

Jet processes at higher perturbative orders

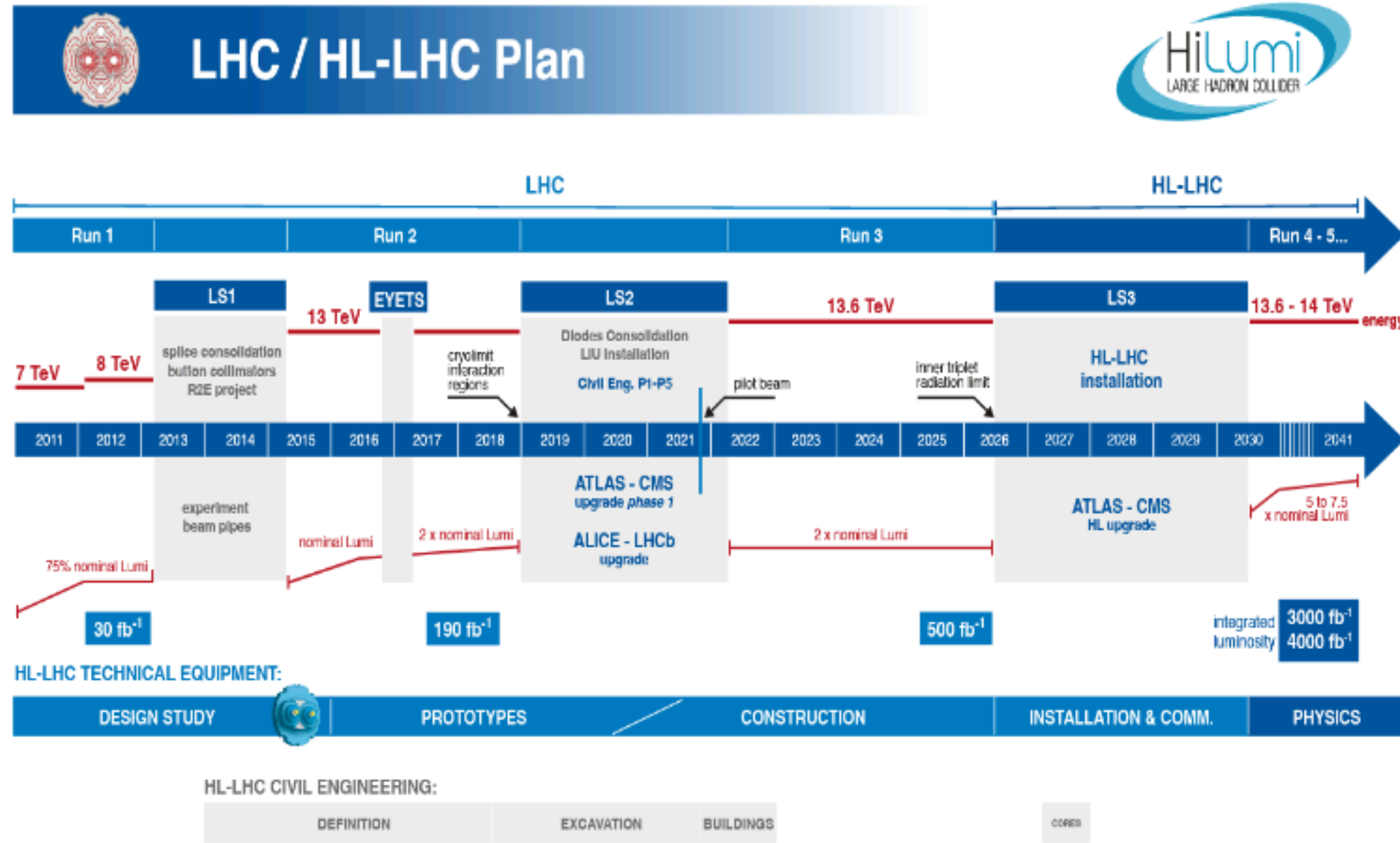
Giovanni Stagnitto



LoopFest 2026, BNL, May 28th

Many Challenges

[S. Dawson]

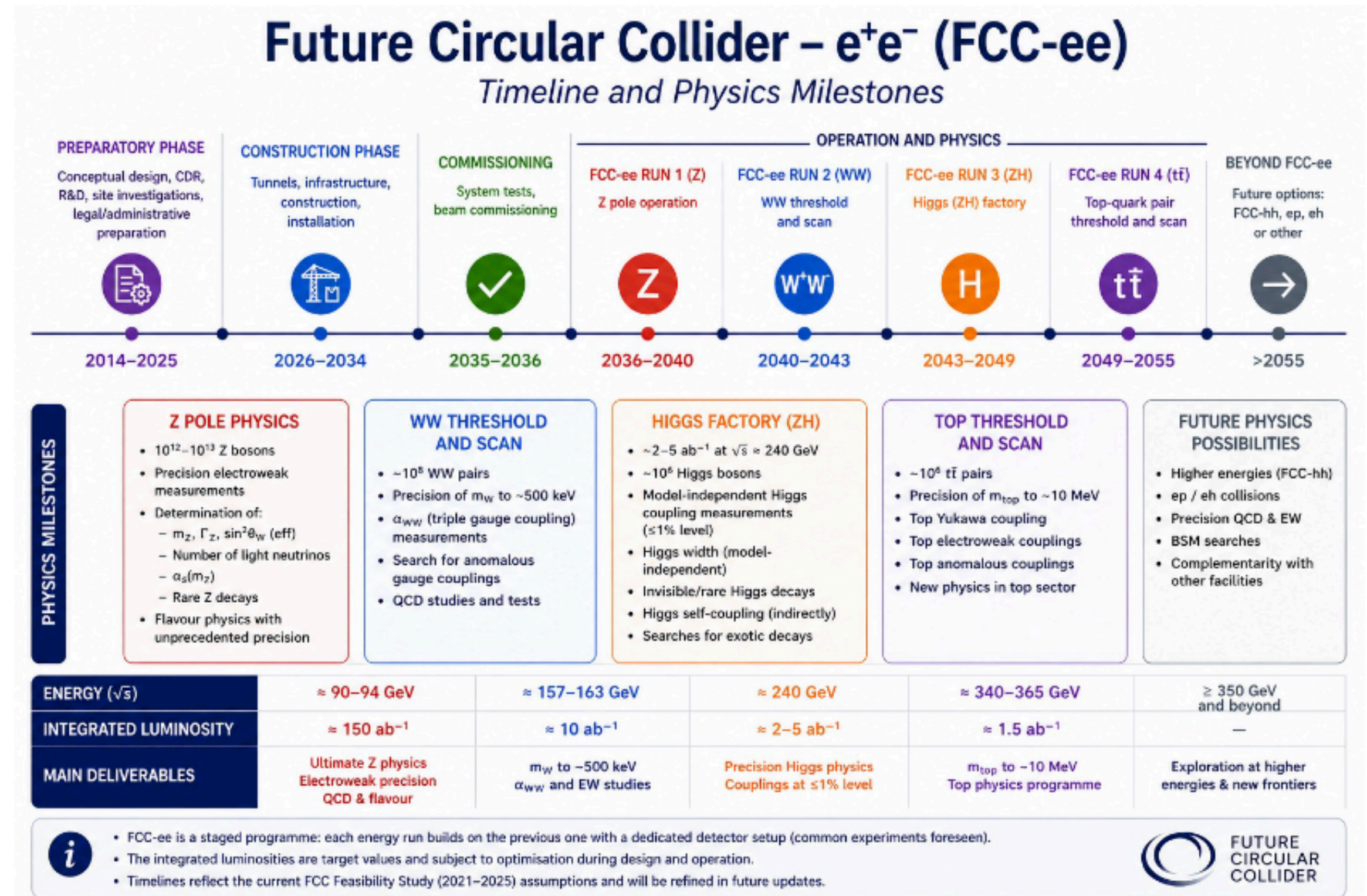


Luminosity!

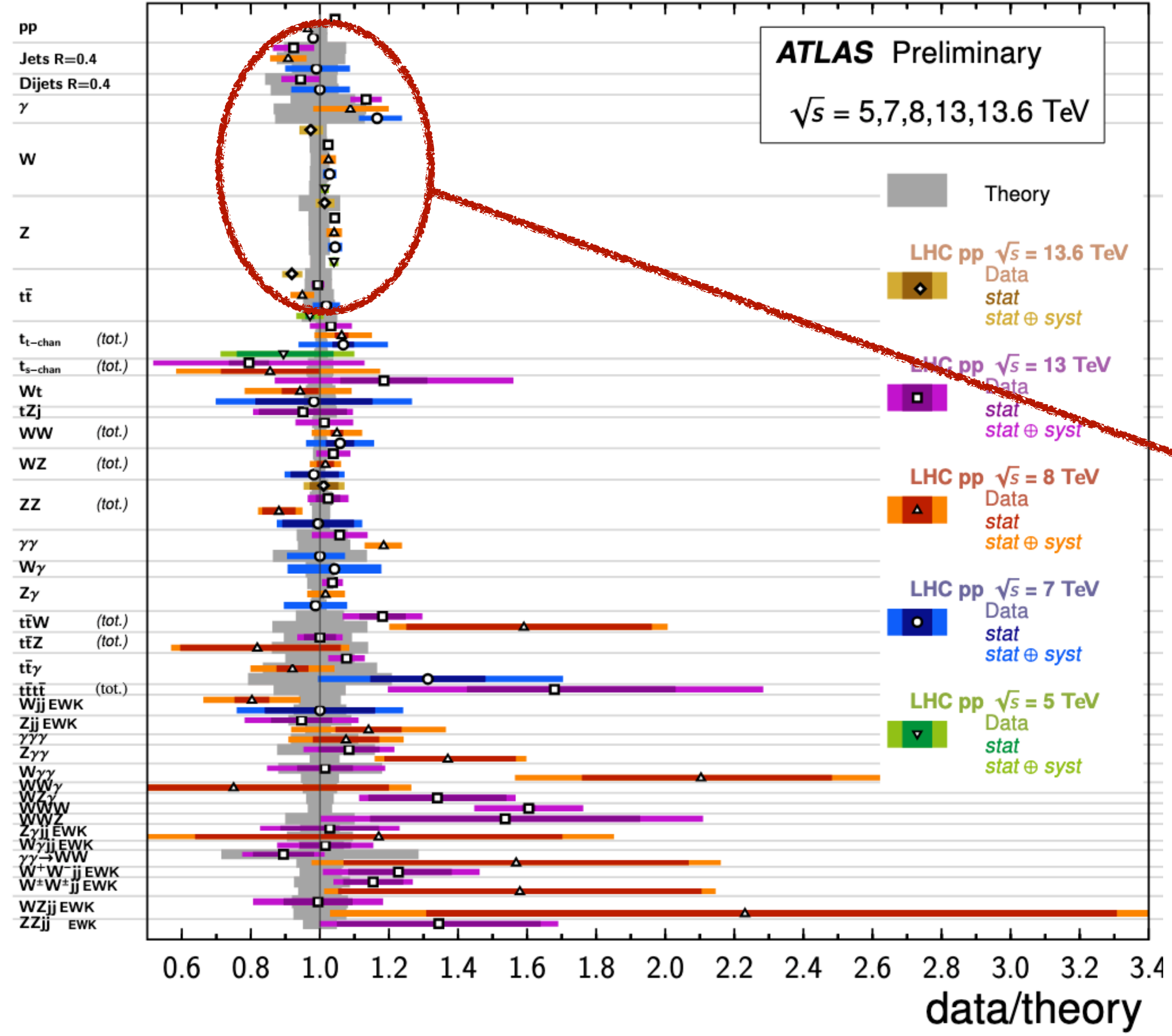
(HL-)LHC

Theory precision MUST match experimental precision

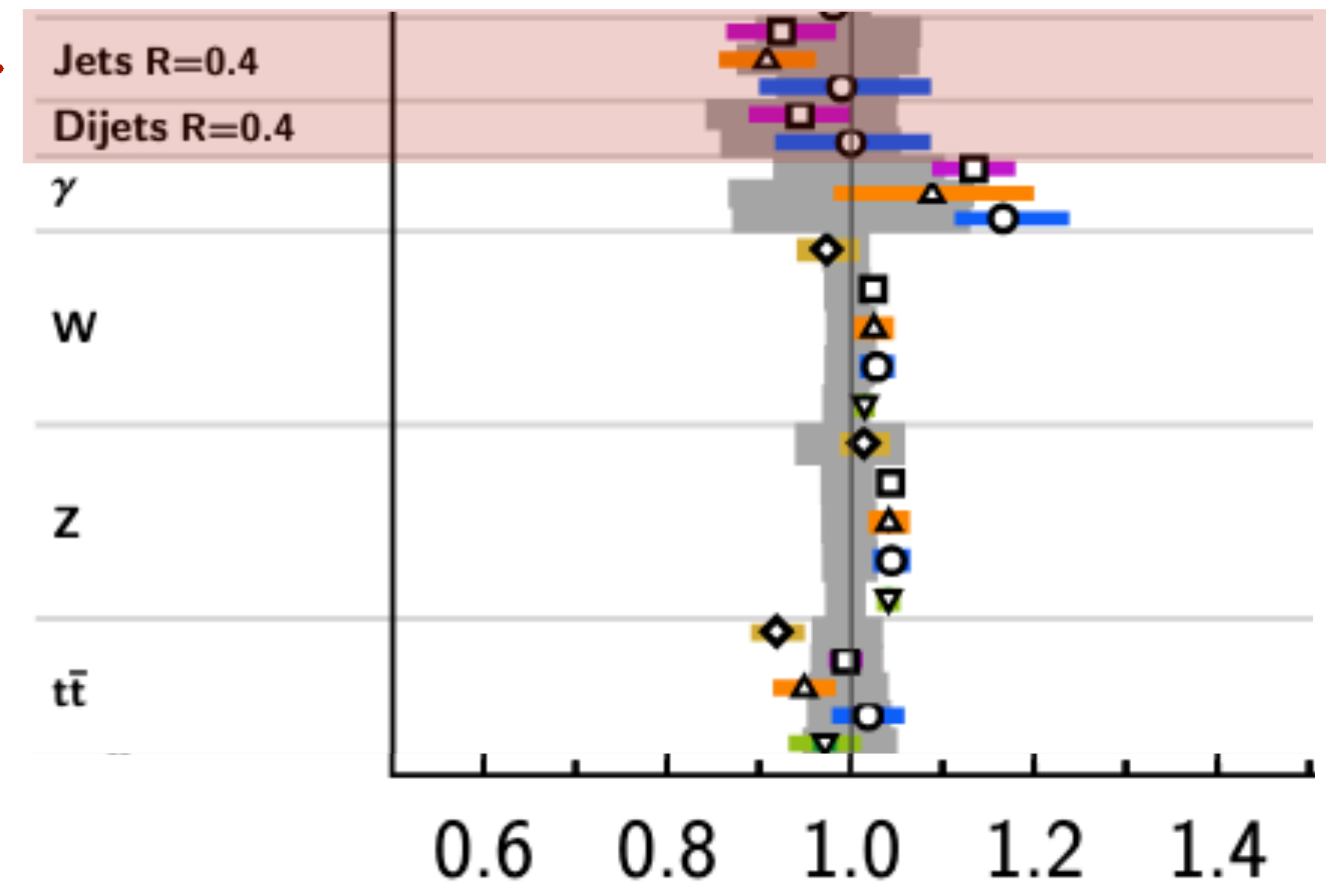
FCC-ee



Standard Model Production Cross Section Measurements Status: June 2024



(HL-)LHC



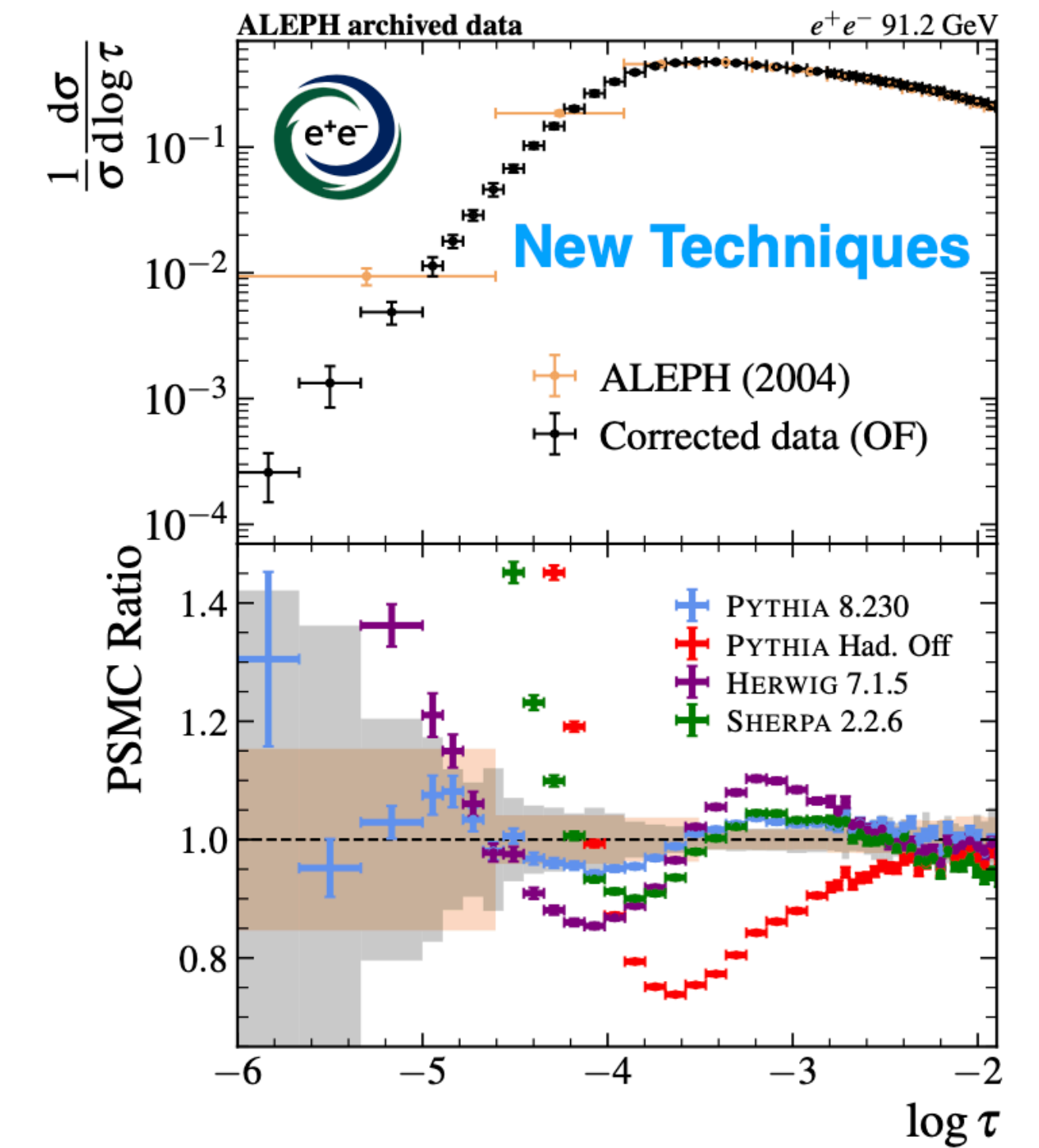
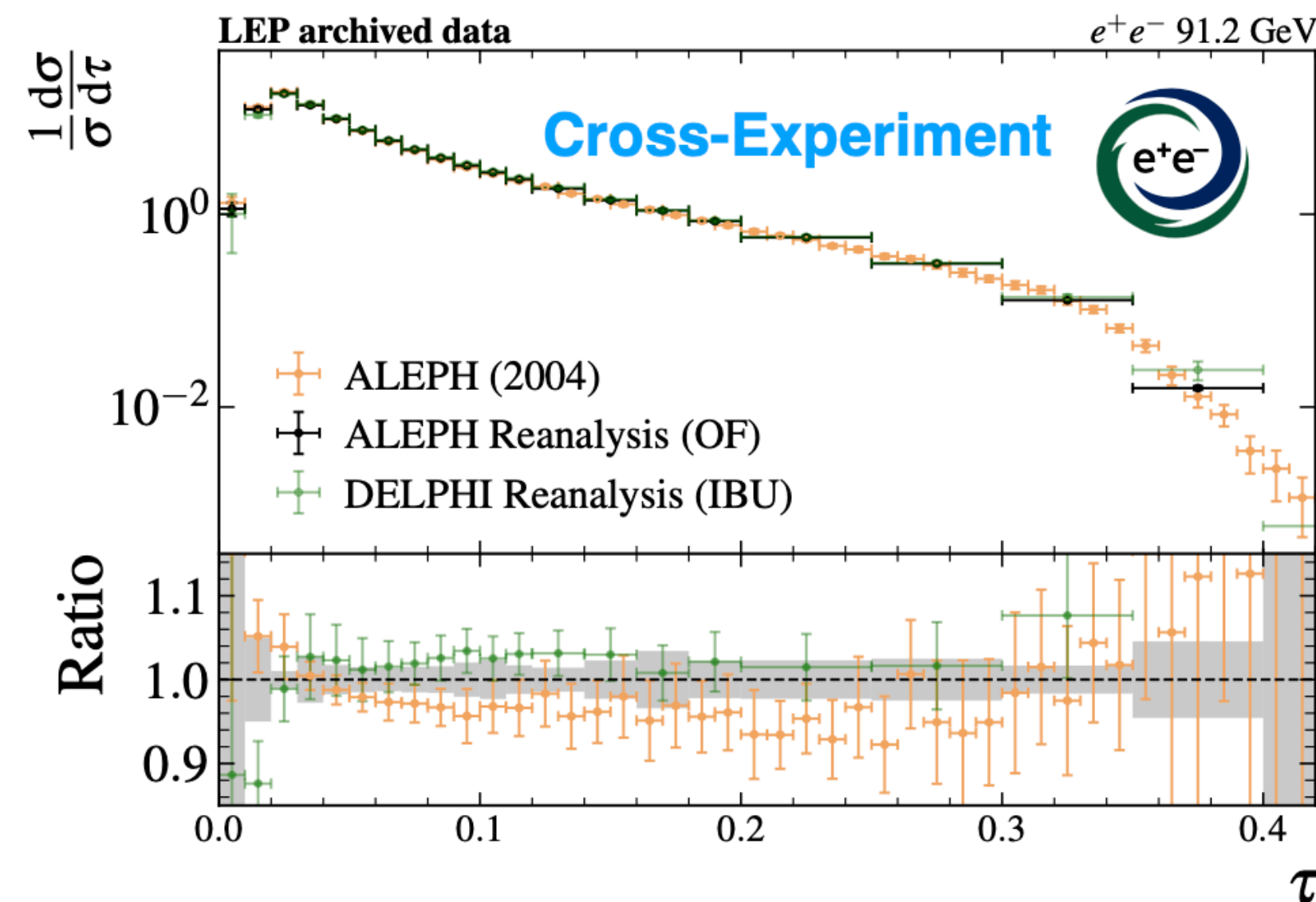
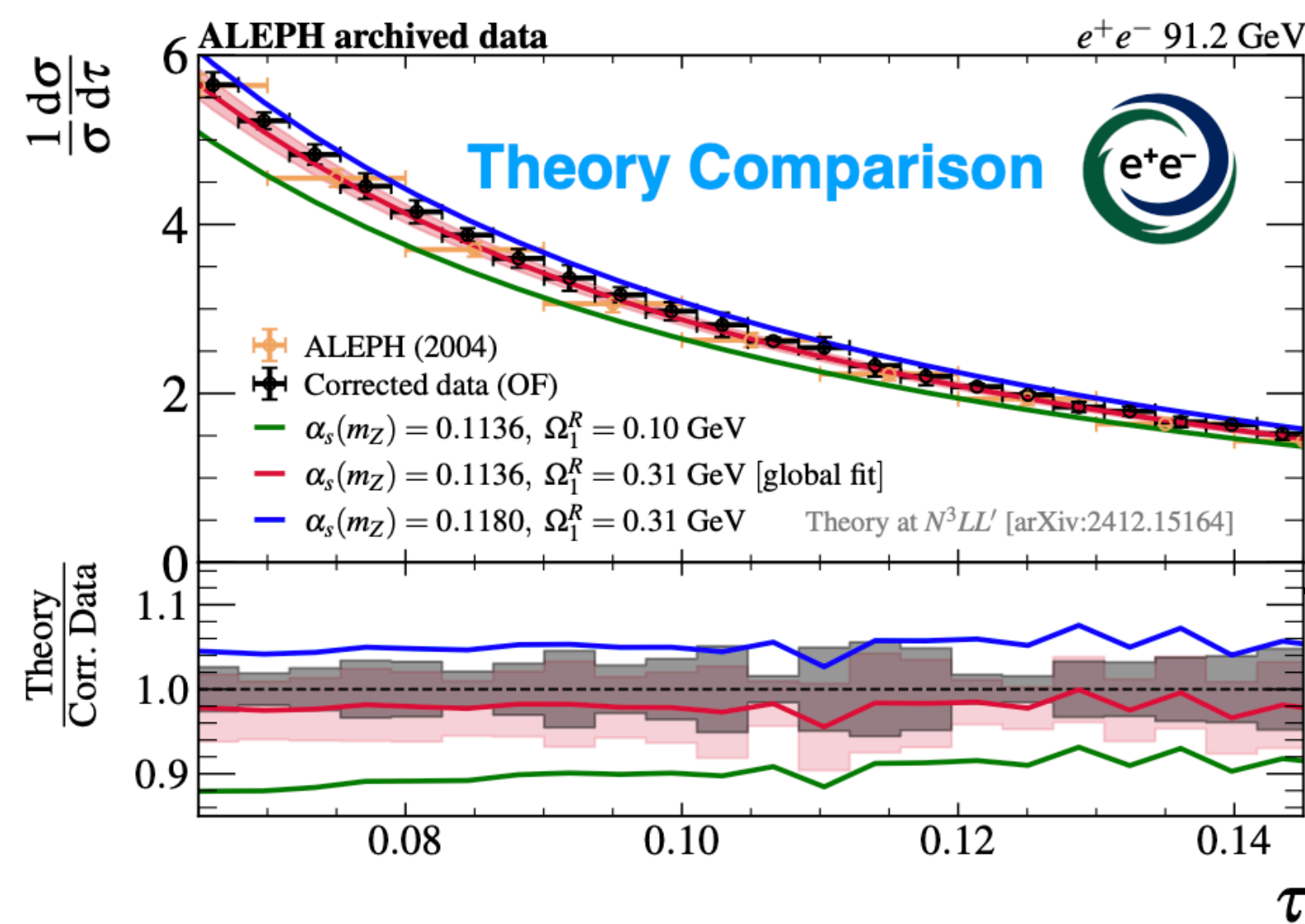
[FCC-ee Physics Briefing Book 2511.03883]

Table 4.2: Wish-list for calculations of missing higher-order perturbative QCD $\mathcal{O}(\alpha_s^n)$ and/or EW $\mathcal{O}(\alpha^n)$ to match the expected experimental uncertainty at future e^+e^- and ep colliders.

Observable	Missing higher-order & power-suppressed corrections
Hadronic Z width	$\mathcal{O}(\alpha_s^5)$, $\mathcal{O}(\alpha_s^6)$, $\mathcal{O}(\alpha^3)$, $\mathcal{O}(\alpha_s \alpha^3)$, $\mathcal{O}(\alpha_s^2 \alpha^2)$
Hadronic W width	$\mathcal{O}(\alpha_s^5)$, $\mathcal{O}(\alpha^2)$, $\mathcal{O}(\alpha_s^2 \alpha)$
Hadronic τ width	$\mathcal{O}(\alpha_s^5)$
Hadronic event shapes (Z, W, H decays)	N^3 LO differential, $N^{3,4}$ LL resummation, power corrections
Inclusive jet rates	3-jet cross-sections at N^3 LO, 4-jets at N^2 LO, 5-jets at NLO
Lattice QCD results (α_s extr.; quark masses m_c, m_b)	$\mathcal{O}(\alpha_s^6)$ β -function; $\mathcal{O}(\alpha_s^5)$ heavy quark decoupling; $\mathcal{O}(\alpha_s^4)$ static potential $\mathcal{O}(\alpha_s^3)$ lattice perturbation theory matching (lattice coupling to $\alpha_s^{\overline{MS}}$ etc.)
$\sigma(e^+e^- \rightarrow W^+W^-)$ vs. \sqrt{s}	EW N^2 LO: $\mathcal{O}(\alpha^2)$, Mixed EW-QCD: $\mathcal{O}(\alpha_s \alpha^2)$, $\mathcal{O}(\alpha_s^2 \alpha)$
$\sigma(e^+e^- \rightarrow t\bar{t})$ vs. \sqrt{s}	NRQCD: $\mathcal{O}(\alpha_s^5)$, Non-resonant: $\mathcal{O}(\alpha_s^5)$, $\mathcal{O}(\alpha_s^3)$ differential; QED: $\mathcal{O}(\alpha^3)$ at NNLL
$H \rightarrow b\bar{b}$ width	N^4 LO ($m_b \neq 0$); N^4 LO differential ($m_b = 0$)
$H \rightarrow gg$ width	N^5 LO (heavy-top limit), N^4 LO ($m_t \neq 0$); N^4 LO differential, N^3 LO differential ($m_t \neq 0$)
MC simulations for $e^+e^- \rightarrow X$ processes	$N^{2,3}$ LO matched to $N^{2,3}$ LL PS. Per mille control of non-pQCD effects (hadronization, CR, ...)
$ep \rightarrow$ hadrons (PDF and α_s determ.)	$N^{3,4}$ LO evolution equations and inclusive cross-sections
$ep \rightarrow$ jets (α_s determ.)	N^3 LO cross-sections

While waiting for the FCC-ee... Reanalysis of archived ALEPH data and DELPHI open data

e.g. thrust distribution



Join the *fcc-ped-physicsgroup-qcd* on
<https://groups-portal.web.cern.ch>

Conveners:

A. Badea, S. Kluth, R. Poncelet, GS

We plan to organise online topical meetings
and in-person workshops

Example of topics:

- fragmentation functions and track-based observables
- jets and their substructure at FCC-ee
- mass effects in QCD observables
- α_s extraction from event shapes



Outline

Phenomenology of jet processes at hadron colliders

Recent progresses in slicing techniques

Towards N³LO jet production at lepton colliders

*Disclaimer: not a comprehensive review, referencing is minimal
Just fixed-order in perturbative QCD*

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Recent progresses in slicing techniques

Towards N³LO jet production at lepton colliders

Single-jet and di-jet production @ NNLO

First results at leading colour
(antenna subtraction, NNLOJET)

[Currie, Gehrmann-De Ridder,
Gehrmann, Glover, Huss, Pires '17-'19]

Single-jet at full colour

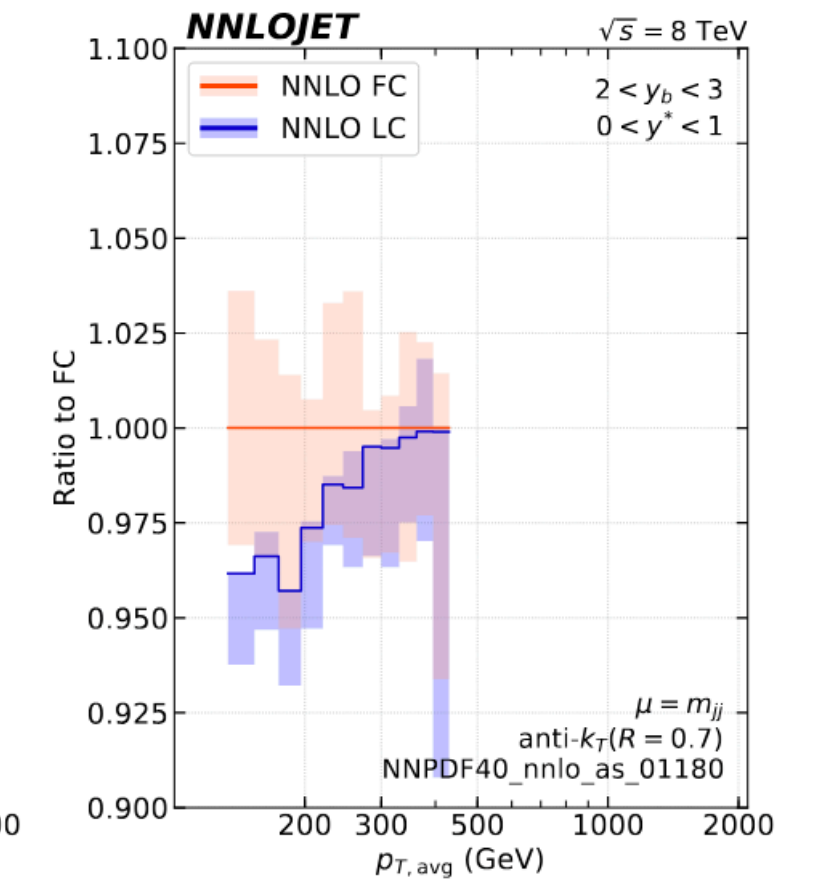
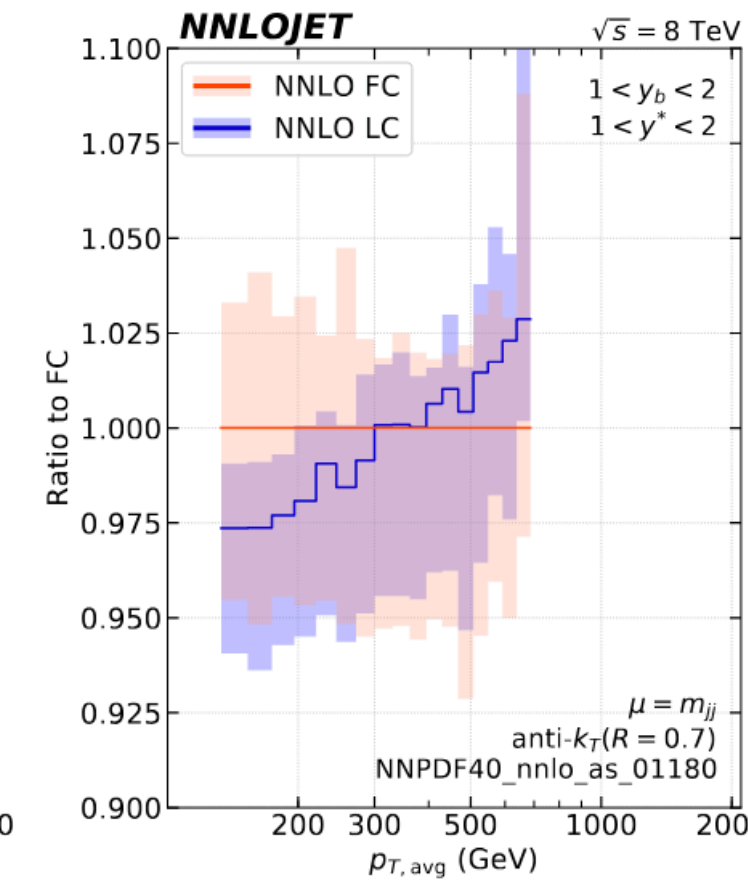
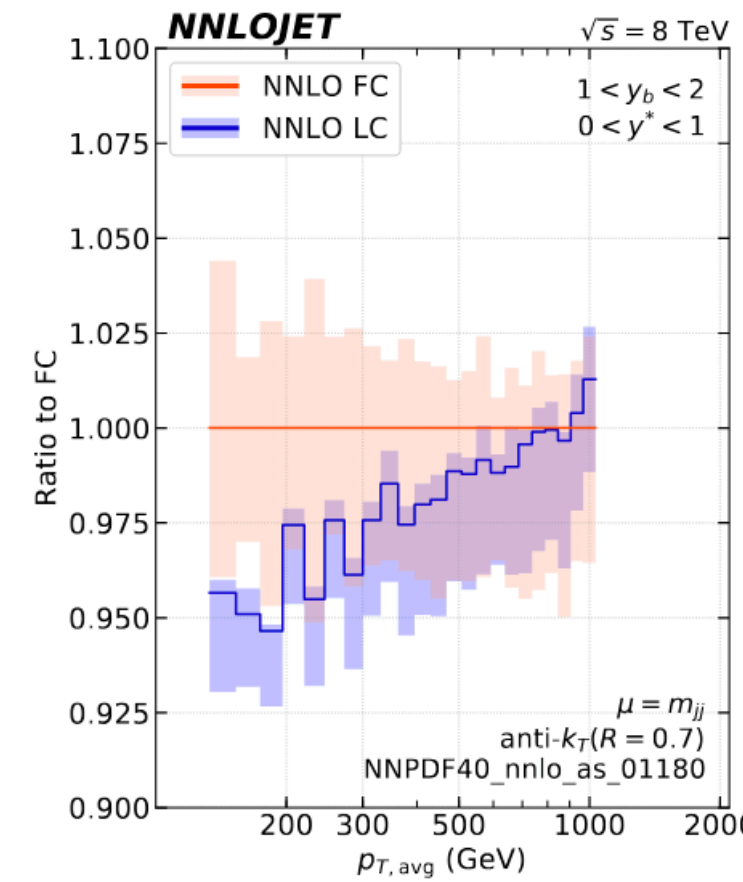
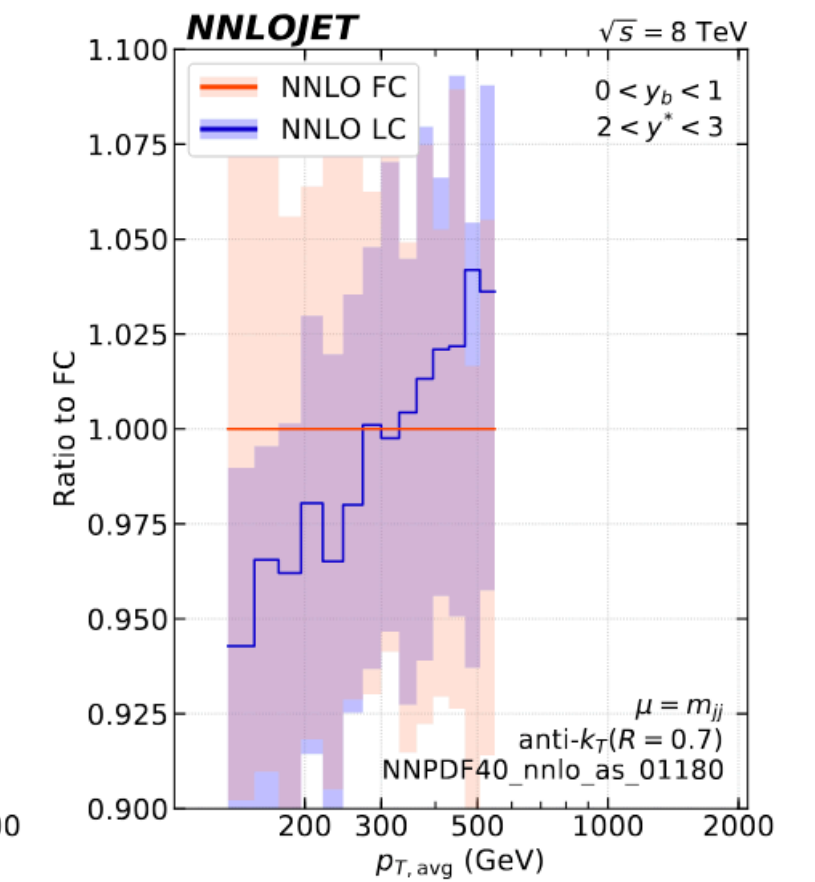
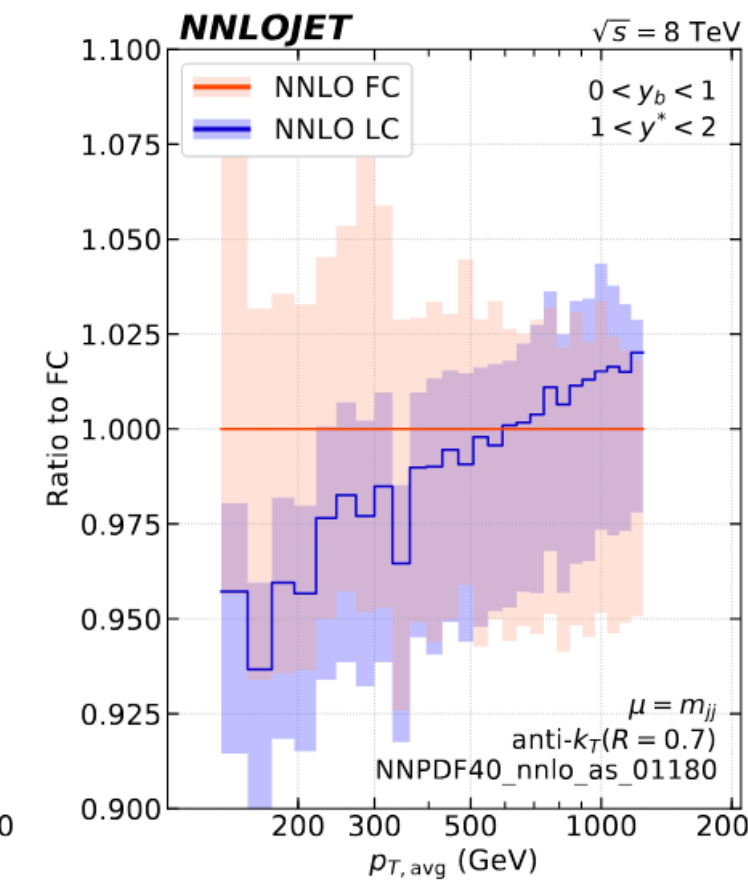
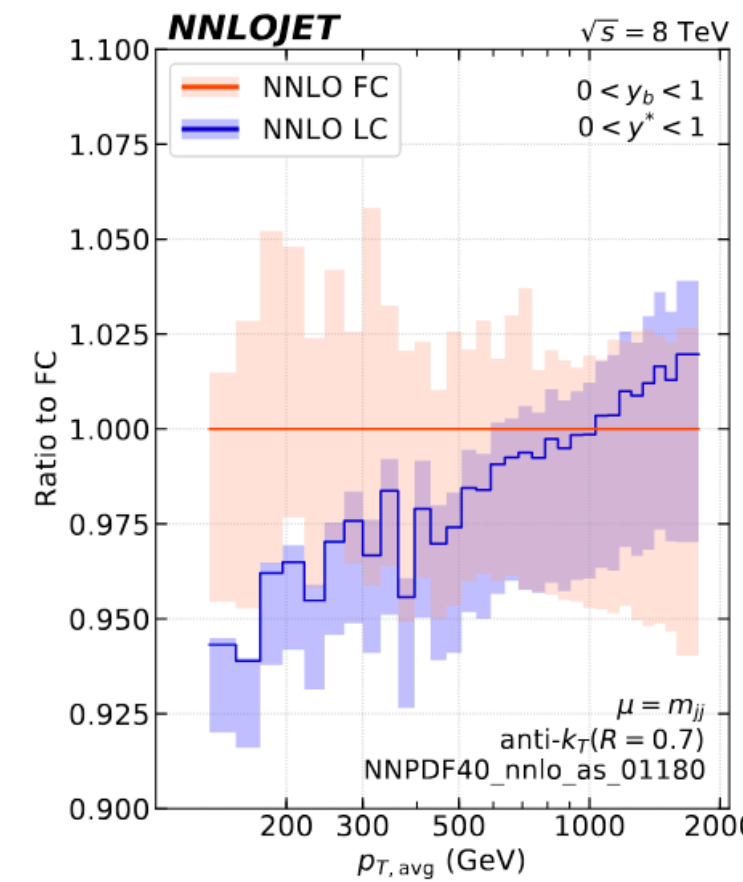
(sector-improved residue subtraction, STRIPPER)

[Czakon, van Hameren, Mitov, Poncelet '19]

Calculation in NNLOJET extended to full colour

[Chen, Gehrmann, Glover, Huss, Mo '22]

Subleading-colour effects particularly relevant for
dijet triple differential distributions, with non-trivial
kinematics dependence.



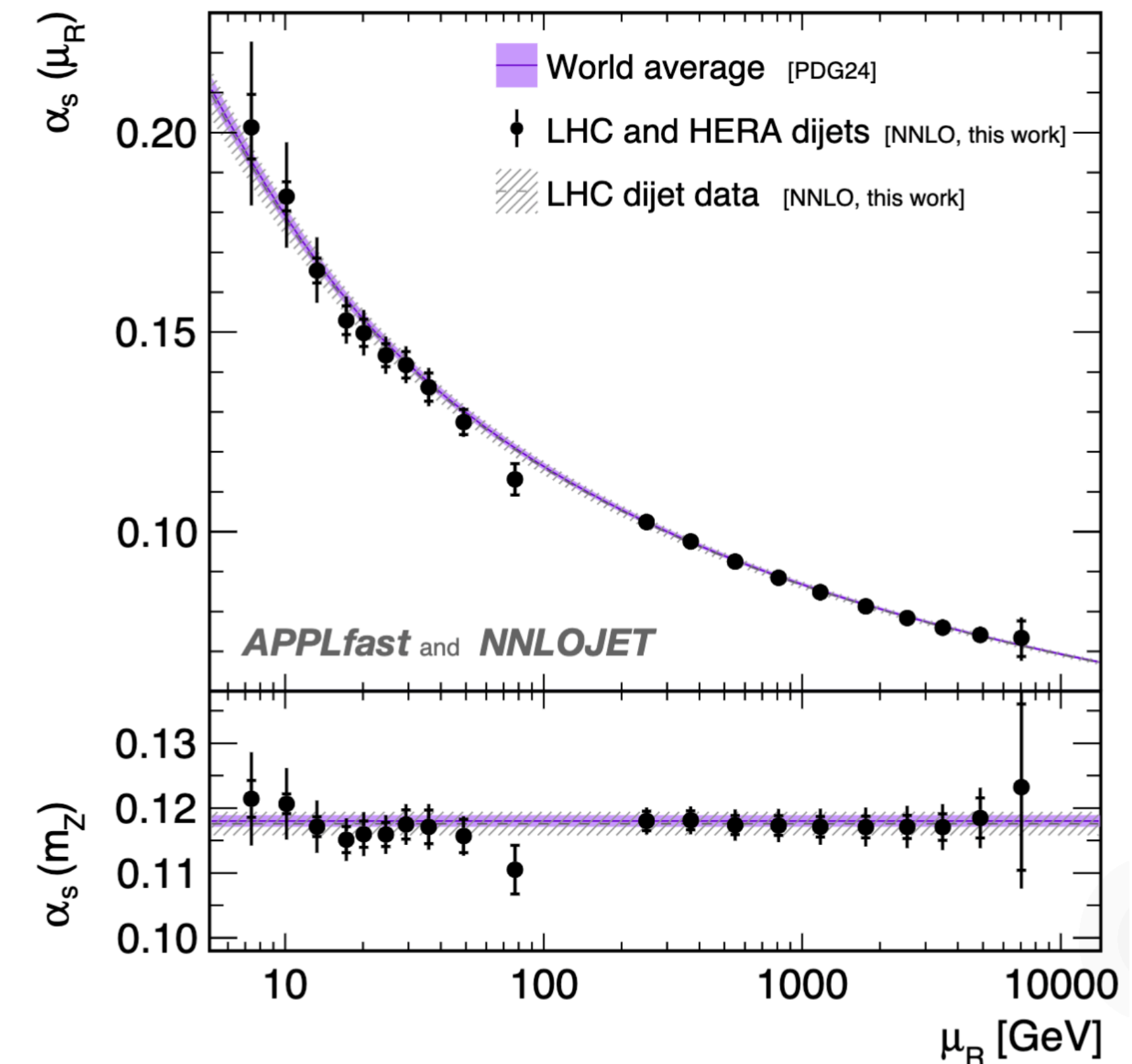
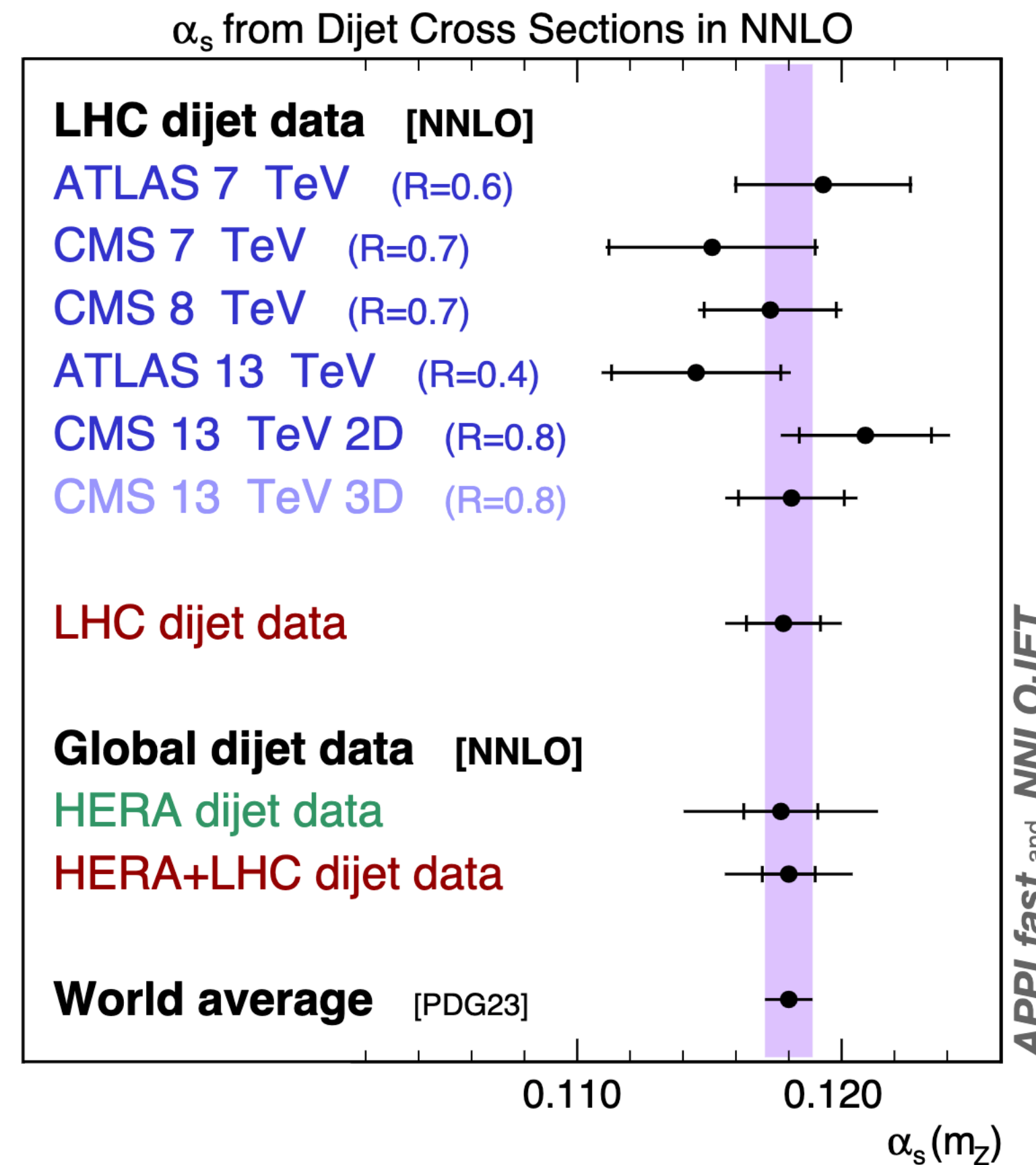
$$\frac{d\sigma}{dp_{T,avg} dy_b dy^*}$$

Grids and fit of α_s

Interfacing NNLOJET to interpolation grids has allowed a rich phenomenology
 e.g. recent fit of α_s with jet data from LHC and HERA

Data set	χ^2/n_{dof}	$\alpha_s(m_Z)$
ATLAS 7 TeV	74.7/ 77	0.1193 (33) (4) (6)
ATLAS 13 TeV	87.7/106	0.1145 (32) (4) (16)
CMS 7 TeV	50.7/ 45	0.1151 (39) (1) (9)
CMS 8 TeV	37.0/ 56	0.1173 (25) (1) (11)
CMS 13 TeV (2D)	71.6/ 78	0.1209 (25) (2) (20)
CMS 13 TeV (3D)	137.7/112	0.1181 (20) (1) (15)
LHC dijets (CMS13-2D)	335.3/366	0.1178 (14) (1) (17)
LHC dijets (CMS13-3D)	397.9/400	0.1172 (14) (1) (14)
HERA	92.8/118	0.1177 (14) (1) (34)
LHC+HERA (CMS13-2D)	428.4/485	0.1180 (10) (1) (22)
LHC+HERA (CMS13-3D)	491.0/519	0.1177 (10) (1) (24)

$$\alpha_s(m_Z) = 0.1180 (10)_{(\text{fit,PDF})} (1)_{(\mu_0)} (22)_{(\mu_R, \mu_F)}$$





NNLOJET is public!
<https://nnlojet.hepforge.org/>



Please send comments, questions and suggestions to nnlojet-support@cern.ch

```

PROCESS GG
  collider = LHC
  sqrts = 13000.0
  photon_isolation = frix[R=0.1, ETmax_epsilon=0.15, n=1] + etsum[R=0.2, ETmax_epsilon=0.09]
  decay_type = 0
END_PROCESS

RUN ATLAS_GG
  PDF = NNPDF30_nnlo_as_0118[0]
  tcut = 1d-9
  reset_vegas_grid = .true.
  angular_average = .false.
  multi_channel = .false.
  lips_reduce = .true.
  warmup = 10000[10]
  production = 10000[10]
END_RUN

PARAMETERS
  SCHEME[GV] = CMS
  SCHEME[alpha] = Gmu
  hard_photon_alpha0 = .true.
END_PARAMETERS

SELECTORS
  select dr_g1g2 min = 0.4 ! Angular separations for photons
  select ptg1 min = 40
  select ptg2 min = 30 ! Leading- and sub-leading-pt cuts for gammas.
  select photons_abs_y max = 2.37
  reject photons_abs_y min = 1.37 max = 1.52 ! The upper bound for nontrivial cut
END_SELECTORS

```

#	LO	R	V	RR	RV	VV
1	PRD A[0] D[1]	PRD A[0] D[1]	PRD A[0] D[1]	WRM A[0] D[4]	PRD A[1] D[0]	PRD A[0] D[1]
2	PRD A[0] D[1]	PRD A[1] D[0]	PRD A[0] D[1]	PRD A[0] D[1]	WRM A[1] D[3]	PRD A[0] D[1]
3	-	PRD A[0] D[1]	PRD A[0] D[1]	PRD A[0] D[0]	WRM A[1] D[3]	PRD A[0] D[1]
4	-	PRD A[0] D[1]	PRD A[0] D[1]	PRD A[0] D[1]	PRD A[0] D[1]	PRD A[0] D[1]
5	-	PRD A[0] D[1]	PRD A[0] D[1]	PRD A[0] D[0]	PRD A[0] D[1]	PRD A[0] D[1]
6	-	PRD A[0] D[0]	PRD A[0] D[1]	PRD A[0] D[1]	PRD A[0] D[1]	PRD A[0] D[1]
7	-	-	-	PRD A[0] D[0]	PRD A[0] D[1]	PRD A[0] D[1]
8	-	-	-	PRD A[1] D[0]	PRD A[0] D[1]	PRD A[0] D[1]
9	-	-	-	PRD A[0] D[0]	PRD A[1] D[0]	PRD A[0] D[1]
10	-	-	-	PRD A[1] D[0]	PRD A[0] D[1]	PRD A[0] D[1]
11	-	-	-	PRD A[0] D[0]	PRD A[0] D[1]	PRD A[0] D[1]
12	-	-	-	PRD A[0] D[1]	-	-
13	-	-	-	PRD A[0] D[0]	PRD A[0] D[1]	PRD A[0] D[1]
14	-	-	-	PRD A[0] D[0]	-	-
15	-	-	-	PRD A[0] D[1]	WRM A[1] D[3]	PRD A[0] D[1]
16	-	-	-	PRD A[1] D[0]	WRM A[1] D[3]	PRD A[0] D[1]
17	-	-	-	PRD A[0] D[0]	WRM A[1] D[3]	PRD A[0] D[1]
18	-	-	-	PRD A[1] D[0]	-	-
19	-	-	-	PRD A[0] D[1]	-	-
20	-	-	-	PRD A[0] D[1]	-	-
21	-	-	-	WRM A[1] D[3]	-	PRD A[0] D[1]
22	-	-	-	PRD A[0] D[1]	-	PRD A[0] D[1]
23	-	-	-	PRD A[1] D[0]	-	PRD A[0] D[1]
24	-	-	-	WRM A[1] D[3]	-	PRD A[0] D[1]
25	-	-	-	WRM A[1] D[3]	-	-
26	-	-	-	PRD A[1] D[0]	-	-
27	-	-	-	PRD A[0] D[1]	-	-

In first release: jets at $e^+e^-/ep/pp$ colliders; Drell-Yan, $V+j$, $H+j$, $\gamma+j$, $\gamma\gamma$ at the LHC

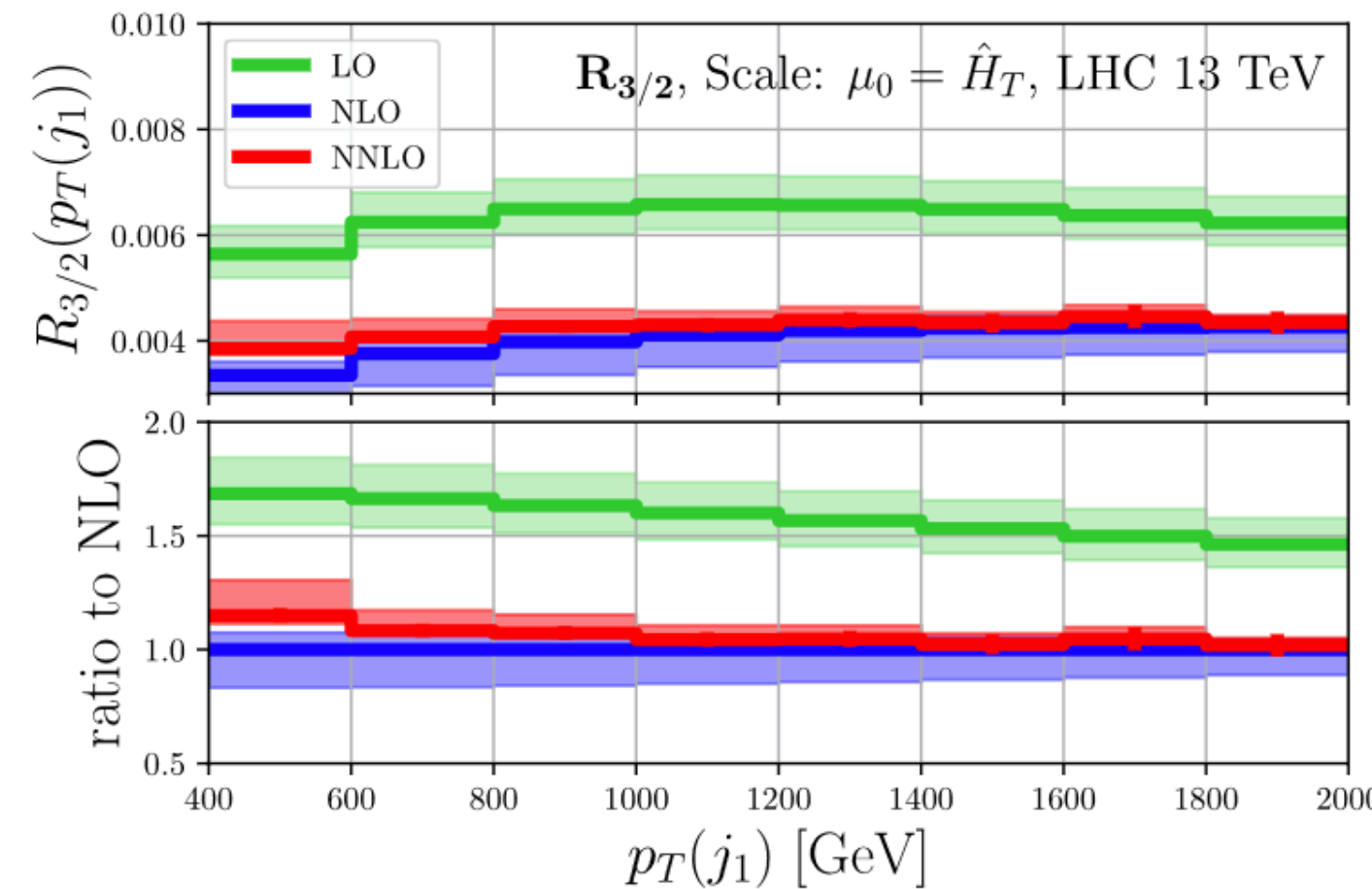
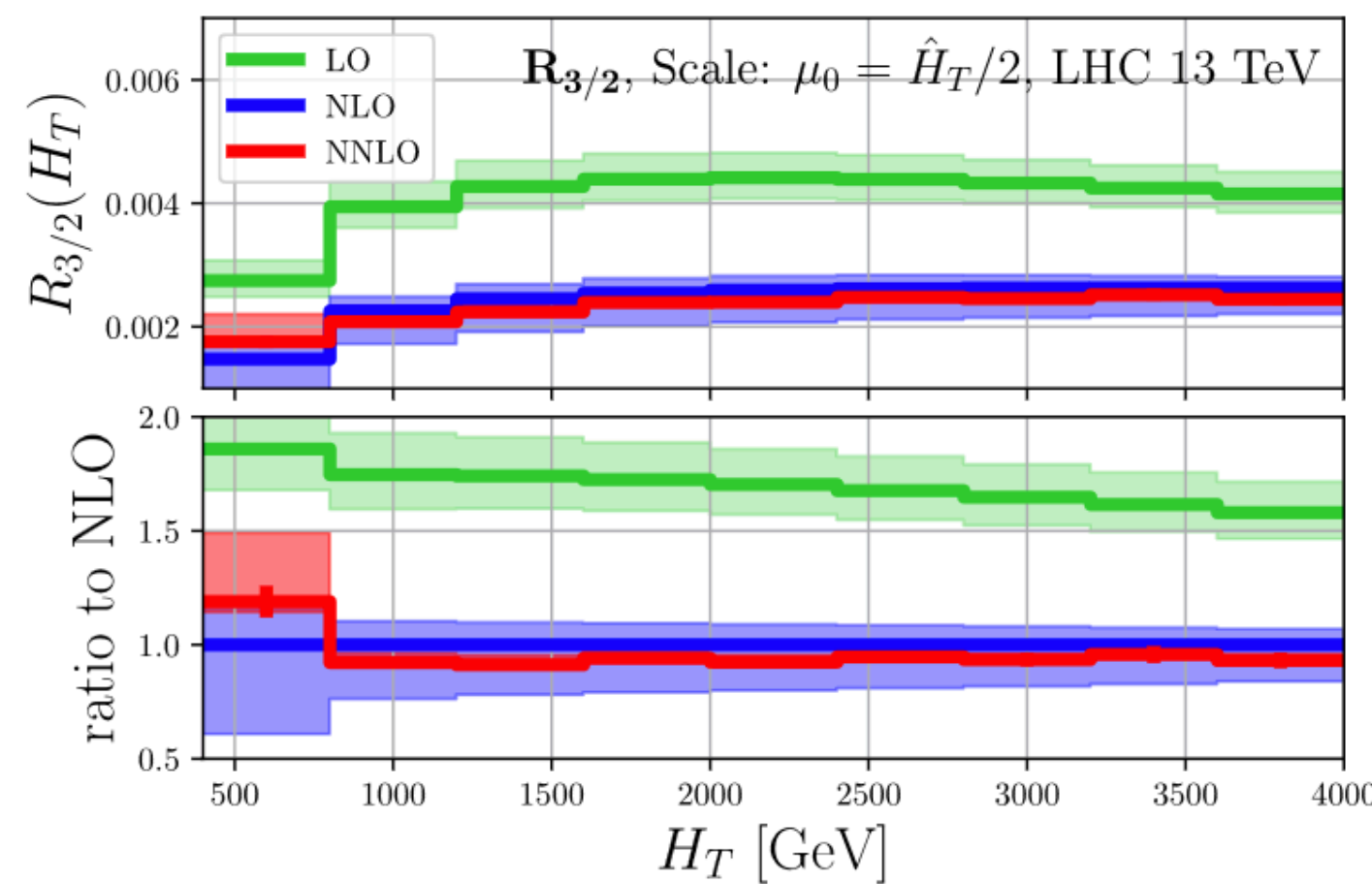
Next major release: more processes, flavour tagging, APPLfast/PineAPPL grid support

Three-jet production @ NNLO

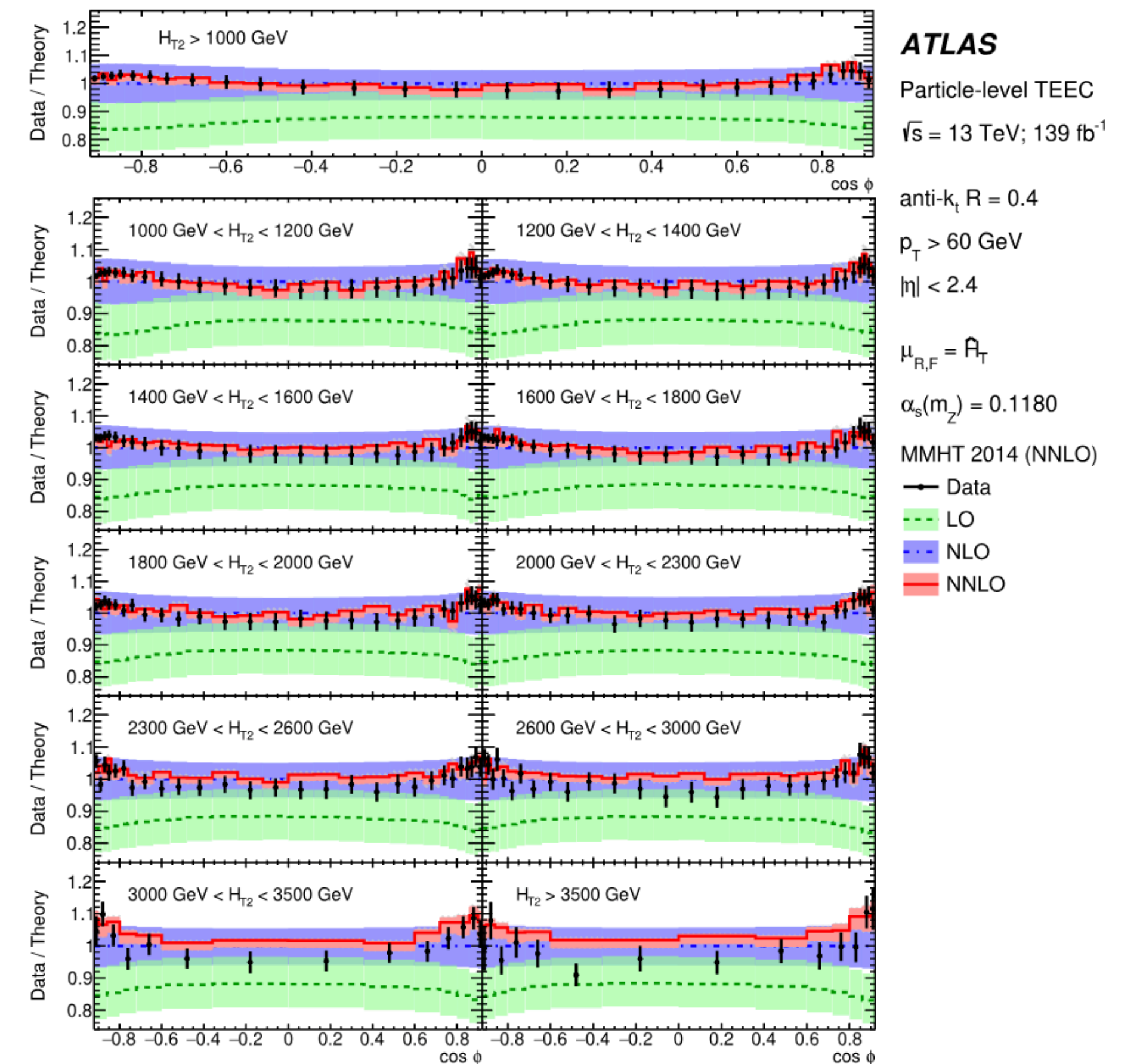
Impressive calculation with STRIPPER

[Czakon, Mitov, Poncelet '21]

With two-loop finite reminder at leading-colour



$$\frac{1}{\sigma} \frac{d\Sigma}{d \cos \phi} \equiv \frac{1}{\sigma} \sum_{ij} \int \frac{d\sigma}{dx_{T_i} dx_{T_j} d \cos \phi} x_{T_i} x_{T_j} dx_{T_i} dx_{T_j}$$



Measurement of event-shapes in three-jet events

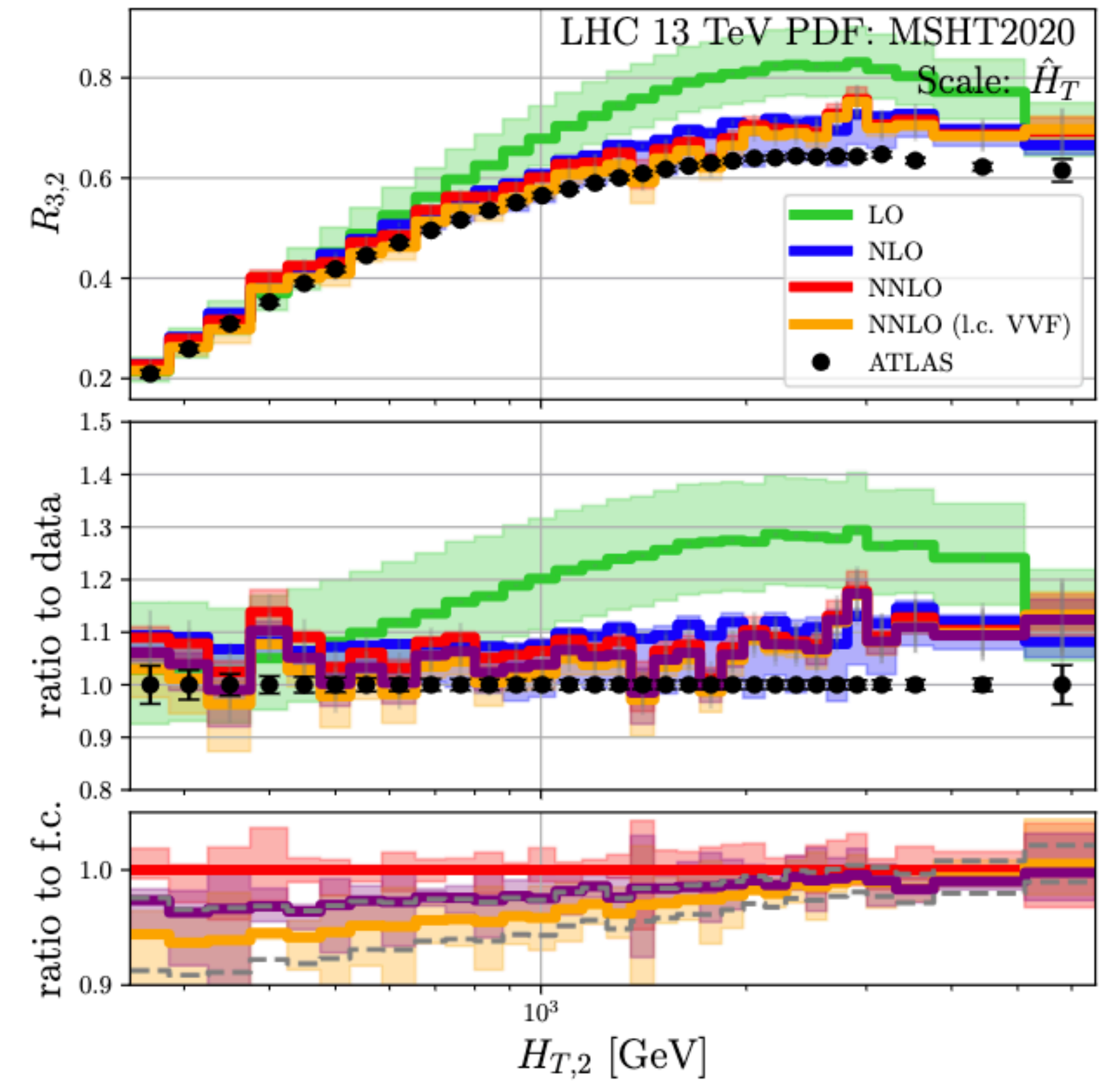
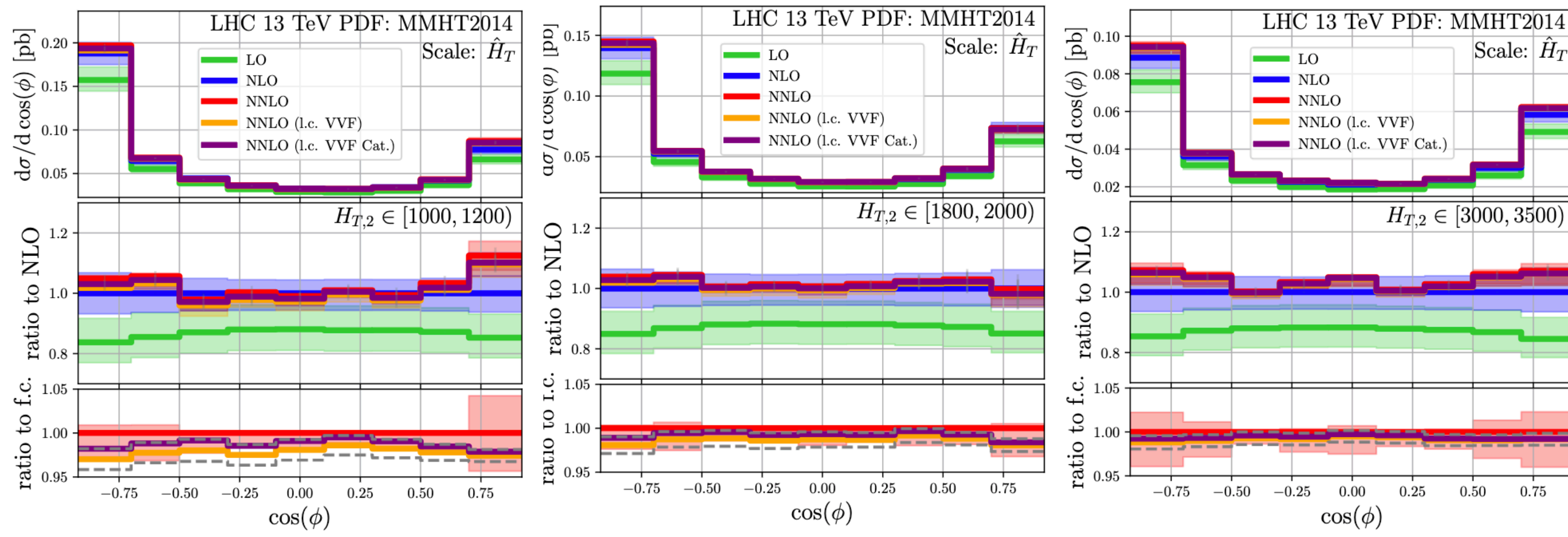
allows for an extraction of α_s [ATLAS '23]

[Alvarez, Cantero, Czakon, Lorente, Mitov, Poncelet '23]

$$\alpha_s(m_Z) = 0.1175 \pm 0.0006(\text{exp.})_{-0.0017}^{+0.0034}(\text{theo.})$$

Impact of subleading-colour contributions

Detailed study in [Czakon, Poncelet '25] after inclusion of full-colour results from
 [De Laurentis, Ita, Klinkert, Sotnikov '23]
 [Agarwal, Buccioni, Devoto, Gambuti, von Manteuffel, Tancredi '23]



- Small effect for TEEC (up to 2%)
 with a cut on $H_{T,2} = p_{T,1} + p_{T,2} > 1$ TeV
- Larger effect (up to 5%) for ratio 3/2 jets
 probing a larger range of $H_{T,2}$ values

Recent developments in other subtraction schemes

- Local Analytic Sector Subtraction (LASS) [Bertolotti, Magnea, Pelliccioli, Signorile-Signorile, Torrielli, Uccirati '18-...]: general formulation at NNLO with massless final-state particles, initial-state + masses at NLO. Numerical implementation of NNLO in progress by [Chargeishvili, Bevilacqua, Kardos, Moch, Trócsányi '24]
- NNLOLOCAL code implementing CoLoRFulNNLO for color-singlet processes [Del Duca, Duhr, Fekeshazy, Guadagni, Mukherjee, G. Somogyi, Tramontano, Van Thurenhout '24]
- Nested soft-collinear [Caola, Melnikov, Röntsch '17-...]: implementation in the history code [Klein, Simon '26] for colour-singlet processes. Reformulation of the method in [Devoto, Melnikov, Röntsch, Signorile-Signorile, Tagliabue, Tresoldi '24-26]: general formulation at NNLO at hadron colliders

→ see this afternoon's parallel session

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Recent progresses in slicing techniques

Towards N³LO jet production at lepton colliders

N -jettiness for slicing...

Introduced in [Stewart, Tackmann, Waalewijn '10]

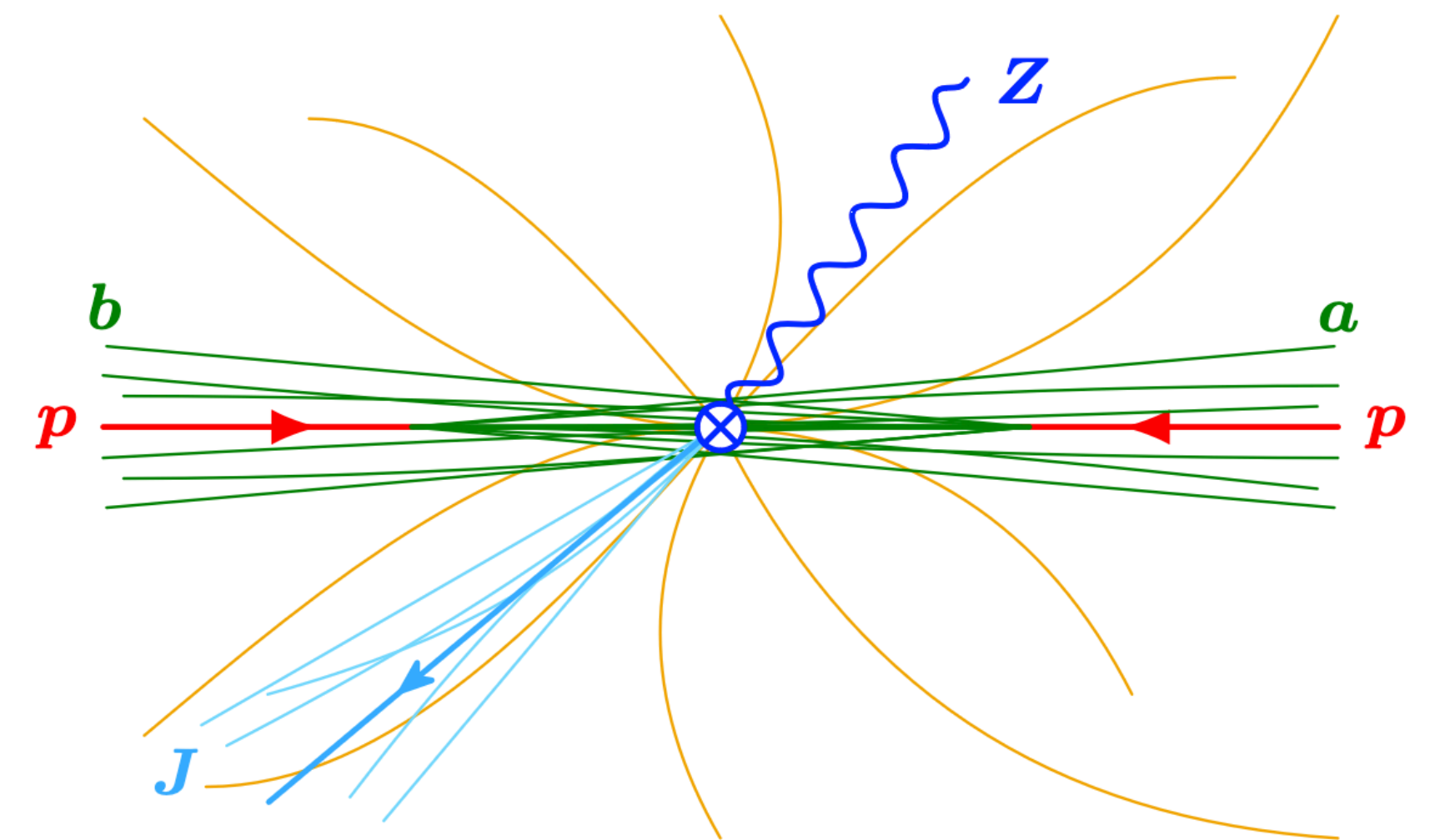
1-jettiness \mathcal{T}_1 used in first seminal NNLO calculations of V/H+jet [Boughezal, Focke, Giele, Liu, Petriello '15]

Nasty ingredient of factorization formula is the soft function:

now available NNLO evaluations for any \mathcal{T}_N

[Bell, Dehnadi, Mohrmann, Rahn '23] [Agarwal, Melnikov, Pedron '24]

$$\mathcal{T}_1 = \sum_k \min \left\{ \frac{2q_a \cdot k}{Q_a}, \frac{2q_b \cdot k}{Q_b}, \frac{2q_J \cdot k}{Q_J} \right\}$$

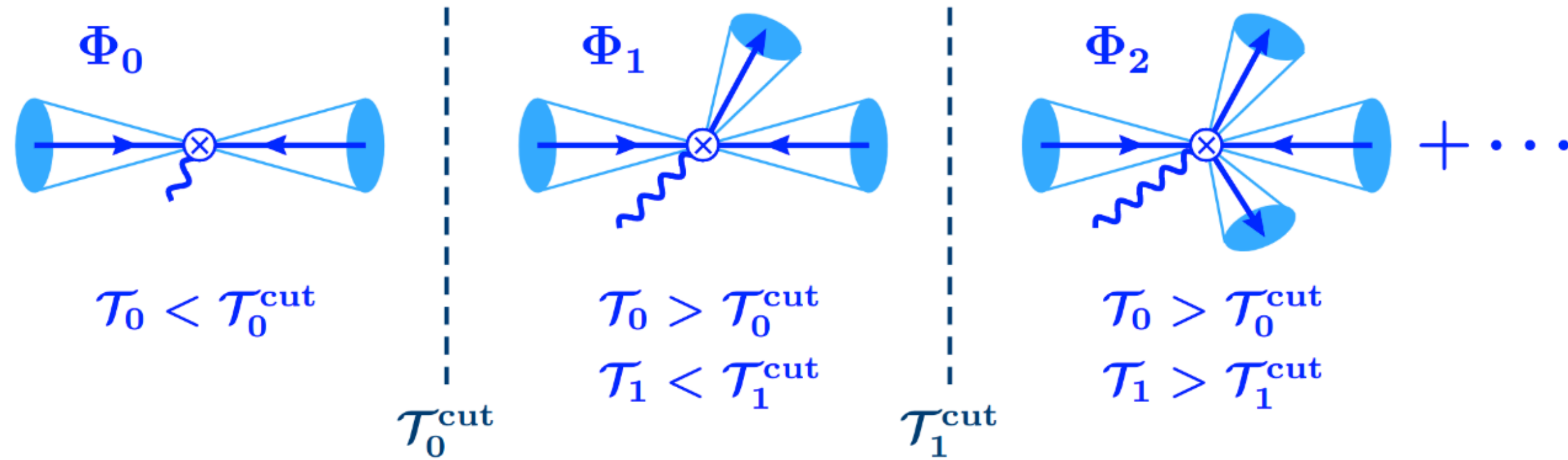


Recently, new method based on separation of contributions, particularly suited to N3LO extension [Buonocore, Delto, Melnikov, Monni, Pikelner, Vita '26]

Dipole contribution as sum of an analytically calculable inclusive soft function and a remainder which is absent at NLO, finite at NNLO and requires NLO-like subtraction at N3LO.

$$\tilde{\mathcal{S}}_{\mathcal{T}_N, \{i,j\}}^{(n)} = \tilde{\mathcal{S}}_{\mathcal{T}_N^{\text{inc.}}, \{i,j\}}^{(n)} + \Delta \tilde{\mathcal{S}}_{\mathcal{T}_N, \{i,j\}}^{(n)}$$

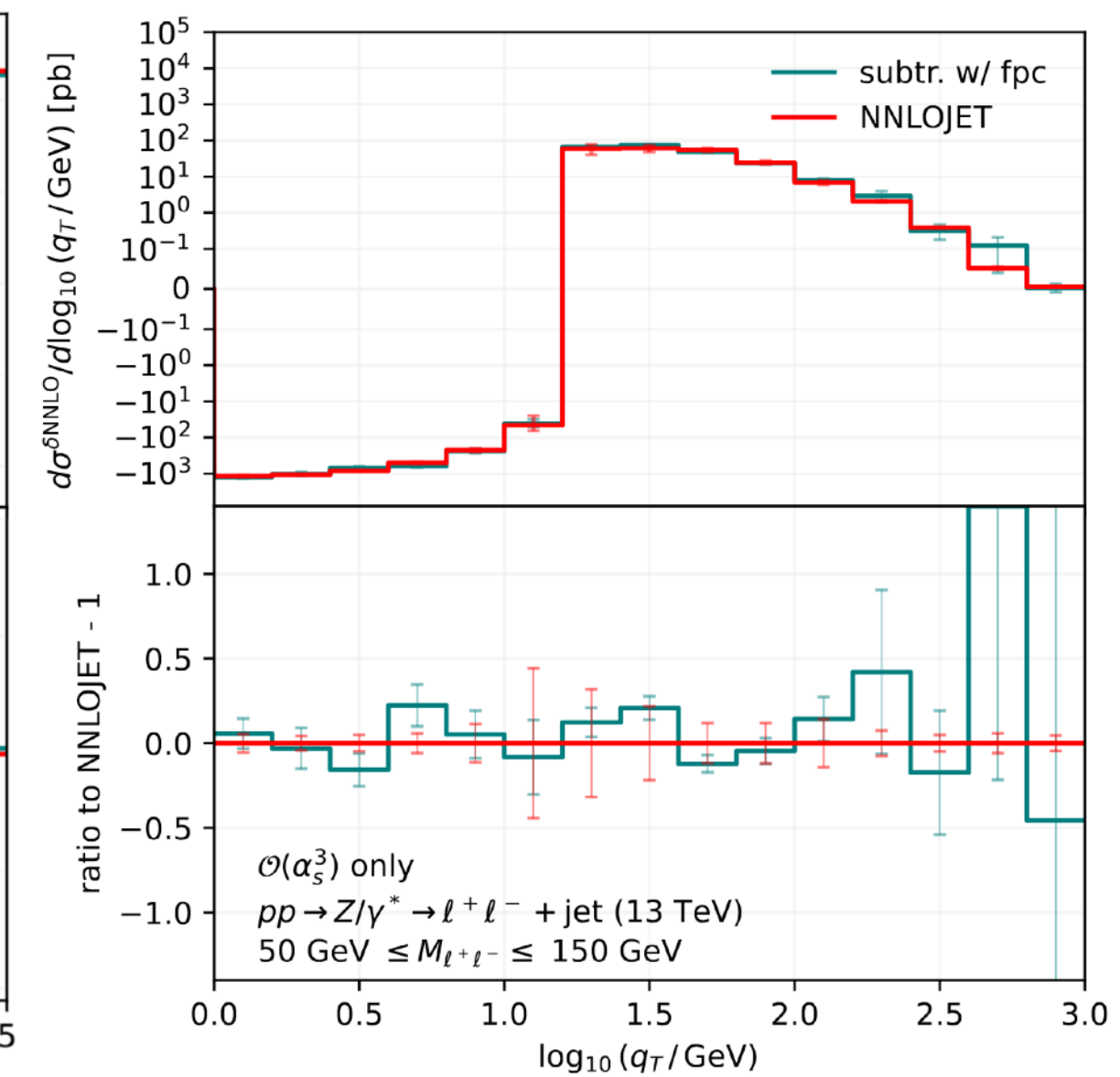
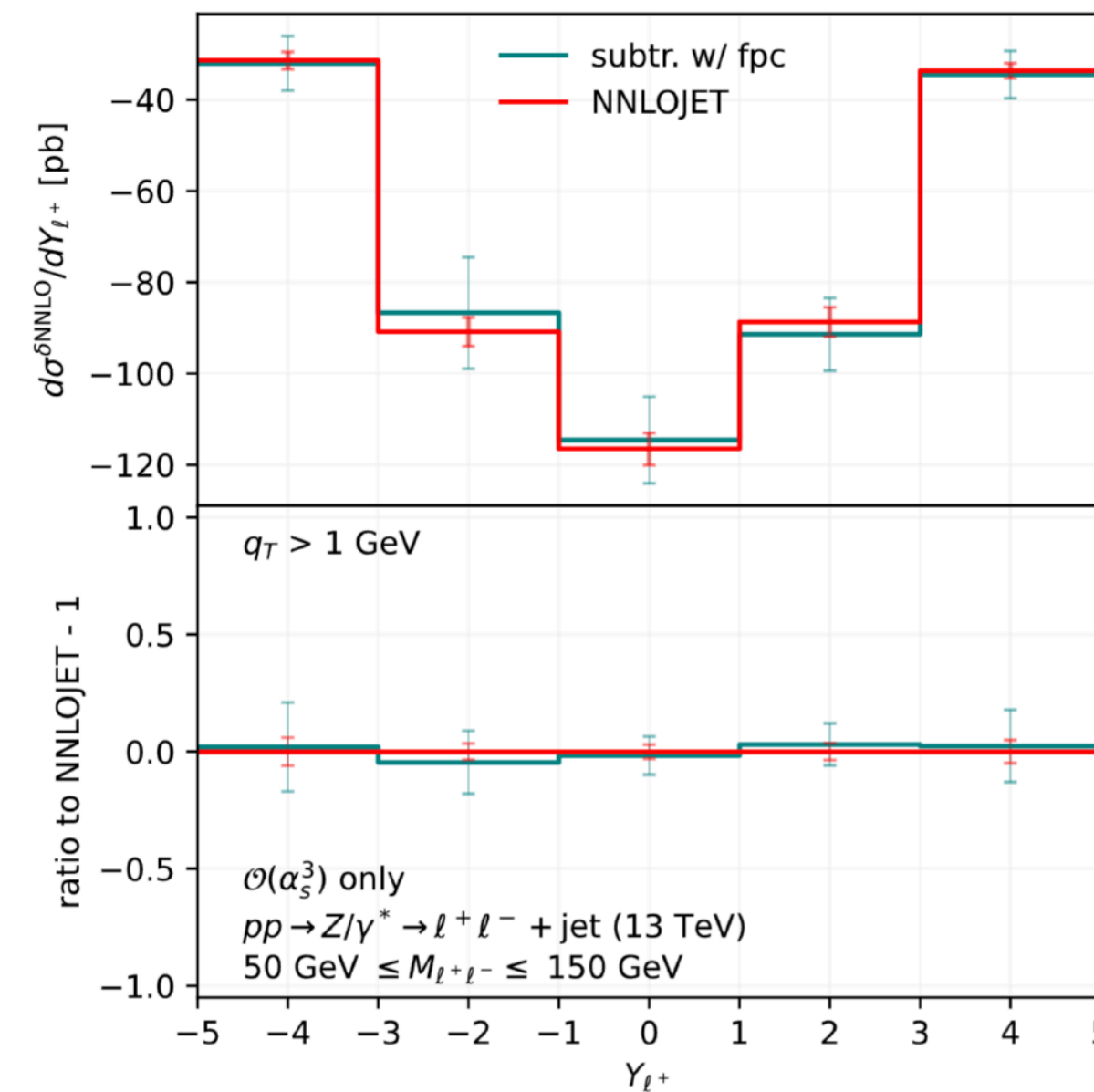
... and matching to parton showers in GENEVA



GENEVA method based on introduction of **resolution variable(s)** \mathcal{T}_N to provide IR-safe definition of events.

As a step towards NNLO+PS for Z+jet, recast NNLO prediction in GENEVA language [Alioli, Billis, Broggio, GS '25]. Inclusion of fiducial power corrections with the Projection-to-Born method

$$\begin{aligned} \mathcal{O}_{\delta\text{NNLO}}(\Phi_N) &= \frac{d\Sigma_N^{\delta\text{NNLO}}}{d\Phi_N}(\mathcal{T}_N^{\text{cut}}) \mathcal{O}(\Phi_N) \\ &+ \int \frac{d\Phi_{N+1}}{d\Phi_N} \frac{d\sigma_{N+1}^{\delta\text{NLO}}}{d\Phi_{N+1}} \mathcal{O}(\Phi_N) \theta(\mathcal{T}_N(\Phi_{N+X}) > \mathcal{T}_N^{\text{cut}}) \\ &+ \int \frac{d\Phi_{N+1}}{d\Phi_N} \frac{d\sigma_{N+1}^{\delta\text{NLO}}}{d\Phi_{N+1}} \left[\mathcal{O}(\Phi_{N+X}) - \mathcal{O}(\Phi_N) \right], \end{aligned}$$



q_T -like variables for slicing with jets

[Fu, Rahn, Shao, Waalewijn, Wu '24-'26]

q_T of colour-singlet unsuited if jets in the final state.

Proposal: use WTA recombination scheme in definition of variable

$$\vec{q}_T \equiv \vec{p}_{1,T}^{\text{WTA}} + \vec{p}_{2,T}^{\text{WTA}}, \quad \delta\phi \equiv |\pi - |\phi_1^{\text{WTA}} - \phi_2^{\text{WTA}}||.$$

azimuthal decorrelation $\delta\phi$:

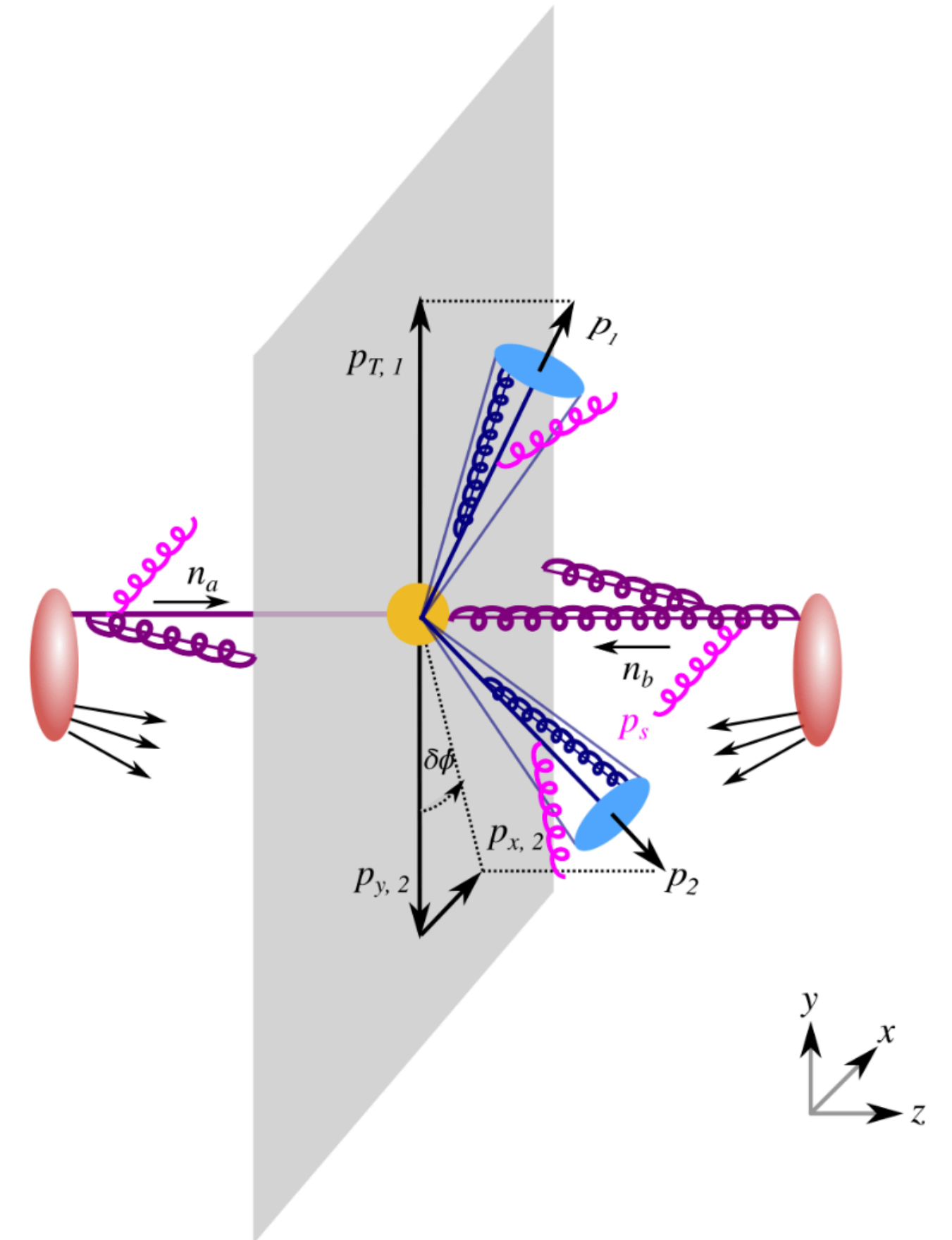
simpler factorization, but limited to planar Born configurations

transverse momentum imbalance q_T :

works in general, but more complicated soft function

Resummation for $pp \rightarrow$ dijets achieved recently:

$\delta\phi$: NNLL resummation; q_T : NNLL for global logs, LL for NGLs in small-R limit



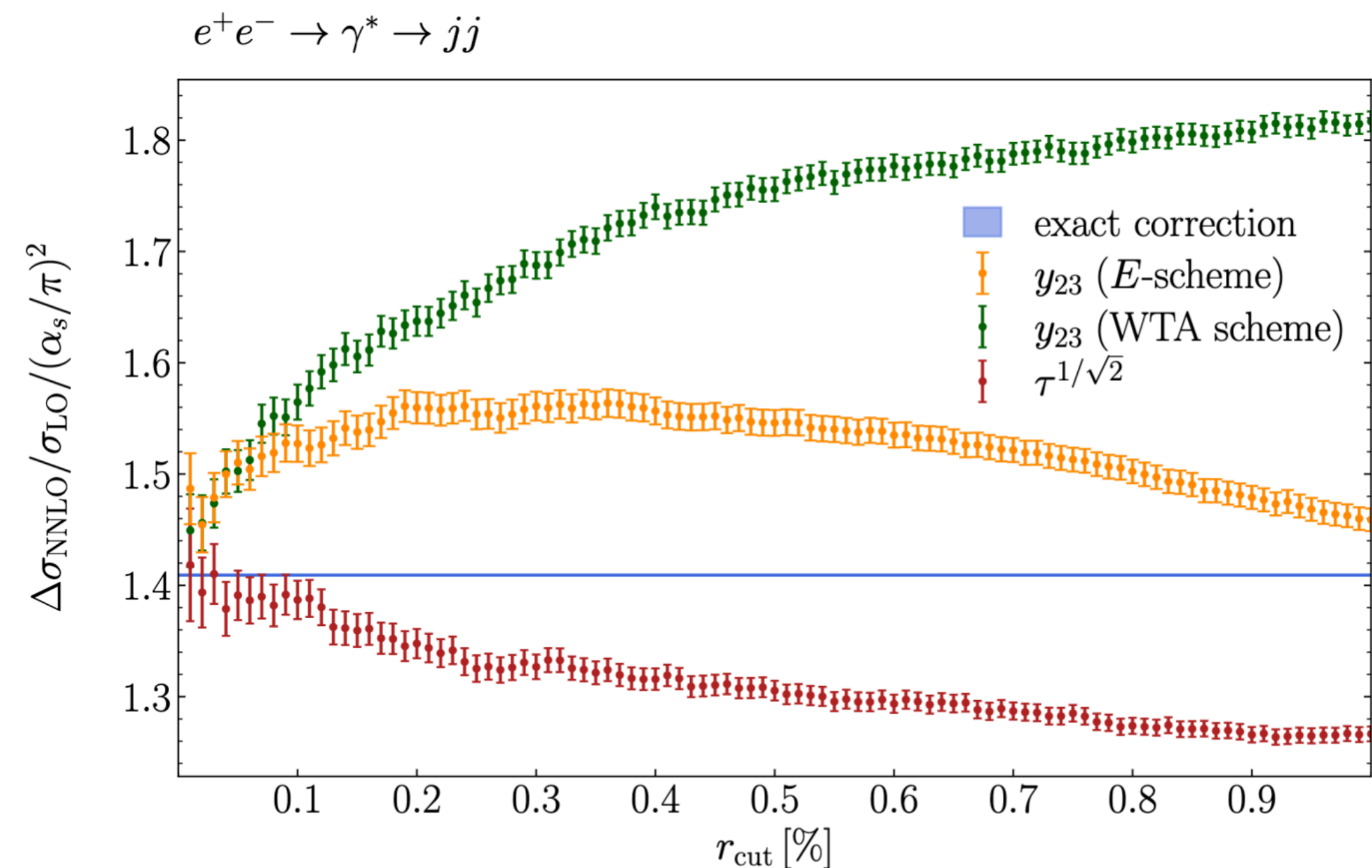
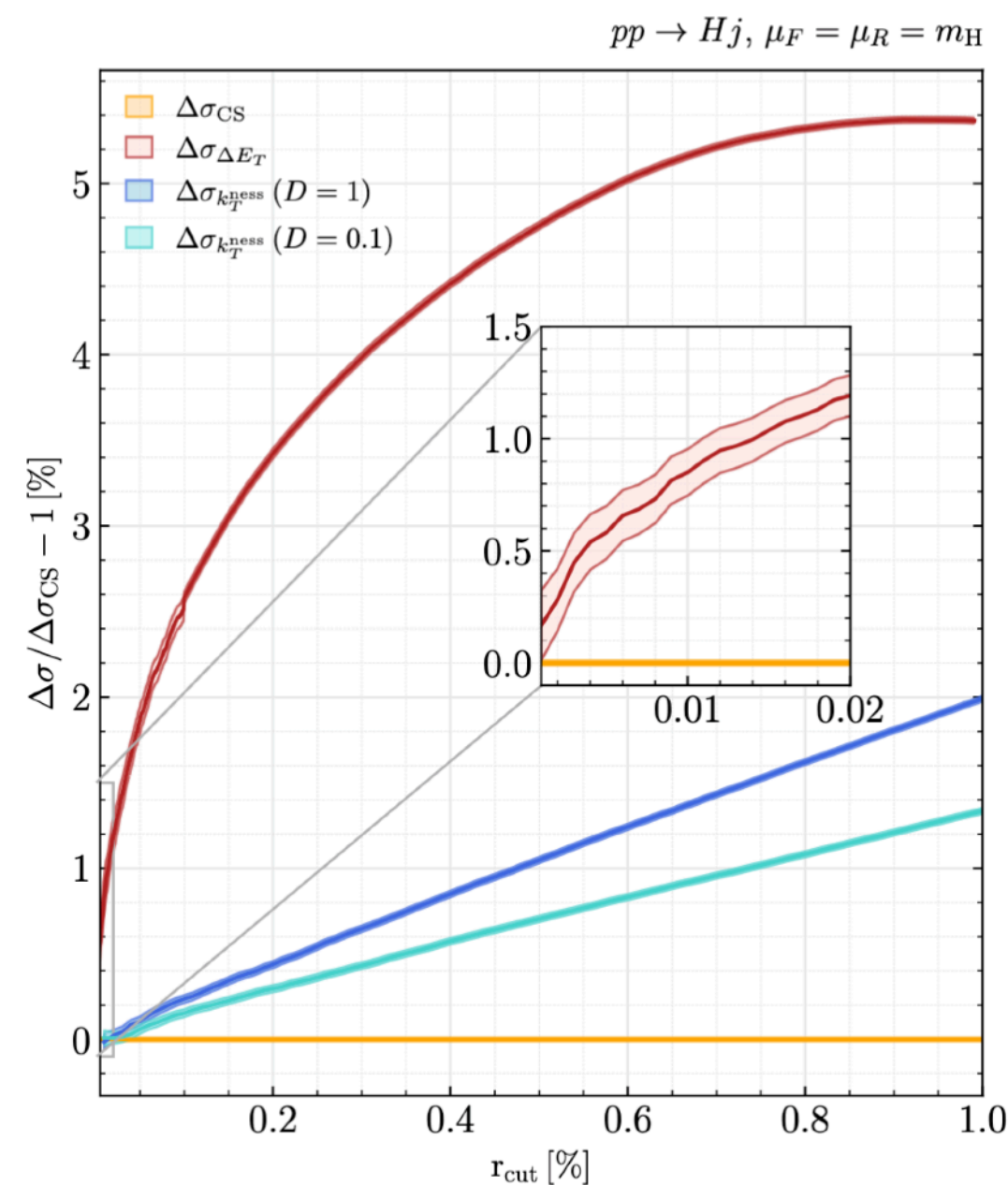
q_T -like variables for slicing with jets

[Buonocore, Grazzini, Guadagni, Haag, Kallweit, Rottoli, Savoini '22-'25]

New observable k_T^{ness} based on exclusive k_T -clustering algorithm (in e^+e^- equivalent to Durham y_{ij})

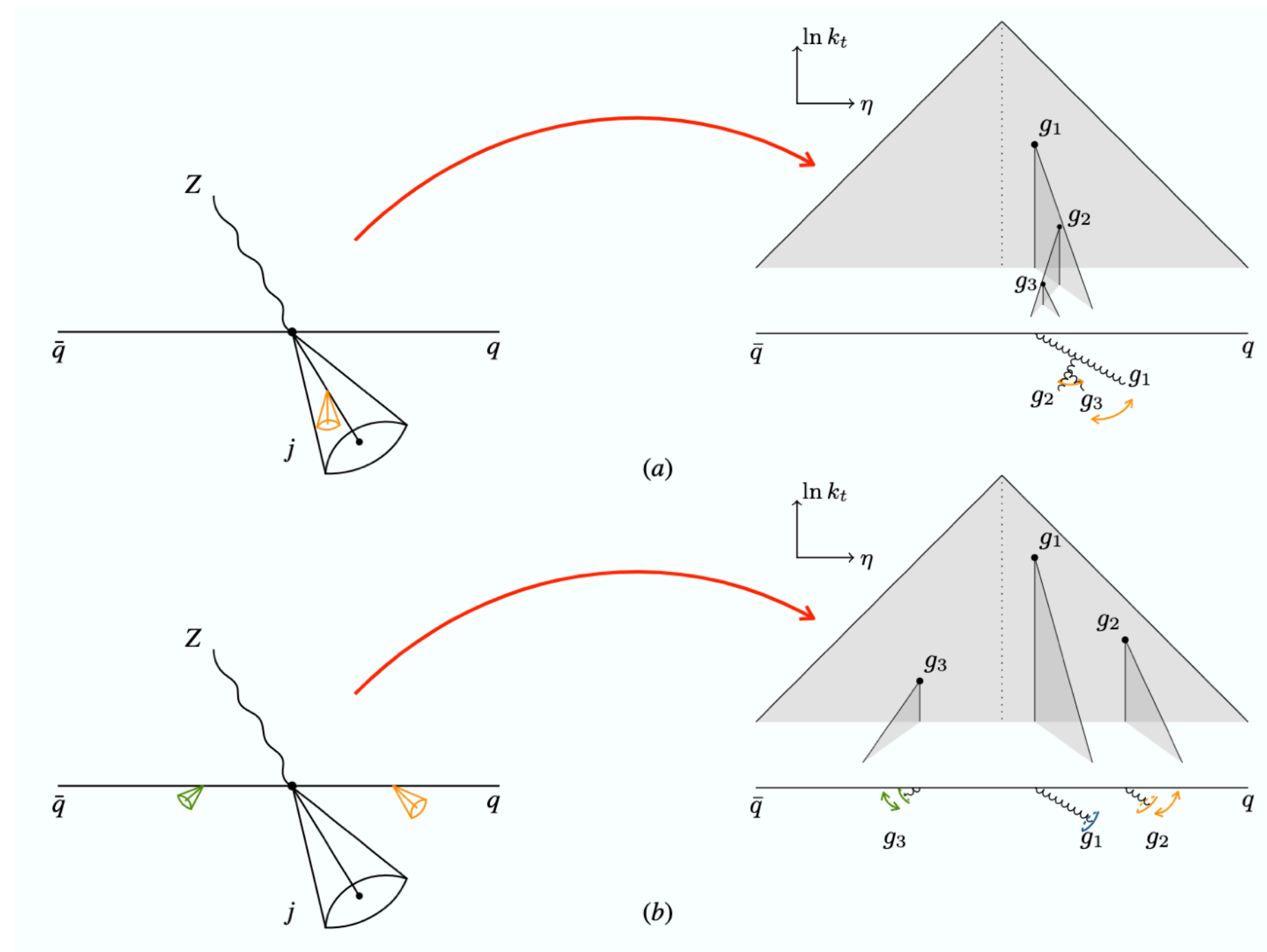
Formulation and first applications at NLO at hadron colliders

Calculation of quark jet function at NNLO, followed by results for $e^+e^- \rightarrow 2$ jets at NNLO



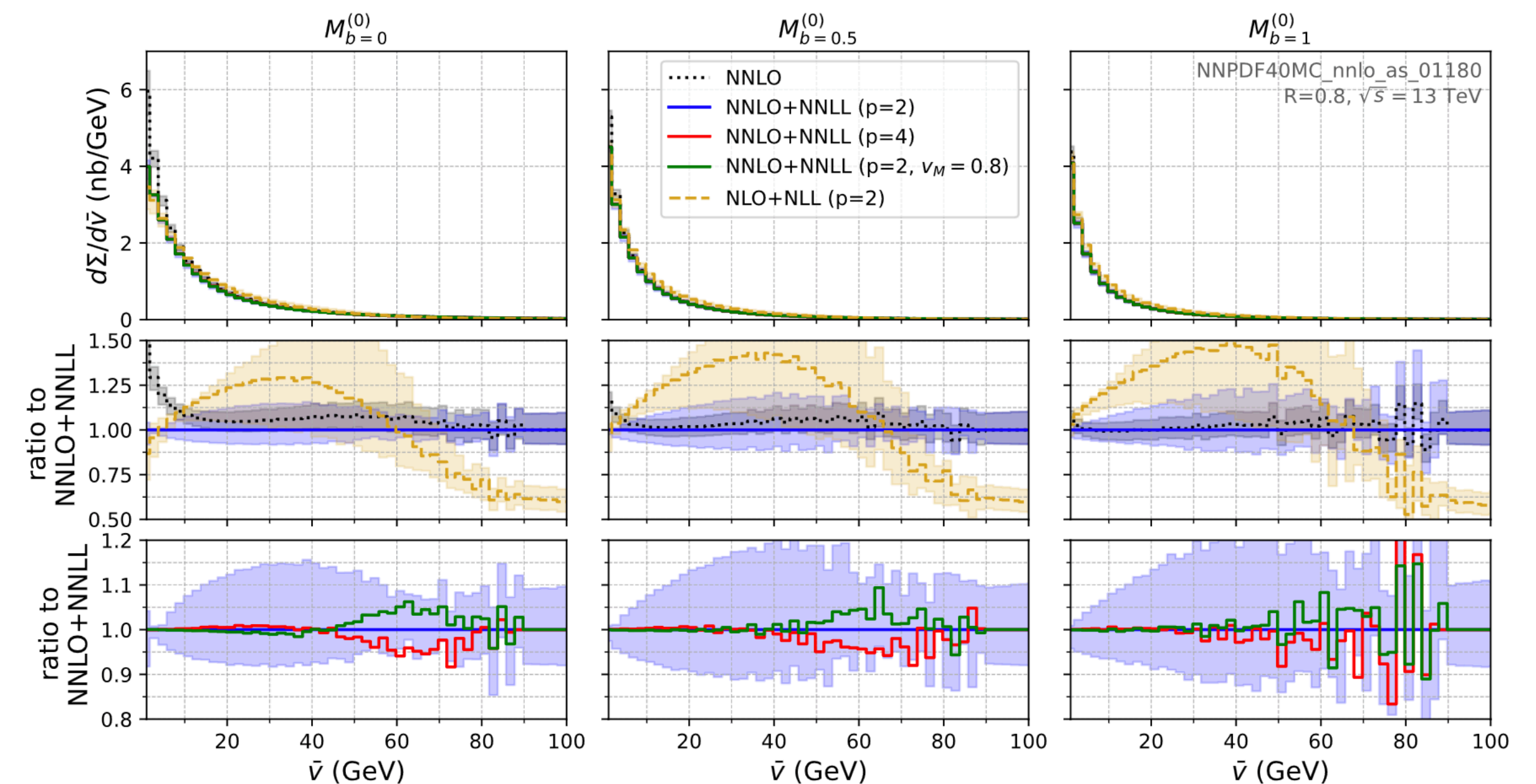
Lund-Tree Shapes (LTS)

[van Beekveld, Buonocore, Ferrario Ravasio, Monni, Soto-Ontoso, Soyez '25]



Observable constructed from Lund plane declustering variables in angular-ordered history
 Suitable for any number of (groomed) final-state jets

Can be used for slicing, matching, PS validation
 Results at NNLO+NNLL for two-leg processes



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COUPLING



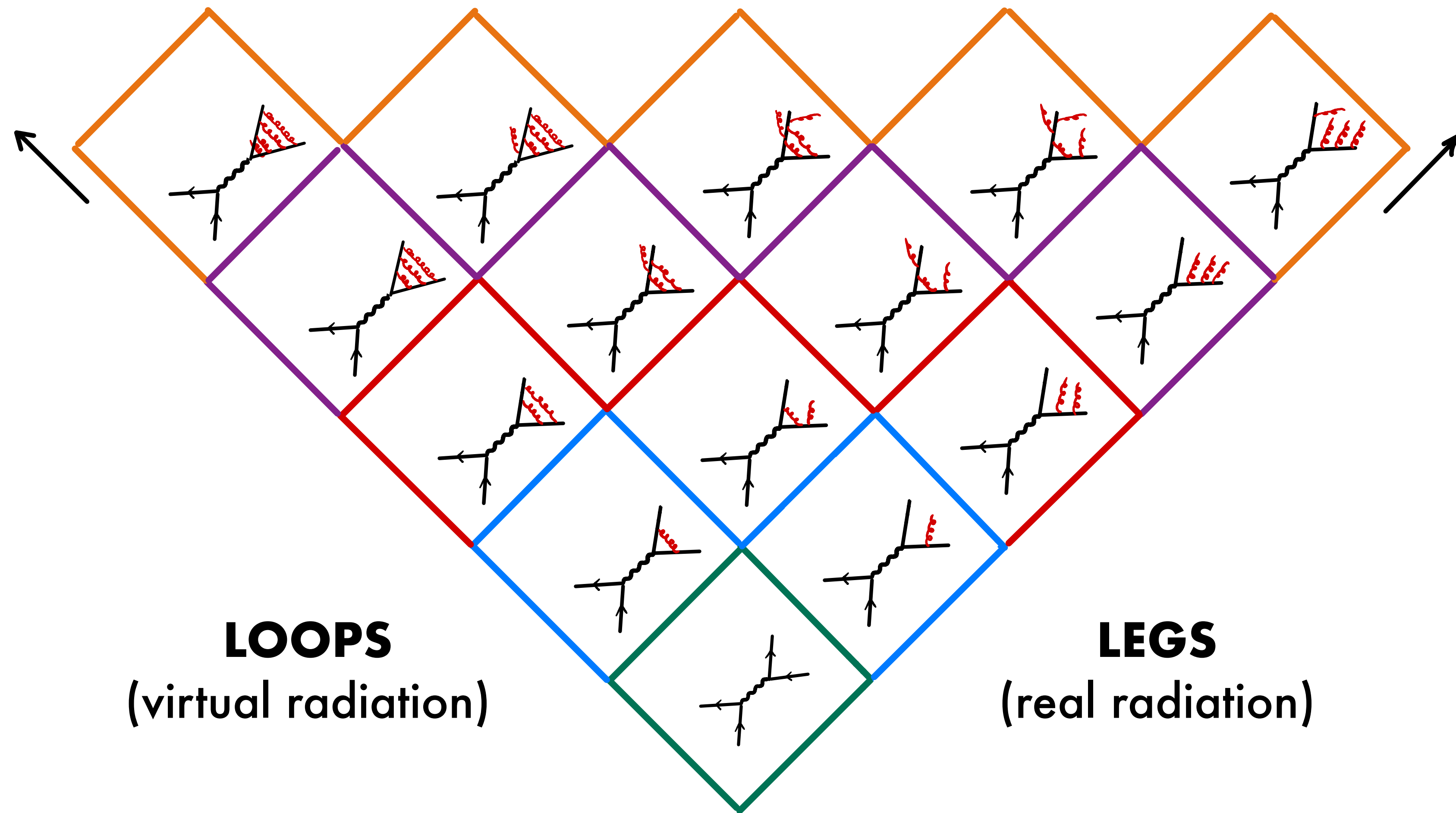
$\mathcal{O}(\alpha_s^4)$

$\mathcal{O}(\alpha_s^3)$

$\mathcal{O}(\alpha_s^2)$

$\mathcal{O}(\alpha_s^1)$

$\mathcal{O}(\alpha_s^0)$

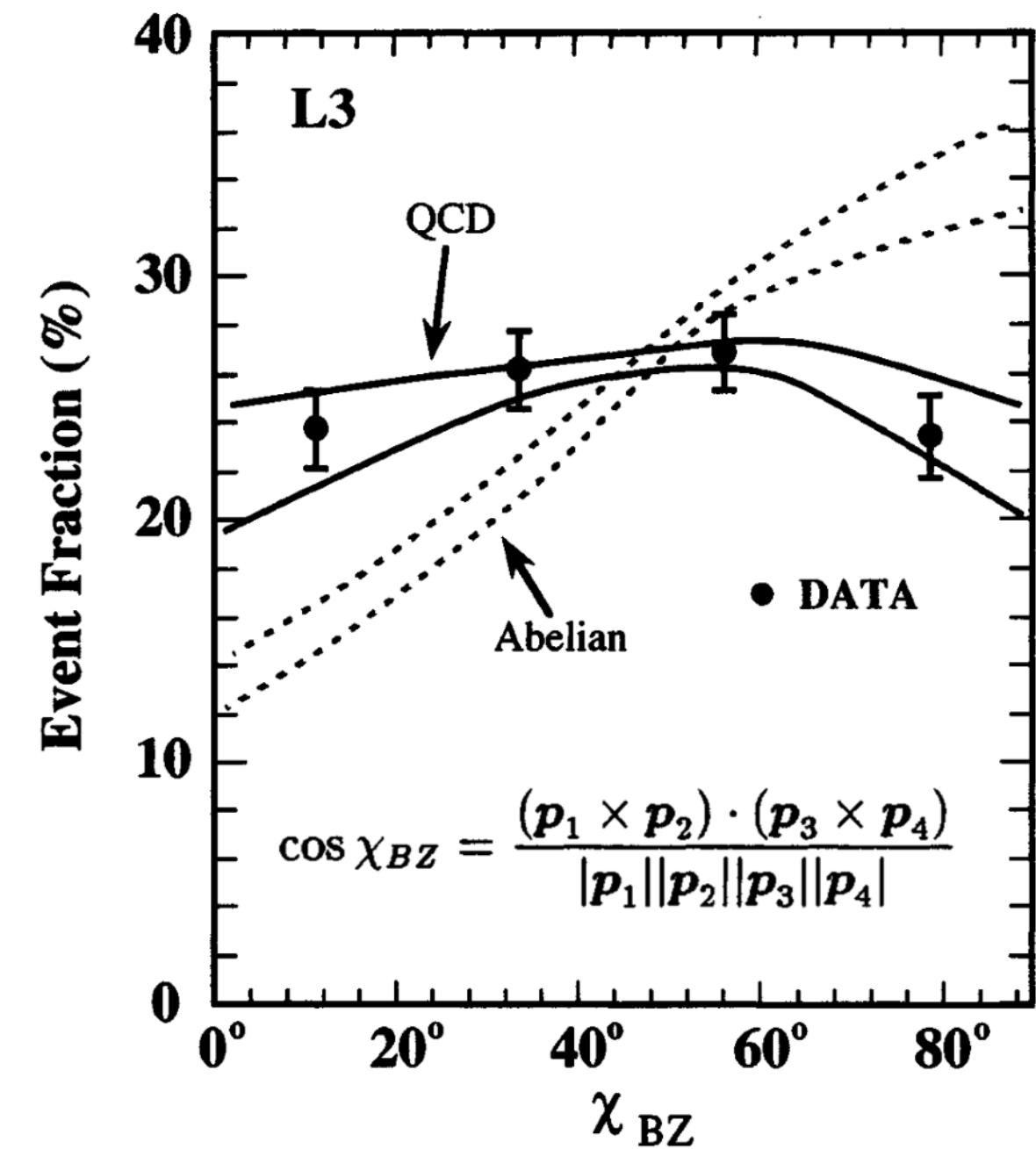
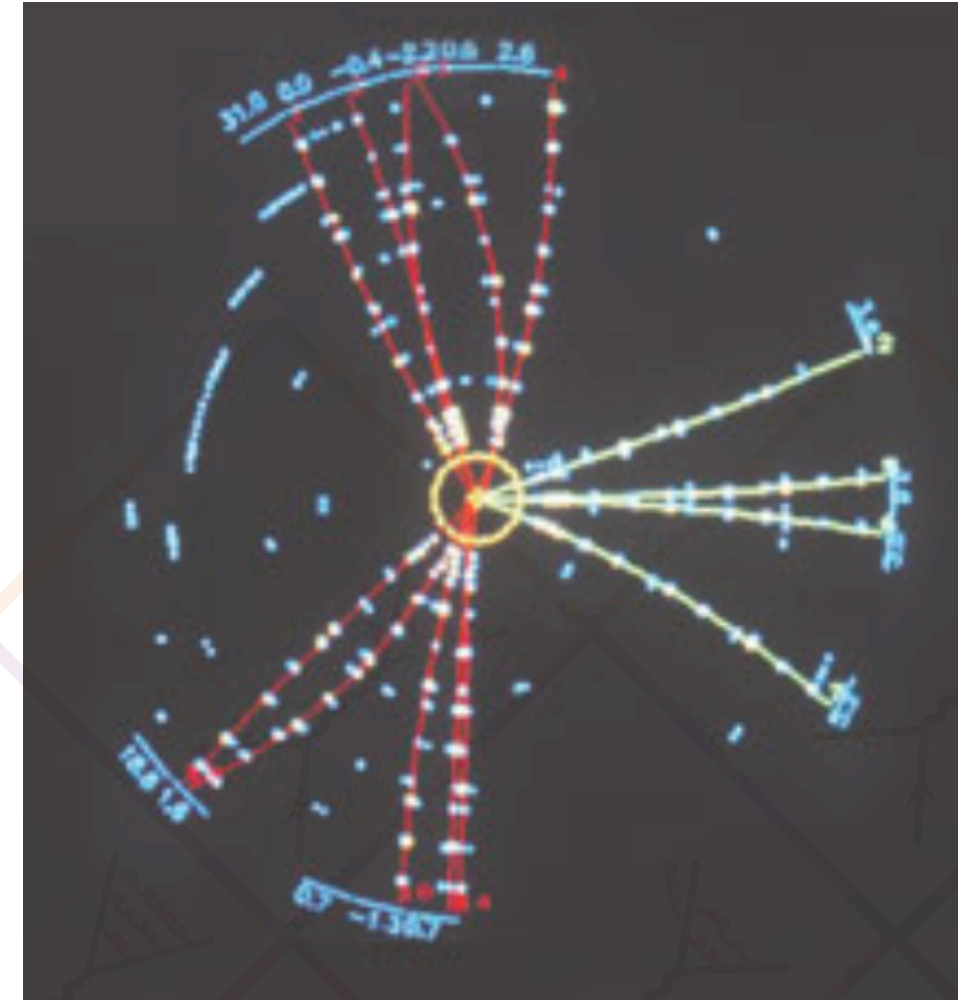


LOOPS
(virtual radiation)

LEGS
(real radiation)

First seminal tests of QCD...

Discovery of the gluon



Quarks have spin 1/2

$$\frac{d\sigma(e^+e^- \rightarrow \gamma^* \rightarrow q\bar{q})}{d \cos \vartheta} = \frac{3}{8} \sigma_0 (1 + \cos^2 \vartheta)$$

Evidence for Jet Structure in Hadron Production by e^+e^- Annihilation*

G. Hanson, G. S. Abrams, A. M. Boyarski, M. Breidenbach, F. Bulos,
 W. Chinowsky, G. J. Feldman, C. E. Friedberg, D. Fryberger, G. Goldhaber,
 D. L. Hartill,† B. Jean-Marie, J. A. Kadyk, R. R. Larsen, A. M. Litke,
 D. Lüke,‡ B. A. Lulu, V. Lüth, H. L. Lynch, C. C. Morehouse,
 J. M. Paterson, M. L. Perl, F. M. Pierre,§ T. P. Pun, P. A. Rapidis,
 B. Richter, B. Sadoulet, R. F. Schwitters, W. Tanenbaum,
 G. H. Trilling, F. Vannucci,|| J. S. Whitaker,
 F. C. Winkelmann, and J. E. Wiss

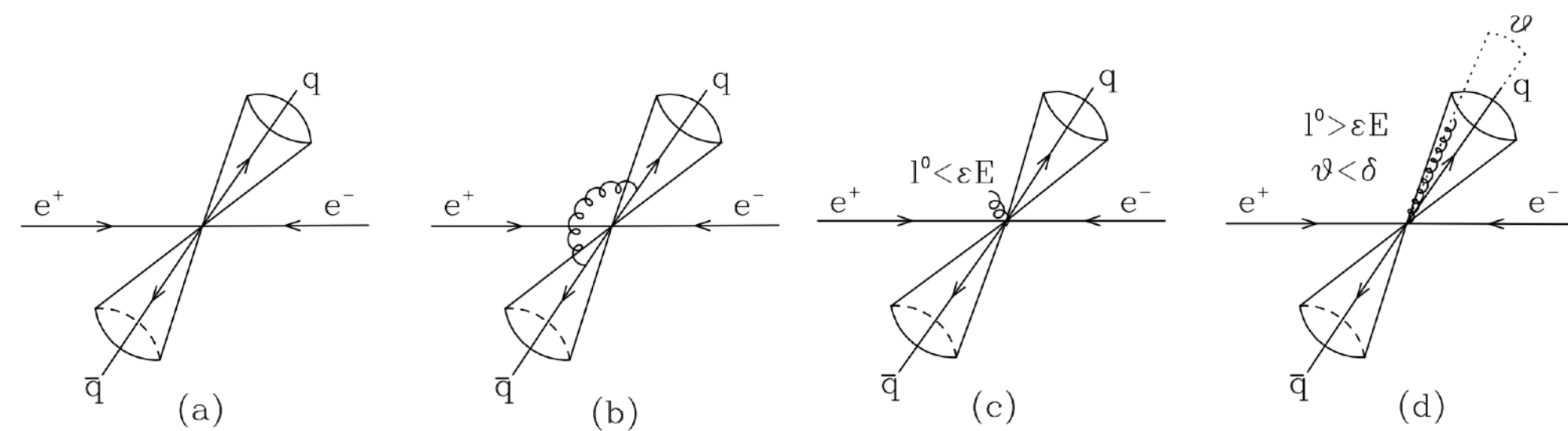
Lawrence Berkeley Laboratory and Department of Physics, University of California, Berkeley, California 94720,
 and Stanford Linear Accelerator Center, Stanford University, Stanford, California 94305
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We have found evidence for jet structure in $e^+e^- \rightarrow$ hadrons at center-of-mass energies of 6.2 and 7.4 GeV. At 7.4 GeV the jet-axis angular distribution integrated over azimuthal angle was determined to be proportional to $1 + (0.78 \pm 0.12)\cos^2\theta$.

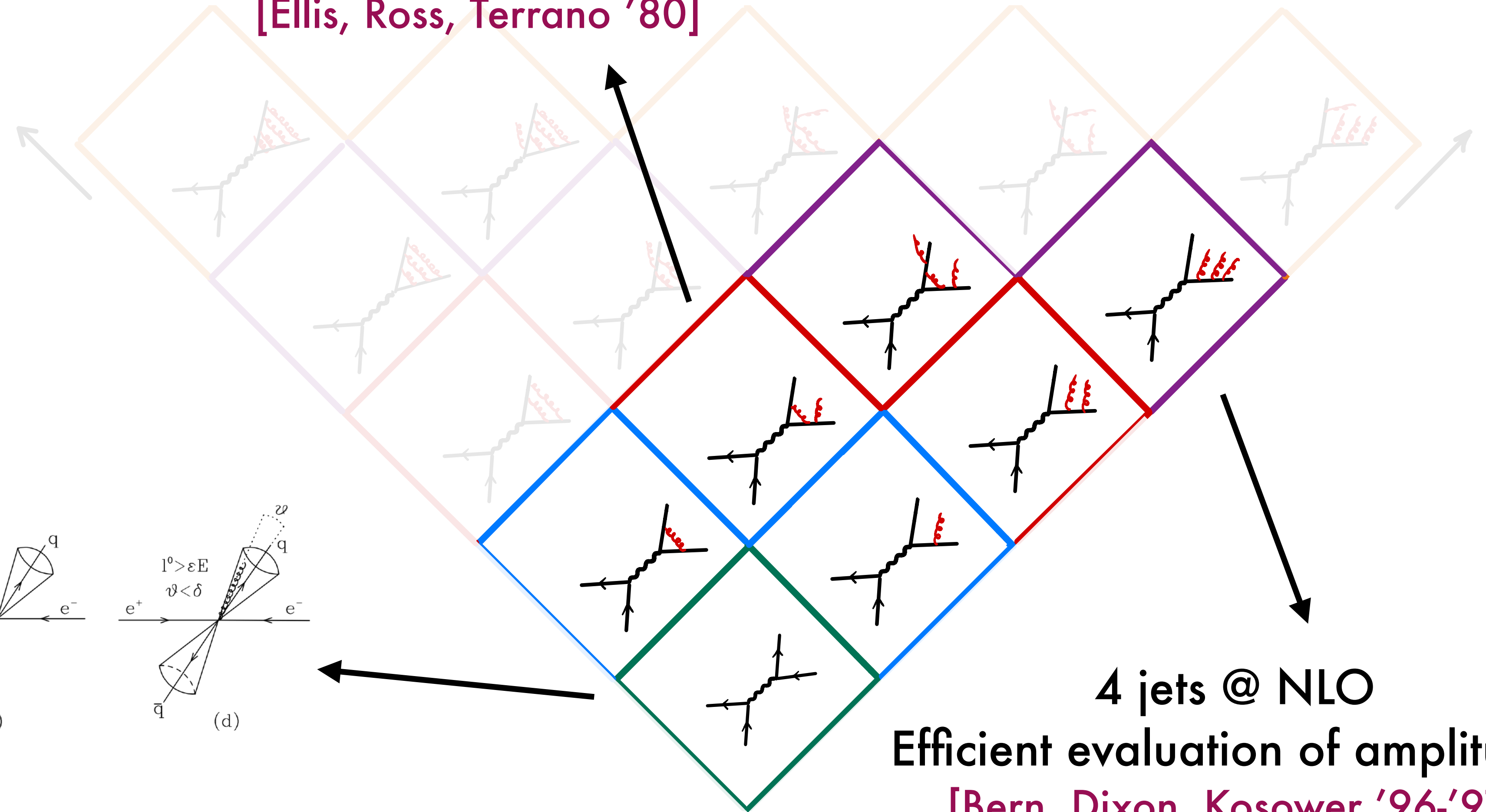
QCD is a non-abelian theory

... and ideal playground for theorists

2 jets @ NLO
 Jets and infrared safety
 [Sterman, Weinberg '77]

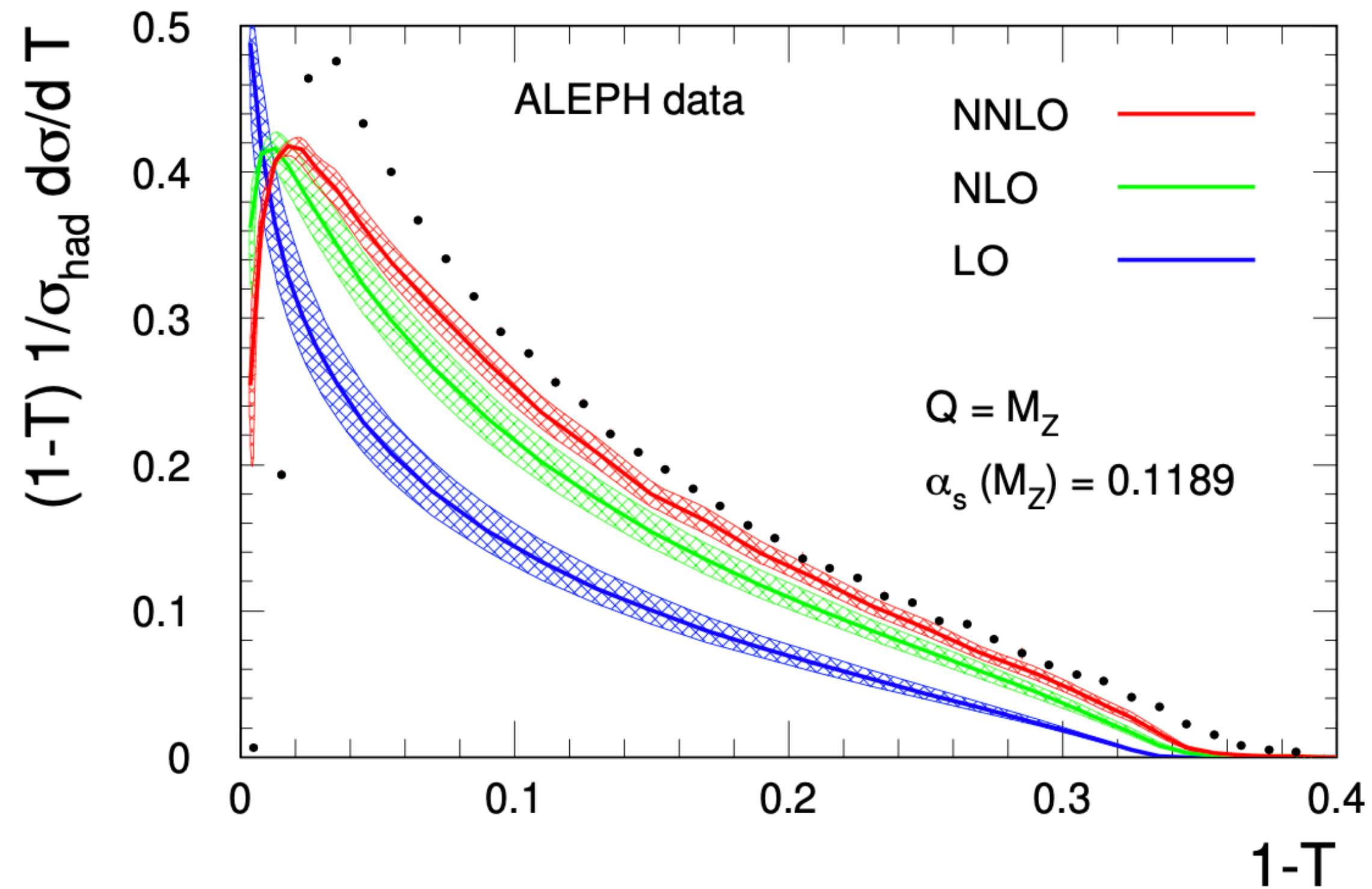


3 jets @ NLO
 First subtraction methods
 [Ellis, Ross, Terrano '80]



4 jets @ NLO
 Efficient evaluation of amplitudes
 [Bern, Dixon, Kosower '96-'97]

3 jets @ NNLO

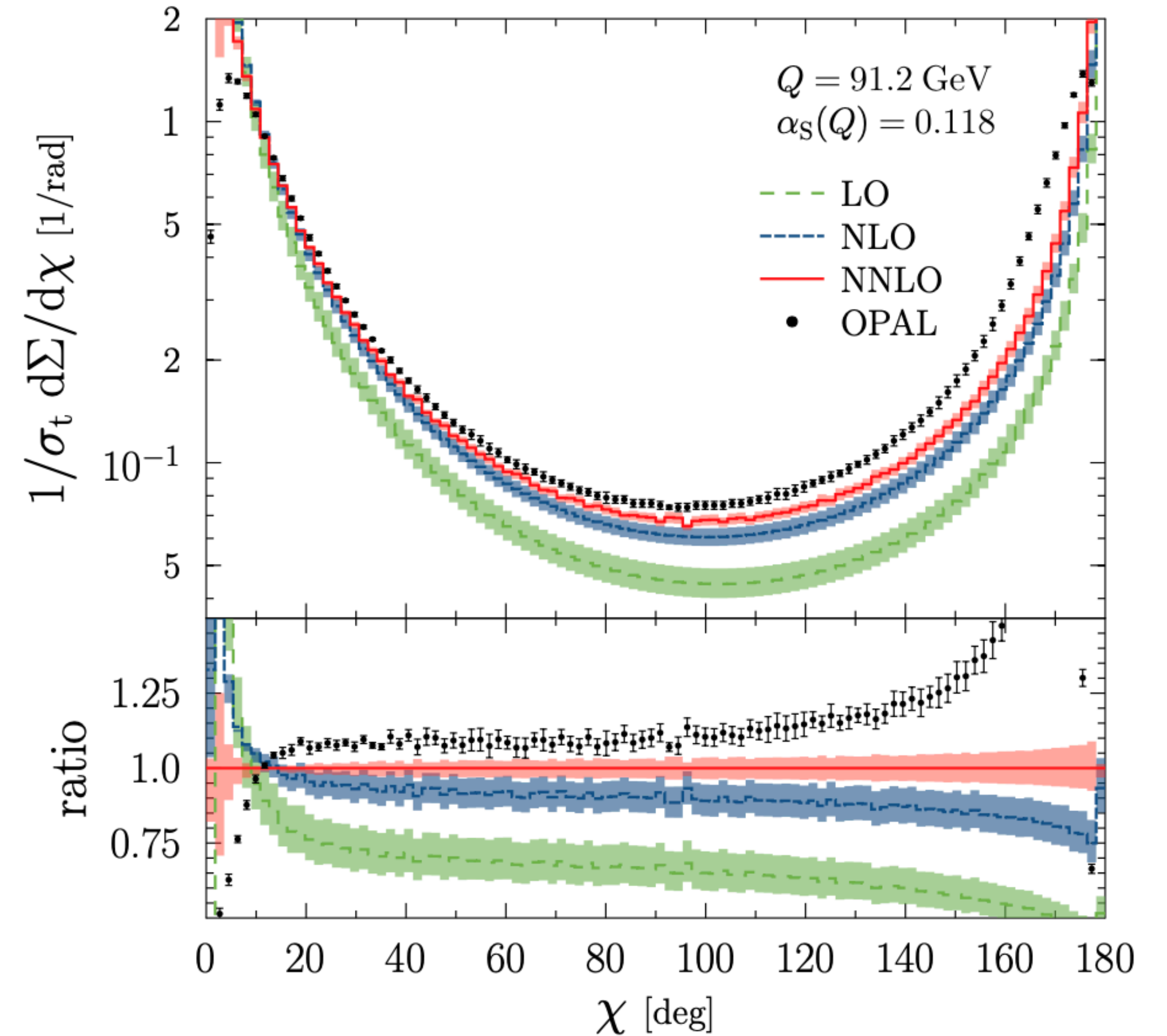


Antenna subtraction

[Gehrmann-De Ridder, Gehrmann, Glover, Heinrich '07-'09]

[Weinzierl '07-'09]

(EERAD, now also in NNLOJET)



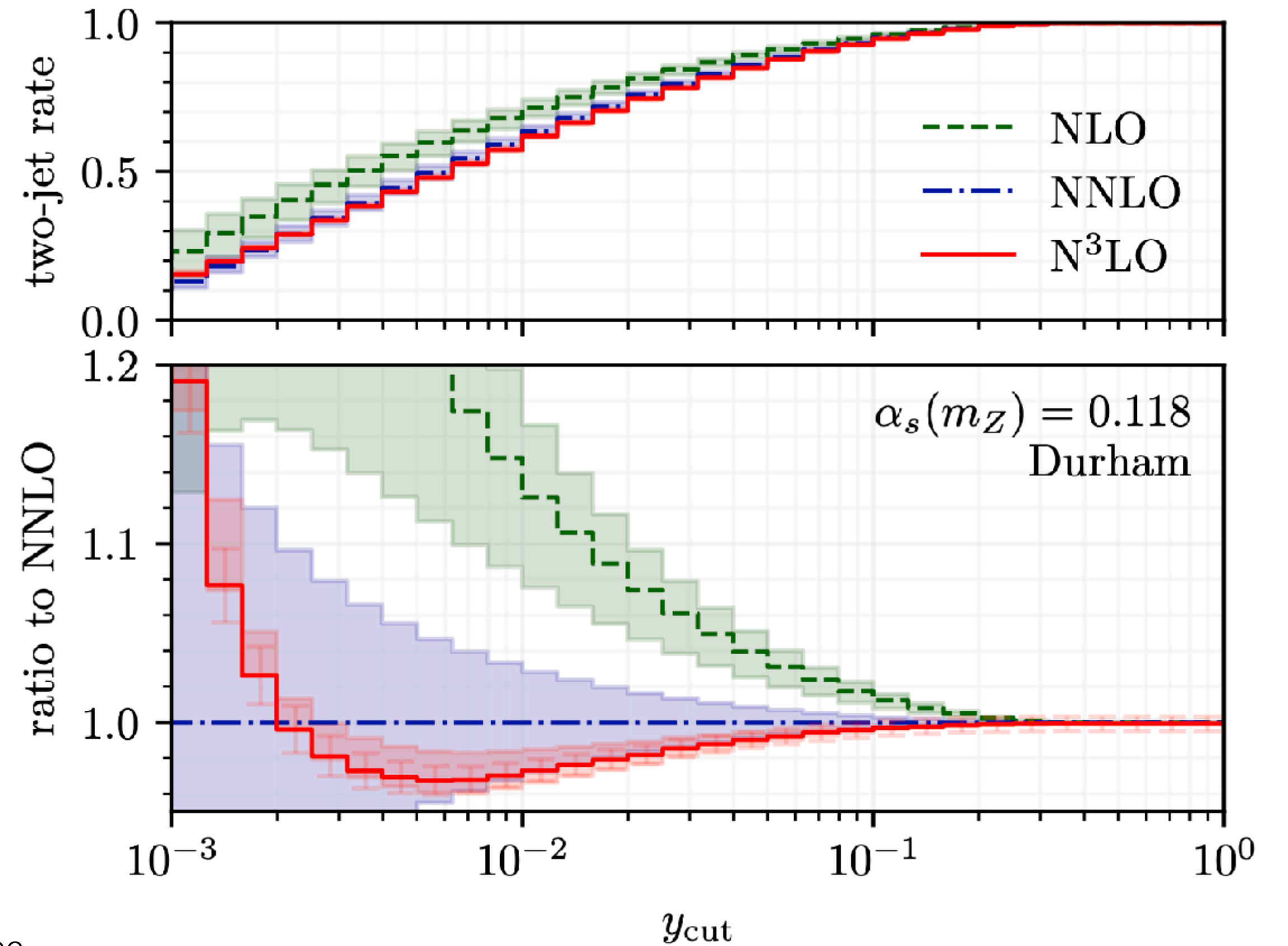
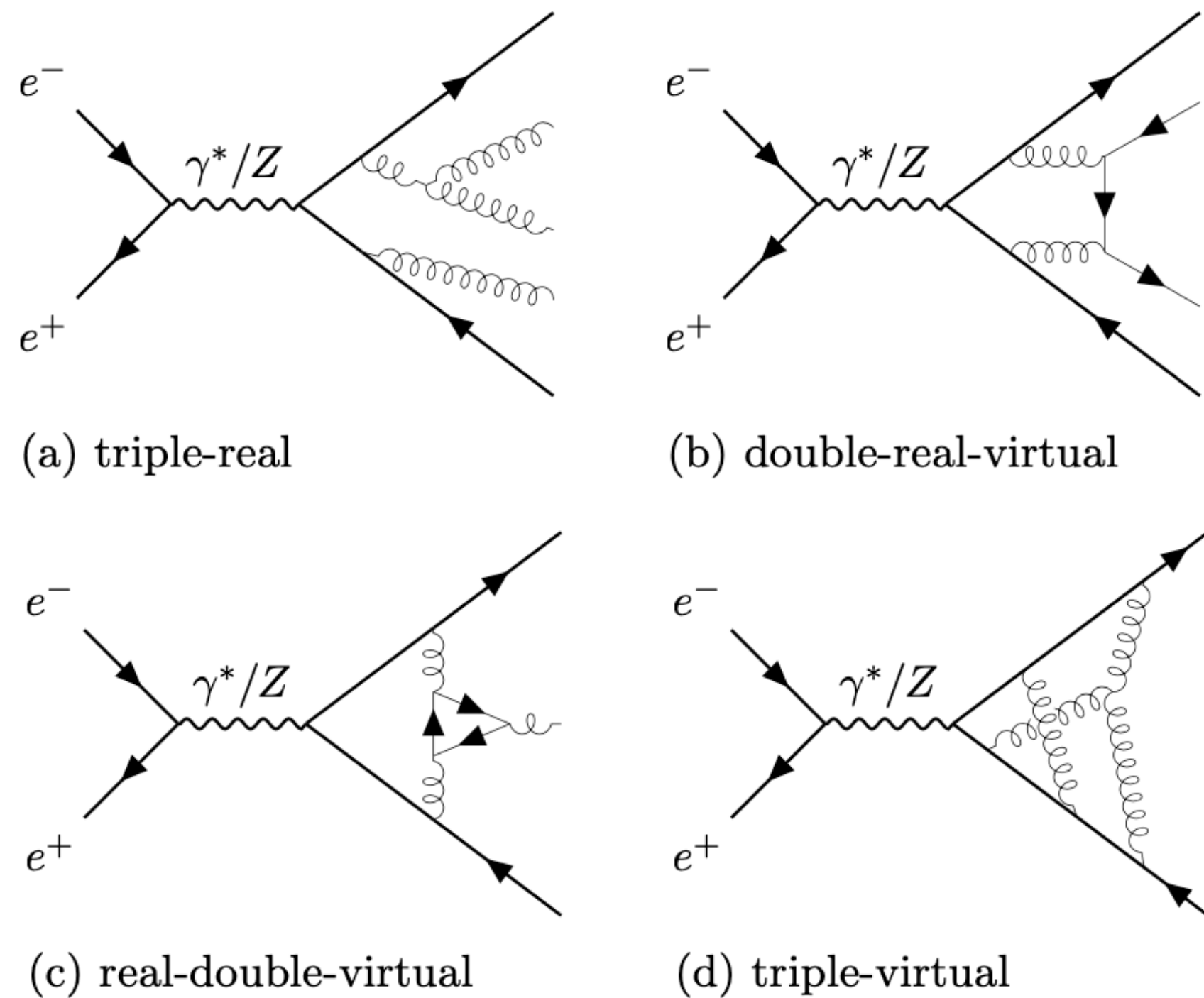
CoLoRFuNNLO

[Del Duca, Duhr, Kardos, Somogyi, Trócsányi '16]

2 jets @ N3LO

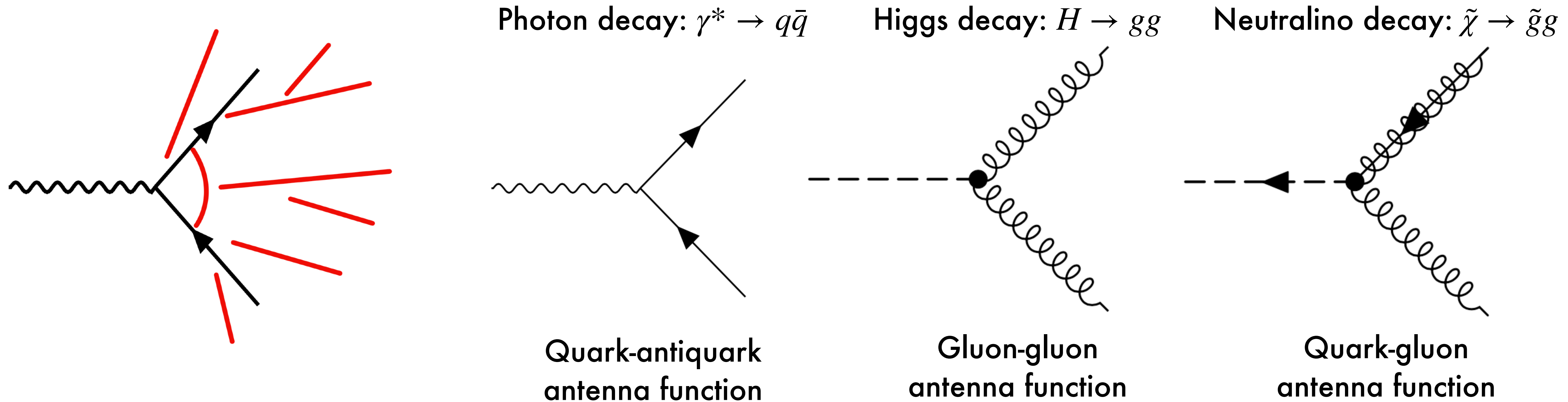
First differential N³LO calculation with a local subtraction method
[Chen, Jakubčík, Marcoli, GS '25]

Proof-of-principle calculation (trivial Born kinematics)



Antenna subtraction in a nutshell

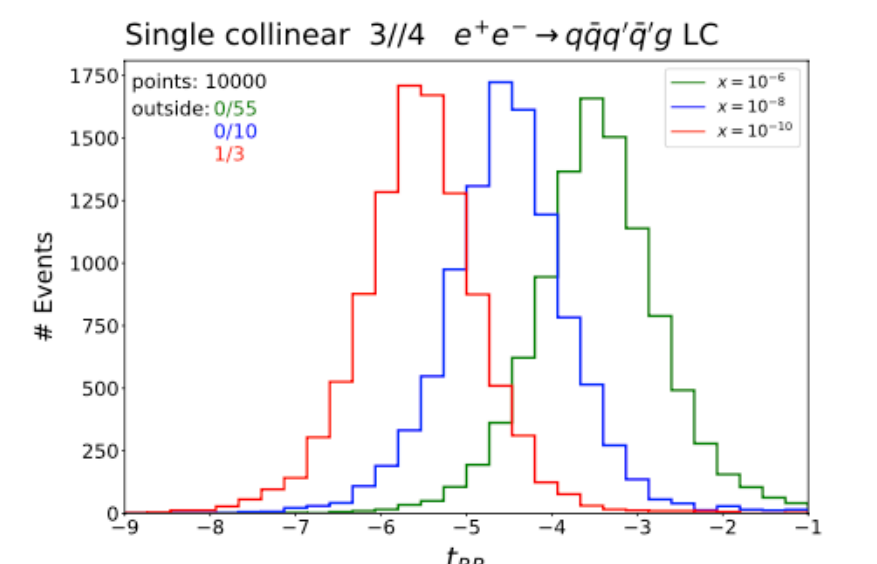
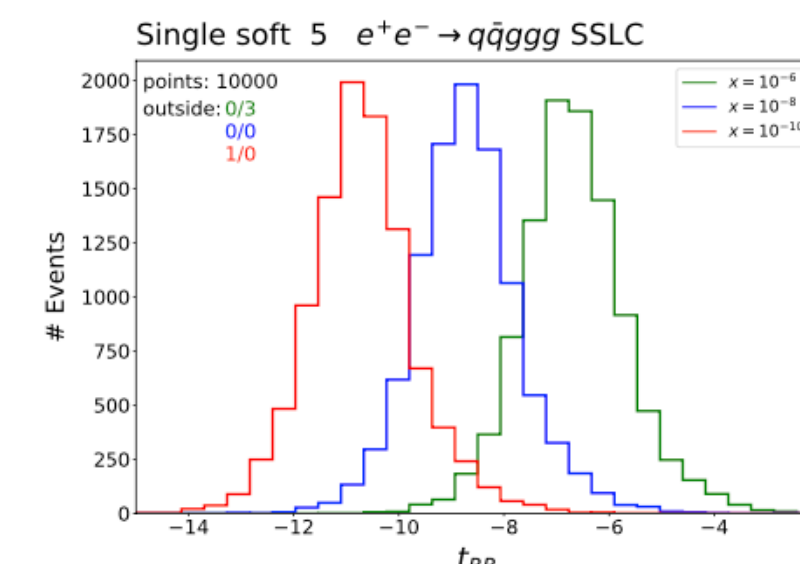
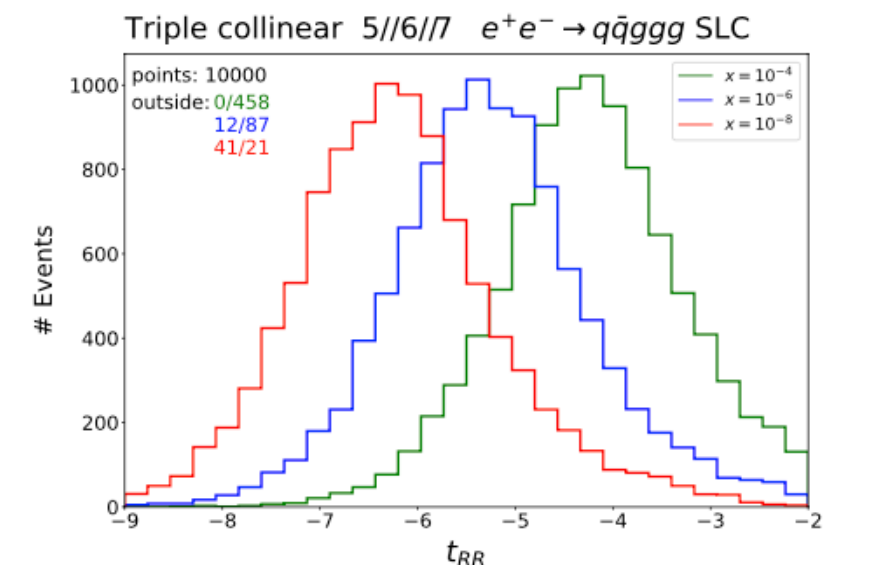
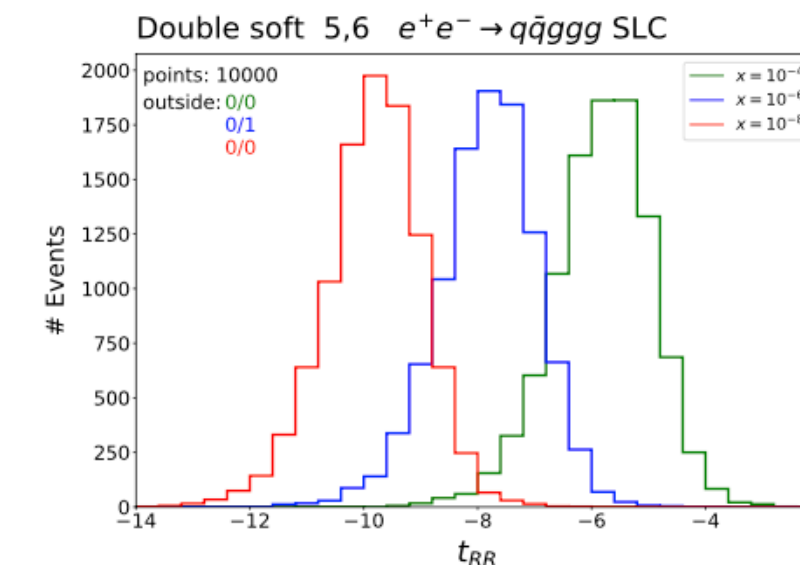
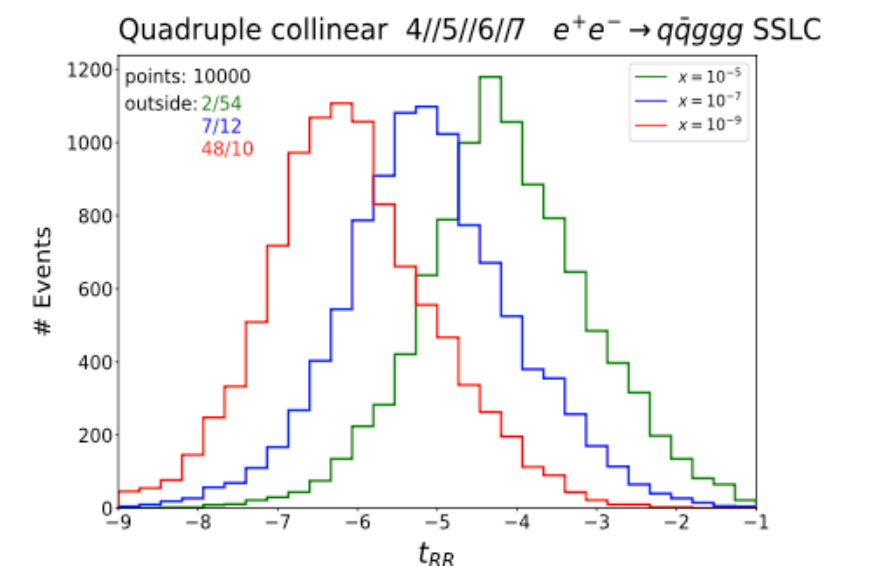
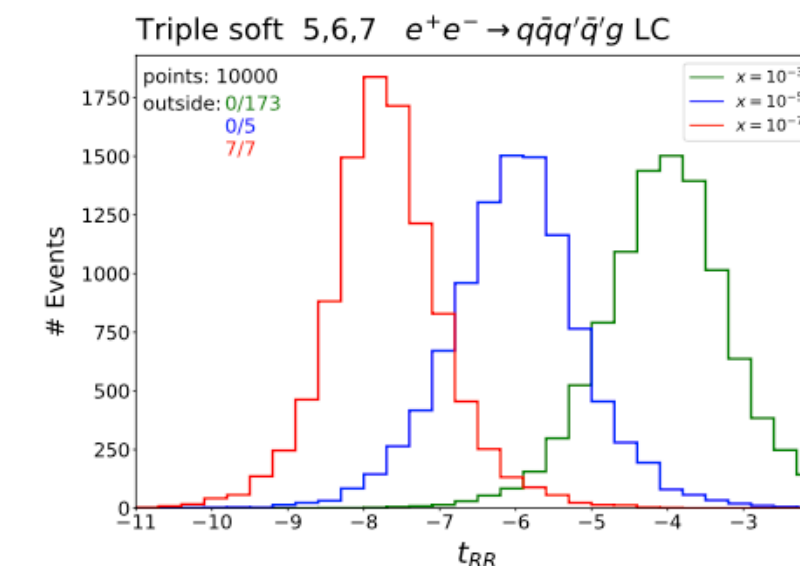
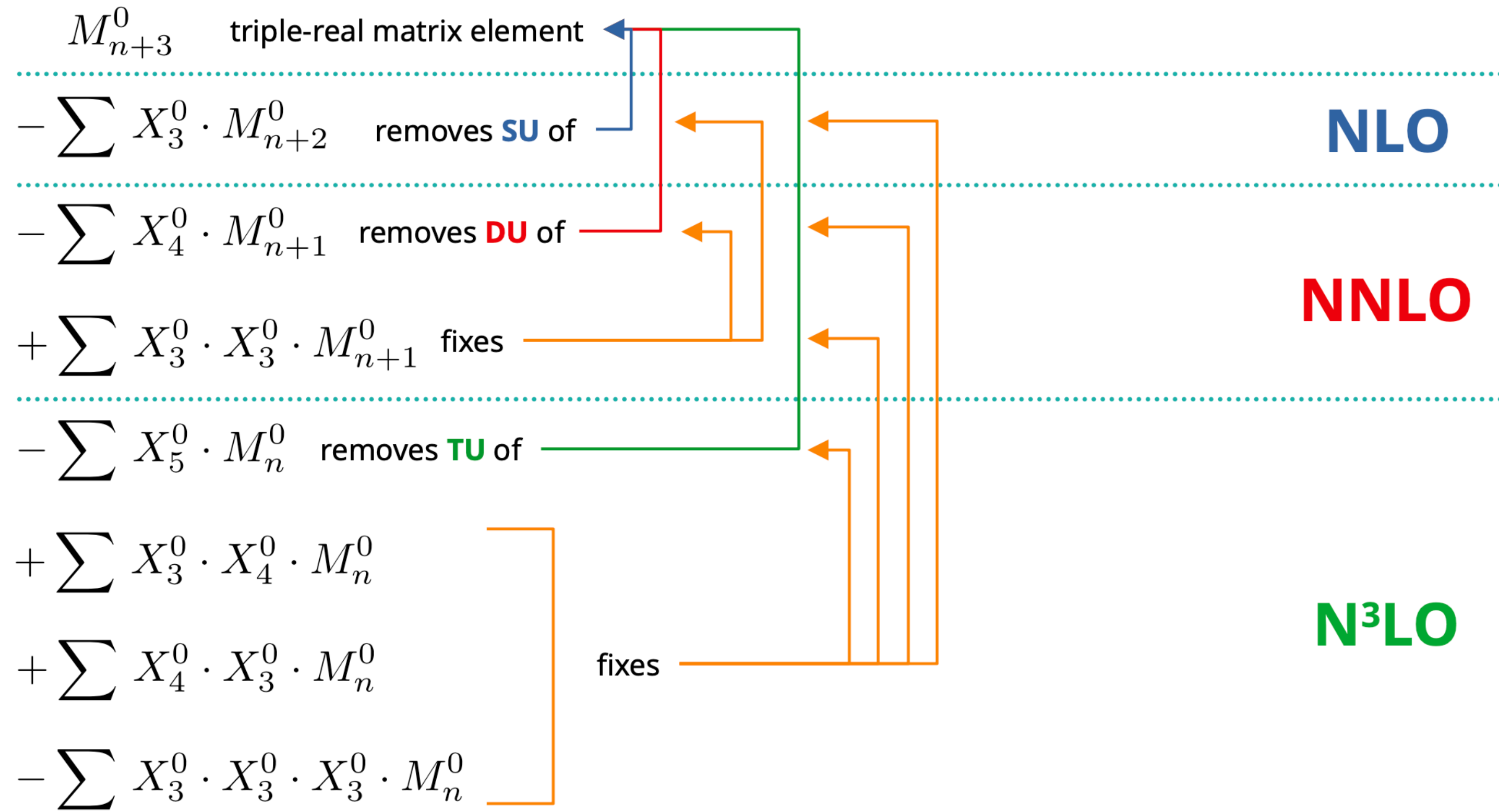
l -loop n -parton antenna functions $X_{n'}^l$ derived from matrix elements of physical processes to encapsulate all unresolved (real and virtual) radiation between pair of hard radiators



To obtain virtual subtraction terms, integration over phase space of unresolved radiation.
Up to N3LO, all relevant antenna functions integrated in [Chen, Jakubčík, Marcoli, GS '22-'23]

Challenges in computation of 2 jets @ N3LO

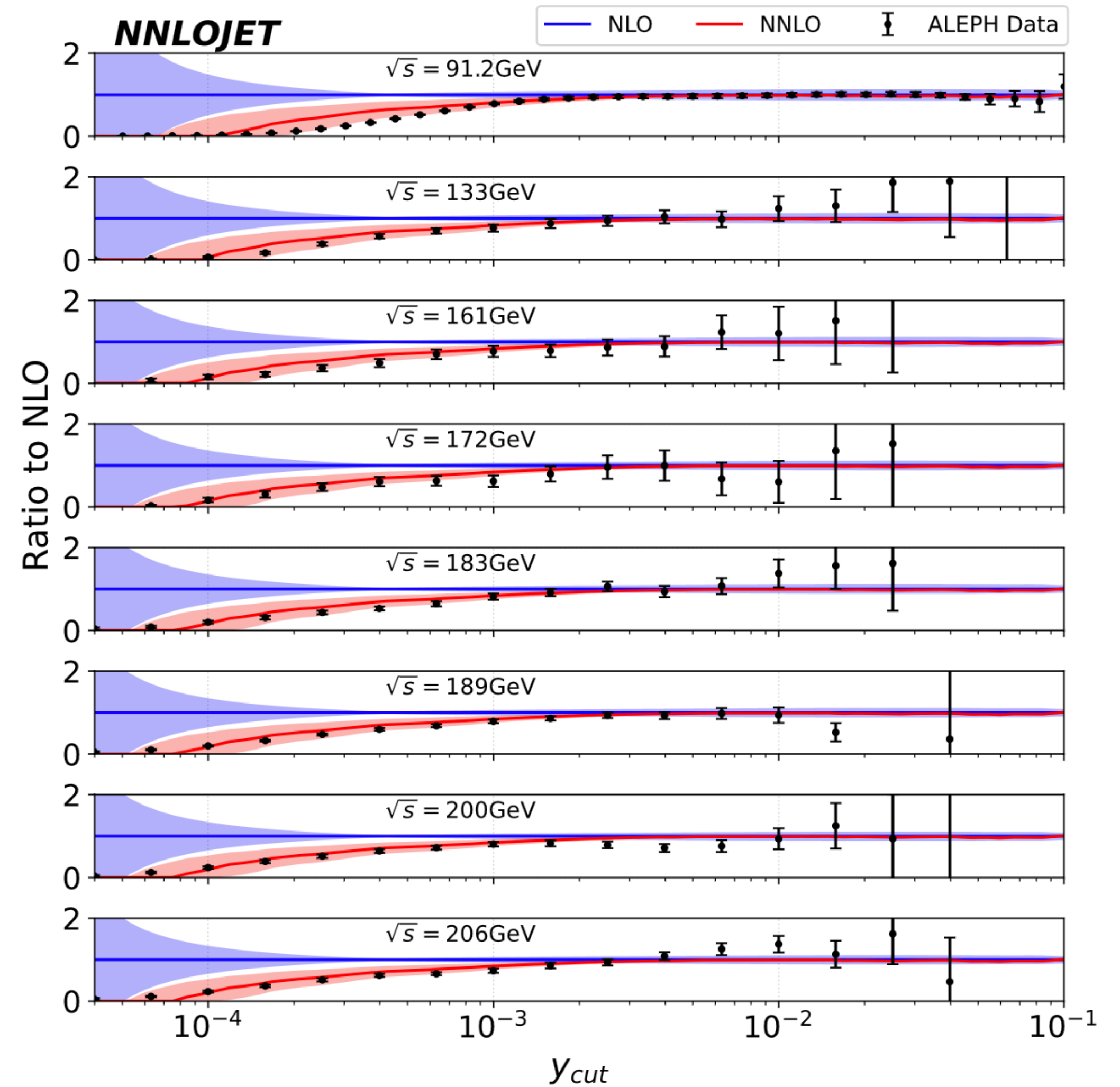
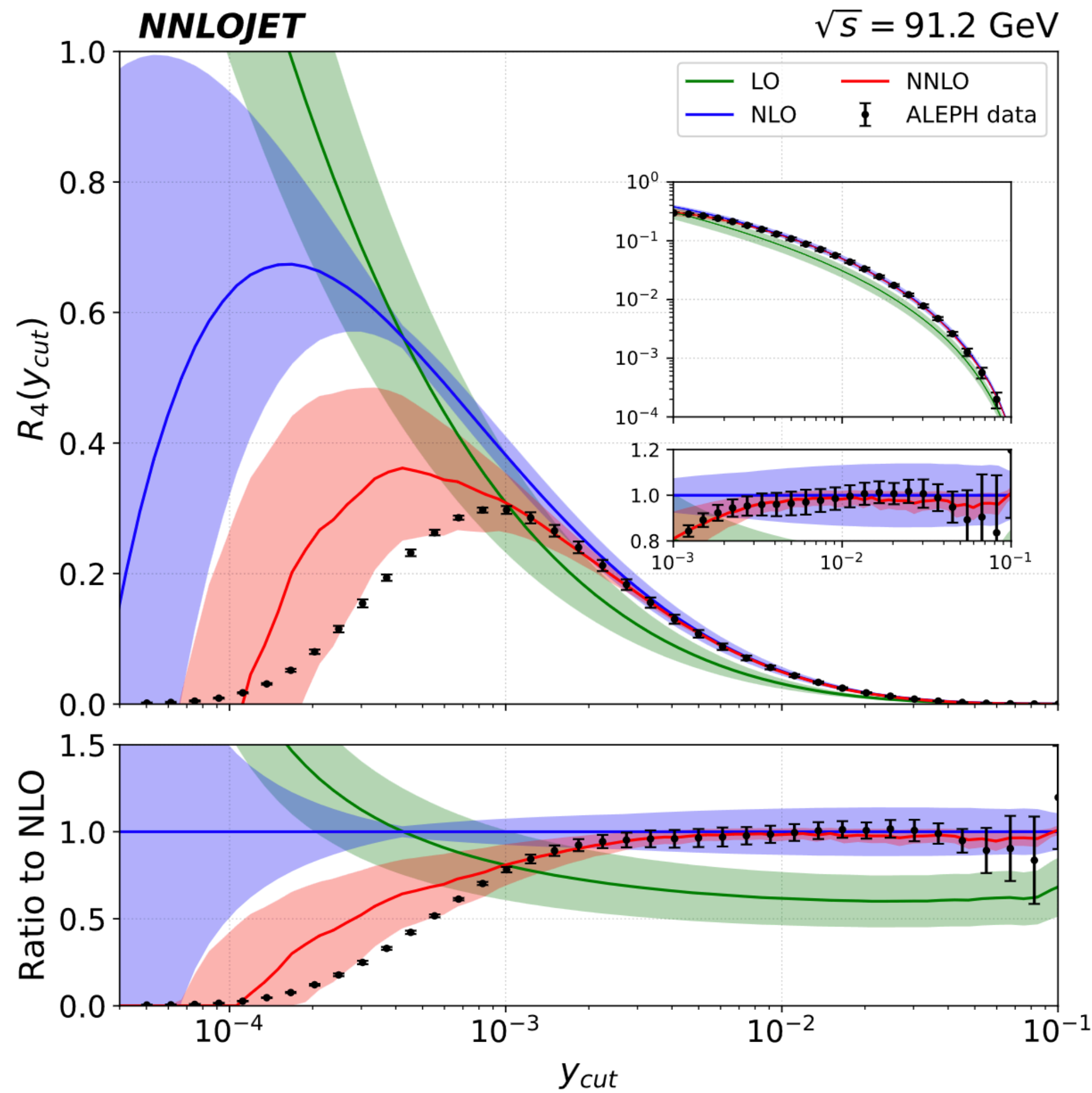
Real subtraction up to 3 unresolved emissions = intricate interplay of cancellations



+ numerical stability of virtual matrix elements
(switch to quadruple precision, use suitable Taylor expansions...)

4 jets @ NNLO

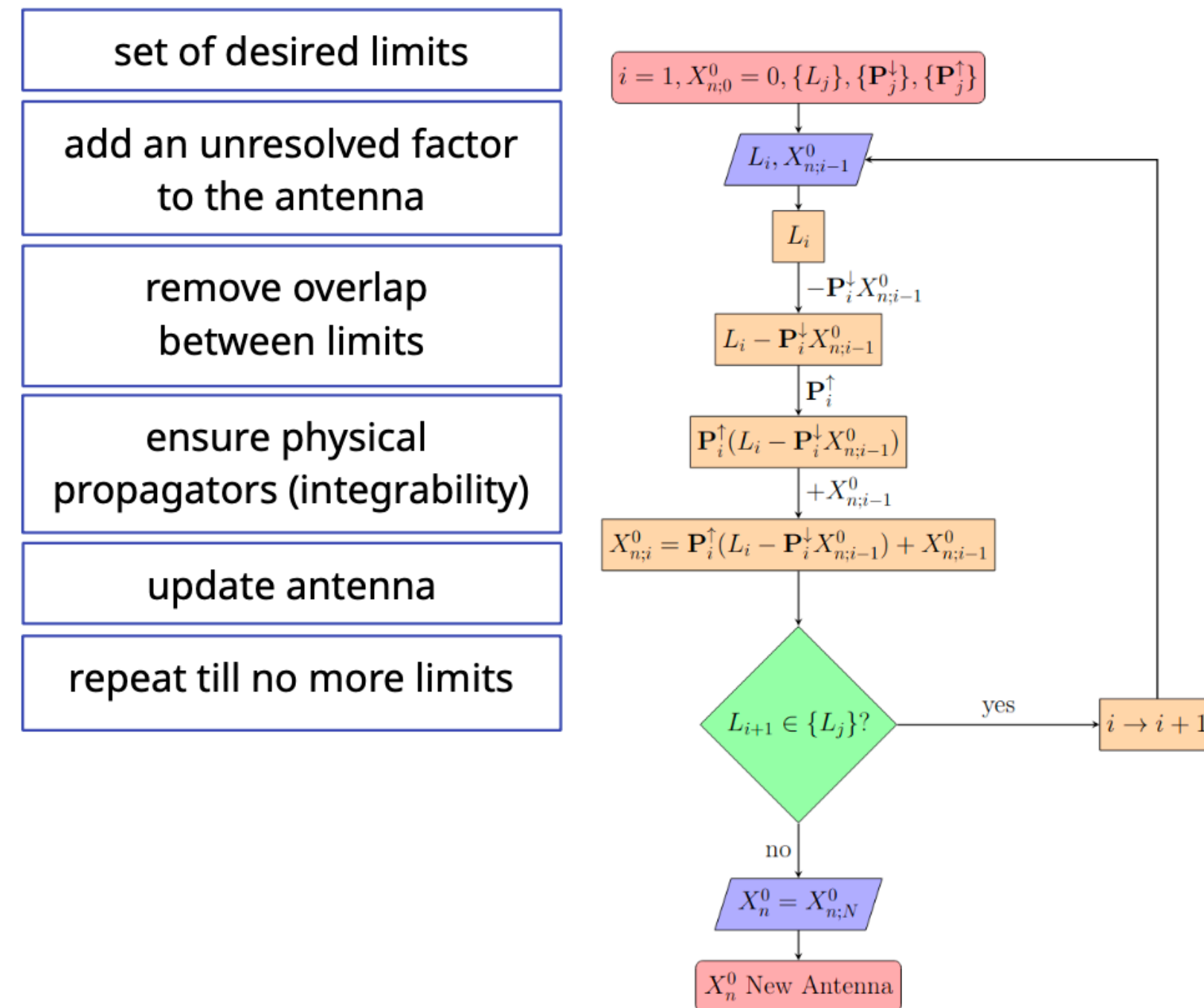
[Chen, Chicherin, Fox, Glover, Marcoli, Sotnikov, Sun, Zhang, Zoia '26]



Calculation made possible by refinements of antenna subtraction at NNLO

Designer antenna functions:

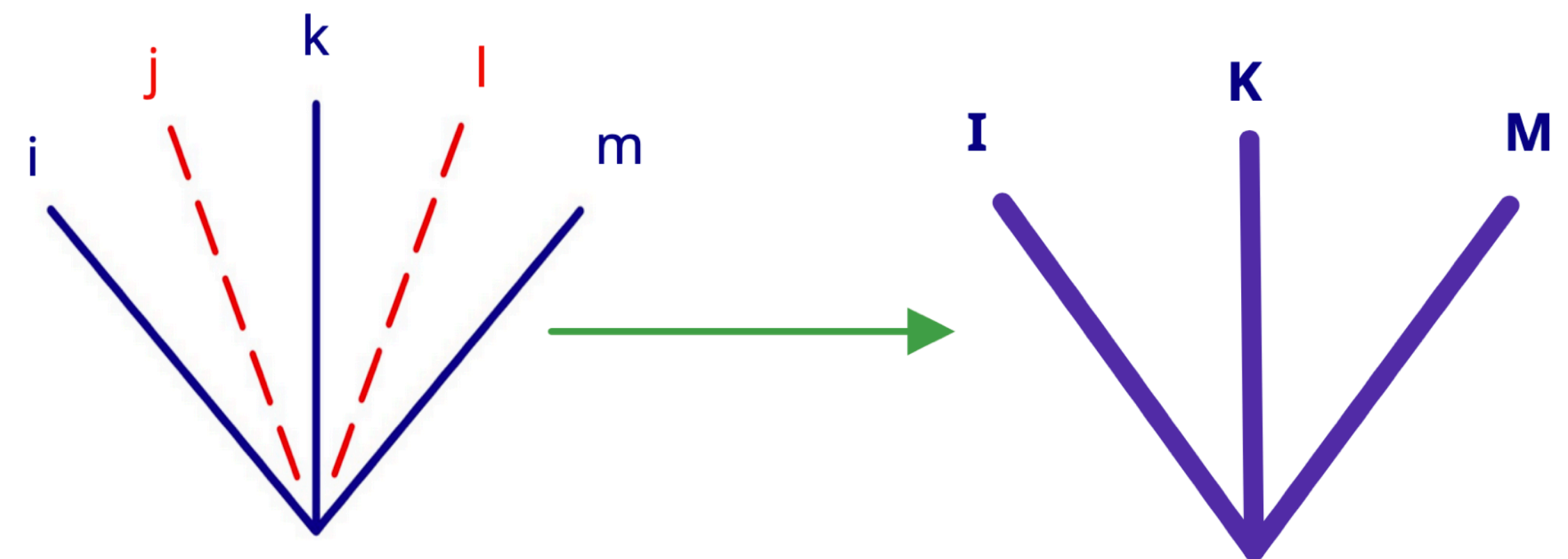
build antenna directly from infrared limits \rightarrow
simpler expressions, less spurious singularities



[Braun-White, Glover, Preuss '22-'23]

Generalised antenna functions:

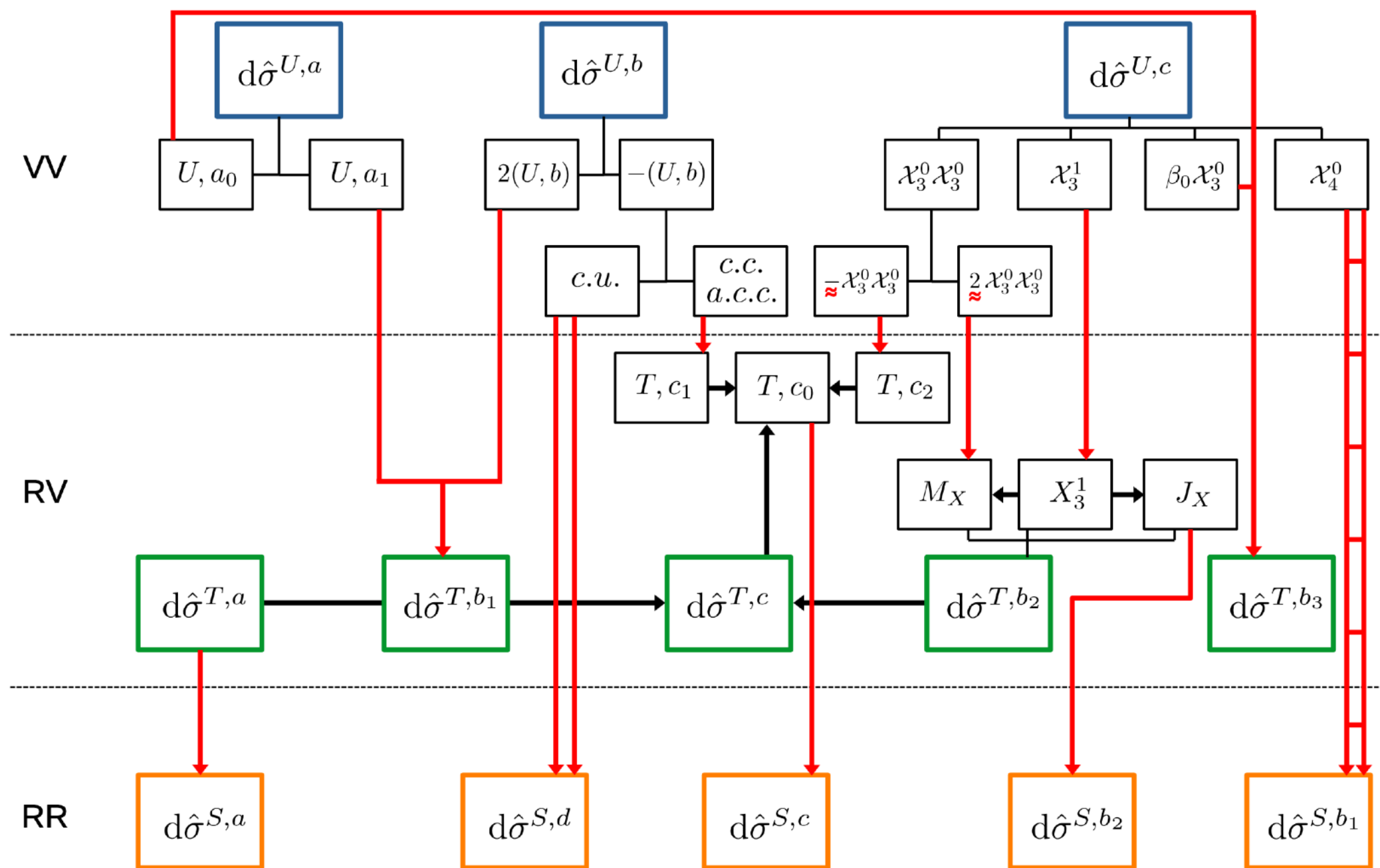
three hard radiators to better deal with almost
colour-connected emissions \rightarrow
simplify subtraction terms



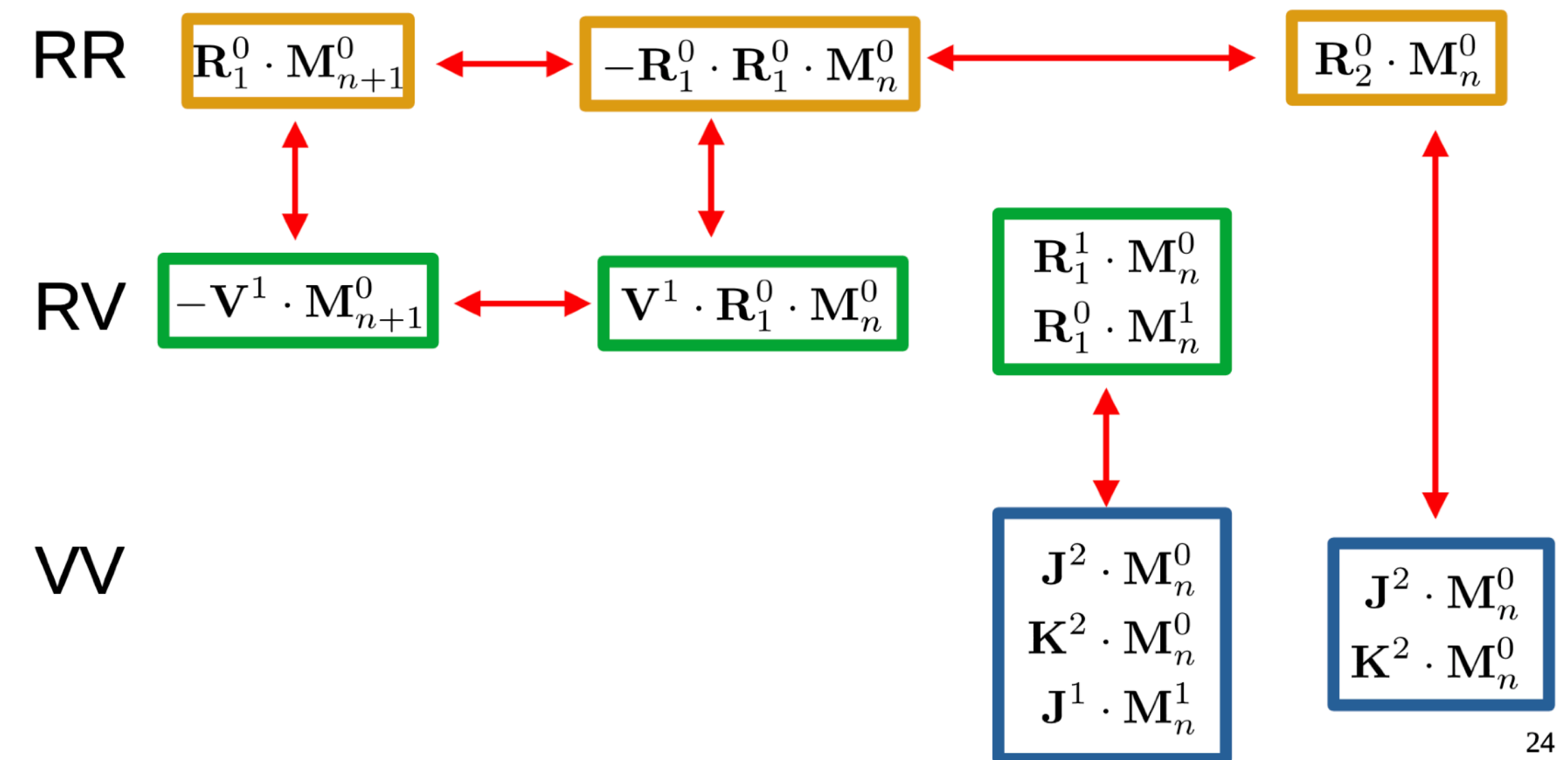
[Fox, Glover, Marcoli '24]

Interdependency of subtraction terms

Before



Now



3 jets @ N3LO ?

The way forward for antenna subtraction:
exploit recent advances at NNLO and extend them to N3LO

Should be feasible on a short timescale. In addition:

- automation of subtraction-term construction
- stability of matrix elements in IR limits

Other NNLO local subtraction methods are mature enough (eg. STRIPPER) or rapidly maturing (eg. nested soft-collinear, local analytic sector subtraction) to motivate their extension to N3LO. Loop-Tree Duality could emerge as competitive approach.

To address the computational burden, new technologies will be relevant
eg. machine learning to speed up evaluation of matrix elements
and/or improve phase space sampling in Monte Carlo integration