Three-loop rapidity anomalous dimension and N^{3}LL resummation of jet-vetoed Higgs cross section

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Outline

- Three-loop rapidity anomalous dimension — Calculation
- Three-loop rapidity anomalous dimension — Checks
- N^3LL resummation of jet-vetoed Higgs cross section — Preliminary results
- Summary and outlook

Motivation

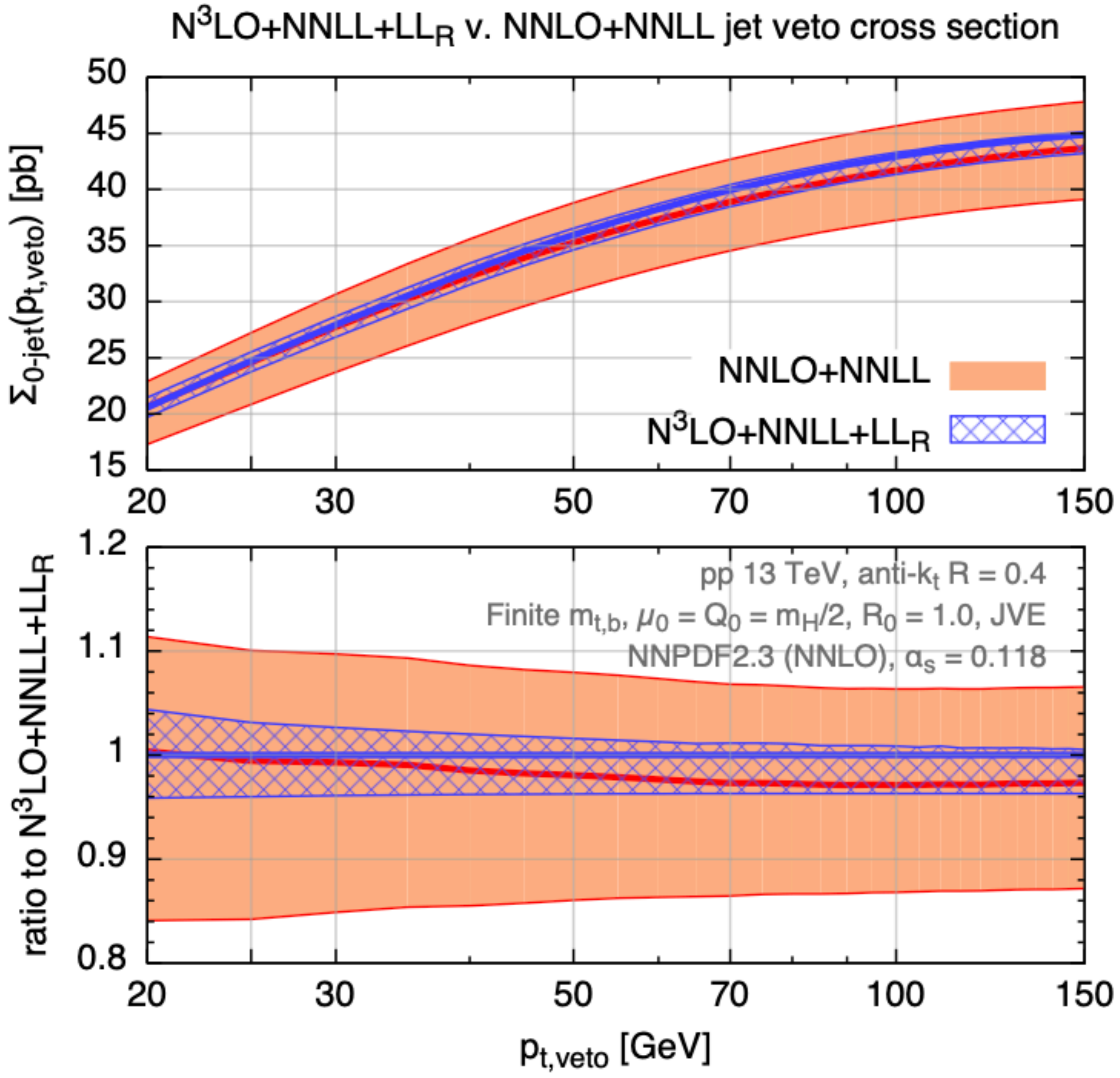
- Study important Higgs decays channels such as $H \to WW^*$ and $H \to \tau\tau$: reduce top-quark and other QCD background by imposing a veto on jet transverse momentum and **isolate zero-jet events**

- **New scale** in the problem — p_T^{veto}

- Large **scale hierarchy** requires resummation

[Banfi, Monni, Salam, Zanderighi 12;
Becher, Neubert 12;
Becher, Neubert, Rothen 13;
Stewart, Tackmann, Walsh, Zuberi 13]

- Can we **push it to N^{3}LO/N^{3}LL**?



[Banfi, Caola, Dreyer, Monni Salam, Zanderighi, Dulat 15]

Calculation

Factorisation à la SCET

- Colour-singlet production with jet veto $p_T^{\text{jet}} < p_T^{\text{veto}}$
  - Potentially large logarithms $\log(p_T^{\text{veto}}/Q)$

- In the limit $p_T^{\text{veto}} \ll Q$

$$\frac{d\sigma(p_T^{\text{veto}})}{d\sigma_{\text{Born}}} = \mathscr{H}(Q;\mu)\,\mathscr{B}_n(x_1,Q,p_T^{\text{veto}},R^2;\mu,\nu)\,\mathscr{B}_{\bar{n}}(x_2,Q,p_T^{\text{veto}},R^2;\mu,\nu)\,\mathscr{S}(p_T^{\text{veto}},R^2;\mu,\nu)$$

  - **Hard function**: large scales $\mu \sim Q$, only virtual contributions $\Rightarrow$ independent of p_T^{veto}
  - **Beam functions**: collinear dynamics of radiation along n and $\bar{n}$
  - **Soft function**: soft radiation emitted off initial state partons
  - μ and ν are the renormalisation and **rapidity regularisation** scales

Evolution equations (for ν)

- **Beam functions**

$$\frac{d}{d\ln\nu}\ln\mathscr{B}_n(x,Q,p_T^{\text{veto}},R^2;\mu,\nu) = 2\int_{p_T^{\text{veto}}}^{\mu}\frac{d\mu'}{\mu'}\Gamma_{\text{cusp}}(\alpha_s(\mu')) - \frac{1}{2}\gamma_\nu(p_T^{\text{veto}},R^2)$$

- **Soft functions**

$$\frac{d}{d\ln\nu}\ln\mathscr{S}(p_T^{\text{veto}},R^2;\mu,\nu) = -4\int_{p_T^{\text{veto}}}^{\mu}\frac{d\mu'}{\mu'}\Gamma_{\text{cusp}}(\alpha_s(\mu')) + \gamma_\nu(p_T^{\text{veto}},R^2)$$

[Becher, Neubert, Rothen 13;
Stewart, Tackmann, Walsh, Zuberi 13]

- **N^{3}LL resummation**
  - Two-loop beam and soft function

[Abreu, Gaunt, Monni, Rottoli, Szafron 22, Bell, Brune et al 24]
[Abreu, Gaunt, Monni, Szafron 22]

  - Three-loop rapidity anomalous dimension $\gamma_\nu(p_T^{\text{veto}},R^2)$: this talk!

Rapidity anomalous dimension

$$\frac{d}{d\ln\nu}\ln\mathcal{S}(p_T^{\rm veto},R^2;\mu,\nu) = -4\int_{p_T^{\rm veto}}^{\mu}\frac{d\mu'}{\mu'}\Gamma_{\rm cusp}(\alpha_s(\mu')) + \gamma_\nu(p_T^{\rm veto},R^2)$$

- $\ln\nu$ derivative of three-loop soft function

$$\mathcal{S}(p_T^{\rm veto},R^2;\mu,\nu) = \frac{1}{d_F}\sum_{X_s}\mathrm{Tr}\left\{\mathcal{M}(p_T^{\rm veto},R^2)\,\langle 0|\,Y_n^\dagger Y_{\bar n}\,|X_s\rangle\langle X_s|\,Y_{\bar n}^\dagger Y_n\,|0\rangle\right\}$$

- **Measurement function**: clusters particles and applies p_T veto, depends on jet algorithm and R

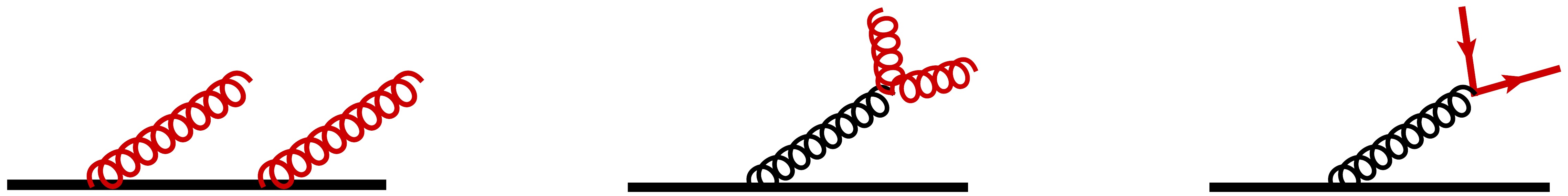
$$\mathcal{M}(p_T^{\rm veto},R^2) = \Theta(p_T^{\rm veto} - p_t^{j_1}(p_1,\ldots,p_n))$$

- Divergences that need to be regulated
  - **UV**: regulated in dimensional regularisation, $D \to 4-2\epsilon$
  - **Rapidity**: exponential regulator $$\prod_i d^dk_i\delta(k_i^2)\theta(k_i^0) \to \prod_i d^dk_i\delta(k_i^2)\theta(k_i^0)\exp\left[\frac{-e^{-\gamma_E}}{\nu}(n\cdot k_i + \bar n\cdot k_i)\right]$$

[Li, Neill, Zhu 16]

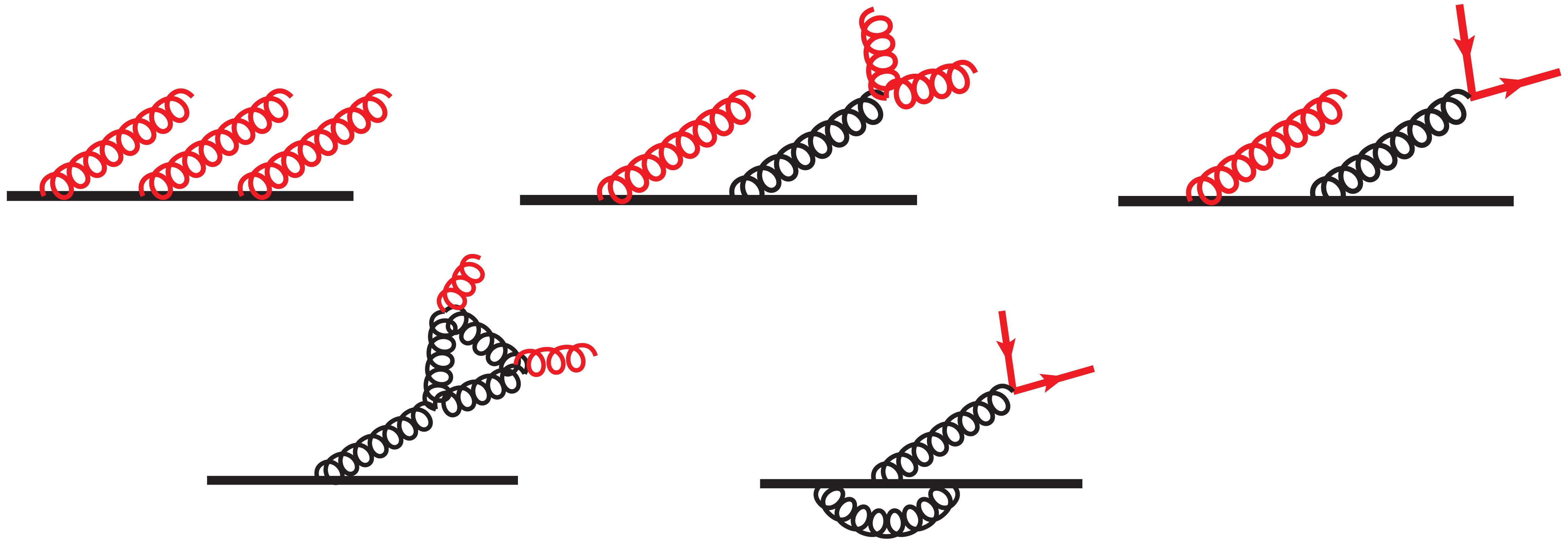
Ingredients: soft amplitudes

- **Two-loops**: tree amplitudes for double soft emission



[Campbell, Glover 98, Dokshitzer, Lucenti, Marchesini, Salam 97, Catani, Grazzini 99]

- **Three-loops**: tree amplitudes for triple soft emission, and one-loop for double soft emission



[Catani, Colferai, Torrini 20; Catani, Cieri, Colferai, Coradeschi 22]

[Zhu, 20; Czakon, Eschment, Schellenberger 22]

Challenges

- Complicated integrands
    - Square amplitudes + colour algebra ⇒ relatively **large expressions to handle**

- Complicated integration measure and measurement function

$$\int [dp_1][dp_2][dp_3]\mathcal{M}(p_T^{\text{veto}}, R^2) \qquad [dp_i] = d^D p_i \delta(p_i^2)\theta(p_i^0)\exp\left[-\frac{e^{-\gamma_E}}{\nu}(n\cdot p_i + \bar{n}\cdot p_i)\right]$$

- Regulate divergences
    - **Dimensional regularisation**: integrate in $4-2\epsilon$ dimensions
    - Rapidity divergence: handle **non-trivial exponential regulator**

Simplifying the calculation 1

- Define **independent** and **correlated** contributions. Eg, for two soft-gluon squared amplitude

$$\mathscr{A}_{gg}^{(0)}(p_1, p_2) = \mathscr{A}_{g}^{(0)}(p_1)\,\mathscr{A}_{g}^{(0)}(p_2) + \tilde{\mathscr{A}}_{gg}^{(0)}(p_1, p_2)$$

  - Independent emissions $\mathscr{A}_g(p_1)$: soft single-gluon squared amplitude
  - Correlated terms $\tilde{\mathscr{A}}_{gg}(p_1, p_2)$: p_1 and p_2 have commensurate rapidities
  - Generalise trivially to more emissions

- Helps deal with **rapidity divergences**
  - Correlated contributions **vanish if any particle is far in rapidity**
  - **Collinear singularity mapped onto a single particle** of the correlated cluster

Simplifying the calculation 2

- Define **Reference Simplified Variable (RSV)**

$$\mathcal{S}(p_T^{\text{veto}}, R^2; \mu, \nu) = \mathcal{S}_{\text{RSV}}(p_T^{\text{veto}}; \mu, \nu) + \Delta\mathcal{S}(p_T^{\text{veto}}, R^2; \mu, \nu)$$

➡ Compute $\mathcal{S}_{\text{RSV}}$ and $\Delta\mathcal{S}$ to determine $\mathcal{S}(p_T^{\text{veto}}, R^2; \mu, \nu)$

$$\mathcal{M}_{\text{RSV}}(p_T^{\text{veto}}) = \prod_i \Theta(p_T^{\text{veto}} - p_t^{\text{clust}_i}) = \Theta(p_T^{\text{veto}} - \max\{p_t^{\text{clust}_i}\})$$

- **Factorisation of phase-space** integrals over clusters
- RSV trivially related to TMD soft function $\mathcal{S}_\perp(p_T^{\text{veto}}; \mu, \nu)$: **computed analytically**
- $\Delta\mathcal{S}$ vanishes for single emission: much **simpler singularity structure**
  - Finite at two loops
  - One-loop like divergences (double poles) at three loops

Example @ two loops

$$\Delta\gamma_\nu^{(2)}(R^2) = \Delta\gamma_{\nu,\mathrm{CC}}^{(2)}(R^2) + \Delta\gamma_{\nu,\mathrm{II}}^{(2)}(R^2)$$

- **II term** $$\int [dp_1][dp_2]\,\mathscr{A}_g^{(0)}(p_1)\,\mathscr{A}_g^{(0)}(p_2)\Big(\Theta(p_T^{\mathrm{veto}} - p_t^{j_1}(p_1,p_2)) - \Theta(p_T^{\mathrm{veto}} - \max\{p_{t1},p_{t2}\})\Big)$$
    - Measurement vanishes when p_1 and p_2 are far in rapidity
    - Only **one source of rapidity divergence**
- **CC term** $$\int [dp_1][dp_2]\left(\tilde{\mathscr{A}}_{q\bar{q}}^{(0)}(p_1,p_2) + \frac{1}{2!}\tilde{\mathscr{A}}_{gg}^{(0)}(p_1,p_2)\right)\Big(\Theta(p_T^{\mathrm{veto}} - p_t^{j_1}(p_1,p_2)) - \Theta(p_T^{\mathrm{veto}} - |\vec{p}_{t1}+\vec{p}_{t2}|)\Big)$$
    - Amplitude vanishes when p_1 and p_2 are far in rapidity
    - Only **one source of rapidity divergence**
- Greatly simplifies rapidity integration — **only retain $\ln\nu$ terms**
- Remaining integrations: Monte Carlo because of non-trivial jet algorithm

Three-loop decomposition

$$\Delta\gamma_\nu^{(3)}(R^2) = \Delta\gamma_{\nu,\mathrm{III}}^{(3)}(R^2) + \Delta\gamma_{\nu,\mathrm{CCI}}^{(3)}(R^2) + \Delta\gamma_{\nu,\mathrm{CCC}}^{(3)}(R^2)$$

$$\gamma_\nu^{(3)}(R^2) = C_R^3\,\gamma_\nu^{(3),\,C_R^3}(R^2)$$
$$+C_R^2 C_A\,\gamma_\nu^{(3),\,C_R^2 C_A}(R^2) + C_R^2 T_R n_f\,\gamma_\nu^{(3),\,C_R^2 T_R n_f}(R^2)$$
$$+C_R C_F T_R n_f\,\gamma_\nu^{(3),\,C_R C_F T_R n_f}(R^2) + C_R C_A T_R n_f\,\gamma_\nu^{(3),\,C_R C_A T_R n_f}(R^2) + C_R C_A^2\,\gamma_\nu^{(3),\,C_R C_A^2}(R^2) + C_R T_R^2 n_f^2\,\gamma_\nu^{(3),\,C_R T_R^2 n_f^2}(R^2)$$

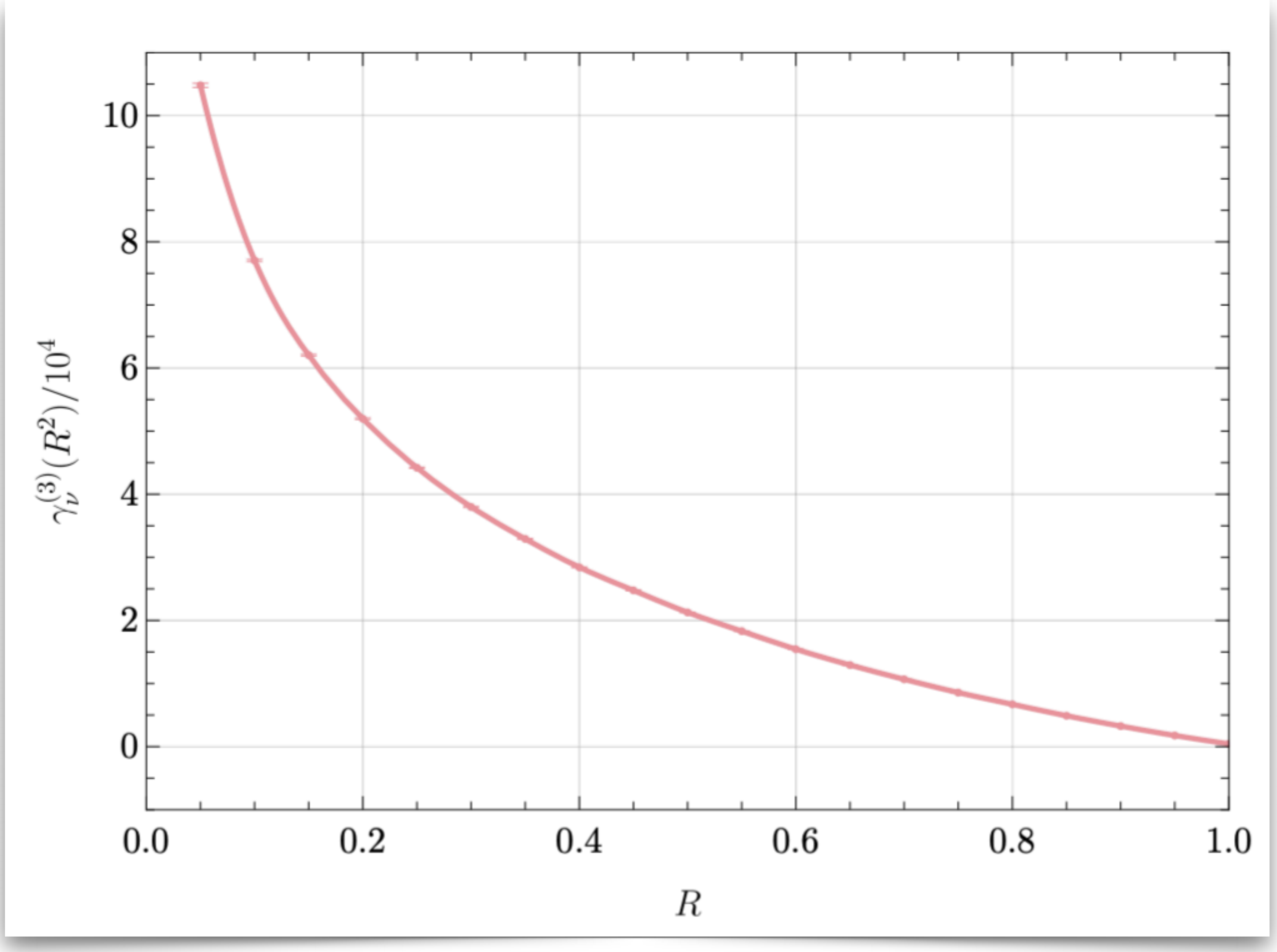
- **Challenges**
  - Calculation in $4-2\epsilon$ dimensions: combine real and virtual singularities
  - Many integrals to compute numerically: heavy Monte Carlo integration
- Computed all numerical channels in **two independent codes**: Vegas in fortran and Tensor Flow

[Carrazza, Cruz-Martinez 20]

- Sub per-mil precision for all contributions

[Abreu, Gaunt, Monni, Rottoli, Szafron, TO APPEAR]

Three-loop rapidity anomalous dimension



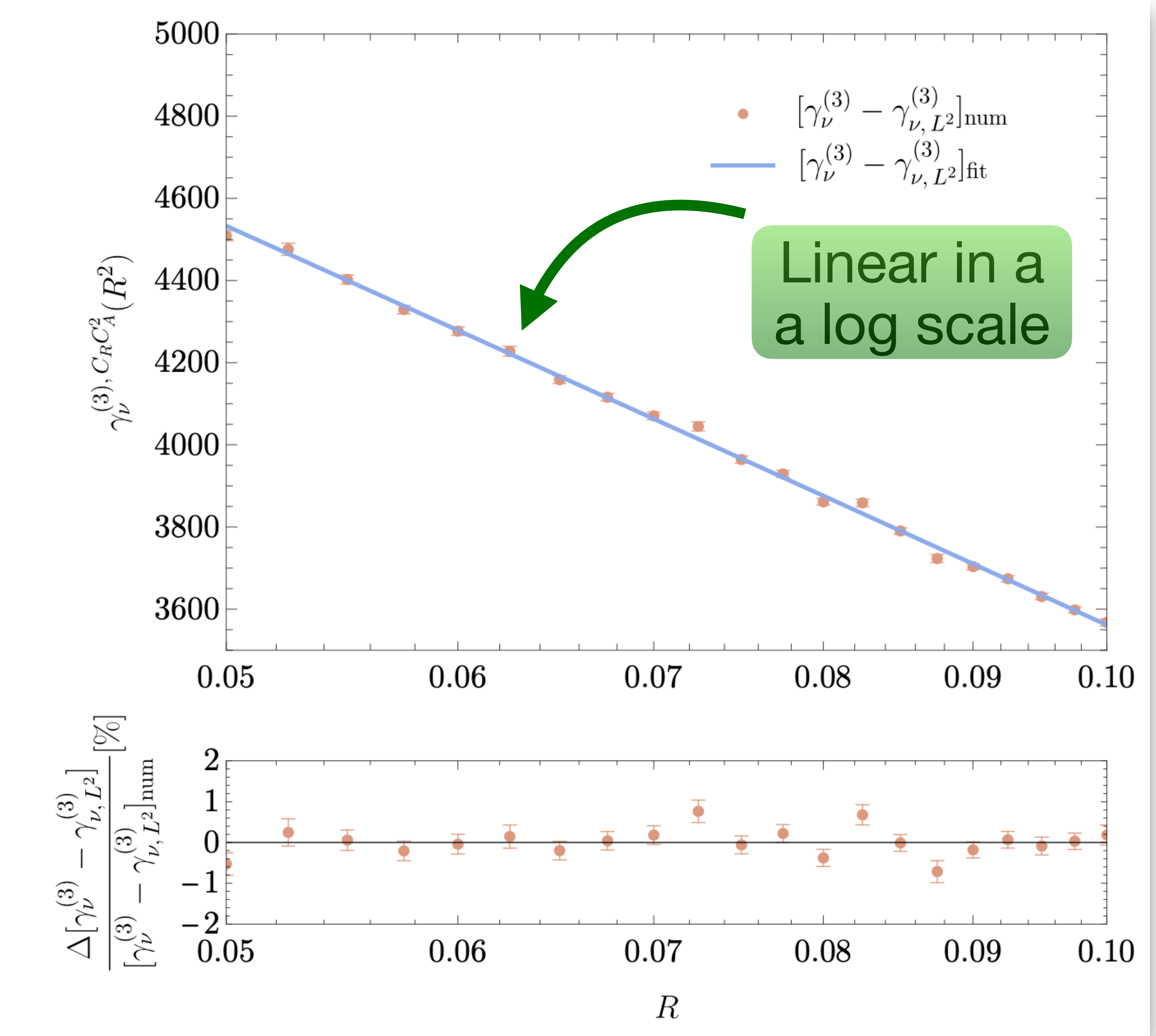
- Fit over **20 values of R**
- Negligible error bars
- @ Three loops: **jet-algorithm dependent**
- Results for anti-k_t, k_t, Cambridge/Aachen

- It's **quite large**!

[Abreu, Gaunt, Monni, Rottoli, Szafron, TO APPEAR]

Checks

Check 1: Small-R behaviour



- The three-loop soft anomalous dimension has at most $\ln^2 R^2$ behaviour at small R

$$\gamma_\nu^{(3)}(R^2) = c_{-2}\ln^2 R^2 + c_{-1}\ln R^2 + \sum_i c_i R^{2i}$$

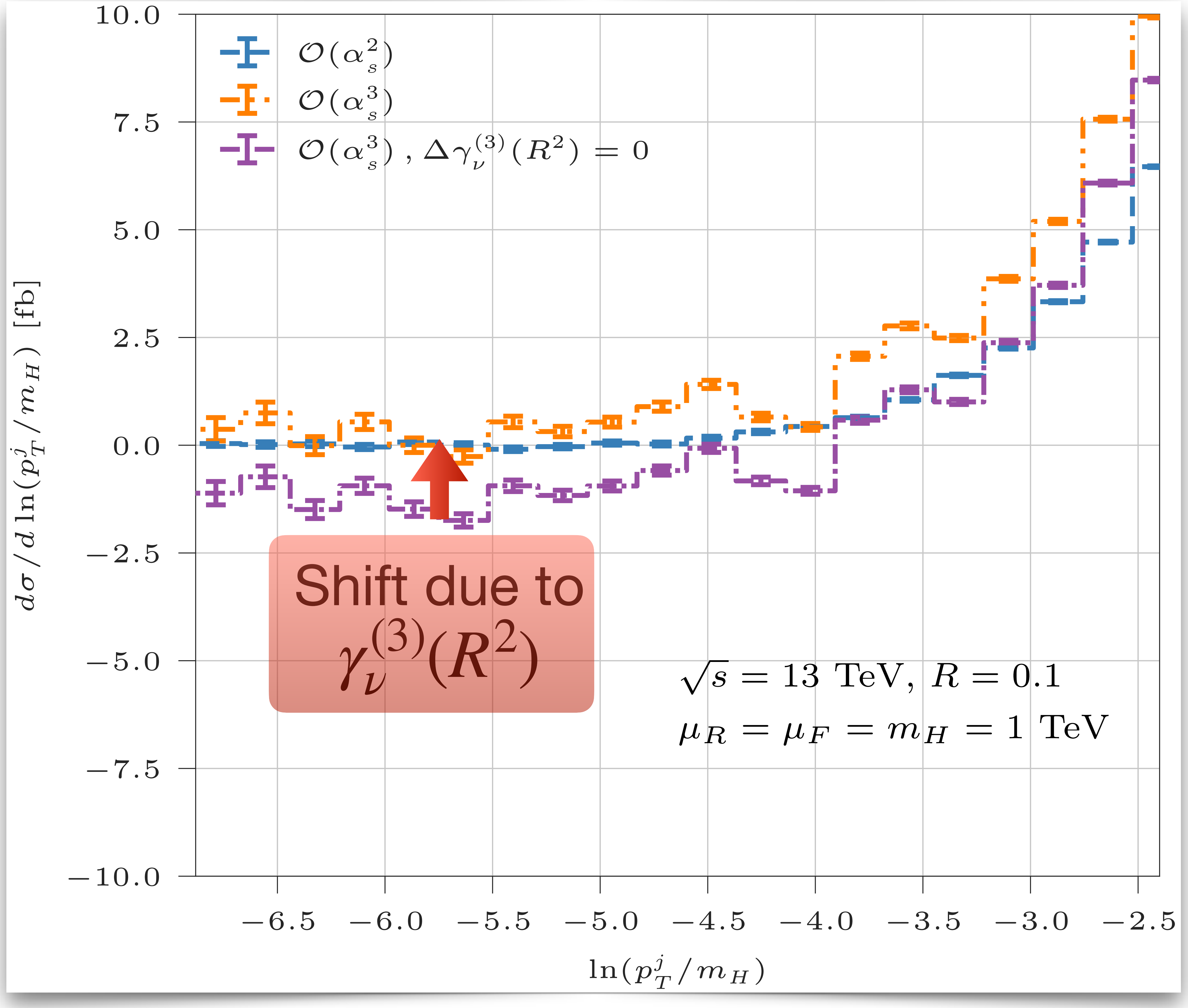
  - c_{-2} predicted from small R resummation ⇒ remaining dependence is logarithmic

$$f_{\rm fit}(R^2) = c_{-1}^L \ln R^2 + c_0 + \sum_{i=1}^{i_{\max}} c_i (R^2)^i$$

  - same agreement for other 3 CCC channels
  - large c_{-1} coefficient!

[Abreu, Gaunt, Monni, Rottoli, Szafron, TO APPEAR]

Check 2: leading-jet p_T slicing



$$d\sigma = \mathbb{H}_{\text{veto}} \otimes d\sigma_{\text{Born}} + \lim_{p_{T,\text{cut}}^{\text{jet}} \to 0} \int_{p_{T,\text{cut}}^{\text{jet}}}^{+\infty} dp_T^{\text{jet}} \left(\frac{d^2\sigma^{H+\text{jet}}}{dp_T^{\text{jet}}} - \frac{d^2\sigma(p_T^{\text{veto}})}{dp_T^{\text{veto}}} \bigg|_{p_T^{\text{veto}} = p_T^{\text{jet}}} \right)$$

- Logarithmic dependence at small p_T captured by $\gamma_\nu(R^2)$
- Implemented in RadISH

[Bizon, Monni, Re, Rottoli, Torrielli 18]

- H+jet cross-section from NNLOJET

[Huss et al 25]

- Consistent with our result for $\gamma_\nu^{(3)}(R^2)$

[Abreu, Gaunt, Monni, Rottoli, Szafron, TO APPEAR]

N^3LL resummation of jet-vetoed Higgs cross section

!! Preliminary !!

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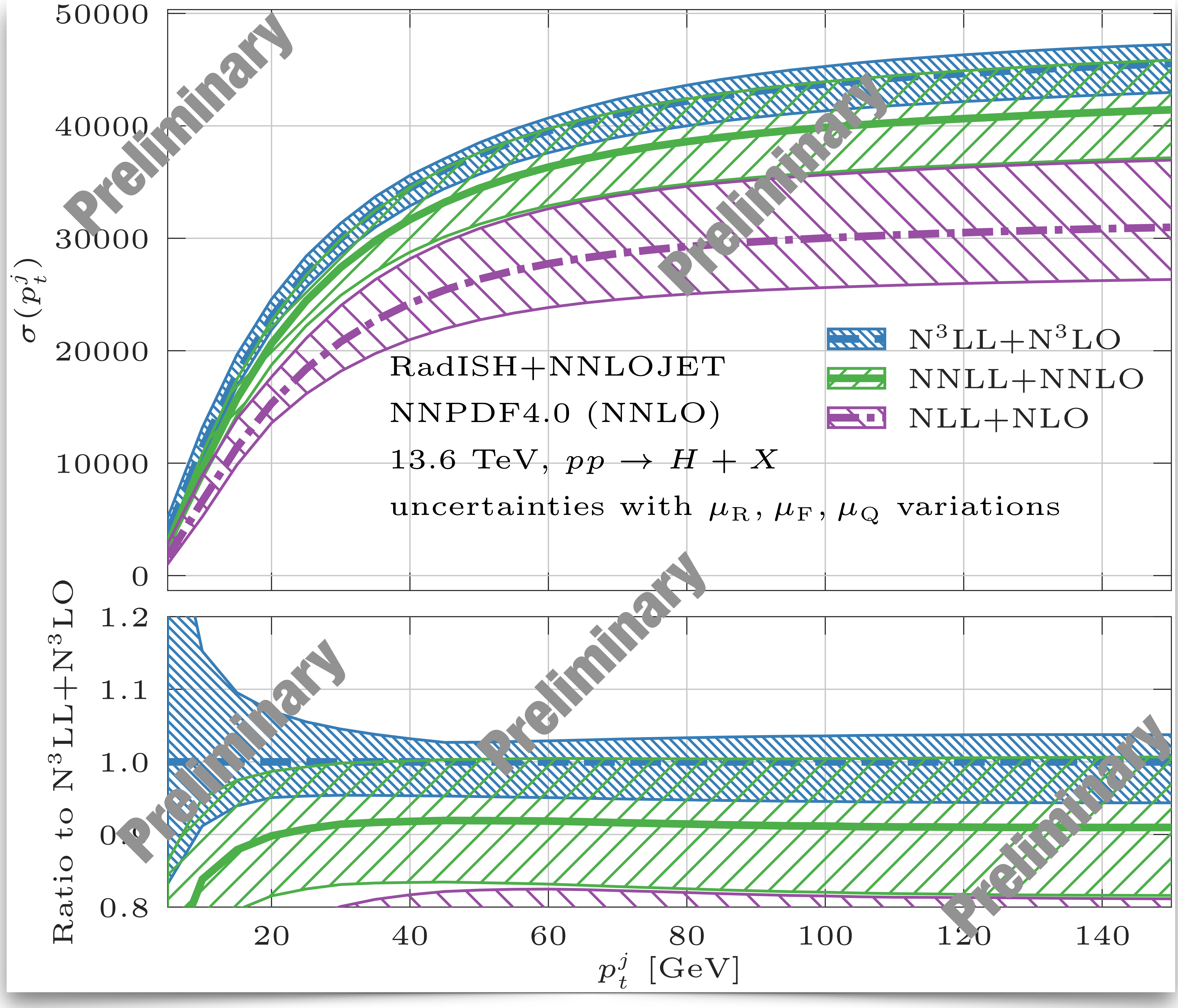
N^3LO/N^3LL Higgs jet-veto cross section



[Abreu, Gaunt, Monni, Rottoli, Szafron, TO APPEAR]

- **Fixed-order ingredients** from NNLOJET and Higgs N^3LO cross section
  [Huss et al 25]
  [Anastasiou et al 15, 16; Mistlberger 18]
- **Resummation** performed with RadISH
  [Bizon, Monni, Re, Rottoli, Torrielli 18]
- Very **good perturbative convergence**
- Very **large jump from N^2LO/N^2LL to N^3LO/N^3LL**
- What explains this large jump? Could it be large logarithms of R?
  ➡ Require small-R resummation?

Summary and outlook

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- Completed the calculation of the **three-loop rapidity anomalous dimension**
- Passed **several non trivial checks**
- Used in obtaining the **N^3LO/N^3LL Higgs jet-veto cross section**
- Very **large jump from N^2LO/N^2LL to N^3LO/N^3LL**

- Do we need **small-R resummation?** Need to go beyond leading log…
- Can the same computational setup be applied to other observables?

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Thank you