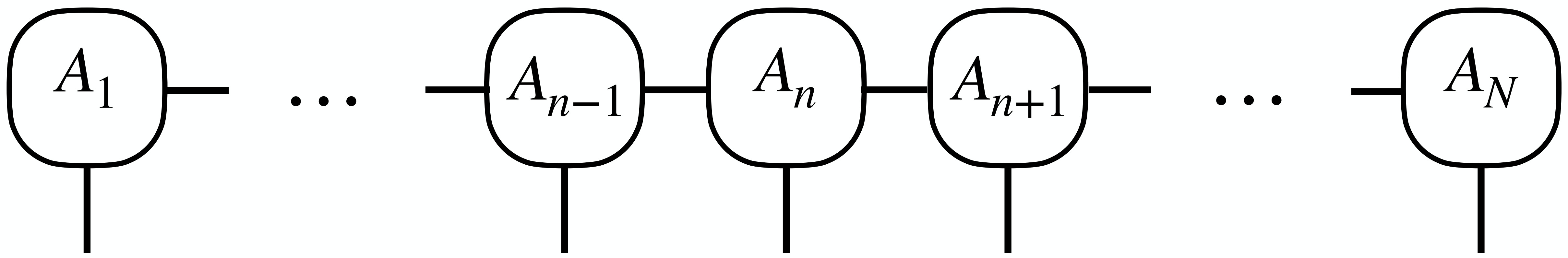
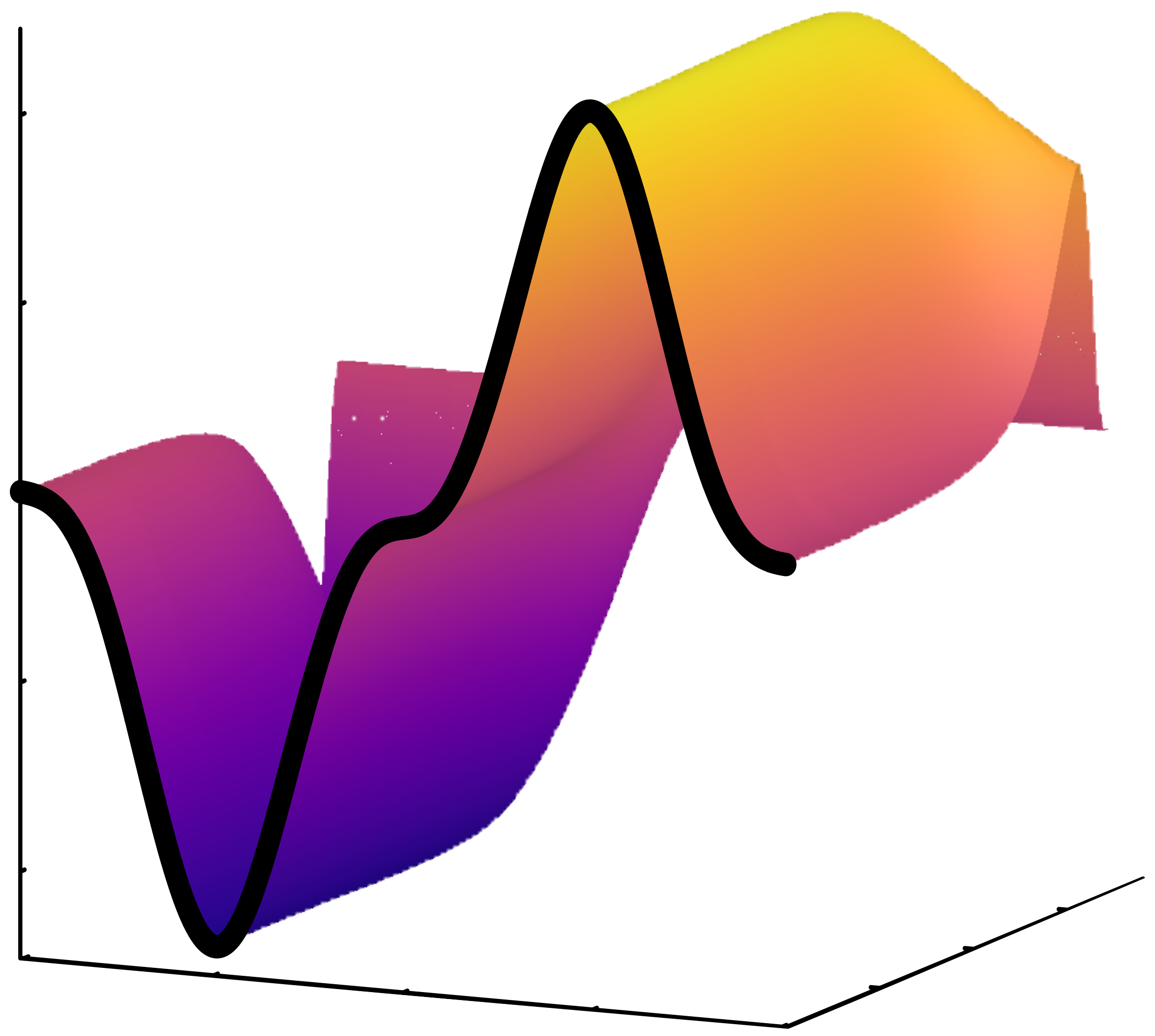
Quasi-PDFs for the Schwinger model using tensor networks

Jake Montgomery

Stony Brook University





LoopFest XXIV, May 28, 2026

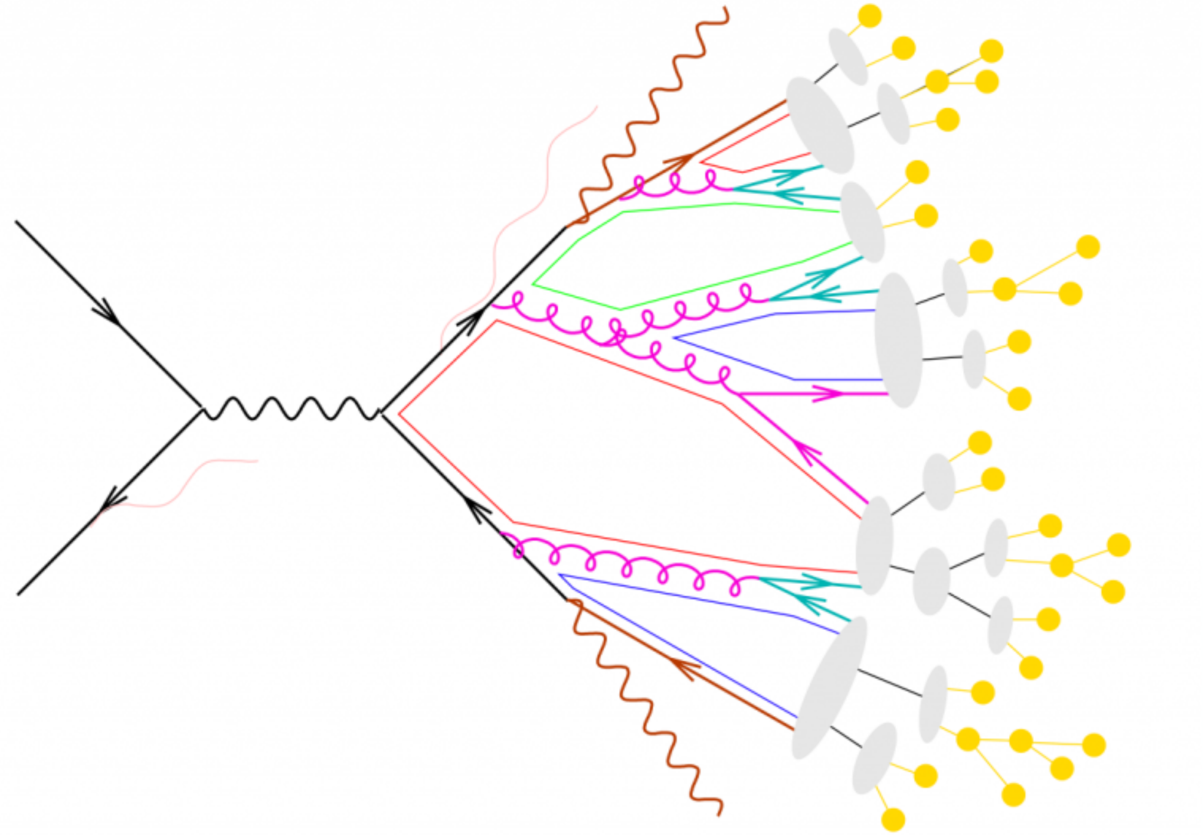
Brookhaven National Laboratory

Real-time dynamics of QFTs

$$\sim \int d^4x \, e^{ip\cdot x} \langle P_f | \, T\{\hat{O}(x)\hat{O}(0)\} \, | P_i \rangle$$

- Want to study real time scattering of QCD in 3+1D
  - Need understanding of the partonic content of the theory

$$\frac{d\sigma}{dxdQ^2} = \sum_i f_{i/A}(x) \otimes F(x, Q^2)$$



  - Nonperturbative at low energies:

    Simulations on lattices in Euclidean time

    Use Hamiltonian formulation?

- Very difficult: use the Schwinger model as test bed

- Want to do these simulations on a quantum hardware

  Jordan, Lee, Preskill

  → PDFs are near-term goal

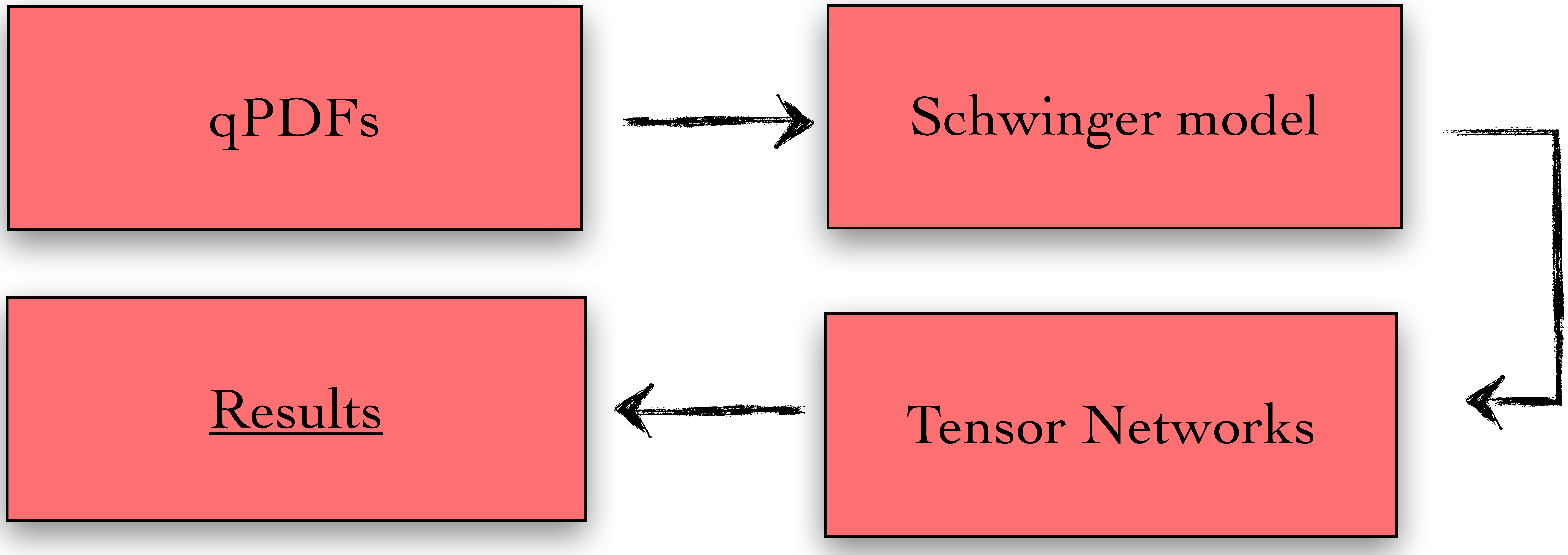
- In collaboration with:

Sebastian Grieninger

Felix Ringer

Ismail Zahed

Outline



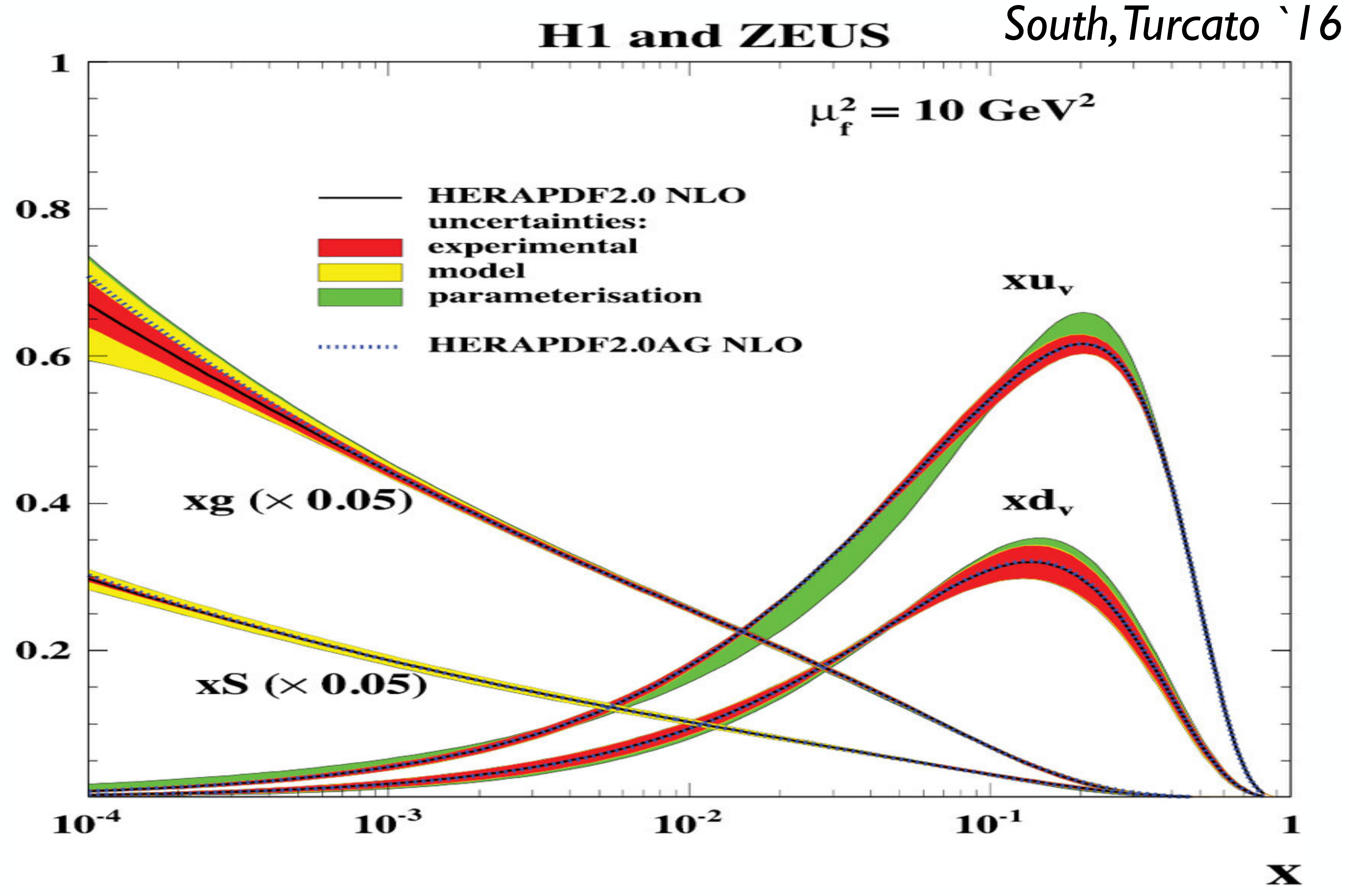
Quasi-PDFs

- PDFs: $$\frac{d\sigma}{dxdQ^2} = \sum_i f_{i/A}(x) \otimes F(x, Q^2)$$ $$x = \frac{k^+}{P^+}$$

Collins, '11

$$f_i(x) = \int_{-\infty}^{\infty} \frac{dz^-}{2\pi} e^{-ixz^-P^+} \langle \eta \,|\, \overline{\psi}_i(0,z^-) W(z^-,0)\, \gamma^+ \psi_i(0,0) \,|\, \eta \rangle$$

South, Turcato `16



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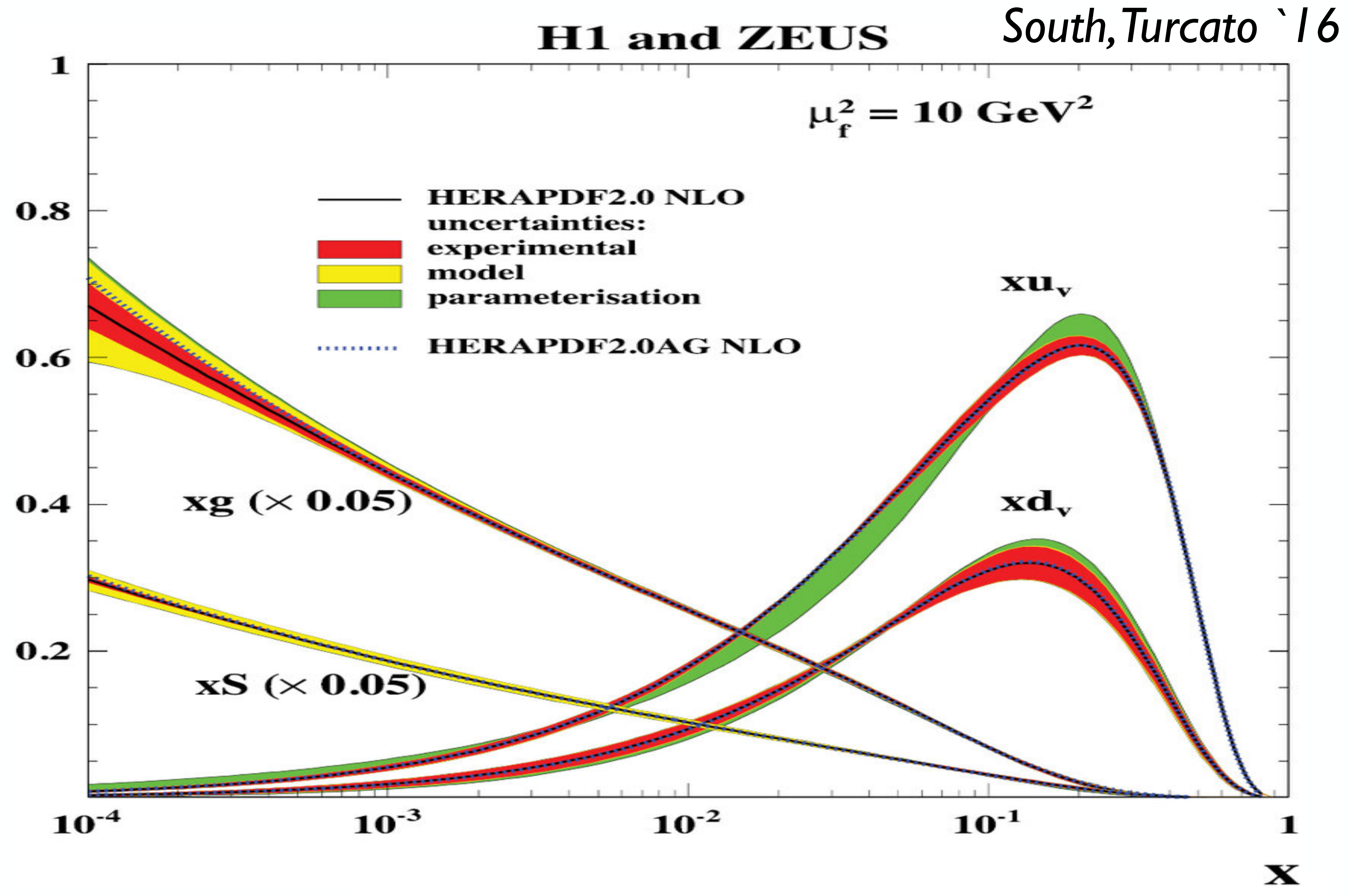
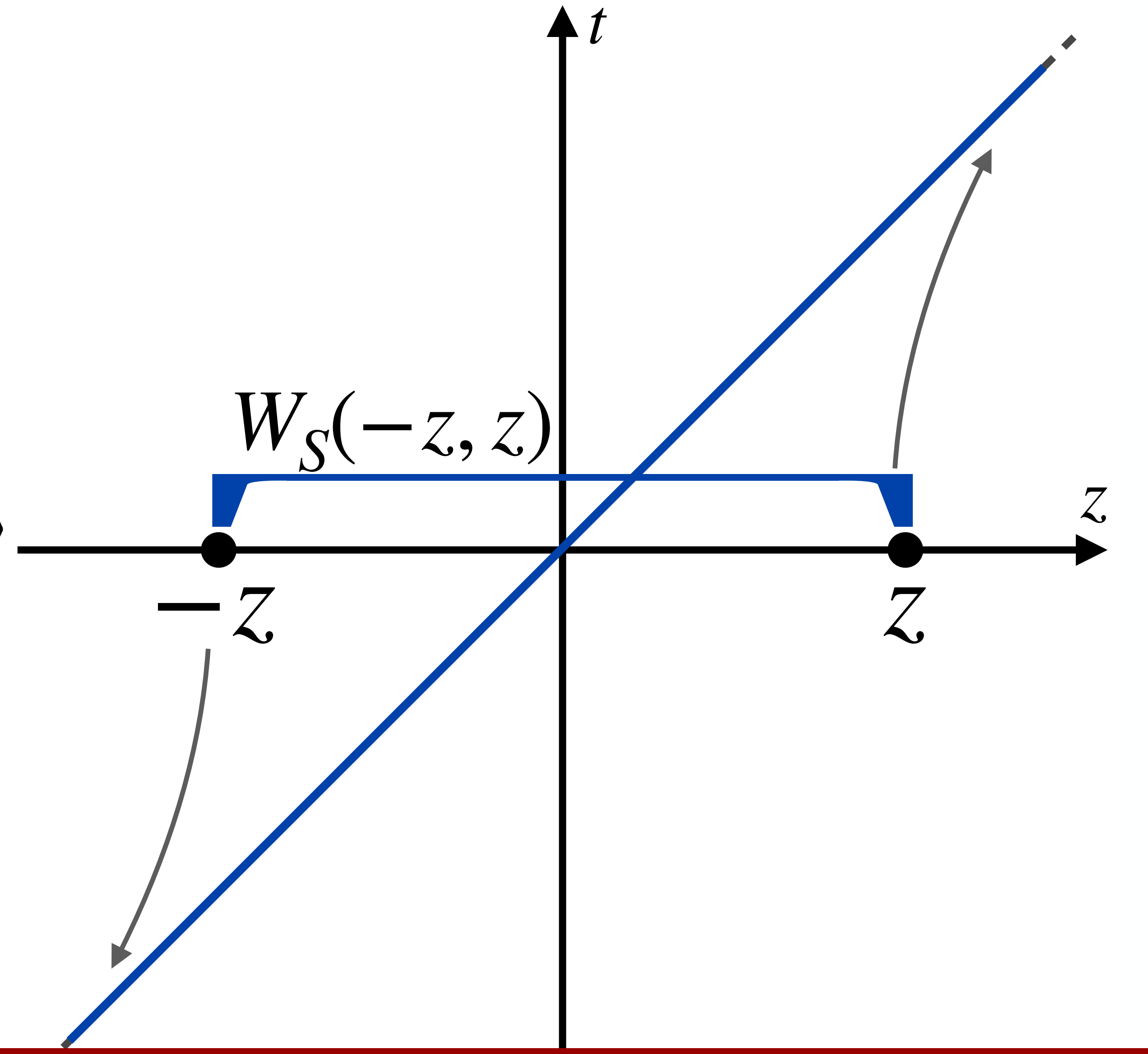


- For velocities $|v| \leq 1$ $\quad \chi = \frac{1}{2} \ln\left(\frac{1+v}{1-v}\right)$

- qPDFs (accessible through lattice MC) Ji. '13

$$q_\eta(x,\chi) = \frac{1}{2} \int_{-\infty}^{\infty} \frac{dz}{2\pi} e^{-izxP^1(\chi)} \langle \eta | e^{-i\chi\hat{K}} \overline{\psi}(-z) W_S(-z,z)\, \gamma^0 \psi(z) e^{i\chi\hat{K}} | \eta \rangle$$



Schwinger Model

- QED in 1+1D: $$\mathscr{L} = \bar{\psi}(i\gamma^\mu D_\mu - m)\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$ *Schwinger `50s*
    - Confining → toy model for QCD

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$$\hat{\psi}_n = \begin{pmatrix} \hat{\varphi}_n \\ \hat{\varphi}_{n+1} \end{pmatrix}$$

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Kogut, Susskind `70s

$$\mathbb{H} = \frac{ag^2}{2}\sum_{n=0}^{N-2}\left(\frac{1}{2}\sum_{m=0}^{n}(-1)^m + Z_m\right)^2 + \frac{1}{4a}\sum_{n=0}^{N-2}\left(X_{n+1}X_n + Y_{n+1}Y_n\right) + \frac{m}{2}\sum_{n=0}^{N-1}(-1)^n Z_n = \sum_n h_n$$

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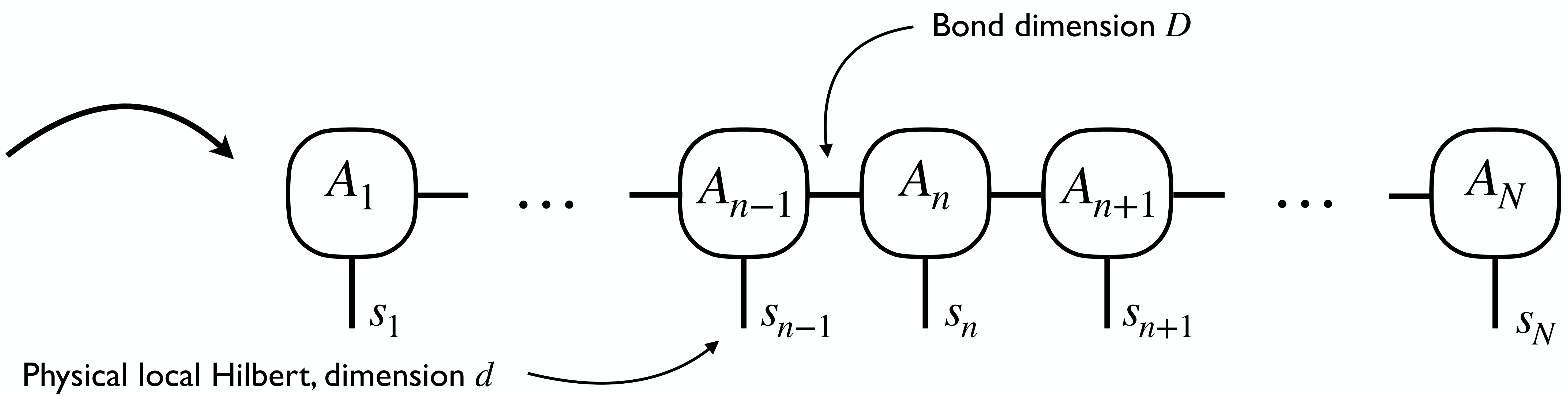
$$\mathbb{K} = \int dx\, x\mathscr{H}(x) \to \sum_n a n h_n$$

$$\mathbb{P} = \frac{-1}{4a}\sum_{n=0}^{N-2}\left(X_{n+2}Z_{n+1}Y_n - Y_{n+2}Z_{n+1}X_n\right)$$

Tensor Networks

- Matrix product states (MPS)

$$|\Psi(A)\rangle = \sum_{\{s\}} \left(\prod_{n=1}^{N} A_n^{s_n} \right) |s_1, s_2, \ldots, s_N\rangle$$



Penrose graphical notation

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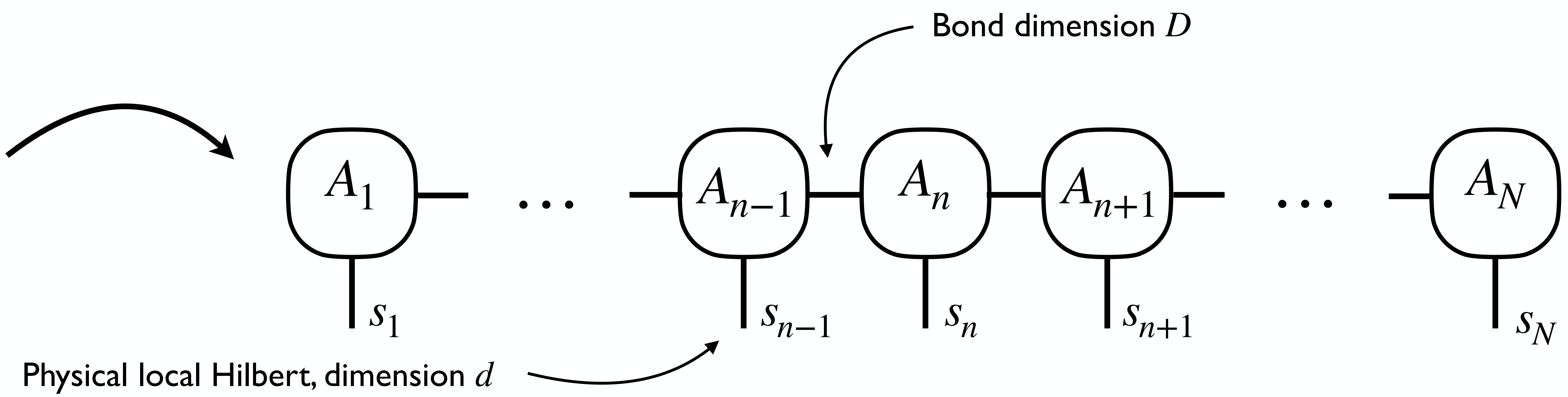

Penrose graphical notation

- Vacuum and hadron state preparation

$$|\Omega\rangle \approx |\Psi(A)\rangle = \underset{|\Psi(A)\rangle}{\operatorname{argmin}} \left[\frac{\langle \Psi(A) | \mathbb{H} | \Psi(A) \rangle}{\langle \Psi(A) | \Psi(A) \rangle} \right]$$

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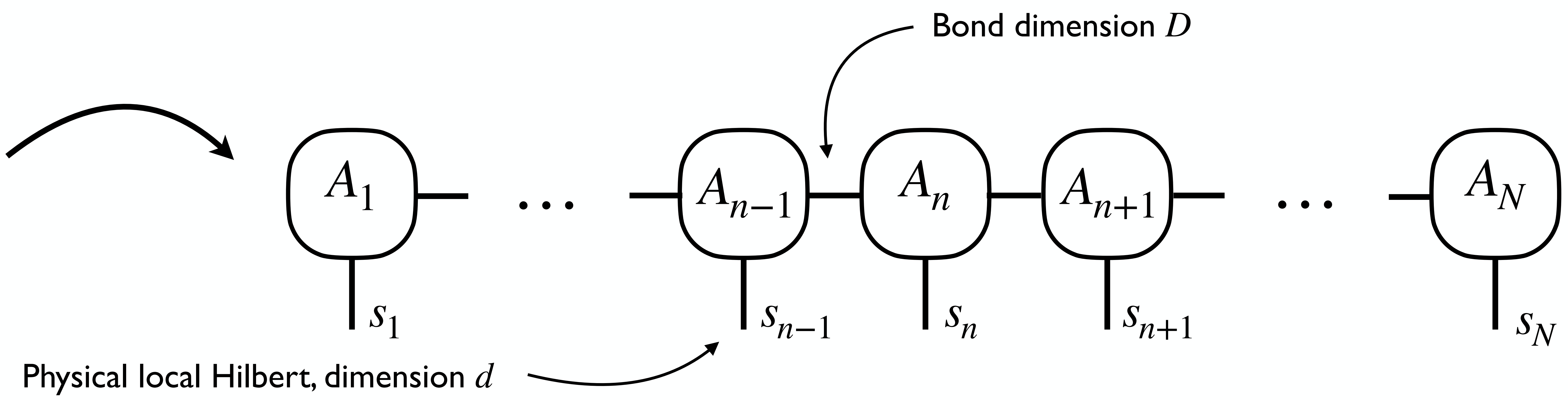

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- Unitary evolution

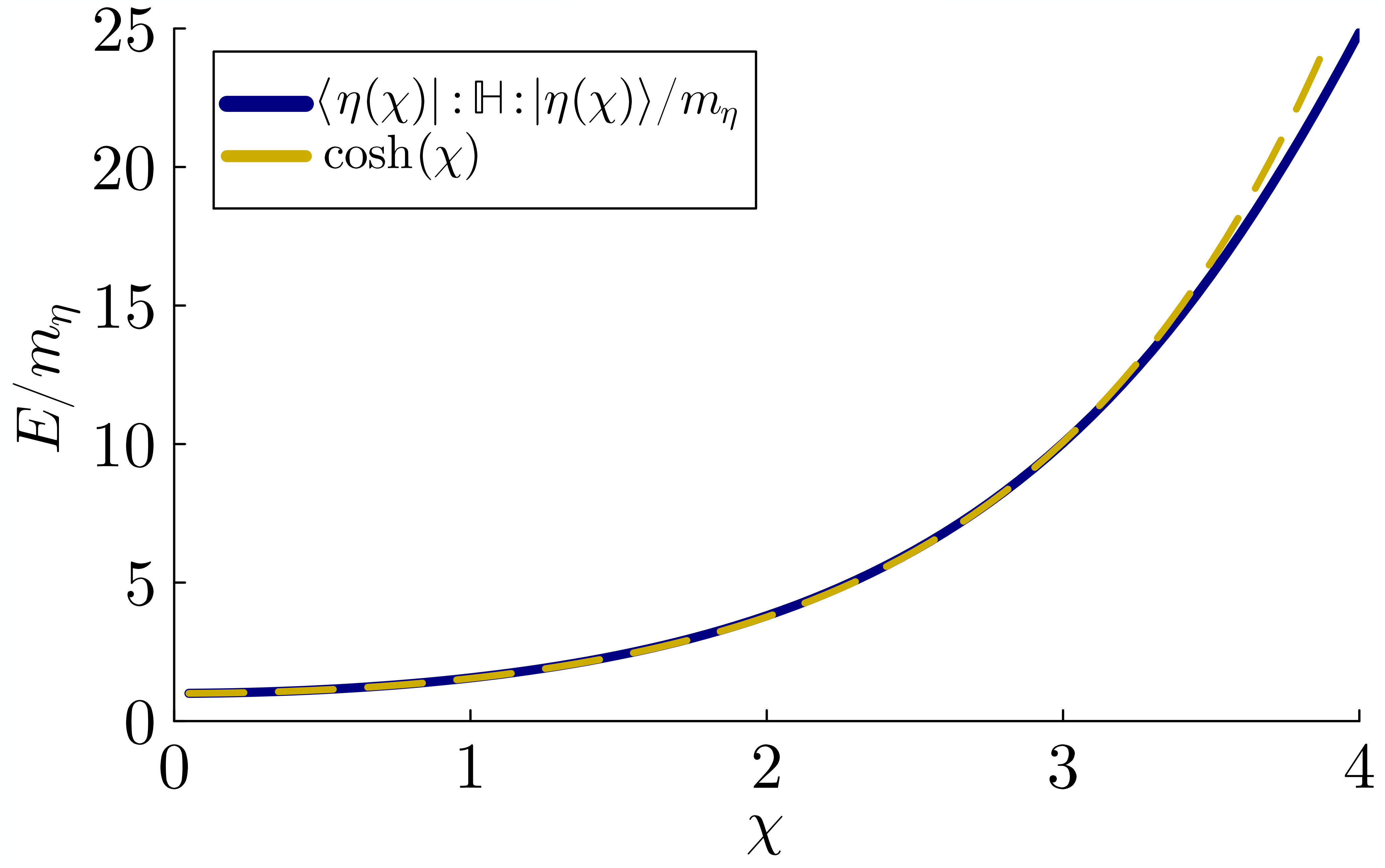
$$\frac{d}{d\chi} |\eta(\chi)\rangle = -i\mathscr{P}_A \mathbb{K} |\eta(\chi)\rangle$$

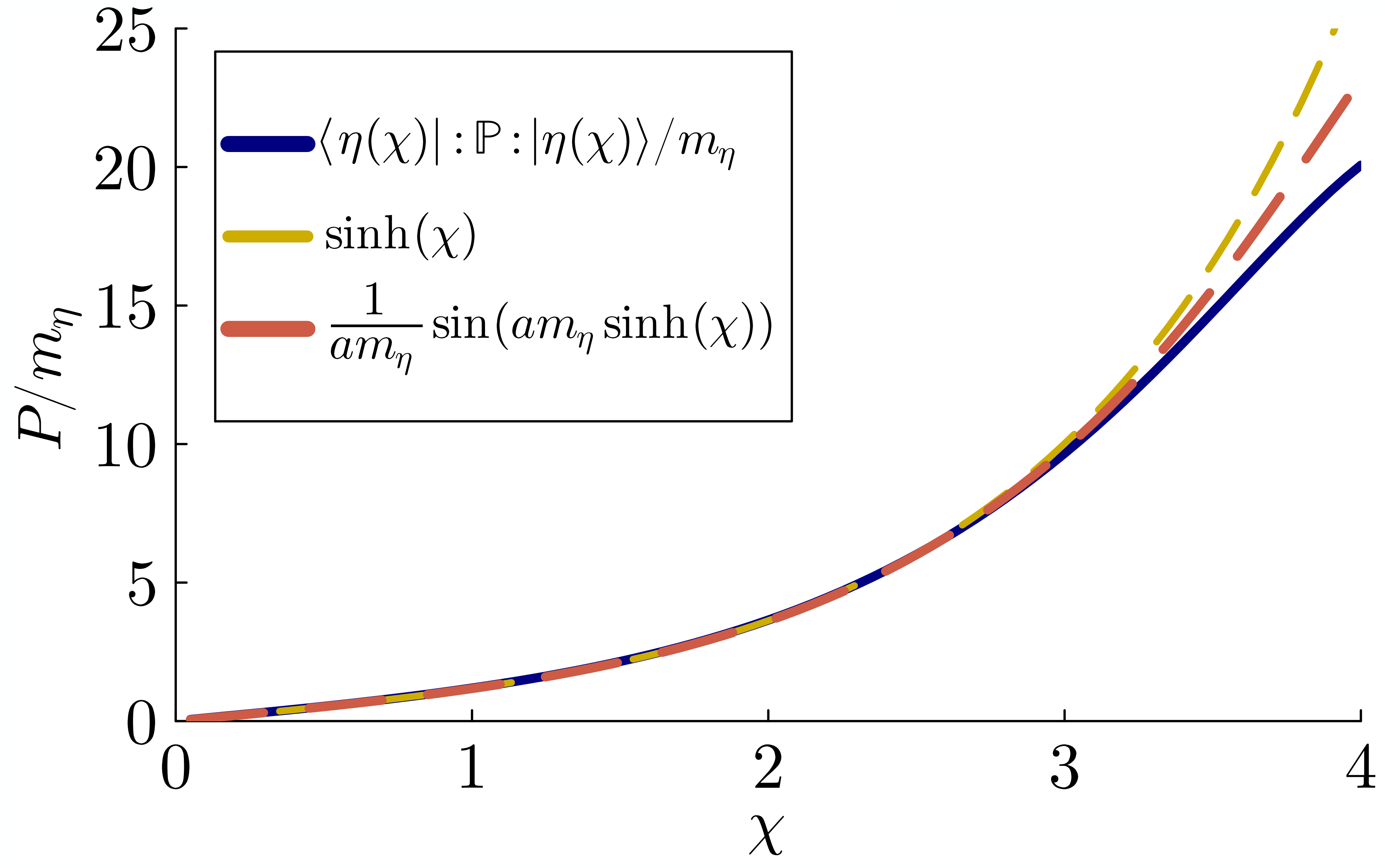
Approaching the lightcone

- State preparation: small variances achievable $\delta E_\Omega / m_\eta \sim \delta E_\eta / m_\eta \sim 10^{-4}$
- In continuum:

$$\langle \eta | e^{i\hat{K}\chi} : \hat{H} : e^{-i\hat{K}\chi} | \eta \rangle = m_\eta \cosh(\chi) \qquad \langle \eta | e^{i\hat{K}\chi} : \hat{P} : e^{-i\hat{K}\chi} | \eta \rangle = m_\eta \sinh(\chi)$$

- For $N = 400$, $a = 0.01$, $m/g = 0.2$:





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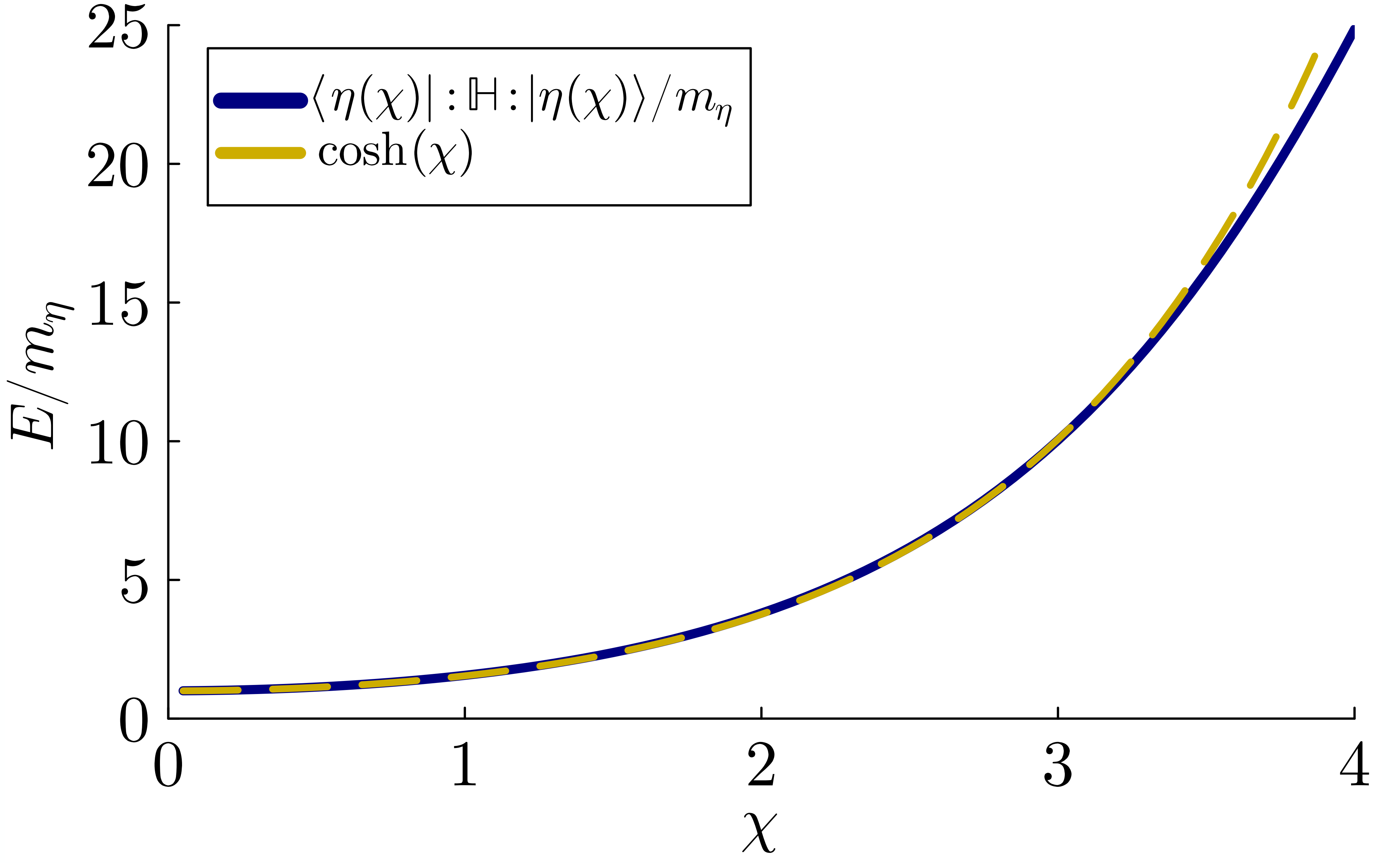
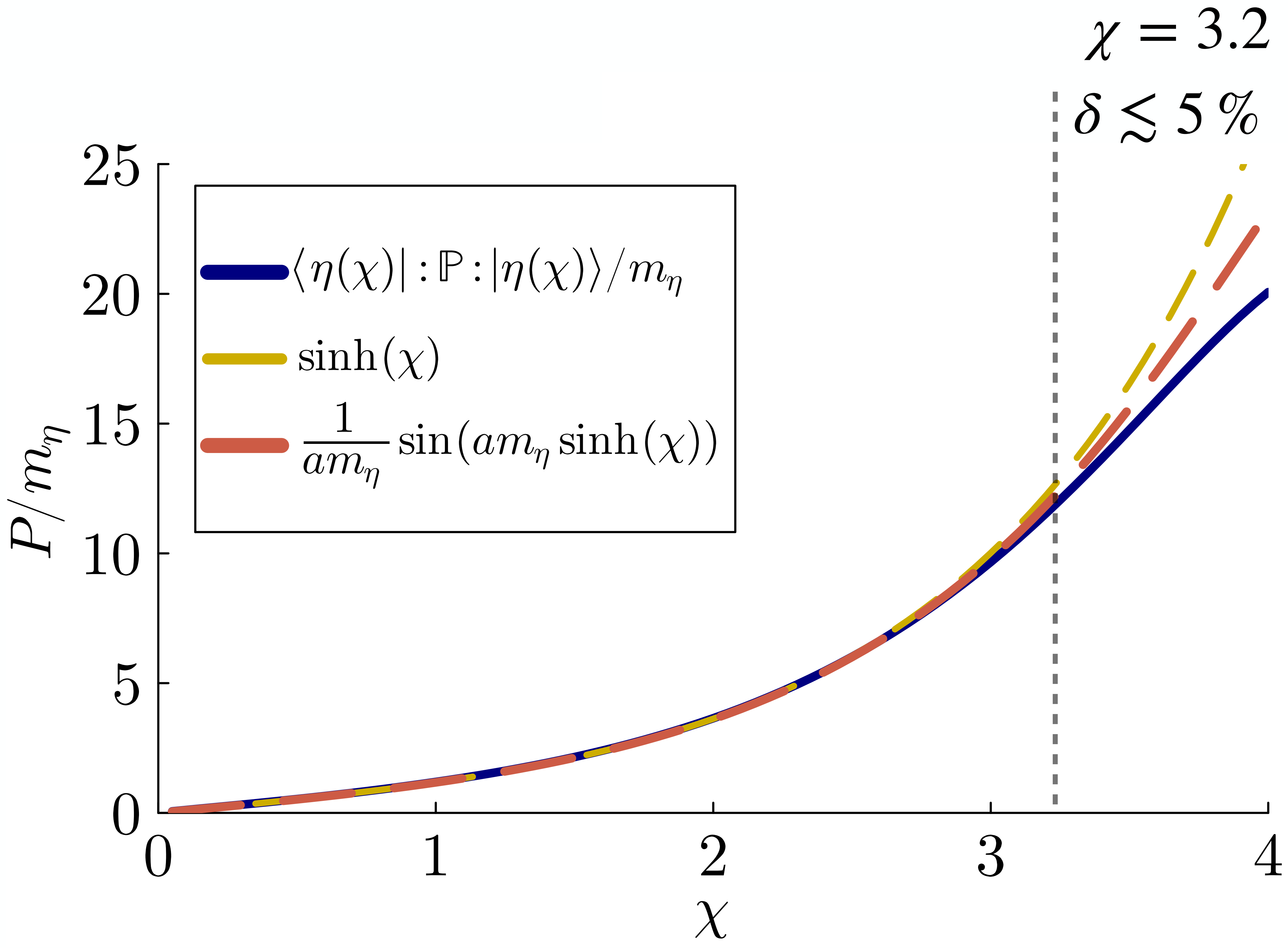
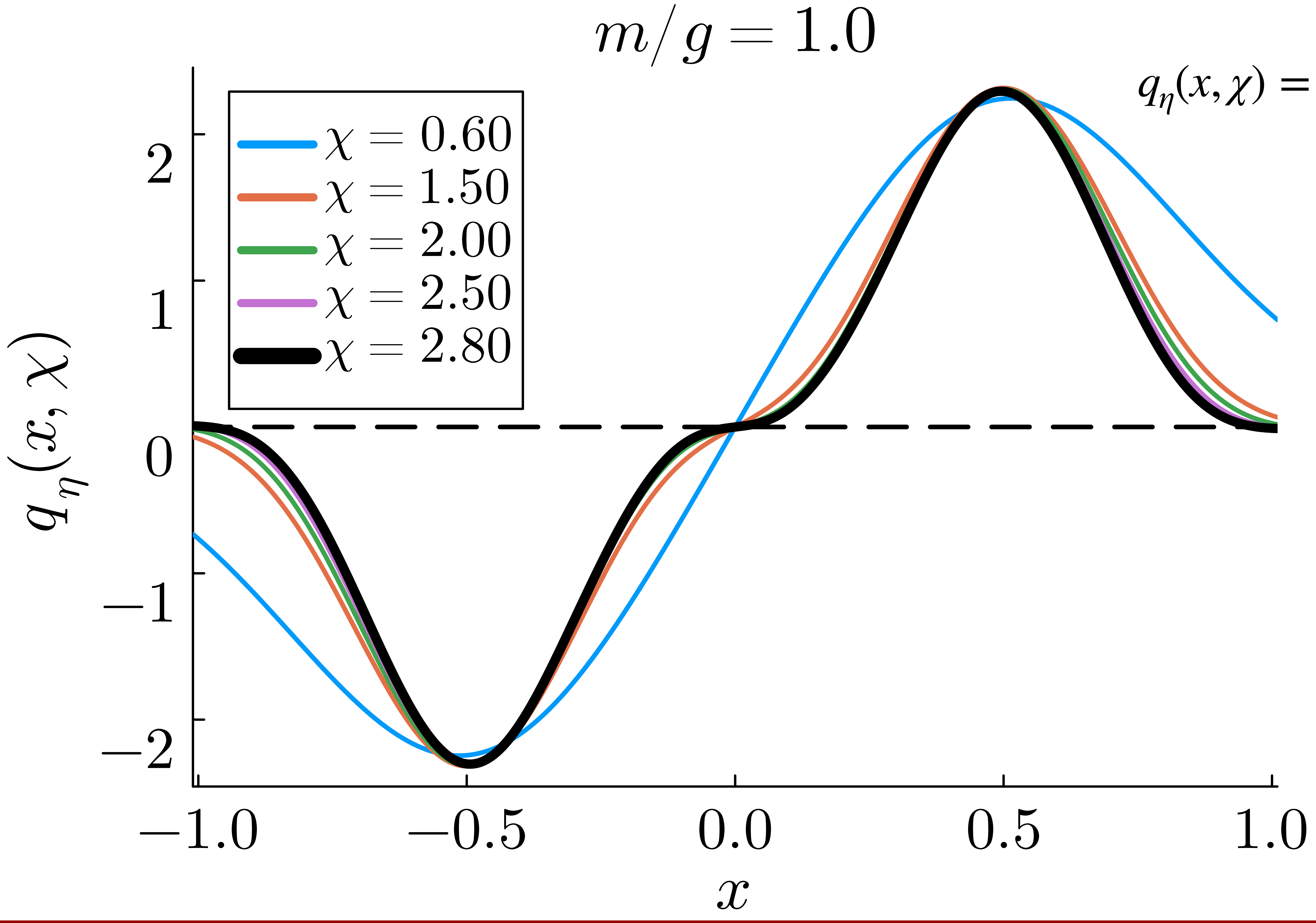




qPDFs



$$q_\eta(x,\chi) = \frac{2a}{2\pi}\sum_z e^{ixzP^1(\chi)} \times$$
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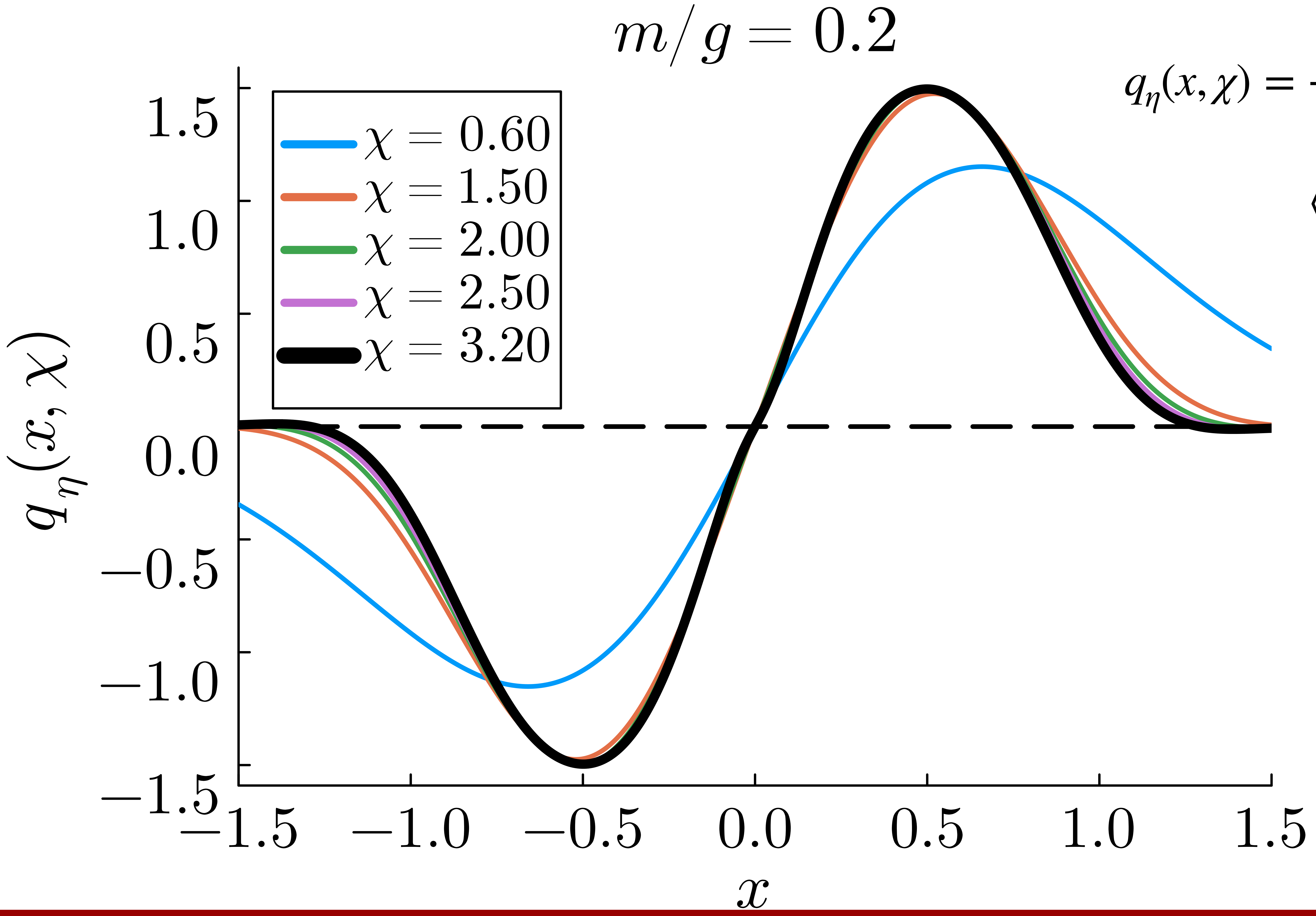
qPDFs



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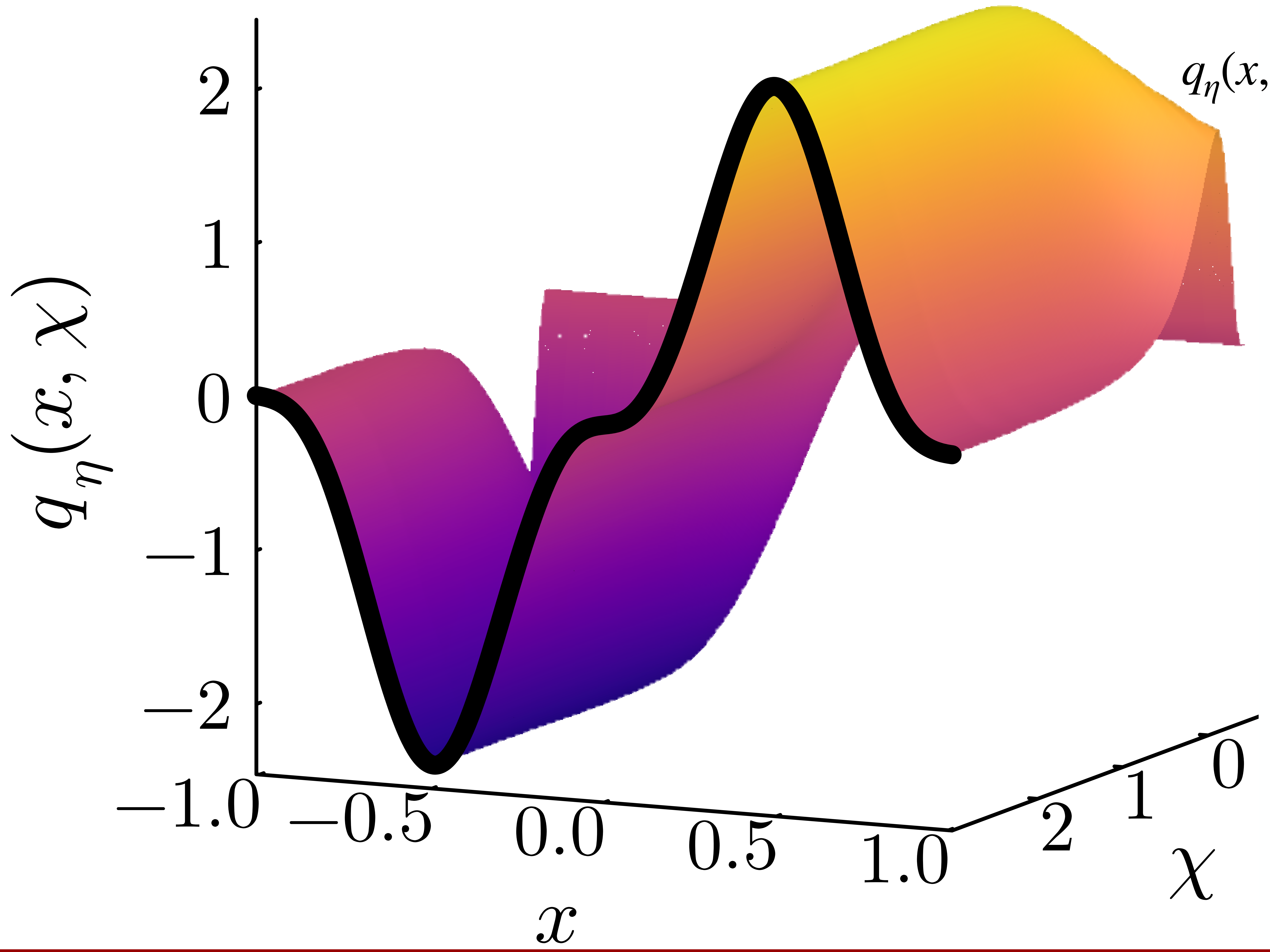

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qGPDs

$$\xi = \frac{(p-p')^+}{(p+p')^+}$$

- GPDs: $H(x, \xi, t)$ → $t = (p-p')^2$

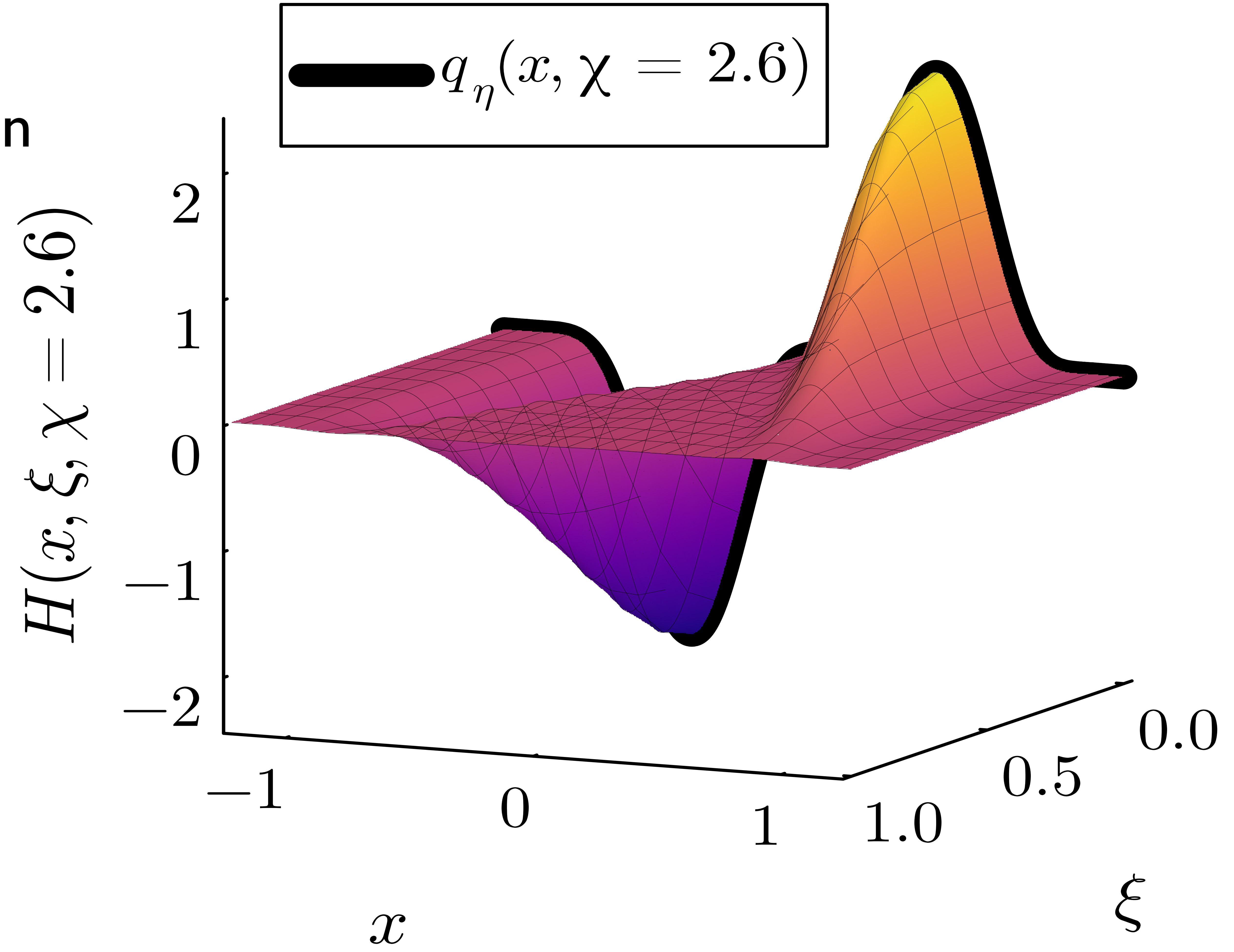
  extension to include transverse information

  for exclusive processes

- qGPDs:

$$H_\eta(x, \xi, \chi) = \frac{2a}{2\pi} \sum_z e^{ixzP^1(\chi)} \times$$

$$\langle \eta(\chi_+) | \overline{\psi}(-z) W_S(-z, z) \gamma^0 \psi(z) \; | \eta(\chi_-) \rangle$$



Conclusion

- Quasi-PDFs for Schwinger model accessible using tensor networks
    - Convergence at modest χ
    - Finite volume effects dominant
    - Provide benchmark for future quantum simulations (in progress…)
    - Can be crosschecked with Euclidean time MC methods
- Quasi-GPDs are also accessible