

Higher order corrections for FCC-ee

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LoopFest XXIV, BNL



Scope of this presentation

- Demands for SM theory inputs from FCC-ee physics case
- Focus on Z-pole and Higgs physics
(W/top/... physics left out for time reasons)
- Theory uncertainty estimations
- Brief discussion of calculational methods

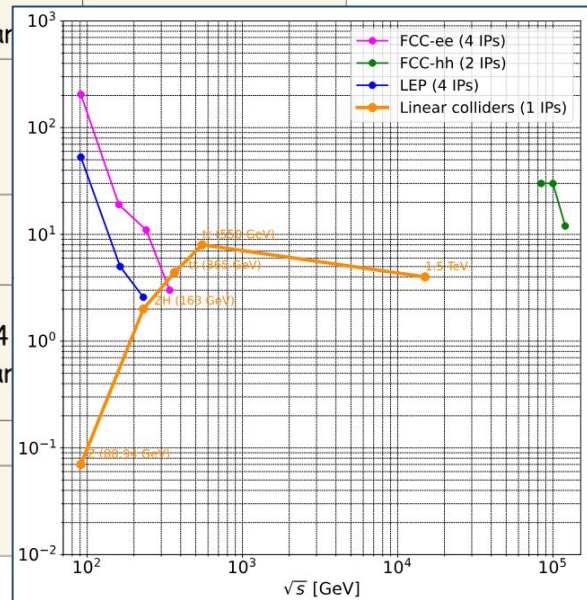
Major future facilities

Project	IP	Z-pole (91.2 GeV)	WW (160 GeV)	Higgs (230–250 GeV)	Top (365 GeV)	Higher Energy
FCC-ee	4	205 ab ⁻¹ 4 year	19 ab ⁻¹ 2 year	11 ab ⁻¹ 3 year	3 ab ⁻¹ 5 year	—
FCC-hh	2	—	—	—	—	84.6 TeV: 0.6 ab ⁻¹ / year /IP
LEP3	2	53 ab ⁻¹ 5 years	5 ab ⁻¹ 4 years	2.5 ab ⁻¹ 6 years	—	—
Linear colliders	1	0.07 ab ⁻¹ 1 year	—	2 ab ⁻¹ 3 years	CLIC: 4.4 ab ⁻¹ 10 years	550 GeV: 8 ab ⁻¹ 1.5 TeV: 4 ab ⁻¹ 10 years
LHeC	1	—	—	—	—	1 TeV for 6 years
Muon Collider	2	—	—	—	—	10 TeV: 1.1 ab ⁻¹ 8 year

Also: HL-LHC, 14 TeV, 3 ab⁻¹

Major future facilities

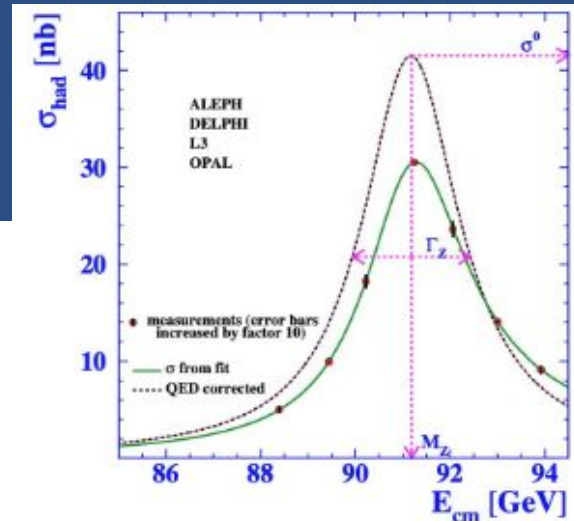
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FCC-hh	2	—	—	—	—	
LEP3	2	53 ab ⁻¹ 5 years	5 ab ⁻¹ 4 years	2.5 ab ⁻¹ 6 years	—	
Linear colliders	1	0.07 ab ⁻¹ 1 year	—	2 ab ⁻¹ 3 years	CLIC: 4.4 10 year	
LHeC	1	—	—	—	—	
Muon Collider	2	—	—	—	—	



Also: HL-LHC, 14 TeV, 3 ab⁻¹

Z lineshape

- **Z mass/width:**
 - main systematic: beam energy calibration (absolute / point-to-point)
 - $\sim 0.1 / 0.012$ MeV at circular colliders through res. depolarization
 - up to $\sim 0.2 / 0.12$ MeV at LC using J/ψ and K_S mass calibration
- **Branching ratios** ($R_\ell = \Gamma_{\text{had}}/\Gamma_\ell$, $R_{b,c} = \Gamma_{b,c}/\Gamma_{\text{had}}$)
 - main systematics: angular acceptance, flavor tagging
 - metrology target for FCC-ee: 5-10 μm ($\sim 100 \mu\text{m}$ for LC)
 - modern ML taggers improve purity and thus syst. calibration from data



	current	LCF	FCC-ee
m_Z [MeV]	2.1	0.2	0.004 (0.1)
Γ_Z [MeV]	2.3	0.12	0.004 (0.012)
R_μ [10^{-3}]	33	2 (2)	0.05 (0.05)
R_c [10^{-5}]	300	5.9	0.05
R_b [10^{-5}]	66	1.5 (1.3)	0.025 (0.035)

stat. \oplus sys. unless noted stat. (sys.)

Z asymmetries

- **Left-right asymmetry:**

- if longitudinal beam pol. available (LCF)

$$A_{LR} = \frac{1}{P_{\text{eff}}} \frac{N_L - N_R}{N_L + N_R} \approx A_e$$

- main systematic: polarization calibration

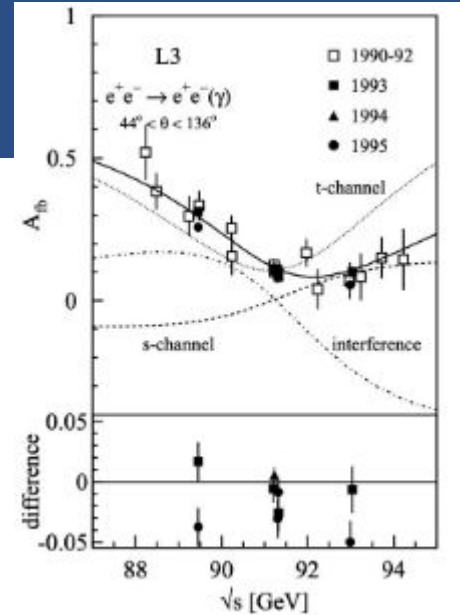
- **Forward-backward asymmetry:**

$$A_{FB} = \frac{N(c_\theta > 0) - N(c_\theta < 0)}{N(c_\theta > 0) + N(c_\theta < 0)} \approx \frac{3}{4} A_e A_f$$

- main systematics:
angular acceptance, beam energy, ISR

- also: $A_{LR,FB}, A_{\text{pol}}^r$

$$A_f = \frac{2(1 - 4\sin^2 \theta_{\text{eff}}^f)}{1 + (1 - 4\sin^2 \theta_{\text{eff}}^f)^2}$$



	current	LCF	FCC-ee
$A_e [10^{-5}]$	130	4	1.4
$A_b [10^{-5}]$	2150	48	10

stat. ⊕ sys.

Need for theory input

- Comparison of EWPOs/HPOs with SM to **probe new physics**
→ multi-loop corrections in full SM
- Extraction of EWPOs/HPOs (**pseudo-observables**) from **real observables**
→ backgrounds (in full SM), QED/QCD, MC tools
- “Other” electroweak parameters (“**input**” parameters)
→ m_t , α_s , etc. extracted from other processes

Pseudo-observables: $\Gamma_{Z \rightarrow ff}$, $\sin^2 \theta_{\text{eff}}^f$, $g_{L,\text{eff}}^{Zff}$, $g_{R,\text{eff}}^{Zff}$, g_{eff}^{Hff} , C_i^{SMEFT} , ...

Theory/modelling uncertainties

Examples of theory uncertainties are:

- **Missing higher-order uncertainties** (MHOUs) in perturbative calculations.
- **Non-perturbative** (NP) effects (e.g. hadronization, running couplings, ...); usually implemented via physics inspired modelling, and requires input from data.

Estimation of theory uncertainties is subject to a **high degree of arbitrariness**.

- MHOUs:**
- Count prefactors (e.g. $g^2/(4\pi^2)*N_f$ for ew. loop order)
 - Extrapolation of perturbative series (e.g. assuming geometric series)
 - Renormalization scale dependence
 - Renormalization scheme dependence (e.g. OS vs. MS)

NP modeling: Variation of models and parameters, lattice computations with error evaluation, ...

Future uncertainty estimates

- Update of theory uncertainty estimates as part of ESPPU 2026 ([2511.03883](#))
- Attempt to provide estimates for as many sources as possible, *but not comprehensive*
- **Three scenarios:**
 - a) Current status
 - b) “Conservative” future theory developments
 - c) “Aggressive” future theory developments
- *Not predictions for what is likely to be achieved!*

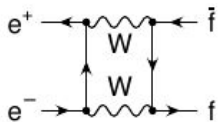
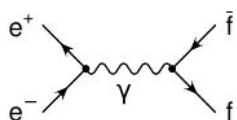
Z-pole (extraction of POs)

- Subtraction of γ -exchange, γ - Z interference, box contributions:

$$\sigma_{\text{hard}} = \sigma_Z + \underbrace{\sigma_\gamma + \sigma_{\gamma Z} + \sigma_{\text{box}}}_{\text{computed in SM}}$$

- Z -pole contribution:

$$\sigma_Z = \frac{R}{(s - M_Z^2)^2 + M_Z^2 \Gamma_Z^2} + \sigma_{\text{non-res}}$$

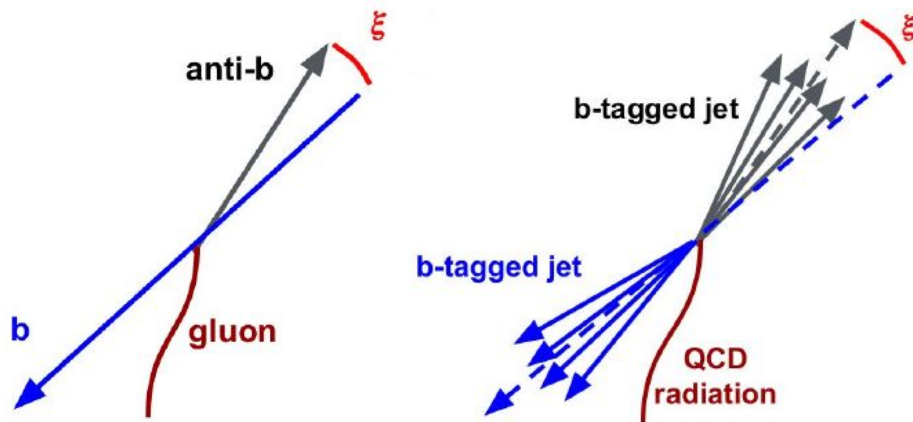


Scenarios:

- Current: NLO corrections
- Future (conservative): NNLO
- Future (aggressive): undetermined (neglect)

Z-pole (extraction of POs)

- Background (γ exchange, box diagrams, non-resonant contributions)
- Need Monte-Carlo (MC) simulations to account for acceptances, cuts, particle ID, etc.
- **QED** [[1903.09895](#)]:
Future scenario includes full $O(\alpha^2)$, log-enhanced $O(\alpha^3)$, and fermion-pair production
- **QCD** [[2010.08604](#), [2209.08078](#)]:
dominated by gluon splitting, NP hadronization
Conservative (aggressive) future scenario assumes fixed-order NNLO (N3LO) QCD corrections, and factor 5 (50) NP improvement



Z-pole (extraction of POs)

- Background (non-resonant contributions)
- Monte-Carlo (MC) simulations to account for acceptances, cuts, particle ID, etc. (QED+QCD)
- In some cases no estimate for aggressive scenario available (neglect in fits)

	current	future (conservative)	future (aggressive)	FCC-ee exp. goal
Γ_z [MeV]	0.23	0.035	—	0.013
m_z [MeV]	0.3	0.03	—	0.1
σ_{had} [pb]	25	1.7	—	
R_ℓ [10^{-3}]	12	0.4	—	0.07
R_c [10^{-5}]	170	17	3.4	0.05
R_b [10^{-5}]	44	4.4	0.9	0.043
A_{FB}^ℓ [10^{-5}]	60	4.3	—	0.33
A_{FB}^b [10^{-5}]	15	3.2	0.28	0.6
A_{FB}^c [10^{-5}]	11	2.3	0.21	0.7

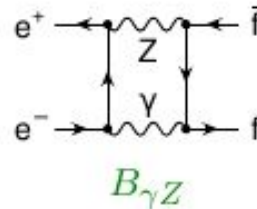
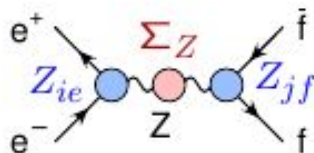
Pole expansion (for “background” subtraction)

Expand amplitude for $e^+e^- \rightarrow f\bar{f}$ about **complex pole** $s_0 \equiv \overline{M}_Z^2 + i\overline{M}_Z\overline{\Gamma}_Z$:

→ All terms are individually gauge-invariant

$$\mathcal{M}_{ij} = \frac{R_{ij}}{s - s_0} + \mathcal{M}_{ij}^{\text{non-res}} + \dots \quad (i, j = V, A)$$

$$R_{ij} = \left. \frac{Z_{ie}Z_{jf}}{1 + \Sigma'_Z} \right|_{s=s_0} + B_{\gamma Z, ij}^R$$



Current state of art: R @ NNLO + leading higher orders

$\mathcal{M}_{ij}^{\text{non-res}}$ @ NLO

$$\overline{M}_Z = M_Z^{\text{exp}} / \sqrt{1 + \Gamma_Z^2 / M_Z^2} \approx M_Z^{\text{exp}} - 34 \text{ MeV}$$

$$\overline{\Gamma}_Z = \Gamma_Z^{\text{exp}} / \sqrt{1 + \Gamma_Z^2 / M_Z^2} \approx \Gamma_Z^{\text{exp}} - 0.9 \text{ MeV}$$

Pole expansion (for “background” subtraction)

Express R_{ij} in terms of $\sin^2 \theta_{\text{eff}}^f$ and F_A^f (accurate up to NNLO):

$$\sin^2 \theta_{\text{eff}}^f = \frac{1}{4|Q_f|} \left[1 - \text{Re} \frac{Z_{Vf}}{Z_{Af}} \right]_{s=\overline{M}_Z^2}$$

$$\Gamma_{Z \rightarrow ff} = \frac{N_c^f M_Z}{12\pi} F_A^f \left[(1 - 4|Q_f| \sin^2 \theta_{\text{eff}}^f)^2 + \dots \right]$$

$$F_A^f = \left[\frac{|Z_{Af}|^2}{1 + \text{Re} \Sigma'_Z} - \frac{1}{2} M_Z \Gamma_Z |Z_{Af(0)}|^2 \text{Im} \Sigma''_{Z(1)} \right]_{s=\overline{M}_Z^2} + \mathcal{O}(\alpha^3)$$

$$\Rightarrow R_{ij} = \frac{Z_{ie} Z_{jf}}{1 + \Sigma'_Z} \Big|_{s=s_0}$$

$$= 4I_e^3 I_f^3 \sqrt{F_A^e F_A^f} \left[Q_i^e Q_j^f + \dots \right],$$

$$Q_V^f = 1 - 4|Q_f| \sin^2 \theta_{\text{eff}}^f,$$

$$Q_A^f = 1$$

terms related to
 $\text{Im} Z_{ie}$, $\text{Im} Z_{jf}$, $\text{Im} \Sigma_Z$
 and the γZ box

(n) = loop order

Implemented in **GRIFFIN**
[\[2211.16272\]](#)

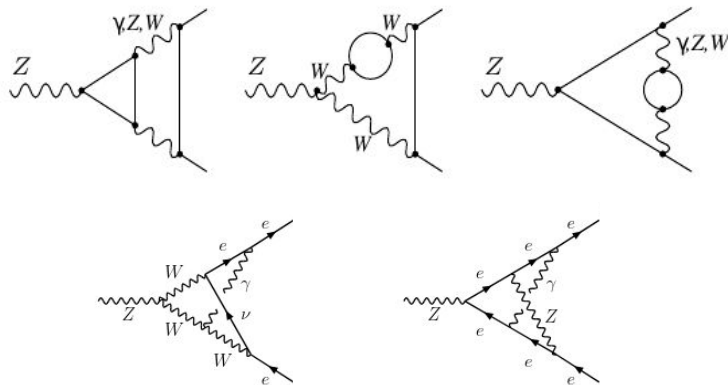


Z-pole (SM predictions for POs)

- Comparison of POs extracted from data with SM predictions used to test SM and probe BSM physics
- EW SM corrections are relatively large
→ need multi-loop contributions
- *Current*: NNLO + m_t -enhanced N3LO*
- *“Conservative”*: N3LO with $N_f \geq 2$ or $N(\alpha_S) \geq 1$ + m_t -enhanced N4LO + α_S^5 FSR
- *“Aggressive”*: N4LO with $N_f \geq 2$ or $N(\alpha_S) \geq 2$ + partial m_t -enhanced N5LO + α_S^6 FSR

* also $O(\alpha_t \alpha_S^3)$

For past work see: [1809.01830](#), [1906.05379](#)



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	current	future (conservative)	future (aggressive)
σ_{had} [pb]	6	1.6	0.3
R_ℓ [10^{-3}]	6	1.2	0.2
R_c [10^{-5}]	5	1	0.2
R_b [10^{-5}]	10	2	0.35
Γ_Z [MeV]	0.4	0.08	0.016
$\sin^2 \theta_{\text{eff,lept}}$ [10^{-5}]	4.5	0.7	0.06
m_W [MeV]	4	1	0.1

Higgs decays

- Uncertainties dominated by MHOU
- Also some parametric uncertainties from strong coupling, W/Z/t/b/c masses

See also: [1404.0319](#), [1906.05379](#), [2206.08326](#)

α = ew loop, α_t = ew loop with top Yukawa, α_s = QCD loop

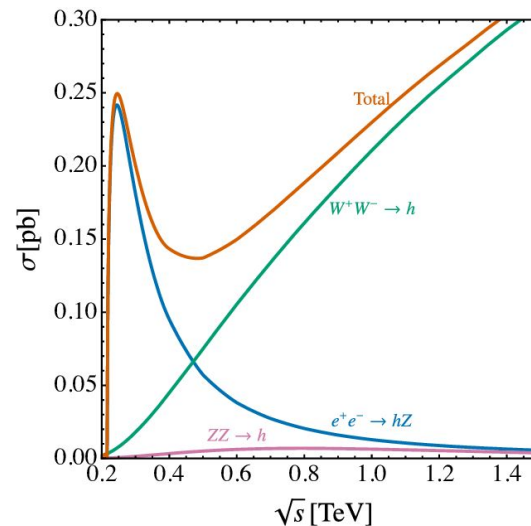
	current		future (conservative)		future (aggressive)		FCC-ee exp. goal
	th. err. estimate	available orders	th. err. estimate	additional orders	th. err. estimate	add'l orders	
H → bb / cc [%]	< 0.4	$\alpha_s^4 + \alpha + \alpha_t^2 + \alpha_t \alpha_s$	0.2	$\alpha^2 + \alpha \alpha_s$	0.1	α_s^5	0.3
H → $\tau\tau$ / $\mu\mu$ [%]	< 0.3	$\alpha + \alpha_t^2 + \alpha_t \alpha_s$	< 0.1	$\alpha^2 + \alpha \alpha_s$			0.4
H → WW / ZZ [%]	0.5	$\alpha_s + \alpha + \alpha_t^2 + \alpha_t \alpha_s$	0.3	α_s^2			0.25
H → gg [%]	2.3	$\alpha_s^3 + \alpha$	1.0	α_s^4	0.5	$\alpha_s^5 + \alpha \alpha_s$	0.45
H → $\gamma\gamma$ [%]	< 1.0	$\alpha_s^2 + \alpha$					1.1
H → Z γ [%]	1.5	$\alpha_s + \alpha$					3.5

Higgs production

- H+Z dominant @ 240/250 GeV (for current theory prediction see [2305.16547](#))
- WW fusion more important at higher energies (NLO known for hadron colliders, [0710.4749](#))
- At very high energies, corrections dominated by $\log^2(E/m_W)$ contributions (apply naive scaling)

α = ew loop, α_f = ew fermion loop, α_s = QCD loop

	collider energy	current		future (conservative)		FCC-ee exp. goals
		th. err. estimate	available orders	th. err. estimate	add'l orders	
HZ [%]	240 GeV	0.3	$\alpha + \alpha\alpha_s + \alpha\alpha_f$	< 0.1	α^2	0.31
WW fusion [%]	< 500 GeV	1	α	0.1	$\alpha^2 + \alpha\alpha_s$	0.52
	1 TeV	2		0.2		
	3 TeV	4		0.4		



Fixed-order theory calculations

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- **Higgs and WW physics:**
 - Full NNLO for $2 \rightarrow 2$ processes
 - $O(\alpha_s^4)$ QCD corrections
 - Matching to Monte-Carlo tools
 - Also need $O(\alpha)$ (or better ?) corrections for backgrounds: $e^+e^-b\bar{b}$, $\nu\bar{\nu}b\bar{b}$, etc.
→ Technology exists, but work needed
- **For Z-pole:**
 - N3LO EW and mixed EW-QCD corrections for Zff vertices
 - Leading 4-loop effects
 - Initial-final QED effects / merging multi-loop and Monte-Carlo

Reduction to master integrals (MIs):

- Public programs: Reduze, FIRE, LiteRed, KIRA, ...
von Manteuffel, Studerus '12; Smirnov '13,14; Lee '13; Maierhoefer, Usovitsch, Uwer '17
- Large computing time and memory
- Possible solutions:
 - Numerical IBP reduction
 - Optimize generation of lin. equation system

Guan, Liu, Ma, Wu '24;
Wu, Boehm, Ma, Xu, Zhang '23; ...

General techniques

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Solutions for MIs:

- a) Approaches based on differential eqs. (needs IBPs) [Kotikov '91](#); [Remiddi '97](#); [Smirnov '01](#)
- Complicated functions needed:
Goncharov polylogs, iterated elliptic integrals, Calabi-Yau geometries, ...
 - Solve DEs with Frobenius series or numerical integration
e.g. [Liu, Ma, Wang '17](#); [Moriello '19](#); [Hidding '20](#)
- b) Compute numerically with sector decomposition or other representations
e.g. [Borowka et al. '17](#); [Heinrich et al. '23](#); [Huang, Huang, Ma '24](#)

NNLO calculations using dispersion relations

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- **Basic idea:** use dispersion relation for sub-loop

$$\begin{aligned}
 \text{---} \bigcirc \text{---} &= B_0(p^2, m_1^2, m_2^2) = \int_{(m_1+m_2)^2}^{\infty} d\sigma \frac{\Delta B_0(\sigma, m_1^2, m_2^2)}{\sigma - p^2 - i\epsilon}, \\
 \Delta B_0(\sigma, m_1^2, m_2^2) &\equiv \frac{1}{\pi} \text{Im} B_0(\sigma, m_1^2, m_2^2)
 \end{aligned}$$

$$\text{---} \text{---} \begin{array}{c} \diagup \quad \diagdown \\ \diagdown \quad \diagup \end{array} \text{---} \text{---} = - \int d\sigma \Delta B_0(\sigma, \dots) D_0(\dots, \sigma)$$

- For full diagram including numerator terms:

$$\begin{aligned}
 \text{---} \text{---} \begin{array}{c} \diagup \quad \diagdown \\ \diagdown \quad \diagup \end{array} \text{---} \text{---} &= - \int d\sigma [c_0 \Delta B_0 + c_1 \Delta B_1 + c_{00} \Delta B_{00} + \dots] \\
 &\times [a_1 D_0 + a_2 D_1 + \dots + a_n C_0 + a_{n+1} C_1 + \dots]
 \end{aligned}$$

(coefficients depend on masses, external momenta and σ)

NNLO calculations using dispersion relations

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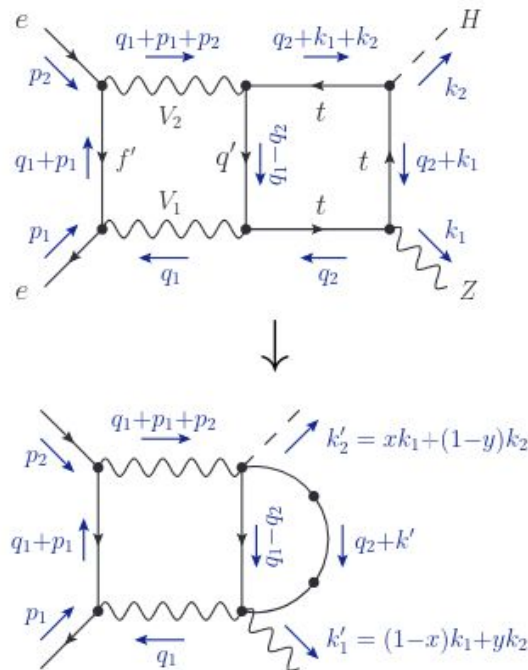
- Introduce Feynman parameters

- q_2 loop =
$$\int dx dy \frac{\partial^2}{\partial(m'^2)^2} \int_{\sigma_0}^{\infty} d\sigma \frac{\Delta B_0(\sigma, m'^2, m_{q'}^2)}{\sigma - \tilde{q}_1^2}$$

$$\tilde{q}_1 = q_1 + k' + i\epsilon,$$

$$m'^2 = m_t^2 - xy(k_1 + k_2)^2 - (1-x-y)(xk_1^2 + yk_2^2).$$

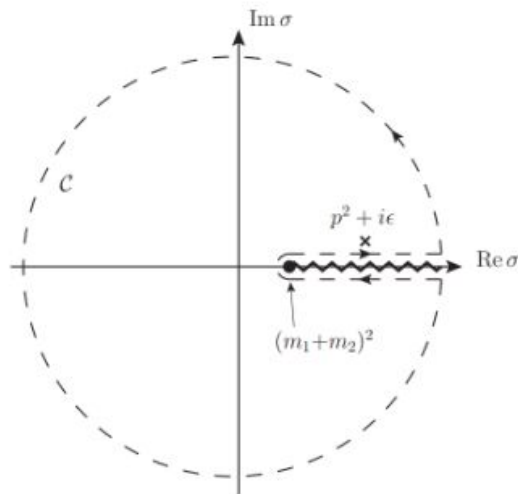
- Similar for other box/vertex topologies
- **Problem:** m'^2 can become negative!



NNLO calculations using dispersion relations

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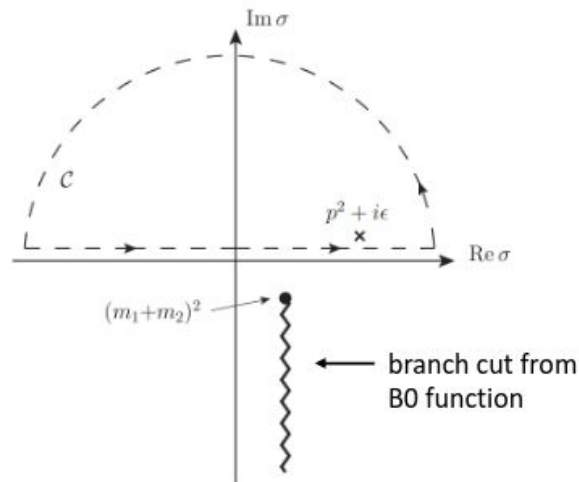
$$m_1^2 \geq 0, m_2^2 \geq 0$$



$$\begin{aligned} B_0(p^2, m_1^2, m_2^2) &= \frac{1}{2\pi i} \oint d\sigma \frac{B_0(\sigma, m_1^2, m_2^2)}{\sigma - p^2 - i\epsilon} \\ &= \int_{(m_1+m_2)^2}^{\infty} d\sigma \frac{\Delta B_0(\sigma, m_1^2, m_2^2)}{\sigma - p^2 - i\epsilon} \end{aligned}$$

$$m_1^2 > 0, m_2^2 < 0$$

integration contours



formulas

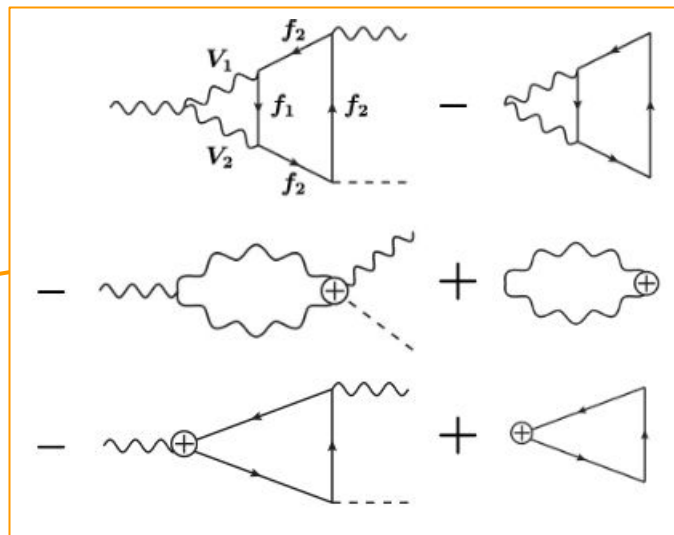
$$\begin{aligned} B_0(p^2, m_1^2, m_2^2) &= \frac{1}{2\pi i} \oint_C d\sigma \frac{B_0(\sigma, m_1^2, m_2^2)}{\sigma - p^2 - i\epsilon} \\ &= \frac{1}{2\pi i} \int_{-\infty}^{+\infty} d\sigma \frac{B_0(\sigma, m_1^2, m_2^2)}{\sigma - p^2 - i\epsilon} \end{aligned}$$

NNLO calculations using dispersion relations

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- Need to subtract terms to make integral UV finite
- Subtraction terms simple enough to integrate analytically and add back

$$\begin{aligned}
 |M_0 M_2^*| &\sim \int dx \int dy \int d\sigma \times \underbrace{[\text{integrand}]}_{\text{UV div}} \\
 &= \int dx \int dy \int d\sigma \times \underbrace{[\text{integrand} - I_{\text{subtra}}]}_{\text{UV finite, integrate numerically}} \\
 &+ \int dx \int dy \int d\sigma \times \underbrace{[I_{\text{subtra}}]}_{\text{UV div, integrate analytically}}
 \end{aligned}$$



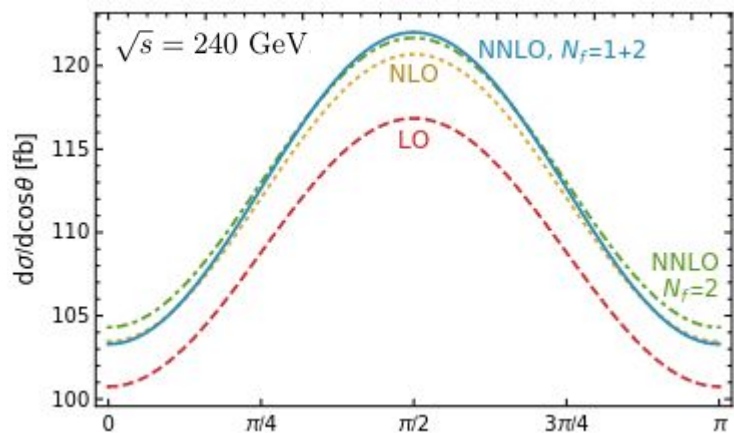
NNLO calculations using dispersion relations

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Applications:

Fermionic NNLO corrections to $ee \rightarrow HZ$

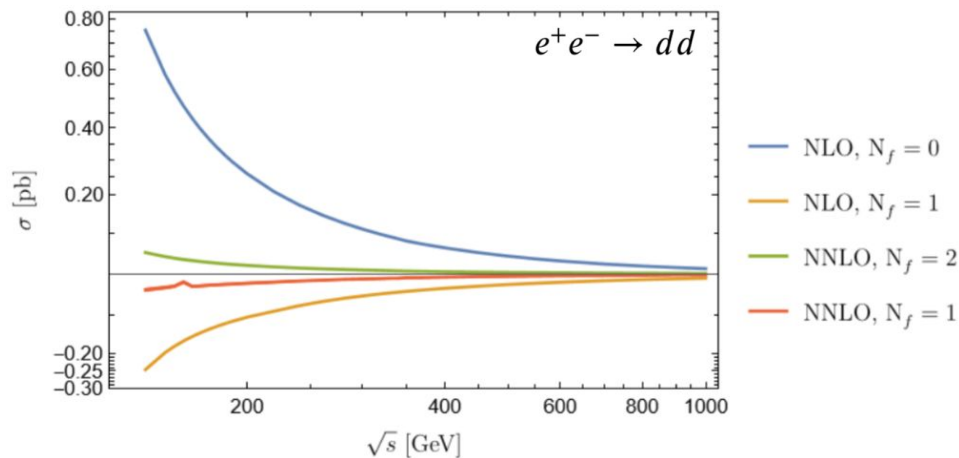
[\[2209.07612, 2305.16547\]](#)



(also see [\[2209.14953\]](#))

Fermionic NNLO corrections to $ee \rightarrow ff$

[\[2512.15700\]](#)



(also see [\[2511.20365\]](#))

Conclusions and Outlook

- Future e^+e^- colliders have enormous potential to improve precise understanding of EW physics through precision measurements
- Improved experimental precision (1-2 orders of magnitude) call for significant advances in theory calculations, simulation tools, and control of uncertainties
- Many areas require 1–2 perturbative orders improvement (fixed order and resummed), as well as non-perturbative contributions (hadronization, running couplings, ...) and advanced simulation frameworks
- Much progress in recent years (driven by LHC physics), but may need development of entirely new ideas and methods

Backup

Theory for e^+e^- colliders

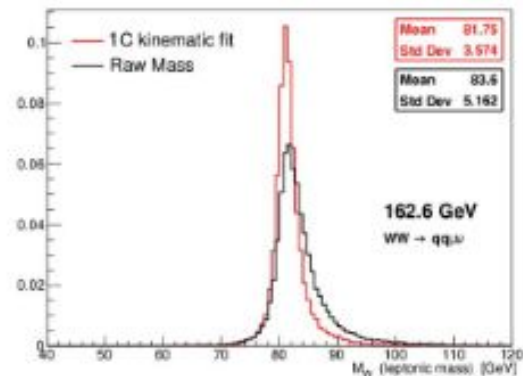
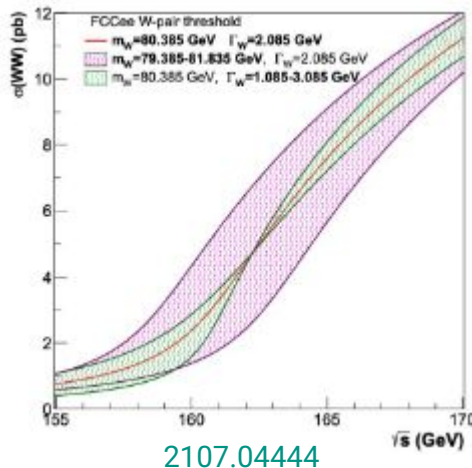
Experimental precision goals of e^+e^- colliders:

(from ESPPU submissions;
with some adjustments for consistency)

	current	LCF	FCC-ee
σ_{had} [pb]	33		0.8
R_μ [10^{-3}]	33	3	0.07
R_c [10^{-5}]	300	5.9	0.05
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Γ_Z [MeV]	2.3	0.12	0.013
A_e [10^{-5}]	130	4	1.4
A_b [10^{-5}]	2150	48	10
m_W [MeV]	10	1.7	0.24
$B(W \rightarrow e\nu)$ [%]	1.5		0.015

W mass/width

- **From threshold scan**
(preferred for circ. colliders):
 - main systematic:
beam energy calibration
- **From kinematic fit**
(preferred for LC)
 - $qq\ell\nu$ most sensitive
(avoid color reconn.)
 - main systematics:
energy calibration (from constrained kinematics, using meson masses as inputs);
beam energy spread



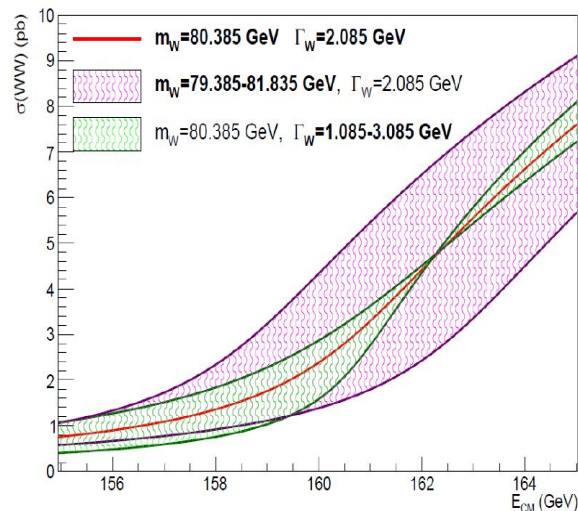
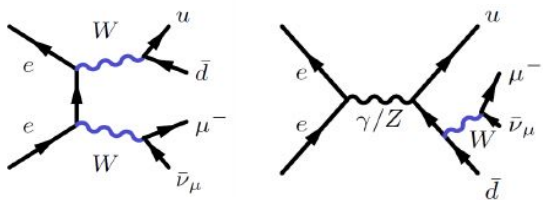
[CERN-THESIS-2019-291](#)

	current	LCF	FCC-ee
m_W [MeV]	9.9	0.5 (1.6)	0.4 (0.7)
Γ_W [MeV]	42	2	0.7 (0.5)

stat. \oplus sys. unless noted stat. (sys.)

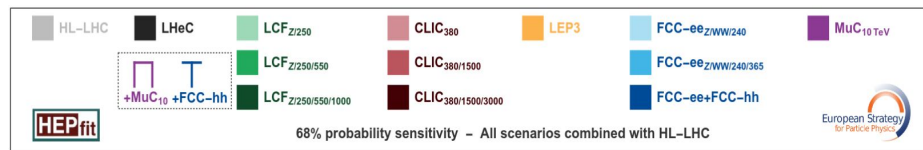
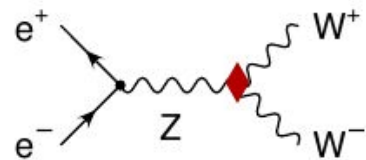
WW threshold

- Full $\mathcal{O}(\alpha)$ calculation of $e^+e^- \rightarrow 4f$
Denner, Dittmaier, Roth, Wieders '05
- EFT expansion in $\alpha \sim \Gamma_W/M_W \sim \beta^2$
Beneke, Falgari, Schwinn, Signer, Zanderighi '07
 - NLO corrections with NNLO Coulomb correction
($\propto 1/\beta^n$): $\delta_{\text{th}}M_W \sim 3 \text{ MeV}$
Actis, Beneke, Falgari, Schwinn '08
 - Adding NNLO corrections to $ee \rightarrow WW$ and $W \rightarrow f\bar{f}$ and NNLO ISR: $\delta_{\text{th}}M_W \lesssim 0.6 \text{ MeV}$



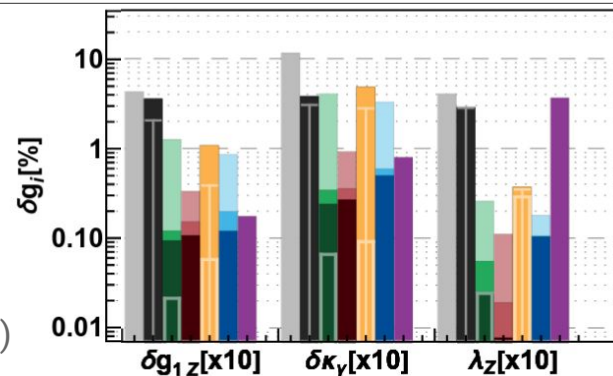
W production and decay

- $ee \rightarrow WW$ useful to constrain anomalous gauge-boson couplings (aGC) (current SM predictions: [hep-ph/0502063](https://arxiv.org/abs/hep-ph/0502063))
- W decays useful, e.g., to constrain α_s (current SM predictions: [2005.04545](https://arxiv.org/abs/2005.04545))
- main systematic: luminosity determination, $\mathcal{O}(10^{-4})$



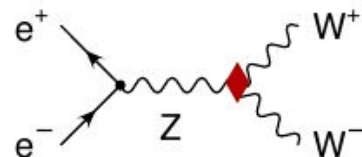
Observable	present		FCC-ee	
	value	\pm uncertainty	FCC-ee Stat.	FCC-ee syst.
$\mathcal{B}(W \rightarrow e\nu_e) \times 10^4$	1071	± 16	0.13	0.10
$\mathcal{B}(W \rightarrow \mu\nu_\mu) \times 10^4$	1063	± 15	0.13	0.10
$\mathcal{B}(W \rightarrow \tau\nu_\tau) \times 10^4$	1138	± 21	0.13	0.15

([2511.03883](https://arxiv.org/abs/2511.03883))



W production and decay

- $ee \rightarrow WW$ useful to constrain anomalous gauge-boson couplings (aGC) (current SM predictions: [hep-ph/0502063](https://arxiv.org/abs/hep-ph/0502063))
- W decays useful, e.g., to constrain α_s (current SM predictions: [2005.04545](https://arxiv.org/abs/2005.04545))
- Assume $\sqrt{s} = 240$ GeV



$\alpha = \text{ew loop}, \alpha_s = \text{QCD loop}$

	current		future (conservative)	
	th. err. estimate	available orders	th. err. estimate	additional orders
$\sigma(ee \rightarrow WW)$ [%]	0.4	α	0.07	$\alpha^2 + \alpha\alpha_s$
$\Gamma(W \rightarrow \text{lept.})$ [%]	0.1	α	0.013	$\alpha^2 + \alpha\alpha_s$
$\Gamma(W \rightarrow \text{had.})$ [%]	0.1	$\alpha + \alpha\alpha_s + \alpha_s^4$	0.015	$\alpha^2 + \alpha\alpha_s^2$
had./lept. BR [%]	0.015	$\alpha + \alpha\alpha_s + \alpha_s^4$	<0.01	$\alpha^2 + \alpha\alpha_s^2$

SM parameters

- SM predictions require precise inputs for m_t , m_W , $\alpha_s(m_Z)$, $\Delta\alpha = 1 - \alpha(0)/\alpha(m_Z)$ [also $m_{b,c}$]
- Can be extracted from e+e- data, but requiring additional theory input

	current	future (conservative)	future (aggressive)	Comments
m_t [MeV]	35	30		from tt threshold; th. error based on 1711.10429 , 1906.05379 , 2503.18713
m_W [MeV]	3	0.6		from WW threshold; th. error for EW/QCD corrections from 1906.05379 , for QED MC based on 1903.09895
$\alpha_s(m_Z)$ [10^{-3}]	1	0.3	0.1	mostly based on lattice improvements (see QCD group)
$\Delta\alpha$ [10^{-5}]	<10	<5	1–3	“conservative” from e+e- data (based on doi:10.23731/CYRM-2020-003.9), “aggressive” from direct Z-pole measurement (1512.05544 , 2501.05508)

“Model-independent” fit

LEP1 era fit parameters:

Schael et al. '05

- 9-parameter fit: $M_Z, \Gamma_Z, \sigma_{\text{had}}^0 \approx \frac{12\pi}{M_Z^2} \frac{\Gamma_{ee}\Gamma_{ff}}{\Gamma_Z^2}, R_e, R_\mu, R_\tau, A_{\text{FB}}^e, A_{\text{FB}}^\mu, A_{\text{FB}}^\tau$
- 5-parameter fit: $M_Z, \Gamma_Z, \sigma_{\text{had}}^0 \approx \frac{12\pi}{M_Z^2} \frac{\Gamma_{ee}\Gamma_{ff}}{\Gamma_Z^2}, R_\ell, A_{\text{FB}}^\ell$

Heavy-flavor pseudo-observables ($R_{b,c}, A_{\text{FB}}^{b,c}, \dots$) handled separately

- Underlying assumption: Can trust SM predictions for
 - a) non-resonant contributions (beyond leading Z pole)
 - affected e.g. by Z' bosons or 4f operators
 - b) $\text{Im } Z_{if}, \text{Im } \Sigma_Z$
 - affected e.g. by light BSM particles in loops
- Assumption probably ok for LEP1 given constraints from BSM searches
 - doubtful if precision is increased by factor 10-100

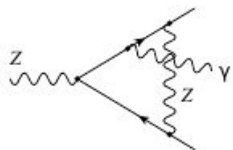
FSR QED/QCD radiation

Factorization of massive and QED/QCD FSR:

$$\bar{\Gamma}_f \approx \frac{N_c \bar{M} Z}{12\pi} \left[\mathcal{R}_V^f F_V^f + \mathcal{R}_A^f F_A^f \right]_{s=\bar{M}_Z^2}$$



Additional non-factorizable contributions, e.g.



→ Known at $\mathcal{O}(\alpha\alpha_s)$

Czarnecki, Kühn '96

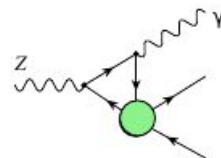
Harlander, Seidensticker, Steinhauser '98

→ Currently not consistently implemented $\mathcal{O}(\alpha^2)$ and beyond

→ Small ($< \mathcal{O}(0.1\%)$) but not negligible contribution

→ Additional matching terms in MC simulations

→ Influenced by different SMEFT ops. than Zff vertex



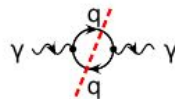
Running electromagnetic coupling

- $\Delta\alpha \equiv 1 - \frac{\alpha(0)}{\alpha(M_Z)} \approx 0.059 = 0.0315_{\text{lept}} + 0.0276_{\text{had}}$

a) $\Delta\alpha_{\text{had}}$ from $e^+e^- \rightarrow \text{had.}$ using dispersion relation

→ Current precision $\sim 10^{-4}$

Davier et al. '19; Jegerlehner '19; Keshavarzi, Nomura, Teubner '19



b) $\Delta\alpha_{\text{had}}$ from Lattice QCD

→ Much progress, recent precision $\sim 5 \times 10^{-5}$

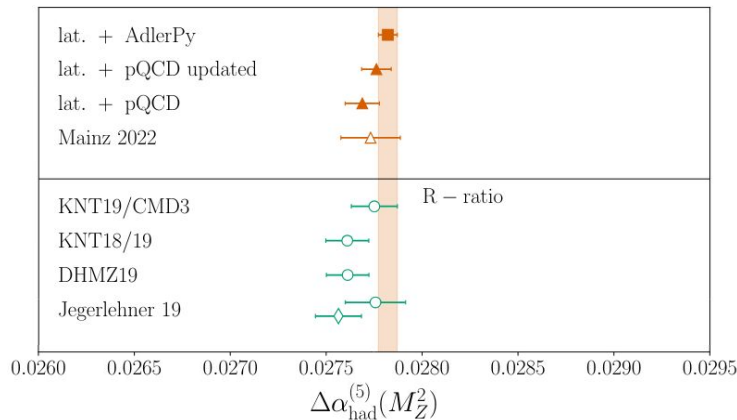
Burger et al. '15

Cè et al. '22

Conigli et al. '26

Future improvements for methods (a) and (b):

- More precise exp./lattice data
- Full 4-loop pQCD for R-ratio / Adler function (for $|Q^2| \gg \Lambda_{\text{QCD}}$)
- More precise inputs for m_b, m_c, α_s



Running electromagnetic coupling

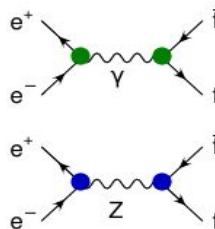
- c) Direct det. of $\alpha(M_Z)$ from $e^+e^- \rightarrow \mu^+\mu^-$ off the Z peak
 ($\sqrt{s} = M_Z \pm \text{few GeV}$)

Janot '15

$$|\mathcal{M}_{ij}|^2 \propto |g_i^\ell|^2 |g_j^\ell|^2 + (s - M_Z^2) \alpha(M_Z) |g_{i,j}^\ell|^2 + \dots$$

$$\rightarrow \delta(\alpha(M_Z)) \sim 3 \times 10^{-5}$$

determined
from Z pole

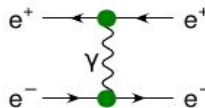


- d) "Semi"direct det. of $\alpha(M_Z)$ from angular dist.
 of $\underbrace{e^+e^- \rightarrow \mu^+\mu^-}_{\text{dominated by Z res.}}$ vs. $\underbrace{e^+e^- \rightarrow e^+e^-}_{\text{dom. by photon t-channel in forward region}}$ on Z peak

Riembeau '25

\rightarrow translate $\alpha(t)$ to $\alpha(M_Z)$ with theory

$$\rightarrow \delta(\alpha(M_Z)) \sim 10^{-5}$$



Methods (c) and (d) require 2/3-loop corrections for $e^+e^- \rightarrow \mu^+\mu^-/e^+e^-$

Open issues for e^+e^- colliders

No available theory uncertainty estimates for:

- EWPOs from radiative return $ee \rightarrow \gamma Z$
- Luminosity determination from $ee \rightarrow ee$, $ee \rightarrow \gamma\gamma$ (see e.g. [2506.15390](#))
- MC generator effects for W/Higgs/top production and decay
- Unique challenges in cross-over region between threshold and continuum for WW and $t\bar{t}$ production, e.g. $\sqrt{s} \sim 360\text{--}365$ GeV for $ee \rightarrow t\bar{t}$ (see [1712.02220](#), [2209.14259](#))