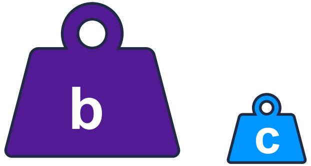
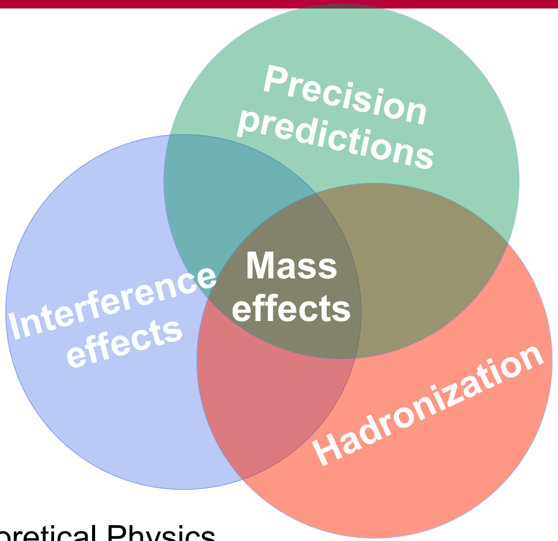
UNIVERSITÄT BERN

Heavy Quark Effects in Soft-Collinear Effective Theory

Rebecca von Kuk

LoopFest 2026





Albert Einstein Center for Fundamental Physics — Institute for Theoretical Physics

May 27 2026

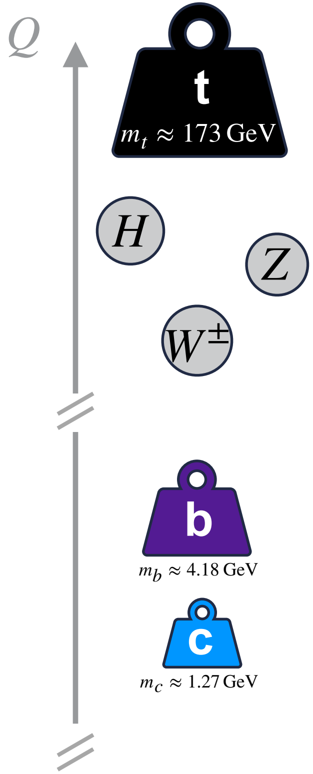
Introduction

Heavy Quarks

UNIVERSITÄT BERN

What are Heavy Quarks?

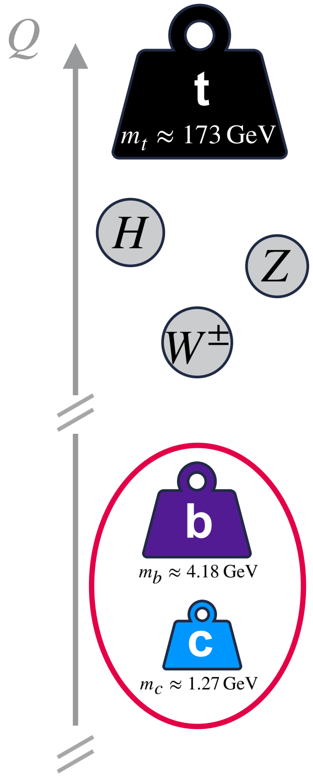
- QCD: usually consider 5 massless flavors and the massive top quark
  - m_t can only be neglected at very high energies $Q \gg m_t$
  - expansion in heavy-top limit (HTL) $m_t \to \infty$
- Lighter quark masses often neglected
  - typical hard scattering energies at the LHC: $m_q \ll Q \lesssim 100\,\text{GeV}$
- Precision era: theory and experiments archive impressive precision
  - bottom and charm mass effects become relevant!
- **Here**: heavy quarks = bottom and charm



Heavy Quarks

What are Heavy Quarks?

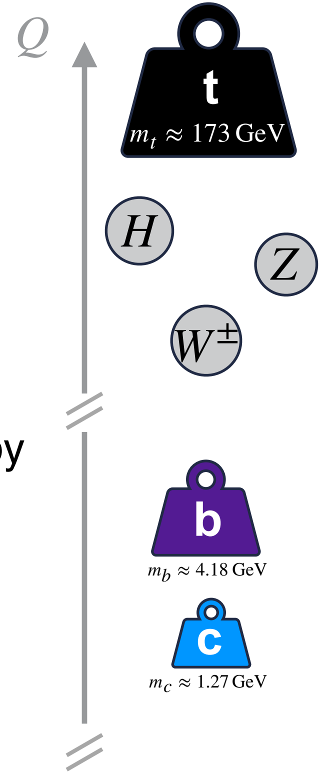
- QCD: usually consider 5 massless flavors and the massive top quark
  - m_t can only be neglected at very high energies $Q \gg m_t$
  - expansion in heavy-top limit (HTL) $m_t \to \infty$
- Lighter quark masses often neglected
  - typical hard scattering energies at the LHC: $m_q \ll Q \lesssim 100\,\mathrm{GeV}$
- Precision era: theory and experiments archive impressive precision
  - bottom and charm mass effects become relevant!
- **Here**: heavy quarks = bottom and charm



Heavy Quarks

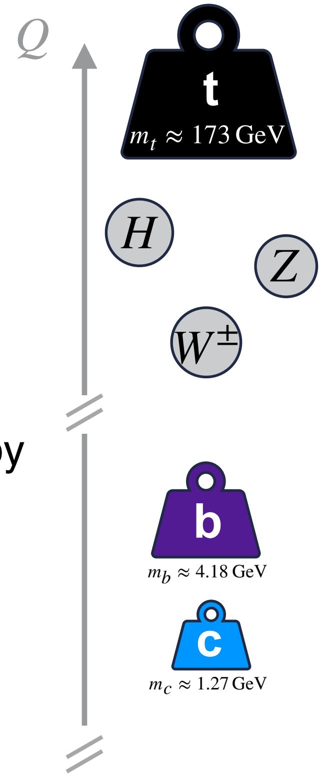
Where can we see Heavy Quarks effects ?

- Validity of factorization theorems in high-order predictions
  - Factorization separates physics at different scale, e.g. $q_T \ll Q$
  - Standard factorization theorems assume massless quarks
- Interference effects in loop-induced processes, e.g. gluon fusion
  - usually only top-quark loop considered but lighter contribution enhanced by interference
- Fragmentation processes: $Q \sim 10\,\mathrm{GeV}$
  - m_Q is no longer small →must be resummed and matched properly!



Outline

- **Motivation** ✓
- **Validity of factorization theorems in high-order predictions**
  - Factorization separates physics at different scale, e.g. $q_T \ll Q$
  - Standard factorization theorems assume massless quarks
- **Interference effects in loop-induced processes**, e.g. gluon fusion
  - usually only top-quark loop considered but lighter contribution enhanced by interference
- **Fragmentation processes**: $Q \sim 10\,\mathrm{GeV}$
  - m_Q is no longer small →must be resummed and matched properly!
- **Outlook and Summary**



Mass effects in q_T resummation

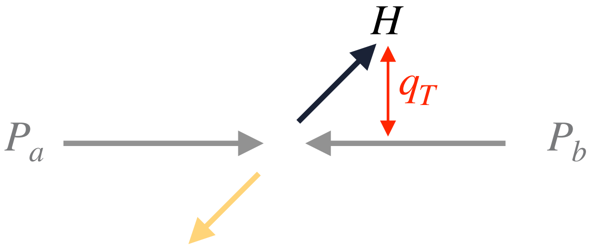
Factorization and resummation in SCET

q_T factorization in SCET: $q_T \ll Q$

$$\log\left(\frac{q_T}{Q}\right) = \log\left(\frac{\mu}{Q}\right) + \log\left(\frac{q_T}{\mu}\right)$$

- $\alpha_s \log(q_T/Q) \approx 1$ for $q_T \to 0$ spoil convergence and require resummation
- Factorization in soft-collinear effective theory (SCET)

$$\frac{d\sigma}{dq_T} = H(Q,\mu) \times B(q_T,\mu) \otimes B(q_T,\mu) \otimes S(q_T,\mu)\left[1 + \mathcal{O}\left(\frac{q_T^2}{Q^2}, \frac{m_q^2}{q_T^2}\right)\right]$$



Factorization and resummation in SCET

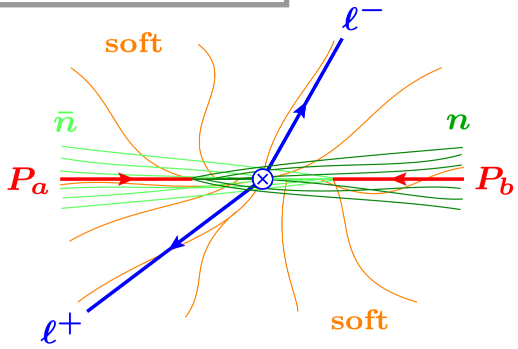
$$\log\left(\frac{q_T}{Q}\right) = \log\left(\frac{\mu}{Q}\right) + \log\left(\frac{q_T}{\mu}\right)$$

q_T factorization in SCET: $q_T \ll Q$

- $\alpha_s \log(q_T/Q) \approx 1$ for $q_T \to 0$ spoil convergence and require resummation
- Factorization in soft-collinear effective theory (SCET)

$$\frac{d\sigma}{dq_T} = H(Q,\mu) \times B(q_T,\mu) \otimes B(q_T,\mu) \otimes S(q_T,\mu)\left[1 + \mathcal{O}\left(\frac{q_T^2}{Q^2}, \frac{m_q^2}{q_T^2}\right)\right]$$

- Hard function: virtual contributions at hard scale Q
- Beam function: collinear radiation
- Soft function: soft, isotropic radiation

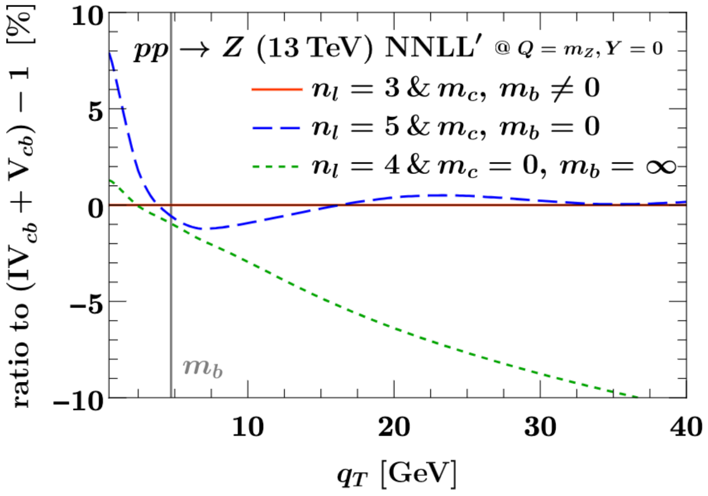


- Solve RGEs for $H(Q,\mu)$, $B(q_T,\mu)$and $S(q_T,\mu)$ to resum large logs

Drell-Yan and W mass measurement

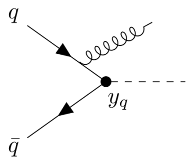
- Z and W q_T spectra are high precision observables
- aN⁴LL prediction available for $m_Q = 0$ [Billis, Michel, Tackmann '24]
    - but mass effects are important for m_W measurements
- Factorization theorem already worked out [Pietrulewicz, Samitz, Spiering, Tackmann '17]
    - three different regimes: $q_T \ll m$, $q_T \sim m$ and $m \ll q_T$
- Implementation remains challenging
    - First results for results for Z and W q_T spectra

[Dehnadi, Plößl, Tackmann *work in progress*]



The Higgs q_T spectrum

$q\bar{q} \to H$

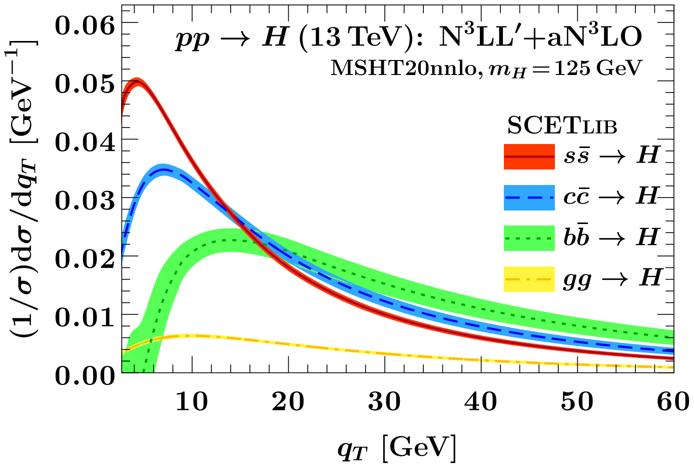


- Higgs q_T spectrum allows to access Yukawa coupling from Higgs production
  - gluon fusion and quark flavors exhibit different shapes

[Ebert et al. '16, Bishara et al. '16]

- N^{3}LL$'$ +a N^{3}LO prediction available
  - treat quarks as massless but $y_b \neq 0$
- **b-quark mass effects become relevant!**
  - ➡plot is cut off a 5 GeV
- **Outlook:** include mass effects and fit Yukawa couplings from $q\bar{q} \to H$

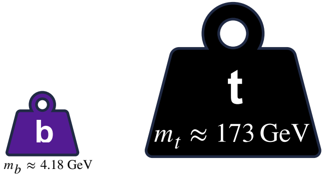
[Cal, RvK, Lim, Tackmann '23]



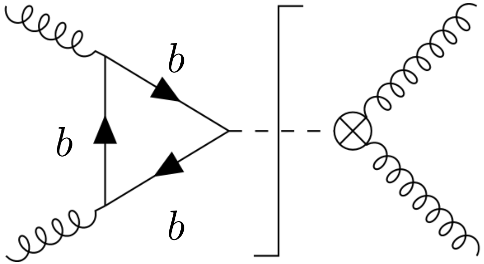
$y_b y_t$ interference in the Higgs q_T spectrum

UNIVERSITÄT BERN

Bottom mass effects in gluon fusion



- Consider $gg \to H$ with massive bottom-quark loop
- Usually consider top-quark loop as $m_t \gg m_b$
- $\mathcal{O}(5-10\%)$ contribution from interference with top-quark to total cross section [Liu, Penin ‘17 and ‘18]
- Lighter quarks only make up for a few percent of the Higgs cross section
- **Challenging problem already at 1-loop!**



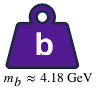
$y_b y_t$ interference in the Higgs q_T spectrum

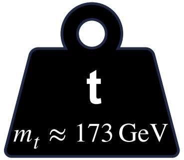
UNIVERSITÄT BERN

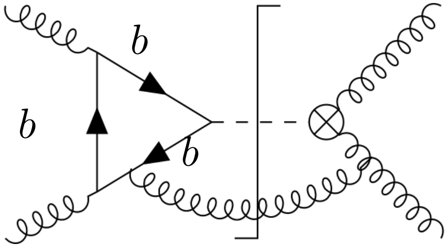
[Caola, Lindert, Melnikov, Monni, Tancredi, Wever '18]

so far: interference effects at NLO+NNLL

- NLO+NNLL prediction for Higgs q_T spectrum for $m_b \lesssim q_T \lesssim m_H$ including b mass effects
- Mass effects enter only through hard matrix elements
    - no genuine mass-dependent resummation
- No rigorous factorization theorem or all-order resummation





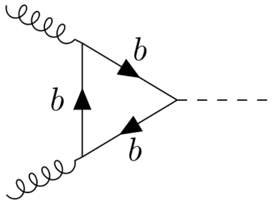


UNIVERSITÄT BERN

so far: form factor $F(m_b, m_H)$

- subleading power factorization and resummation of form factor for $m_b \ll Q$

[Liu, Neubert ‘19,Liu, Mecaj, Neubert, Wang ’20, Liu, Neubert, Schnubel, Wang ‘22]

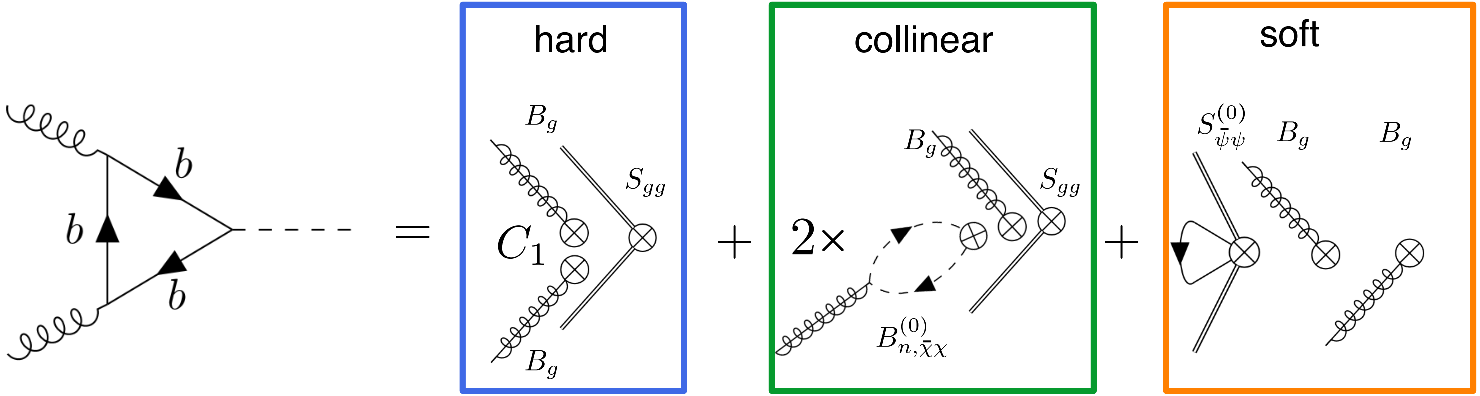


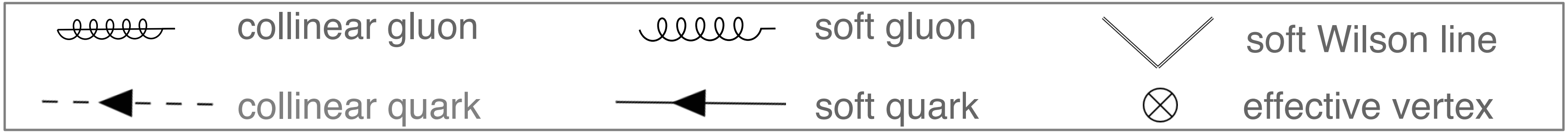
- $F(m_b, m_H)$depends on two scales
- Treatment of endpoint divergences is active field of research [Beneke, Ji, Wang ‘24]
- Cancellation crucial to recover full QCD result

$y_b y_t$ interference in the Higgs q_T spectrum

Notation: leading-order NLP contribution

factorization in SCET: $H\, B \otimes B \otimes S$

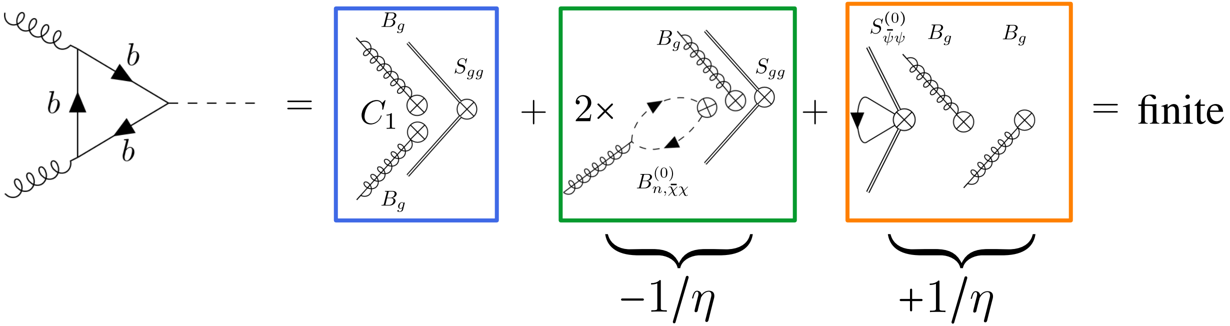




Endpoint divergences

Similar to dim. reg.
$\nu \leftrightarrow \mu$, $\epsilon \leftrightarrow \eta$

- Subtlety at sub-leading power: endpoint divergences in soft and collinear sectors
- Endpoint divergences appear when soft and collinear regions overlap
- Introduce regulator e.g. abs. value $|\ell_z/\nu|^{-\eta}$



- The endpoint divergences have to cancel in the sum to recover full QCD result

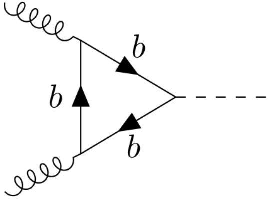
$y_b y_t$ interference in the Higgs q_T spectrum

UNIVERSITÄT BERN

so far: form factor $F(m_b, m_H)$

- subleading power factorization and resummation of form factor for $m_b \ll Q$

[Liu, Neubert ‘19,Liu, Mecaj, Neubert, Wang ’20, Liu, Neubert, Schnubel, Wang ‘22]

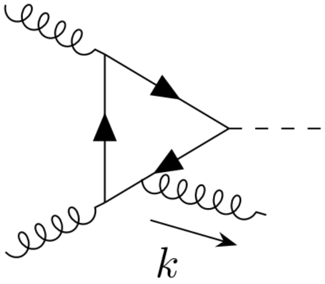


- $F(m_b, m_H)$ depends on two scales
- Treatment of endpoint divergences is active field of research [Beneke, Ji, Wang ‘24]
- Cancellation crucial to recover full QCD result

now: spectrum $\mathrm{d}\sigma(q_T, m_b, m_H)$

- q_T measurement adds additional scale

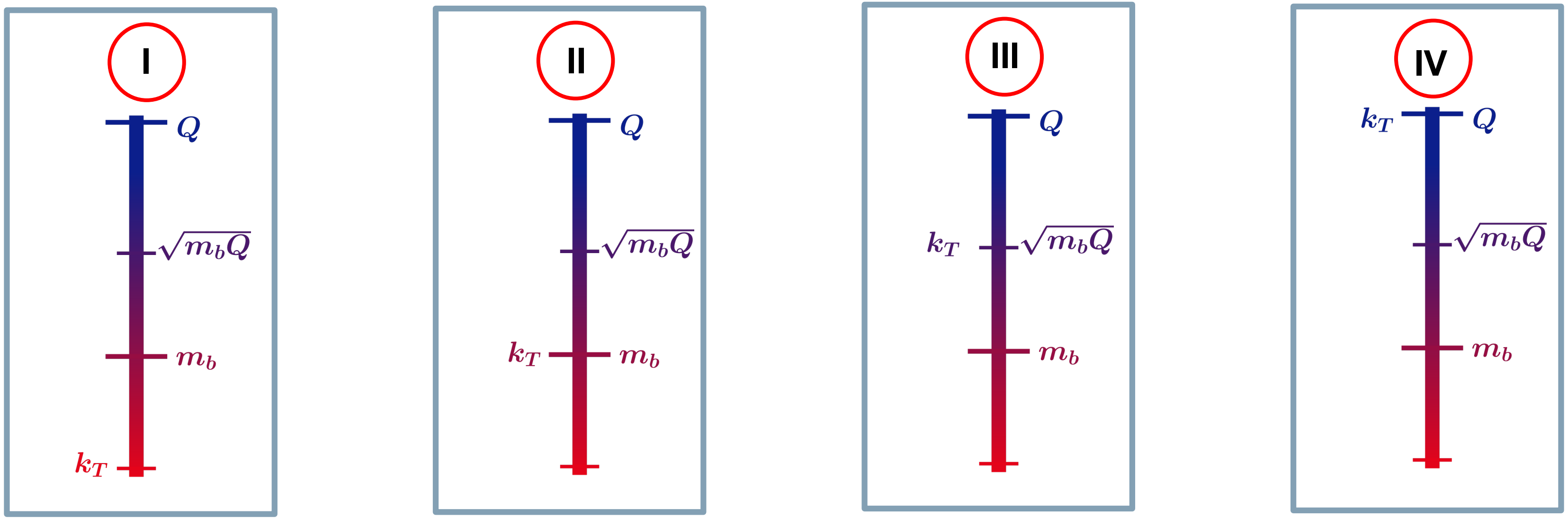
➡three scale problem!



- add emission with $\vec{k}_T = -\vec{q}_T$
- still have $m_b \ll Q$, but k_T can have different scalings

Different scalings of k_T

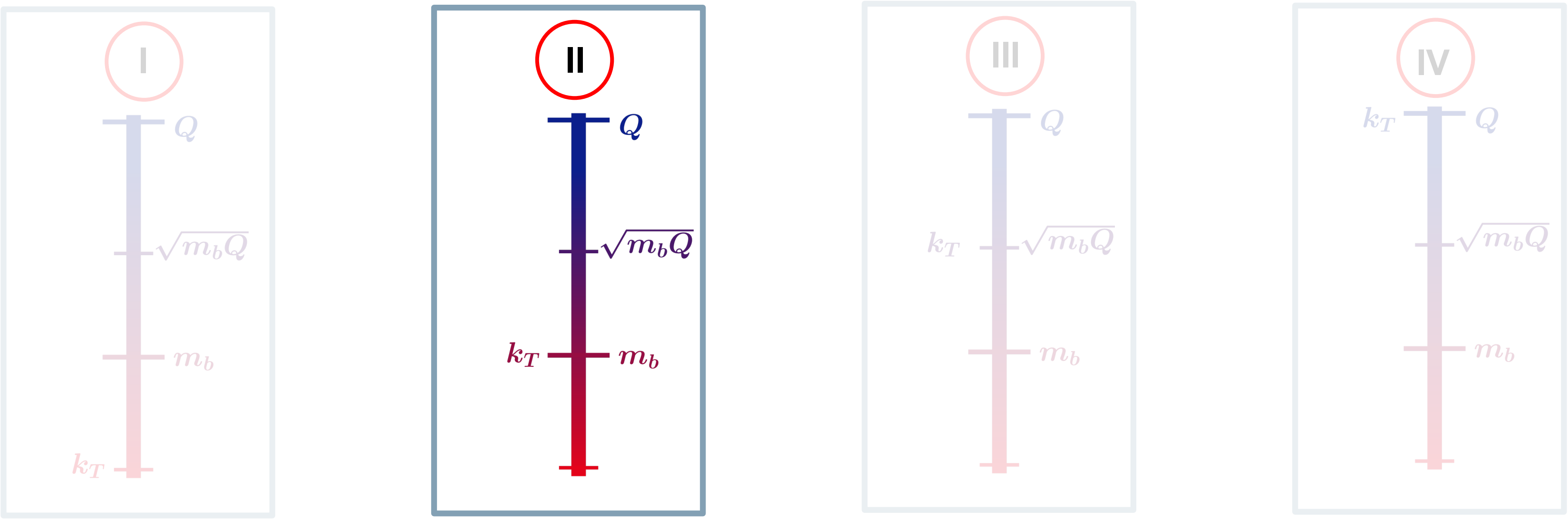
- Emission k_T introduces additional scale to the calculation



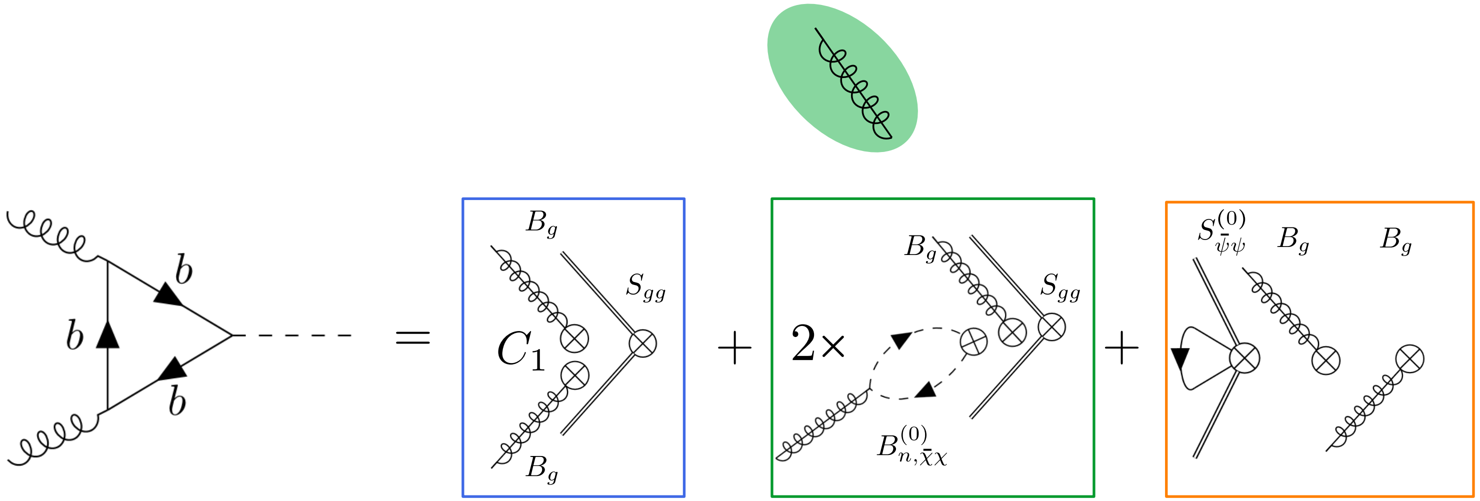
UNIVERSITÄT BERN

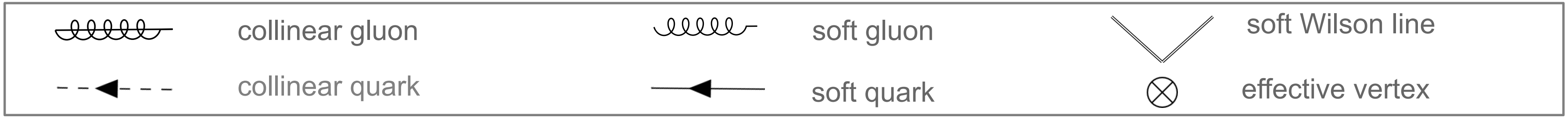
Different scalings of k_T

- Emission k_T introduces additional scale to the calculation



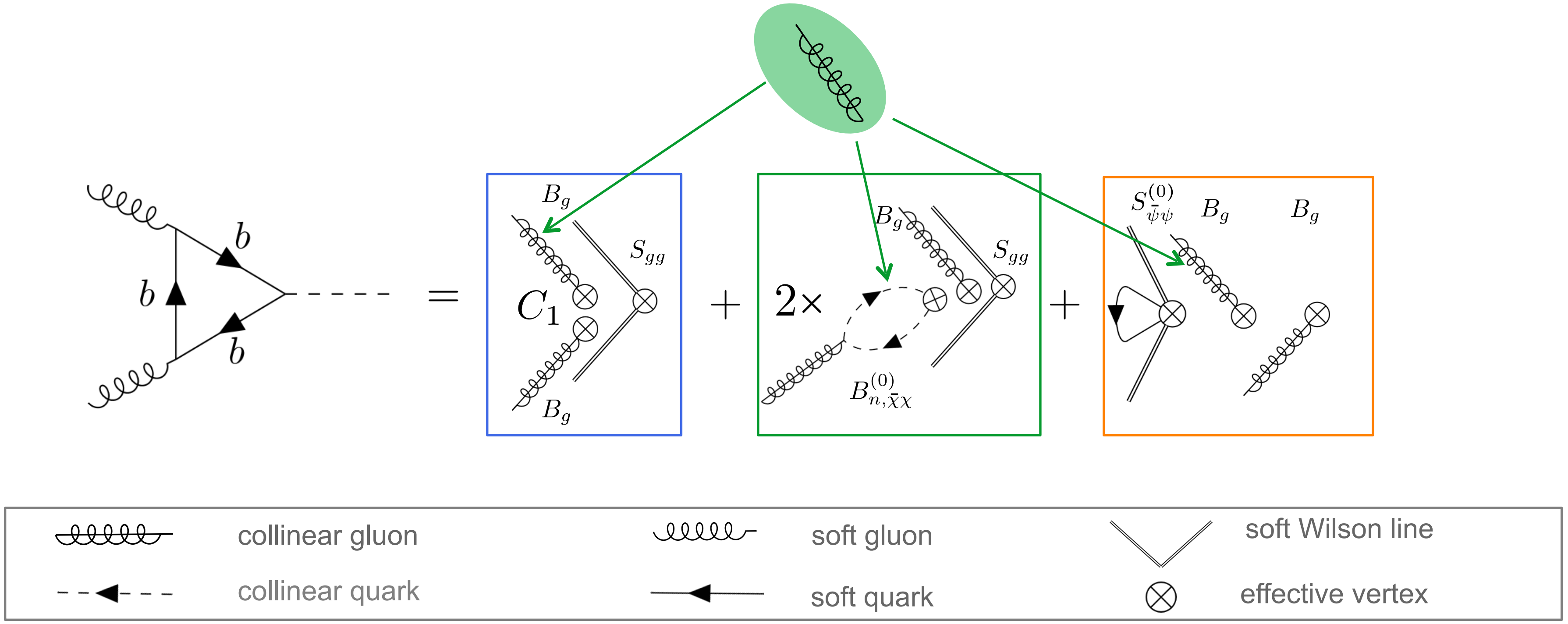
NLO NLP contribution





$y_b y_t$ interference in the Higgs q_T spectrum

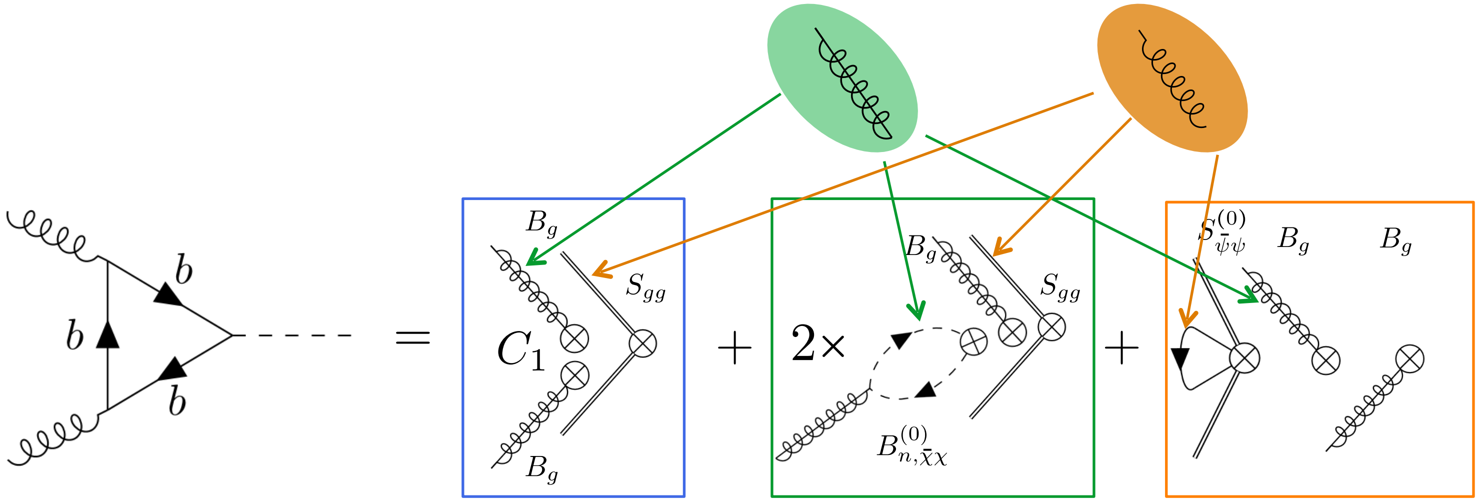
NLO NLP contribution

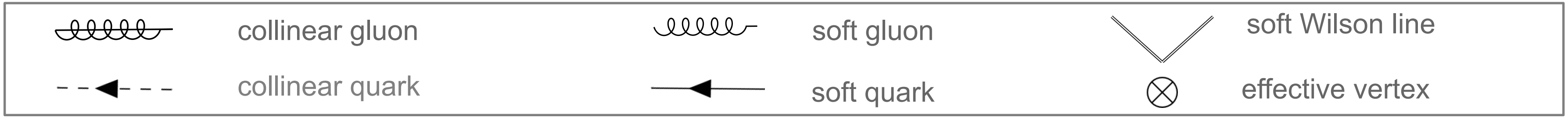


$y_b y_t$ interference in the Higgs q_T spectrum

UNIVERSITÄT BERN

NLO NLP contribution





$y_b y_t$ interference in the Higgs q_T spectrum

UNIVERSITÄT BERN

- Expect endpoint divergences in soft and collinear contribution
- Form factor $F(m_b, m_H)$ depends on two scales
- The spectrum introduces an additional scale k_T

$$\frac{1}{\eta} f_n\left(\frac{m}{k_T}\right) \longleftrightarrow \frac{1}{\eta} f_s\left(\frac{m}{k_T}\right)$$

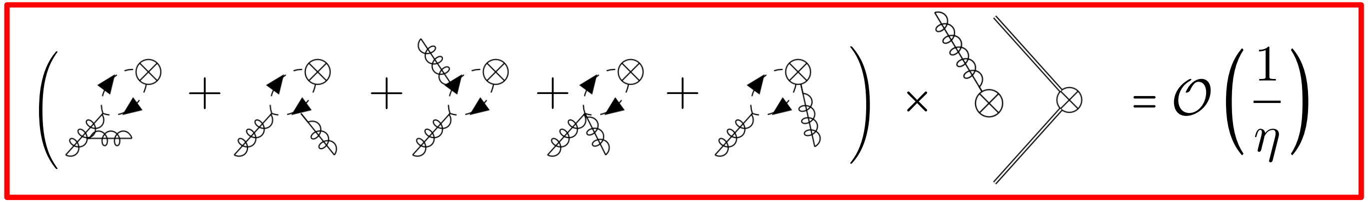
- **How does the additional scale affect the structure of the endpoint divergences?**
- In general $f_n(m/k_T)$ and $f_s(m/k_T)$ can be non-trivial functions of m/k_T

$y_b y_t$ interference in the Higgs q_T spectrum

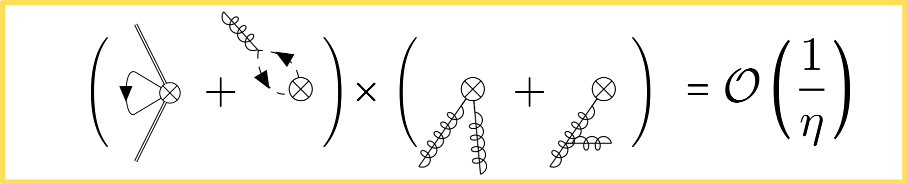
collinear NLP one-gluon contribution

[RvK, Michel, Stewart, Sun, Tackmann *work in progress*]

- Now: add emission k_T to contribution from collinear loop



- There can also be collinear emissions from collinear LP beam function



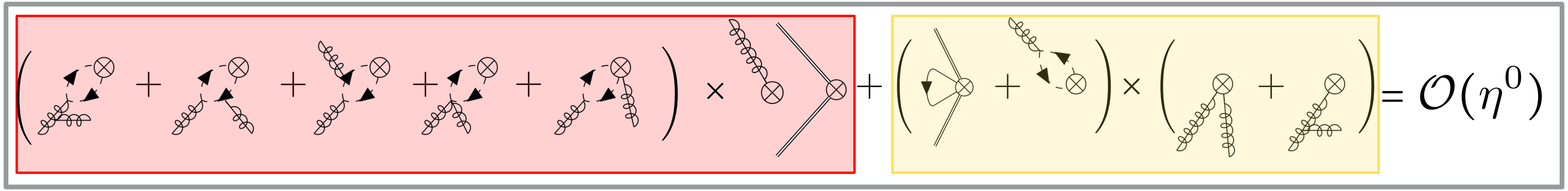
$y_b y_t$ interference in the Higgs q_T spectrum

UNIVERSITÄT BERN

collinear NLP one-gluon contribution

[RvK, Michel, Stewart, Sun, Tackmann *work in progress*]

- The endpoint divergences cancel in the sum!



- **Non-trivial result:** NLP emission diagrams cancel against LP emission!

➡Mass and transverse-momentum dependence factorizes at this order!

$y_b y_t$ interference in the Higgs q_T spectrum

UNIVERSITÄT BERN

Summary

- Factorization theorem for different regimes ✓
- All endpoint divergences cancel in a non-trivial way ✓
- m and k_T dependence factorizes at this order! ✓

Next steps

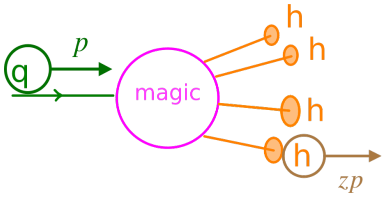
- Coming soon: NLP factorization and NLP beam and soft functions of $y_b y_t$ interference
- Resummed prediction for $gg \to H$ including m_b effects

Heavy Quarks in TMDs

Heavy Quark TMDs

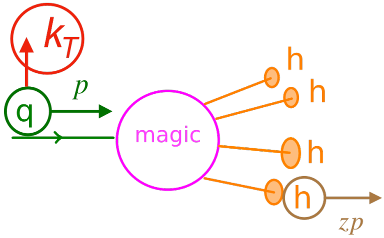
Hadronization

- Measured final states rarely correspond directly to the predicted distribution
- Collinear fragmentation: parton fragments into final state hadron
- **But**: so far only at a 1D snapshot of longitudinal momentum distributions!



Transverse momentum dependent (TMD) FFs

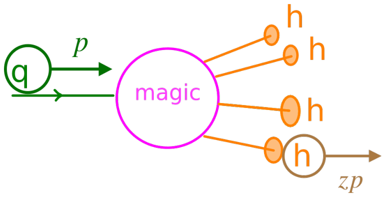
- Allows for extraction of 3D structure of the hadronization cascade
- **Here:** heavy quark TMD FFs
  - ➡Two different regimes: $\Lambda_{\mathrm{QCD}} \sim k_T \ll m_Q$ vs. $\Lambda_{\mathrm{QCD}} \ll m_Q \lesssim k_T$



Heavy Quark TMDs

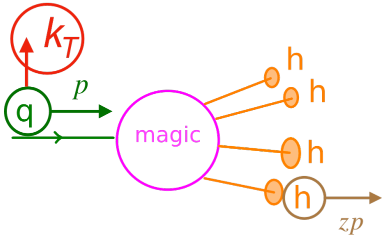
Hadronization

- Measured final states rarely correspond directly to the predicted distribution
- Collinear fragmentation: parton fragments into final state hadron
- **But**: so far only at a 1D snapshot of longitudinal momentum distributions!



Transverse momentum dependent (TMD) FFs

- Allows for extraction of 3D structure of the hadronization cascade
- **Here:** heavy quark TMD FFs
  - ➡Two different regimes: $\Lambda_{\mathrm{QCD}} \sim k_T \ll m_Q$ vs. $\boxed{\Lambda_{\mathrm{QCD}} \ll m_Q \lesssim k_T}$



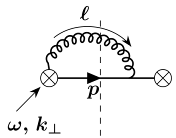
Fragmentation functions

- Considered all heavy-quark TMD FFs for unpolarized hadrons
  - Unpolarized TMD FF $D_{1,H/Q}$: unpol. quark fragments into unpol. hadron
- Use bHQET to show factorization in terms of novel matrix elements and Wilson coefficients

$$D_{1\,H/Q}(z_H, b_T, \mu, \zeta) = d_{1\,Q/Q}(z_H, b_T, \mu, \zeta)\chi_H \qquad (\Lambda_{\rm QCD} \ll m_Q \lesssim k_T)$$

[RvK, Michel, Sun '23]

- $d_{1,Q/Q}$ partonic heavy-quark TMD FF: perturbative Wilson coefficient



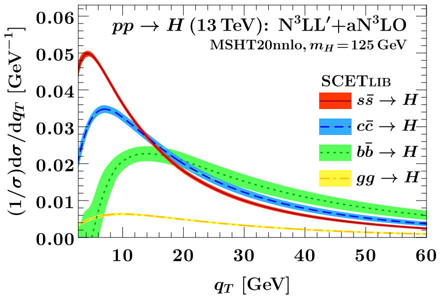
➡Calculated to NLO [RvK, Michel, Sun '24]

➡Application: mass effects for EEC in back-to-back limit in e^+e^- collisions

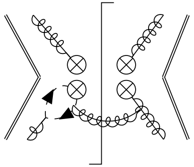
Outlook and summary

Outlook

- **Mass effect in q_T resummation**
  - Treatment of mass effects in m_W measurements
  - Yukawa fits with mass effects for $q\bar{q} \to H$ to extract y_q from initial state

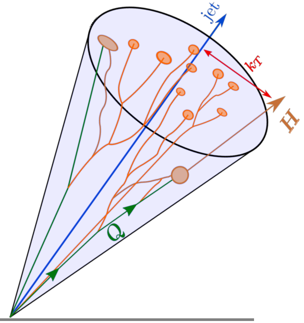


- **$y_b y_t$ interference in the Higgs q_T spectrum**
  - Resummed prediction for gluon fusion including m_b effects



- **Heavy quark TMD FFs**
  - Include polarized hadrons!
  - HQ TMDs within jets: NLO factorization theorem for TMD FFs within jets in pp collisions

[Kang, Liu, Ringer, Xing '17]

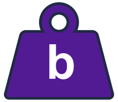


- **And many more!**

Summary

UNIVERSITÄT BERN

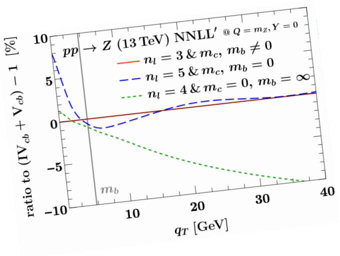
- Quark mass effects can no longer be neglected and enter in numerous processes



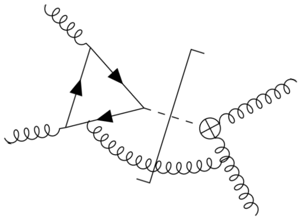
$m_b \approx 4.18\,\mathrm{GeV}$

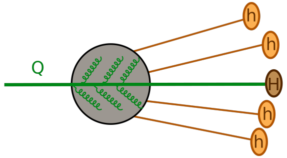
$m_c \approx 1.27\,\mathrm{GeV}$

- **Mass effect in q_T resummation**
  - first results for Z and W q_T are on the horizon

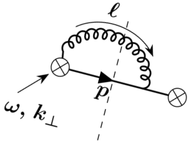


- **$y_b y_t$ interference in the Higgs q_T spectrum**
  - up to $\mathcal{O}(20\%)$ effect from $y_b y_t$ interference
  - emission k_T adds extra scale to the problem
  - m_b and k_T dependence factorize at this order



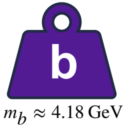


- **Heavy quark TMD FFs**
  - Heavy quarks are ideal to study hadronization
  - Novel TMD sum rules and factorization in terms of novel Wilson coeffs. and matrix elements

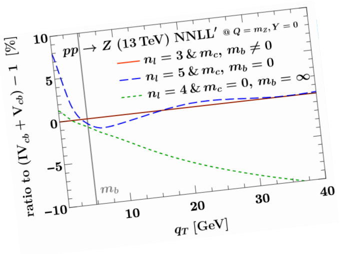


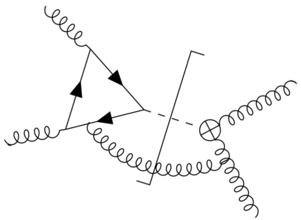
Summary

- Quark mass effects can no longer be neglected and enter in numerous processes

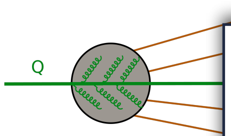


- **Mass effect in q_T resummation**
  - first results for Z and W q_T are on the horizon





- **$y_b y_t$ interference in the Higgs q_T spectrum**
  - up to $\mathcal{O}(20\%)$ effect from $y_b y_t$ interference
  - emission k_T adds extra scale to the problem
  - m_b and k_T dependence factorize at this order

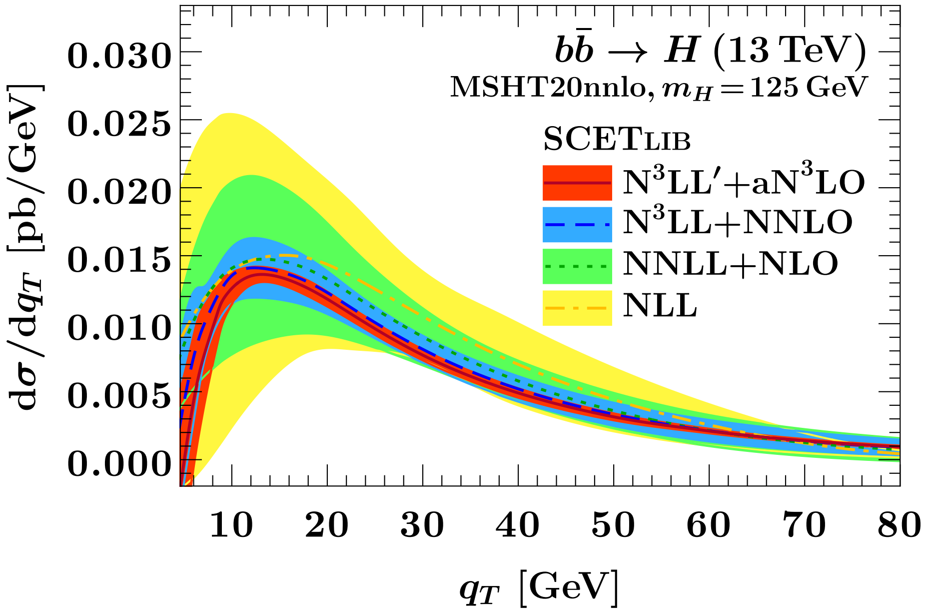


Thank you!

- **Heavy quark TMD FFs**
  - Heavy quarks are ideal to study hadronization
  - Novel TMD sum rules and factorization in terms of novel Wilson coeffs. and matrix elements

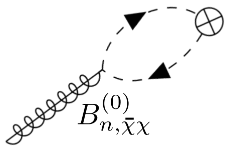
Back up

Endpoint divergences



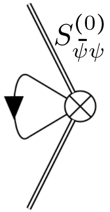
Endpoint divergences

$$\int \mathrm{d}\xi \left(\frac{1}{\xi}\left|\frac{\xi\omega_n}{\nu}\right|^{-\eta} + \frac{1}{1-\xi}\left|\frac{(1-\xi)\omega_n}{\nu}\right|^{-\eta}\right)$$



$\times 2$

$$+\int \mathrm{d}\ell^+ \mathrm{d}\ell^- \frac{1}{\ell^+\ell^-}\left|\frac{\ell^+\ell^-}{\nu}\right|^{-\frac{\eta}{2}} |\sinh y_\ell|^{-\eta}$$



$= \mathcal{O}(\eta^0)$

- all endpoint divergences cancel between soft and collinear contributions!

Regime I

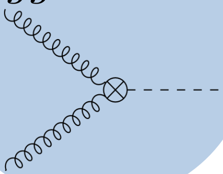
UNIVERSITÄT BERN

Factorization theorem

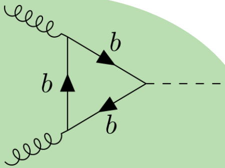
- only valid in a very small region of the q_T spectrum
- use standard factorization for q_T resummation with $n_f = 4$ massless flavors

$$\frac{d\sigma_{y_t y_b}}{dq_T} = 2\mathrm{Re}[C^*_{ggt}(m_H)\, C_{ggb}(m_b, m_H)]\, B_g(q_T) \otimes B_g(q_T) \otimes S_{gg}(q_T)$$

$gg \to H$

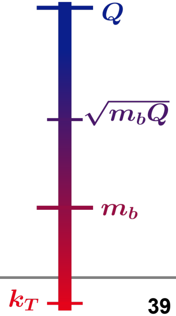


from



form factor

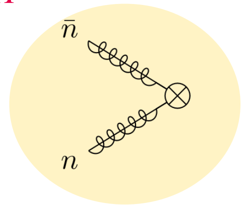
q_T resummation



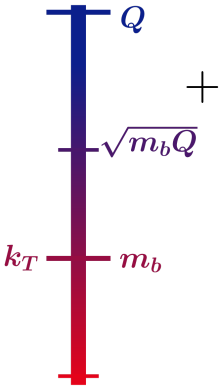
Regime II

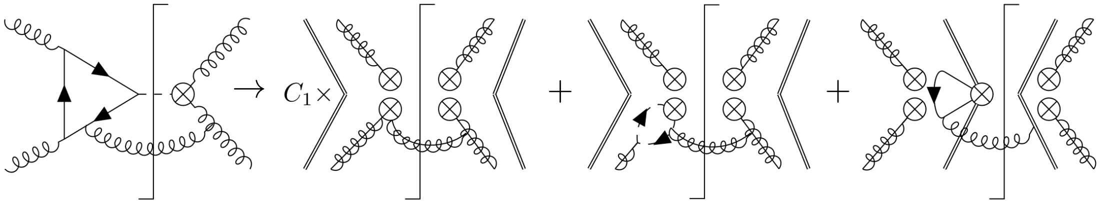
Bare factorization theorem $k_T \sim m_b \ll m_H$

operators: [M. Stahlhofen's SCET talk ' 15, Liu, Neubert '19]



$$\frac{d\sigma_{y_t y_b}}{dq_T} = H_{gg}(m)\, B_g(q_T) \otimes B_g(q_T) \otimes S_{gg}(q_T)$$
$$+ \int d\xi H_{bbg}(\xi)\, [B_{n,\bar{\chi}\chi}(\xi, q_T, m) \otimes B_g(q_T) \otimes S_{gg}(q_T)$$
$$+ B_g(q_T) \otimes B_{\bar{n},\bar{\chi}\chi}(\xi, q_T, m) \otimes S_{gg}(q_T)]$$
$$+ \int d\ell^+ d\ell^- H_{bbgg} \mathcal{J}(\ell^+)\mathcal{J}(\ell^-)\, B_g(q_T) \otimes B_g(q_T) \otimes S_{\bar{\psi}\psi}(\ell^+, \ell^-, q_T, m)$$

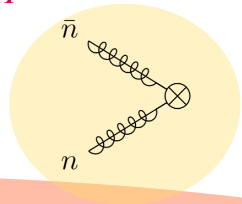


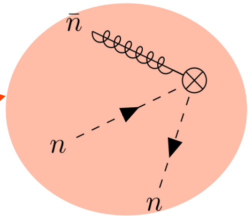


Regime II

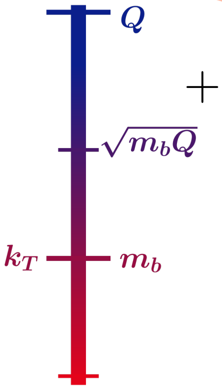
Bare factorization theorem $k_T \sim m_b \ll m_H$

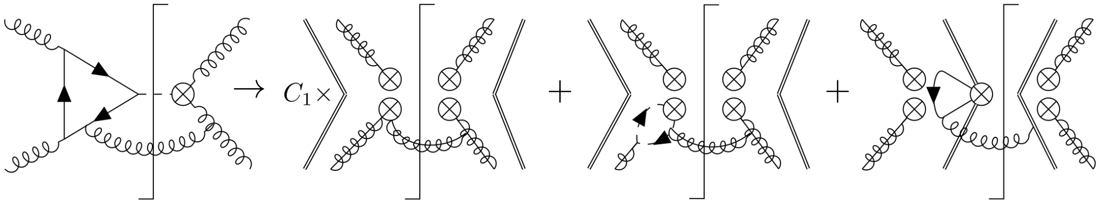
operators: [M. Stahlhofen's SCET talk ' 15, Liu, Neubert '19]

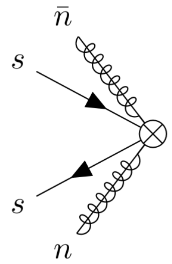




$$
\begin{aligned}
\frac{d\sigma_{y_t y_b}}{dq_T} = & \; H_{gg}(m)\, B_g(q_T) \otimes B_g(q_T) \otimes S_{gg}(q_T) \\
& + \int d\xi\, H_{bbg}(\xi)\, \big[B_{n,\bar{\chi}\chi}(\xi, q_T, m) \otimes B_g(q_T) \otimes S_{gg}(q_T) \\
& \qquad + B_g(q_T) \otimes B_{\bar{n},\bar{\chi}\chi}(\xi, q_T, m) \otimes S_{gg}(q_T) \big] \\
& + \int d\ell^+ d\ell^-\, H_{bbgg}\, \mathcal{J}(\ell^+) \mathcal{J}(\ell^-)\, B_g(q_T) \otimes B_g(q_T) \otimes S_{\bar{\psi}\psi}(\ell^+, \ell^-, q_T, m)
\end{aligned}
$$





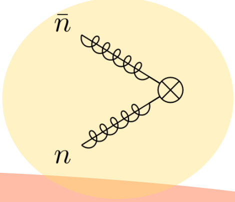


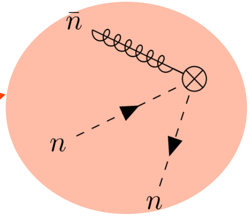
Regime II

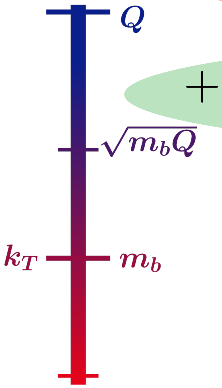
Bare factorization theorem $k_T \sim m_b \ll m_H$

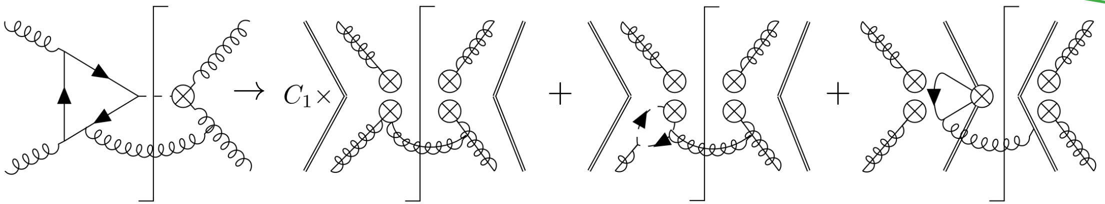
operators: [M. Stahlhofen's SCET talk ' 15, Liu, Neubert '19]

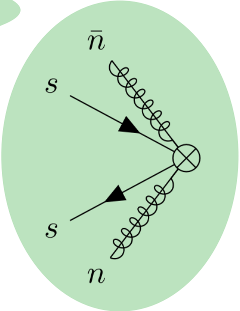
$$
\begin{aligned}
\frac{\mathrm{d}\sigma_{y_t y_b}}{\mathrm{d}q_T} &= H_{gg}(m)\, B_g(q_T) \otimes B_g(q_T) \otimes S_{gg}(q_T) \\
&+ \int \mathrm{d}\xi\, H_{bbg}(\xi)\, \big[B_{n,\bar{\chi}\chi}(\xi, q_T, m) \otimes B_g(q_T) \otimes S_{gg}(q_T) \\
&\qquad + B_g(q_T) \otimes B_{\bar{n},\bar{\chi}\chi}(\xi, q_T, m) \otimes S_{gg}(q_T)\big] \\
&+ \int \mathrm{d}\ell^+ \mathrm{d}\ell^-\, H_{bbgg}\, \mathcal{J}(\ell^+)\mathcal{J}(\ell^-)\, B_g(q_T) \otimes B_g(q_T) \otimes S_{\bar{\psi}\psi}(\ell^+, \ell^-, q_T, m)
\end{aligned}
$$











Regime II

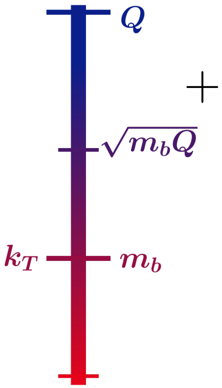
Bare factorization theorem $k_T \sim m_b \ll m_H$

operators: [M. Stahlhofen's SCET talk ' 15, Liu, Neubert '19]

$$\frac{d\sigma_{y_t y_b}}{dq_T} = H_{gg}(m)\, B_g(q_T) \otimes B_g(q_T) \otimes S_{gg}(q_T)$$
$$+ \int d\xi\, H_{bbg}(\xi)\, \left[B_{n,\bar{\chi}\chi}(\xi, q_T, m) \otimes B_g(q_T) \otimes S_{gg}(q_T) + B_g(q_T) \otimes B_{\bar{n},\bar{\chi}\chi}(\xi, q_T, m) \otimes S_{gg}(q_T)\right]$$
$$+ \int d\ell^+ d\ell^- H_{bbgg} \mathcal{J}(\ell^+) \mathcal{J}(\ell^-)\, B_g(q_T) \otimes B_g(q_T) \otimes S_{\bar{\psi}\psi}(\ell^+, \ell^-, q_T, m)$$

$$\frac{1}{\xi} + \frac{1}{1-\xi}$$

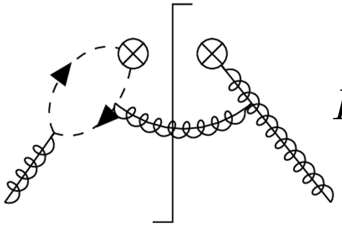
$$\frac{1}{\ell^+ \ell^-}$$



lead to endpoint divergences! (just as for form factor)

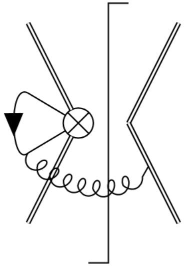
Matrix element definition

Beam function



$$B_{n,\chi\bar{\chi}}\left(x = \frac{\omega}{P_N^-}\right) = \sum\!\!\!\!\!\!\!\int \langle N|\, \mathcal{B}_\perp^{a,\mu}\, \delta^2(k_\perp - P_{X,\perp})\, |X\rangle \langle X|\, \bar{\chi}_{n,\omega z} T^a \gamma_{\perp\mu} \chi_{n,\omega(1-z)}\, |N\rangle$$

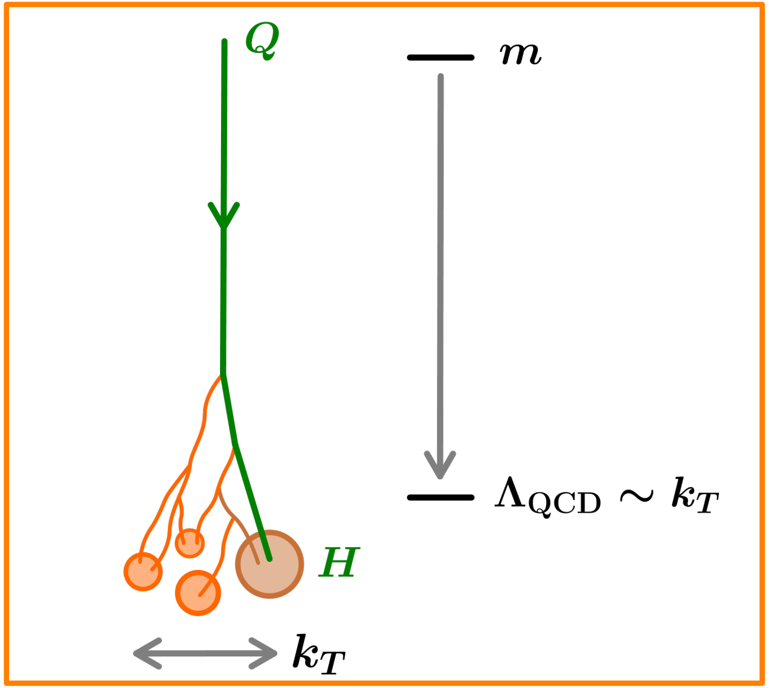
Soft function

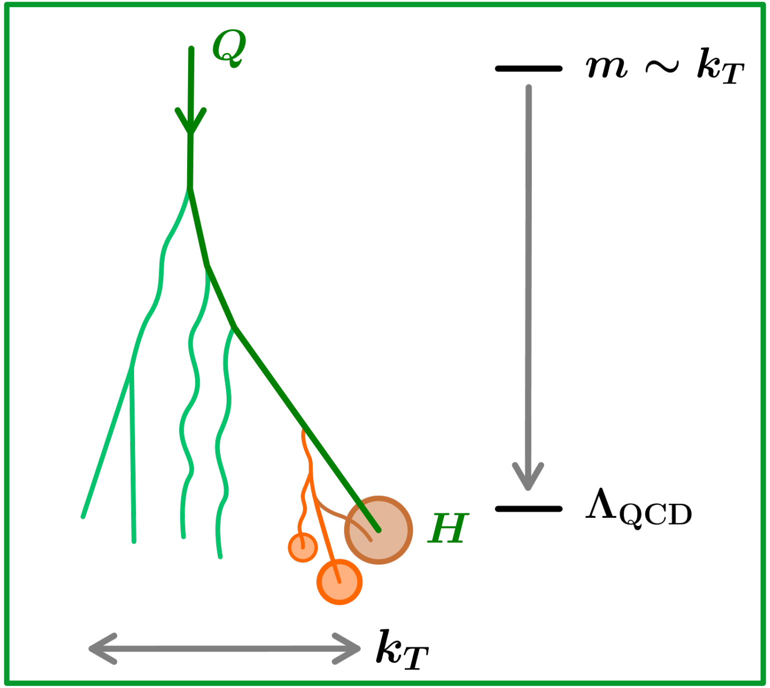


$$\mathcal{O}^{ab}_{S,\bar{\psi}\psi}(\ell^+\ell^-) = \frac{1}{\ell^+\ell^-}[\bar{\psi} S_n \delta(\ell^+ - n\cdot\mathcal{P}])[\gamma_{\perp,\mu} T^a \frac{\bar{\not{n}}\not{n}}{4} S_n^\dagger S_{\bar{n}} \gamma_\perp^\mu T^b][S_{\bar{n}}^\dagger \psi \delta(\ell^- - \bar{n}\cdot\mathcal{P}])$$

$$S_{\bar{\psi}\psi}(\ell^+, \ell^-, k_T, m) = \sum\!\!\!\!\!\!\!\int \frac{1}{N_c^2 - 1} \langle 0|\, \mathcal{O}_s^{(0)\,ab}\, \delta^2(k_\perp - P_{X,\perp})\, |X\rangle \langle X|\, \mathcal{O}^{ba}_{s,\bar{\psi}\psi}\, |0\rangle$$

Different regimes





HQ TMDs

Regime 1 $\Lambda_{QCD} \simeq k_T \ll m$

- Evaluate TMD FFs

$$D_{1\,H/Q}(z_H, b_T, \mu, \zeta) = \delta(1 - z_H)\, C_m(m, \mu, \zeta)\, \chi_{1,H}\Big(b_T, \mu, \frac{\sqrt{\zeta}}{m}\Big)$$

$$b_T M_H\, H^{\perp(1)}_{1\,H/Q}(z_H, b_T, \mu, \zeta) = \delta(1 - z_H)\, C_m(m, \mu, \zeta)\, \chi^{\perp}_{1,H}\Big(b_T, \mu, \frac{\sqrt{\zeta}}{m}\Big)$$

- New scalar bHQET TMD fragmentation factors:

$$\chi_{1,H}(b_T) = \frac{1}{2}\,\mathrm{tr}\, F_H(b_\perp) \qquad \chi^{\perp}_{1,H}(b_T) = \frac{1}{2}\,\mathrm{tr}\Big[\frac{b_\perp}{b_T}\, z\, F_H(b_\perp)\Big]$$

- $\chi_{1,H}$ conditional probability to produce *H* given k_T
- $\chi_{1,H}^{\perp}$ conditional density of quark spin w.r.t magnetization axis given by *Q* and k_T

Step 1: tree-level matching to HQET

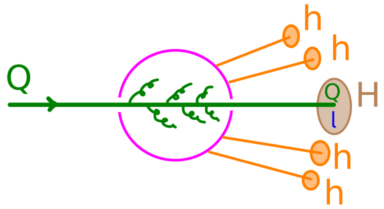
[Isgur, Wise '89 & '90; Eichten, Hill '90; Grinstein '90; Georgi '90; Korner,Thompson '91; Mannel, Roberts Ryzak '92; Fleming, Hoang, Mantry, Stewart '08]

- TMD quark-quark correlator

$$\Delta_{H/Q}(z_H, b_\perp) = \frac{1}{2 z_H N_c} \int \frac{b^+}{4\pi} e^{b^+(P_H^-/z_H)/2} \, \mathrm{Tr} \sum\!\!\!\!\!\!\int_X \langle 0| W^\dagger(b)\, \psi_Q(b) \,|HX\rangle \langle HX| \bar{\psi}_Q W |0\rangle$$

HQET matching

$$\Delta_{H/Q}(z_H, b_\perp) = \frac{\delta(1-z_H)}{\bar{n}\cdot v} F_H(b_\perp)$$



- $F_H(b_T)$: Heavy quark TMD FF correlator — carries all non-perturbative dynamics!

HQET: $\psi_Q(x) = e^{-mv\cdot x} h_v(x)$ $\quad |H, h_H; X\rangle = \sqrt{m}\, |H_v, h_H; X\rangle$

Step 2: decouple light degrees of freedom

[Korchemsky and Radyushkin '92; Bauer, Pirjol and Stewart '02]

$$F_H(b_\perp) = \frac{1}{2}\sum_{h_H}\sum_{h_Q,h'_Q}\sum_{h_\ell,h'_\ell} \underbrace{u(v,h_Q)\,\bar u(v,h'_Q)}_{\text{heavy quark spinors}} \overbrace{\langle s_Q,h_Q;s_\ell,h_\ell|s_H,h_H\rangle\langle s_H,h_H|s_Q,h'_Q;s_\ell,h'_\ell\rangle}^{\text{Clebsch-Gordan coefficients}}$$

$$\times \underbrace{\frac{1}{N_c}\,\mathrm{Tr}\sum\!\!\!\!\!\!\!\int_X \langle 0|\,W^\dagger(b_\perp)\,Y_v(b_\perp)\,|s_\ell,h_\ell,f_\ell;X\rangle\langle s_\ell,h'_\ell,f_\ell;X|\,Y_v^\dagger(0)\,W(0)\,|0\rangle}_{\rho_{\ell,h_\ell h'_\ell}(b_\perp)}$$

- $\rho_\ell(b_\perp)$: light spin density matrix — carries all non-perturbative dynamics!

decoupling: $h_v(x) = Y_v(x)\,h_v^{(0)}(x)$

HQ TMDs

UNIVERSITÄT BERN

Regime 1 $\Lambda_{QCD} \simeq k_T \ll m$

$$h_v(x) = Y_v(x)\, h_v^{(0)}(x)$$

- Decouple light degrees of freedom

$$F_H(b_\perp) = \frac{1}{2} \sum_{h_H} \sum_{h_Q, h'_Q} \sum_{h_\ell, h'_\ell} u(v, h_Q)\, \bar{u}(v, h'_Q) \overbrace{\langle s_Q, h_Q; s_\ell, h_\ell | s_H, h_H \rangle \langle s_H, h_H | s_Q, h'_Q; s_\ell, h'_\ell \rangle}^{\text{Clebsch-Gordan-Coefficients}}$$
$$\times \underbrace{\frac{1}{N_c} \mathrm{Tr} \sum\!\!\!\!\!\!\int_X \langle 0 | W^\dagger(b_\perp)\, Y_v(b_\perp)\, | s_\ell, h_\ell, f_\ell; X \rangle \langle s_\ell, h'_\ell, f_\ell; X | Y_v^\dagger(0)\, W(0)\, | 0 \rangle}_{\rho_{\ell, h_\ell h'_\ell}(b_\perp)}$$

- $\rho_{\ell, h\ell\, h\ell'}(b_\perp)$ light spin density matrix, encodes all non-perturbative physics

UNIVERSITÄT BERN

TMD definitions

- TMD quark-quark correlator

$$\Delta_{H/Q}(z_H, b_\perp) = \frac{1}{2 z_H N_c} \int \frac{b^+}{4\pi} e^{b^+(P_H^-/z_H)/2} \, \mathrm{Tr} \sum\!\!\!\!\!\!\int_X \langle 0| W^\dagger(b)\, \psi_Q(b)\, |HX\rangle \langle HX| \, \bar{\psi}_Q \, W\, |0\rangle$$

- k_T: transverse momentum of heavy quark, b_T : its Fourier conjugate
- z_H: fraction of the quarks lightcone momentum retained by the HQ
- P^-_H: large lightcone momentum of the heavy hadron

Unpol TMD FF:

$$D_{1\,h/q}(z_H, b_T) = \mathrm{tr}\Big[\frac{\bar{n}\!\!\!/}{2} \Delta_{h/q}(z_H, b_\perp)\Big]$$

Collins TMD FF:

$$H^{\perp(1)}_{1\,h/q}(z_H, b_T) = \mathrm{tr}\Big[\frac{\bar{n}\!\!\!/}{2} \frac{b\!\!\!/_\perp}{M_H b_T^2} \Delta_{h/q}(z_H, b_\perp)\Big]$$

Results for unpolarized TMD FF

- Performing trace sets $h_Q=h_Q'$ and $h_\ell=h_\ell'$

$$D_{1\,H/Q}(z_H, b_T, \mu, \zeta) \propto \chi_{1,H}(b_T) = \frac{1}{2}\sum_{h_H}\sum_{h_Q}\sum_{h_\ell} |\langle s_Q, h_Q; s_\ell, h_\ell | s_H, h_H\rangle|^2 \rho_{\ell, h_\ell h_\ell}(b_\perp)$$

D (s_ℓ =1/2, s_H =0)

$$\chi_{1,D}(b_T) = \frac{1}{4}\left[\rho_{\ell,++}(b_\perp) + \rho_{\ell,--}(b_\perp)\right]$$

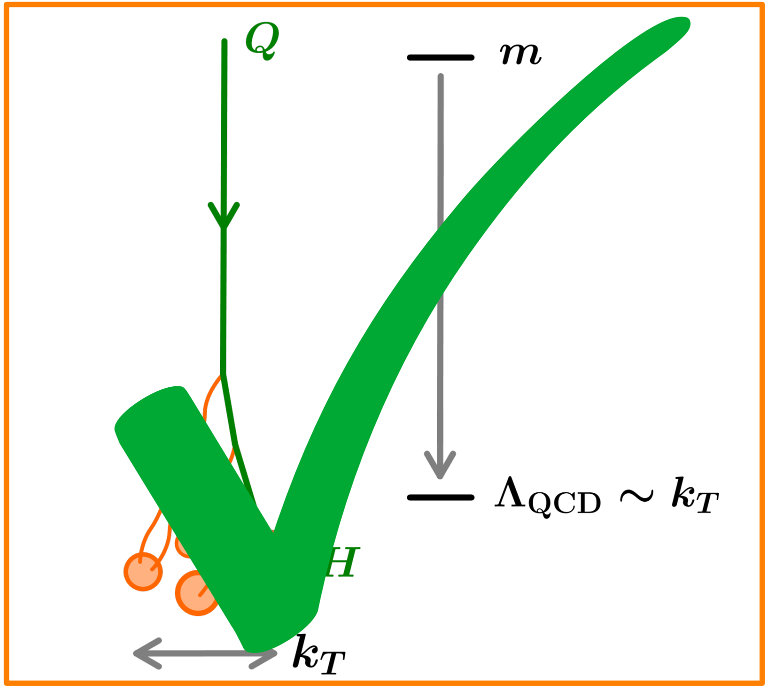
D* (s_ℓ =1/2, s_H =1)

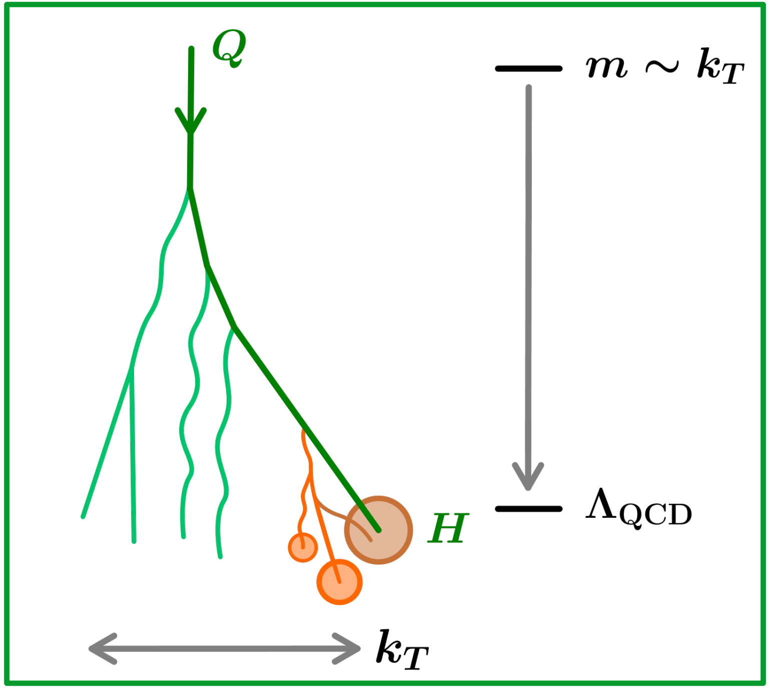
$$\chi_{1,D^*}(b_T) = \frac{3}{4}\left[\rho_{\ell,++}(b_\perp) + \rho_{\ell,--}(b_\perp)\right]$$

- **Three times as likely to produce D* than D!**
- for the first time: show that relations hold point by point in k_T

[proven for inclusive fragmentation Falk and Peskin '94, Manohar and Wise '00]

Different regimes





HQ TMDs

Results for unpolarized TMD FF in Regime II

- Match into bHQET at μ ~ m ~ k_T [Nadolsky, Kidonakis, Olness and Yuan '03]

$$D_{1\,H/Q}(z_H, b_T, \mu, \zeta) = d_{1\,Q/Q}(z_H, b_T, \mu, \zeta)\chi_H + \mathcal{O}\left(\frac{\Lambda_{\rm QCD}}{m}\right) + \mathcal{O}(\Lambda_{\rm QCD} b_T)$$

- New perturbative matching coefficient:

$$d_{1\,Q/Q}(z_H, b_T, \mu, \zeta) = \mathrm{tr}\Big[\frac{\not{n}}{2}\Delta_{Q/Q}(z_H, b_\perp)\Big] = \delta(1 - z_H) + \mathcal{O}(\alpha_s)$$

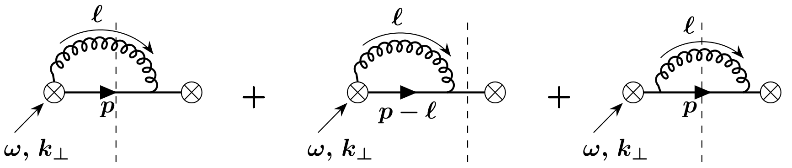
- χ_H total probability of Q to fragment into *H*
- **symmetry relations hold for all values of k_T and to all orders in α_s !**

Heavy Quark TMDs

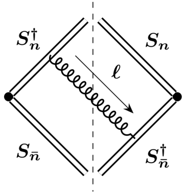
Results for unpolarized TMD FF at NLO

- Calculate $d_{1Q/Q}(z_H, b_T, \mu, \zeta)$ to NLO [RvK, Michel, Sun '24]

$$d_{1\,Q/Q}(z_H, b_T, \mu, \zeta) =$$

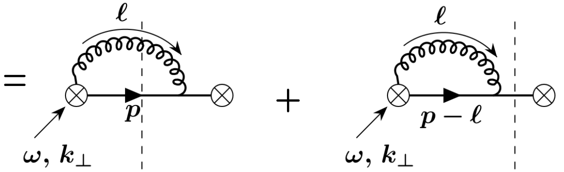


$$+\ \delta(1 - z_H)$$

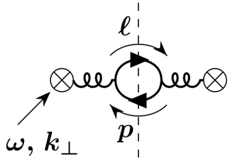


- Also consider $d_{1g/Q}(z_H, b_T, \mu, \zeta)$ and $d_{1Q/g}(z_H, b_T, \mu, \zeta)$

$$d_{1\,g/Q}(z_H, b_T, \mu, \zeta) =$$



$$d_{1\,Q/g}(z_H, b_T, \mu, \zeta) =$$



➡Application: mass effects for EEC in back-to-back limit in e^+e^- collisions

HQ TMD PDFs

Setup

- Consider production of a heavy quark *Q* with from light partons within a (polarized) nucleon *N*
- Heavy quarks is pair produced in initial state gluon splitting at $\mu \sim m$
- **TMD PDFs can be calculated by matching them onto leading collinear PDFs**
- TMD PDF decomposition: [Bacchetta, Diehl, Goeke, Metz, Mulders and Schlegel ’07, Ebert, Gao and Stewart ’22]

$$\Phi_{Q/N}(x, k_\perp) = \left\{ f_{1\,Q/N} + g_{1L\,Q/N} S_L \gamma_5 + h^\perp_{1L\,Q/N} S_L \gamma_5 \frac{\not{k}_\perp}{M_N} + h^\perp_{1\,Q/N} \frac{\not{k}_\perp}{M_N} + \left(\text{terms} \propto S_\perp\right) \right\} \frac{\not{n}}{4}$$

- S_L : longitudinal nucleon polarization, M_N : the nucleon mass
- all terms $\propto S_\perp$ vanish at order α_s

HQ TMD PDFs

University of Bern

Setup

- TMD PDF decomposition:

$$\Phi_{Q/N}(x,k_\perp) = \left\{ f_{1\,Q/N} + g_{1L\,Q/N} S_L \gamma_5 + h_{1L\,Q/N}^{\perp} S_L \gamma_5 \frac{\not{k}_\perp}{M_N} + h_{1\,Q/N}^{\perp} \frac{\not{k}_\perp}{M_N}) \right\} \frac{\not{n}}{4}$$

- Collinear PDF decomposition: [Collins '11]

$$\Phi_{g/N}^{\mu\nu}(x) = -\frac{g_\perp^{\mu\nu}}{2} f_{g/N}(x) + \frac{\epsilon_\perp^{\mu\nu}}{2} g_{g/N}(x) S_L$$

- unpolarized f_1 and Boer-Mulders $h_1^{\perp}$ match onto unpolarized PDF f_g
- Helicity $g_{1,L}$ and Worm-gear $h_{1,L}^{\perp}$ match onto helicity PDF g_g

HQ TMD PDFs

Results

- unpolarized quark from unpolarized gluon:

$$C^{(1)}_{Q/g}(z,k_T,m) = T_F\,\Theta(z)\Theta(1-z)\,\frac{2}{\pi}\,\frac{k_T^2(1-2z+2z^2)+m^2}{(k_T^2+m^2)^2}$$

[Nadolsky, Kidonakis, Olness, Yuan ’02, Pietrulewicz, Samitz, Spiering, Tackmann ’17]

- **NEW:** longitudinally polarized quark from longitudinally polarized gluon:

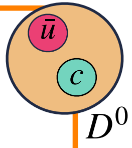
$$C^{(1)}_{Q_\parallel/g_\parallel}(z,k_T,m) = T_F\,\Theta(z)\Theta(1-z)\,\frac{2}{\pi}\,\frac{k_T^2(2z-1)+m^2}{(k_T^2+m^2)^2}$$

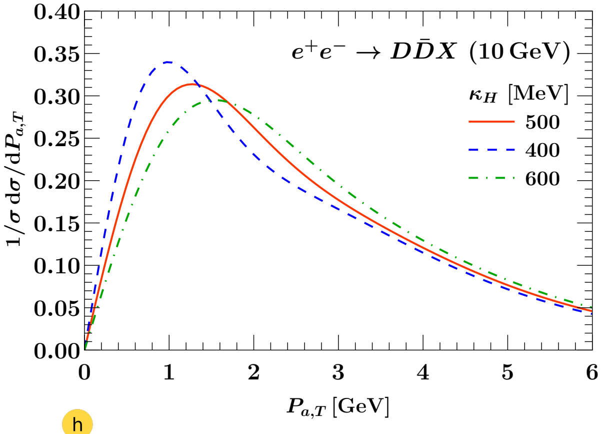
- **NEW:** transversely polarized quark from longitudinally polarized gluon:

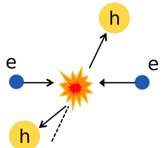
$$C^{(1)}_{Q_\perp/g_\parallel}(z,k_T,m) = T_F\,\Theta(z)\Theta(1-z)\,\frac{4}{\pi}\,\frac{m k_T(z-1)}{(k_T^2+m^2)^2}$$

Predictions for Belle II and the future EEC

HQ TMDs in e^+e^-

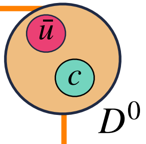


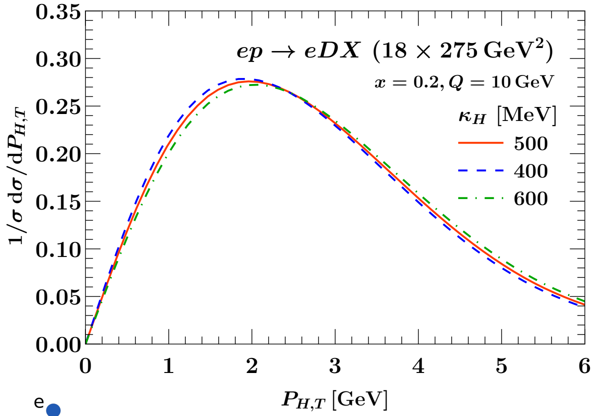


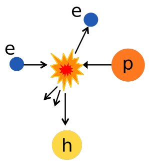


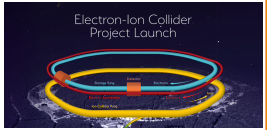
Belle II

HQ TMDs in SIDIS





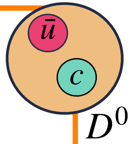


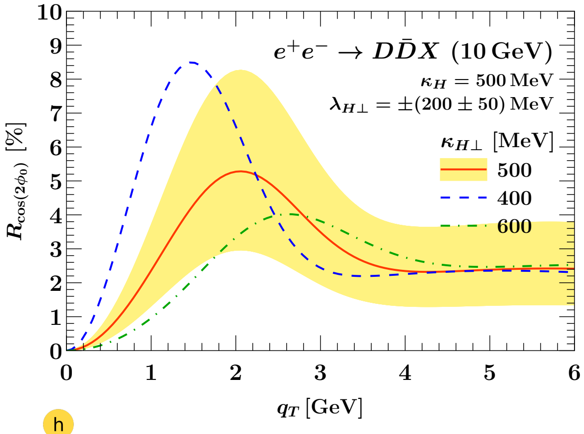


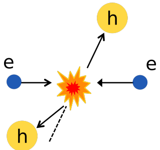
[RvK, Michel, Sun ’23]

Predictions for Belle II and the future EEC

HQ TMDs in e^+e^-

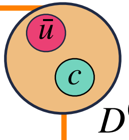


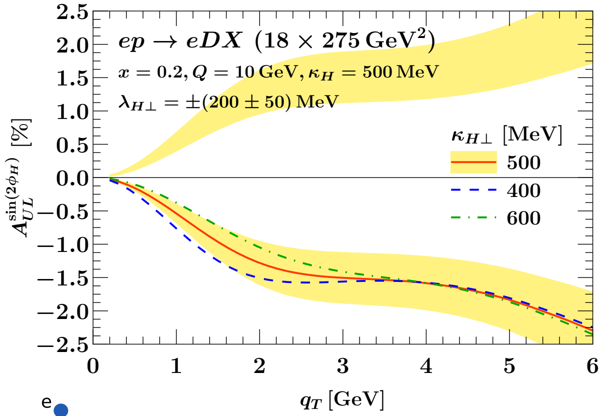


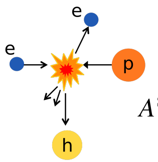


$$R_{\cos(2\phi_0)} = \frac{H_1^{T,(1)} \otimes H_1^{T,(1)}}{D_1 \otimes D_1}$$

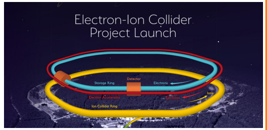
HQ TMDs in SIDIS







$$A^{\sin(2\phi_0)} = \frac{h_{1,L}^{\perp} \otimes H_1^{T,(1)}}{f_1 \otimes D_1}$$



[RvK, Michel, Sun '23]