Precision matching kernels for lattice PDFs and TMDs in the HL-LHC era

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Southern Methodist University

LoopFest 2026 @ Brookhaven National Laboratory

What you'll take away

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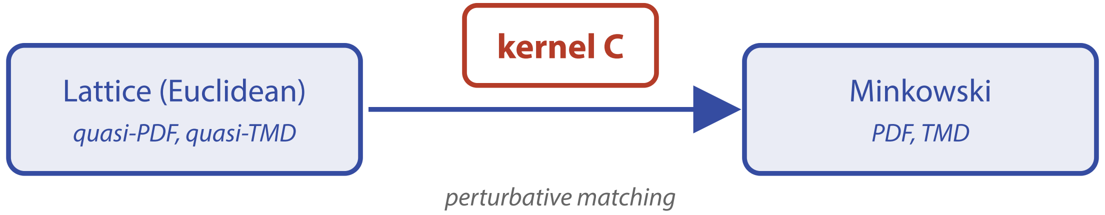

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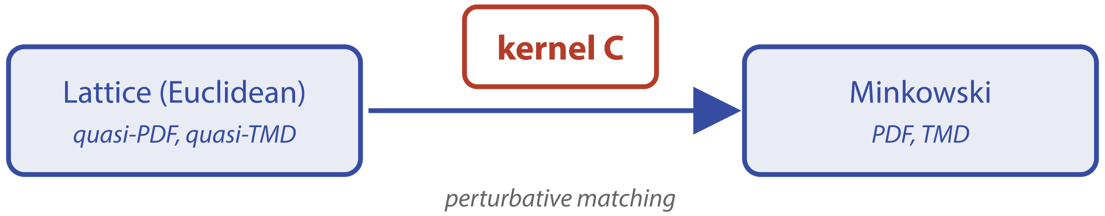

The matching kernel is the bridge — and a bottleneck.

Why we need lattice PDFs

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- **di-Higgs:** PDF$+\alpha_s$ uncertainty $\approx 3\%$ same size as N^{3}LO QCD truncation, gluon-dominated

  Di Micco et al. '19 (LHCHXSWG); N^{3}LO: Chen, Li, Shao, Wang '19

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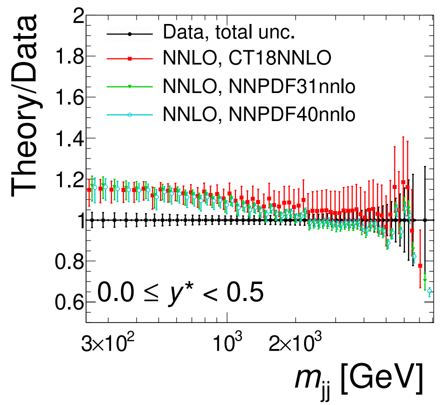
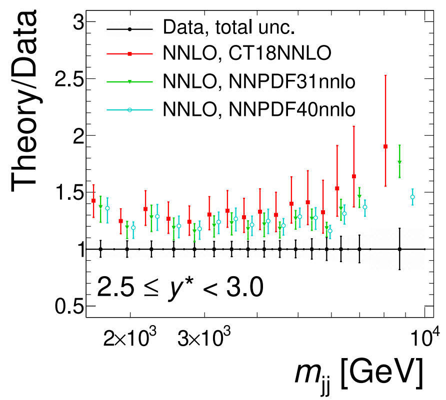
  ATLAS '25





ATLAS dijet $\sqrt{s} = 13$ TeV: central (top), forward (bottom). ATLAS '25, Fig. 11a,f

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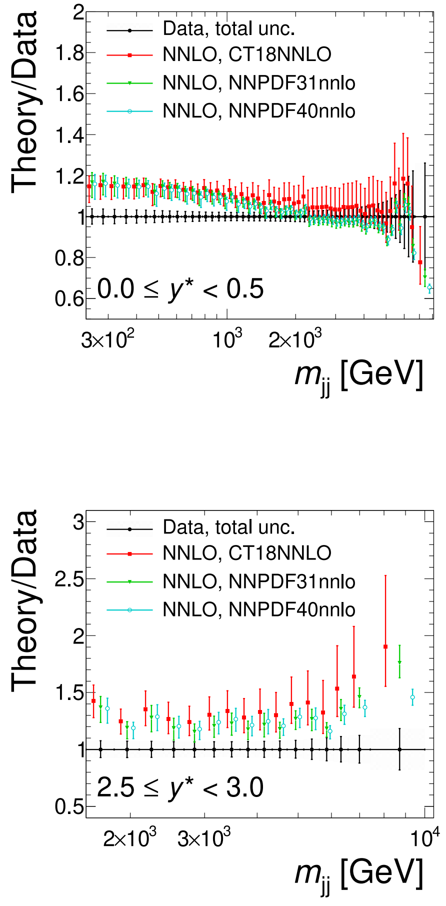
  CMS '24; ATLAS '24



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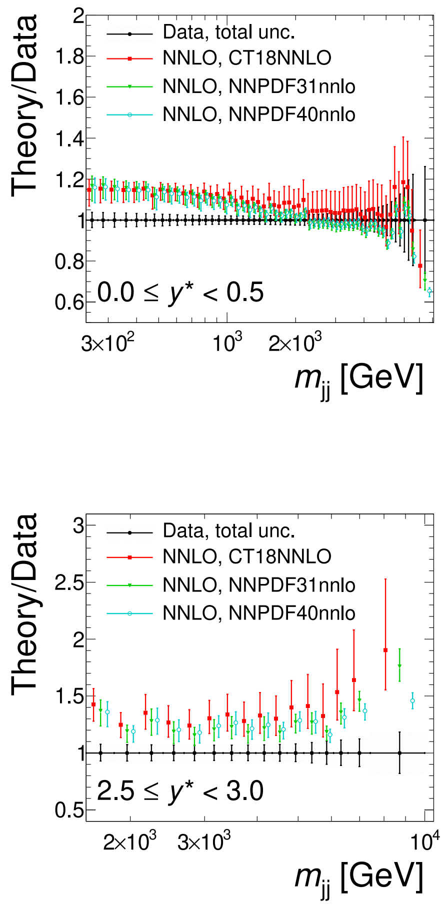

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PDFs and our nucleon parametrizations need a first-principles complement.

The Euclidean–Minkowski bridge

Minkowski collider

$$\sigma = f \otimes \hat{\sigma}$$

universal PDF $\otimes$ partonic cross section

$$\Longleftrightarrow$$

Euclidean lattice

$$\sigma_E = f \otimes C$$

same physical PDF $\otimes$ **perturbative matching** C

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The matching coefficient C is what this talk is about.

LaMET in one slide

Ji '13, Ji '14; review: Ji et al. '20

Quasi-PDF: equal-time spatial correlator at large P^z:

$$\tilde{f}(x, P^z, \mu) \;=\; \int \frac{dz}{4\pi}\, e^{ixzP^z}\, \langle P|\bar{\psi}(z)\gamma^z W(z,0)\,\psi(0)|P\rangle$$

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Factorization (LaMET):

$$\tilde{f}(x, P^z, \mu) \; = \; \int \frac{dy}{|y|} \, C(x/y, \mu/(yP^z)) \, f(y, \mu) \; + \; \mathcal{O}\left(\frac{1}{(xP^z)^2}, \frac{1}{((1-x)P^z)^2}\right)$$

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Foundational lineage: Radyushkin '17 (pseudo-PDF); Lin et al. '14 (first numerical extraction); Izubuchi, Ji, Jin, Stewart, Zhao '18 (1-loop factorization); Ishikawa, Ma, Qiu, Yoshida '17 (renormalizability); Wang, Zhang, Zhao, Zhu '19 (1-loop complete matching), Constantinou, Panagopoulos '17 (lattice PT)

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Active community: LPC, BNL/ANL, various MIT collaborations, ETMC, HadStruc, MSULat

"The lattice machinery is in place"

Quark matching: NNLO and N^{3}LO

Chen, Wang, Zhu '20: direct canonical

PRL 126.072002; masters: JHEP 10 (2020) 079; earlier non-diag.: PRD 102.011503

- 96 quasi-master integrals
- Canonical basis, rational alphabet
- Result $\rightarrow$ multiple polylogs
- Physical kernel: harmonic polylogs only

Li, Ma, Qiu '20: reconstruction

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- 21 NNLO master integrals via IBP reduction
- Differential equations + harmonic-polylog ansatz (Moch–Vermaseren)
- Closed-form non-singlet kernel

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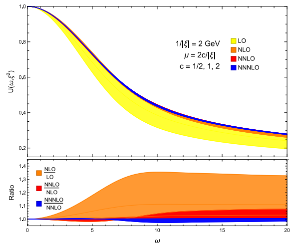
Cheng, Huang, Li, Ma '24: first N^{3}LO quark quasi-PDF kernel

PRL 134.251902

Higher-loop continuation of Li-Ma-Qiu



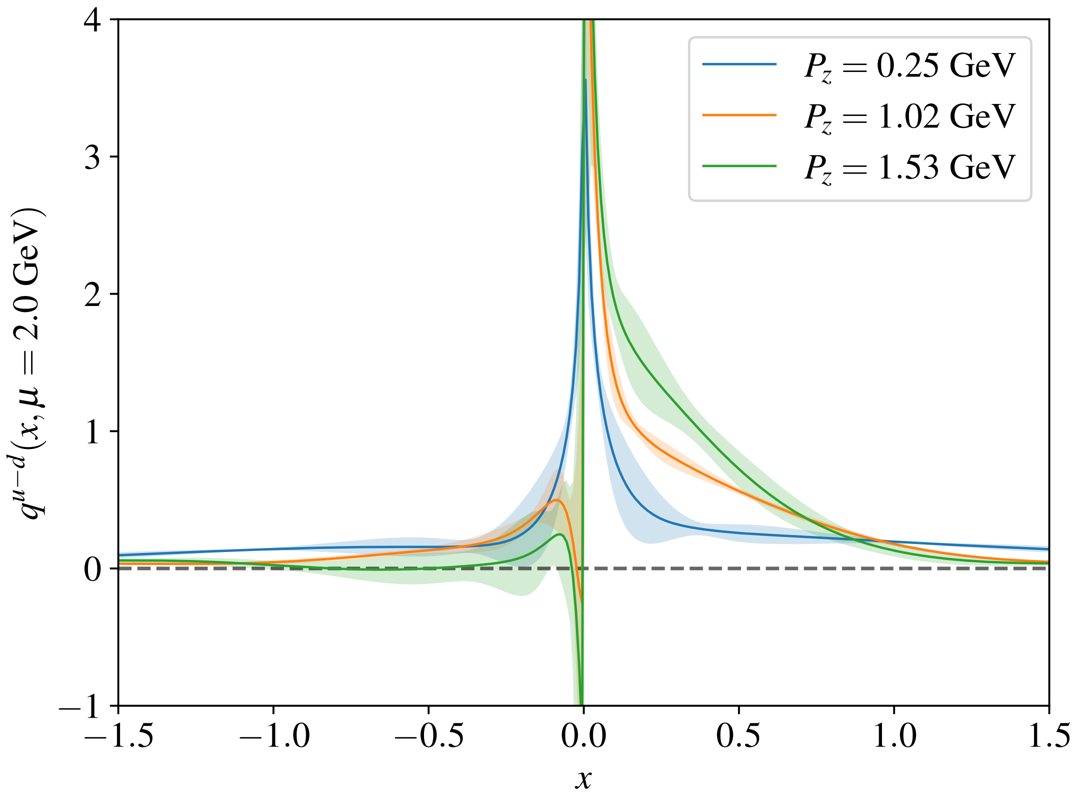
NLO/LO~30% $\longrightarrow$ NNLO/NLO~7% $\longrightarrow$ N^{3}LO/NNLO~1%

See also e.g. Braun, Chetyrkin, Kniehl '20 (3-loop renormalisation of the straight non-local quark/gluon op)

Impact of NNLO: P^z dependence

LPC '22 (Gao et al.): u-d isovector quark distribution

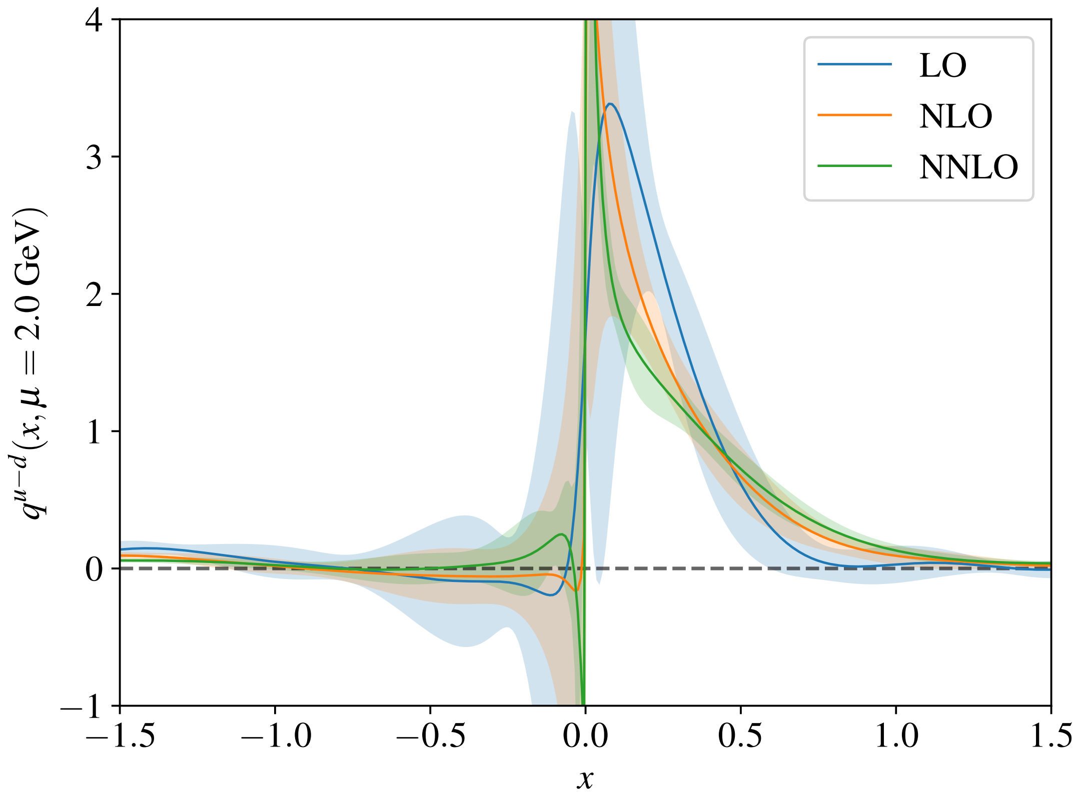
- Three boost momenta:
  $P^z = 0.25,\ 1.02,\ 1.53$ GeV
  NNLO matching throughout
- Visible drift with P^z: $1/(xP^z)^2$ power corrections still significant; asymptotic limit not yet reached at this ceiling



Impact of NNLO: LO $\rightarrow$ NLO $\rightarrow$ NNLO

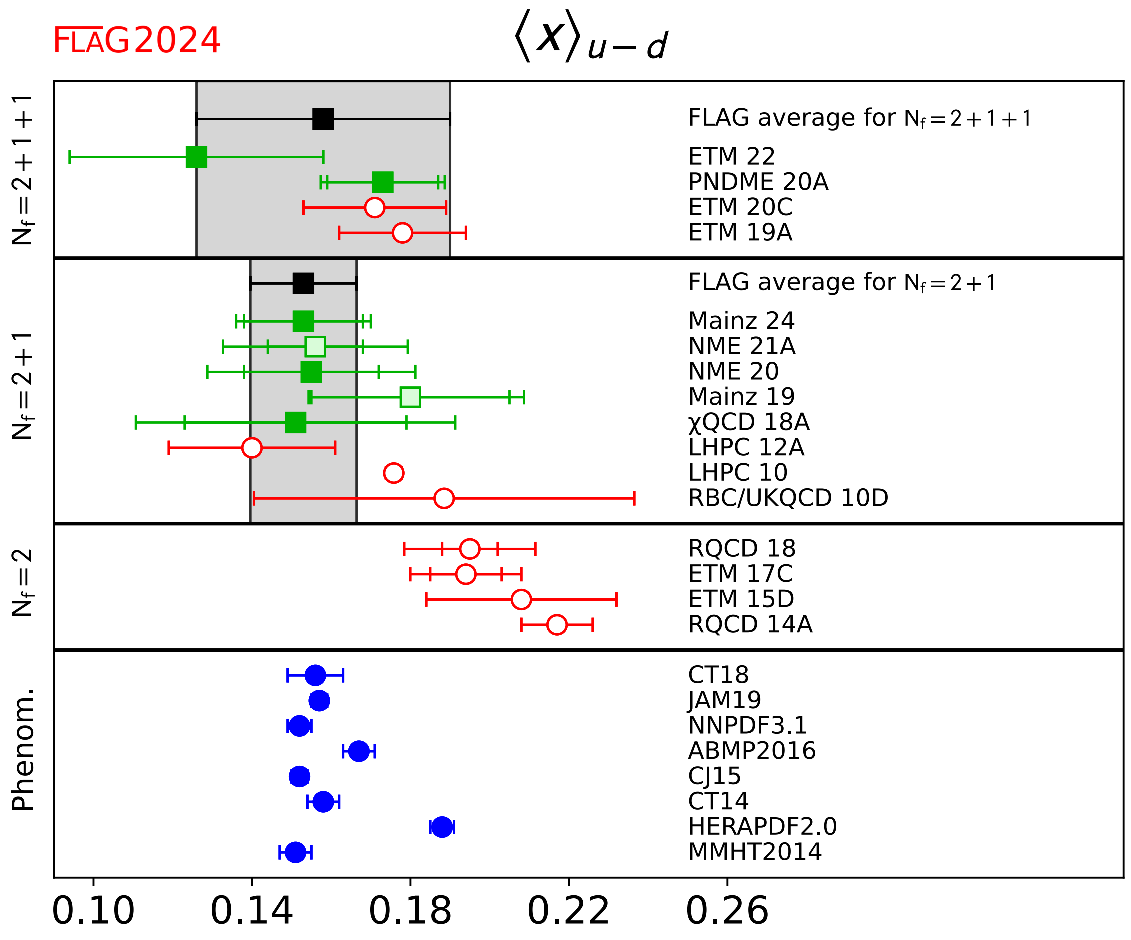
LPC '22 (Gao et al.): u-d isovector quark distribution

- Mid-x region **stabilizes** at NNLO
- Uncertainty band shrinks order by order: controlled perturbative expansion
- NNLO matching kernel turns LaMET from idea into **precision-science program**



The gluon grand challenge

The well-studied quark case for context:

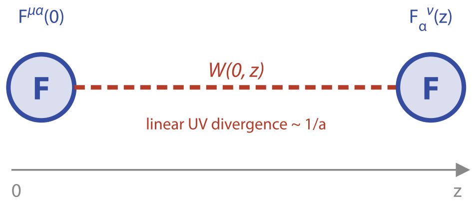


$\langle x \rangle_{u-d}$; lattice (red/green) vs phenomenology (blue, bottom row). FLAG 2024

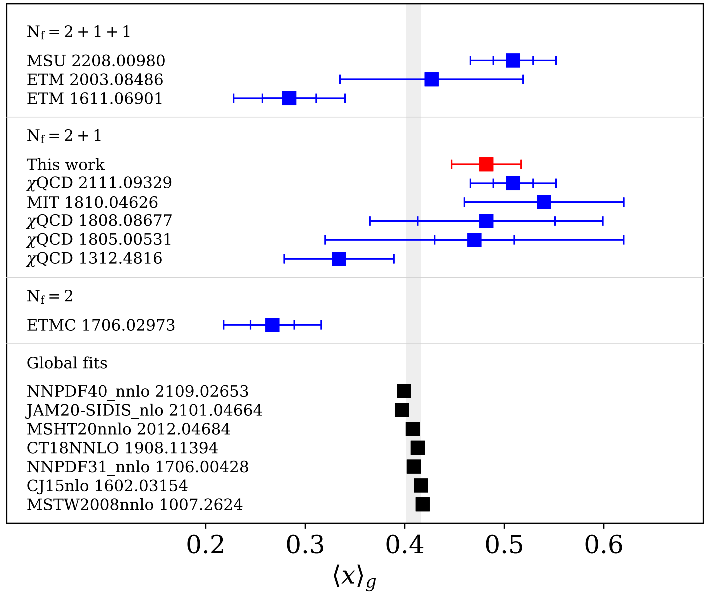
Horizontal range ~ 0.1

The gluon grand challenge

Two lattice difficulties: limiting in ways that motivate new approaches.



$$\mathcal{O}_g^{\mu\nu\rho\sigma}(z) = F^{\mu\nu}(0)\, W(0,z)\, F^{\rho\sigma(z)}$$



$\langle x \rangle_g$: lattice (blue) vs phenomenology (black); grey band $\langle x \rangle_g^{\text{exp.}} = 0.409(7)$. HadStruc/Edwards et al. '26

Horizontal range $\sim$ 0.4: 4× the quark case.

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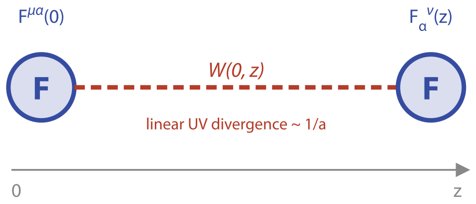

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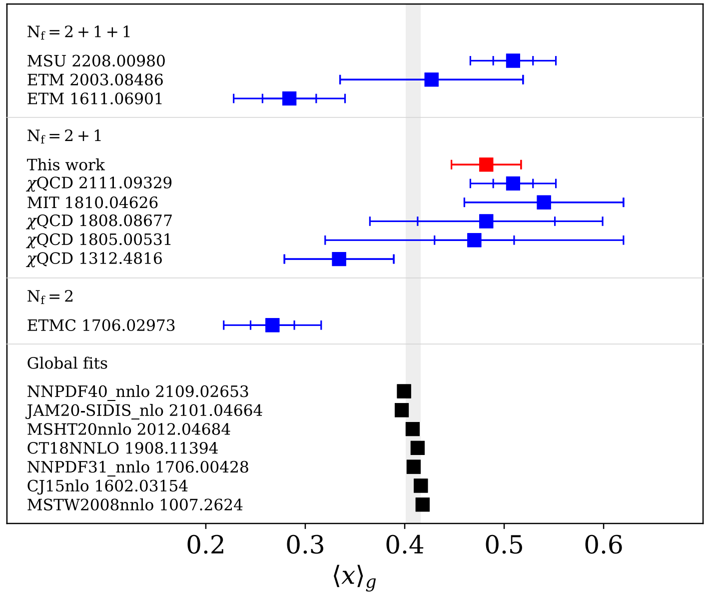

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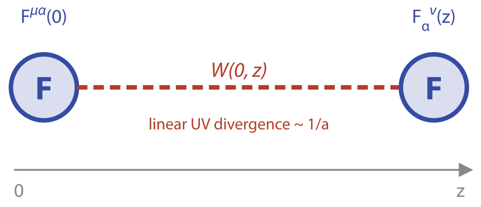

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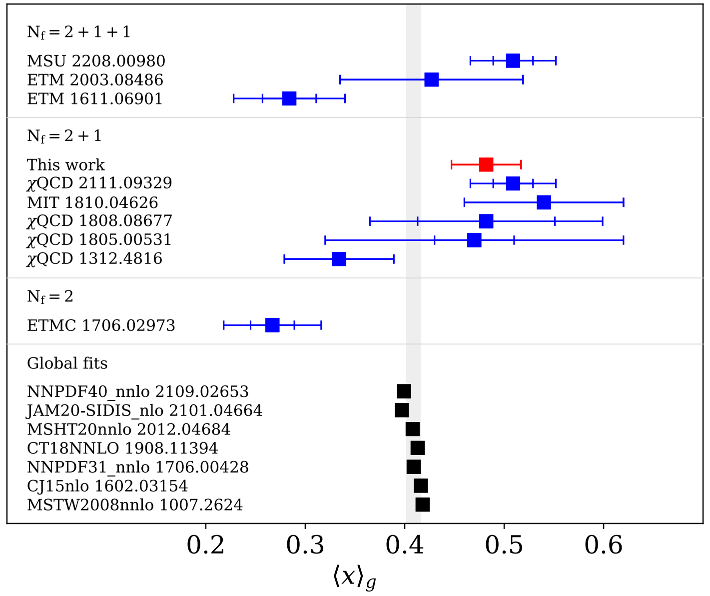

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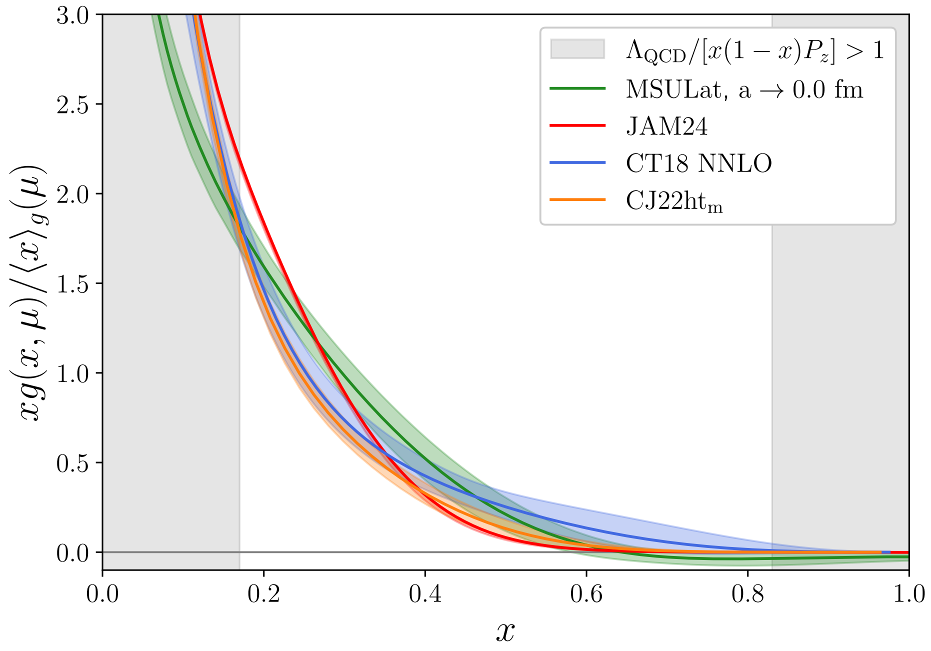
- **Linear power divergence.** The spatial Wilson line generates a $\sim 1/a$ self-energy in the bare matrix element: requires non-perturbative subtraction

Lattice gluon PDF: the field is moving

2025 frontier: continuum limit reached, NLO matching, bulk-x agreement with global fits.



Continuum-limit MSULat unpolarized gluon PDF at $P^z = 2.12$ GeV vs CT18 NNLO, JAM24, CJ22
NieMiera, Good, Lin, Yao '25; Good, Yao, Lin '25

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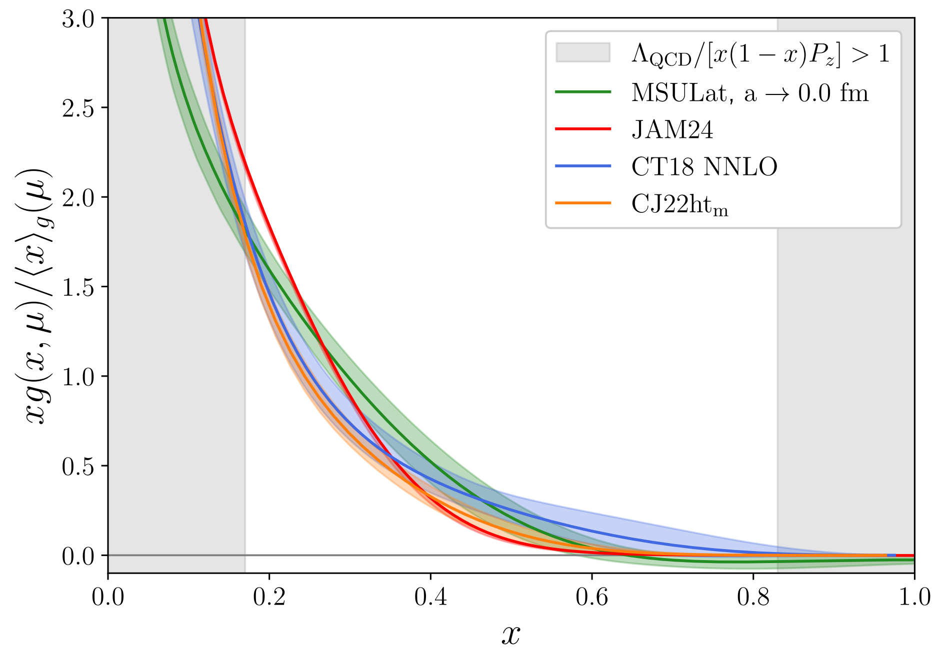

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What's needed moving forward: smarter lattice constructions that tame the lattice side.
Each demands its own matching kernel at the order we need.

Matching machinery: multi-channel, multi-gauge

With Christopher Monahan (Colorado College); paper this summer

Operators:

quark $\mathcal{O}_q^{\Gamma}(z) = \bar{q}(z)\,\Gamma\,W(z,0)\,q(0)$ · gluon $\mathcal{O}_g^{\mu\nu\rho\sigma}(z) = F^{\mu\nu}(z)\,W(z,0)\,F^{\rho\sigma}(0)$

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Unpolarized quark + gluon $\times$ quasi + pseudo, all $\overline{\mathrm{MS}} \to \overline{\mathrm{MS}}$.

Lattice-side bridge to $\overline{\mathrm{MS}}$ via self-renormalisation (Huo et al. '21; NieMiera, Good, Lin, Yao '25)

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Tensor decomposition: Six gluon form factors with purely-numerical 6×6 UV mixing matrix

Any operator-index choice is a numerical recombination; no recalculation per index.

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Automatized: Fully analytic results, multiple gauges (R_ξ and general axial)

Kernels, cross-checks

With Christopher Monahan (Colorado College); paper this summer

Kernels $\overline{\mathrm{MS}} \to \overline{\mathrm{MS}}$:

- $C_{qq},\ C_{qg},\ C_{gq},\ C_{gg}$ for quasi-PDF
- $C_{qq},\ C_{qg},\ C_{gq},\ C_{gg}$ for pseudo-PDF

- e.g. gq mixing in gluon PDF: $\sim 10\%$ effect, currently below the $30–50\%$ statistical floor of pseudo-PDF extractions

- Comprehensive Quasi-PDF $\pm\infty$ boundary-term discussion

Cross-checks

Comparison	Result
R_ξ vs axial	✓ gauge-independent final kernel
Quark vs Izubuchi '18, Chou '22	✓ confirmed, but without shortcuts
Gluon vs Wang '17, Balitsky '19	✓ where regulator allows
Two-loop vacuum Braun '20	✓ confirmed

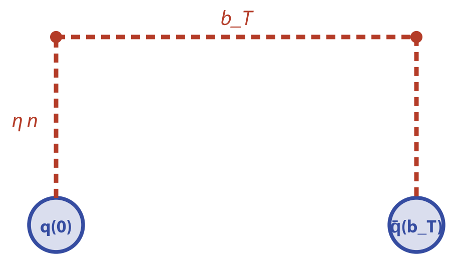
Why TMDs are next

Why TMDs at all

- Small-q_T Drell–Yan: M_W, $Z\,p_T$, intrinsic transverse momentum
- Cross-validate phenomenological CS kernel against first principles
- Now reachable: quasi-TMD operators

TMD quark operator

$$\mathcal{O}_q^{\mathrm{TMD}}(b_T) = \bar{q}(b_T)\,\Gamma\,W_{\sqcap}^{n,\eta}(b_T,\,0)\,q(0)$$



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Quasi-TMD: n spatial, η finite.
Extra non-perturbative scale b_T.

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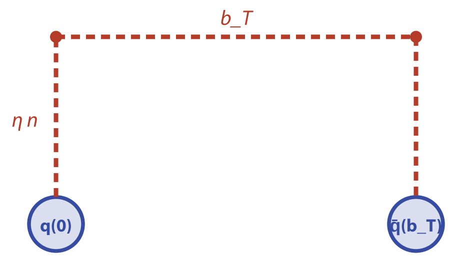

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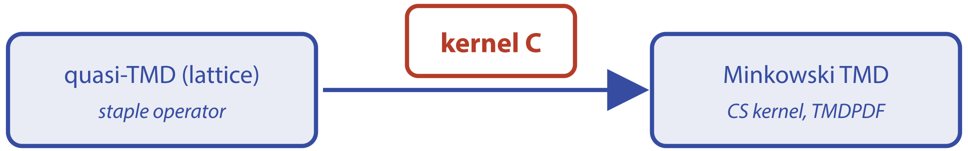

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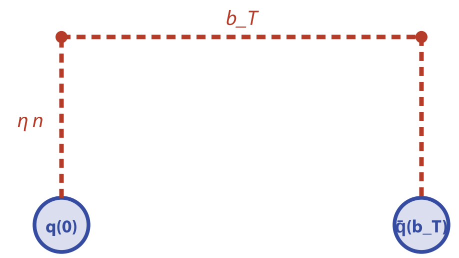

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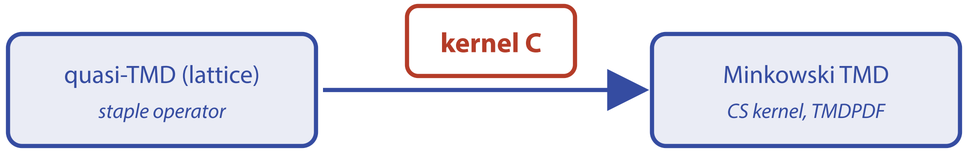

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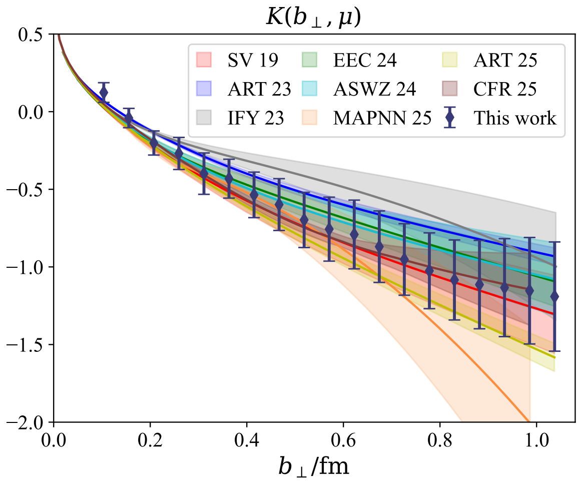
TMD matching status

Quark Collins–Soper kernel



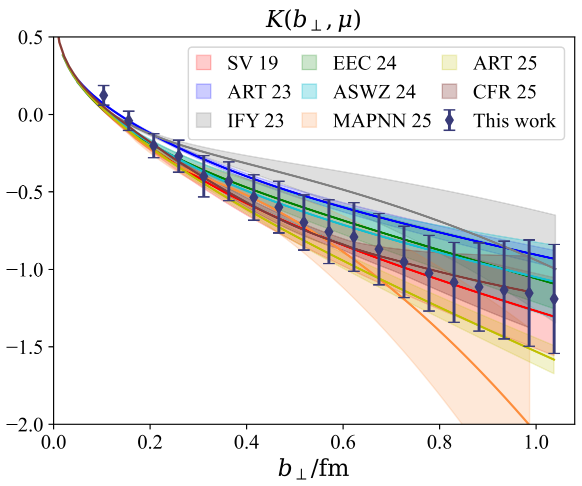
Lattice (continuum + physical pion mass): Tan et al. '25.
Other lattice: ASWZ24, LPC23, BGMZ24.
Phenomenology: ART23, MAP22, SV19.
NLO matching anchor: Deng, Wang, Zeng '22.

TMDPDF matchingO

- **NNLO:** del Rio, Vladimirov '23, quasi-TMDPDF at two loops in the Collins TMD scheme
  (see also Ji, Liu, Su '23 for the same NNLO coefficient via threshold resummation)
- **Factorisation framework:** Ebert, Schindler, Stewart, Zhao '22: all-orders quasi-TMDPDF factorisation

TMD matching status

Quark Collins–Soper kernel



Lattice (continuum + physical pion mass): Tan et al. '25.
Other lattice: ASWZ24, LPC23, BGMZ24.
Phenomenology: ART23, MAP22, SV19.
NLO matching anchor: Deng, Wang, Zeng '22.

TMDPDF matchingO

- **NNLO:** del Rio, Vladimirov '23, quasi-TMDPDF at two loops in the Collins TMD scheme
  (see also Ji, Liu, Su '23 for the same NNLO coefficient via threshold resummation)
- **Factorisation framework:** Ebert, Schindler, Stewart, Zhao '22: all-orders quasi-TMDPDF factorisation

Gluon TMDPDF at NNLO: does not exist yet

Coulomb gauge: the Wilson line is gone

Gauge-fixing eliminates the Wilson-line connector at the operator level.

Quasi-TMD operator in CG

$$\mathcal{O}_q^{\mathrm{TMD}}(b_T) = \bar{q}(b_T)\,\Gamma\,W_{\sqcap}^{n,\eta}(b_T,\,0)\,q(0)$$

$\downarrow$ Coulomb gauge: $\partial_i A_i = 0$

$$\mathcal{O}_q^{\mathrm{CG}}(b_T) = \bar{q}(b_T)\,\Gamma\,q(0)\,\Big|_{\partial_i A_i = 0}$$

Boost limit: CG $\to$ light-cone gauge $A^+ = 0$, where the Wilson line is trivial. Bare CG bilinear and staple share a universality class (Ji '13; Hatta–Ji–Zhao '14). No Wilson-line self-energy ⇒ no linear UV divergence.

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What's missing and critical:

- **One-loop CG gluon matching kernel**
- **Two-loop CG quark matching kernel**: non-covariant gluon propagator

Gradient flow: regulation via gauge field smearing

Flow time regulates the Wilson-line linear UV divergence at the operator level.

Quasi-TMD operator under gradient flow

$$\mathcal{O}_q^{\mathrm{TMD}}(b_T) = \bar{q}(b_T)\,\Gamma\, W_{\sqcap}^{n,\eta}(b_T,\,0)\, q(0)$$

$\downarrow$ gradient flow: $\dot{B}_\mu = D_\nu G_{\nu\mu}, \quad t > 0$

$$B_\mu^a(t,x) = \int d^4y\, K_t(x-y)\, A_\mu^a(y) + \mathcal{O}(g)$$

$$K_t(x) = \frac{1}{(4\pi t)^2}\, e^{-x^2/(4t)} \quad \Longleftrightarrow \quad \tilde{B}_\mu^a(t,p) = e^{-tp^2}\, \tilde{A}_\mu^a(p) + \mathcal{O}(g)$$

$$\mathcal{O}_q^{\mathrm{GF}}(t,\,b_T) = \bar{\chi}(b_T)\,\Gamma\, W_{\sqcap}^{(t)}\, \chi(0)$$

Leading order: Gaussian convolution at scale $\sqrt{8t}$.

The momentum-space factor $\exp(-tp^2)$ is the operator-level UV regulator

SFTX: $\mathcal{O}^{\mathrm{GF}}(t) = \sum_k c_k(t, \mu)\, \mathcal{O}_k(\mu)$: matching to unflowed operators with perturbative Wilson coefficients

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- Auxiliary heavy-field formalism for straight off-lightcone Wilson lines
  Brambilla, Wang '23
- UV renormalon analysis
  Zhang '25
- NNLO GF matching for non-singlet PDF moments
  Harlander, Kohnen, Shindler '25
- Multi-loop GF toolchain:
  Harlander, Neumann '16; Artz et al. '19

What's next

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Collinear PDF + TMD 2-loop matching in Coulomb-gauge and gradient-flow schemes

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Targeted immediate consumers: lattice PDF/TMD groups (BNL/ANL, LPC, MIT, HadStruc, ...)
Where it ultimately lands: HL-LHC precision: M_W and the $Z\,p_T$ spectrum (TMD side); high-mass Drell–Yan and dijet large-m_{jj} (large-x collinear side)