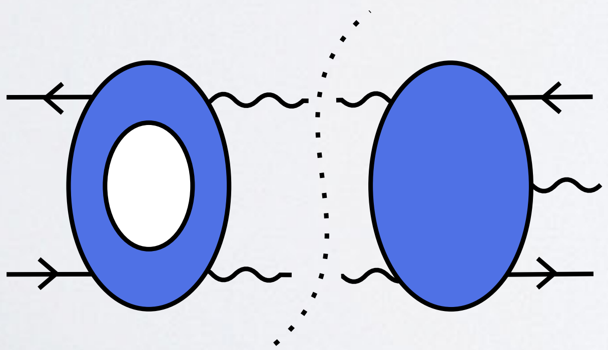


e^+
e^-
μ^+
μ^-
=
+
$\frac{1}{2!}$
$\frac{1}{3!}$

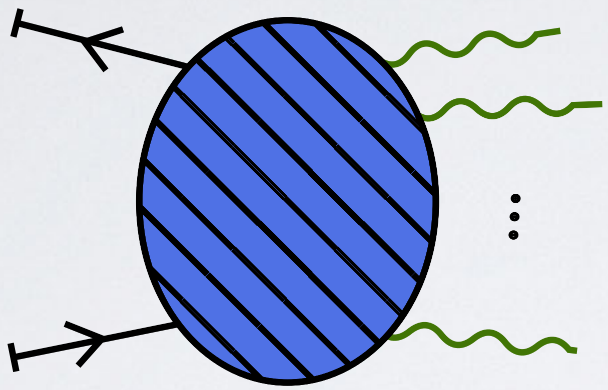
BUILDING-BLOCK AMPLITUDES

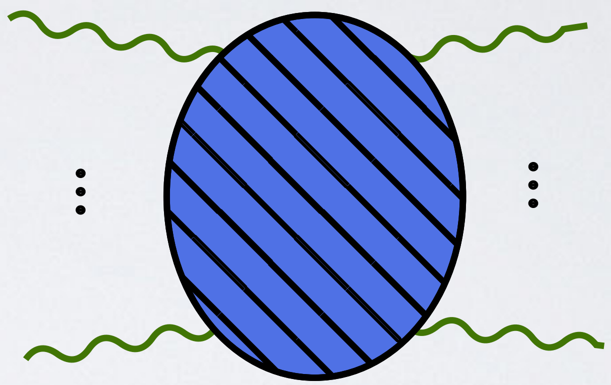




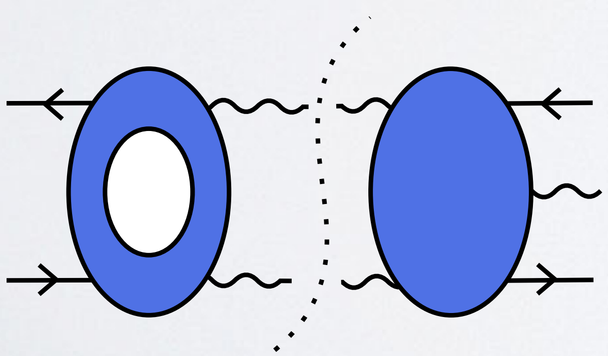


BUILDING-BLOCK AMPLITUDES

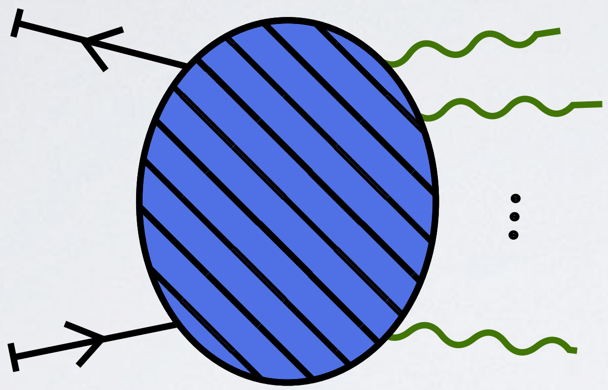


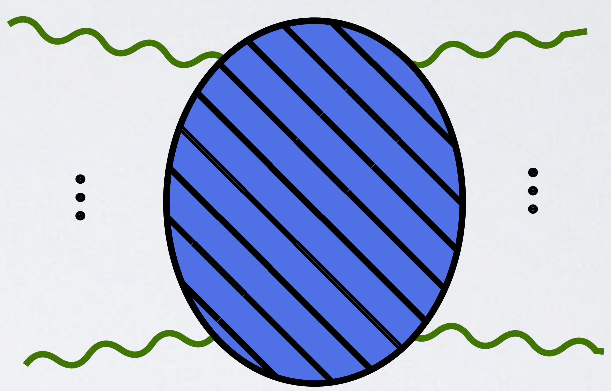


No new collinear
divergences in sewing:



BUILDING-BLOCK AMPLITUDES





No new collinear
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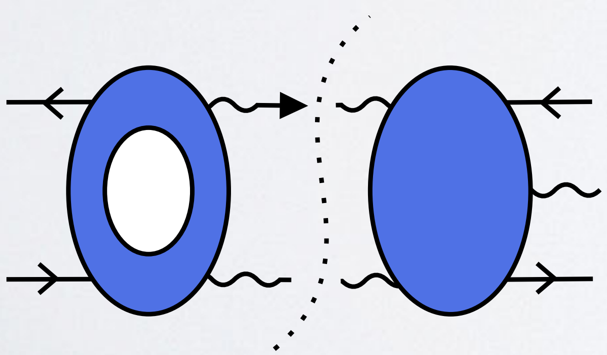
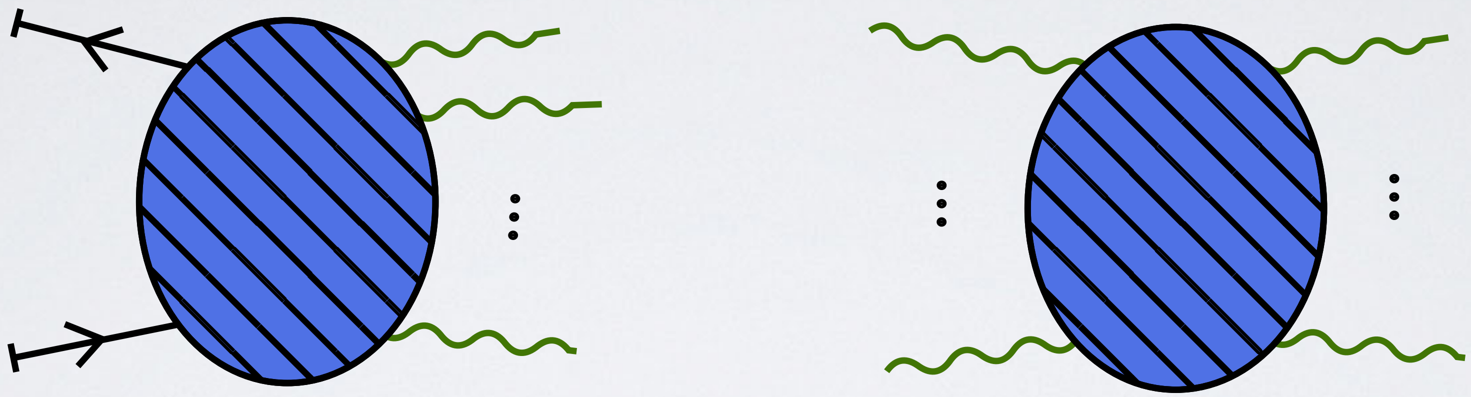
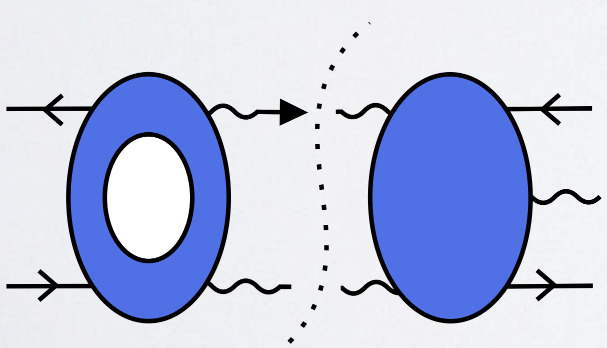


BUILDING-BLOCK AMPLITUDES

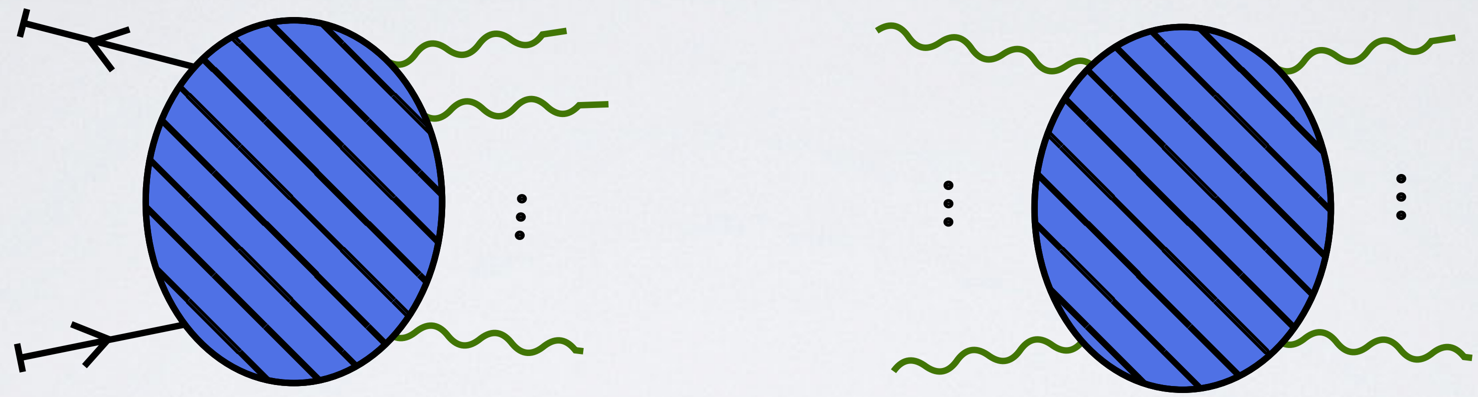


No new collinear
divergences in sewing:

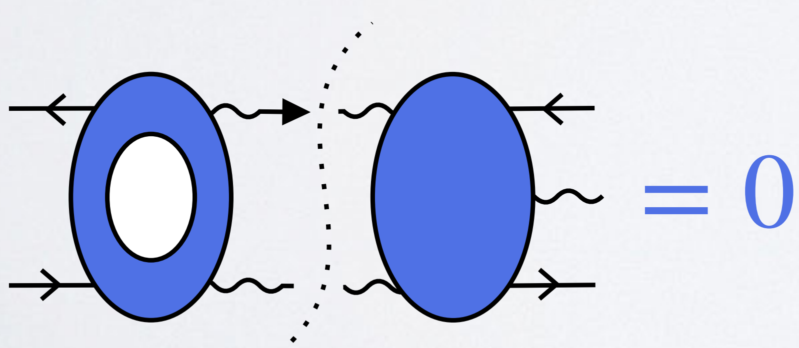


$= 0$

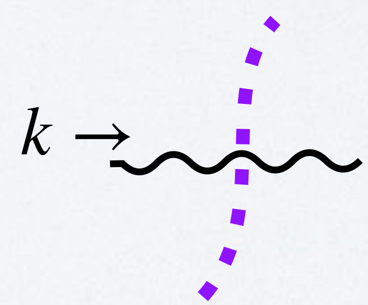
BUILDING-BLOCK AMPLITUDES



No new collinear divergences in sewing:



A sewing propagator:



$$\longrightarrow \frac{-i}{k^2}\left(g_{\mu\nu} - 2\frac{k_\mu \rho_\nu + k_\nu \rho_\mu}{(k+\rho)^2 - \rho^2}\right)$$

LOCALLY FINITE REMAINDERS

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$$F = \exp(-\hat{\Gamma}) \circ A$$

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$$\hat{\Gamma} = \alpha\,\hat{\Gamma}^{(1)} + \alpha^2\,\hat{\Gamma}^{(2)} + \dots$$

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Soft and collinear operators:

$$\hat{\Gamma}^{(i)} = \hat{S}^{(i)} + \hat{C}^{(i)}$$

LOCALLY FINITE REMAINDERS

$$F = \exp(-\hat{\Gamma}) \circ A$$

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Soft and collinear operators:

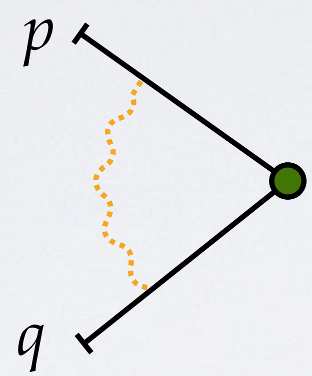
$$\hat{\Gamma}^{(i)} = \hat{S}^{(i)} + \hat{C}^{(i)}$$

$$F^{(1)} = A^{(1)} - \hat{\Gamma}^{(1)} \circ A^{(0)}$$

$$F^{(2)} = A^{(2)} - \hat{\Gamma}^{(1)} \circ F^{(1)} - \left(\frac{1}{2}\hat{\Gamma}^{(1)} \circ \hat{\Gamma}^{(1)} + \hat{\Gamma}^{(2)}\right) \circ A^{(0)}$$

1-LOOP SUBTRACTION OPERATORS

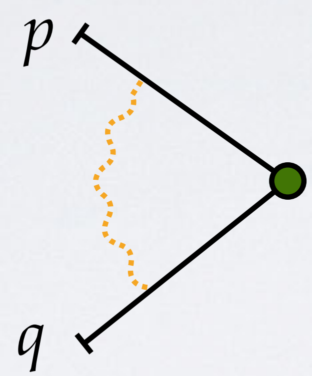
$\hat{S}^{(1)} \longrightarrow$



$$\equiv \frac{-ie_q e_p(2p \cdot 2q)}{k^2(p-k^2)(q+k)^2}$$

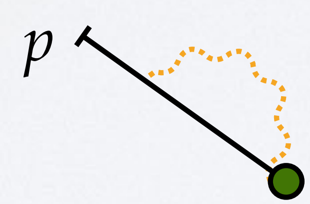
1-LOOP SUBTRACTION OPERATORS

$\hat{S}^{(1)} \longrightarrow$



$$\equiv \frac{-ie_q e_p(2p \cdot 2q)}{k^2(p-k^2)(q+k)^2}$$

$\hat{C}^{(1)} \longrightarrow$

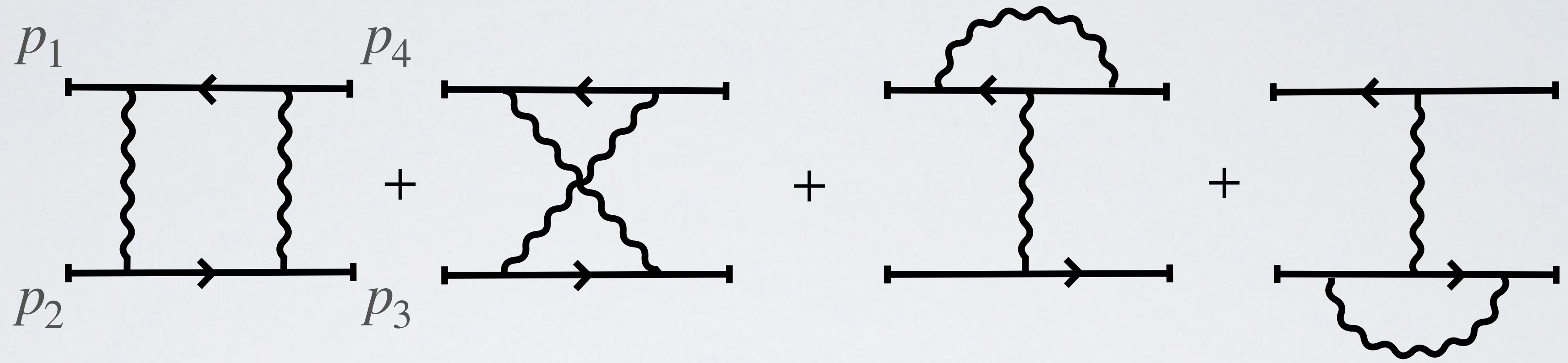


$$\equiv \frac{2ie_p^2}{k^2(p-k^2)}$$

ONE-LOOP IR-FINITE REMAINDER

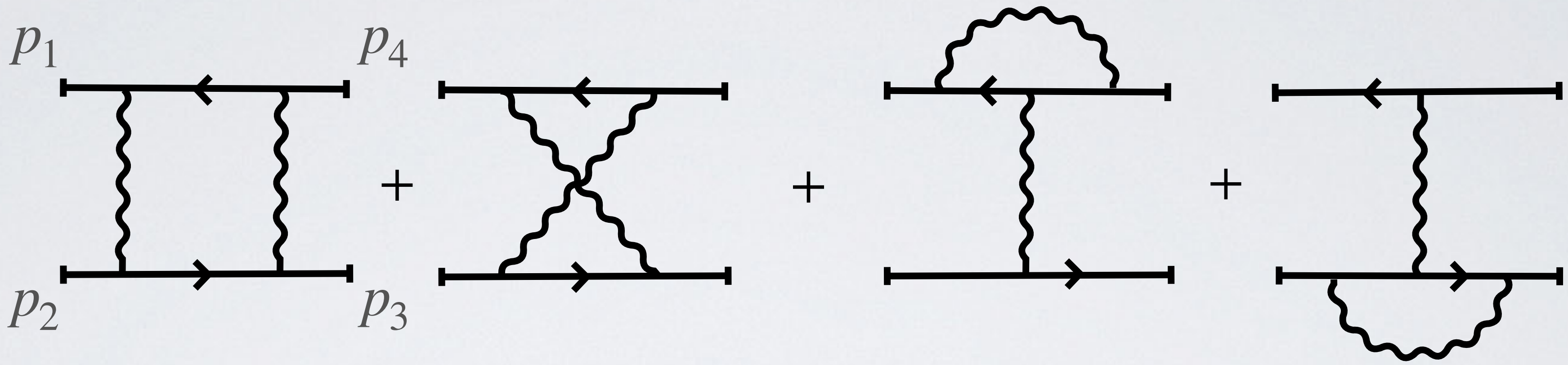
$A^{(1)} =$

ONE-LOOP IR-FINITE REMAINDER

$A^{(1)} =$ 

ONE-LOOP IR-FINITE REMAINDER

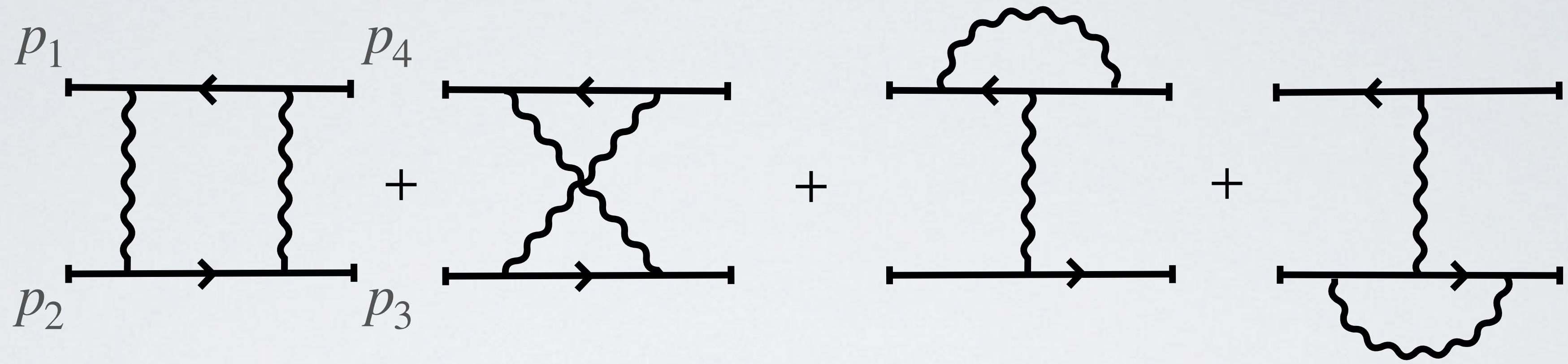
$A^{(1)} =$



$$F^{(1)} = A^{(1)} - \left(\hat{S}^{(1)} + \hat{C}^{(1)}\right) \circ A^{(0)}$$

ONE-LOOP IR-FINITE REMAINDER

$A^{(1)} =$

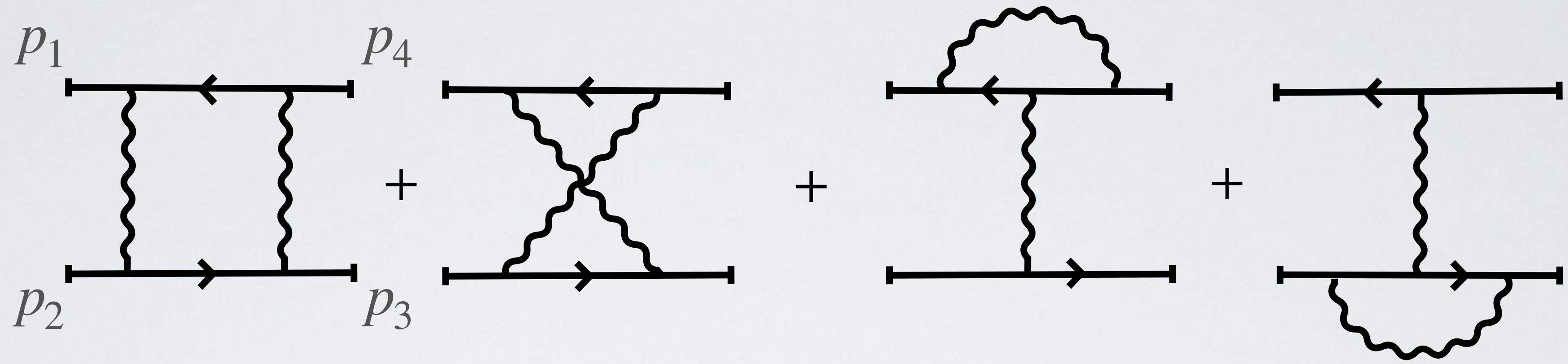


$$F^{(1)} = A^{(1)} - \left(\hat{S}^{(1)} + \hat{C}^{(1)}\right) \circ A^{(0)}$$

soft and collinear finite!

ONE-LOOP IR-FINITE REMAINDER

$A^{(1)} =$



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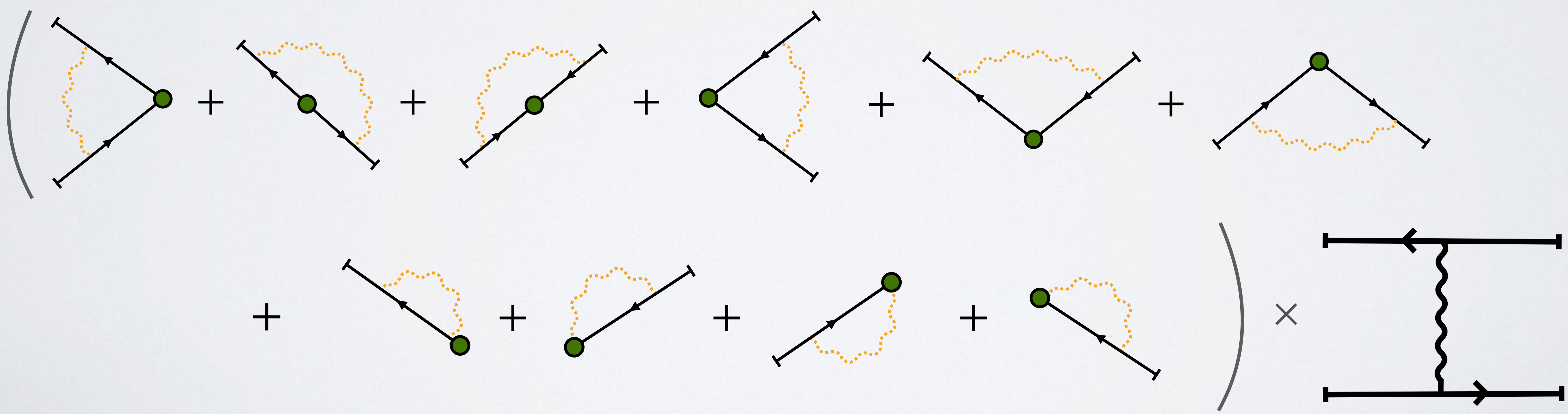
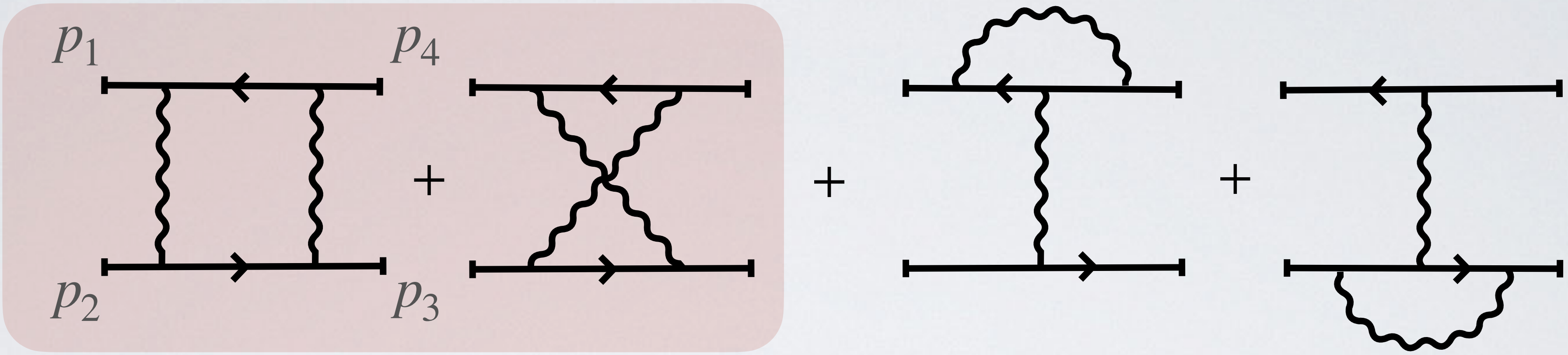


ONE-LOOP IR-FINITE REMAINDER

$A^{(1)} =$



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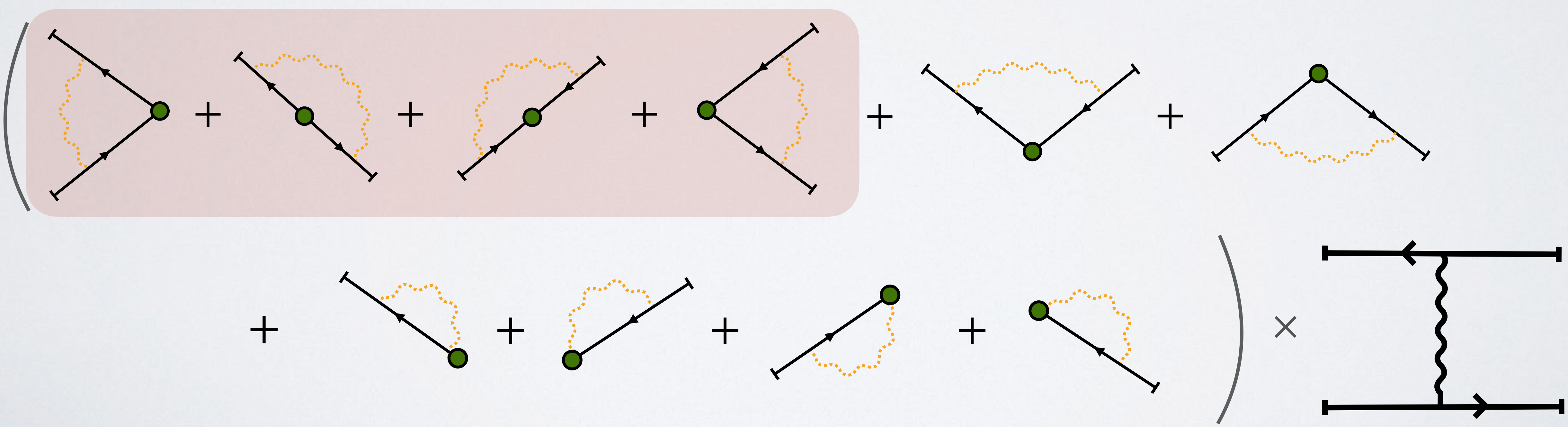
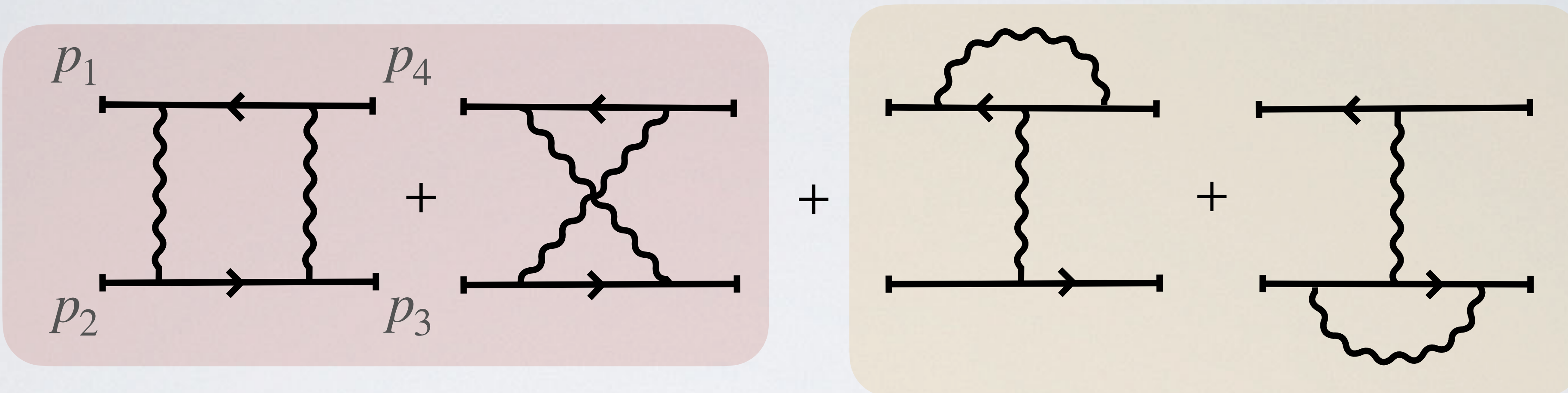


ONE-LOOP IR-FINITE REMAINDER

$A^{(1)} =$



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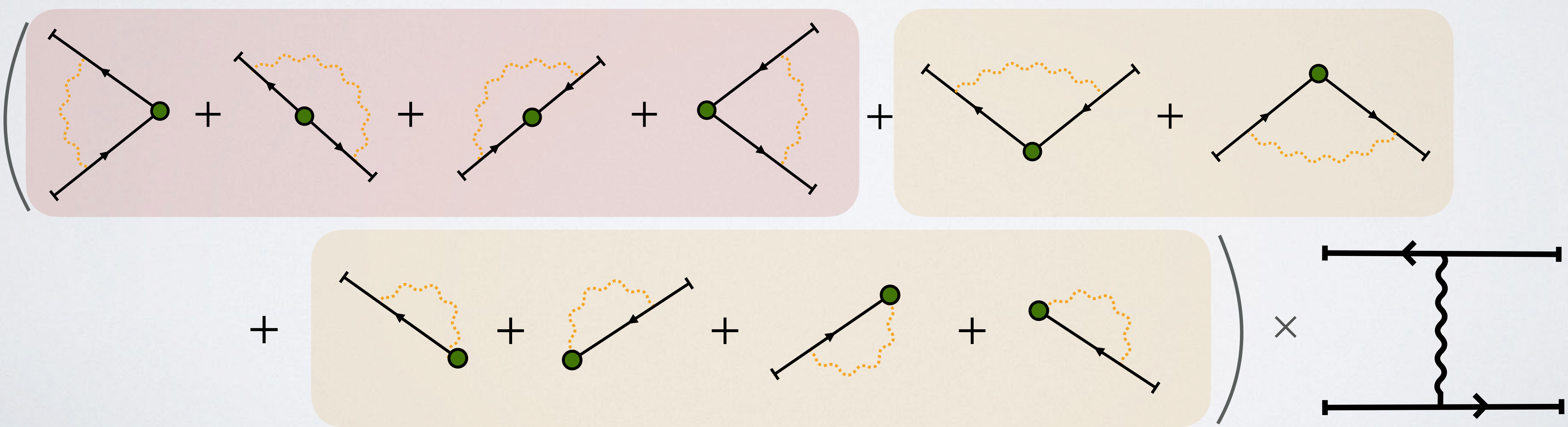
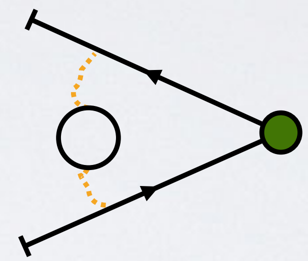
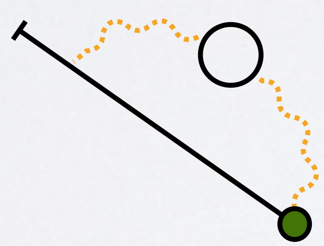
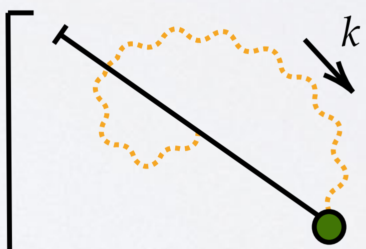
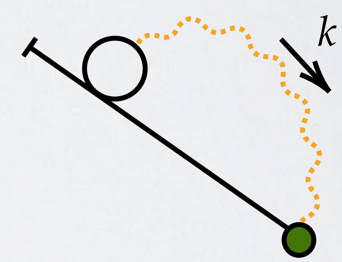


2-LOOP SUBTRACTION OPERATORS

$$\hat{S}^{(2)} \quad \rightarrow \quad \frac{4(1-\epsilon)}{3-2\epsilon}$$ 

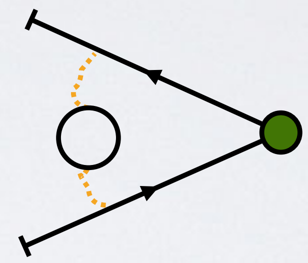
$$\hat{C}^{(2)} \quad \rightarrow \quad \frac{4(1-\epsilon)}{3-2\epsilon}$$  $+ \; 2(1-\epsilon)\,\frac{(p-k)\cdot\eta}{p\cdot\eta}\Bigg[$ 

$-$ 

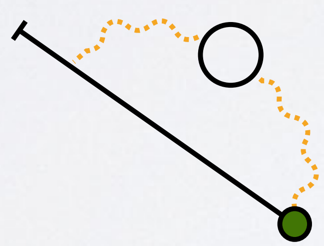
$\Bigg]$

2-LOOP SUBTRACTION OPERATORS

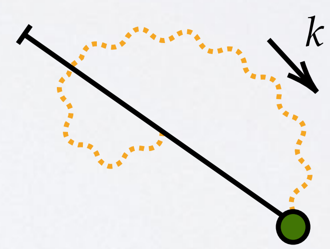
$\hat{S}^{(2)} \quad \to \quad \frac{4(1-\epsilon)}{3-2\epsilon}$



$\hat{C}^{(2)} \quad \to \quad \frac{4(1-\epsilon)}{3-2\epsilon}$



$+ \quad 2(1-\epsilon) \; \frac{(p-k)\cdot\eta}{p\cdot\eta} \Bigg[$



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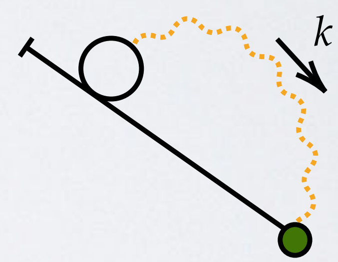
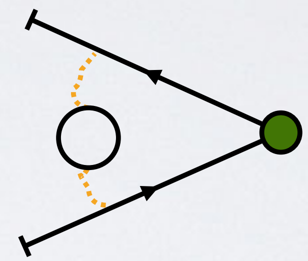
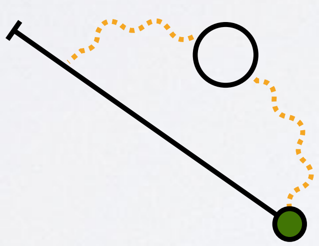
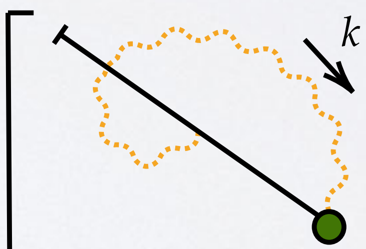


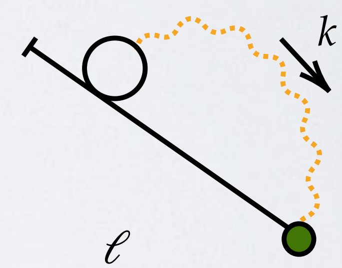
$\Bigg]$

$+ \quad \delta\hat{C}^{(2)}_{\text{NLP double collinear}}$

2-LOOP SUBTRACTION OPERATORS

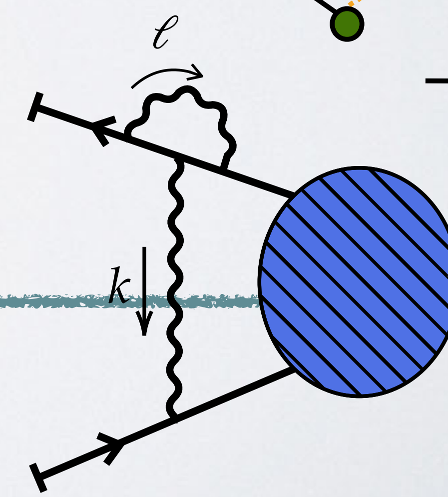
$$\hat{S}^{(2)} \quad \rightarrow \quad \frac{4(1-\epsilon)}{3-2\epsilon}$$ 

$$\hat{C}^{(2)} \quad \rightarrow \quad \frac{4(1-\epsilon)}{3-2\epsilon}$$  $$+ \; 2(1-\epsilon) \; \frac{(p-k)\cdot\eta}{p\cdot\eta} \Bigg[$$ 

$-$ 

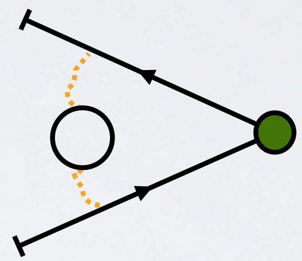
$\Bigg]$

$$+ \quad \delta\hat{C}^{(2)}_{\text{NLP double collinear}}$$

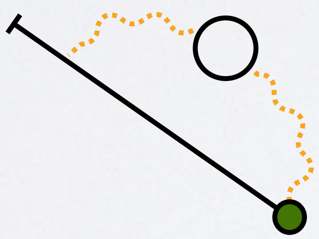


2-LOOP SUBTRACTION OPERATORS

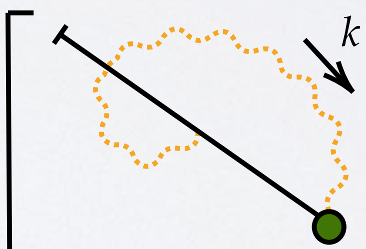
$$\hat{S}^{(2)} \quad \to \quad \frac{4(1-\epsilon)}{3-2\epsilon}$$



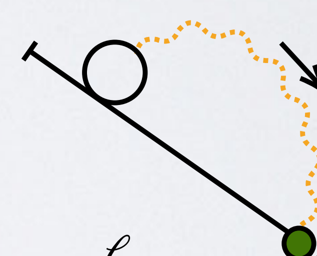
$$\hat{C}^{(2)} \quad \to \quad \frac{4(1-\epsilon)}{3-2\epsilon}$$



$$+ \quad 2(1-\epsilon) \; \frac{(p-k)\cdot\eta}{p\cdot\eta} \Bigg[$$



$$-$$



$$\Bigg]$$

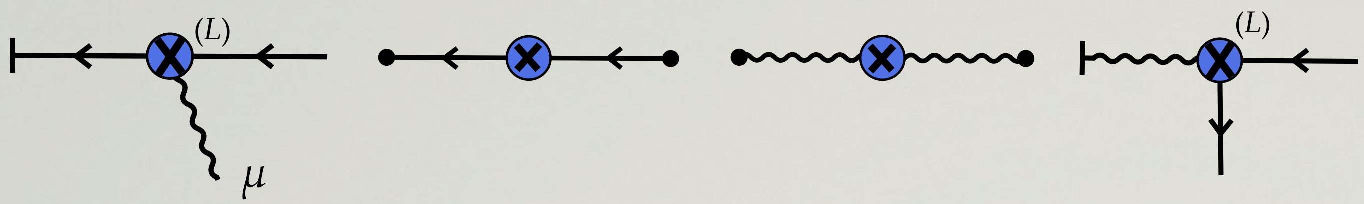
$$+ \quad \delta\hat{C}^{(2)}_{\text{NLP double collinear}}$$

WORK IN PROGRESS

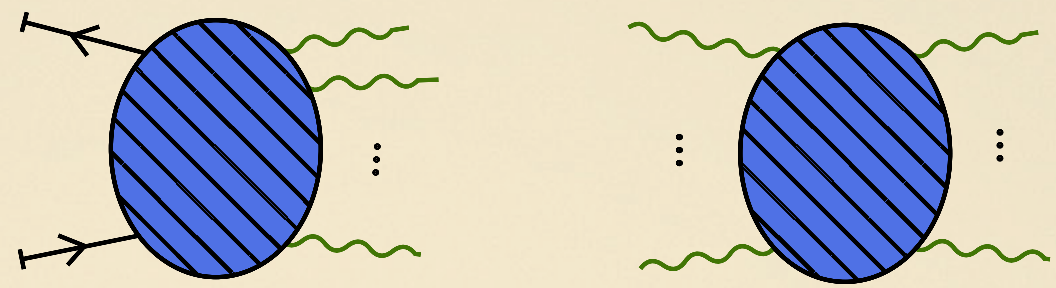
NEXT STEPS

- QED practical applications
- Generalisation to QCD (colour = routings)
- Generalisation to 3+ loops (all orders?)

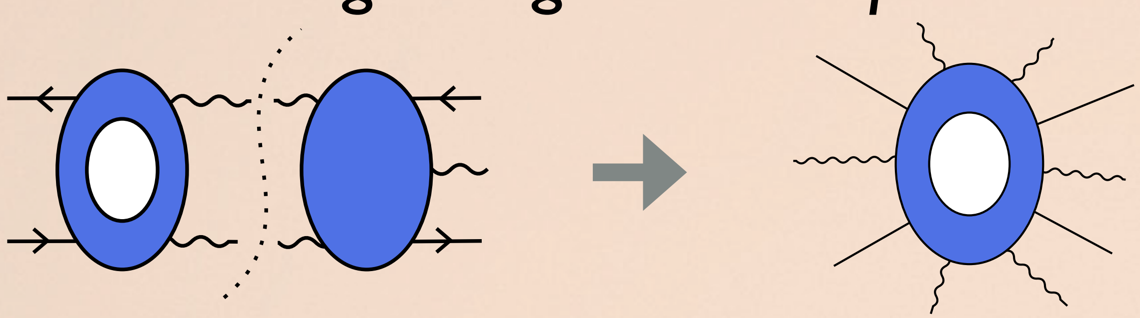
local counterterms



locally factorising building blocks



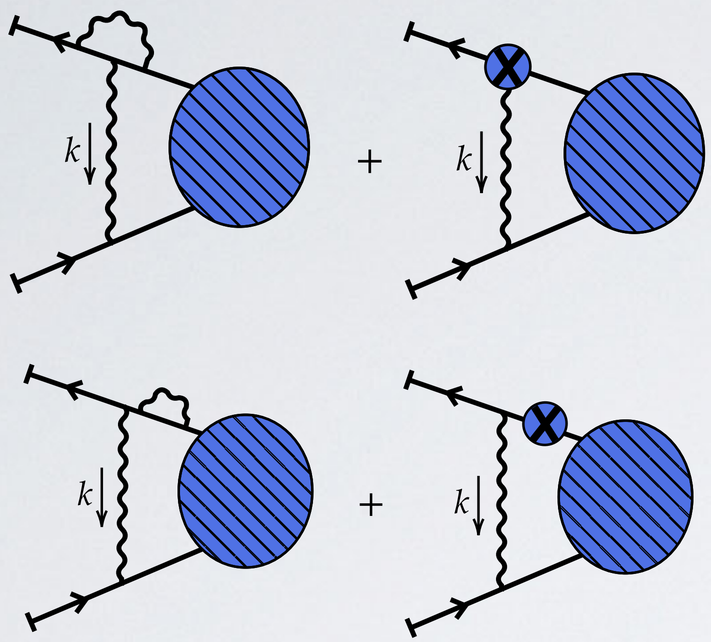
sewing into general amplitudes

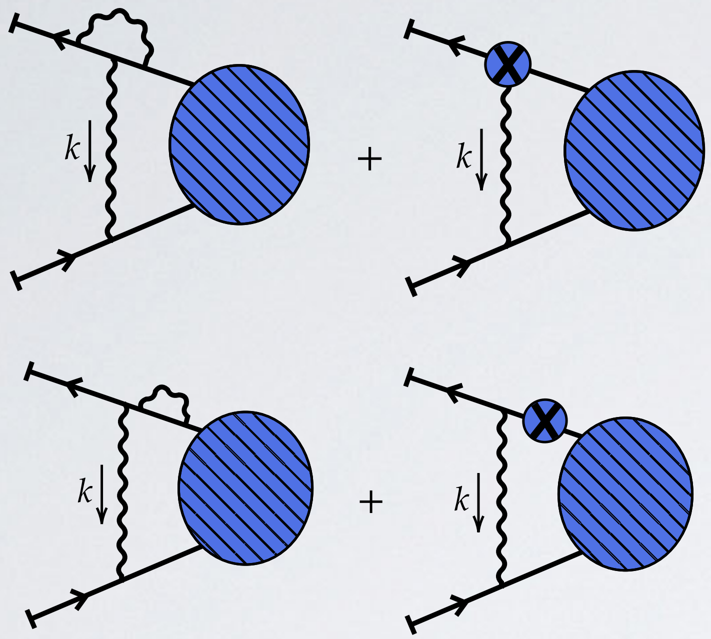


Integrand soft and collinear factorisation!

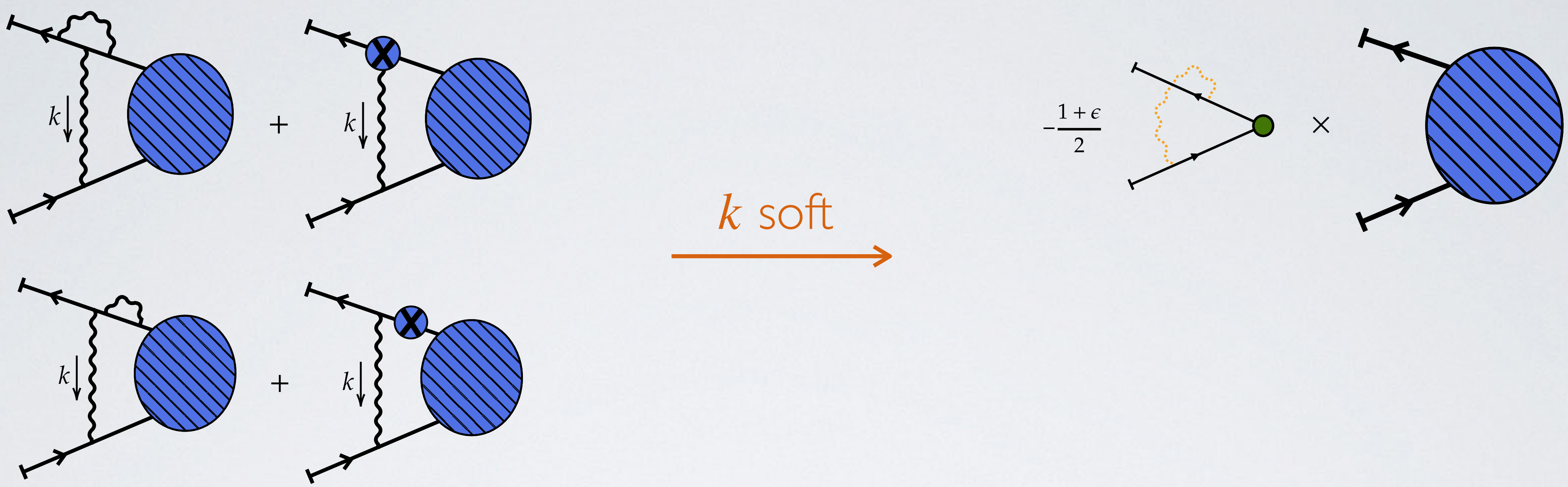
$$F^{(1)} = A^{(1)} - \hat{\Gamma}^{(1)} \circ A^{(0)}$$

$$F^{(2)} = A^{(2)} - \hat{\Gamma}^{(1)} \circ F^{(1)} - \left(\frac{1}{2}\hat{\Gamma}^{(1)} \circ \hat{\Gamma}^{(1)} + \hat{\Gamma}^{(2)}\right) \circ A^{(0)}$$

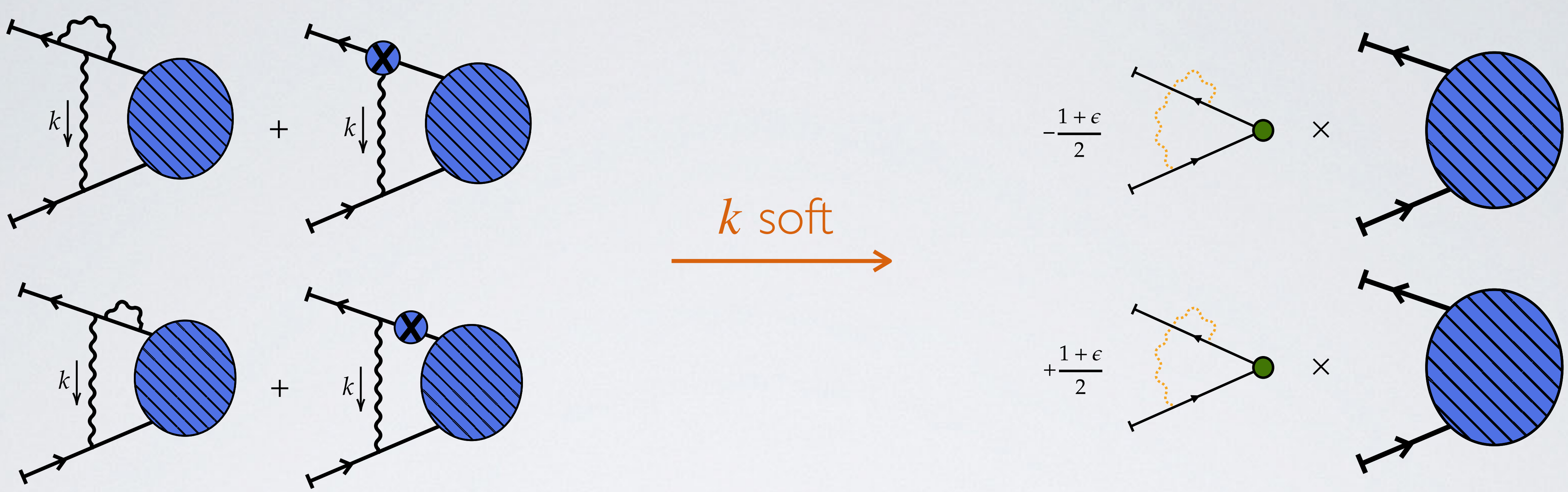

k
+
k
k
+
k



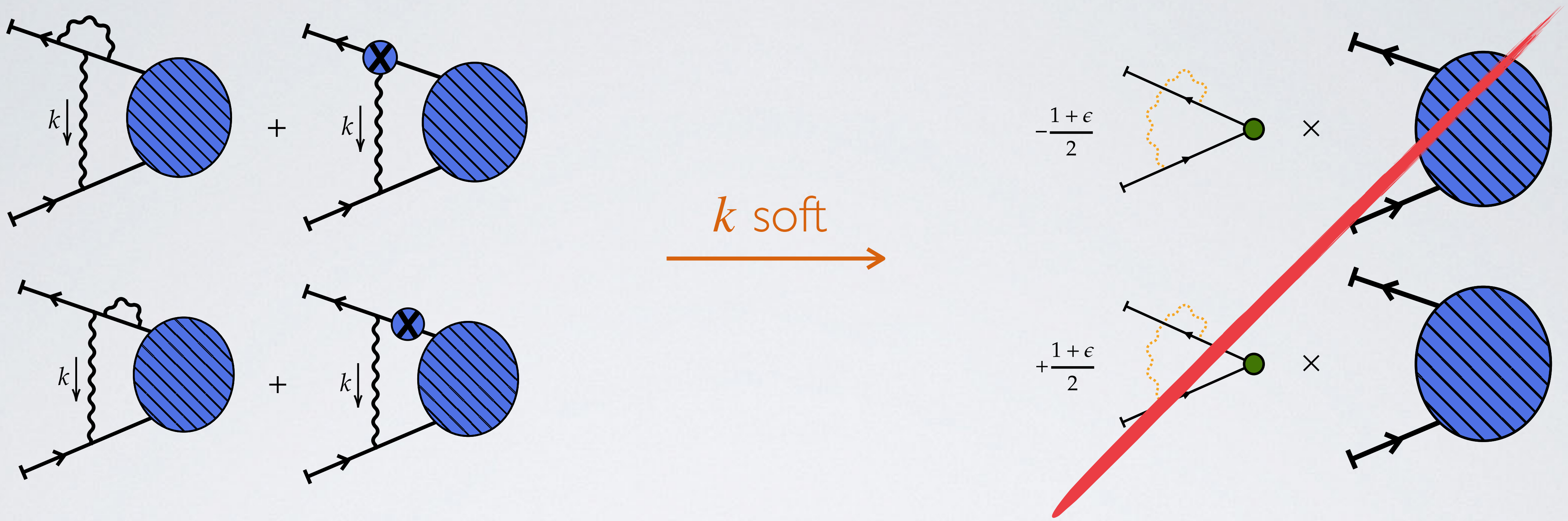
k soft



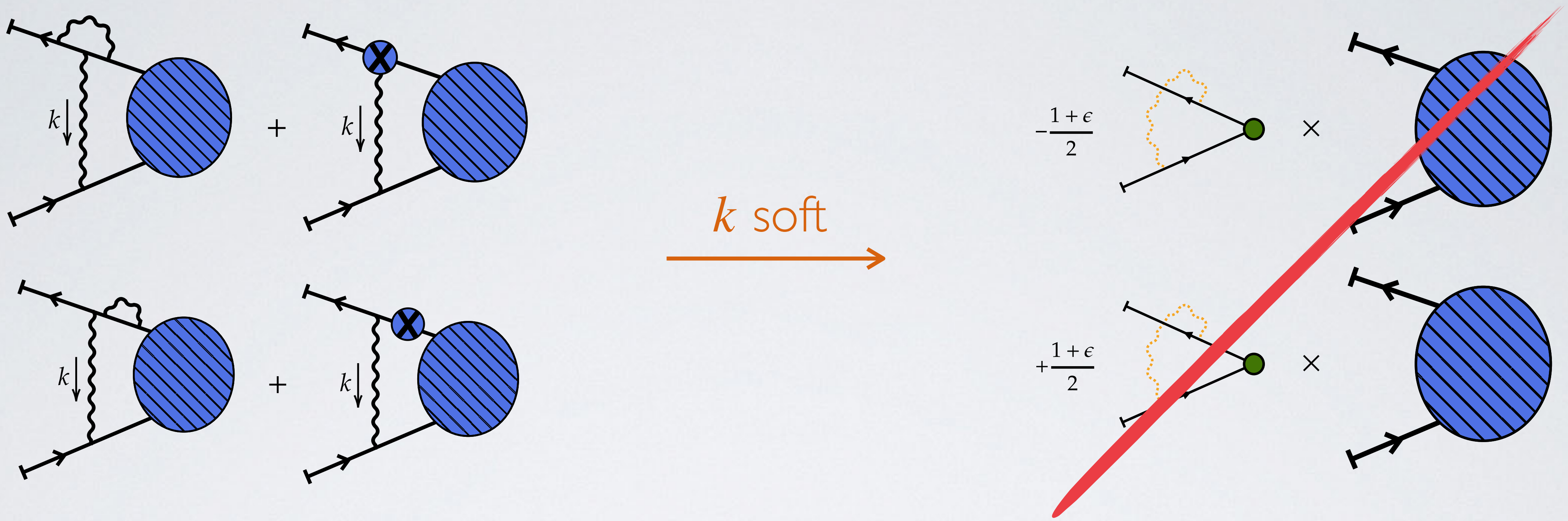
k
+
k
k
+
k
k soft
$-\frac{1+\epsilon}{2}$
×

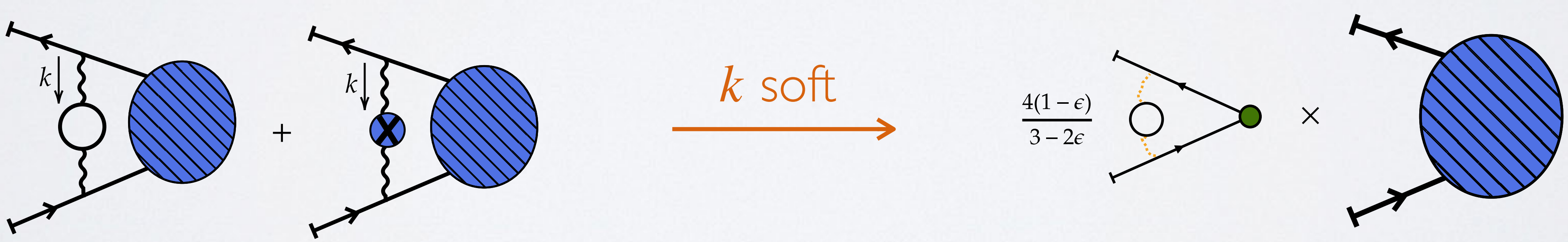


k
+
k
k
+
k
k soft
$-\frac{1+\epsilon}{2}$
×
$+\frac{1+\epsilon}{2}$
×

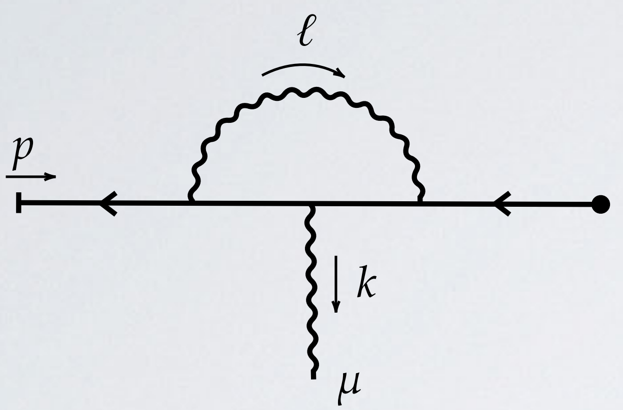


k
+
k
k
+
k
k soft
$-\frac{1+\epsilon}{2}$
×
$+\frac{1+\epsilon}{2}$
×


k
+
k soft
$-\frac{1+\epsilon}{2}$
×
$+\frac{1+\epsilon}{2}$
×

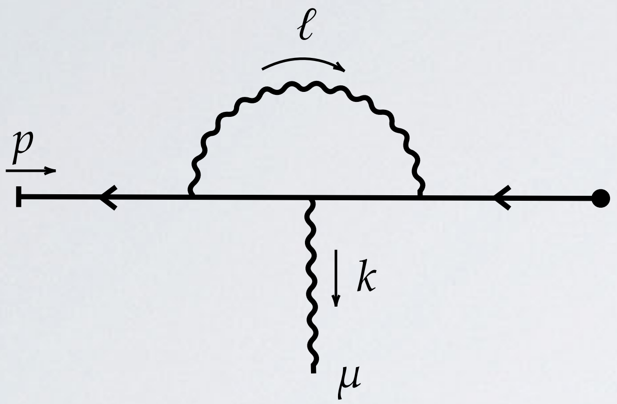

k
+
k soft
$\frac{4(1-\epsilon)}{3-2\epsilon}$
×

THERE ARE NO LOOP POLARISATIONS!



$$= \quad \bar{u}(p)\, \not{\Gamma}^{\mu}(p,k)\, \frac{i(\not{k}-\not{p})}{(k-p)^2}$$

THERE ARE NO LOOP POLARISATIONS!

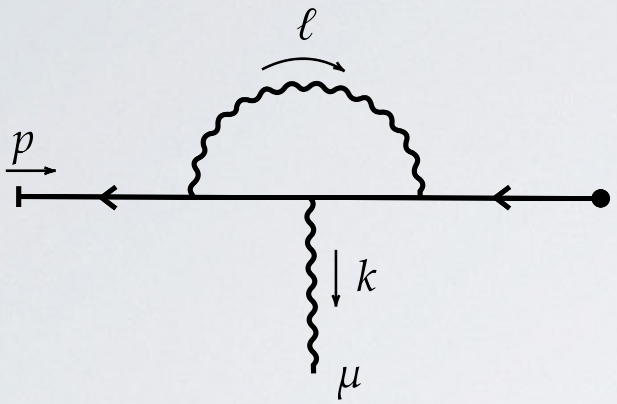


$$= \quad \bar{u}(p)\, \not{V}^{\mu}(p,k)\, \frac{i(\not{k}-\not{p})}{(k-p)^2}$$

tensor reduction

$$\not{V}^{\mu}(p,k) \doteq \not{V}_p\, p^{\mu} + \not{V}_k\, k^{\mu}$$

THERE ARE NO LOOP POLARISATIONS!



$$= \quad \bar{u}(p)\, \not{V}^{\mu}(p,k)\, \frac{i(\not{k} - \not{p})}{(k-p)^2}$$

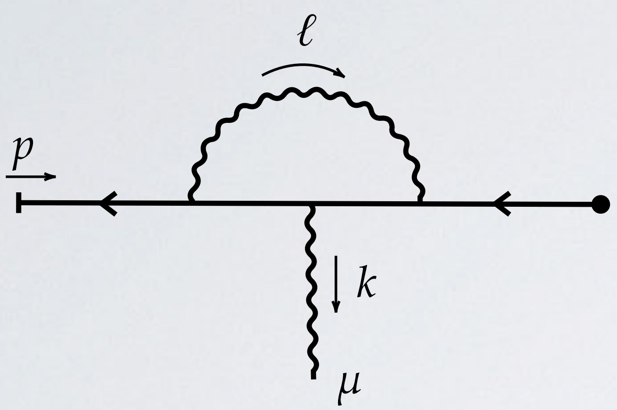
tensor reduction

$$\not{V}^{\mu}(p,k) \overset{\int}{=} \not{V}_p\, p^{\mu} + \not{V}_k\, k^{\mu}$$

<u>all loop polarisation can be removed via boundary terms!</u>

$$\not{V}^{\mu}(p,k) \quad \rightarrow \quad \not{V}^{\mu}(p,k) \quad + \quad \delta\not{V}^{\mu}(p,k)$$

THERE ARE NO LOOP POLARISATIONS!



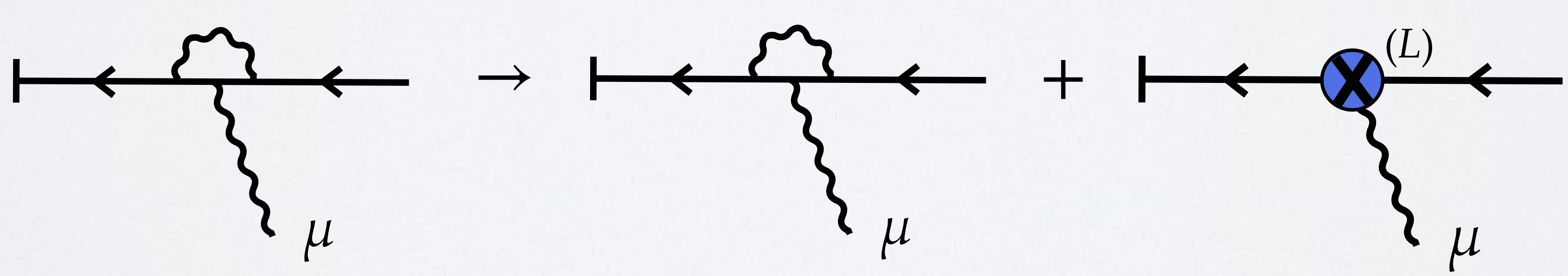
$$= \quad \bar{u}(p)\, \not{\Gamma}^{\mu}(p,k)\, \frac{i(\not{k}-\not{p})}{(k-p)^2}$$

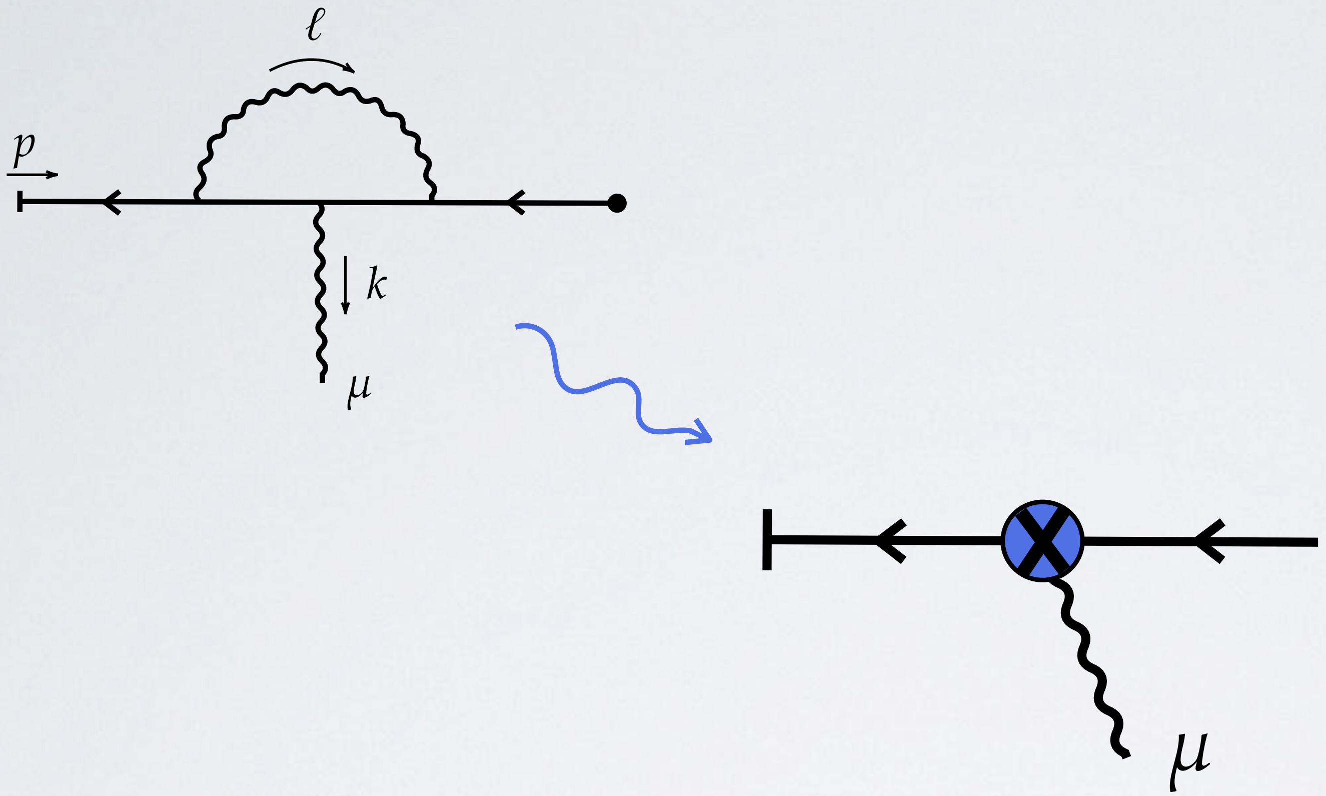
tensor reduction

$$\not{\Gamma}^{\mu}(p,k) \overset{\int}{=} \not{\Gamma}_p\, p^{\mu} + \not{\Gamma}_k\, k^{\mu}$$

all loop polarisation can be removed via boundary terms!

$$\not{\Gamma}^{\mu}(p,k) \quad \rightarrow \quad \not{\Gamma}^{\mu}(p,k) \quad + \quad \delta\not{\Gamma}^{\mu}(p,k)$$



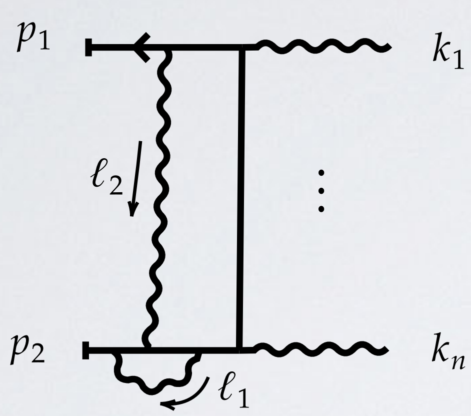


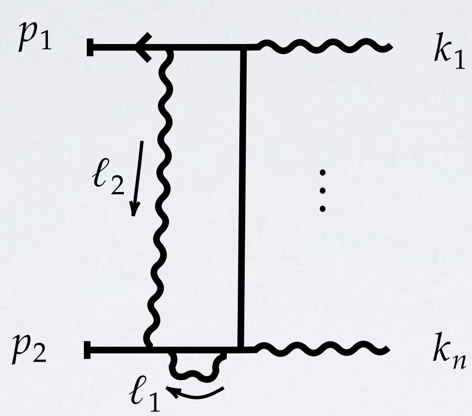
$$\delta \Gamma^{\mu}(p,k;\ell) \equiv ie^2(2-d)\frac{\not{\eta}}{2p\cdot\eta}\left[\frac{(p-2\ell-k)^{\mu}}{\ell^2(p-\ell-k)^2} - \frac{(2p-2\ell-k)^{\mu}}{(p-\ell)^2(p-\ell-k)^2}\right]$$

$$\eta^2 = 0$$

AMPLITUDE COUNTERTERMS

AMPLITUDE COUNTERTERMS





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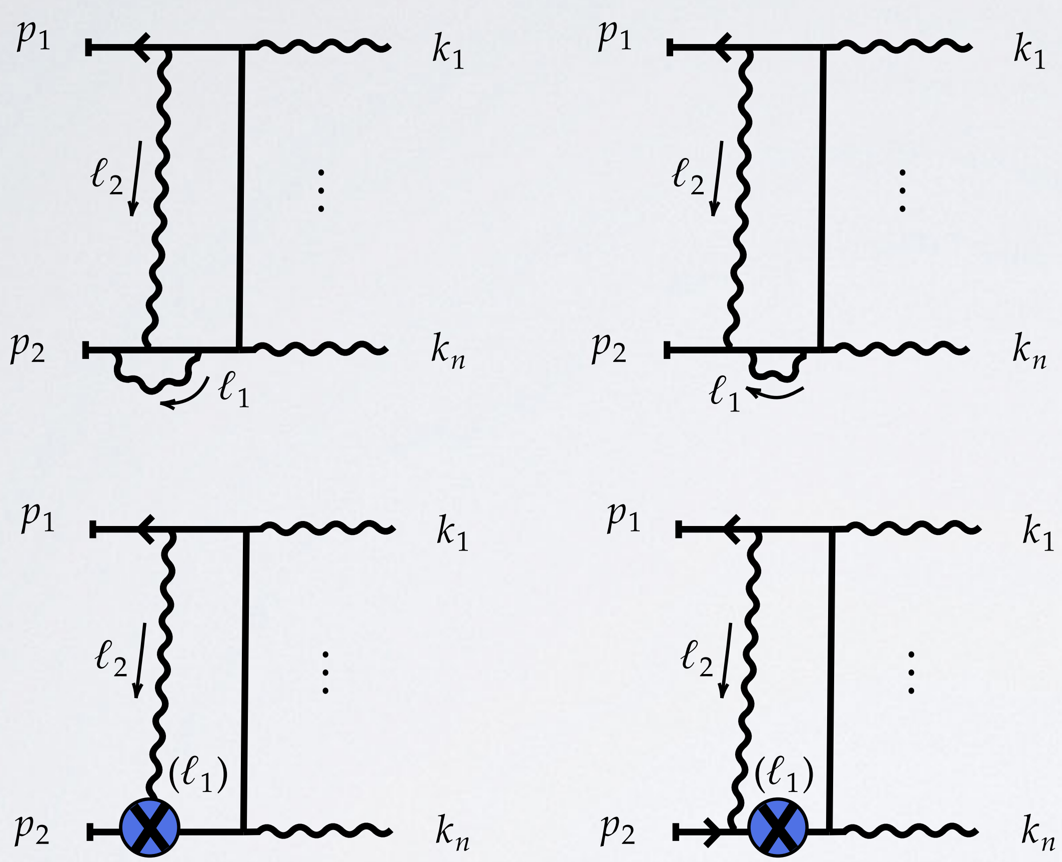

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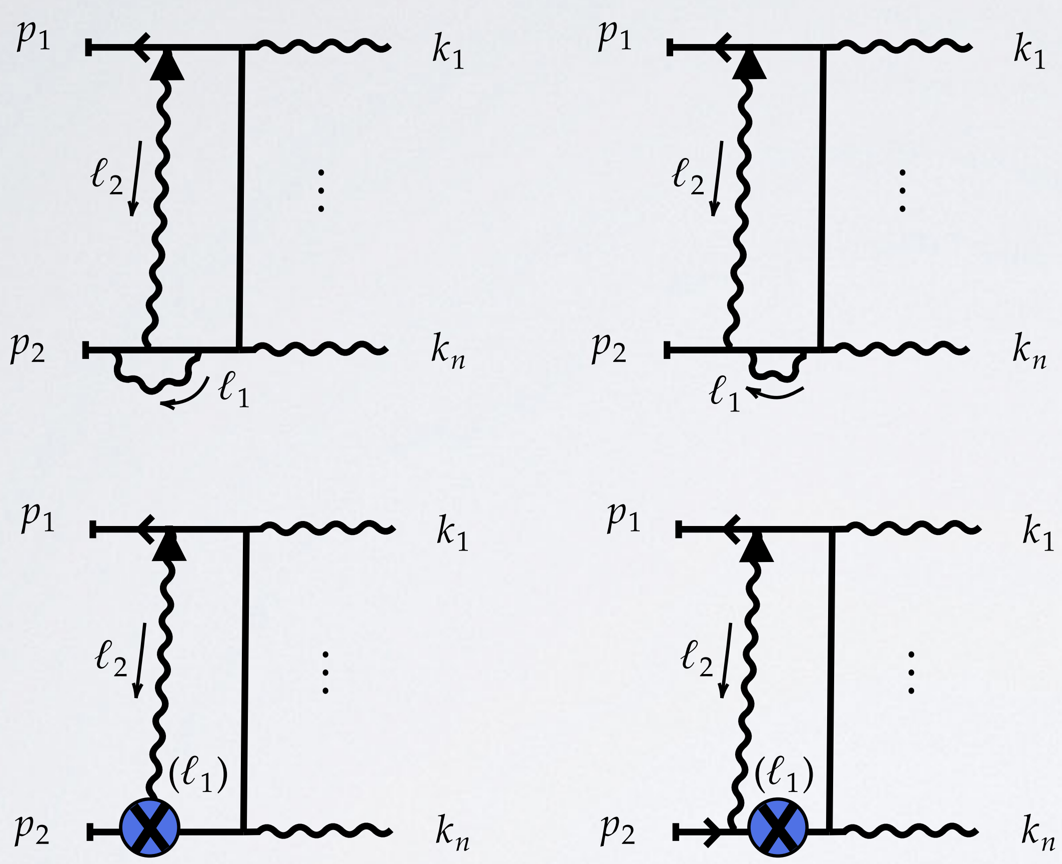

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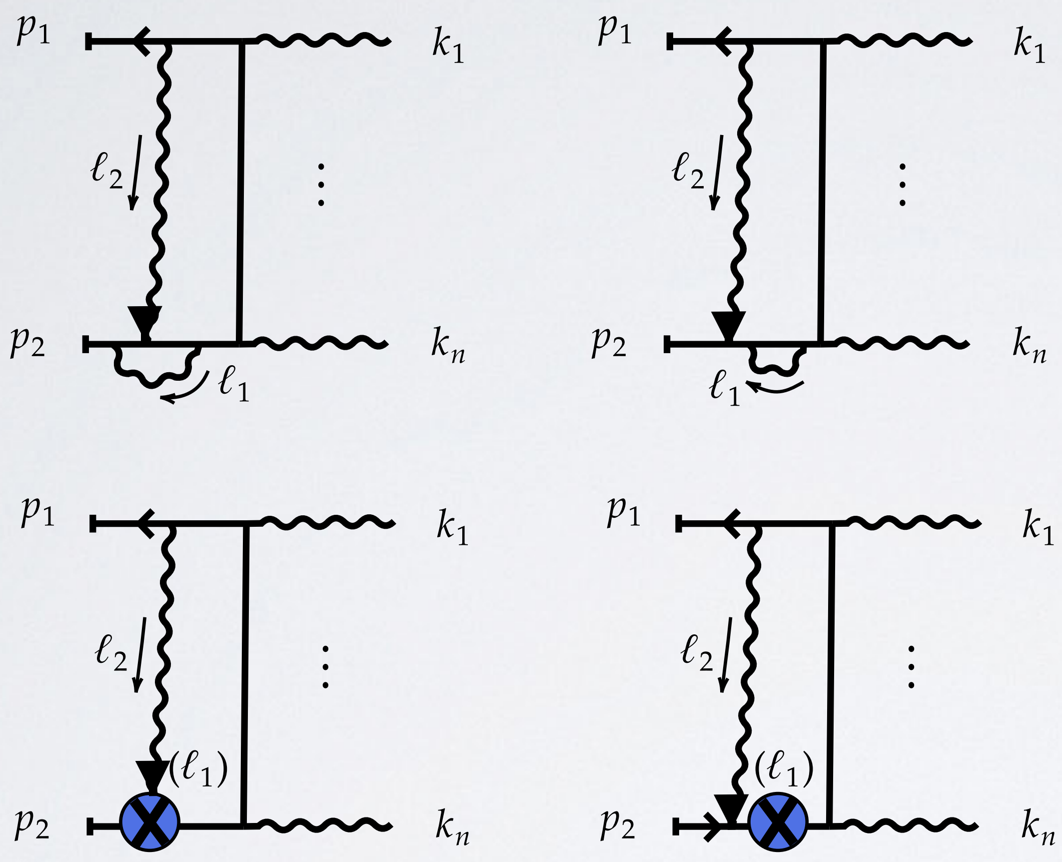

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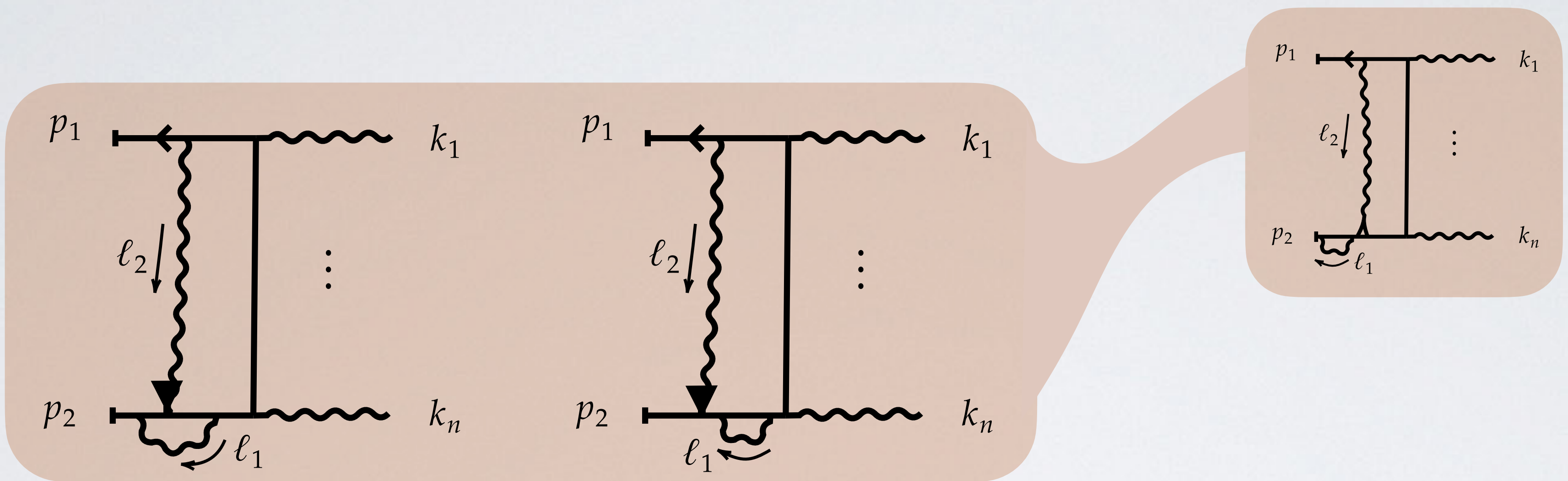

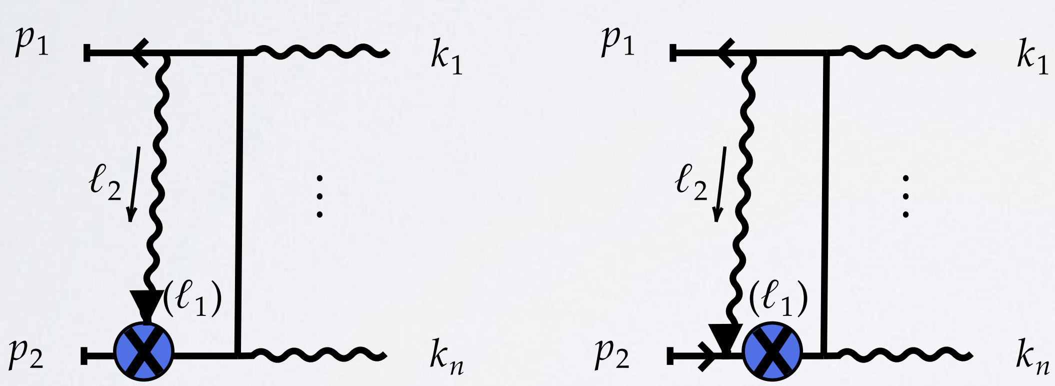

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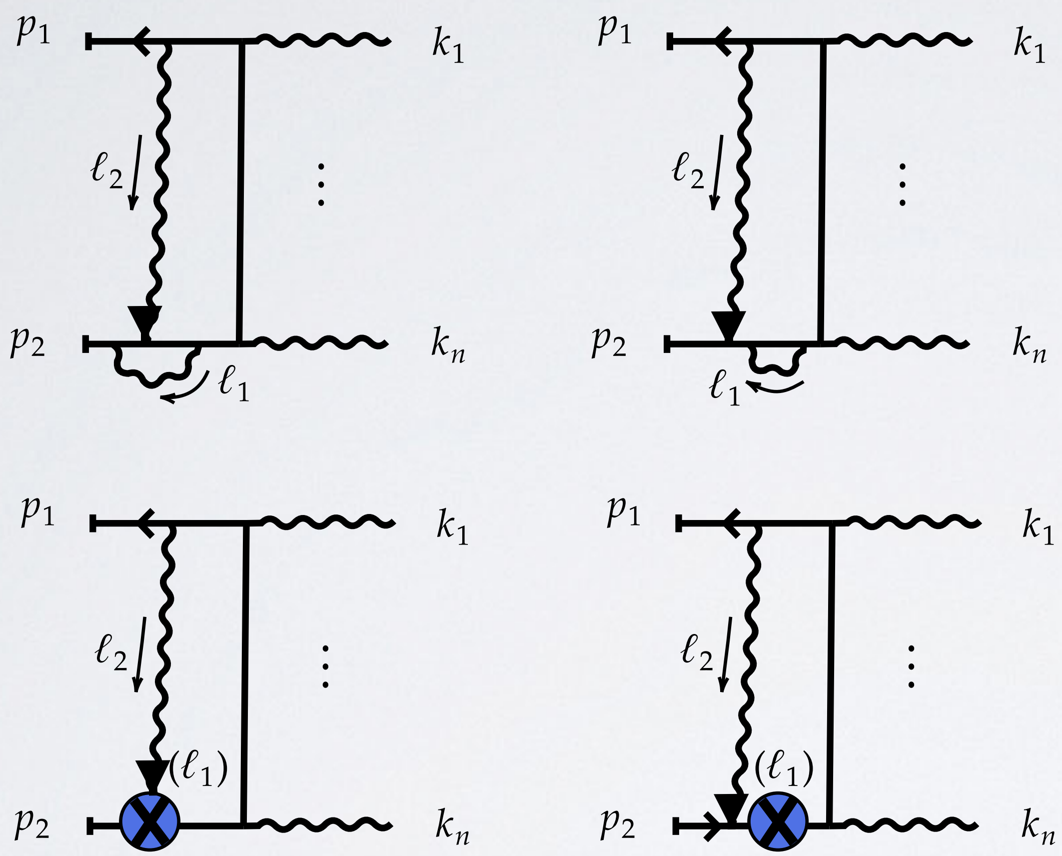

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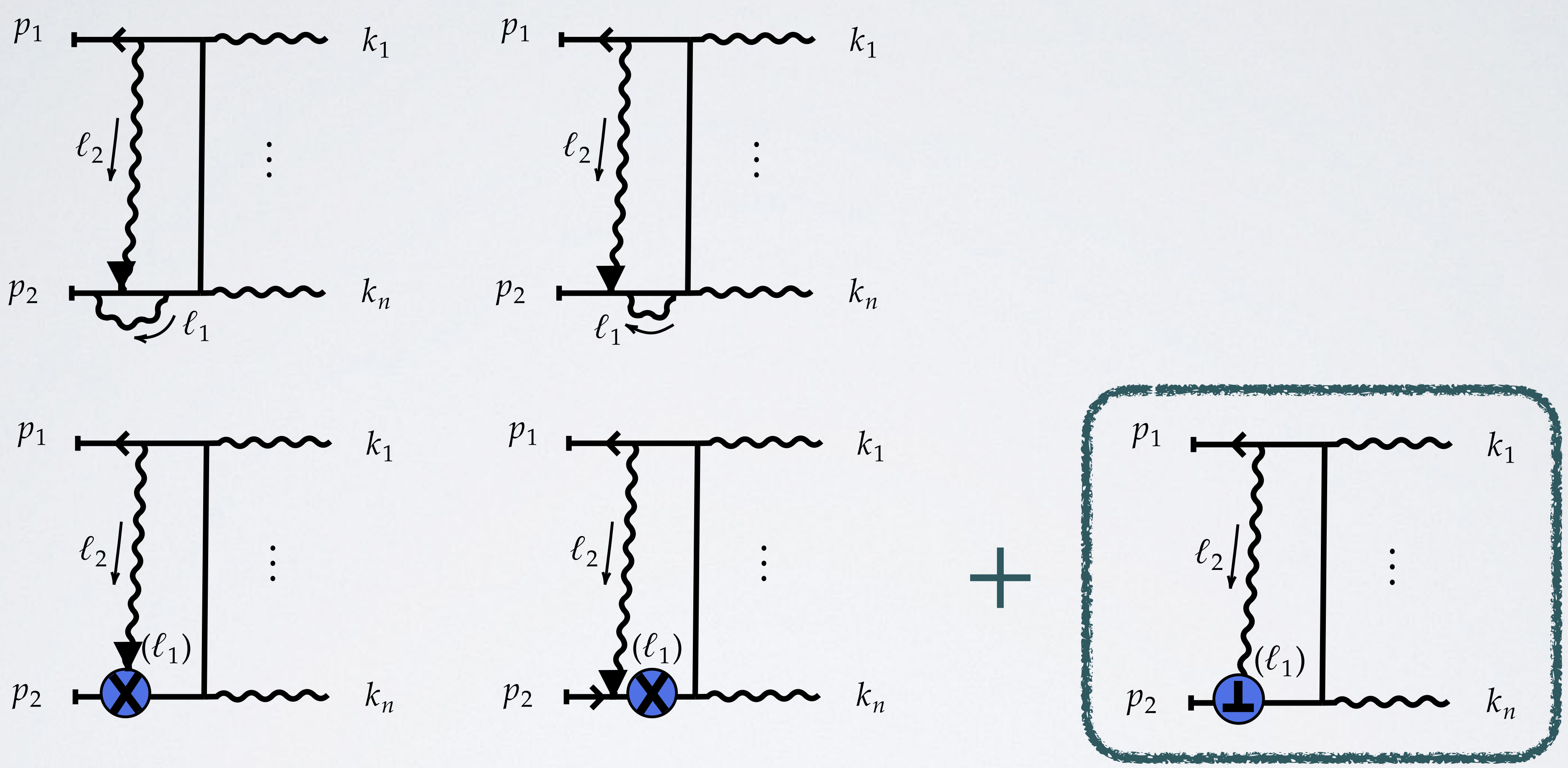

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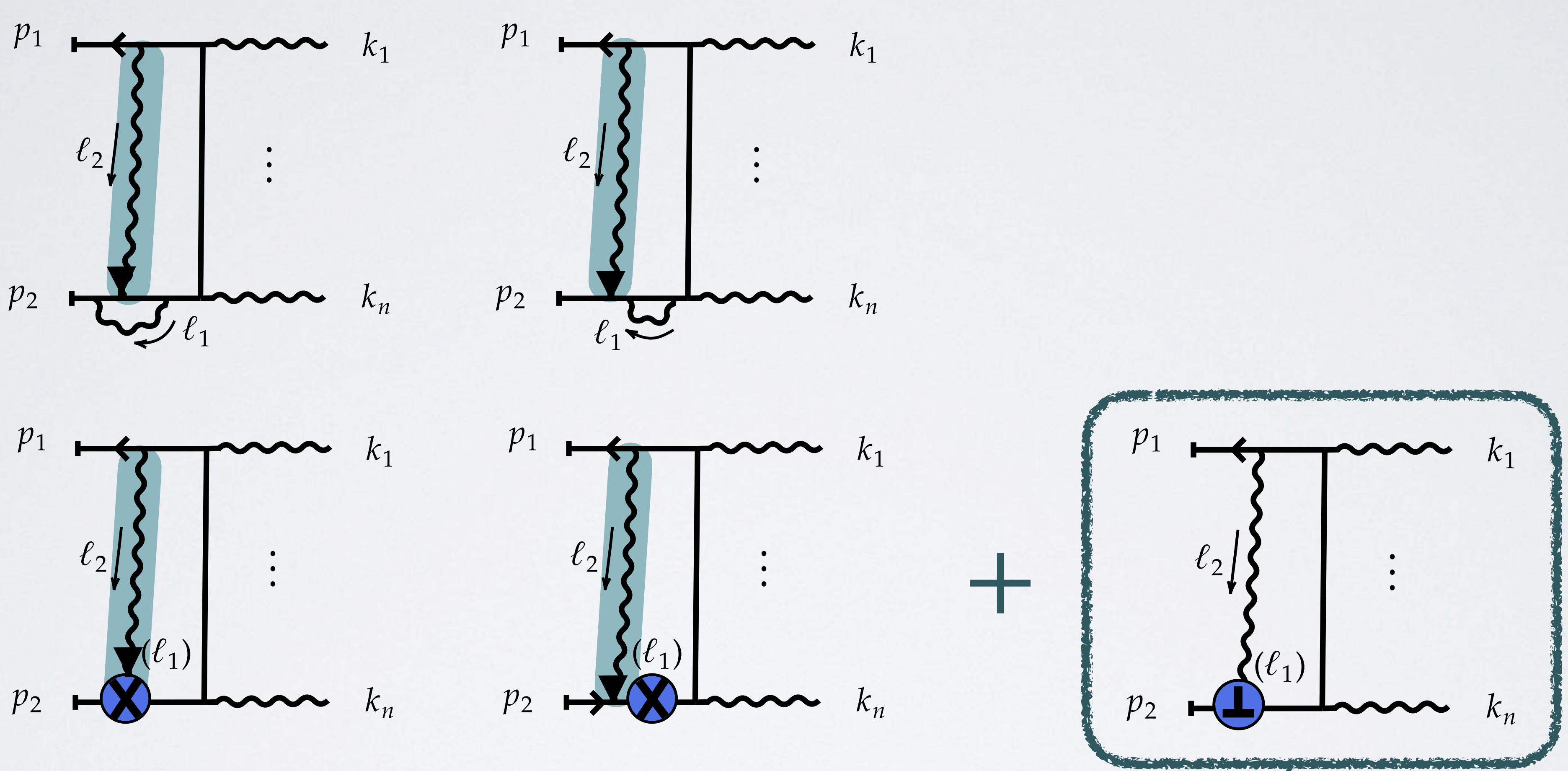
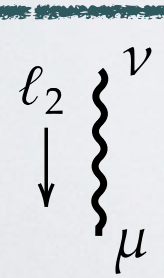
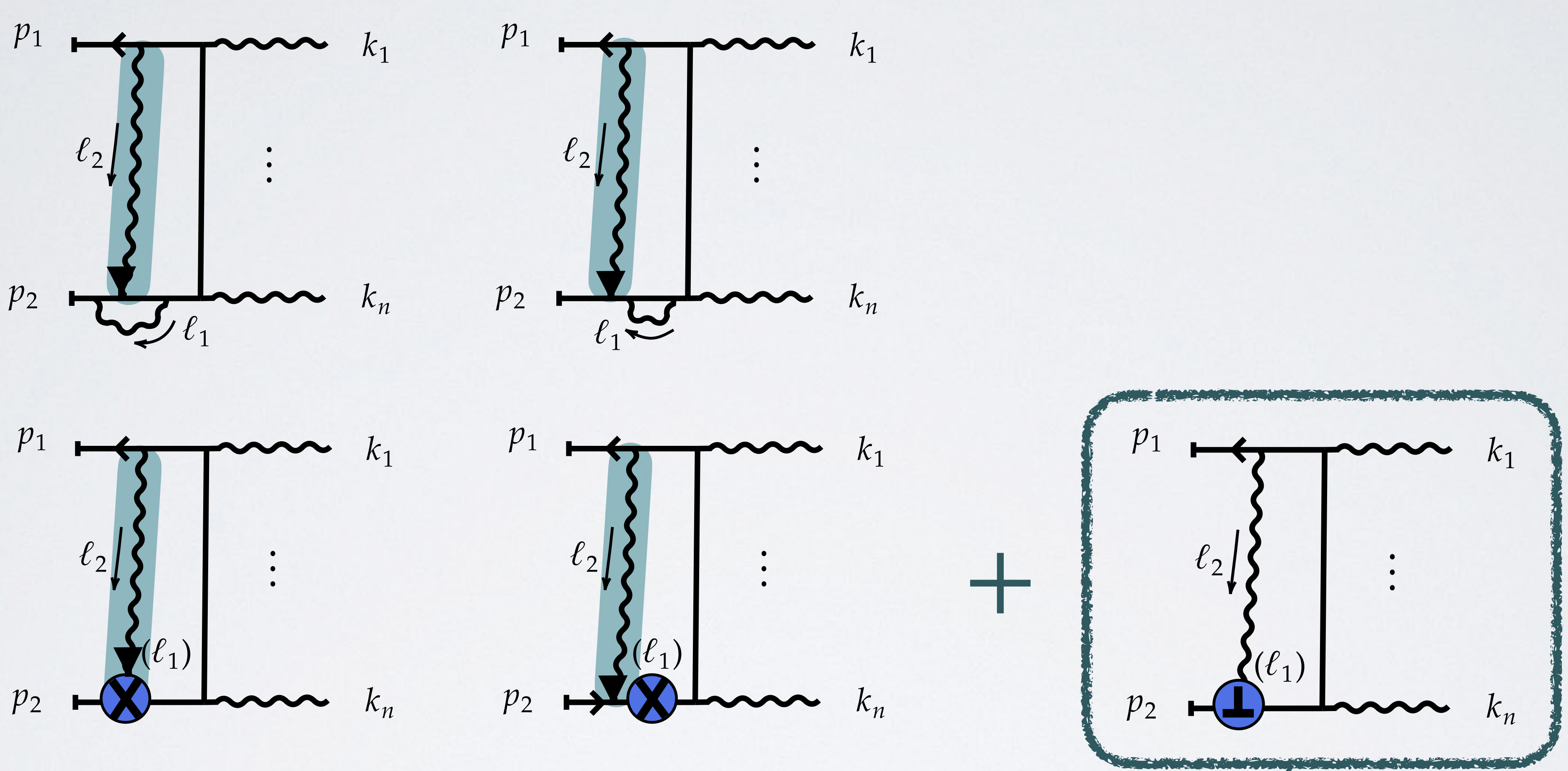
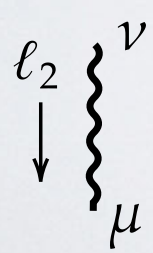




$$\frac{-ig^{\mu\nu}}{{\ell_2}^2} \to \frac{-i}{{\ell_2}^2}\left(g^{\mu\nu} - 2\frac{{\ell_2}^\mu \rho^\nu}{(\ell_2+\rho)^2-\rho^2}\right)$$

AMPLITUDE COUNTERTERMS





$$\frac{-ig^{\mu\nu}}{\ell_2^{\,2}} \to \frac{-i}{\ell_2^{\,2}}\left(g^{\mu\nu} - 2\frac{\ell_2^{\,\mu}\rho^{\nu}}{(\ell_2+\rho)^2-\rho^2}\right)$$

fully automated integrand generation