EFFICIENT COMPUTATION OF SOFT AND BEAM FUNCTIONS AT NNLO

Emmet Byrne (The University of Manchester)

Based on work performed in collaboration with:
Thomas Clark (The University of Manchester)
Jonathan Gaunt (University of Cyprus)
Alexander Bennett (Durham University)
Elsa Lang (Trinity College Dublin)

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AS SIMPLE AS POSSIBLE…

Over the years a series of techniques has been maturing, enabling the computation of many new soft and beam functions at N^2LO

[1102.4344] Jouttenus, Stewart, Tackmann, Waalewijn
[1206.4312] Tackmann, Walsh, Zuberi
[1412.2126] Banfi, McAslan, Monni, Zanderighi
[1512.00857] Kasemets, Waalewijn, Zeune
[1608.01999] Gangal, Gaunt, Stahlhofen, Tackmann
[2012.09213] Bauer, Manohar and Monni
[2204.02987] Abreu, Gaunt, Monni, Szafron
[2207.07037] Abreu, Gaunt, Monni, Rottoli, Szafron

- Compute new soft and beam functions as corrections to existing results
- Use non-Abelian exponentiation where possible
- For the remaining colour channels compute a simplified observable to capture (most of) the singularities
- Compute the remainder (mostly) numerically

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Recent directions

Higer multiplicity
[2604.13167] Buonocore, Delto, Melnikov, Monni, Pikelner, Vita
N-Jettiness Soft Functions Made Simple

Higher loops
See Samuel's talk
Abreu, Gaunt, Monni, Rottoli, Szafron
Three-loop rapidity anomalous dimension for jet-veto cross sections

More general observables
Part 1: Analytic soft function for generalised event shapes at NNLO

More differential observables
Part 2: NNLO double-differential soft functions for joint resummation of two towers of logs

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There is an existing automated, numerical code to compute soft functions at N^2LO [SoftServe: Bell, Rahn, Talbert], but here our aim is to make N^2LO as simple as possible, and in a manner which can be readily extended to higher orders.

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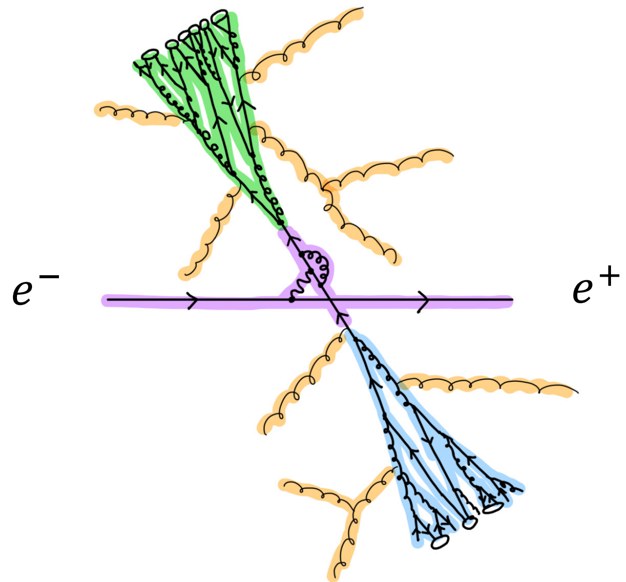
PART 1: Analytic results for C-angularity soft function at NNLO

Based on [2512.23398] with
Alexander Bennett (Durham University), Jonathan Gaunt (University of Cyprus) and Elsa Lang (Trinity College Dublin)

DIJET EVENT SHAPES

Dijet event shapes, τ_e , are distributions that are dominated by two almost back-to-back collimated sets of particles in the limit $\tau_e \to 0$. A wide class of event shapes in $e^+e^- \to X$ is given by

$$\tau_e(X) = \frac{1}{Q}\sum_{i\in X} f_e(\eta_i)|\vec{p}_{i\perp}|$$



Factorisation as $\tau_e \to 0$ [0801.4569] Bauer, Fleming, Lee, Sterman

$$\frac{1}{\sigma^{(0)}}\frac{\mathrm{d}\sigma}{\mathrm{d}\tau_e} = H(Q,\mu)\int \mathrm{d}\tau_1\mathrm{d}\tau_2\mathrm{d}\tau_s\delta(\tau-\tau_1-\tau_2-\tau_s)J_1(\tau_1,\mu)J_2(\tau_2,\mu)S(\tau_s,\mu)$$

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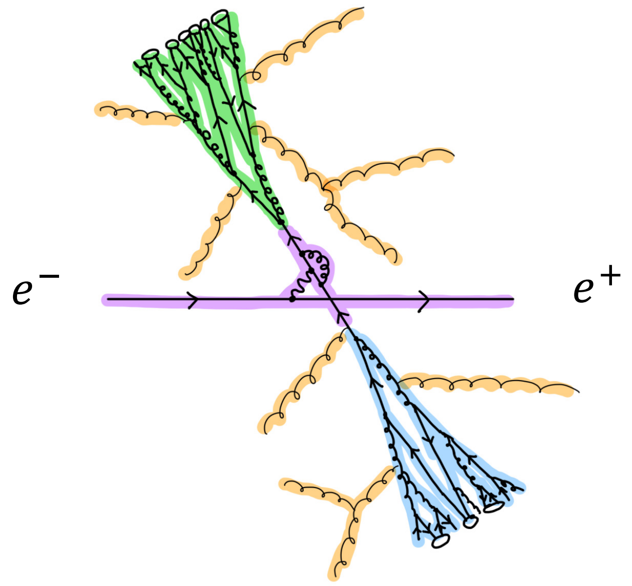

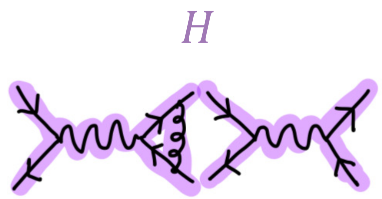

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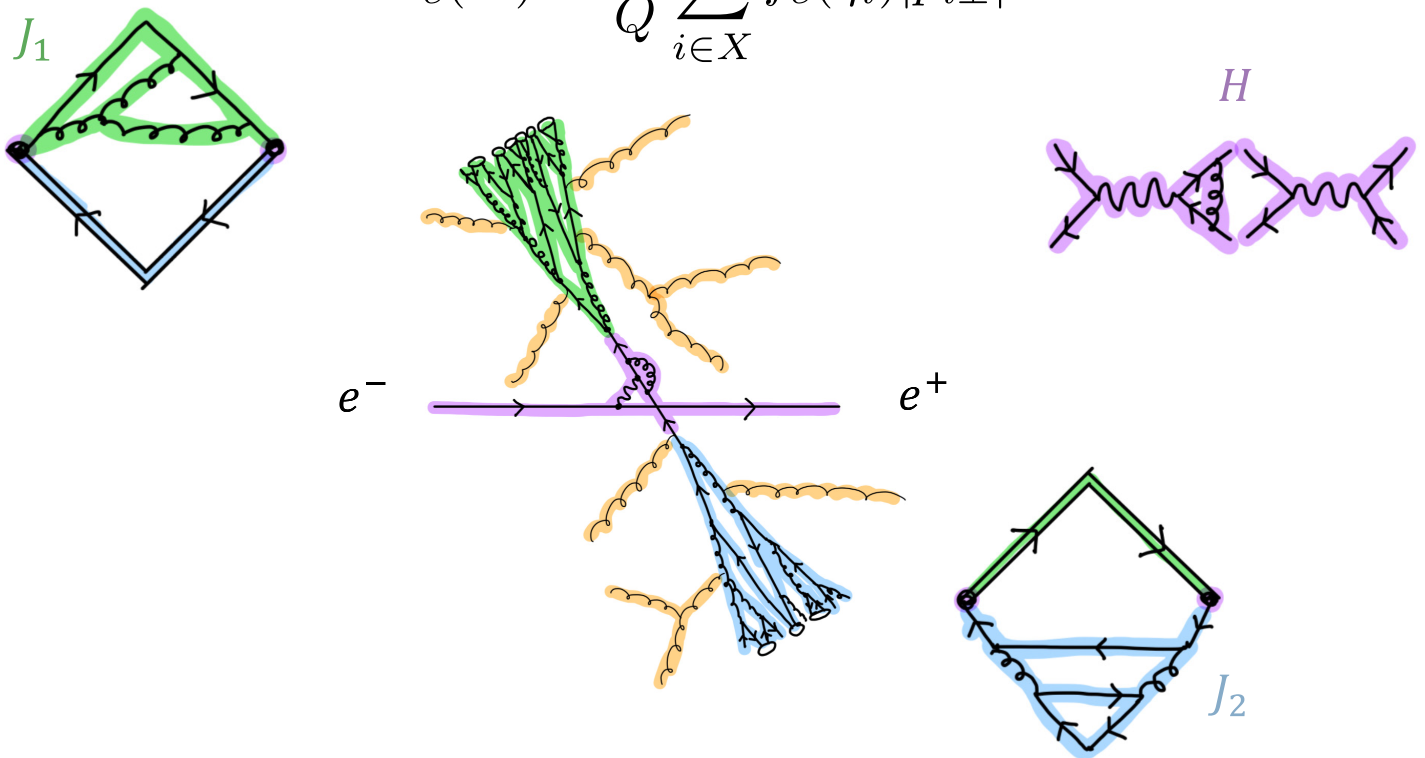

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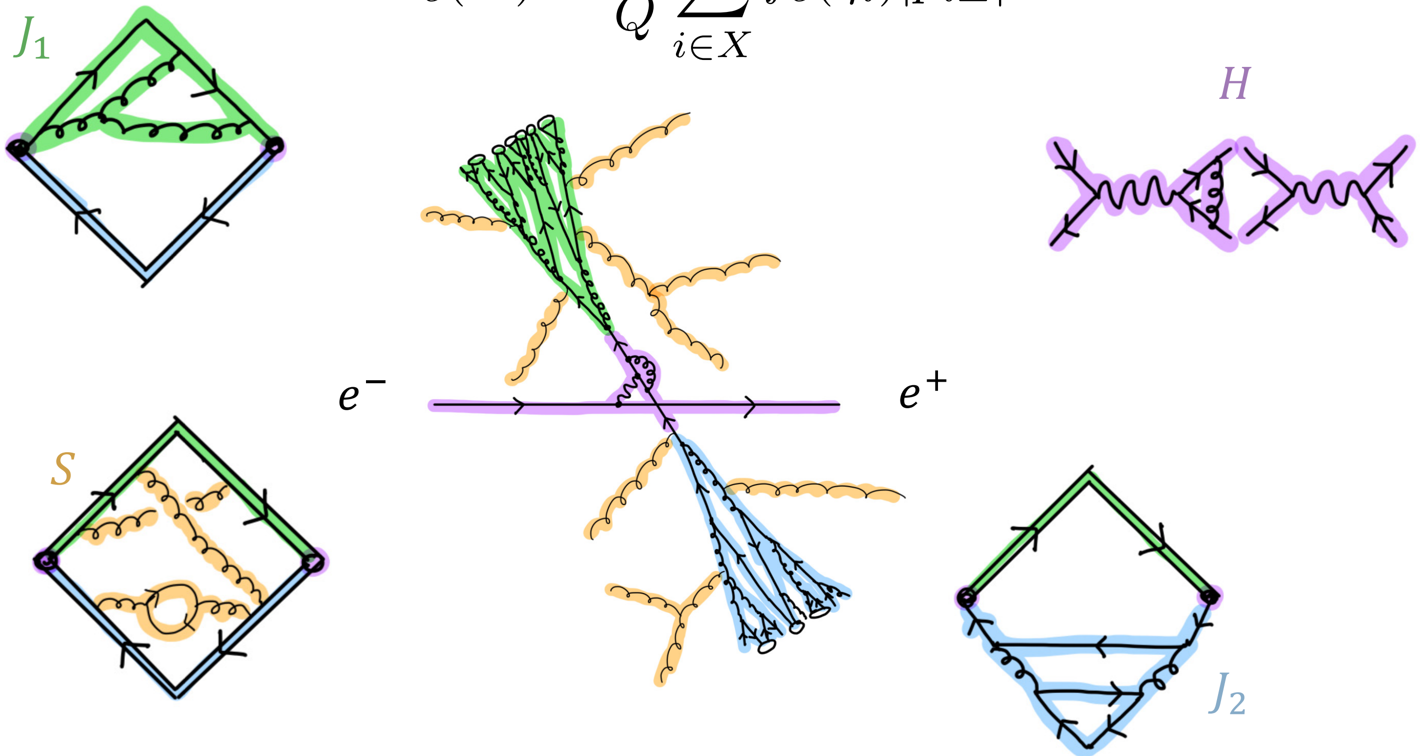

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DIJET EVENT SHAPE LANDSCAPE

Why study e^+e^- event shapes in 2026 ?

- Future colliders: FCC-ee, CEPC, CLIC, ILC
- "Recycling frontier": New measurements of thrust using archived DELPHI [2510.18762] and ALEPH [2507.14349] data
- Tension in α_S extracted from thrust [2412.15164], [2501.18173], [2603.06091],...
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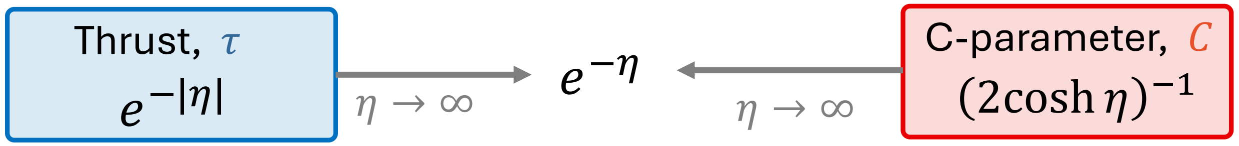

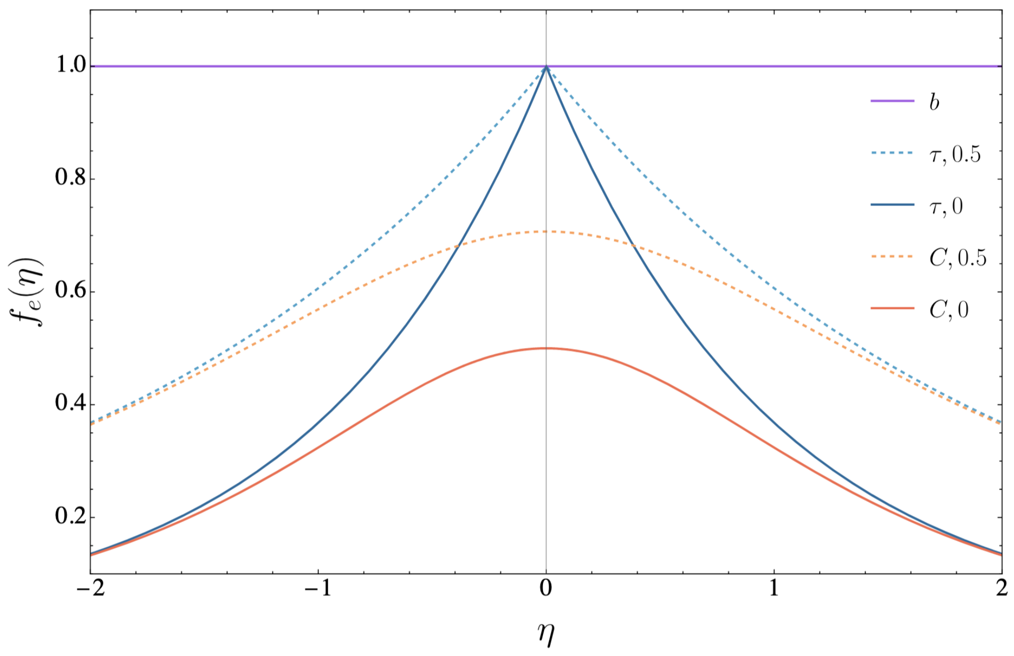

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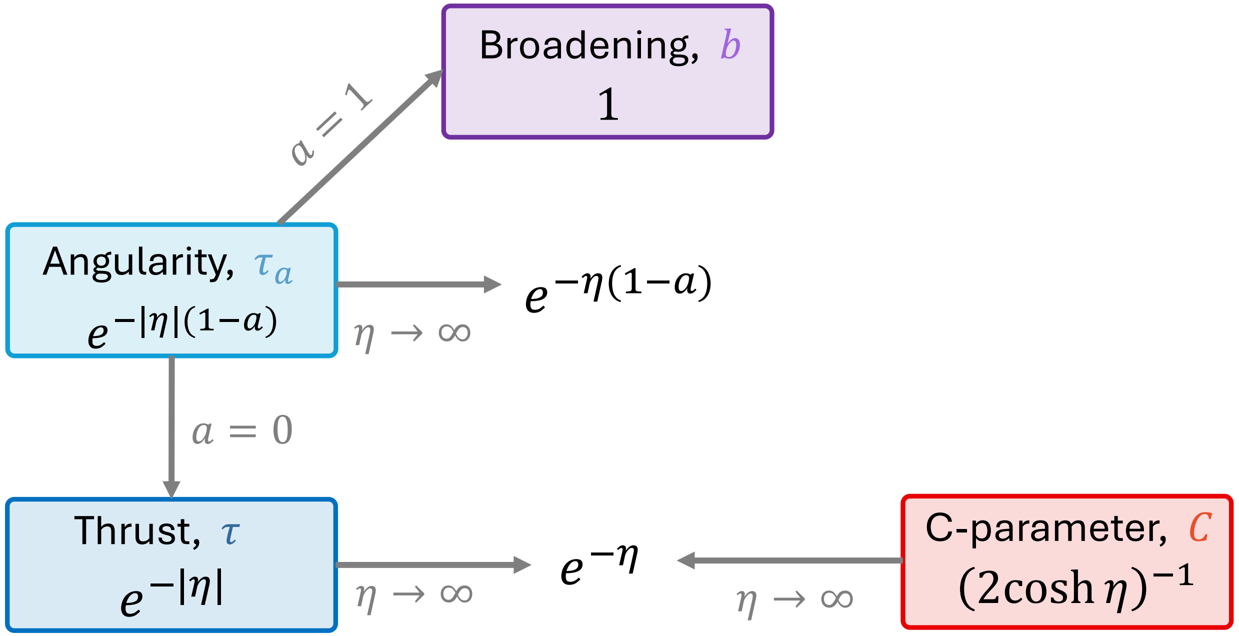

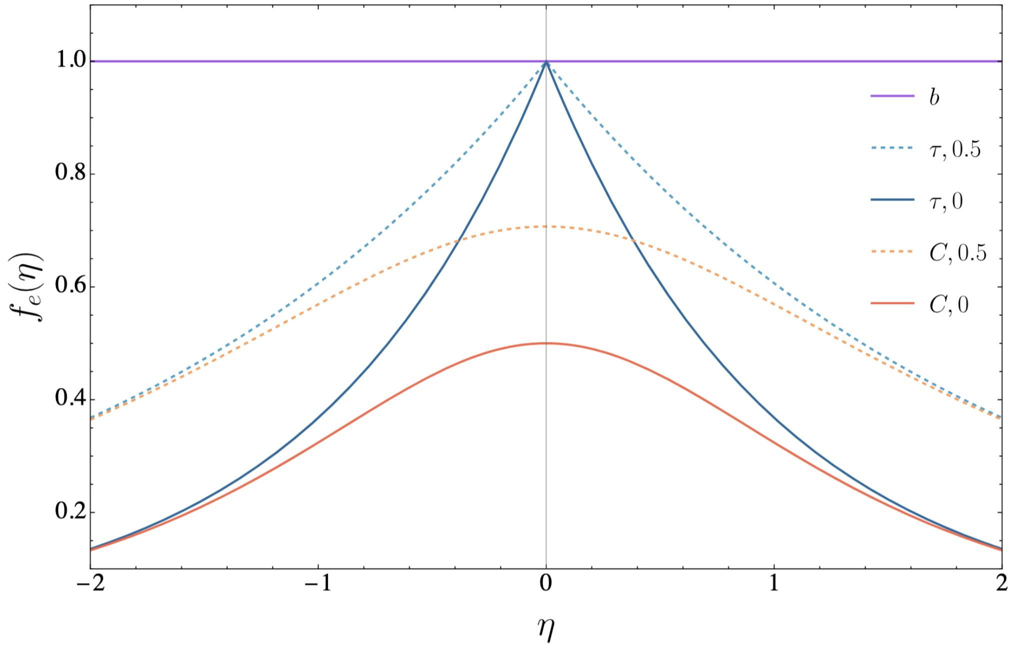

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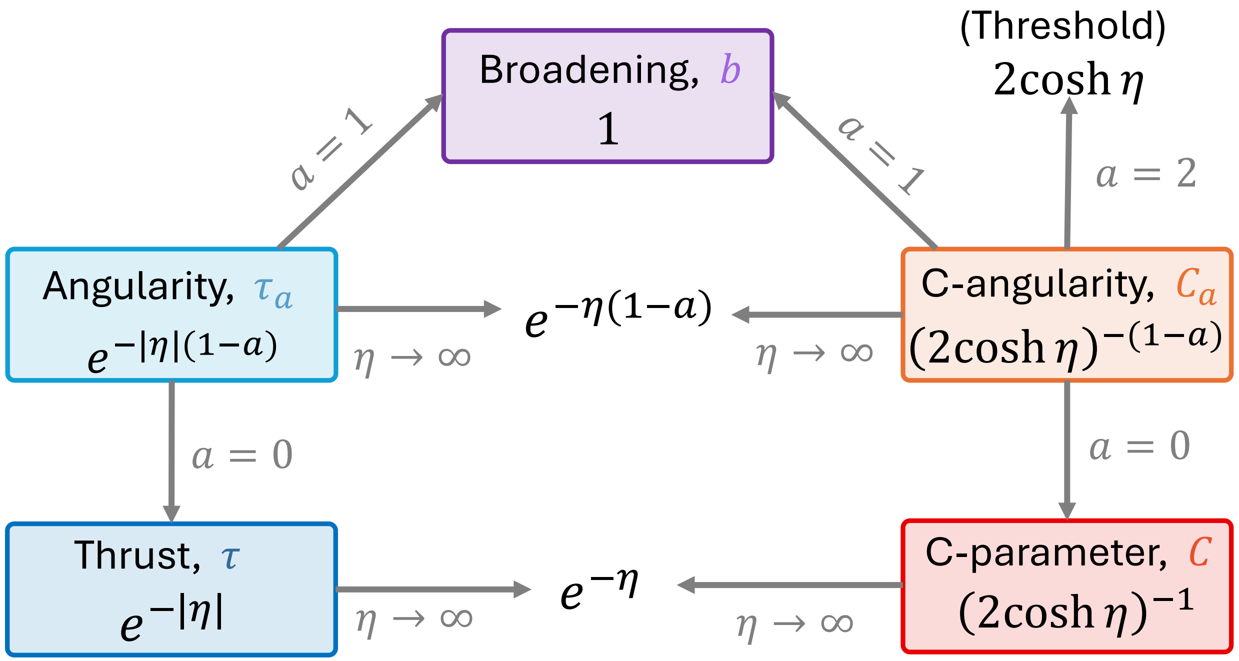

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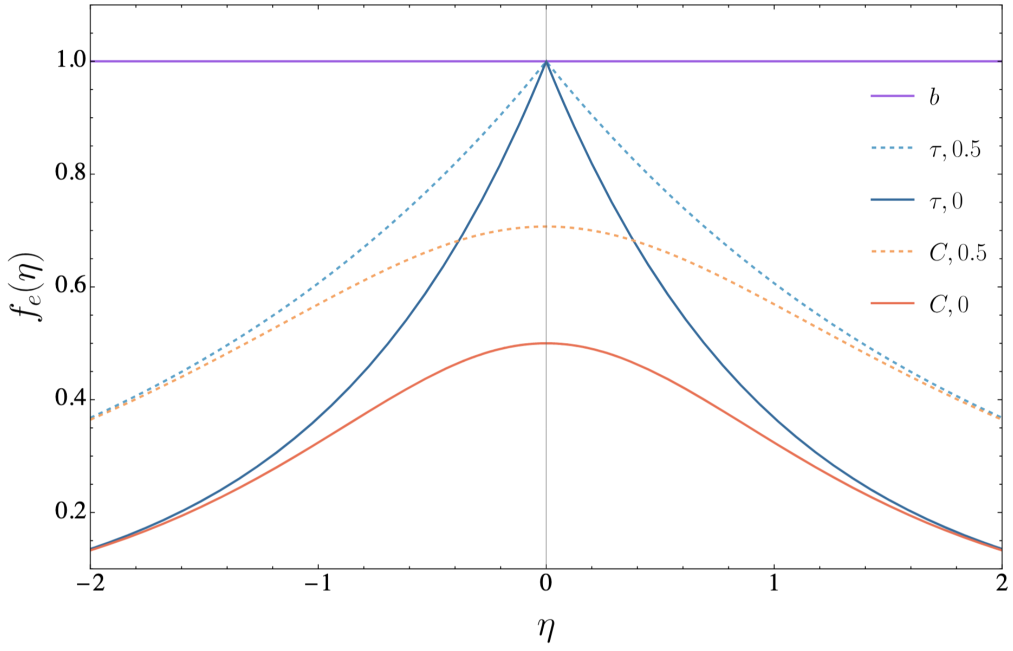

"C-angularity" was briefly mentioned in [hep-ph/0611061] Lee & Sterman.

SOFT FUNCTIONS FOR DIJET EVENT SHAPES AT NLO

At NLO, the soft function for a dijet event shape is given by

$$S_e^{\text{bare}(1)}(\mathcal{T}) = \frac{8C_R e^{\epsilon\gamma_E}}{\Gamma(1-\epsilon)} F_e(\epsilon) \frac{1}{\mu} \left(\frac{\mathcal{T}}{\mu}\right)^{-1-2\epsilon}$$

where the only dependence on the observable is given by the integral

$$F_e = \int_{-\infty}^{\infty} \mathrm{d}\eta \big(f_e(\eta)\big)^{2\epsilon}$$

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$$F_{\tau,a} = \frac{1}{(1-a)\epsilon}$$

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For C-angularity, we have

$$F_{C,a} = \frac{\Gamma(\epsilon(1-a))^2}{2\Gamma(2\epsilon(1-a))} = \frac{1}{\epsilon(1-a)} - \epsilon\zeta_2(1-a) + \epsilon^2(1-a)^2\zeta_3 + \mathcal{O}(\epsilon^3)$$

As an aside, the observable-dependence of the non-perturbative shape function, can be immediately obtained by setting $\epsilon = 1/2$ [Lee & Sterman, hep-ph/0611061, hep-ph/0603066]

SOFT FUNCTIONS FOR DIJET EVENT SHAPES AT NLO

The renormalised soft function satisfies the RGE

$$\mu \frac{\mathrm{d}}{\mathrm{d}\mu} S_e(\mathcal{T},\mu) = \left[\Gamma_S \, \mathcal{L}_0(\mathcal{T},\mu) + \hat{\gamma}_S \delta(\mathcal{T})\right] \otimes_{\mathcal{T}} S_e(\mathcal{T},\mu)$$

$$\mathcal{L}_n(x,\mu) = \left[\frac{\theta(x)}{x} \ln^n \frac{x}{\mu}\right]_+^\mu$$

We expand the renormalized soft function in plus distributions

$$S_e^{(1)}(\mathcal{T}) = -\Gamma_S^0 \, \mathcal{L}_1(\mathcal{T}) - \hat{\gamma}_S^0 \, \mathcal{L}_0(\mathcal{T}) + S_{e,\delta}^{(1)} \, \delta(\mathcal{T})$$

In terms of the integrated shape function,

$$F_e = \int_{-\infty}^{\infty} \mathrm{d}\eta \big(f_e(\eta)\big)^{2\epsilon}$$

the coefficients of these distributions are given by

$$\Gamma_S^0 = 4 F_e^{[-1]} \Gamma_{\mathrm{cusp}}^0 \qquad \hat{\gamma}_S^0 = -8 C_R F_e^{[0]} \qquad S_{e,\delta}^{(1)} = 2 C_R \left[\zeta_2 F_e^{[-1]} - 2 F_e^{[1]}\right]$$

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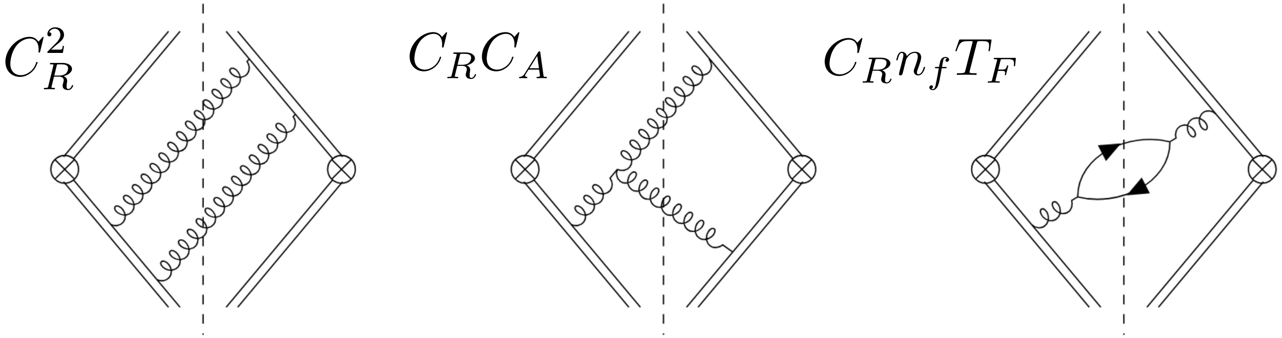
At NNLO, there are three colour factors. Representative diagrams for these colour factors are:



The coefficients $F_e^{[-1]}, F_e^{[0]}, F_e^{[1]}$ fully determine the C_R^2 term, which is given by "squaring" the NLO result.

$$S_e^{(2,C_R^2)}(\mathcal{T}) = \frac{1}{2} S_e^{(1)}(\mathcal{T}) \otimes_{\mathcal{T}} S_e^{(1)}(\mathcal{T})$$

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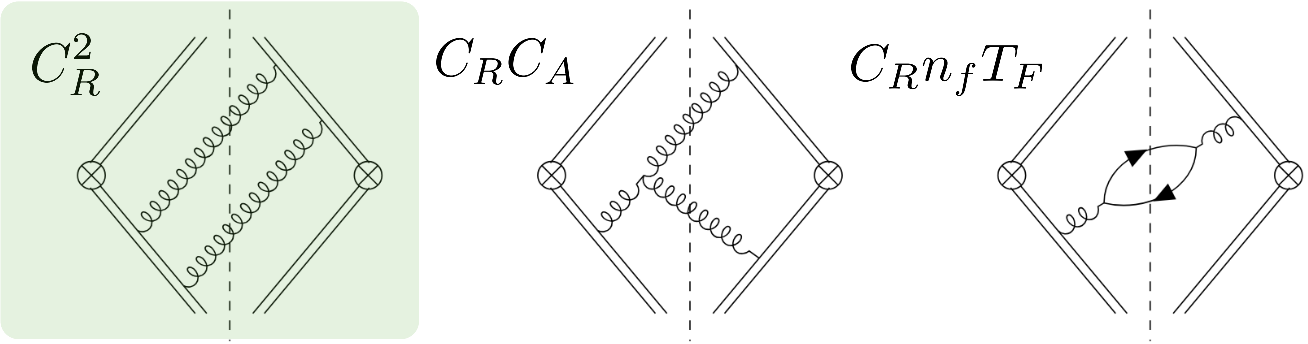

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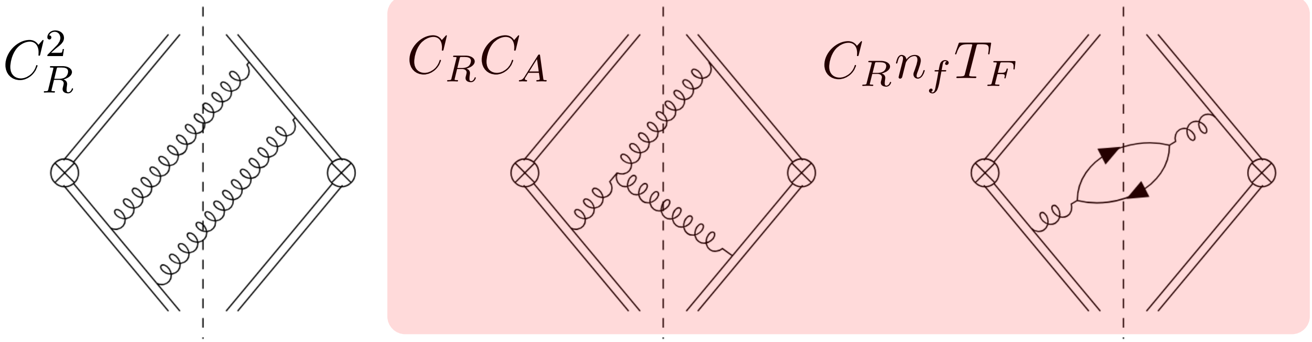

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For the non-exponentiating colour factors, it is useful to consider the *inclusive* analogue of the observable

$$\mathcal{M}_e(\mathcal{T}_{\text{cut}}) = \theta\Big(\sum_i \mathcal{T}_e(k_i) < \mathcal{T}_{\text{cut}}\Big) \qquad \mathcal{M}_{e_I}(\mathcal{T}_{\text{cut}}) = \theta\Big(\mathcal{T}_e(\sum_i k_i) < \mathcal{T}_{\text{cut}}\Big)$$

See also [2012.09213], [2604.13167]

We then write the full soft function as the inclusive soft function, plus a correction

$$S_e^{\text{bare}(2,c)}(\mathcal{T}_{\text{cut}}) = S_{e_I}^{\text{bare}(2,c)}(\mathcal{T}_{\text{cut}}) + S_{e-e_I}^{\text{bare}(2,c)}(\mathcal{T}_{\text{cut}}) \qquad c \in \{C_A, n_f T_F\}$$

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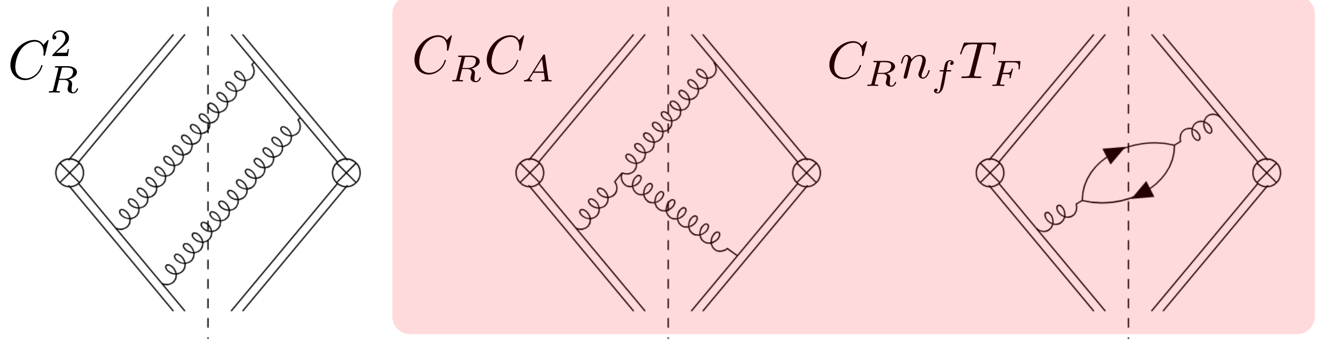

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$$\mathcal{T}_{C,a}(k_i) = \frac{(k_i^+ k_i^-)^{1-\frac{a}{2}}}{(k_i^+ + k_i^-)^{1-a}}$$

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INCLUSIVE SOFT FUNCTIONS FOR DIJET EVENT SHAPES AT NNLO

Similar to what we did at NLO,

$$\Gamma_S^0 = 4F_e^{[-1]}\Gamma_{\text{cusp}}^0 \qquad \hat{\gamma}_S^0 = -8C_R F_e^{[0]} \qquad S_{e,\delta}^{(1)} = 2C_R\left[\zeta_2 F_e^{[-1]} - 2F_e^{[1]}\right]$$

solving the RGE lets us read off the 2-loop anomalous dimensions and constant term in terms of F_e. The cusp is simply

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The non-cusp anomalous dimension only needs the ϵ^0 coefficient:

$$\hat{\gamma}_{S_{eI}}^{(1,n_f T_F)} = \left(\frac{448}{27} - \frac{16}{3}\right) F_e^{[-1]} + \frac{160}{9} F_e^{[0]}$$

$$\hat{\gamma}_{S_{eI}}^{(1,n_f T_F)} = \left(-\frac{1616}{27} + \frac{44}{3}\zeta_2 + 56\zeta_3\right) F_e^{[-1]} + \left(-\frac{536}{9} + 16\zeta_2\right) F_e^{[0]}$$

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Finally, the boundary terms need the ϵ^1 and ϵ^2 coefficient of F_e:

$$S_{eI,\delta}^{(2,n_f T_F)} = \left(\frac{656}{81} - \frac{20}{3}\zeta_2 - \frac{40}{9}\zeta_3\right) F_e^{[-1]} + \left(\frac{224}{27} - \frac{16}{3}\zeta_2\right) F_e^{[0]} + \frac{80}{9} F_e^{[1]} + \frac{16}{3} F_e^{[2]}$$

$$S_{eI,\delta}^{(2,n_f T_F)} = \left(-\frac{2428}{81} + \frac{67}{3}\zeta_2 + \frac{110}{9}\zeta_3 + 10\zeta_4\right) F_e^{[-1]} + \left(-\frac{808}{27} + \frac{44}{3}\zeta_2 + 28\zeta_3\right) F_e^{[0]} + \left(-\frac{268}{9} + 8\zeta_2\right) F_e^{[1]} - \frac{44}{3} F_e^{[2]}$$

CORRECTION FROM INCLUSIVE TO PARTONIC

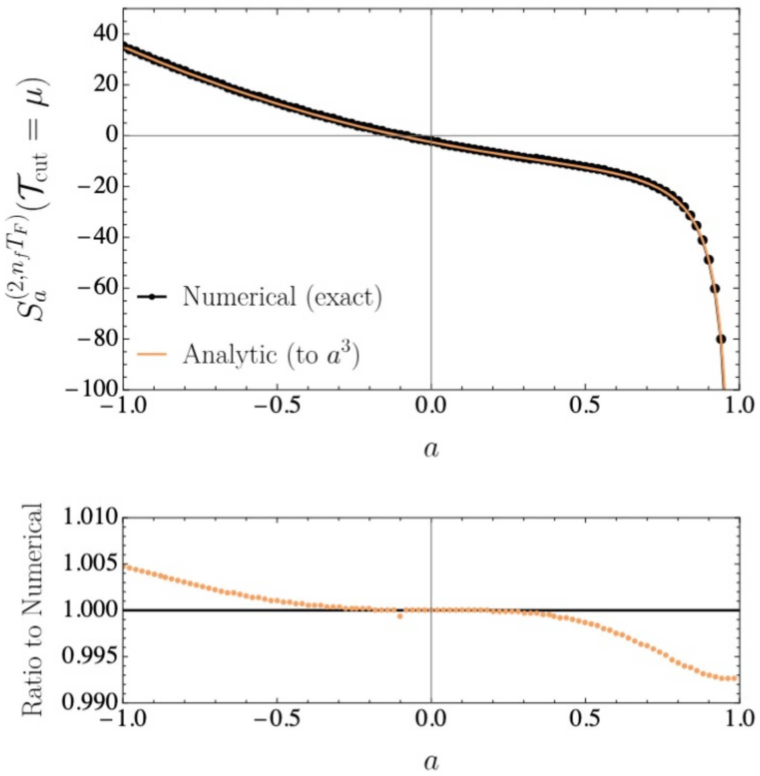
Recall our split $$S_e^{\mathrm{bare}(2,c)}(\mathcal{T}_{\mathrm{cut}}) = S_{e_I}^{\mathrm{bare}(2,c)}(\mathcal{T}_{\mathrm{cut}}) + S_{e-e_I}^{\mathrm{bare}(2,c)}(\mathcal{T}_{\mathrm{cut}})$$

The correction from the inclusive can be written in terms of finite integrals

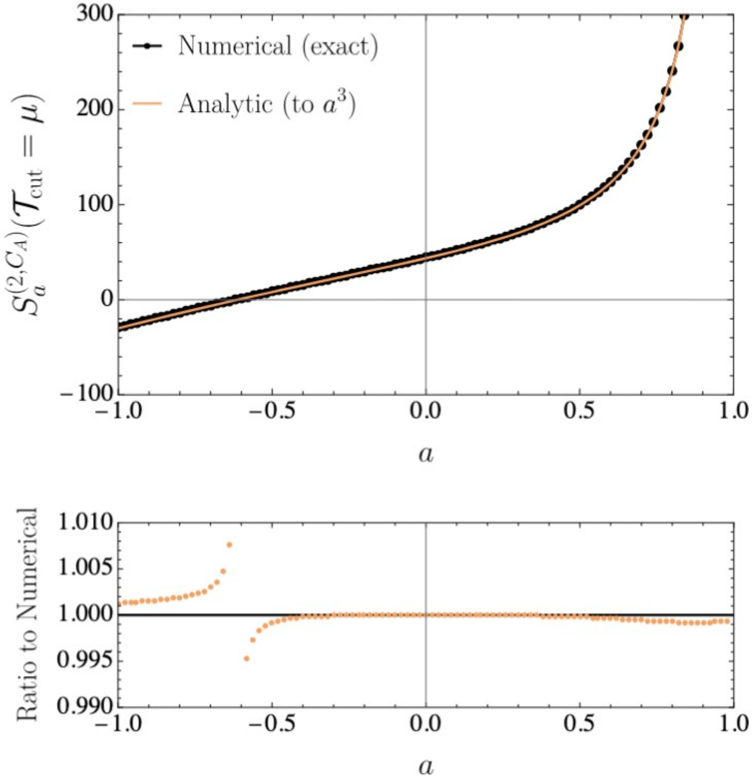
$$S_{e-e_I}^{\mathrm{bare}(2,c)}(\mathcal{T}_{\mathrm{cut}}) = \frac{1}{2(1-a)\epsilon} I_{\mathrm{div.}}^{c,0} + 2 I_{\mathrm{reg.}}^{c,0} + \frac{1}{2(1-a)} \left(4 I_{\mathrm{div.}}^{c,0} \log\left(\frac{\mu}{\mathcal{T}_{\mathrm{cut}}}\right) + I_{\mathrm{div.}}^{c,1} \right)$$

which can be computed numerically, or analytically as an expansion in a.

$C_R n_f T_F$



$C_R C_A$



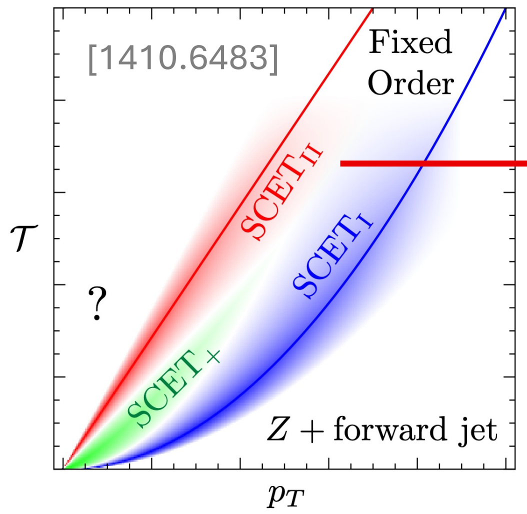
PART 2: Double-differential soft functions

Work with Jonathan Gaunt, to appear soon.

RECENT PROGRESS IN JOINT RESUMMATION

[1401.4458] Larkoski, Moult, Neill, [1806.10622] Procura, Waalewijn, Zeune: Two angularities.

[1410.6483] Procura, Waalewijn, Zeune: $pp \to Z + 0j$ where the transverse momentum p_T of the Z is measured and jets are vetoed via beam thrust, $\mathcal{T}$. There is an interesting landscape of resummation, but in this talk, we focus on $\mathcal{T} \sim p_T$, where the soft function is sensitive to both scales.



$$\frac{\mathrm{d}^4\sigma}{\mathrm{d}Q^2\,\mathrm{d}Y\,\mathrm{d}p_T^2\,\mathrm{d}\mathcal{T}} = \sum_q \hat{\sigma}_q^0\, H(Q^2,\mu) \int \mathrm{d}^2\vec{k}_{1\perp}\,\mathrm{d}^2\vec{k}_{2\perp}\,\mathrm{d}^2\vec{k}_{\perp} \int \mathrm{d}k^+\,\delta\big(p_T^2 - |\vec{k}_{1\perp}+\vec{k}_{2\perp}+\vec{k}_{\perp}|^2\big)\,\delta(\mathcal{T}-k^+)$$
$$\times \Big[B_q(x_1,\vec{k}_{1\perp},\mu,\nu)\,B_{\bar{q}}(x_2,\vec{k}_{2\perp},\mu,\nu) + (q \leftrightarrow \bar{q})\Big]\, S(k^+,\vec{k}_\perp,\mu,\nu)\,.$$

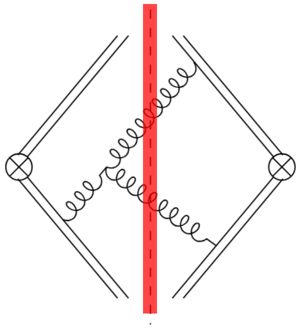
[1901.03331] Lustermans, Michel, Tackmann, Waalewijn: $p_T^Z\ /Q$ and $\mathcal{T}_0\ /Q$ resummed to NNLL&NLO.

[1909.04704] Monni, Rottoli, Torielli: $p_T^H\ /m_H$ and p_T^J/m_H resummed to NNLL.

These double-differential results are also useful in studies of the Underlying Event [1602.08980], and could be used to improve the formal accuracy of the GENEVA generator.

SOFT MEASUREMENT FUNCTIONS

Consider the production of a colour singlet h in addition to a hadronic final state, X, $pp \to h + X$



Transverse momentum of h

$$\mathcal{M}_T(p_{T\mathrm{c}}) = \theta\Big(\big|\sum_i \vec{k}_{iT}\big| < p_{T\mathrm{c}}\Big)$$

TMD N3LO [1604.01404]

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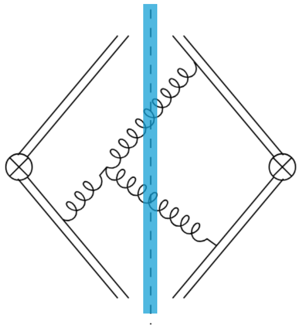

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$$\mathcal{M}_T(p_{Tc}) = \theta\Big(\big|\sum_i \vec{k}_{iT}\big| < p_{Tc}\Big)$$

TMD N3LO [1604.01404]

Jet veto on X

$$\mathcal{M}_e(\mathcal{T}_c) = \theta\Big(\sum_i \mathcal{T}_e(k_i) < \mathcal{T}_{\text{cut}}\Big)$$

Beam thrust N2LO [1105.3676]
'Beam C-parameter' N2LO [1812.08690]

$$\mathcal{M}_{e_J}(\mathcal{T}_c, R) = \prod_{j \in J(R)} \theta\Big(\mathcal{T}_e(k_j) < \mathcal{T}_c\Big)$$

Rapidity-dependent jet vetos [1412.4792]
B_J, C_J N2LO [1608.01999]

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Inclusive angularity [2012.09213]
Inclusive C-angularity [2512.23398]
Inclusive N-jettiness [2604.13167]

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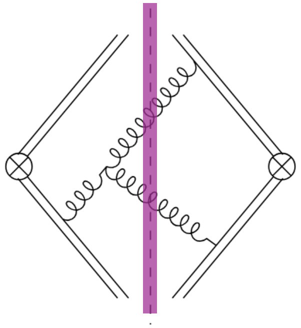

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$=$

Joint measurement [this work]

$$\mathcal{M}_{Te}(p_{Tc}, \mathcal{T}_c) = \mathcal{M}_T(p_{Tc})\mathcal{M}_e(\mathcal{T}_c)$$

$$\mathcal{M}_{Te_J}(p_{Tc}, \mathcal{T}_c, R) = \mathcal{M}_T(p_{Tc})\mathcal{M}_{e_J}(\mathcal{T}_c, R)$$

$$\mathcal{M}_{Te_I}(p_{Tc}, \mathcal{T}_c) = \mathcal{M}_T(p_{Tc})\mathcal{M}_{e_I}(\mathcal{T}_c)$$

Not IR safe, but a useful intermediate step

JOINT SOFT FUNCTION AT NLO

At NLO we have at most one real emission and the jet veto measurements coincide.

$$S^{(1)}_{Te}(\mathcal{T}_c, p_{Tc}, \mu, \nu) = S^{(1)}_{T}(p_{Tc}, \mu, \nu) + \Delta S^{(1)}_{Te}(\mathcal{T}_c, p_{Tc})$$

For $e = B$ (thrust/0-jettiness, B_J etc.) and $e = C$ (C-parameter, C_J etc.) we find a similar structure

$$\Delta S^{(1)}_{TB}(\mathcal{T}_c, p_{Tc}) = -8C_R \theta\left(p_{Tc} > \mathcal{T}_c\right) I^B_0(q_c)$$

$$I^B_0(q) = \frac{1}{2}\ln^2 q$$

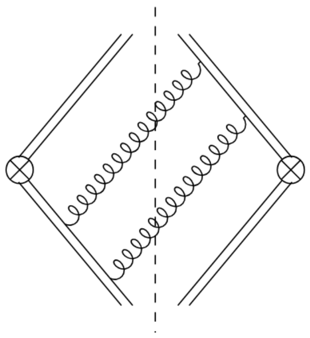
[1410.6483] Procura, Wallewijn, Zeune

$$\Delta S^{(1)}_{TC}(\mathcal{T}_c, p_{Tc}) = -8C_R \theta\left(p_{Tc} > 2\mathcal{T}_c\right) I^C_0(u_c)$$

$$I^C_0(u) = \frac{1}{2}\left[\mathrm{Li}_2\left(\tfrac{1-u}{2}\right) - \mathrm{Li}_2\left(\tfrac{1+u}{2}\right) + \frac{1}{2}\ln\left(\tfrac{1-u}{1+u}\right)\ln\left(\tfrac{1-u^2}{4}\right)\right]$$

The 'natural' variables are $q_c = \dfrac{p_{Tc}}{\mathcal{T}_c}$ and $u_c = \sqrt{1 - \left(\dfrac{\mathcal{T}_c}{2p_{Tc}}\right)^2}$ respectively.

NNLO: C_R^2 CHANNEL



For the event-shape case, exponentiation is a double-convolution in momentum space:

$$S_{Te}^{(2,C_R)}(\vec{p}_T, \mathcal{T}) = \frac{1}{2}[S_{Te}^{(1)} \otimes_{T,\mathcal{T}} S_{Te}^{(1)}](\vec{p}_T, \mathcal{T})$$

The jet-based case is given by a single convolution of the $\mathcal{T}$-cumulant plus an R-dependent correction:

$$S_{Te_J}^{(2,C_R)}(\vec{p}_T, \mathcal{T}_{\text{cut}}, R) = \frac{1}{2}[S_{Te}^{(1)} \otimes_T S_{Te}^{(1)}](\vec{p}_\perp, \mathcal{T}_{\text{cut}}) + \Delta S_{Te_J}^{(2,C_R)}(\vec{p}_\perp, \mathcal{T}_{\text{cut}}, R)$$

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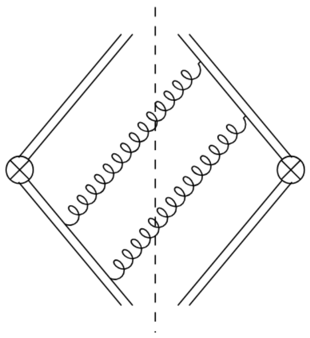

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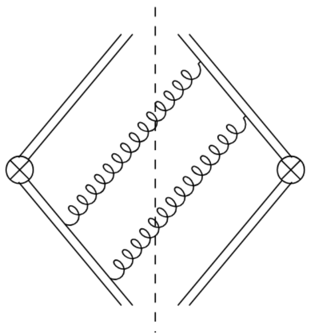
We can perform this correction as a 3-fold numerical integral:

$$\Delta S_{Te_J}^{(2,C_R)}(p_{T\rm c},\mathcal{T}_{\rm c},R) = \int {\rm d}z\,{\rm d}\Delta y\,{\rm d}\Delta\phi \left[\int \frac{{\rm d}T}{T}\frac{{\rm d}Y_t}{Y_t}\mathcal{M}_T(p_{T\rm c})\Big[\mathcal{M}_{e_J}(\mathcal{T}_{\rm c},0)-\mathcal{M}_{e_I}(\mathcal{T}_{\rm c})\Big]\right]\theta(\Delta R<R)\mathcal{A}^{(2,C_R)}(z,\Delta y,\Delta\phi)$$

NNLO: C_R^2 CHANNEL



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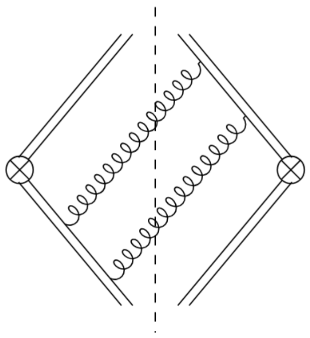
Weight-2 effective measurement

NNLO: C_R^2 CHANNEL



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We can perform this correction as a 3-fold numerical integral:

$$\Delta S_{Te_J}^{(2,C_R)}(p_{T\text{c}},\mathcal{T}_\text{c},R) = \int \mathrm{d}z\,\mathrm{d}\Delta y\,\mathrm{d}\Delta\phi \left[\underbrace{\int \frac{\mathrm{d}T}{T}\frac{\mathrm{d}Y_t}{Y_t}\mathcal{M}_T(p_{T\text{c}})\Big[\mathcal{M}_{e_J}(\mathcal{T}_\text{c},0) - \mathcal{M}_{e_I}(\mathcal{T}_\text{c})\Big]}_{\tilde{\mathcal{M}}_{T[e_J-e_I]}(p_{T\text{c}},\mathcal{T}_\text{c},z,\Delta y,\Delta\phi)}\right]\theta(\Delta R < R)\mathcal{A}^{(2,C_R)}(z,\Delta y,\Delta\phi)$$

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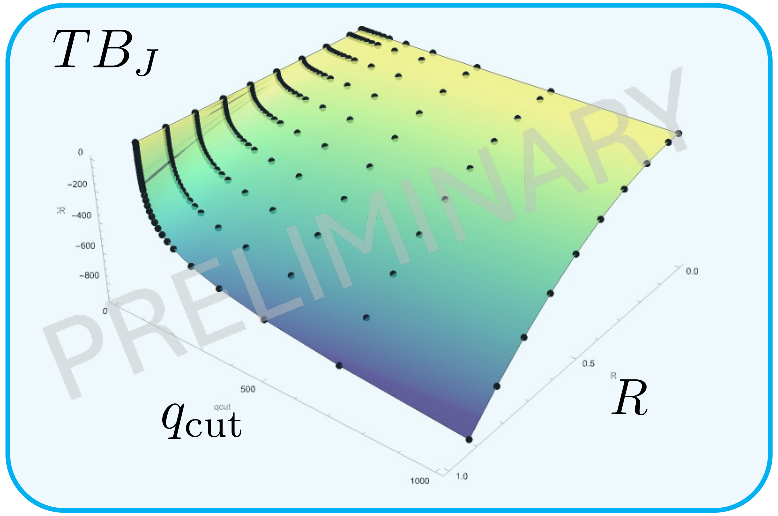
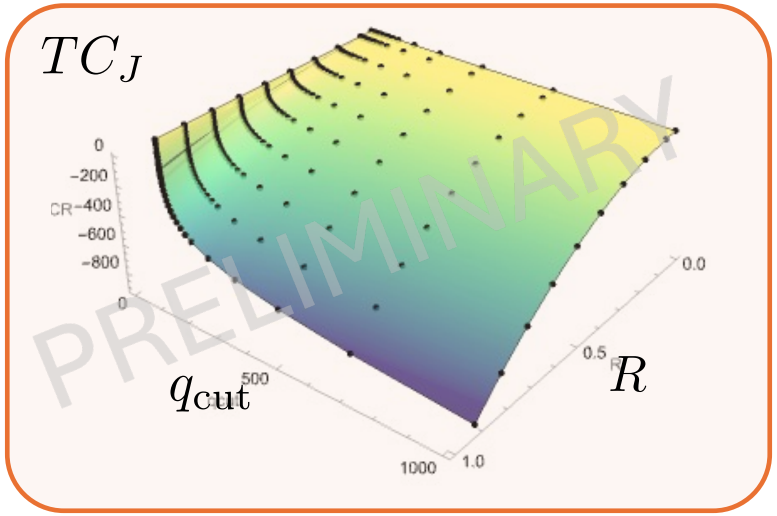
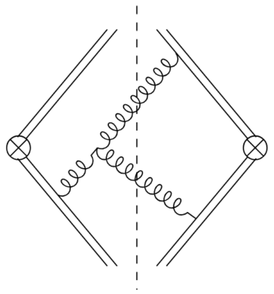
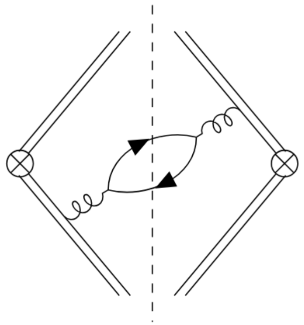




NNLO: $C_R C_A$ & $C_R n_f T_F$





For the $C_R C_A$ and $C_R n_f T_F$, we can obtain the inclusive soft function by integrating the fully-differential soft function

$$\Delta S_{Te_I}^{\mathrm{bare}(c,2)}(p_{T\mathrm{c}}, \mathcal{T}_\mathrm{c}) = \int \mathrm{d}q^+ \mathrm{d}q^- \mathrm{d}^{2-2\epsilon}\vec{q}_T \, \Delta M_{Te_I}(p_{T\mathrm{c}}, \mathcal{T}_\mathrm{c}) \, S_{\mathrm{FD}}^{\mathrm{bare}(2,c)}(q^+, q^-, \vec{q}_T)$$

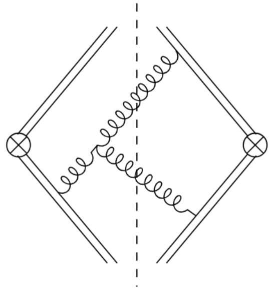
Lee, Mantry, Petriello, [1105.5171]

Then we write the partonic and jet observables as corrections to this inclusive function:

$$\Delta S_{Te}^{(2,c)}(p_{T\mathrm{c}}, \mathcal{T}_\mathrm{c}) = \Delta S_{Te_I}^{(2,c)}(p_{T\mathrm{c}}, \mathcal{T}_\mathrm{c}) + \Delta S_{T[e-e_I]}^{(2,c)}(p_{T\mathrm{c}}, \mathcal{T}_\mathrm{c})$$

$$\Delta S_{Te_J}^{(2)}(p_{T\mathrm{c}}, \mathcal{T}_\mathrm{c}, R) = \Delta S_{Te_I}^{(2)}(p_{T\mathrm{c}}, \mathcal{T}_\mathrm{c}) + \Delta S_{T[e_J-e_I]}^{(2)}(p_{T\mathrm{c}}, \mathcal{T}_\mathrm{c}, R)$$

NNLO: $C_R C_A$ & $C_R n_f T_F$



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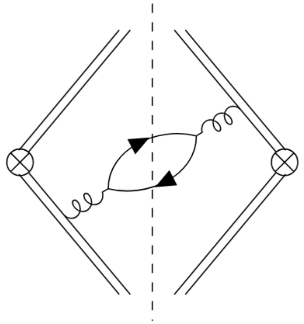

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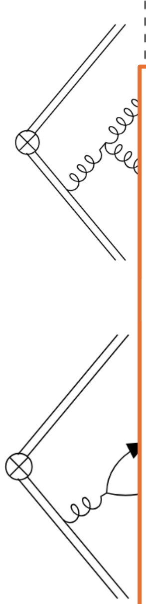
For the jet measurement we can use the previous effective measurement!

$$\Delta S_{T[e_J-e_I]}^{(2,c)}(p_{T\mathrm{c}}, \mathcal{T}_\mathrm{c}) = \int \mathrm{d}z \, \mathrm{d}\Delta y \, \mathrm{d}\Delta\phi \, \theta(\Delta R > R) \tilde{\mathcal{M}}_{T[e_I-e_J]} \mathcal{A}^{(2,c)}(z, \Delta y, \Delta\phi)$$

NNLO: $C_R C_A$ & $C_R n_f T_F$



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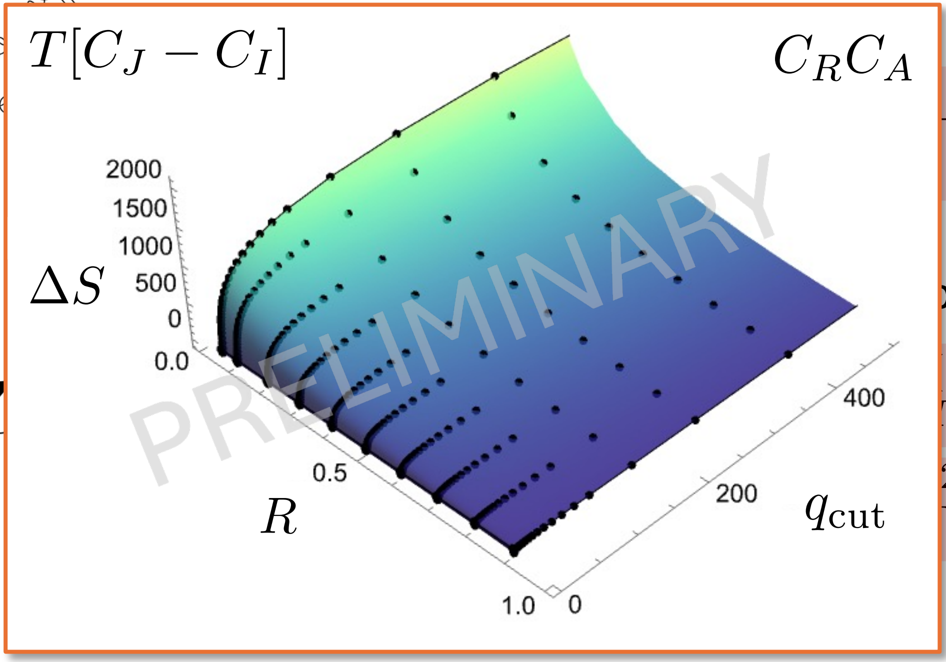

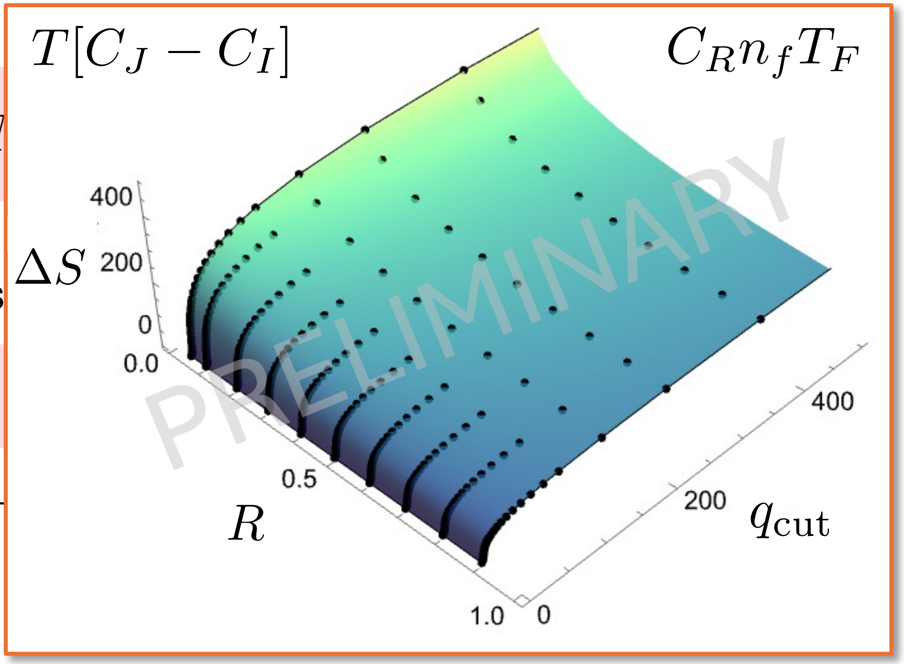

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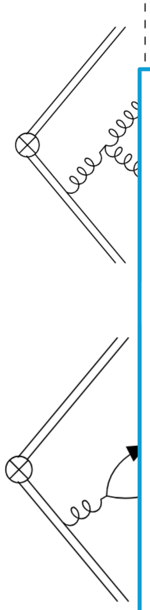
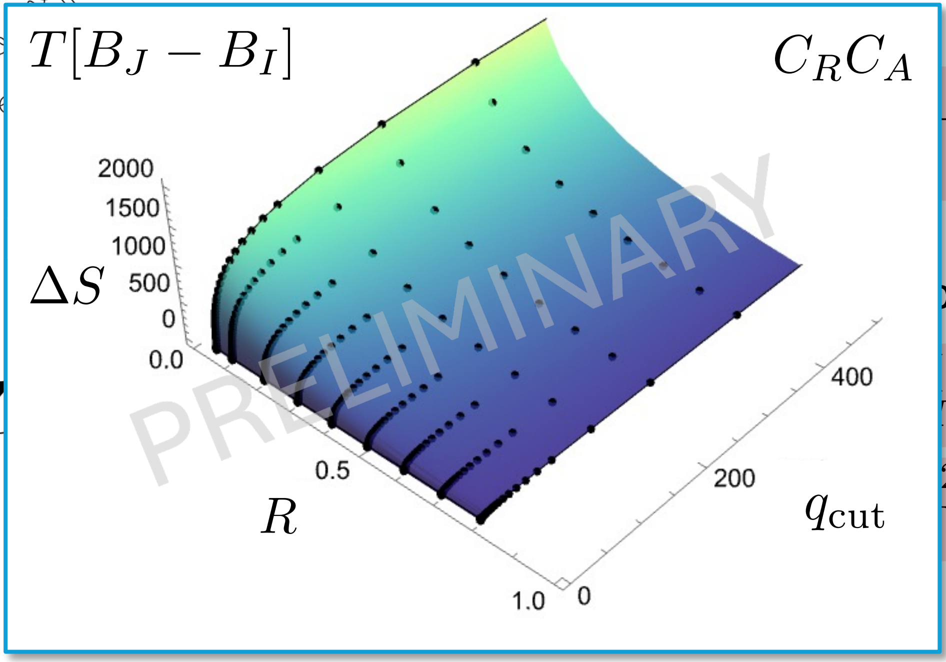


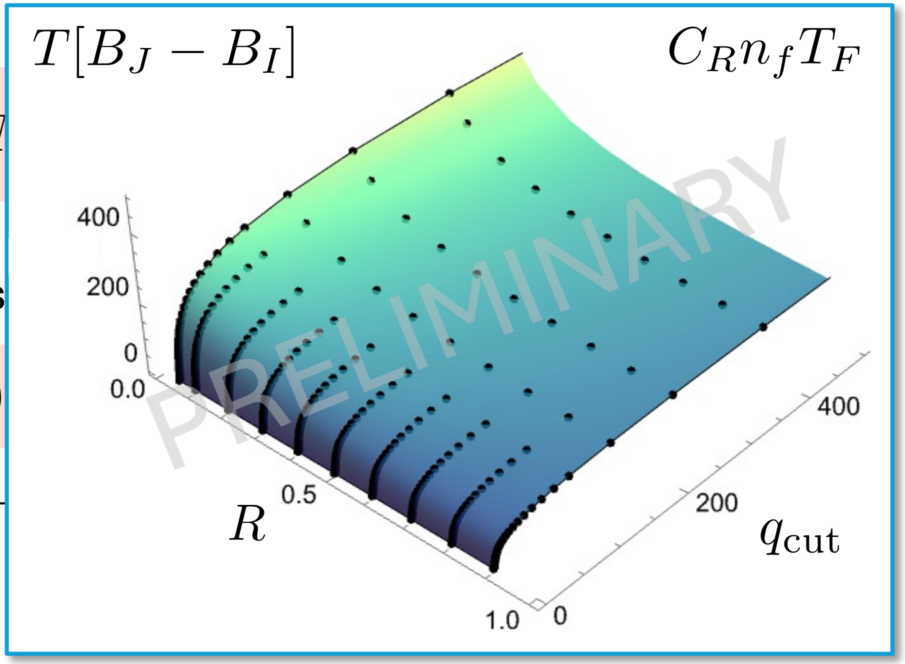
$$\mathrm{d}^{2-2\epsilon}\vec{q}_T \, \Delta$$

...bservables as

$$^{(2,c)}_{e_I}(p_{Tc}, \mathcal{T}_c)$$

$$^{2)}_{e_I}(p_{Tc}, \mathcal{T}_c)$$





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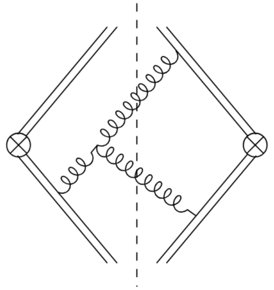
NNLO: $C_R C_A$ & $C_R n_f T_F$



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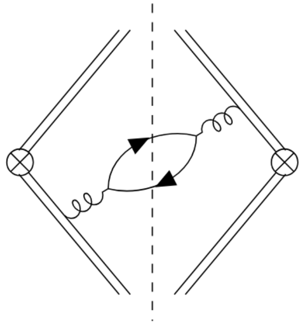
$$\Delta S_{Te_I}^{\mathrm{bare}(c,2)}(p_{T\mathrm{c}}, \mathcal{T}_\mathrm{c}) = \int \mathrm{d}q^+ \mathrm{d}q^- \mathrm{d}^{2-2\epsilon}\vec{q}_T \, \Delta M_{Te_I}(p_{T\mathrm{c}}, \mathcal{T}_\mathrm{c}) \, S_{\mathrm{FD}}^{\mathrm{bare}(2,c)}(q^+, q^-, \vec{q}_T)$$

Lee, Mantry, Petriello, [1105.5171]



Then we write the partonic and jet observables as corrections to this inclusive function:

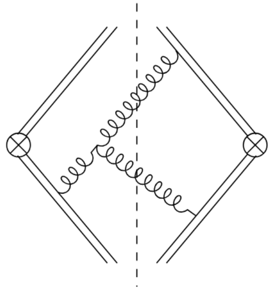
$$\Delta S_{Te}^{(2,c)}(p_{T\mathrm{c}}, \mathcal{T}_\mathrm{c}) = \Delta S_{Te_I}^{(2,c)}(p_{T\mathrm{c}}, \mathcal{T}_\mathrm{c}) + \Delta S_{T[e-e_I]}^{(2,c)}(p_{T\mathrm{c}}, \mathcal{T}_\mathrm{c})$$

$$\Delta S_{Te_J}^{(2)}(p_{T\mathrm{c}}, \mathcal{T}_\mathrm{c}, R) = \Delta S_{Te_I}^{(2)}(p_{T\mathrm{c}}, \mathcal{T}_\mathrm{c}) + \Delta S_{T[e_J-e_I]}^{(2)}(p_{T\mathrm{c}}, \mathcal{T}_\mathrm{c}, R)$$

For the jet measurement we can use the previous effective measurement!

$$\Delta S_{T[e_J-e_I]}^{(2,c)}(p_{T\mathrm{c}}, \mathcal{T}_\mathrm{c}) = \int \mathrm{d}z \, \mathrm{d}\Delta y \, \mathrm{d}\Delta\phi \, \theta(\Delta R > R) \tilde{\mathcal{M}}_{T[e_I-e_J]} \mathcal{A}^{(2,c)}(z, \Delta y, \Delta\phi)$$

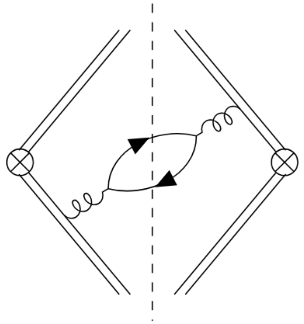
NNLO: $C_R C_A$ & $C_R n_f T_F$



For the $C_R C_A$ and $C_R n_f T_F$, we can obtain the inclusive soft function by integrating the fully-differential soft function

$$\Delta S^{\mathrm{bare}(c,2)}_{Te_I}(p_{Tc}, \mathcal{T}_c) = \int \mathrm{d}q^+ \mathrm{d}q^- \mathrm{d}^{2-2\epsilon}\vec{q}_T \, \Delta M_{Te_I}(p_{Tc}, \mathcal{T}_c) \, S^{\mathrm{bare}(2,c)}_{\mathrm{FD}}(q^+, q^-, \vec{q}_T)$$

Lee, Mantry, Petriello, [1105.5171]



Then we write the partonic and jet observables as corrections to this inclusive function:

$$\Delta S^{(2,c)}_{Te}(p_{Tc}, \mathcal{T}_c) = \Delta S^{(2,c)}_{Te_I}(p_{Tc}, \mathcal{T}_c) + \Delta S^{(2,c)}_{T[e-e_I]}(p_{Tc}, \mathcal{T}_c)$$

$$\Delta S^{(2)}_{Te_J}(p_{Tc}, \mathcal{T}_c, R) = \Delta S^{(2)}_{Te_I}(p_{Tc}, \mathcal{T}_c) + \Delta S^{(2)}_{T[e_J-e_I]}(p_{Tc}, \mathcal{T}_c, R)$$

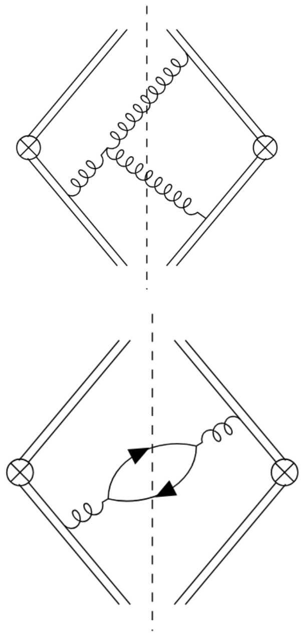
For the jet measurement we can use the previous effective measurement!

$$\Delta S^{(2,c)}_{T[e_J-e_I]}(p_{Tc}, \mathcal{T}_c) = \int \mathrm{d}z \, \mathrm{d}\Delta y \, \mathrm{d}\Delta\phi \, \theta(\Delta R > R) \tilde{\mathcal{M}}_{T[e_I-e_J]} \mathcal{A}^{(2,c)}(z, \Delta y, \Delta\phi)$$

For beam thrust the correction to this inclusive function is

$$\Delta S^{(2,c)}_{T[e-e_I]}(p_{Tc}, \mathcal{T}_c) = \int \mathrm{d}z \, \mathrm{d}\Delta y \, \mathrm{d}\Delta\phi \, \tilde{\mathcal{M}}_{T[e_I-e]} \mathcal{A}^{(2,c)}(z, \Delta y, \Delta\phi)$$

NNLO: $C_R C_A$ & $C_R n_f T_F$

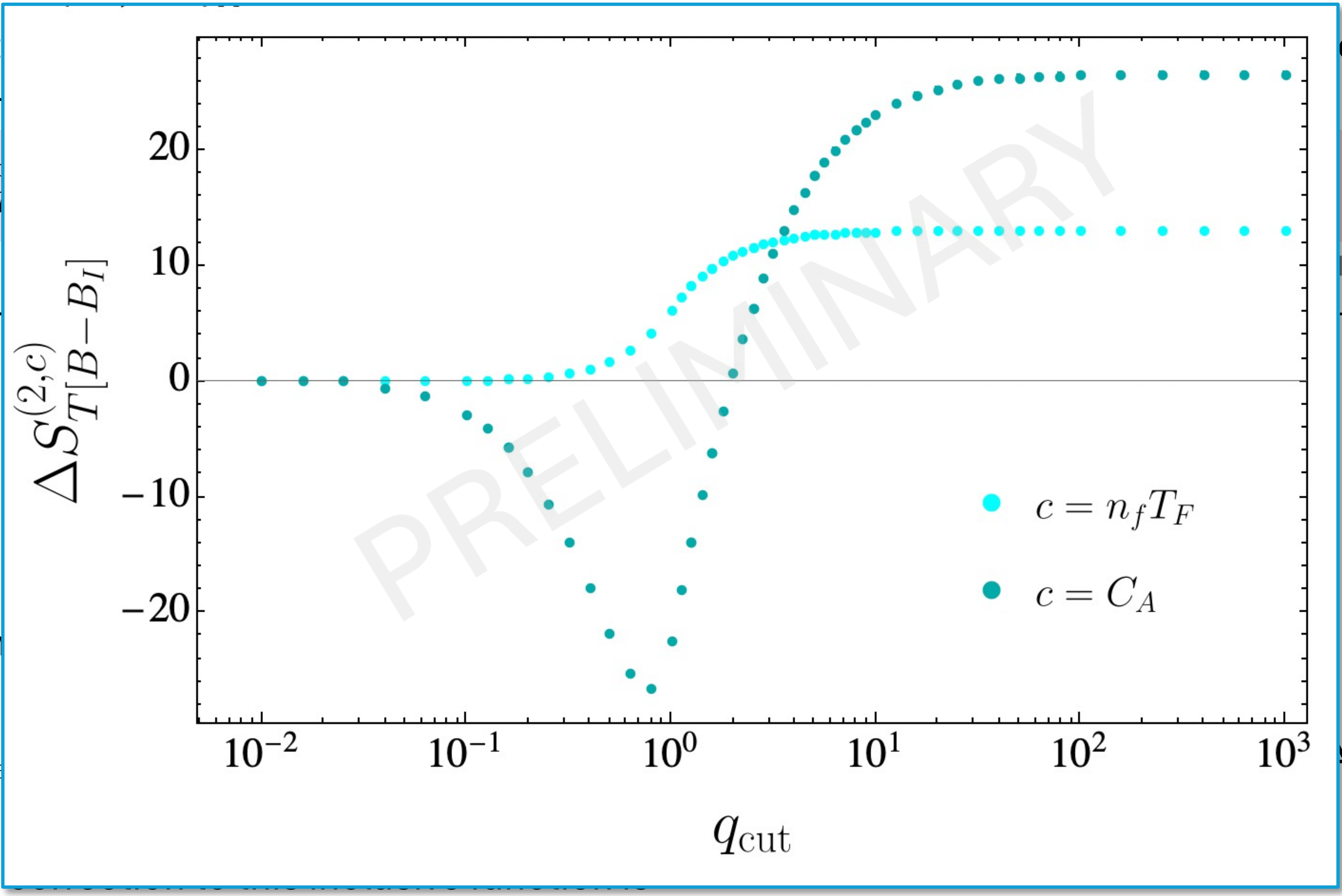


For t ... e
fully- ...

$\Delta S_T^{\mathrm{b}} \ldots , \vec{q}_T)$

... 105.5171]

Then ... tion:



For the jet measuren ...

$\Delta S^{(2,}_{T[e} \ldots \Delta\phi)$

For beam thrust the ...

$$\Delta S^{(2,c)}_{T[e-e_I]}(p_{Tc}, \mathcal{T}_{\mathrm{c}}) = \int \mathrm{d}z\, \mathrm{d}\Delta y\, \mathrm{d}\Delta\phi\, \tilde{\mathcal{M}}_{T[e_I - e]} \mathcal{A}^{(2,c)}(z, \Delta y, \Delta\phi)$$

SUMMARY OF CALCULATION

For p_T^V and beam thrust/0-jettiness the complete NNLO soft function is

$$S^{(2)}_{\mathcal{T}e}(p_{T\mathrm{cut}}, \mathcal{T}_{\mathrm{cut}}) = \frac{1}{2}\int^{\mathcal{T}_{\mathrm{cut}}} \mathrm{d}\mathcal{T} \int_{\vec{p}_T < p_{T\mathrm{cut}}} \mathrm{d}^2\vec{p}_T [S^{(1)}_{\mathcal{T}e} \otimes_{T,\mathcal{T}} S^{(1)}_{\mathcal{T}e}](\vec{p}_T, \mathcal{T})$$

$$+ C_R \sum_{c \in \{C_A, n_f T_F\}} c \left[S^{(2,c)}_T(p_{T\mathrm{c}}) + \Delta S^{(2,c)}_{\mathcal{T}e_I}(p_{T\mathrm{c}}, \mathcal{T}_\mathrm{c}) + \Delta S^{(2,c)}_{T[e-e_I]}(p_{T\mathrm{c}}, \mathcal{T}_\mathrm{c}) \right]$$

For p_T^V and a clustering, rapidity-dependent jet-veto, the complete NNLO soft function is

$$S^{(2)}_{\mathcal{T}e_J}(p_{T\mathrm{cut}}, \mathcal{T}_{\mathrm{cut}}, R) = \frac{1}{2}\int_{\vec{p}_T < p_{T\mathrm{cut}}} \mathrm{d}^2\vec{p}_T [S^{(1)}_{\mathcal{T}e} \otimes_T S^{(1)}_{\mathcal{T}e}](\vec{p}_\perp, \mathcal{T}_{\mathrm{cut}}) + C_R^2 \Delta S^{(2,C_R)}_{\mathcal{T}e_J}(p_{T\mathrm{c}}, \mathcal{T}_\mathrm{c}, R)$$

$$+ C_R \sum_{c \in \{C_A, n_f T_F\}} c \left[S^{(2,c)}_T(p_{T\mathrm{c}}) + \Delta S^{(2,c)}_{\mathcal{T}e_I}(p_{T\mathrm{c}}, \mathcal{T}_\mathrm{c}) + \Delta S^{(2,c)}_{T[e_J-e_I]}(p_{T\mathrm{c}}, \mathcal{T}_\mathrm{c}, R) \right]$$

Both constructions use **existing results**, semi-numerical **convolutions**, analytic calculation of the **inclusive function**, and 3-fold numerical integration of the double-real **amplitudes** and **integrated measurement**.

SUMMARY

- A focus on calculational efficiency and simplicity makes make the computation of many NNLO soft and beam functions highly tractable (and fun) problems
- The general strategy is not confined to NNLO, and the same approach should bring more N^3LO calculations within reach.

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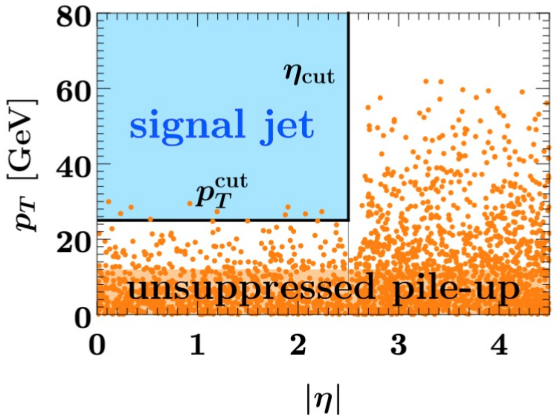

Figure from [1810.12911] Michel, Pietrulewicz, Tackmann

Quick advertisement: With Thomas Clark and Jo Gaunt, we have applied a similar methodology to compute the NNLO quark beam function for a p_T-veto with a rapidity cut on jets,

$$B_q^{(2)}(x, p_T^{\rm cut}, \eta_{\rm cut}, \mu, \nu)$$

This should appear on the arXiv soon.

BACKUP SLIDES

INCLUSIVE SOFT FUNCTIONS FOR DIJET EVENT SHAPES AT NNLO

See also [2604.13167]

The inclusive soft function is constrained by dimensional analysis to have the form

$$S_e^{\mathrm{bare}(l)}(\mathcal{T}) = f_T^{(l)}(\epsilon, N_c, n_f) \int \frac{\mathrm{d}q^+ \mathrm{d}q^-}{q^+ q^-} \left(\frac{\mu^2}{q^+ q^-}\right)^{l\epsilon} \delta(\mathcal{T}_{e_I} - \mathcal{T})$$

where $f_T^{(l)}$ is some scale-less function. At NLO we can simply read off

$$f_T^{(1)}(\epsilon, N_C) = \frac{8 C_R e^{\epsilon \gamma_E}}{\Gamma(1-\epsilon)}$$

At higher loops, a convenient way to determine this function is to note

$$S_e^{\mathrm{bare}(l)}(\mathcal{T}) = \int \mathrm{d}q^+ \mathrm{d}q^- S^{\mathrm{bare}(l)}(q^+, q^-) \delta(\mathcal{T}_{e_I} - \mathcal{T})$$

[1909.00811] Billis, Ebert, Michel, Tackmann

So we can read off

$$S_e^{\mathrm{bare}(l)}(q^+, q^-) = f_T^{(l)}(\epsilon, N_c, n_f) \left(\frac{\mu^2}{q^+ q^-}\right)^{-1+l\epsilon}$$

For illustration, the two-loop $n_f\, T_F$ factor is,

$$f_T^{(2, n_f T_F)}(\epsilon) = \frac{16 e^{2\epsilon\gamma_E}(1-\epsilon)\Gamma(1-\epsilon)\Gamma(-\epsilon)}{(3 - 8\epsilon + 4\epsilon^2)\Gamma(1-2\epsilon)^2}$$

All orders using [1105.5171] Lee, Mantry, Petriello

NNLO: $C_R C_A$ & $C_R n_f T_F$ CHANNELS

See also [2012.09213] ,[2604.13167]

For the non-exponentiating colour factors, it is useful to consider the *inclusive* analogue of the observable

$$\mathcal{M}_e(\mathcal{T}_{\rm cut}) = \theta\left(\sum_i \mathcal{T}_e(k_i) < \mathcal{T}_{\rm cut}\right) \qquad \mathcal{M}_{e_I}(\mathcal{T}_{\rm cut}) = \theta\left(\mathcal{T}_e(\sum_i k_i) < \mathcal{T}_{\rm cut}\right)$$

We then write the full soft function as the inclusive soft function, plus a correction

$$S_e^{\rm bare(2,c)}(\mathcal{T}_{\rm cut}) = S_{e_I}^{\rm bare(2,c)}(\mathcal{T}_{\rm cut}) + S_{e-e_I}^{\rm bare(2,c)}(\mathcal{T}_{\rm cut})$$

To be concrete, for C-angularity, $\mathcal{T}_{C,a}(k_i) = \dfrac{(k_i^+ k_i^-)^{1-\frac{a}{2}}}{(k_i^+ + k_i^-)^{1-a}}$, the measurement for the correction is

$$\mathcal{M}_{e-e_I}(\mathcal{T}_{\rm cut}) = \theta\left(\frac{(k_1^+ k_1^-)^{1-\frac{a}{2}}}{(k_1^+ + k_1^-)^{1-a}} + \frac{(k_2^+ k_2^-)^{1-\frac{a}{2}}}{(k_2^+ + k_2^-)^{1-a}} < \mathcal{T}_{\rm cut}\right) - \theta\left(\frac{((k_1^+ + k_2^+)(k_1^- + k_2^-))^{1-\frac{a}{2}}}{(k_1^+ + k_1^- + k_2^+ + k_2^-)^{1-a}} < \mathcal{T}_{\rm cut}\right)$$

This measurement vanishes in all but two IR limits, meaning the inclusive piece captures almost all of the poles.