

# SMEFT Dimension-8 Renormalization

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# SMEFT Renormalization Status (2026)!

$$L_{\text{SMEFT}} = L_{\text{SM}} + \frac{d_5}{\Lambda} + \frac{d_6}{\Lambda^2} + \frac{d_7}{\Lambda^3} + \frac{d_8}{\Lambda^4} + \dots$$

Gauge symmetry unbroken

## 1-loop

	$d_5$	$d_5^2$	$d_6$	$d_5^3$	$d_5 \times d_6$	$d_7$	$d_5^4$	$d_5^2 \times d_6$	$d_6^2$	$d_5 \times d_7$	$d_8$
$d_{\leq 4}$ (bosonic)			✓						✓		This talk
$d_{\leq 4}$ (fermionic)			✓						✓		X
$d_5$	✓				✓	✓					
$d_6$ (bosonic)		✓	✓					✓	✓	✓	This talk
$d_6$ (fermionic)		✓	✓					X	✓	X	X
$d_7$				✓	✓	✓					
$d_8$ (bosonic)							This talk	This talk	This talk	This talk	This talk
$d_8$ (fermionic)							X	X	✓	X	✓

Blank entries vanish; ✓ → known; X → nothing, or very little, is known. The contribution discussed in this talk is marked by ■.

**Table compiled from: arxiv:2106.05291, 2205.03301, 2301.07151, 2409.15408 . Check these out for Refs.**

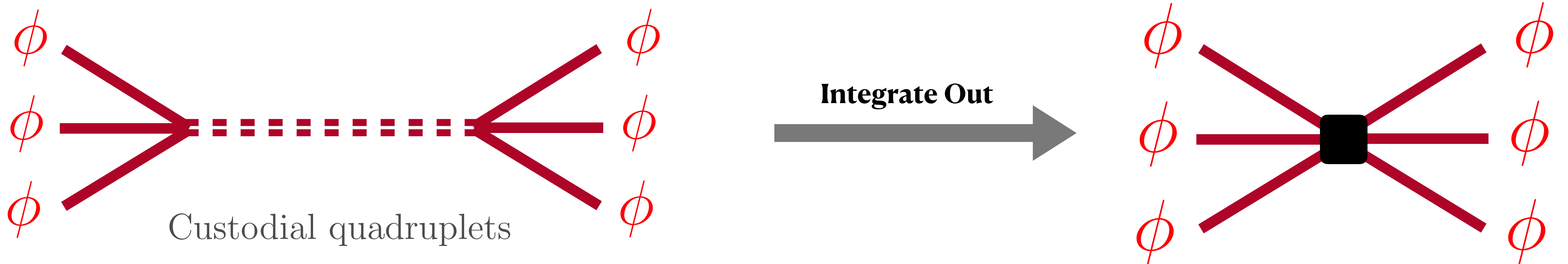
Authors: SDB, M. Chala, A. Diaz-Carmona, G. Guedes, M. Ramos, Z. Ren, J. Santiago, F. Vilches

# Motivations :

- RGEs reveal restrictions on Dim-8 WC space imposed from positivity bounds.

Chala, Xu (2309.16611)

- Custodial symmetry violation absent at tree-level dim-6, dim-8, and 1-loop dim-6.



Chala, Krause, Nardini, (1802.02168);

Durieux, McCullough, Salvioni (2209.00666)

- Also, Dim-8 RGEs computed using multiple computational tools, with agreement in results. These establish validation among these tools.

# SMEFT Dim-8 RGEs

$\Lambda = \text{EFT cut-off scale}$

$$\mathcal{L}_{\text{SMEFT}} = \mathcal{L}_{\text{SM}} + \frac{1}{\Lambda} \sum_i c_i^{(5)} O_i^{(5)} + \frac{1}{\Lambda^2} \sum_i c_i^{(6)} O_i^{(6)} + \frac{1}{\Lambda^3} \sum_j c_j^{(7)} O_j^{(7)} + \frac{1}{\Lambda^4} \sum_j c_j^{(8)} O_j^{(8)} + \dots$$

$$\dot{c}_i^{(8)} \equiv 16\pi^2 \tilde{\mu} \frac{dc_i^{(8)}}{d\tilde{\mu}} = \boxed{\gamma_{ij} c_j^{(8)}} + \gamma'_{ijk} c_j^{(7)} c_k^{(5)} + \gamma''_{ijklm} c_j^{(5)} c_k^{(5)} c_l^{(5)} c_m^{(5)} + \dots$$

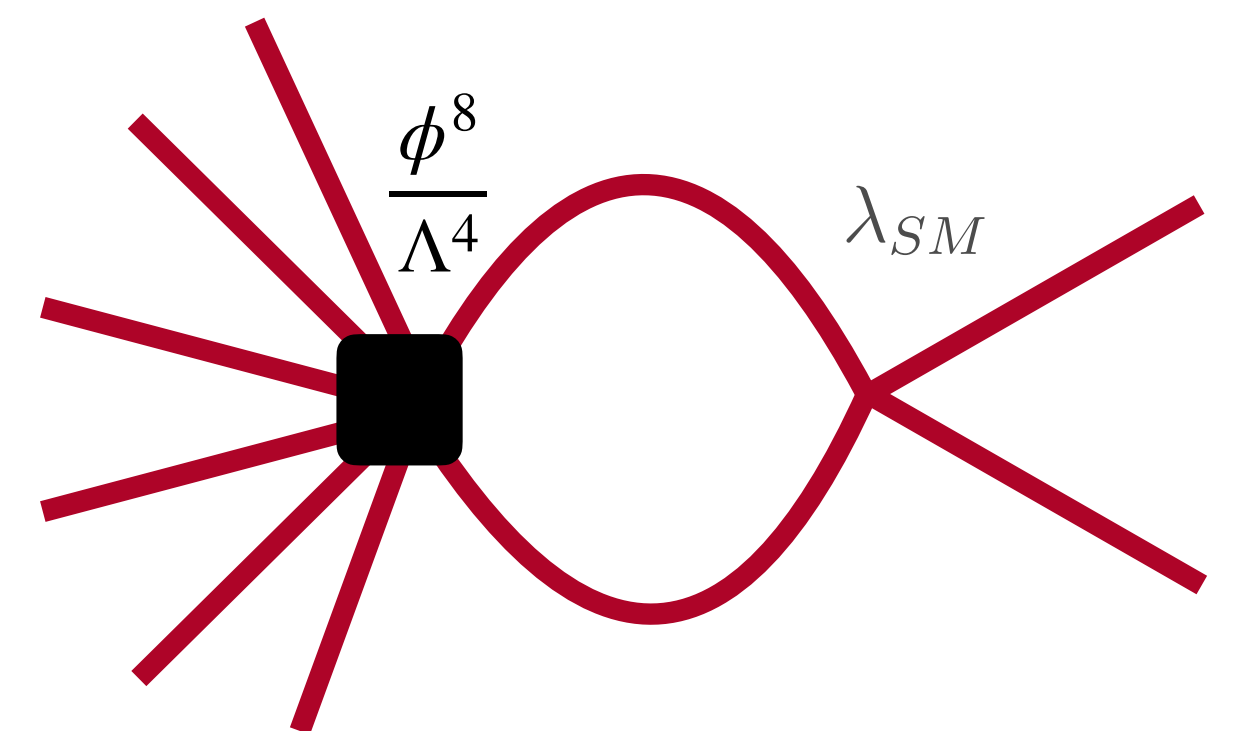
## ❖ One dim-8 operator insertion.

arXiv:2205.03301

Towards the renormalisation of the Standard Model effective field theory to dimension eight: Bosonic interactions II

- SDB, M Chala, Á Díaz-Carmona, G Guedes

e.g. :



# Bosonic SMEFT Dim-8 RGEs

$\Lambda =$  EFT cut-off scale

$$\mathcal{L}_{\text{SMEFT}} = \mathcal{L}_{\text{SM}} + \frac{1}{\Lambda} \sum_i c_i^{(5)} O_i^{(5)} + \frac{1}{\Lambda^2} \sum_i c_i^{(6)} O_i^{(6)} + \frac{1}{\Lambda^3} \sum_j c_j^{(7)} O_j^{(7)} + \frac{1}{\Lambda^4} \sum_j c_j^{(8)} O_j^{(8)} + \dots$$

$$\dot{c}_i^{(8)} \equiv 16\pi^2 \tilde{\mu} \frac{dc_i^{(8)}}{d\tilde{\mu}} = \gamma_{ij} c_j^{(8)} + \boxed{\gamma'_{ijk} c_j^{(7)} c_k^{(5)} + \gamma''_{ijklm} c_j^{(5)} c_k^{(5)} c_l^{(5)} c_m^{(5)}} + \dots$$

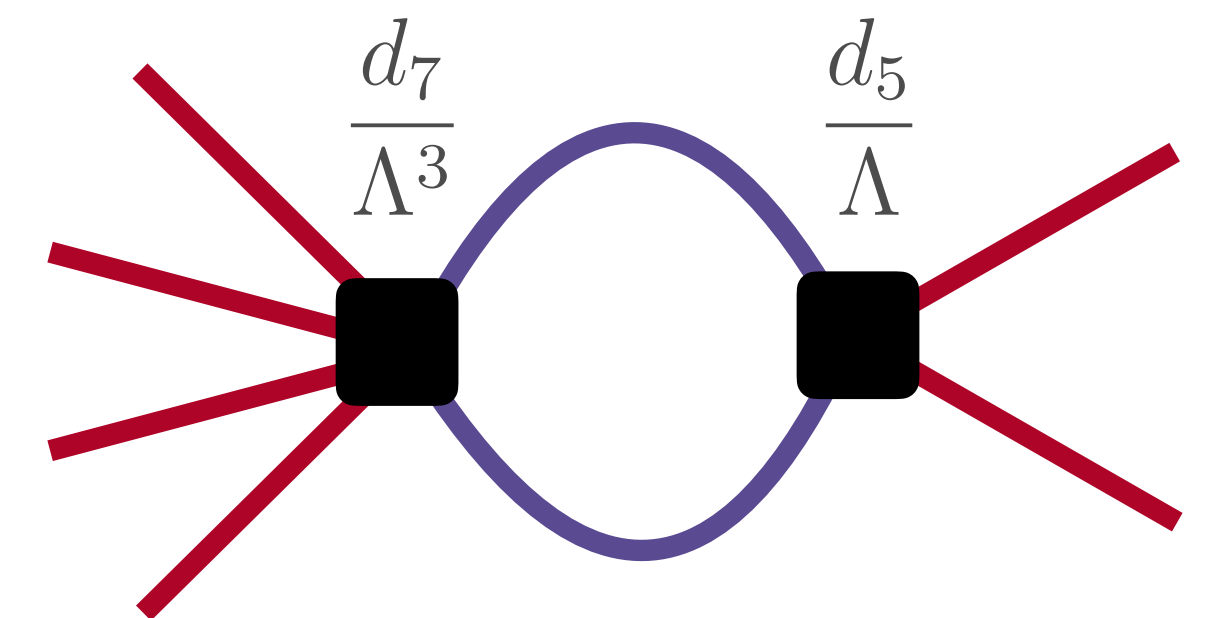
## ❖ Dim-7 and dim-5 operators insertions.

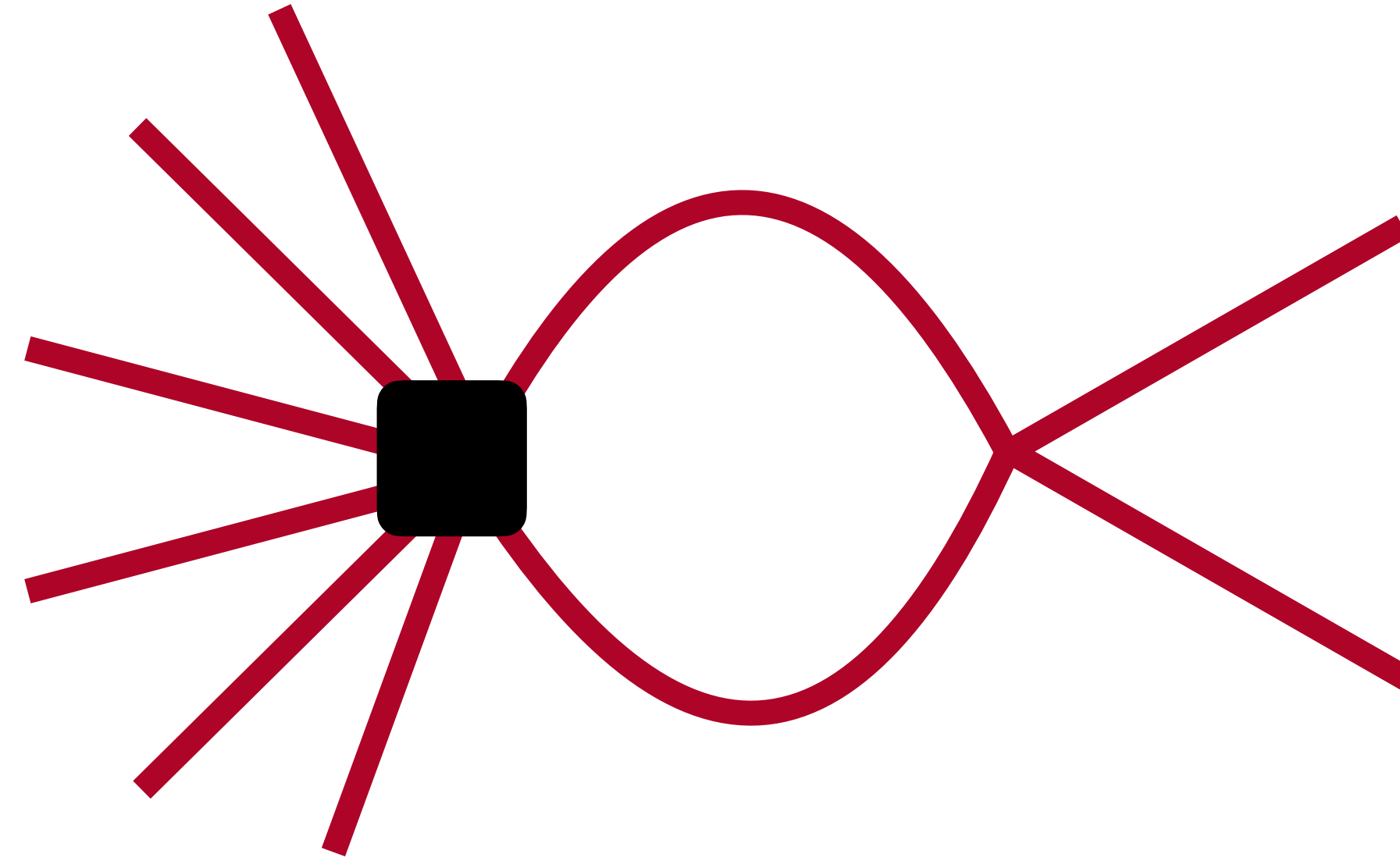
arXiv:2301.07151

Renormalisation of SMEFT bosonic interactions  
up to dimension eight by LNV operators

- SDB, Á Díaz-Carmona

e.g. :





# Part 1 : Dim-8 renormalization by Dim-8

**arXiv:2301.07151**

**arXiv:2409.15408**

**arXiv:2607.xxxxx**

# SMEFT Operator Classes

$$\dot{c}_i^{(8)} \equiv 16\pi^2 \tilde{\mu} \frac{dc_i^{(8)}}{d\tilde{\mu}} = \boxed{\gamma_{ij} c_j^{(8)}} + \gamma'_{ijk} c_j^{(7)} c_k^{(5)} + \gamma''_{ijklm} c_j^{(5)} c_k^{(5)} c_l^{(5)} c_m^{(5)} + \dots$$

- One **tree-level generated** dim-8 operator in one-loop.

Classes of operator that are **tree-level generated**:

arXiv:2001.00017  
- Craig, Jiang, Li, Sutherland

**Bosonic :**  $\{\phi^8, \phi^6 D^2, \phi^4 D^4, X^2 \phi^4, X \phi^4 D^2, X^2 H^2 D^2, X^3 H^2, X^4\}$

**Fermionic :**  $\{\psi^2 X \phi^3, \psi^2 \phi^2 D^3, \psi^2 \phi^5, \psi^2 \phi^4 D, \psi^2 X \phi^2 D, \psi^2 \phi^3 D^2, \psi^2 X^2 \phi, \psi^2 X^2 D, \psi^2 X \phi D^2\}$

**SMEFT Dim-8 on-shell basis :**

arXiv:2005.00059 — C. W. Murphy

**SMEFT Dim-8 Green's basis :**

arXiv:2112.12724 — M. Chala, Á Díaz-Carmona, G. Guedes

# Divergences to RGEs, some details:

- Compute **1-PI loop diagrams**. Use **FeynRules**, **FeynARTs**, and **FormCalc** packages.
- Divergences are captured by the operators of **Green's basis**.

$$16\pi^2 \epsilon \mathcal{L}_{\text{DIV}} = \tilde{K}_\phi (D_\mu \phi)^\dagger (D^\mu \phi) + \tilde{\mu}^2 |\phi|^2 - \tilde{\lambda} |\phi|^4 + \tilde{c}_i^{(6)} \frac{\mathcal{O}_i^{(6)}}{\Lambda^2} + \tilde{c}_j^{(8)} \frac{\mathcal{O}_j^{(8)}}{\Lambda^4}$$

[on RHS we have Green's basis]

arXiv:2112.12724

- M Chala, Á Díaz-Carmona, G Guedes

- **Removing redundant operators** using EOMs (e.g.  $(H^\dagger H)(H^\dagger D^2 H)$  ).

arXiv:2106.05291- M Chala, G Guedes, M Ramos, J Santiago

- **Cross-checks with MatchMakerEFT.** ✓  
H<sup>8</sup> topologies are computed in MM primarily.

arXiv:2112.10787

- A Carmona, A Lazopoulos, P Olgoso, J Santiago

- **Partial cross-checks** with arXiv:2108.03669 (on-shell amplitude methods). ✓

arXiv:2108.03669

- M A Huber, S De Angelis.

and with arXiv:2307.03187 ✓

— Assi, Helset, Manohar, Pagès, Shen

# Bosonic-bosonic RGE:

## Classes of tree-generated bosonic operators

	$\phi^4 D^4$	$B\phi^4 D^2$	$W\phi^4 D^2$	$B^2\phi^4$	$W^2\phi^4$	$WB\phi^4$	$G^2\phi^4$	$\phi^6 D^2$	$\phi^8$
$B^2\phi^2 D^2$	$g_1^2$	0	0	0	0	0	0	0	0
$W^2\phi^2 D^2$	$g_2^2$	0	0	0	0	0	0	0	0
$WB\phi^2 D^2$	$g_1 g_2$	0	0	0	0	0	0	0	0
$G^2\phi^2 D^2$	0	0	0	0	0	0	0	0	0
$W^3\phi^2$	0	0	0	0	0	0	0	0	0
$W^2 B\phi^2$	0	0	0	0	0	0	0	0	0
$G^3\phi^2$	0	0	0	0	0	0	0	0	0
$\phi^4 D^4$	$g_2^2$	0	0	0	0	0	0	0	0
$B\phi^4 D^2$	$g_1 g_2^2$	$\lambda$	0	0	0	0	0	0	0
$W\phi^4 D^2$	$g_2^3$	0	$g_2^2$	0	0	0	0	0	0
$B^2\phi^4$	$g_1^2 g_2^2$	$g_1 \lambda$	$g_1^2 g_2$	$\lambda$	0	$g_1 g_2$	0	0	0
$W^2\phi^4$	$g_2^4$	$g_1 g_2^2$	$g_2^3$	0	$\lambda$	$g_1 g_2$	0	0	0
$WB\phi^4$	$g_1 g_2^3$	$g_2 \lambda$	$g_1 \lambda$	$g_1 g_2$	$g_1 g_2$	$\lambda$	0	0	0
$G^2\phi^4$	0	0	0	0	0	0	$g_3^2$	0	0
$\phi^6 D^2$	$g_2^4$	$g_1 \lambda$	$g_2 \lambda$	0	0	0	0	$\lambda$	0
$\phi^8$	$\lambda^3$	$g_1 \lambda^2$	$g_2 \lambda^2$	$g_1^2 \lambda$	$g_2^2 \lambda$	$g_1 g_2 \lambda$	0	$\lambda^2$	$\lambda$

- Largest contribution from each operator class is shown.
- Loop generated operators that are renormalized by tree-generated operators are in gray.
- Blue entries contribute larger than what expected from naive dimensional analysis.

$$\tilde{\mu} \frac{dc_{\phi^8}}{d\tilde{\mu}} = \frac{1}{16\pi^2} (192\lambda - 6(g_1^2 + 3g_2^2) + \dots) c_{\phi^8}$$

# Fermionic-bosonic RGE:

## Classes of tree-generated fermionic operators

	$\psi^2 B \phi^3$	$\psi^2 W \phi^3$	$\psi^2 G \phi^3$	$\psi^2 \phi^2 D^3$	$\psi^2 \phi^5$	$\psi^2 \phi^4 D$	$\psi^2 B \phi^2 D$	$\psi^2 W \phi^2 D$	$\psi^2 G \phi^2 D$	$\psi^2 \phi^3 D^2$
$B^2 \phi^2 D^2$	0	0	0	$g_1^2$	0	0	0	0	0	0
$W^2 \phi^2 D^2$	0	0	0	$g_2^2$	0	0	0	0	0	0
$WB \phi^2 D^2$	0	0	0	$g_1 g_2$	0	0	0	0	0	0
$G^2 \phi^2 D^2$	0	0	0	$g_3^2$	0	0	0	0	0	0
$W^3 \phi^2$	0	0	0	0	0	0	0	0	0	0
$W^2 B \phi^2$	0	0	0	0	0	0	0	0	0	0
$G^3 \phi^2$	0	0	0	0	0	0	0	0	0	0
$\phi^4 D^4$	0	0	0	$ y^t ^2$	0	0	0	0	0	0
$B \phi^4 D^2$	0	0	0	$g_1  y^t ^2$	0	0	$ y^t ^2$	0	0	$g_1 y^t$
$W \phi^4 D^2$	0	0	0	$g_2  y^t ^2$	0	0	0	$ y^t ^2$	0	$g_2 y^t$
$B^2 \phi^4$	$g_1 y^t$	0	0	$g_1^2  y^t ^2$	0	0	$g_1  y^t ^2$	0	0	$g_1^2 y^t$
$W^2 \phi^4$	0	$g_2 y^t$	0	$g_2^2  y^t ^2$	0	$g_2^2$	0	$g_2  y^t ^2$	0	$g_2^2 y^t$
$WB \phi^4$	$g_2 y^t$	$g_1 y^t$	0	$g_1 g_2  y^t ^2$	0	$g_1 g_2$	$g_2  y^t ^2$	$g_1  y^t ^2$	0	$g_1 g_2 y^t$
$G^2 \phi^4$	0	0	$g_3 y^t$	0	0	0	0	0	0	0
$\phi^6 D^2$	0	0	0	$g_2^2  y^t ^2$	0	$ y^t ^2$	$g_1  y^t ^2$	$g_2  y^t ^2$	0	$y^t  y^t ^2$
$\phi^8$	0	0	0	$\lambda  y^t ^4$	$y^t  y^t ^2$	$\lambda  y^t ^2$	$g_1 \lambda  y^t ^2$	$g_2 \lambda  y^t ^2$	0	$\lambda y^t  y^t ^2$

$$\dot{c}_{W\phi^4 D^2}^{(1)} = 44g_2(c_{q^2\phi^2 D^3}^{(4)})_{\alpha_1, \alpha_2} y_{\alpha_2, \alpha_3}^u (y^u)_{\alpha_3, \alpha_1}^* + 48(c_{q^2 W H^2 D}^{(11)})_{\alpha_1, \alpha_2} y_{\alpha_2, \alpha_3}^u (y^u)_{\alpha_1, \alpha_3}^* + \dots$$

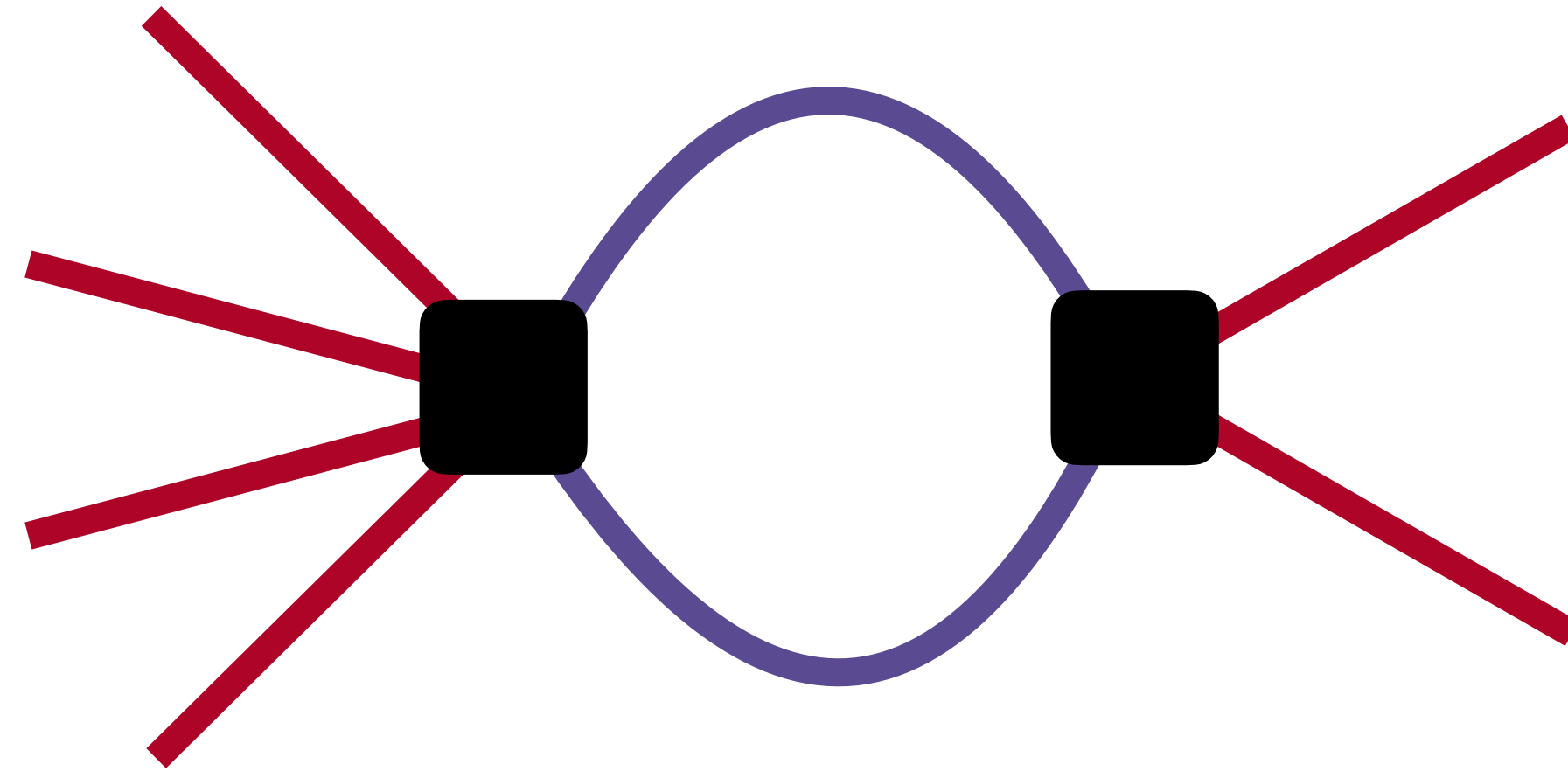
# RGEs of Dim-6,4,2

- Dim-8 operators also induce running of dim-6, dim-4, dim-2 operators.

$\mu^2$  is the squared Higgs mass in the SMEFT.

	$\phi^4 D^4$	$B\phi^4 D^2$	$W\phi^4 D^2$	$B^2\phi^4$	$W^2\phi^4$	$WB\phi^4$	$G^2\phi^4$	$\phi^6 D^2$	$\phi^8$	$\times 1/\Lambda^4$
$\phi^2$	$\mu^6$	0	0	0	0	0	0	0	0	
$\phi^4$	$\lambda\mu^4$	$g_1\mu^4$	$g_2\mu^4$	0	0	0	0	$\mu^4$	0	
$B^2\phi^2$	$g_1^2\mu^2$	$g_1\mu^2$	0	$\mu^2$	0	0	0	0	0	
$W^2\phi^2$	$g_2^2\mu^2$	0	$g_2\mu^2$	0	$\mu^2$	0	0	0	0	
$WB\phi^2$	$g_1g_2\mu^2$	$g_2\mu^2$	$g_1\mu^2$	0	0	$\mu^2$	0	0	0	
$G^2\phi^2$	0	0	0	0	0	0	$\mu^2$	0	0	
$\phi^4 D^2$	$\lambda\mu^2$	$g_1\mu^2$	$g_2\mu^2$	0	0	0	0	$\mu^2$	0	
$\phi^6$	$\lambda^2\mu^2$	$\lambda g_1\mu^2$	$\lambda g_2\mu^2$	$g_1^2\mu^2$	$g_2^2\mu^2$	$g_1g_2\mu^2$	0	$\lambda\mu^2$	$\mu^2$	

Lower dim. classes renormalized by the bosonic dim-8 operators.  
 Similar contributions from two-fermionic dim-8 operators are also present.



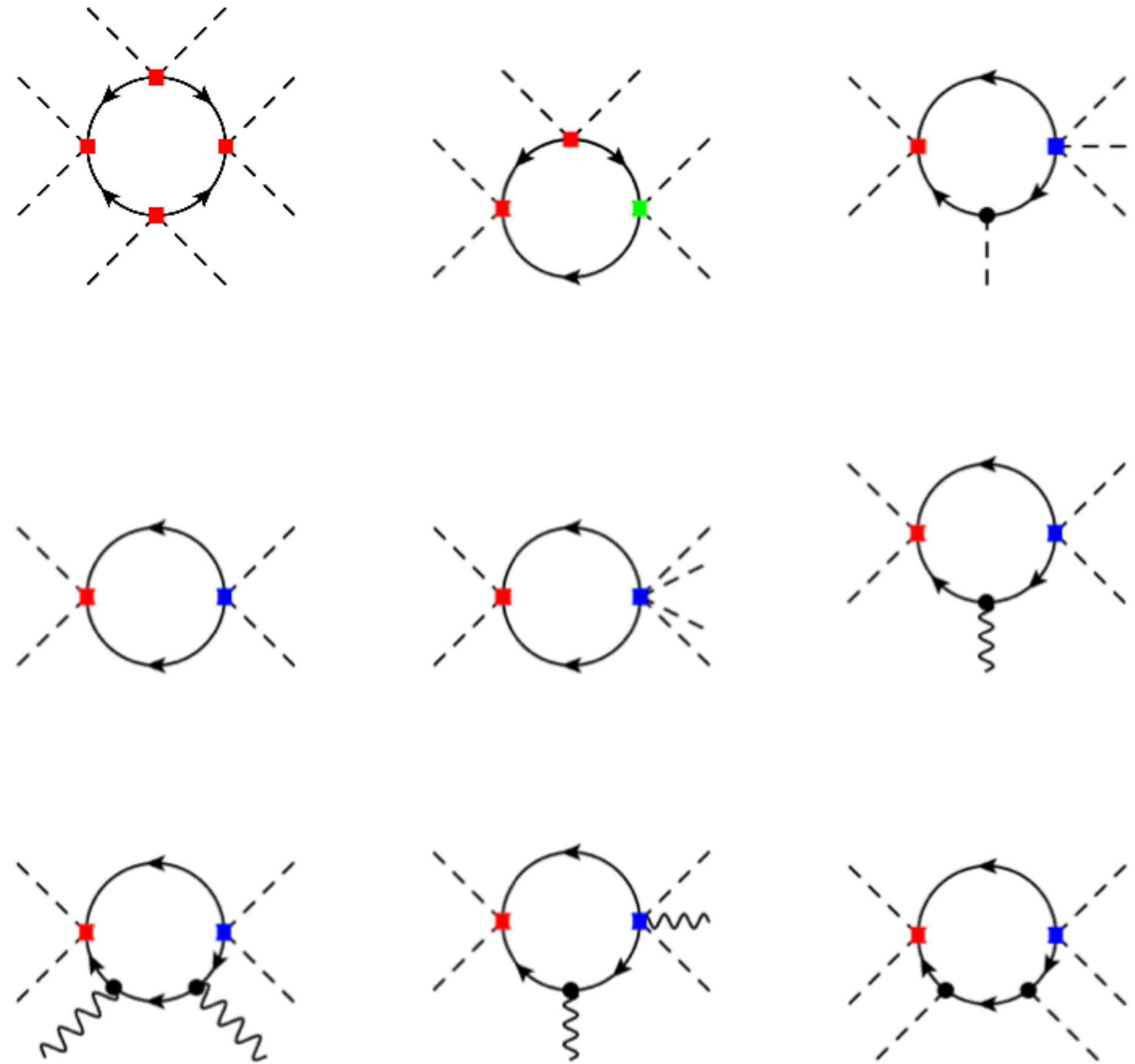
## **Part 2 : Dim-8 renormalization by Dim-5 & Dim-7**

**arXiv:2301.07151**

# LNV contributions (1-loop) to bosonic SMEFT: D5, D6, D7 insertions

$1/\Lambda^4$	$d_5^4$	$d_5^2 \times d_6$	$d_5 \times d_7$
8-Higgs	✓	-	-
6-Higgs	-	✓	✓
4-Higgs	-	-	✓
2-Higgs	-	-	-
0-Higgs	-	-	-

The hyphen stands for the vanishing contributions from LNVs upto  $\Lambda^{-3}$  at 1-loop.



# Bosonic SMEFT RGEs from LNVs :

	$(\alpha_{l\phi})^4$	$(\alpha_{l\phi})^2 \beta_{\phi D}$	$(\alpha_{l\phi})^2 \beta_{\phi l}^{(1)}$	$(\alpha_{l\phi})^2 \beta_{\phi l}^{(3)}$	$\alpha_{l\phi} \omega_{l\phi}$	$\alpha_{l\phi} \omega_{l\phi D}^{(1)}$	$\alpha_{l\phi} \omega_{l\phi D}^{(2)}$	$\alpha_{l\phi} \omega_{l\phi De}$	$\alpha_{l\phi} \omega_{l\phi W}$
$\gamma_{\phi^8}$	16	$8\lambda$	$32\lambda$	$32\lambda$	$16\lambda$	$2\lambda g_2^2$	$\lambda g_2^2$	0	0
$\gamma_{\phi^6}^{(1)}$	0	4	48	64	16	$\frac{14}{3} g_2^2$	$32\lambda$	$4y^e$	0
$\gamma_{\phi^6}^{(2)}$	0	8	32	16	8	$\frac{1}{6} g_2^2$	$16\lambda$	$4y^e$	0
$\gamma_{\phi^4}^{(2)}$	0	0	0	0	0	$\emptyset$	8	0	0
$\gamma_{W\phi^4 D^2}^{(1)}$	0	0	0	0	0	$8g_2$	$4g_2$	0	$\emptyset$
$\gamma_{W\phi^4 D^2}^{(2)}$	0	0	0	0	0	$8g_2$	$4g_2$	0	$\emptyset$
$\gamma_{W\phi^4 D^2}^{(3)}$	0	0	0	0	0	$4g_2$	$2g_2$	0	$\emptyset$
$\gamma_{W\phi^4 D^2}^{(4)}$	0	0	0	0	0	$4g_2$	$2g_2$	0	$\emptyset$
$\gamma_{W^2\phi^4}^{(1)}$	0	0	0	0	0	$\frac{1}{2} g_2^2$	$\frac{1}{4} g_2^2$	0	$4g_2$
$\gamma_{W^2\phi^4}^{(2)}$	0	0	0	0	0	$\frac{1}{2} g_2^2$	$\frac{1}{4} g_2^2$	0	$4g_2$
$\gamma_{W^2\phi^4}^{(3)}$	0	0	0	0	0	$\frac{1}{2} g_2^2$	$\frac{1}{4} g_2^2$	0	$4g_2$
$\gamma_{W^2\phi^4}^{(4)}$	0	0	0	0	0	$\frac{1}{2} g_2^2$	$\frac{1}{4} g_2^2$	0	$4g_2$
$\gamma_{WB\phi^4}^{(1)}$	0	0	0	0	0	$g_1 g_2$	$\frac{1}{2} g_1 g_2$	0	$\emptyset$
$\gamma_{WB\phi^4}^{(2)}$	0	0	0	0	0	$g_1 g_2$	$\frac{1}{2} g_1 g_2$	0	$\emptyset$
$\gamma_{\phi}$	0	$4\mu^2$	$16\mu^2$	$16\mu^2$	$8\mu^2$	$\mu^2 g_2^2$	$16\mu^2 \lambda$	0	0
$\gamma_{\phi\Box}$	0	0	0	0	0	0	$16\mu^2$	0	0
$\gamma_{\phi D}$	0	0	0	0	0	0	$16\mu^2$	0	0
$\gamma_{\lambda}$	0	0	0	0	0	0	$8\mu^4$	0	0

## Simple Application (in context of LNVs) :

$$T = -\frac{1}{2\alpha} \frac{v^2}{\Lambda^2} \left( c_{\phi D} + c_{\phi^6}^{(2)} \frac{v^2}{\Lambda^2} \right) \quad \alpha = \frac{1}{137}$$

Blas et. al. 2017  
arxiv:1611.05354

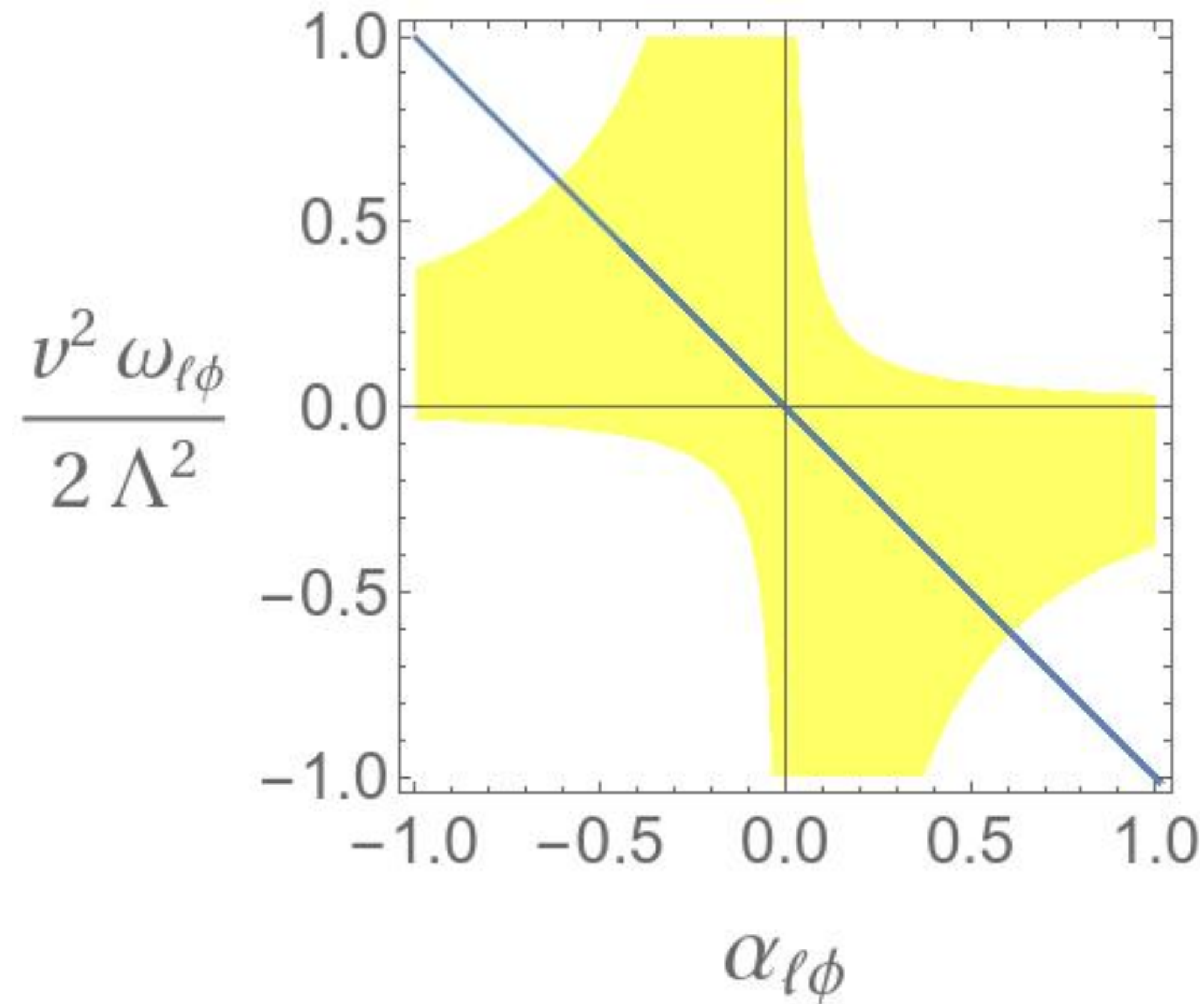
$$\dot{c}_{\phi^6}^{(2)} \supset -8 \operatorname{Re} \left( \operatorname{Tr} \left[ \alpha_{l\phi}^\dagger \omega_{l\phi} \right] \right)$$

$$\therefore T = -\frac{1}{4\pi^2 \alpha} \frac{v^4}{\Lambda^4} \ln \left[ \frac{\Lambda}{\mu} \right] \sum_{mn}^3 \alpha_{l\phi, mn} \omega_{l\phi, mn}$$

## Simple Application :

Neutrino mass bounds:  $(M_N)_{mn} = -\frac{v^2}{\Lambda} \left( \alpha_{\ell\phi, mn} + \frac{v^2}{2\Lambda^2} \omega_{\ell\phi, mn} \right)$

For single non-vanishing flavor direction :



$$\alpha_{\ell\phi} \rightarrow \epsilon_{ij}\epsilon_{mn}(\ell^i C \ell^m)(\phi^j \phi^n)$$

$$\omega_{\ell\phi} \rightarrow \epsilon_{ij}\epsilon_{mn}(\ell^i C \ell^m)(\phi^j \phi^n)(\phi^\dagger \phi)$$

$\Lambda = 1 \text{ TeV}$ , and  $\mu = 246 \text{ GeV}$ ,  
 $T = 0.10 \pm 0.12$ , and  $M_N < 0.081 \text{ eV}$

Blas et. al. 2017  
 arxiv:1611.05354

Loureiro et. al. 2018  
 arxiv:1811.02578

# Summary

- **Renormalization of bosonic SMEFT dim-8 operators discussed.**
  - **By Tree-level generated dim-8 operators.**
  - **By dim-5 & dim-7 LNV operators.**
- **These operators also contribute to the running of lower dimensional operators.**
- **Certain elements contribute larger than what expected from naive dimensional analysis.**
- **T-parameter example: RGEs translate bounds to blind direction of LNV spaces.**

**Thanks!**