UNIVERSITÀ DI PAVIA
INFN

Radiative corrections at e^+e^- colliders

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JHEP 05 (2025) 196
SciPost Phys.Comm.Rep. 9 (2025)
Phys.Rev.D 112 (2025)
JHEP 05 (2026) 221
+ in preparation

Lepton Colliders

QED beyond fixed order

Fixed order calculations increase precision in the perturbative coupling expansion

QED beyond fixed order

Logarithmic expansion in $L^N = N \log Q^2/m^2$

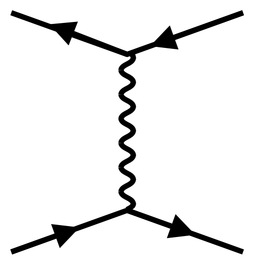
Due to the presence of large logarithms also the log expansion is needed for high precision

$$L = \log \frac{s}{m_e^2} \sim \mathcal{O}(10)$$

QED beyond fixed order

Logarithmic expansion in $L^N = N \log Q^2/m^2$

Coupling expansion in α^N



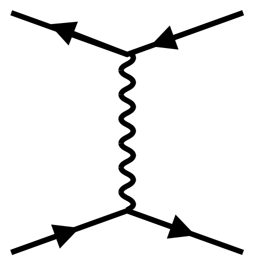
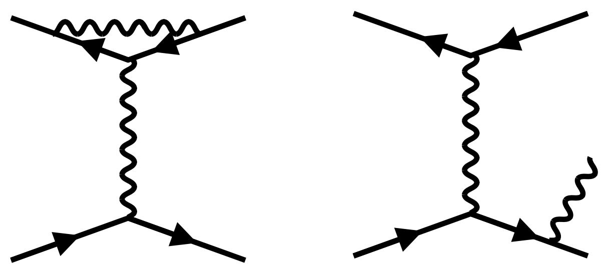
LO

α^0

QED beyond fixed order

Logarithmic expansion in $L^N = N \log Q^2/m^2$

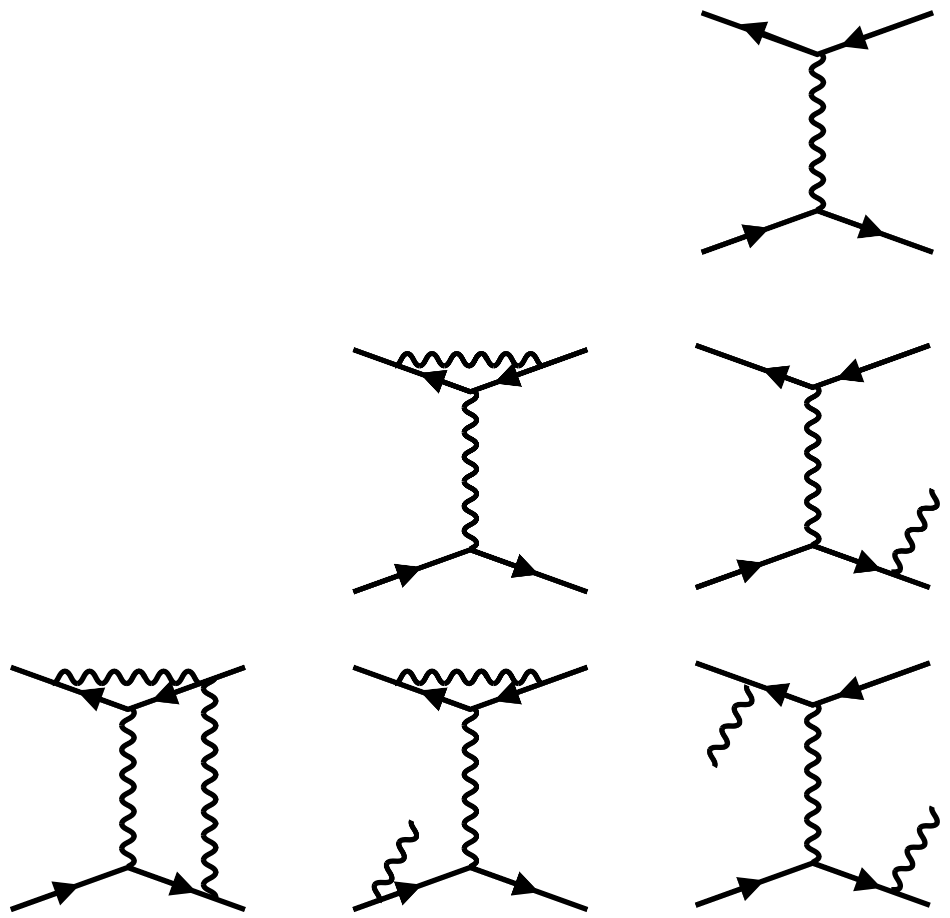
Coupling expansion in α^N

		LL	NLL	NNLL
	LO	α^0		
	NLO	αL	α	

QED beyond fixed order

Logarithmic expansion in $L^N = N \log Q^2/m^2$

Coupling expansion in α^N



	LL	NLL	NNLL
LO	α^0		
NLO	αL	α	
NNLO	$\frac{1}{2}\alpha^2 L^2$	$\frac{1}{2}\alpha^2 L$	$\frac{1}{2}\alpha^2$

QED beyond fixed order

Logarithmic expansion in $L^N = N \log Q^2/m^2$

Coupling expansion in α^N

	LL	NLL	NNLL	
LO	α^0			
NLO	αL	α		
NNLO	$\frac{1}{2}\alpha^2 L^2$	$\frac{1}{2}\alpha^2 L$	$\frac{1}{2}\alpha^2$	
H.O.	$\sum_{n=3}^{\infty} \frac{1}{n!}\alpha^n L^n$	$\sum_{n=3}^{\infty} \frac{1}{n!}\alpha^n L^{n-1}$	$\cdots$	$\cdots$

QED beyond fixed order

Logarithmic expansion in $L^N = N \log Q^2/m^2$

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LO	α^0			
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We want to stop in the α expansion at some order

QED beyond fixed order

Logarithmic expansion in $L^N = N \log Q^2/m^2$

Coupling expansion in α^N

	LL	NLL	NNLL	
LO	α^0			
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NNLO	$\frac{1}{2}\alpha^2 L^2$	$\frac{1}{2}\alpha^2 L$	$\frac{1}{2}\alpha^2$	
H.O.	$\sum_{n=3}^{\infty} \frac{1}{n!}\alpha^n L^n$	$\sum_{n=3}^{\infty} \frac{1}{n!}\alpha^n L^{n-1}$	$\dots$	$\dots$

We want to stop in the α expansion at some order

But we want to resum all the logarithms in a calculable way

QED beyond fixed order

Logarithmic expansion in $L^N = N \log Q^2/m^2$

Coupling expansion in α^N

	LL	NLL	NNLL	
LO	α^0			
NLO	αL	α		
NNLO	$\frac{1}{2}\alpha^2 L^2$	$\frac{1}{2}\alpha^2 L$	$\frac{1}{2}\alpha^2$	
H.O.	$\sum_{n=3}^{\infty} \frac{1}{n!}\alpha^n L^n$	$\sum_{n=3}^{\infty} \frac{1}{n!}\alpha^n L^{n-1}$	$\dots$	$\dots$

This can be done via different approaches

Structure functions
E.A. Kuraev and V.S. Fadin, Sov. J. Nucl. Phys. 41 (1985) 466.
O. Nicrosini and L. Trentadue, Phys. Lett. B 196 (1987) 551.

YFS/CEEX
S. Jadach, B.F.L. Ward and Z. Was, Phys. Rev. D 63 (2001) 113009
S. Jadach, W. Placzek and B.F.L. Ward, Phys. Lett. B 390 (1997) 298

Parton shower
C.M. Carloni Calame et al., Nucl. Phys. B 584 (2000) 459
J. Fujimoto, Y. Shimizu and T. Munehisa, Prog. Theor. Phys. 90(1993)177

QED Structure functions

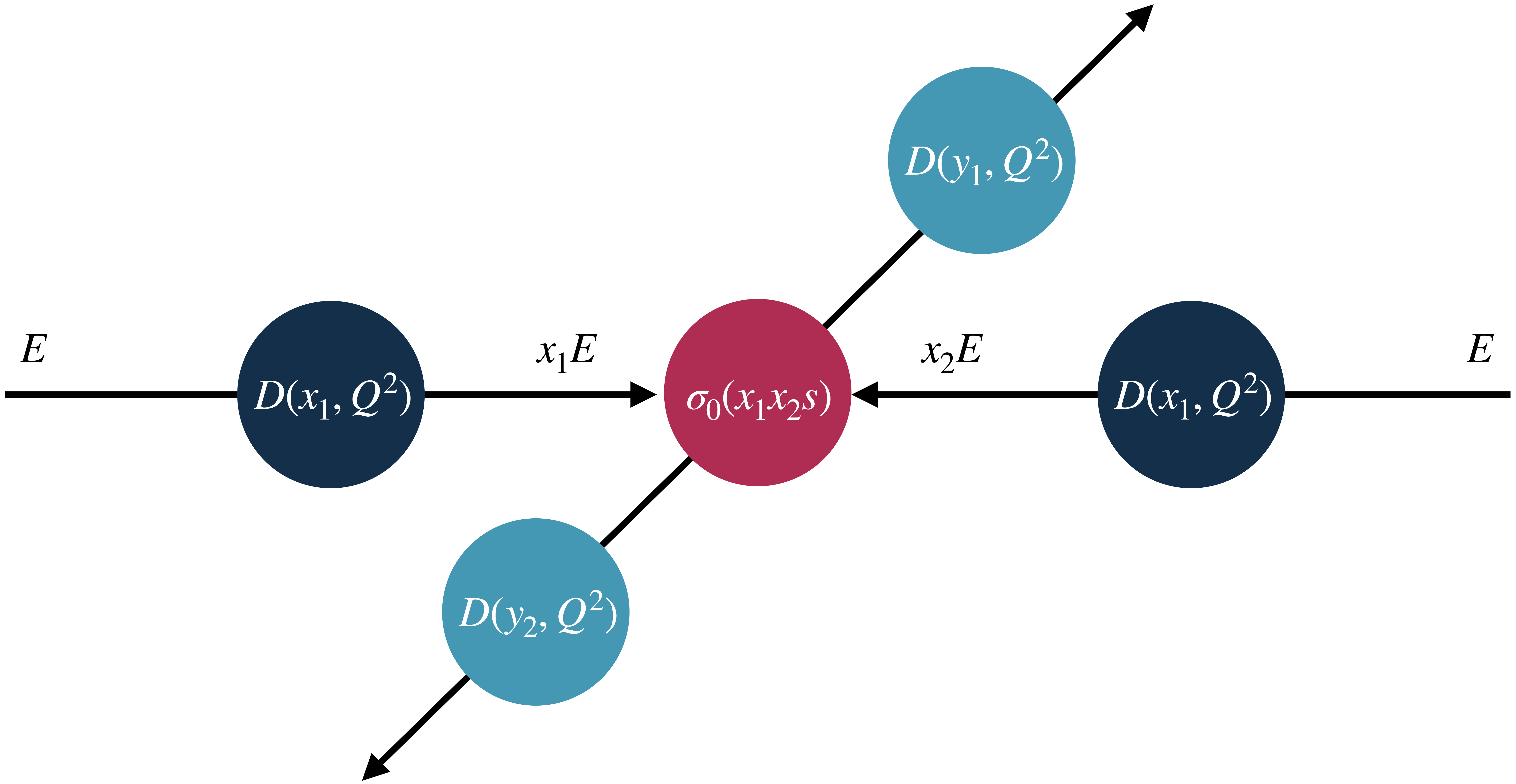
Master Formula

QED corrected cross section

$$\sigma(s) = \int dx_1 dx_2 dy_1 dy_2 \int d\Omega D(x_1, Q^2) D(x_2, Q^2) D(y_1, Q^2) D(y_2, Q^2) \frac{d\sigma_0(x_1 x_2 s)}{d\Omega} \Theta(\text{cuts})$$

Convolution of SFs

Hard-process cross section



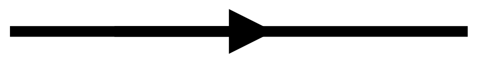
QED Structure functions

DGLAP Equation

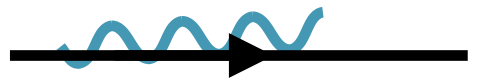
$$Q^2\frac{\partial}{\partial Q^2}D(x,Q^2) = \frac{\alpha}{2\pi}\int_0^1 \frac{ds'}{s'}P_+(s')D\left(\frac{x}{s'},Q^2\right)$$

Structure Functions (FS) are solutions of the DGLAP equation

$$D(x,Q^2) =$$



$$\Pi(Q^2,m^2)\delta(1-x)$$

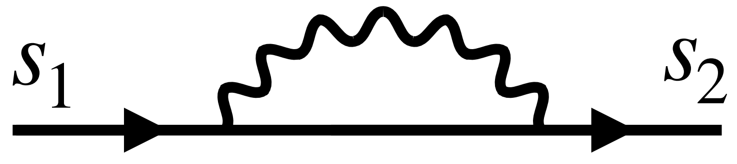


$$+\frac{\alpha}{2\pi}\int_{m^2}^{s}\Pi(Q^2,s')\frac{ds'}{s'}\Pi(s',m^2)\int_0^{x_+}dy\,P(y)\delta(x-y)$$

$+2$ photons…

SF generate all the emissions in collinear approximation

Sudakov form factor

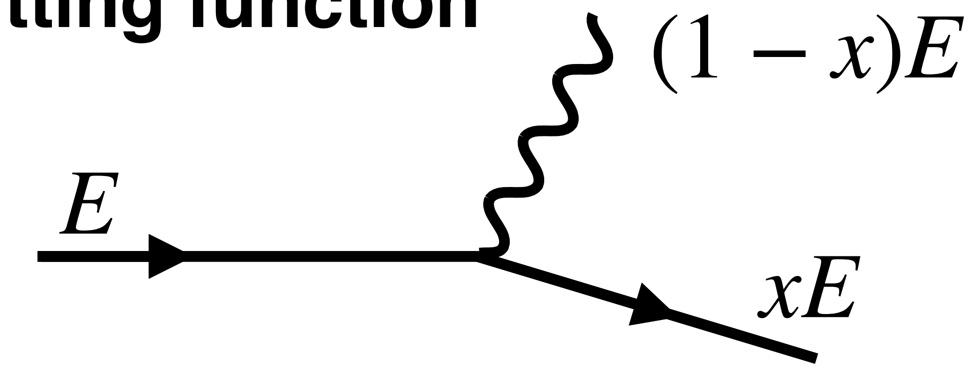


$$\Pi(s_1,s_2) = \exp\left[-\frac{\alpha}{2\pi}\int_{s_1}^{s_2}\frac{ds'}{s'}\int_0^{x_+}dz P(z)\right]$$

Probability that the particle evolves from virtuality s_1 to s_2 without emitting a photon with energy fraction bigger than $\epsilon = 1 - x_+$

Altarelli-Parisi splitting function

$$P(z) = \frac{1+z^2}{1-z}$$



Splitting of a particle of energy E in a daughter of energy xE

QED PS algorithm

The **Parton Shower (PS)** algorithm is a Monte Carlo exact solution of the DGLAP equation

$$d\sigma_{PS}^{LL} = \sum_{n=0}^{\infty} \Pi(\varepsilon, \{p\}) \frac{1}{n!} \left| \mathcal{M}_n^J \right|^2 d\Phi_n(\{p\}, \{k\})$$

$J = 1,2$

The Matrix element is exact

$$\left| \mathcal{M}_1^{LL} \right|^2 = \prod_{i=1}^{n-2} \left[\frac{\alpha}{2\pi} P(z) \mathcal{I}_{SF}(k_i, z_i) \frac{4\pi^2(1-z_i)}{z_i \omega_i^2} \left| \mathcal{M}_2^{ex}(\{\tilde{p}\}, \{\tilde{k}\}) \right|^2 \right]$$

Energy spectrum

Energy generated as the A-P splitting

$$P(z) = \frac{1+z^2}{1-z}$$

Angular spectrum

In the PS approach, you can generate the photon kinematics with $p_\perp \neq 0$

$$\mathcal{I}_{SF}(k) = -\sum_{ij} \eta_i \eta_j \frac{p_i \cdot p_j}{(p_i \cdot k)(p_j \cdot k)} k_0^2 F_{ij}(z)$$

$$\Pi(\epsilon, Q^2) = \exp\left\{ -\frac{\alpha}{2\pi} \int_0^{1-\epsilon} dz P(z) \int d\Omega_k \mathcal{I}(k, \{p\}) \right\}$$

Sudakov FF

The eikonal function $I(k)$ is exponentiated, as it gives the same integral as of the PS kinematics

$$= \exp\left\{ -\frac{\alpha}{2\pi} \quad I_+ \qquad \log\frac{Q^2}{m^2} \right\}$$

QED beyond fixed order

Logarithmic expansion in $L^N = N \log Q^2/m^2$

Coupling expansion in α^N

	LL	NLL	NNLL	
LO	α^0			
NLO	αL	α		
NNLO	$\frac{1}{2}\alpha^2 L^2$	$\frac{1}{2}\alpha^2 L$	$\frac{1}{2}\alpha^2$	
H.O.	$\sum_{n=3}^{\infty}\frac{1}{n!}\alpha^n L^n$	$\sum_{n=3}^{\infty}\frac{1}{n!}\alpha^n L^{n-1}$	$\dots$	$\dots$

Approach	Code
SF + NLO	MCGPJ
YFS + matching	BHWIDE, Sherpa
PS + matching	BabaYaga@NLO
CEEX + NLO	KKMC

QED beyond fixed order

Logarithmic expansion in $L^N = N \log Q^2/m^2$

Coupling expansion in α^N

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Theoretical error = first missing term $\sim 5 \times 10^{-4} - 10^{-3}$

QED beyond fixed order

Logarithmic expansion in $L^N = N \log Q^2/m^2$

Coupling expansion in α^N

	LL	NLL	NNLL	
LO	α^0			
NLO	αL	α		
NNLO	$\frac{1}{2}\alpha^2 L^2$	$\frac{1}{2}\alpha^2 L$	$\frac{1}{2}\alpha^2$	NNLO matching
H.O.	$\sum_{n=3}^{\infty} \frac{1}{n!}\alpha^n L^n$	$\sum_{n=3}^{\infty} \frac{1}{n!}\alpha^n L^{n-1}$	$\dots$	$\dots$

PS NLL

BabaYaga@NLO master formula

BabaYaga@NLO

$$d\sigma_{\rm NLOPS} = \sum_{n=0}^{\infty} \Pi(\varepsilon, \{p\}) F_{\rm SV} \frac{1}{n!} |\mathscr{M}_n^J|^2 d\Phi_n(\{p\},\{k\})$$

	Exact NLO	**LL PS**	**Matched PS**
	virtual and real corrections are exact	virtual and real emissions are approximated	In BabaYaga@NLO, the PS is matched with the NLO calculation via the correction factors
$\mathcal{O}(\alpha)$	$d\sigma_\alpha$	$d\sigma_\alpha^{\rm LL}$	$d\sigma_{\rm NLOPS}^{\alpha}$
Soft+virtual	$(1 + C_\alpha) \vert\mathscr{M}_0\vert^2 d\Phi_0$	$(1 + C_\alpha^{\rm LL}) \vert\mathscr{M}_0\vert^2 d\Phi_0$	$F_{\rm SV} = 1 + (C_\alpha - C_\alpha^{\rm LL})$

$d\Phi_n$ The phase space is exact at all orders

BabaYaga at flavour factories

Logarithmic expansion in $(\log 100\mathrm{GeV}^2/m_e^2)^n$

Coupling expansion in α^N

	LL	NLL	NNLL
LO	$\mathcal{O}(90\%)$		
NLO	$\mathcal{O}(10\%)$	$\mathcal{O}(0.5\%)$	
NNLO	$\mathcal{O}(0.5\%)$	$\mathcal{O}(0.05\%)$	$\mathcal{O}(0.01\%)$
H.O.	$\mathcal{O}(0.01\%)$		

The theoretical error starts at $\mathcal{O}(\alpha^2)$

$d\sigma_{\mathrm{NNLO}} \propto$

$\frac{1}{2} c^{\mathrm{LL}}_{\alpha^2} \alpha^2 L^2$ — ✓ LL is correct resummed in PS approach

$\frac{1}{2} c^{\mathrm{NLL}}_{\alpha^2} \alpha^2 L$ — ~ NLL not exact but part is captured by the product of NLO non-log and LL

$\frac{1}{2} c^{\mathrm{non-log}}_{\alpha^2} \alpha^2$ — ✗ non-log not under control There is no approximation. NNLO matching needed

A very naive estimate of the th. Uncertainty

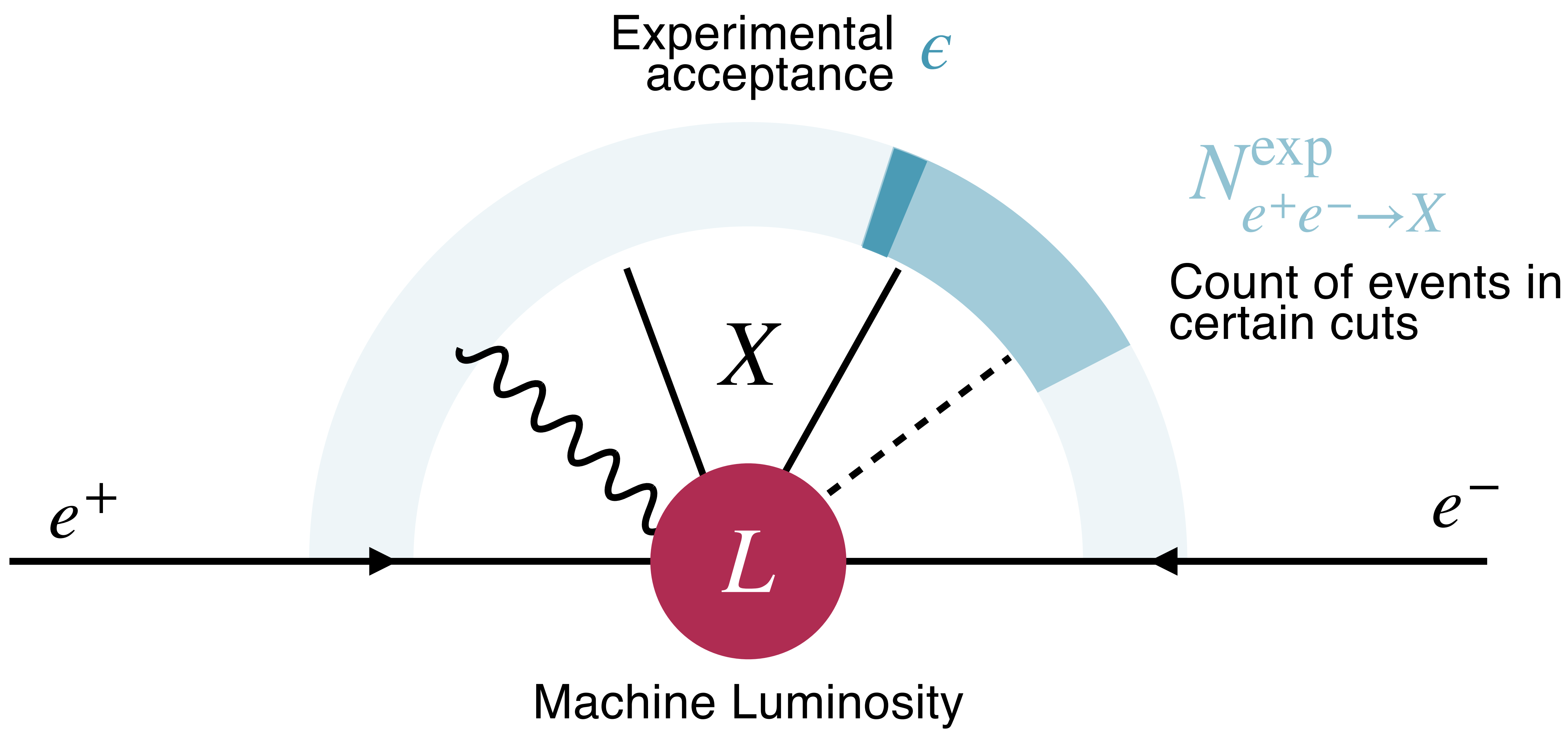
$$\sigma_{\mathrm{NLOPS}} + \left(\sigma_{\mathrm{LO}} C_{\alpha^2 L}, - \frac{1}{2} \sigma_{\mathrm{LO}} C_{\alpha^2 L} \right)$$

$$C_{\alpha^2 L} = \frac{(\sigma_{\mathrm{NLOPS}} - \sigma_{\mathrm{NLO}} - \sigma_{\mathrm{PS}} + \sigma^{\alpha}_{\mathrm{PS}})}{\sigma_{\mathrm{LO}}}$$

Luminosity measurements

*“**Luminosity** converts events into cross sections”*

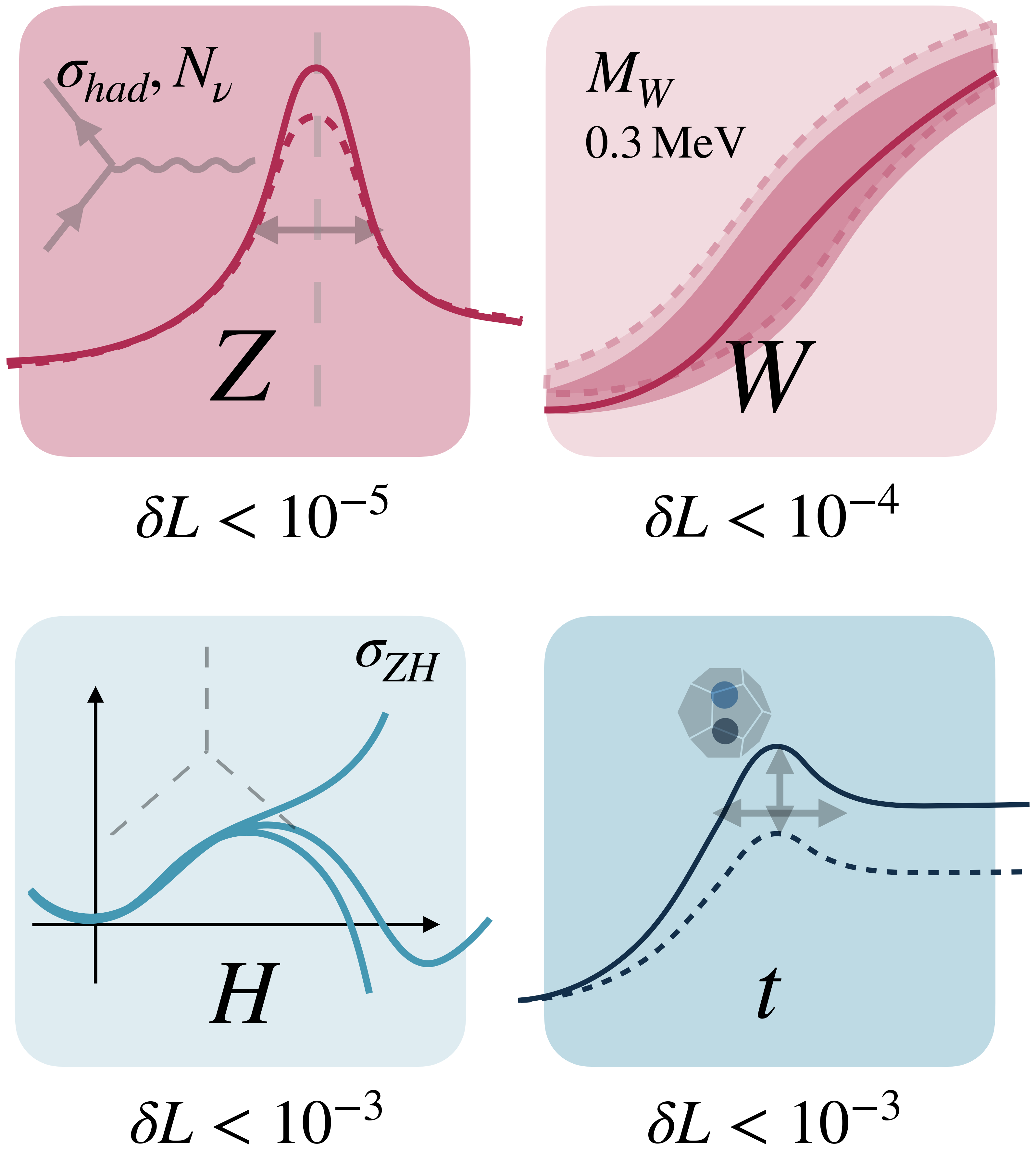
$$\sigma^{\text{exp}}_{e^+e^-\to X} = \frac{1}{\epsilon}\frac{N^{\text{exp}}_{e^+e^-\to X}}{L}$$



At precision machines is important to have a small luminosity uncertainty

$$\frac{\delta\sigma^{\text{exp}}_{e^+e^-\to X}}{\sigma^{\text{exp}}_{e^+e^-\to X}} = \frac{\delta\epsilon}{\epsilon} \oplus \frac{\delta N^{\text{exp}}_{e^+e^-\to X}}{N^{\text{exp}}_{e^+e^-\to X}} \oplus \frac{\delta L}{L}$$

Precision requirements
for Higgs/Top/EW precision program

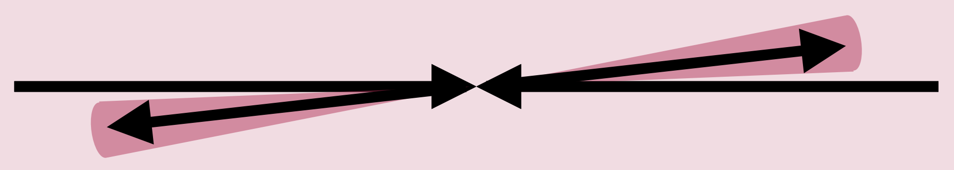


Luminosity measurements

At lepton colliders, the Luminosity is measured through a **benchmark process**

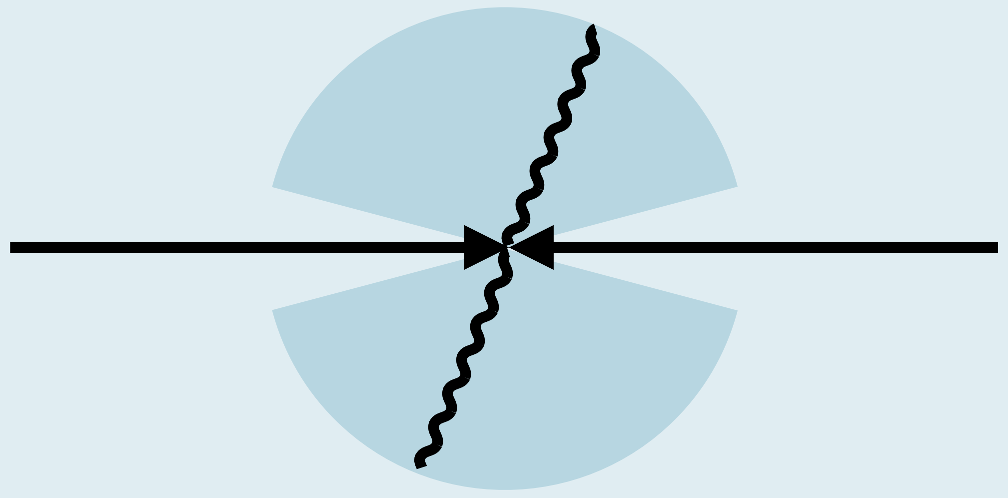
$$L = \int \mathscr{L} \mathrm{d}t = \frac{1}{\epsilon} \frac{N_0}{\sigma_0^{\mathrm{th}}}$$

$e^+e^- \to e^+e^-$



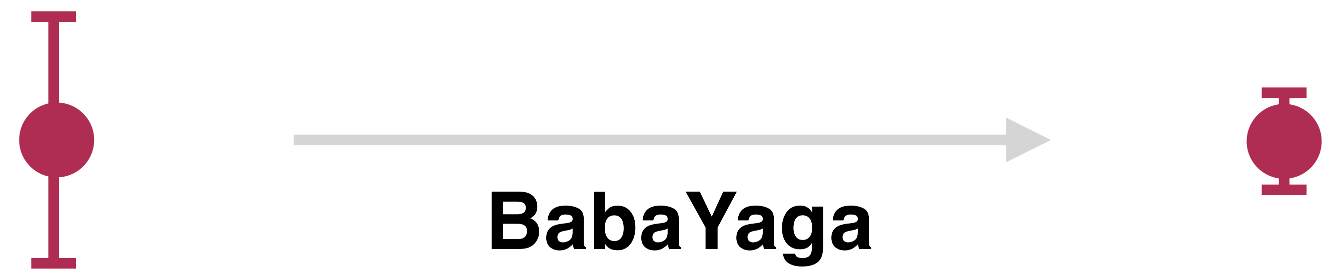
Bhabha (SABS)

$e^+e^- \to \gamma\gamma$



Di-photon

$\simeq 10^{-4}$ $\simeq 10^{-5}$



Current knowledge

Precision target

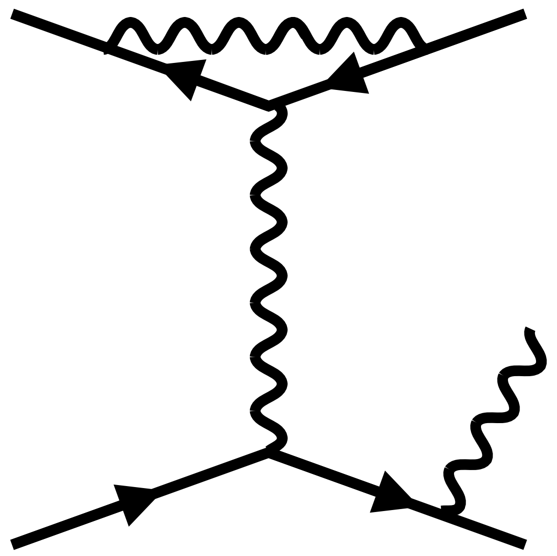
Error	Requests	FCC-ee
$\frac{\delta L}{L}$		$< 10^{-4} \div 10^{-5}$
$=$		
$\frac{\delta \epsilon_{\mathrm{exp}}}{\epsilon_{\mathrm{exp}}}$	Low background, clear exp. signature	$\simeq 10^{-5}$
$\oplus$		
$\frac{\delta N_0}{N_0}$	$\sigma \simeq \mathcal{O}(10^2 - 10^3\,\mathrm{nb})$ Large cross section	$< 10^{-6}$
$\oplus$		
$\frac{\delta \sigma_0^{\mathrm{th}}}{\sigma_0^{\mathrm{th}}}$	$\sigma^{(n)} = \left(\frac{\alpha}{\pi}\right)^n \log^n \frac{Q^2}{m_e^2}$ Calculable at high precision	$< 10^{-4} \div 10^{-5}$

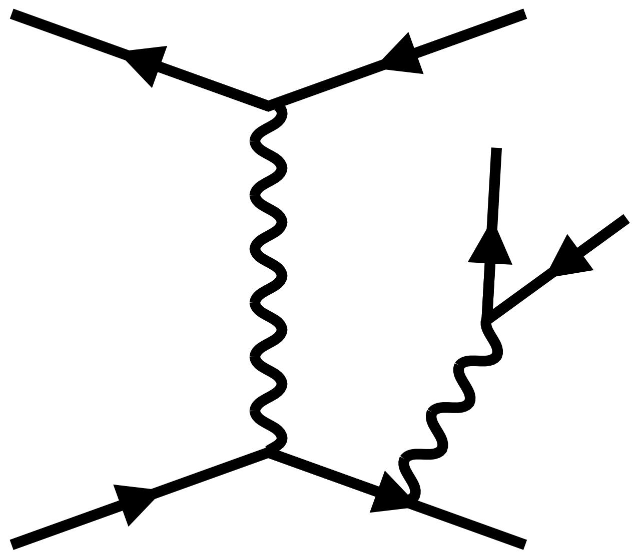
A. Arbuzov et al. *Phys.Lett.B* 383 (1996) 238-242
G Montagna et al. Riv.Nuovo Cim. 21N9 (1998)
S. Jadach et al. Physics Letters B 790 (2019) 314–321

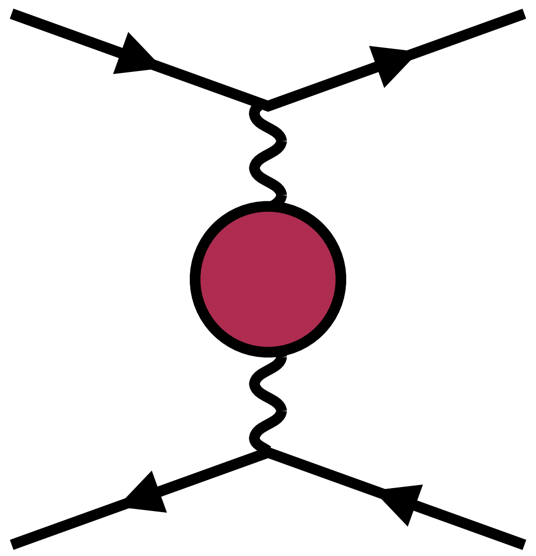
FCC-ee scenario

S. Jadach et al. Physics Letters B 790 (2019) 314–321

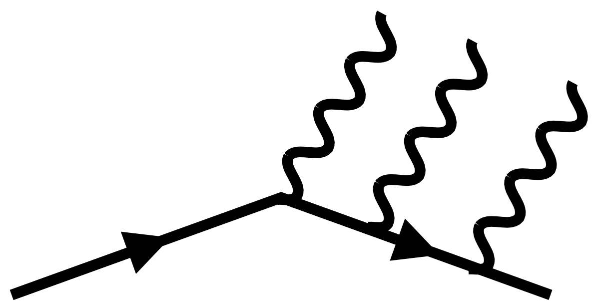
	Photonic	Light Pairs	Vacuum Pol.	Total
2018	0.030%	0.010%	0.014%	0.097%
Foreseen	0.011%	0.005%	0.006%	**0.01%** Target



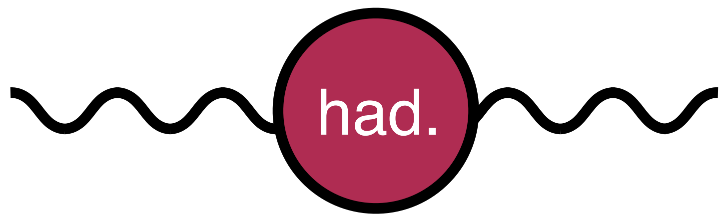




Photonic: $\mathcal{O}\left(\alpha^2 L \oplus \alpha^3 L^3\right)$ (2018) → $\mathcal{O}\left(\alpha^3 L^2 \oplus \alpha^4 L^4\right)$ (Foreseen)



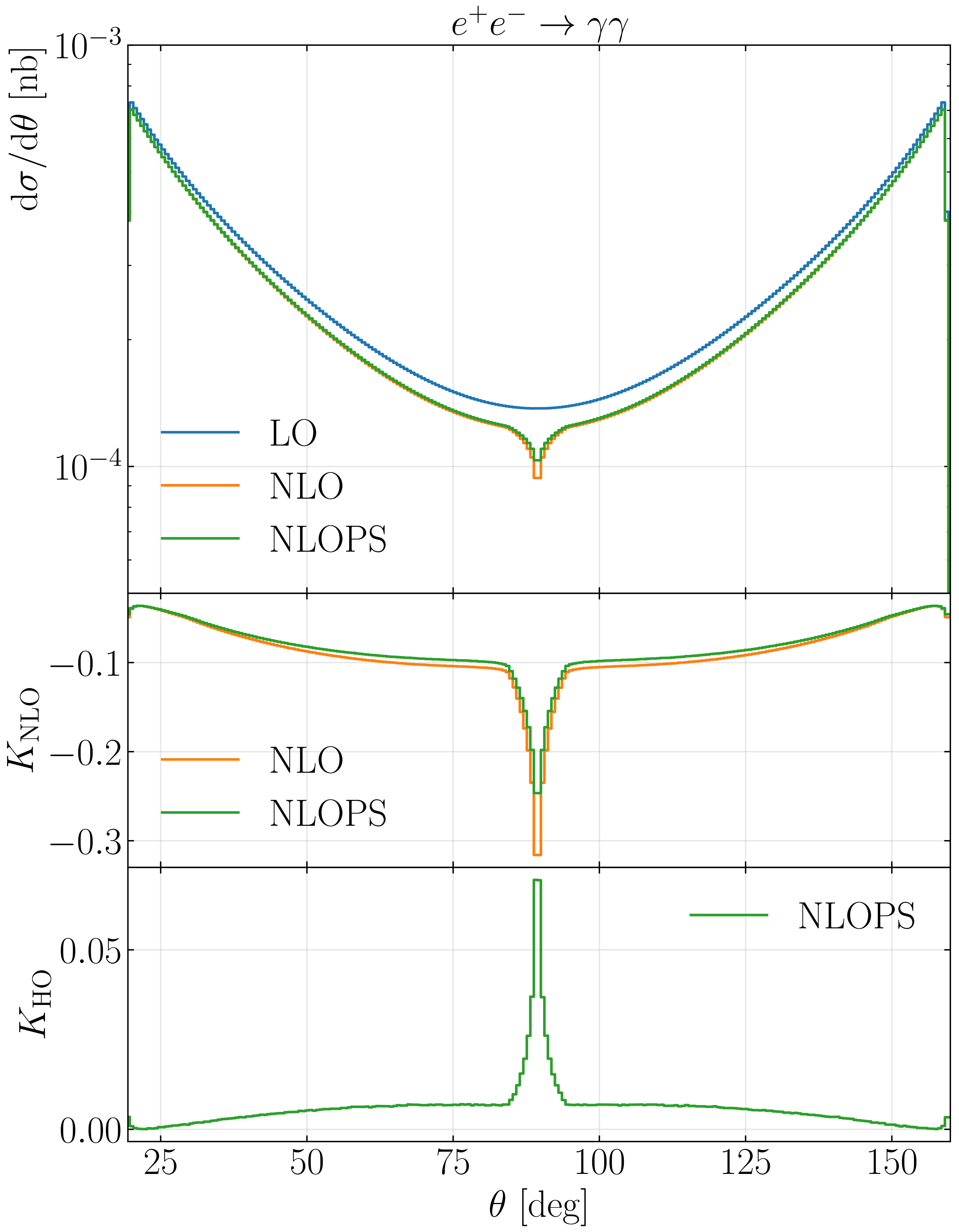
Resummation of Higher orders is essential

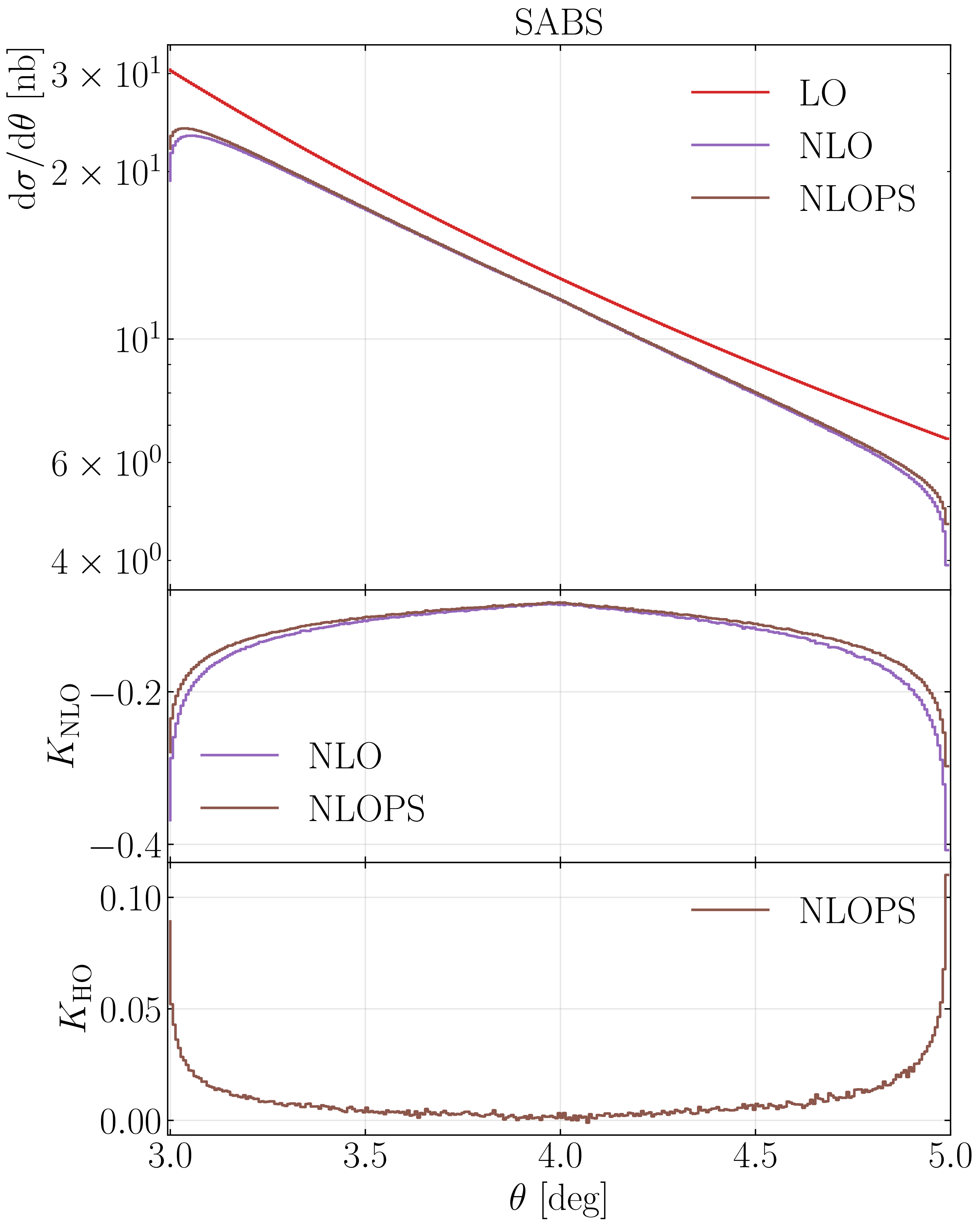


$$\frac{\delta_{VP}\sigma}{\sigma} \simeq 2.4 \times 10^{-4}$$

Janot, P. Direct measurement of αQED(m^2_Z) at the FCC-ee. *J. High Energ. Phys.* 2016, 53 (2016)

Luminosity measurements



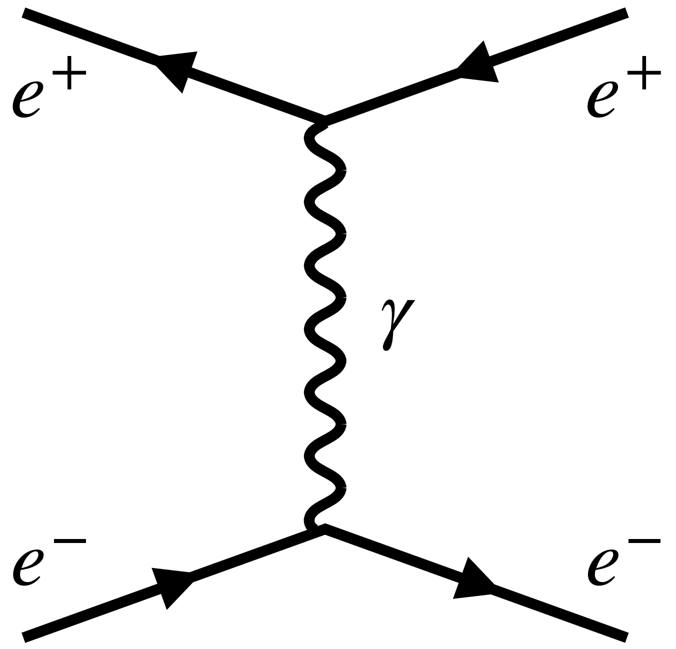


- **LA**: $\theta \in [20°, 160°]$, $E_{\min} \geq 0.25\sqrt{s}$, $\xi \leq 10°$

- **SA**: $\theta \in [3°, 5°]$, $E_{\min} \geq 0.25\sqrt{s}$, $\xi \leq 1°$,

New Physics in Luminosity?

Standard Model



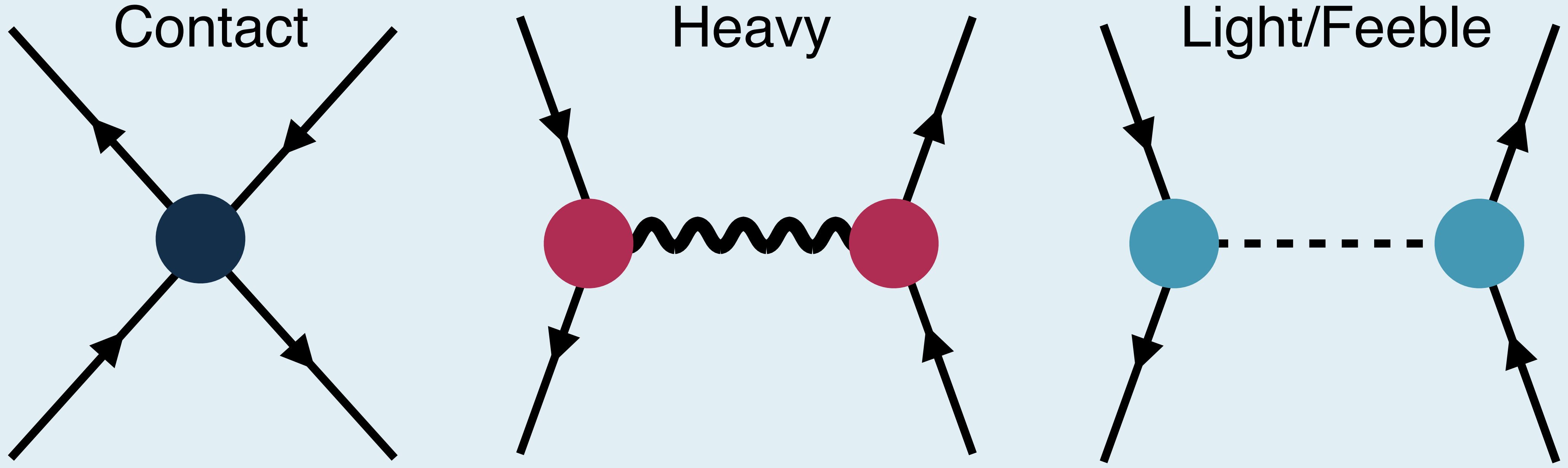
$$\sigma^{(n)} = \left(\frac{\alpha}{\pi}\right)^n \log^n \frac{Q^2}{m_e^2}$$

From the SM side you need **Monte Carlo Generators** able to simulate collisions at this level of precision

$$\left.\frac{\delta\sigma_{\rm SM}}{\sigma_{\rm SM}}\right|_{\rm th}^{\rm FCC} \sim 10^{-5}$$

$\times$

New Physics



$$\frac{\delta\sigma_{\rm NP}}{\sigma_{\rm SM}} \propto \frac{2{\rm Re}\,\mathscr{M}_{\rm SM}\mathscr{M}^\dagger_{\rm NP}}{|\mathscr{M}_{\rm SM}|^2}$$

New Physics d.o.f. could contaminate the SABS

At what level?

$$\frac{\delta\sigma_{\rm NP}}{\sigma_{\rm SM}} \simeq ?$$

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$e^+e^- \to e^+e^-$: Light New Physics

New Physics contamination to precision luminosity measurements at future e+e- colliders

M. Chiesa, C. L. Del Pio, G. Montagna, O. Nicrosini, F. Piccinini, F.P.U.

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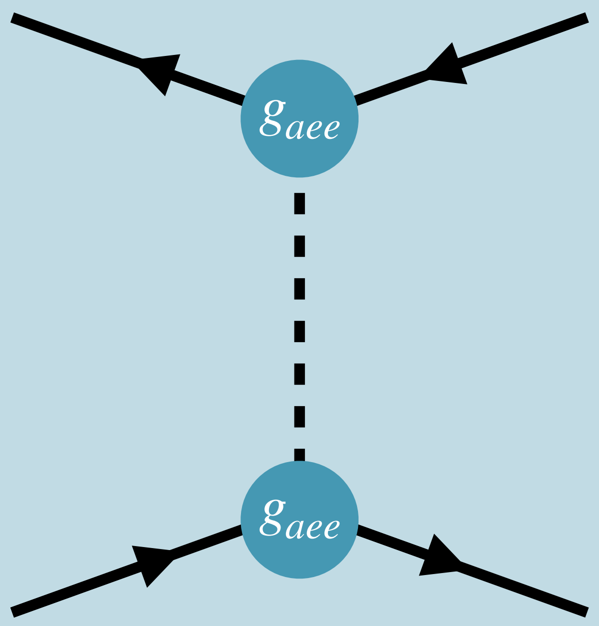
(Pseudo)scalar ALPs

$g_{aee}\,(\bar{e}\, i\gamma_5 e)a$

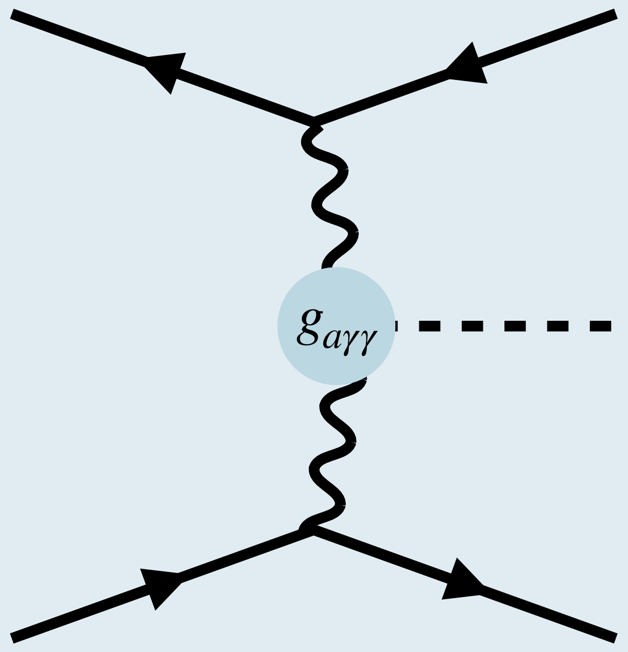


(g_{aee}, m_a)

$\simeq (3\times 10^{-3}, 1\,\mathrm{GeV})$

ALP mixing with photons

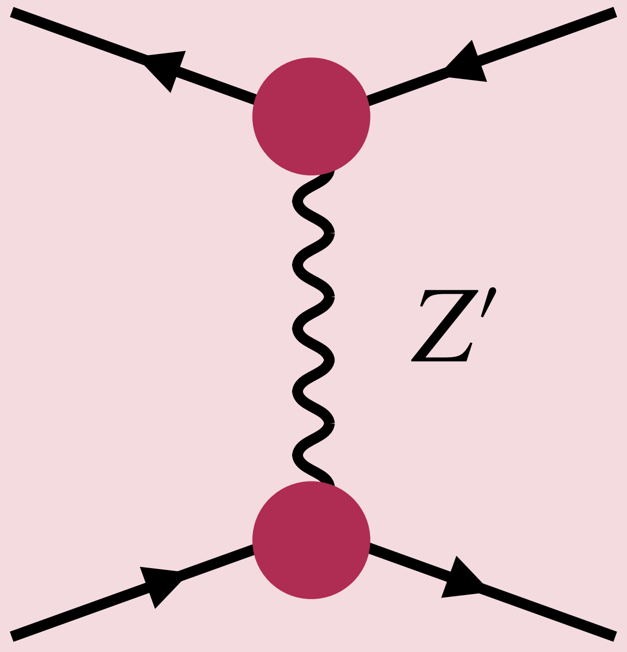
$\frac{1}{4} g_{a\gamma\gamma}(F_{\mu\nu}\tilde{F}^{\mu\nu})a$



$g_{a\gamma\gamma} \simeq 2\times 10^{-4}\,\mathrm{GeV}^{-1}$

Dark Vectors

$g'_A \bar{e}\left(\gamma^\mu\gamma_5\right) e\, V_\mu$

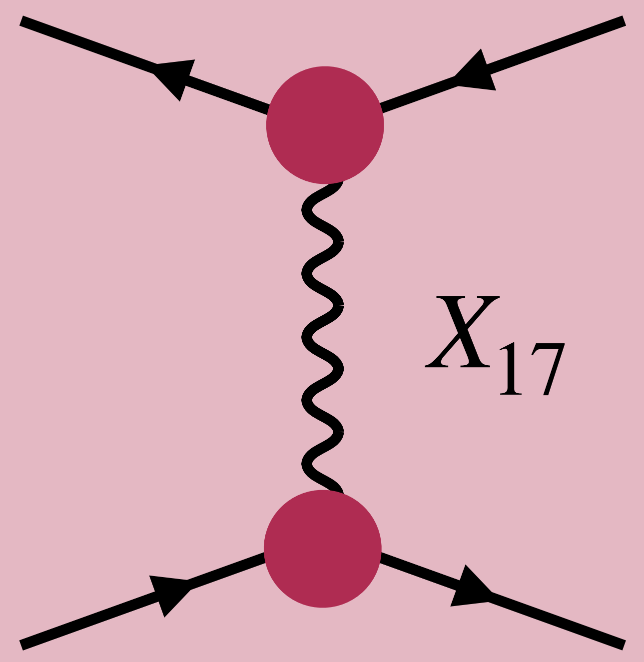


(g'_V, M_V)

$\simeq (3\times 10^{-4}, 1\,\mathrm{GeV})$

X17

$g'_V\left(\bar{e}\,\gamma^\mu\, e\right) V_\mu$



$(g'_V, M_{X_{17}})$

$(6\times 10^{-4}, 17\,\mathrm{MeV})$

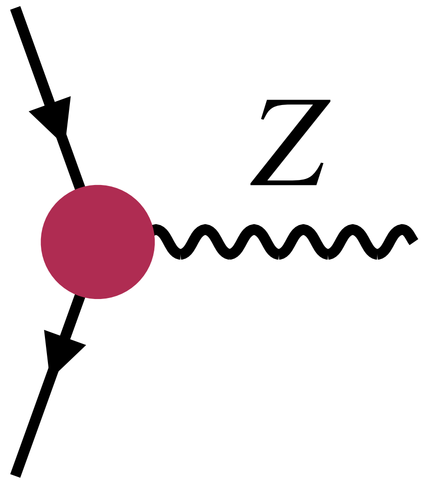
The LNP contribution is **negligible** at 10^{-5} level

$e^+e^- \to e^+e^-$: Heavy New Physics

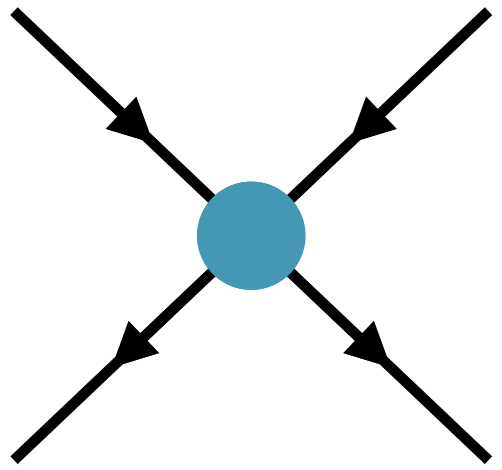
Standard Model Effective Field Theory

$\hat{O}_i^{(6)}$ Operators with same fields and symmetries of the SM

$$\mathscr{L}_{\text{SMEFT}} = \mathscr{L}_{\text{SM}} + \sum_i \frac{C_i}{\Lambda_{\text{NP}}^2} \hat{O}_i^{(6)} + \mathcal{O}(\Lambda_{\text{NP}}^{-4})$$



$\delta g_L^{Ze}, \delta g_R^{Ze}$



C_{ll}, C_{le}, C_{ee}

The deviation is computed taking into account **correlations** between WCs

$$(\delta \pm \Delta\delta)_{\text{SMEFT}} = \frac{1}{\sigma_{\text{SM}}} \left(\sigma^{(6)} \pm \sqrt{\sum_{ij} \sigma_i^{(6)} V_{ij} \sigma_j^{(6)}} \right)$$

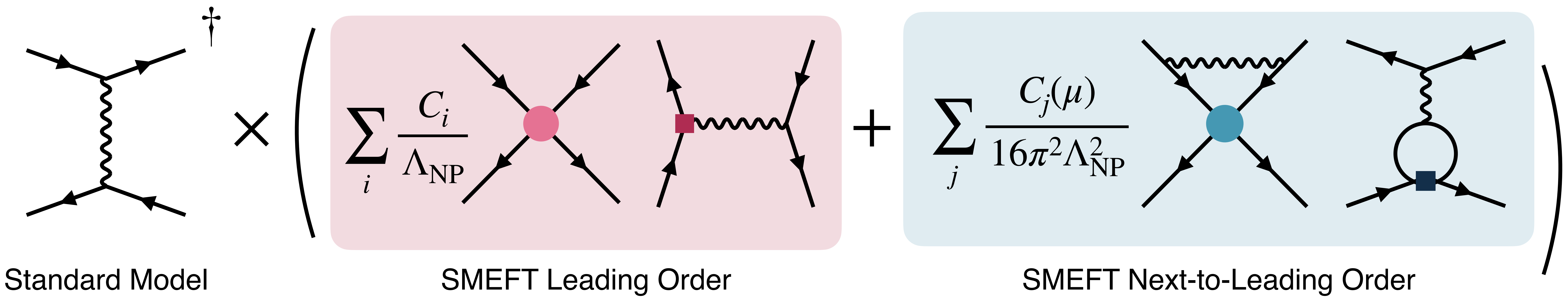
WCs are obtained by **global fits** from LEP data

Bhabha at LEP I-II

C_{ll}, C_{le}, C_{ee} $\delta g_L^{Ze}, \delta g_R^{Ze}$

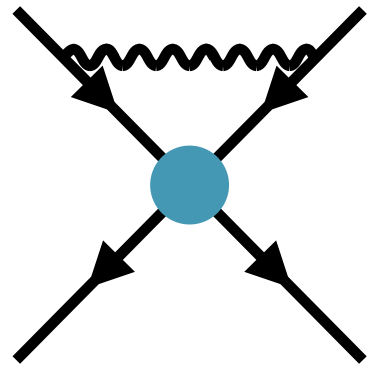
EWPO

$e^+e^- \to e^+e^-$: Heavy New Physics

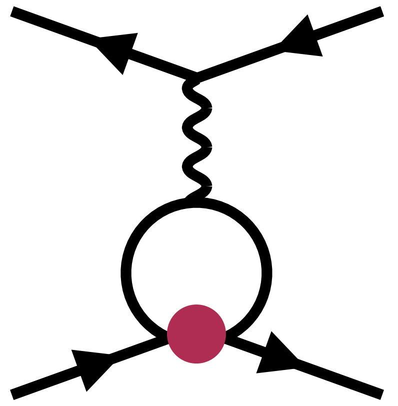


Exp.	$[\theta_{\min}, \theta_{\max}]$	$\sqrt{s}$ [GeV]	$(\delta \pm \Delta\delta)_{\text{SMEFT}}$	$\Delta L/L$
FCC	$[3.7°, 4.9°]$	91	$(-4.2 \pm 1.7) \times 10^{-5}$	$< 10^{-4}$
		160	$(-1.3 \pm 0.5) \times 10^{-4}$	
		240	$(-2.9 \pm 1.2) \times 10^{-4}$	10^{-4}
		365	$(-6.7 \pm 2.7) \times 10^{-4}$	
ILC	$[1.7°, 4.4°]$	250	$(-1.2 \pm 0.5) \times 10^{-4}$	$< 10^{-3}$
		500	$(-4.9 \pm 1.9) \times 10^{-4}$	
CLIC	$[2.2°, 7.7°]$	1500	$(-9.7 \pm 3.9) \times 10^{-3}$	$< 10^{-2}$
		3000	$(-4.2 \pm 1.7) \times 10^{-2}$	

SM NLO corrections to dim-6 operators appearing at LO



$$\frac{\sigma^{(6)}_{\text{NLO},i}}{\sigma^{(6)}_{\text{LO},i}} \simeq \frac{g^2_{\text{SM}}}{16\pi^2} \log \frac{M_Z^2}{|t|} \simeq \mathcal{O}(1\%)$$



Dim-6 operators loop insertions

$$\frac{\sigma^{(6)}_{\text{NLO},j}}{\sigma_{\text{SM}}} \simeq \frac{|t|}{\Lambda^2_{\text{NP}}} \frac{C_j}{16\pi^2} \log \frac{\Lambda^2_{\text{NP}}}{|t|} \simeq 2 \times 10^{-4} C_j$$

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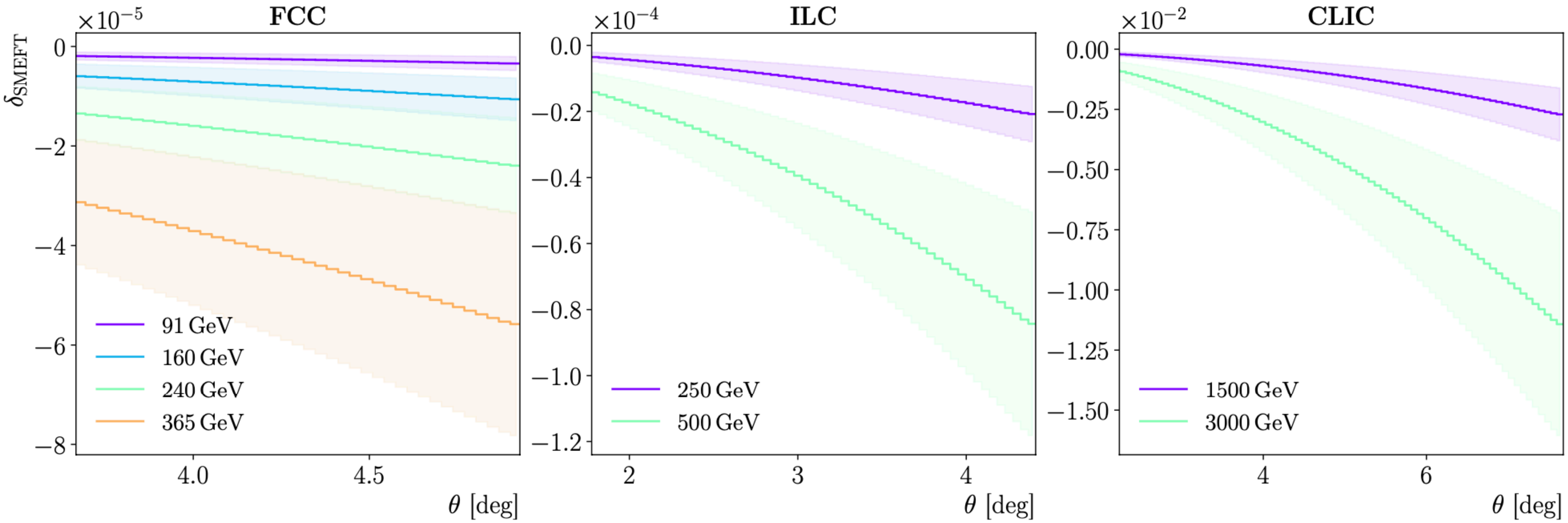
$e^+e^- \to e^+e^-$: Heavy New Physics



$$\vec{C}_{4f} = \{C_{ll}, C_{le}, C_{ee}\}$$

4 fermions WCs
impact the luminosity
in a non-negligible way

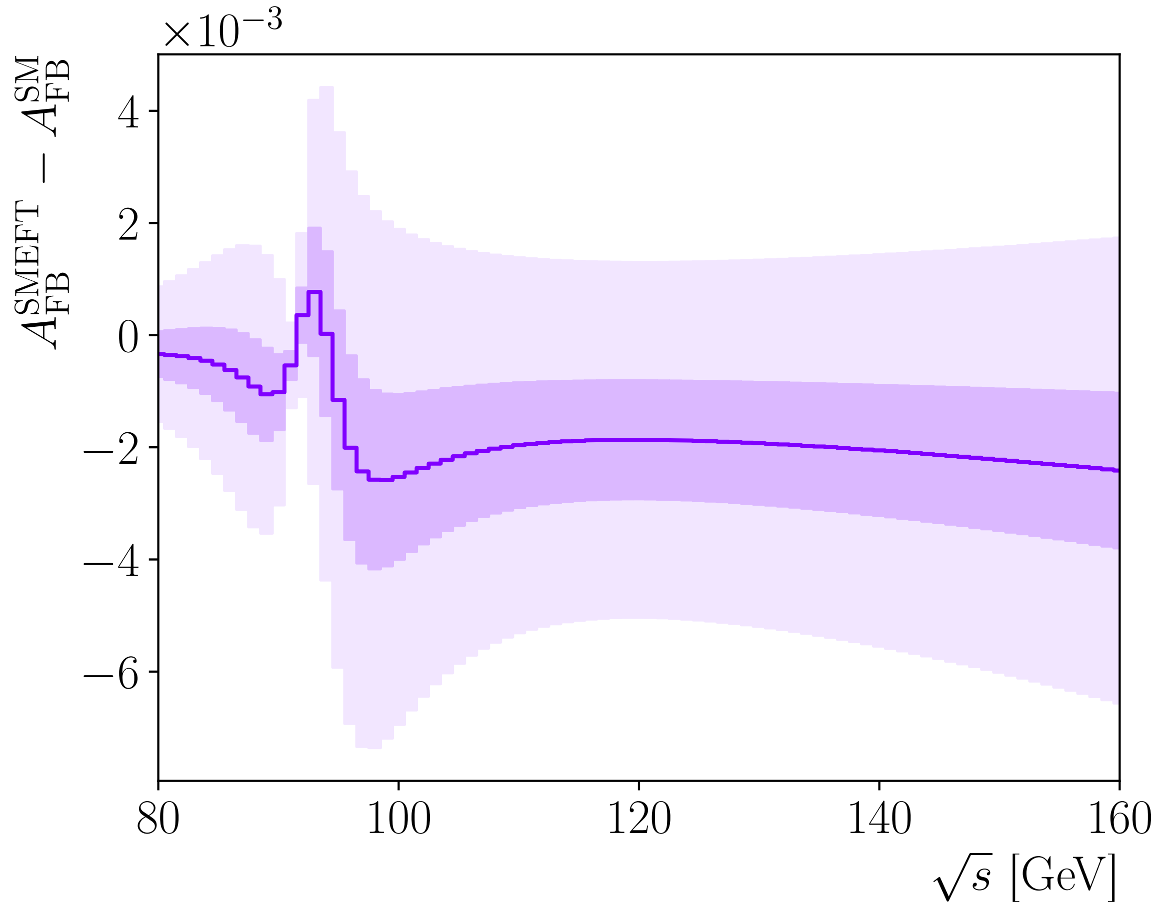
$e^+e^- \to e^+e^-$: Heavy New Physics

Z peak runs — FCC-ee

We use the FB asymmetry as a function of $\sqrt{s_\alpha}$

$$\sum_{i\in 4f} \frac{C_i}{\Lambda^2_{\rm NP}} \left[\frac{(\sigma_{\rm F}-\sigma_{\rm B})_i^{(6)}}{(\sigma_{\rm F}-\sigma_{\rm B})_{\rm SM}} - \frac{(\sigma_{\rm F}+\sigma_{\rm B})_i^{(6)}}{(\sigma_{\rm F}+\sigma_{\rm B})_{\rm SM}} \right]_\alpha = \frac{\Delta A^0_{{\rm FB},\alpha}}{A^0_{{\rm FB},\alpha}},$$

To fit the three WCs we can use three points



$$\sqrt{s_1} = 89\ \text{GeV}$$
$$\sqrt{s_2} = 93\ \text{GeV}$$
$$\sqrt{s_3} = 98\ \text{GeV}$$

In 6 months of run on every point

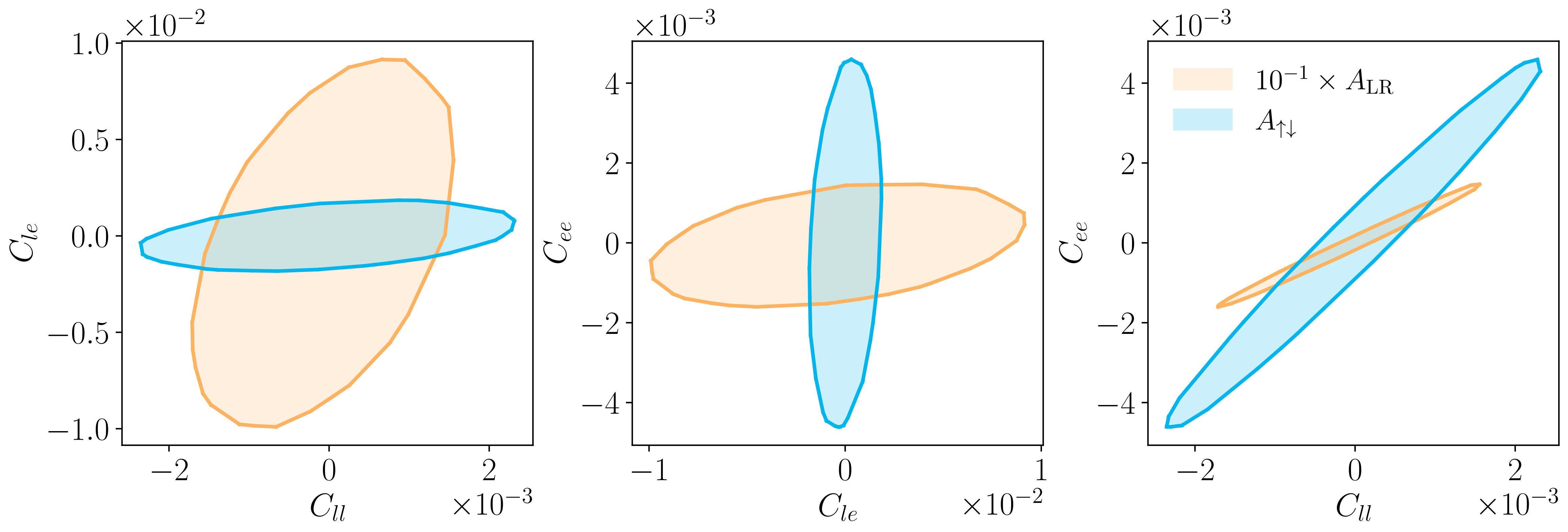
$\Delta C_{4f} < 10^{-2}$ → $\delta_{\rm SMEFT} < 10^{-5}$

250 GeV run — ILC

For polarised beams A_{LR} is not sensitive to all WCs.
We propose another polarisation asymmetry

$$A_{\uparrow\downarrow}^{-}(P_{e^\pm}, \cos\theta) = \frac{d\sigma(P_{e^+},P_{e^-}) - d\sigma(P_{e^+},-P_{e^-})}{d\sigma(P_{e^+},P_{e^-}) + d\sigma(P_{e^+},-P_{e^-})}$$

Up-down asymmetry

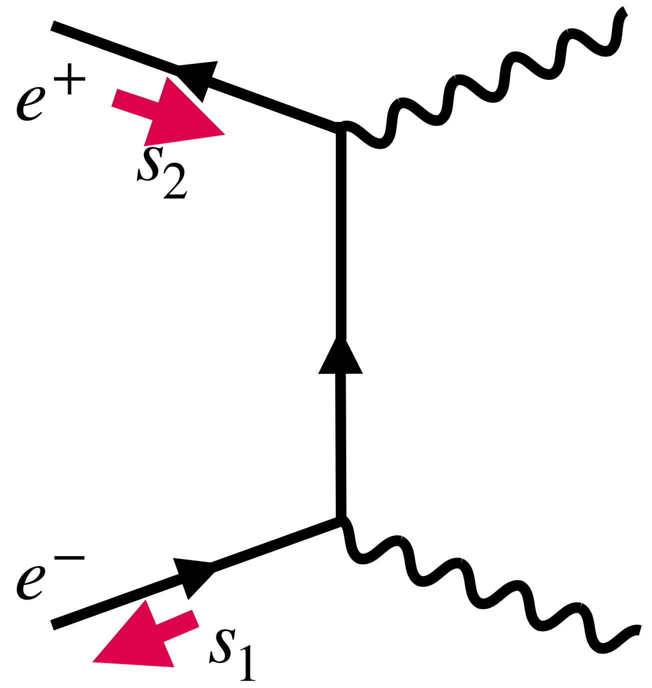


We calculate the 68% CLs using

$$\chi^2 = \sum_{\alpha=1}^{n} \frac{\left(A^0_{\rm pol} - A^{\rm th}_{\rm pol}(\vec{C}_{4f})\right)^2_\alpha}{(\Delta A^0_{\rm pol})^2_\alpha}$$

→ $\delta_{\rm SMEFT} < 10^{-7}$

$e^+e^- \to \gamma\gamma$: Polarised calculation

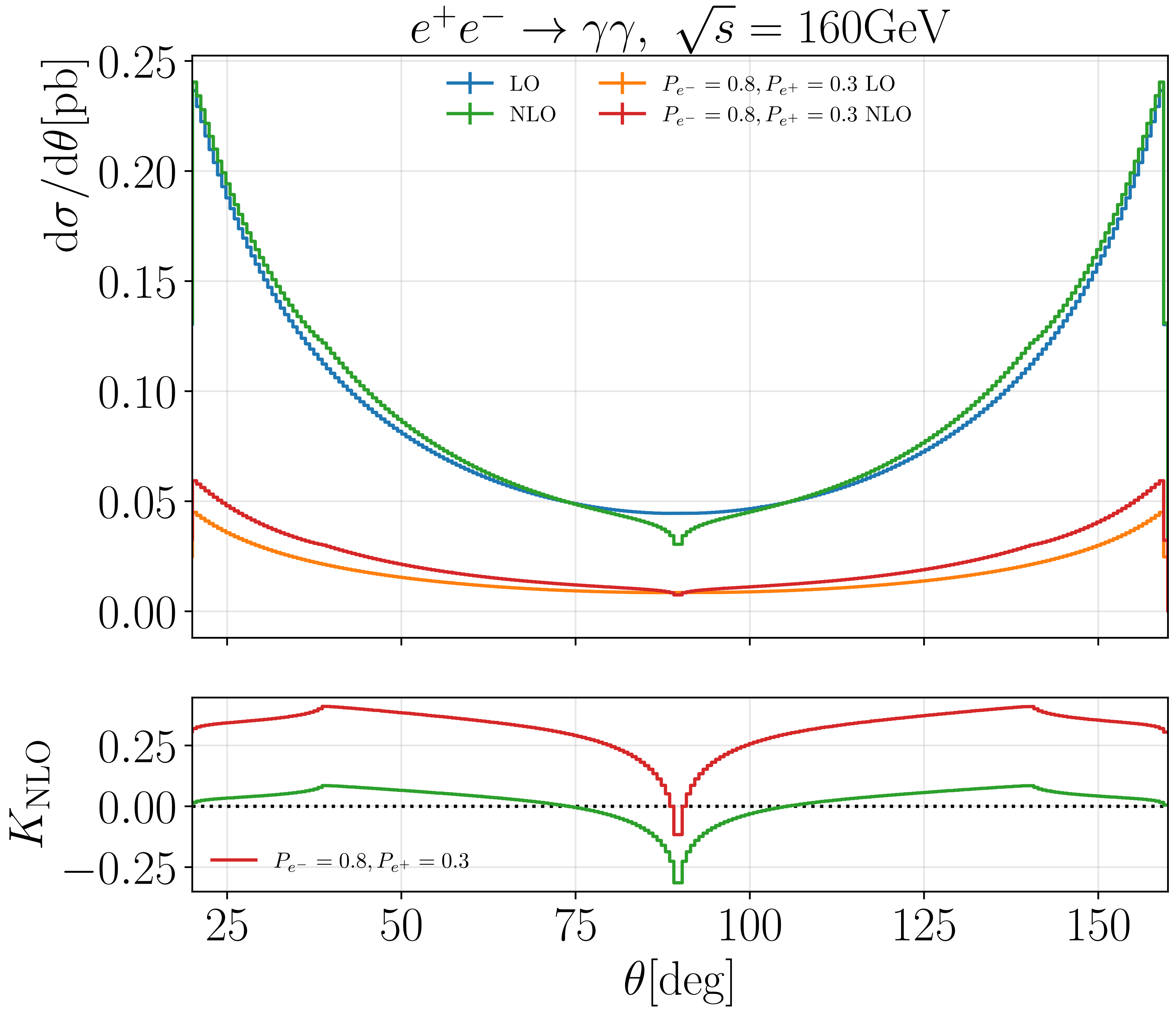


Development of BabaYaga@NLO with longitudinal polarisations

$$(\not{p}_1 - m) \to (\not{p}_1 - m)\frac{1}{2}(1 + \gamma_5 \not{s}_1)$$

$$s_1 = \lambda_- \frac{\sqrt{s}}{2}(\beta,0,0,1)$$

$$\sigma(P_{e^-}, P_{e^+}) = \frac{1}{4}\sum_{\lambda_\pm = \pm 1}(1 + \lambda_- P_{e^-})(1 + \lambda_+ P_{e^+})\sigma_{\lambda_-,\lambda_+}$$



Next step: matching with Parton Shower

$$P(z) = \frac{1+z^2}{1-z}$$

The soft limit is unchanged

$$g_{i,+}^{(\mathrm{out})}(p_i,k) = \frac{1}{p_i k}\left[P_{ff}(z_i) - \frac{m_i^2}{p_i k}\right] - g_{i,-}^{(\mathrm{out})}(p_i,k)$$

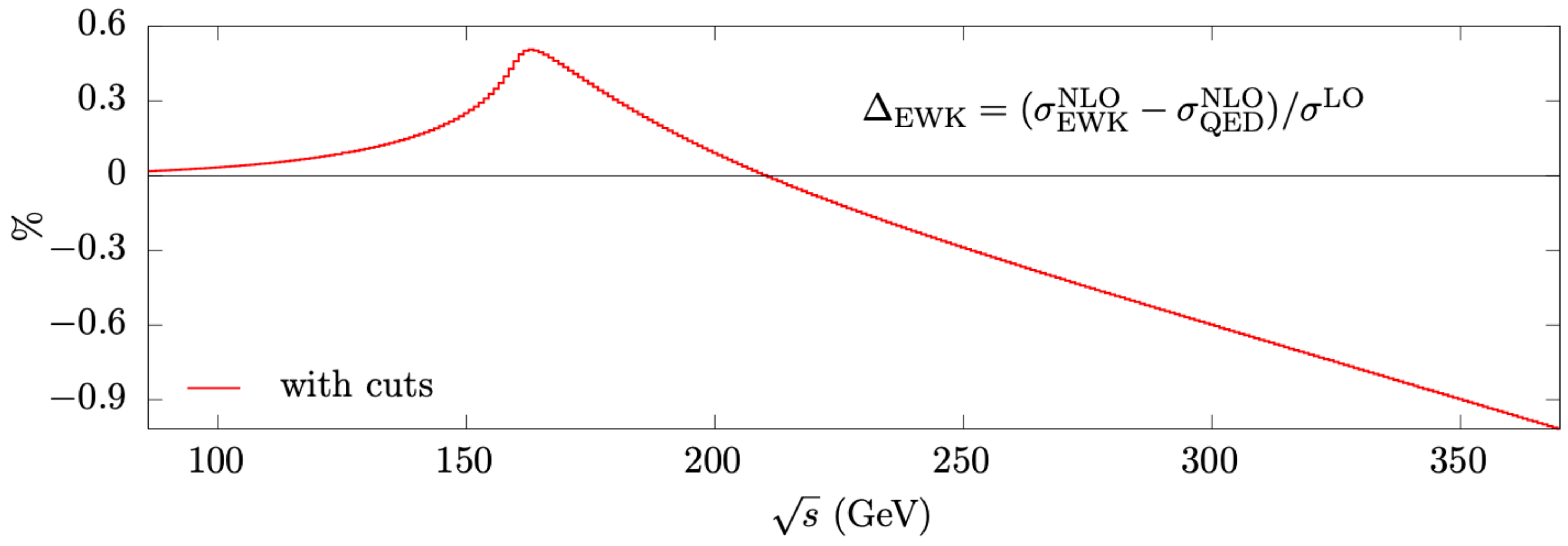
$$g_{i,-}^{(\mathrm{out})}(p_i,k) = \frac{m_i^2}{2(p_i k)^2}\frac{(1-z_i)^2}{z_i},$$

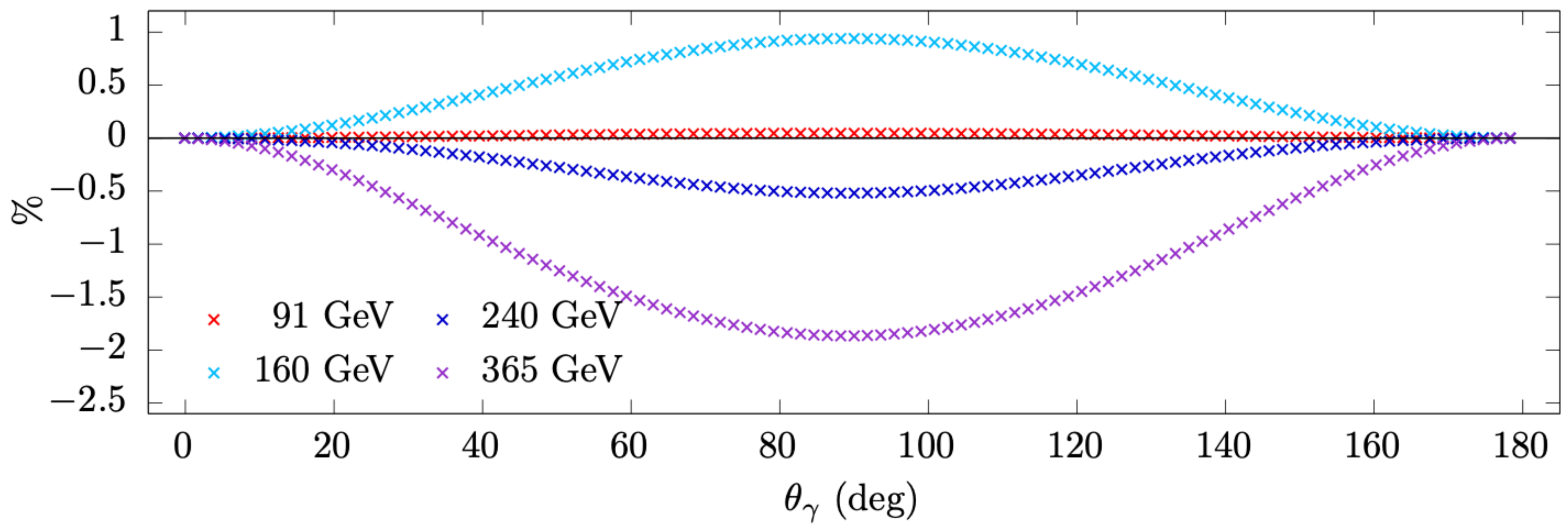
The collinear limit has to take into account the polarisation of electrons

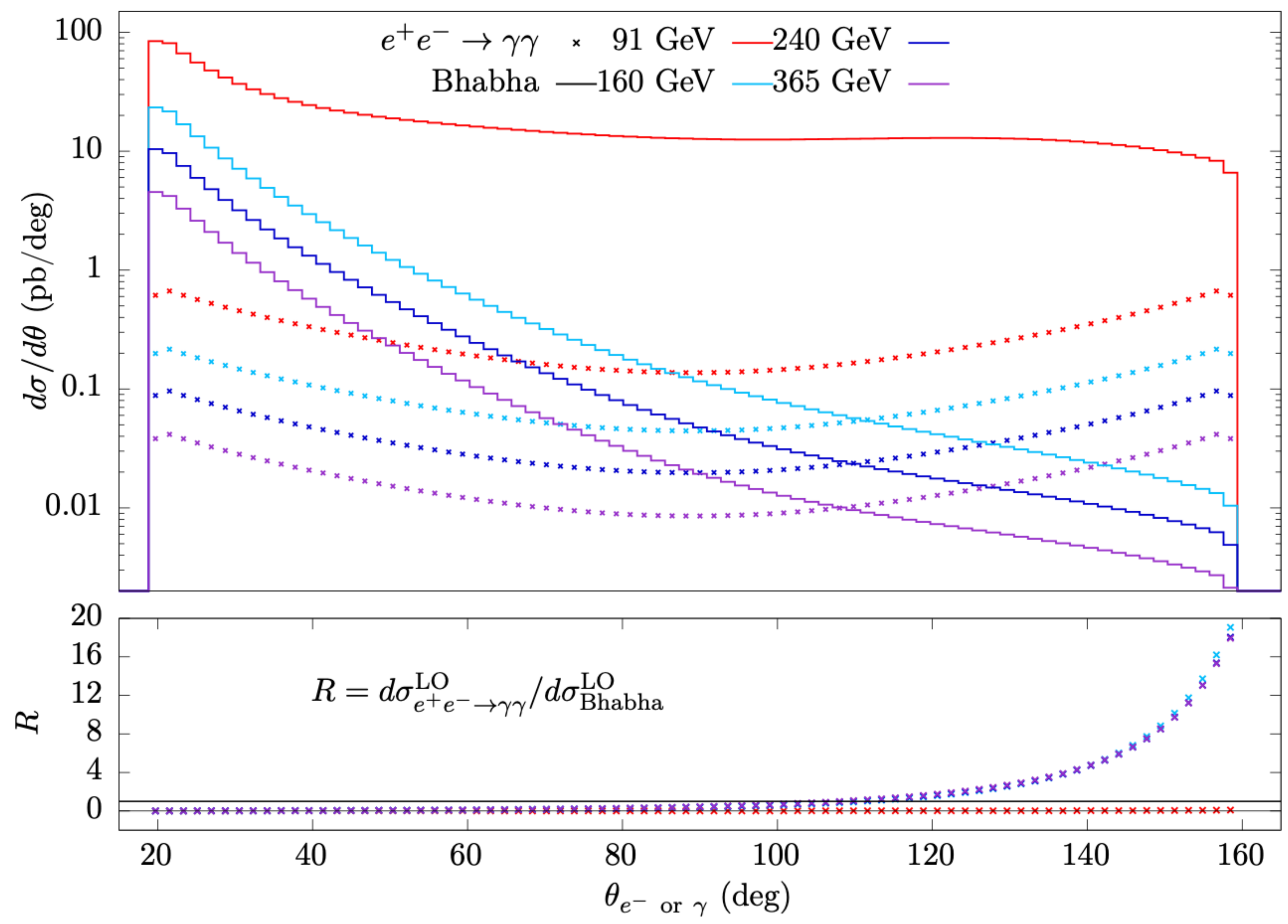
Dittmaier, Nucl. Phys. B,565,69-122,2000

$e^+e^- \to \gamma\gamma$: EW corrections

$\sqrt{s}$ (GeV)	LO (pb)	NLO (pb)	w h.o. (pb)	Bhabha LO (pb)
91	39.821	41.043 [+3.07%]	40.870(4) [−0.43%]	2625.9
160	12.881	13.291 [+3.18%]	13.228(1) [−0.49%]	259.98
240	5.7250	5.9120 [+3.27%]	5.8812(6) [−0.54%]	115.77
365	2.4752	2.5581 [+3.35%]	2.5438(3) [−0.58%]	50.373



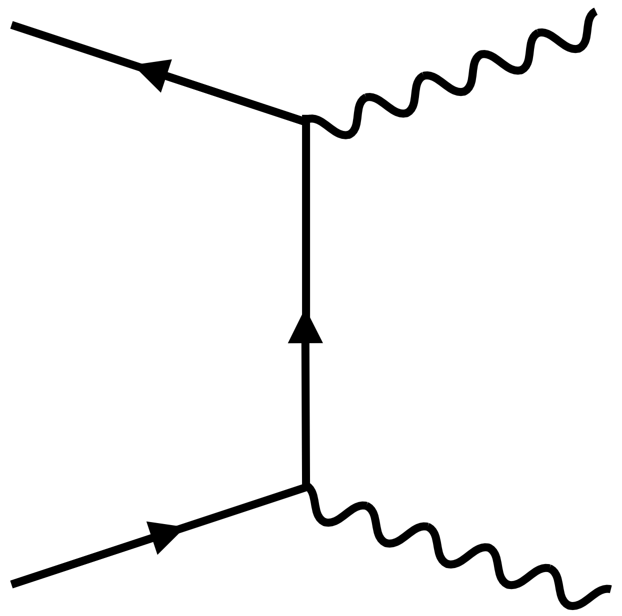




$e^+e^- \to \gamma\gamma$: SMEFT

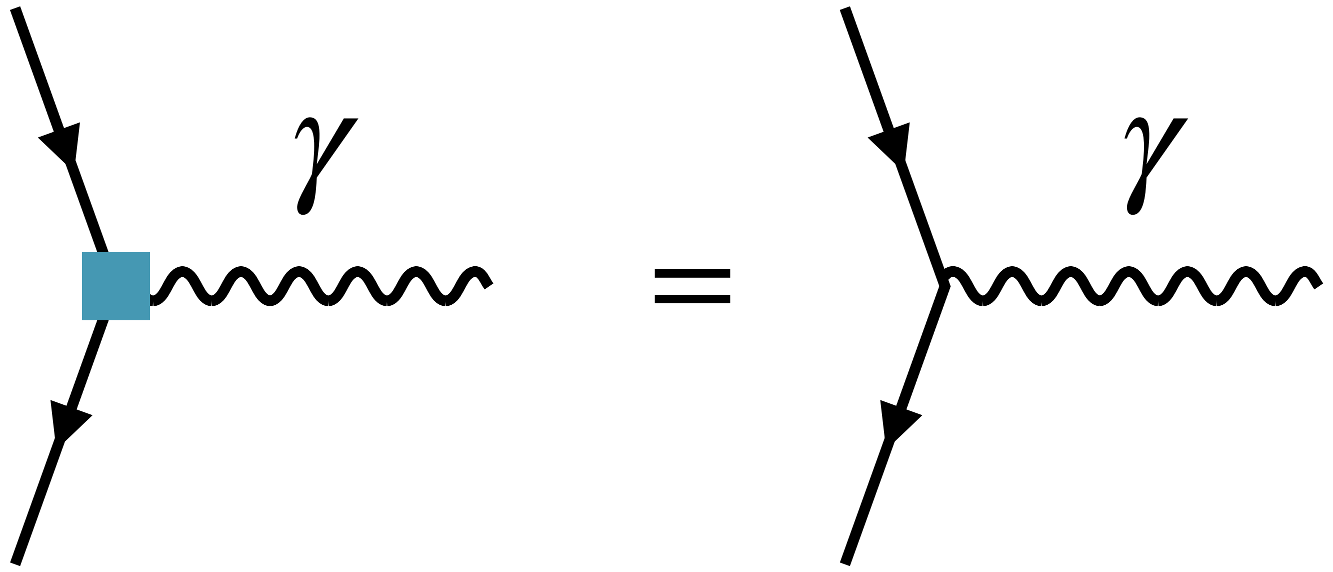
In preparation, with P. P. Giardino, F. Piccinini

Leading order

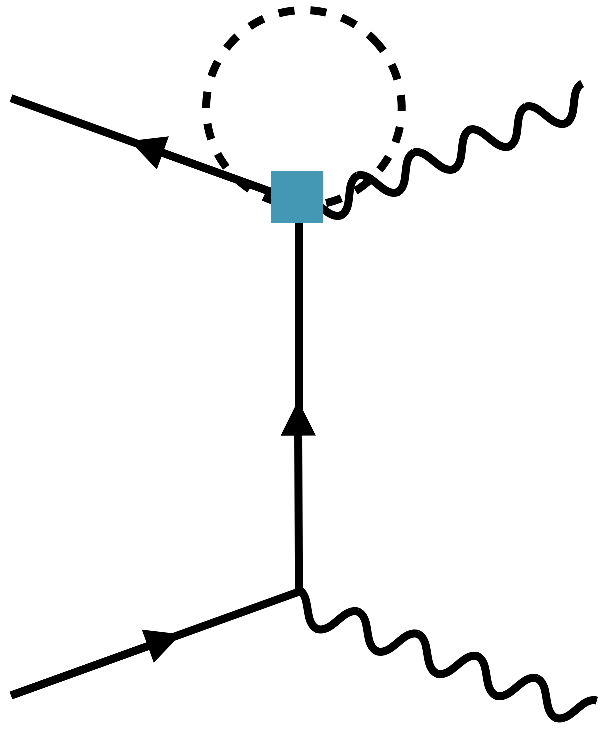


$$i\frac{gg'}{\sqrt{g^2+g'^2}}\left(1-\frac{g^2g'^2v^2}{g^2+g'^2}\frac{C_{\phi WB}}{\Lambda^2}\right)\gamma^\mu \equiv i\sqrt{4\pi\alpha}\gamma^\mu$$

The coupling is redefined, so at tree level it is a QED process



NLO SMEFT



$C_{\phi WB}$	$C_{\phi D}$	C_W	$\tilde{C}_W$
$C_{\phi e}[1,1]$	$C^{(1)}_{\phi l}[1,1]$	$C^{(3)}_{\phi l}[1,1]$	
C^L_{ff}	C^R_{ff}	C^R_{qf}	C^L_{qf}

At NLO, many operators appear

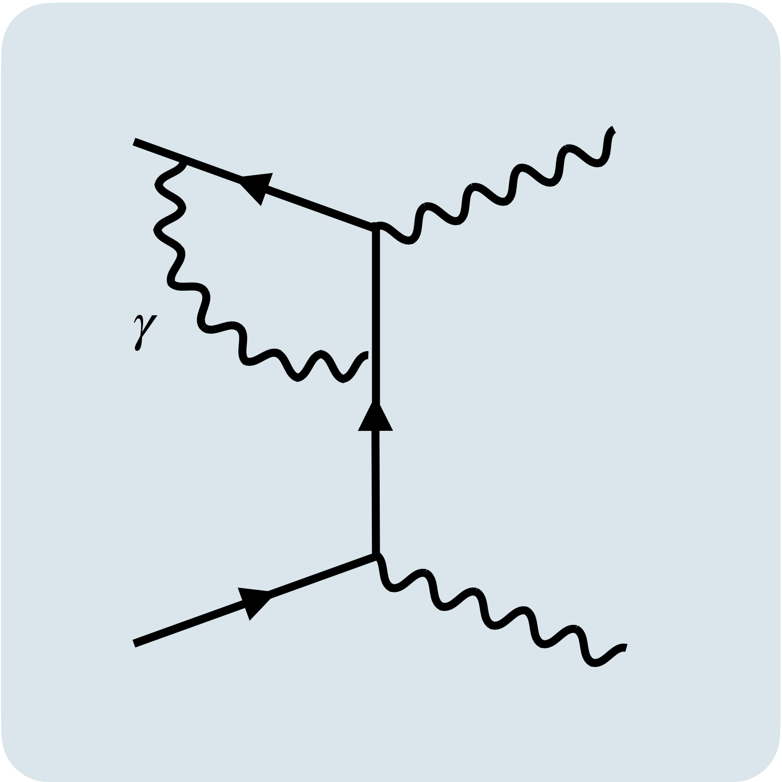
We want to compute the deviation from the SM at $\mathcal{O}(\Lambda^{-2})$

$$\sigma^{\text{SMEFT}}_{\text{NLO}} = \sigma^{\text{SM}}_{\text{NLO}} + \sum_i \frac{C_i}{\Lambda^2 16\pi^2} 2\text{Re}\{\mathcal{M}^\dagger_{\text{LO}}\mathcal{M}^{\text{EFT,i}}_V\}$$

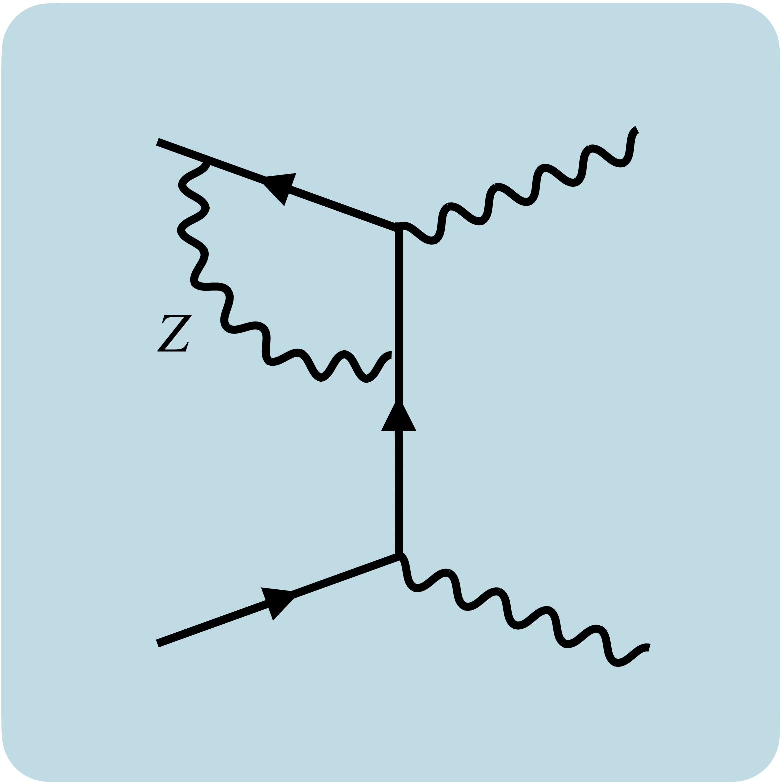
Recomputed with the chain
`FeynArts -> Feyncalc -> PackageX`
in R_ξ gauge and checked against `Recola`

$e^+e^- \to \gamma\gamma$: SMEFT

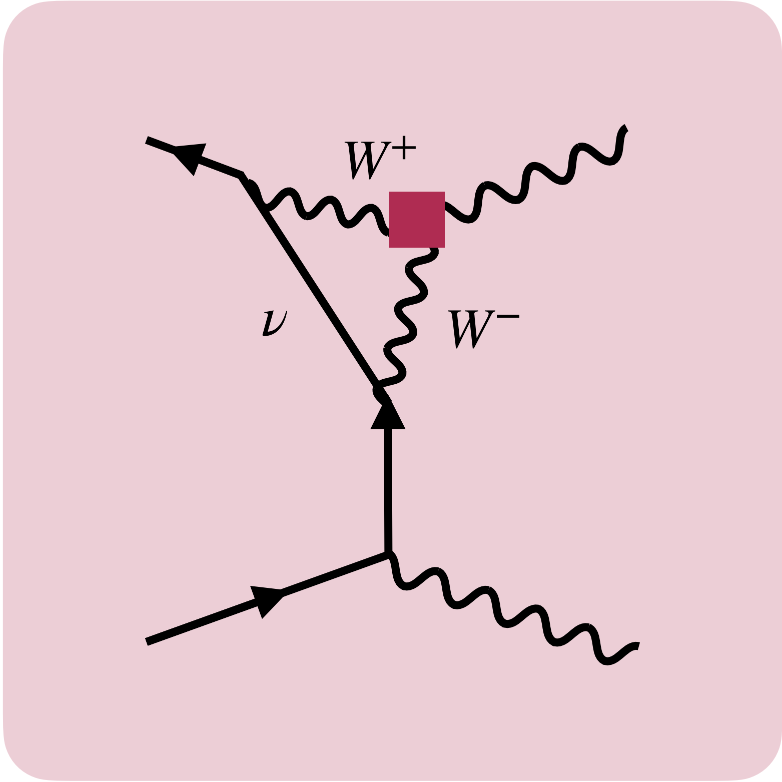
QED



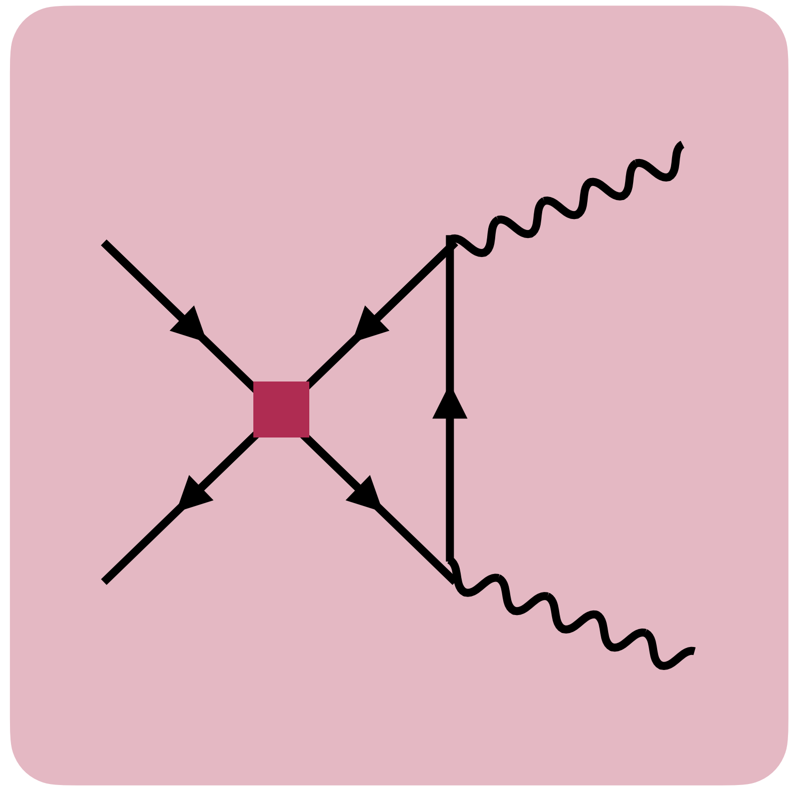
Neutral Current



Charged current



Four Fermions

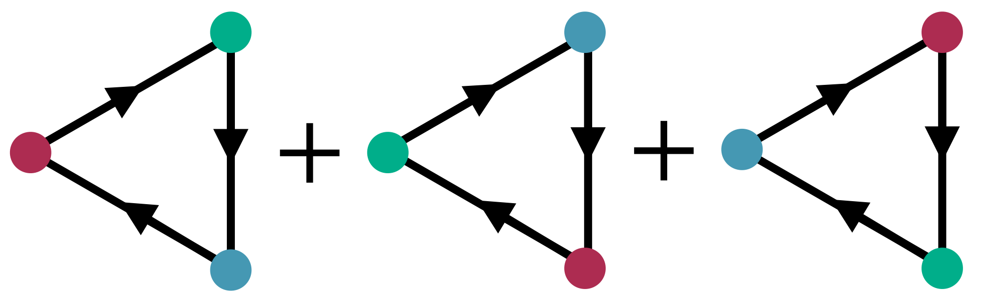


The calculation is conveniently divided in gauge-invariant subsets

Gamma5 in D-dimension

Treated with the Kreimer scheme, anticommuting with other gammas

$$\mathrm{Tr}[\gamma_5\,\gamma_5\,\gamma_5] = \frac{1}{3}\Big[$$



$$\Big]$$

On-shell renormalisation $\{\alpha, M_Z, M_W\}$

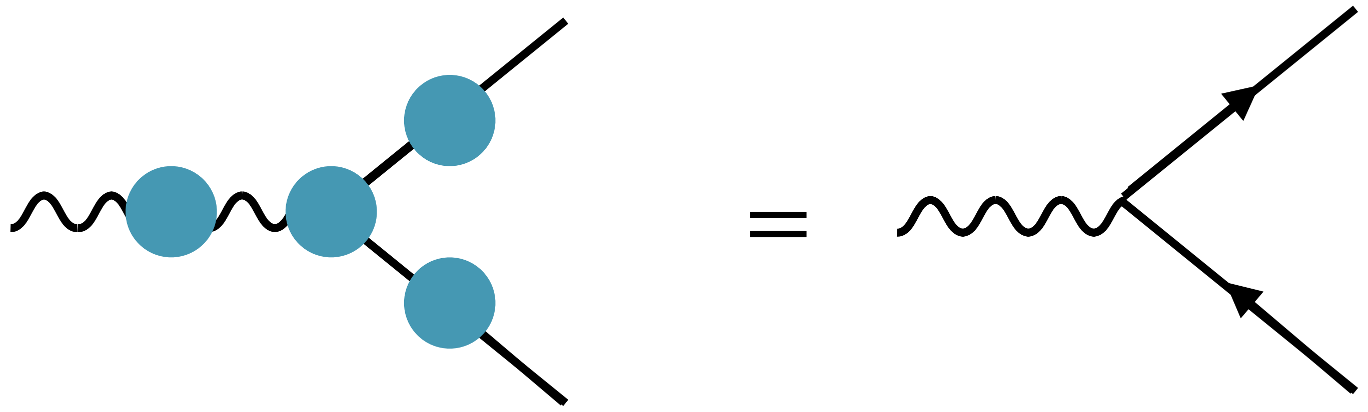
$$\psi_{0,L} \to Z_L^{1/2}\psi_L$$

$$\psi_{0,R} \to Z_R^{1/2}\psi_R$$

$$e_0 \to Z_e e$$

$$\begin{pmatrix} Z_0 \\ A_0 \end{pmatrix} = \begin{pmatrix} 1+\frac{1}{2}\delta Z_{ZZ} & \frac{1}{2}\delta Z_{ZA} \\ \frac{1}{2}\delta Z_{AZ} & 1+\frac{1}{2}\delta Z_{AA} \end{pmatrix} \begin{pmatrix} Z \\ A \end{pmatrix}$$

Thomson limit



$$\bar{u}(p)\Gamma_\mu^{ee\gamma,(6)}u(p)\Big|_{p^2=0} = ie\bar{u}(p)\gamma_\mu u(p)$$

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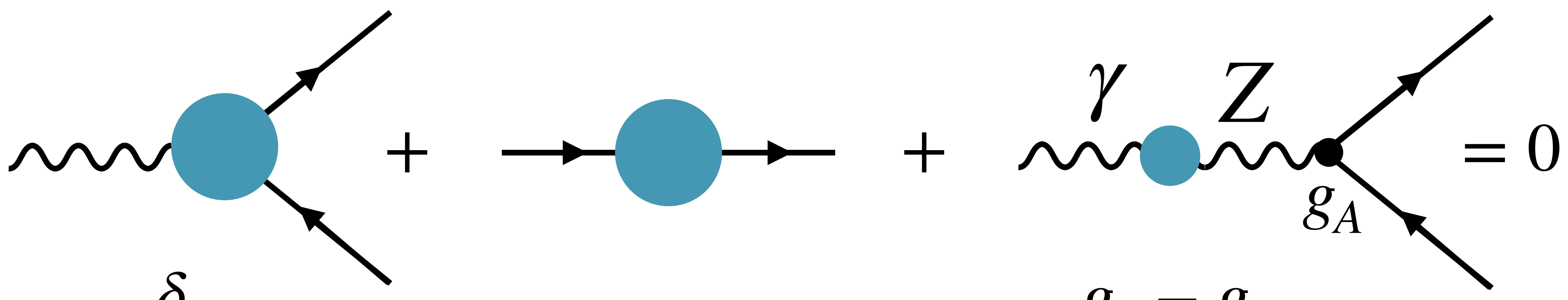
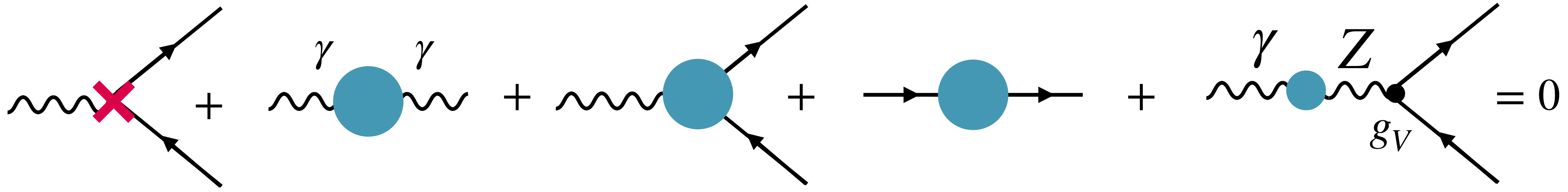
$e^+e^- \to \gamma\gamma$: SMEFT Ward Identity

Axial

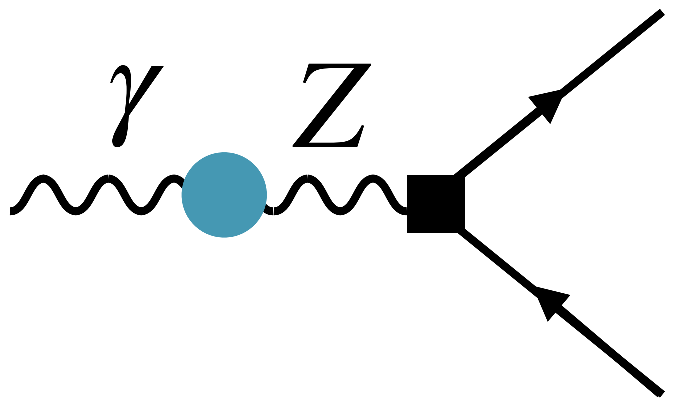


$$\frac{\delta_V}{\sqrt{4\pi\alpha}} \qquad \delta Z_\psi^V \qquad \frac{g_L - g_R}{4\sqrt{4\pi\alpha}}\delta Z_{ZA}$$

Vector



$$\delta Z_e \qquad \frac{1}{2}\delta Z_{AA} \qquad \frac{\delta_V}{\sqrt{4\pi\alpha}} \qquad \delta Z_\psi^V \qquad \frac{g_L + g_R}{4\sqrt{4\pi\alpha}}\delta Z_{ZA}$$



$$-\frac{v^2}{2c_W s_W}\frac{C_{\phi e}[1,1]}{\Lambda^2}$$

Missing term

SMEFT WI

$$\delta Z_e = -\frac{1}{2}\delta Z_{AA} - \left(\frac{g_R}{\sqrt{4\pi\alpha}} - \frac{v^2}{2c_W s_W}\frac{C_{\phi e}[1,1]}{\Lambda^2}\right)\frac{1}{2}\delta Z_{ZA}$$

$e^+e^- \to \gamma\gamma$: SMEFT (preliminary) results

$$\left\{ \frac{\sigma_{\rm NLO}^{SMEFT}}{\sigma_{\rm NLO}^{\rm SM}} - 1 \right\}_{\sqrt{s}=91\rm GeV} = 10^{-6} \frac{1\mathrm{TeV}^2}{\Lambda^2} \Big[19.7 C_{\phi WB} + 6.02 C_{\phi D} + 0.843 C_{\phi l}^{(1)}[1,1] + 7.05 C_{\phi l}^{(3)}[1,1] - 0.67 T_{\phi e}[1,1] \\ 4.72 C_{ff}^L - 4.72 C_{ff}^R + 1.57 C_{qf}^L - 1.57 C_{qf}^R \Big]$$

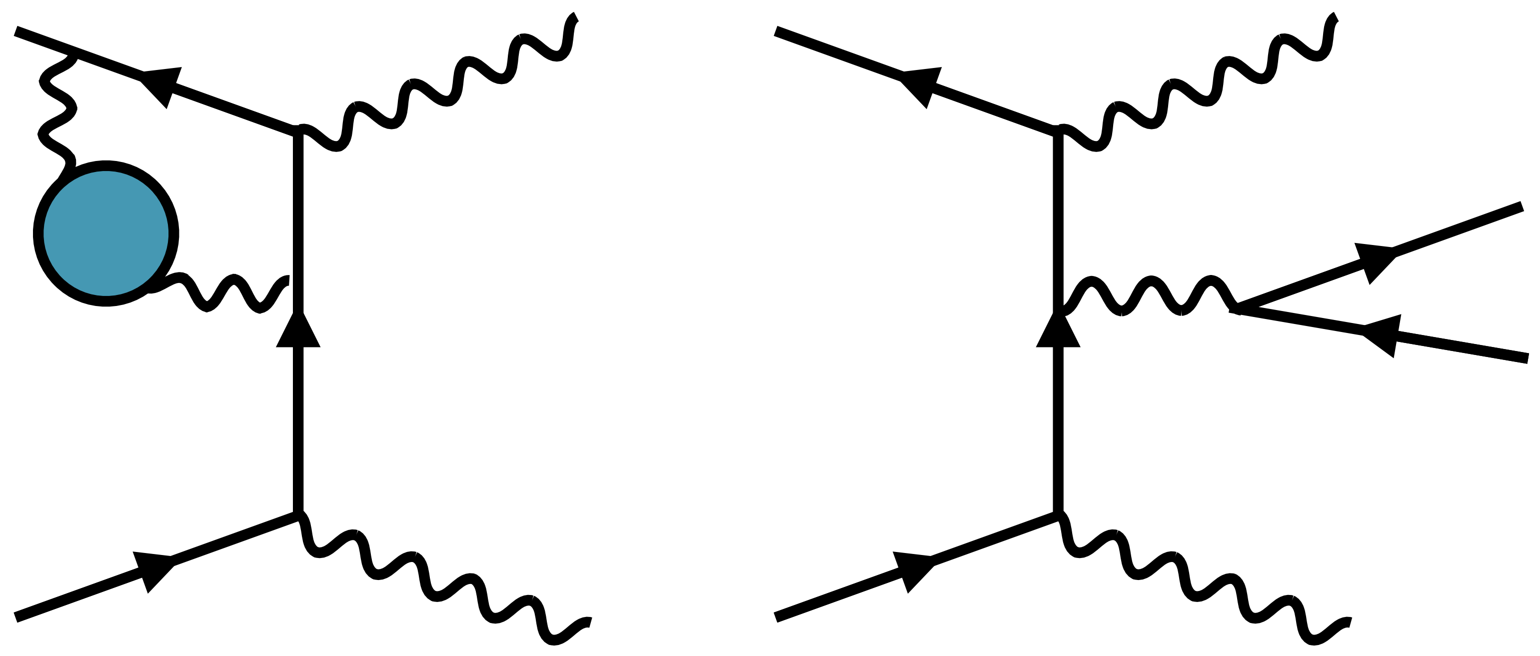
$$\left\{ \frac{\sigma_{\rm NLO}^{SMEFT}}{\sigma_{\rm NLO}^{\rm SM}} - 1 \right\}_{\sqrt{s}=365\rm GeV} = 10^{-4} \frac{1\mathrm{TeV}^2}{\Lambda^2} \Big[8.47 C_{\phi WB} + 2.54 C_{\phi D} + 0.58 C_W - 0.16 C_{\phi l}^{(1)}[1,1] + 2.94 C_{\phi l}^{(3)}[1,1] + 0.13 T_{\phi e}[1,1] \\ 0.75 C_{ff}^L - 0.75 C_{ff}^R + 0.25 C_{qf}^L - 0.25 C_{qf}^R \Big]$$

$e^+e^- \to \gamma\gamma$: NNLO pairs

Lepton and hadron pairs

A subset of the full NNLO correction at $\mathcal{O}(N_f \alpha^2)$

Relevant for experiments, whether the pairs are seen or not



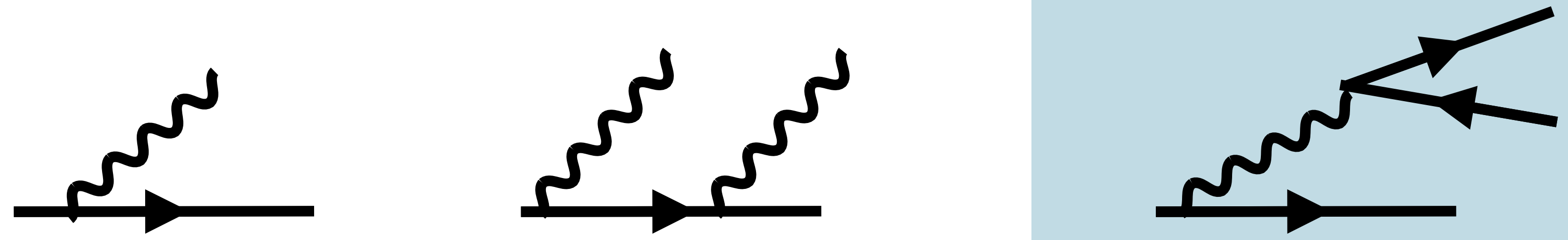
Sudakov

A rough estimation of the order of magnitude can be done considering the running in the SFF

$$\Pi(s_f, s_i) = \exp\left[-\int_{s_i}^{s_f} \frac{ds'}{s'} \frac{\alpha(s')}{2\pi} I_+\right]$$

$$\alpha(s') = \begin{cases} \alpha(0) & \text{No emission} \\ \alpha^{f'}(s') = \dfrac{\alpha}{1 - \frac{\alpha}{3\pi}\log\frac{s'}{m_{f'}^2}\theta(s'-m_{f'})} & \text{Pair of flavour f'} \\ \alpha^{f'f''}(s') = \dfrac{\alpha}{1 - \frac{\alpha}{3\pi}\sum_{f=f',f''}\log\frac{s'}{m_f^2}\theta(s'-m_f)} & \text{Pairs of flavours f' f''} \end{cases}$$

$$\Pi_f^l(s, m_f^2)^{\mathcal{O}\alpha^2} \simeq 1 + \frac{\alpha}{\pi} L_f \log\epsilon + \frac{\alpha^2}{2\pi^2} L_f^2 \log\epsilon^2 + \frac{\alpha^2}{6\pi^2}\sum_l L_l^2 \log\epsilon$$



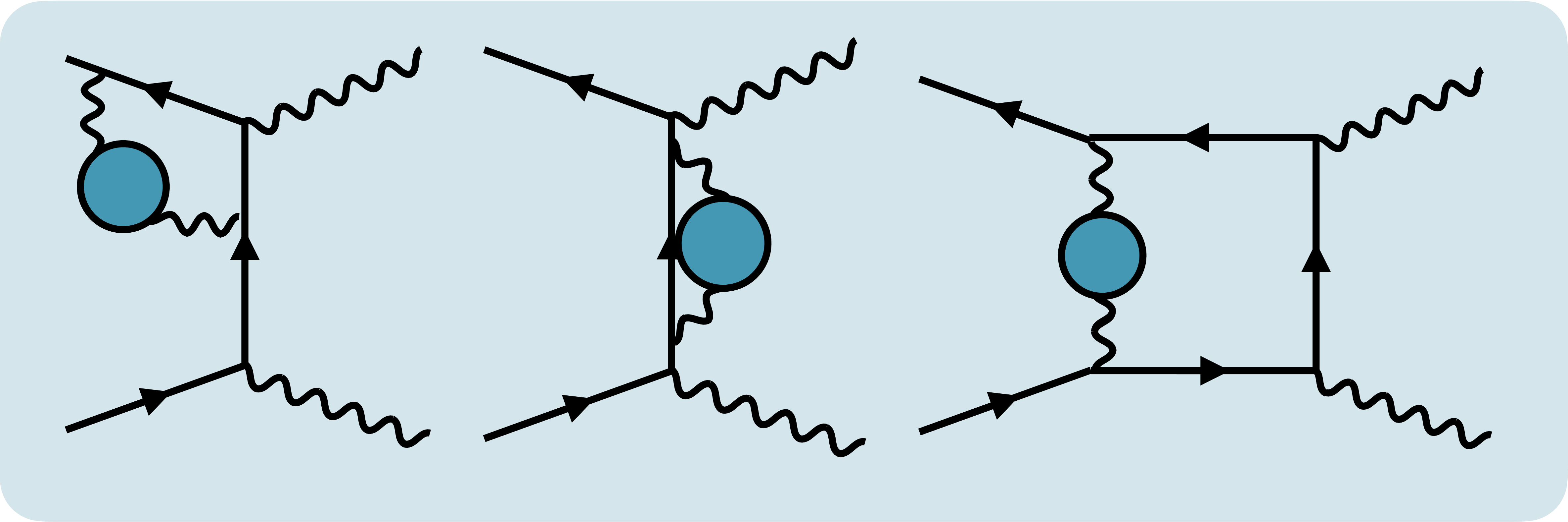
At FCC-ee

$\mathcal{O}(2-3\times 10^{-4})$

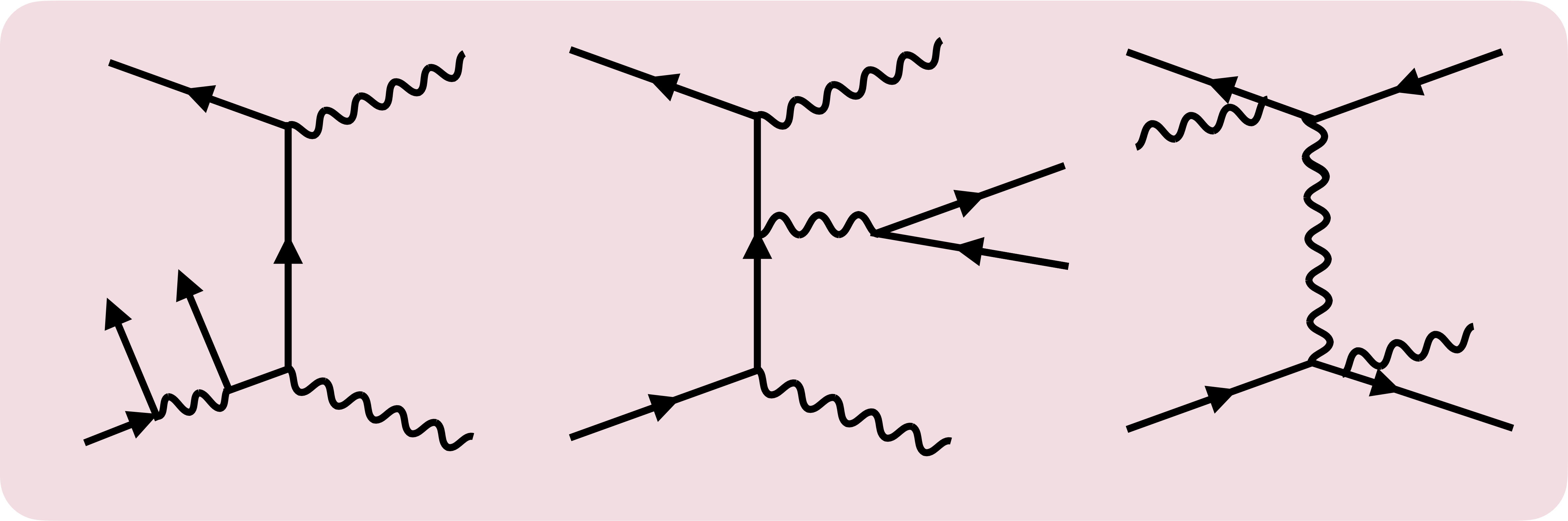
Represents another source of uncertainty at the level of 10^{-5}

$e^+e^- \to \gamma\gamma$: NNLO pairs

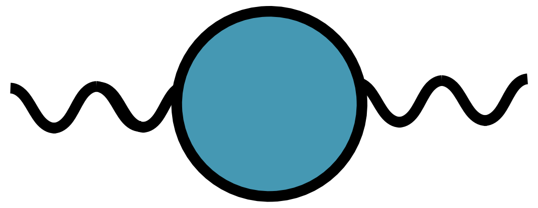
Virtual



Real



Dispersion relation



$$= \int_{4m_\ell^2}^{\infty} \frac{\mathrm{d}s'}{s'} \frac{\Im\Pi(q^2)}{q^2 - s' + i\epsilon}$$

The virtual corrections can be calculated with the NLO kernel with massive photons

$$\mathscr{A}^{N_f}_{\mathrm{NNLO}} = \frac{\alpha}{3\pi} \int_{4m_\ell^2/4m_\pi^2}^{\infty} \frac{\mathrm{d}s'}{s'} R_{\ell/h}(s') \mathscr{A}_{\mathrm{NLO}}(s')$$

Basically a tree level, but the bottleneck is the phase space integration

$$I_k = \frac{1}{N_{\mathrm{ch}}} \int \frac{|\mathscr{M}|^2}{\bar{w}(\vec{y})} \mathrm{d}\tilde{\Phi}(\vec{y})_k$$

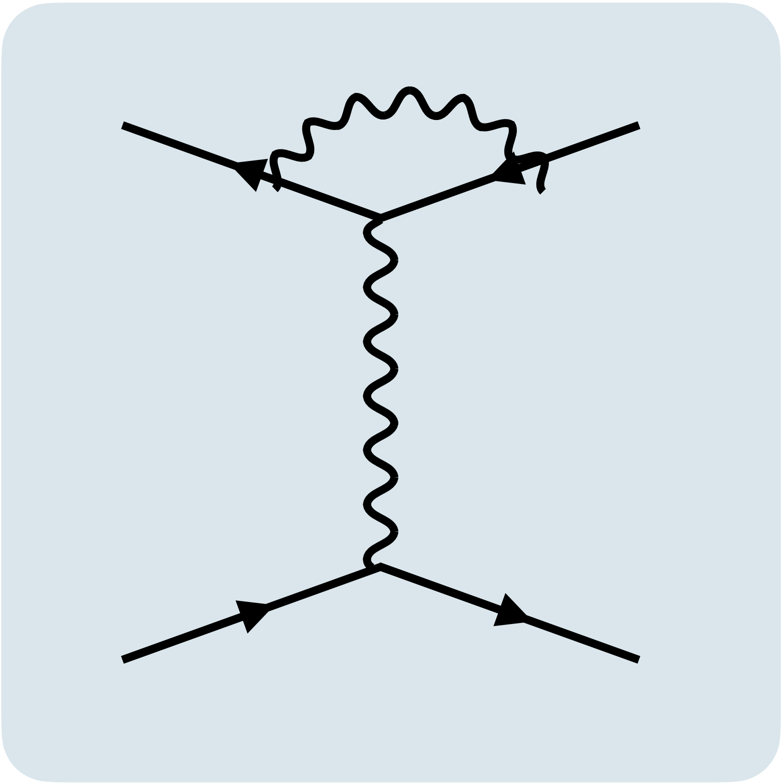
Multichannel sampling

5 different parameterisations of the phase-space **WIP**

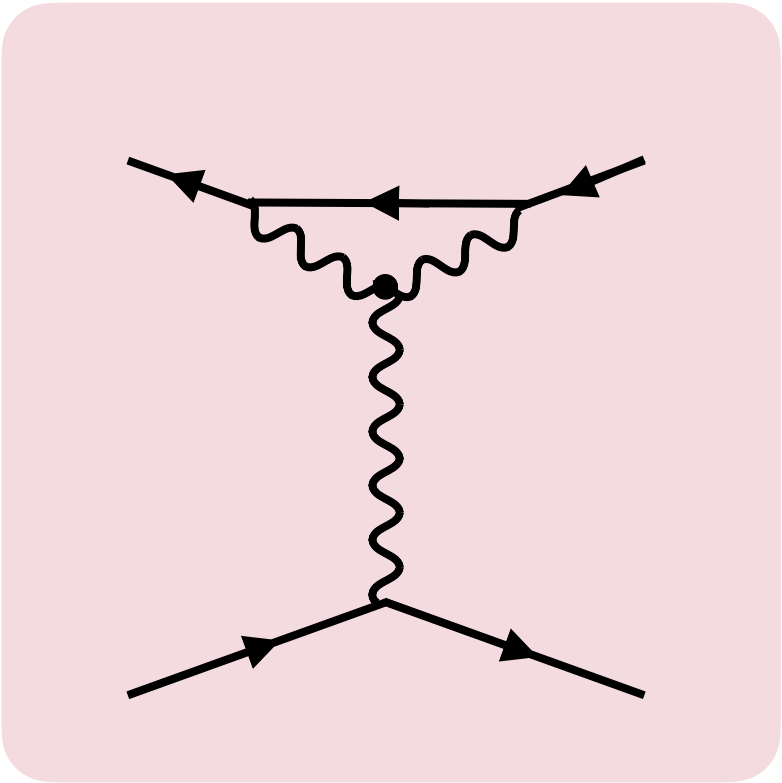
$$\mathrm{d}\Phi_4^{\mathrm{E}} = (2\pi)^3 \mathrm{d}Q_{234}^2 \mathrm{d}\Phi_2(P \to q_1 + Q_{234})$$

$$(2\pi)^3 \mathrm{d}Q_{34}^2 \Phi_2(Q_{234}^* \to q_2^* + Q_{34}^*)\ \mathrm{d}\Phi_2(Q_{34}^* \to p_3 + p_4)$$

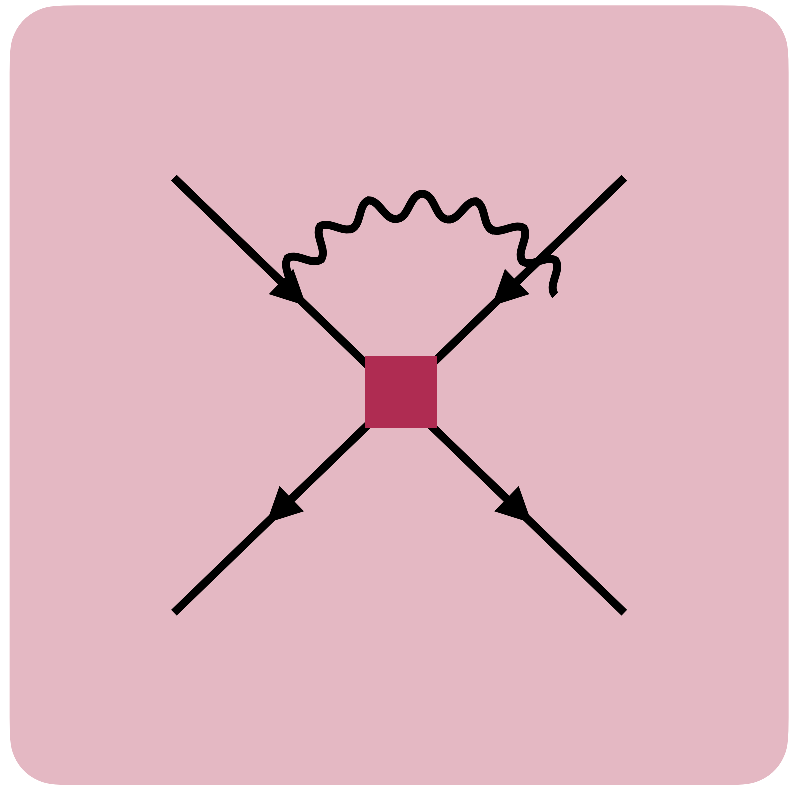
Status and future prospects



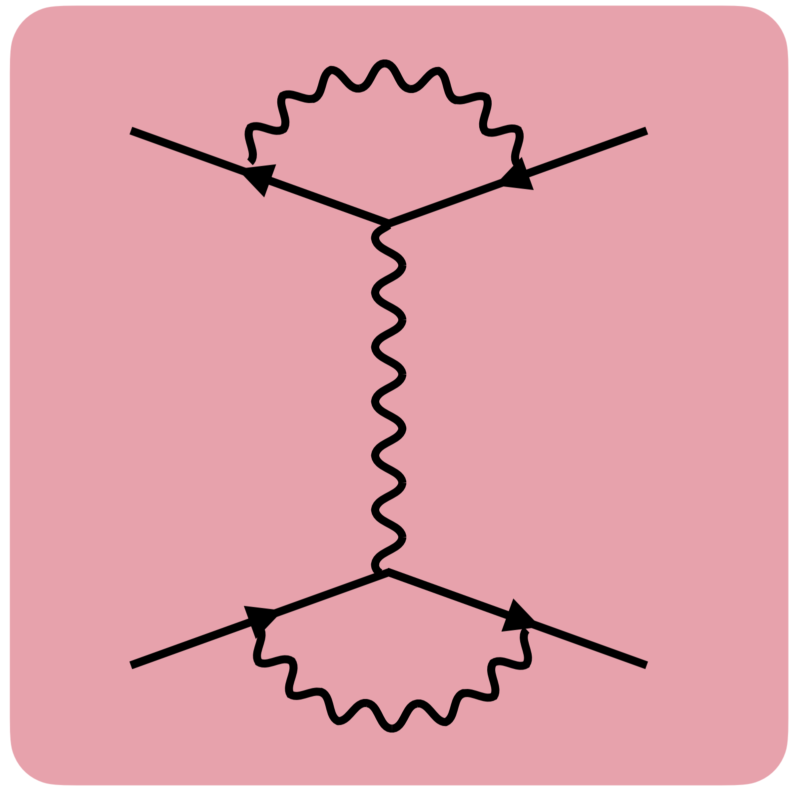
NLOPS QED



NLO EW

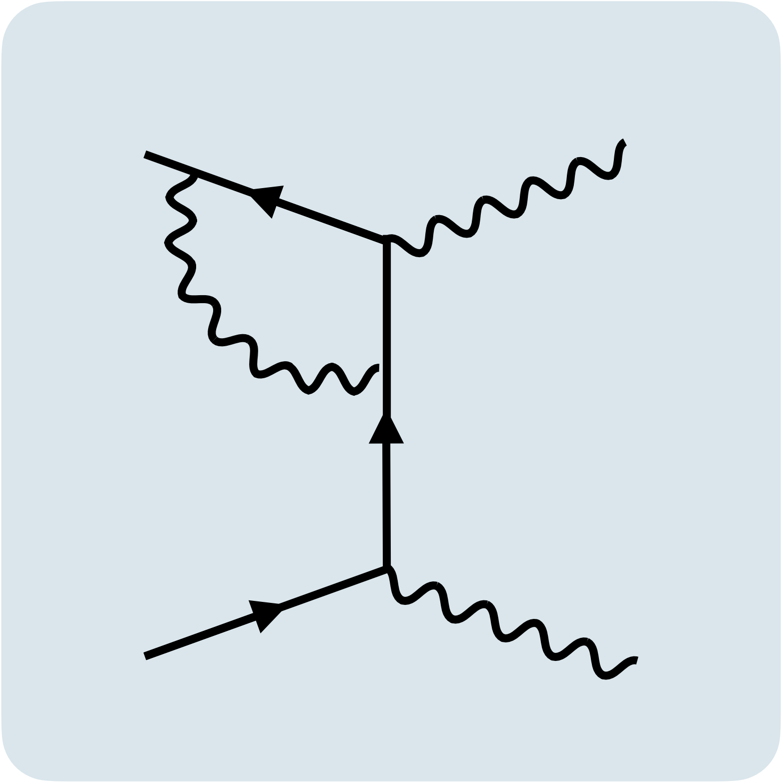


NLO SMEFT

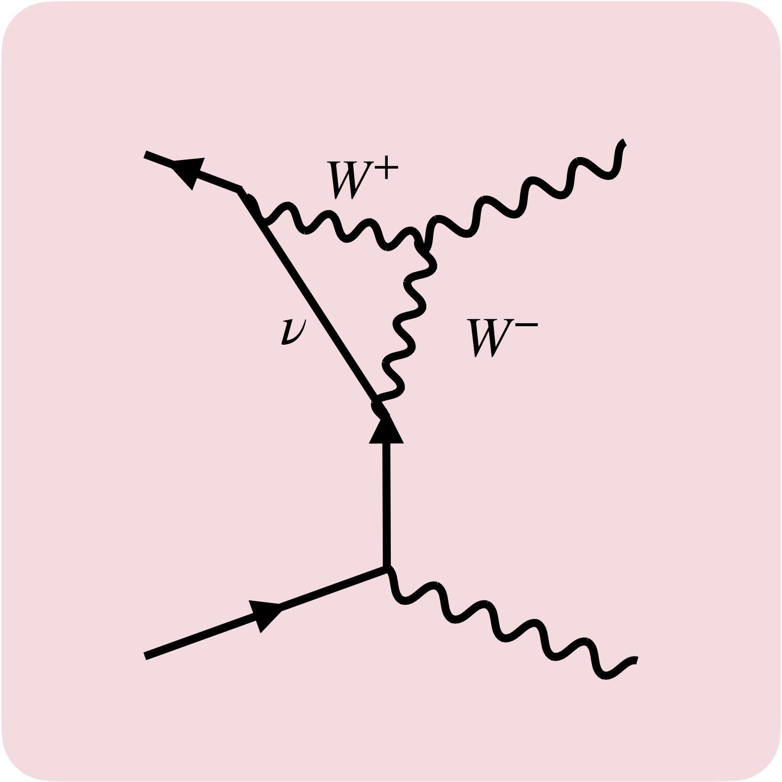


NNLO matching

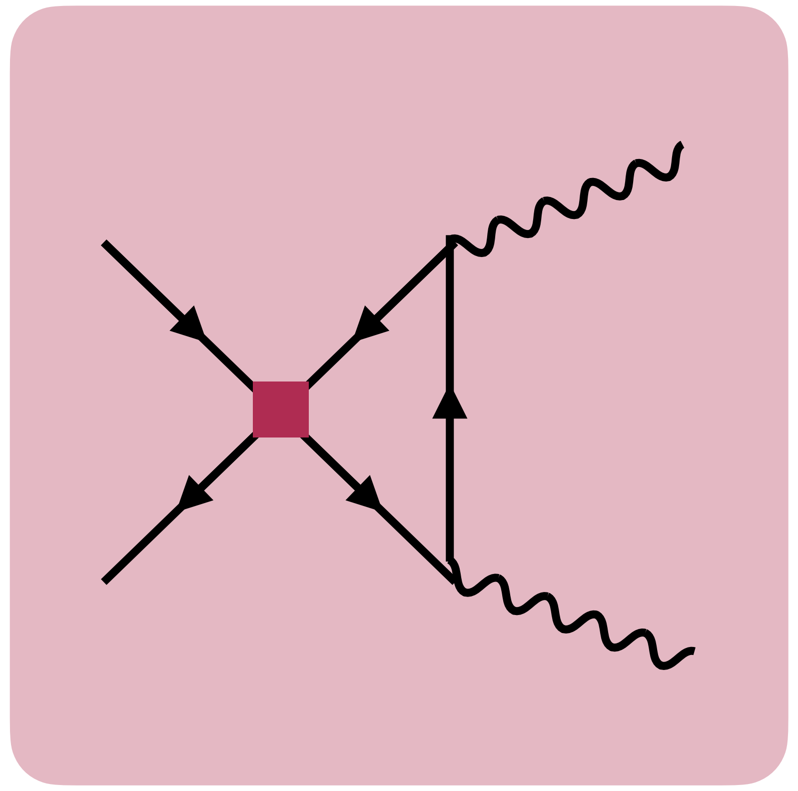
Precision 10^{-5}



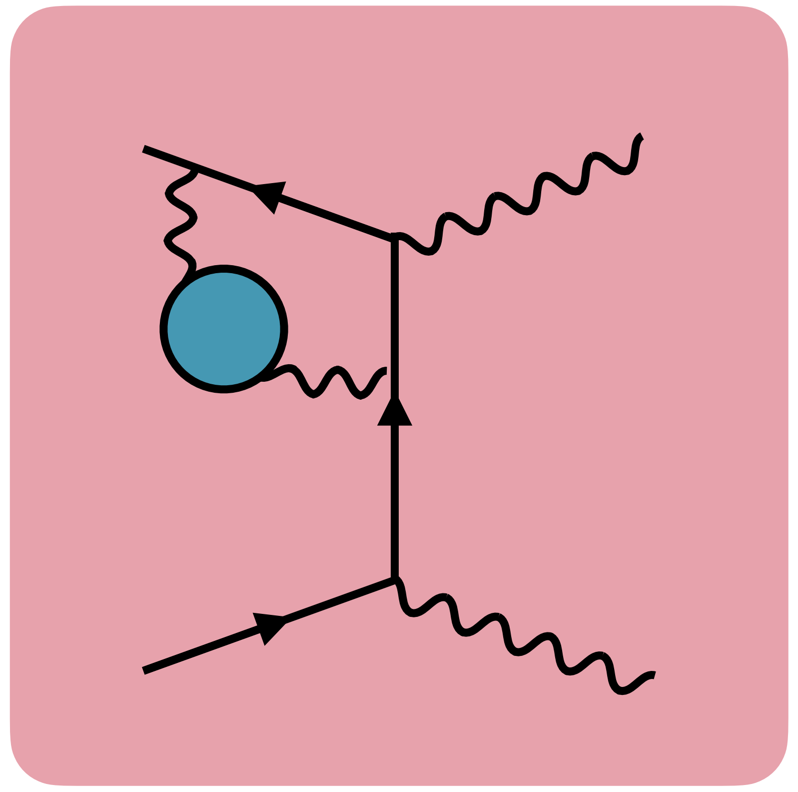
NLOPS QED



NLO EW



NLO SMEFT



NNLO fermionic/ bosonic

Status of BabaYaga

Process	Order	Accuracy
$e^+e^- \to e^+e^-$	NLOPS QED LO EW LO SMEFT	$\mathcal{O}(0.1\%)$
$e^+e^- \to \mu^+\mu^-$	NLOPS QED LO EW	$\mathcal{O}(0.1\%)$
$e^+e^- \to \gamma\gamma$	NLOPS QED NLO EW NLO SMEFT	$\mathcal{O}(0.1\%)$
$e^+e^- \to \pi^+\pi^-$	NLOPS QED FF (FxsQED, GVMD, FSQED)	$\mathcal{O}(0.1\%) \oplus \delta F_\pi$
$e^+e^- \to e^+e^-\gamma$	NLOPS QED	
$e^+e^- \to \mu^+\mu^-\gamma$	NLOPS QED	
$e^+e^- \to \pi^+\pi^-\gamma$	NLOPS QED FF (FxSQED)	

What’s next?

Luminosity

- $e^+e^- \to \gamma\gamma$ NNLO pair corrections

In progress:
- Compute exact NNLO QED virtual and real pair corrections via dispersive methods
- For now, at low energies

Goals:
- Add also hadronic pairs

Next-future:
- Include Z exchange for pair corrections
- Calculate NLO SMEFT corrections for FCC-ee luminometry

- $e^+e^- \to X$ for luminosity

Future plans:
- NNLO matching with ISR

Hadronic channels

- $e^+e^- \to K_L K_S, K^0\bar{K}^0$ at NLOPS accuracy

Future plans:
- Achieve NLOPS accuracy also for other relevant hadronic channels

Other channels

- $e^+e^- \to \tau^+\tau^-(\gamma)$ at NLOPS accuracy

- Tau decays are interesting for polarisation asymmetries and to measure $a_\tau = (g-2)_\tau/2$

Backup

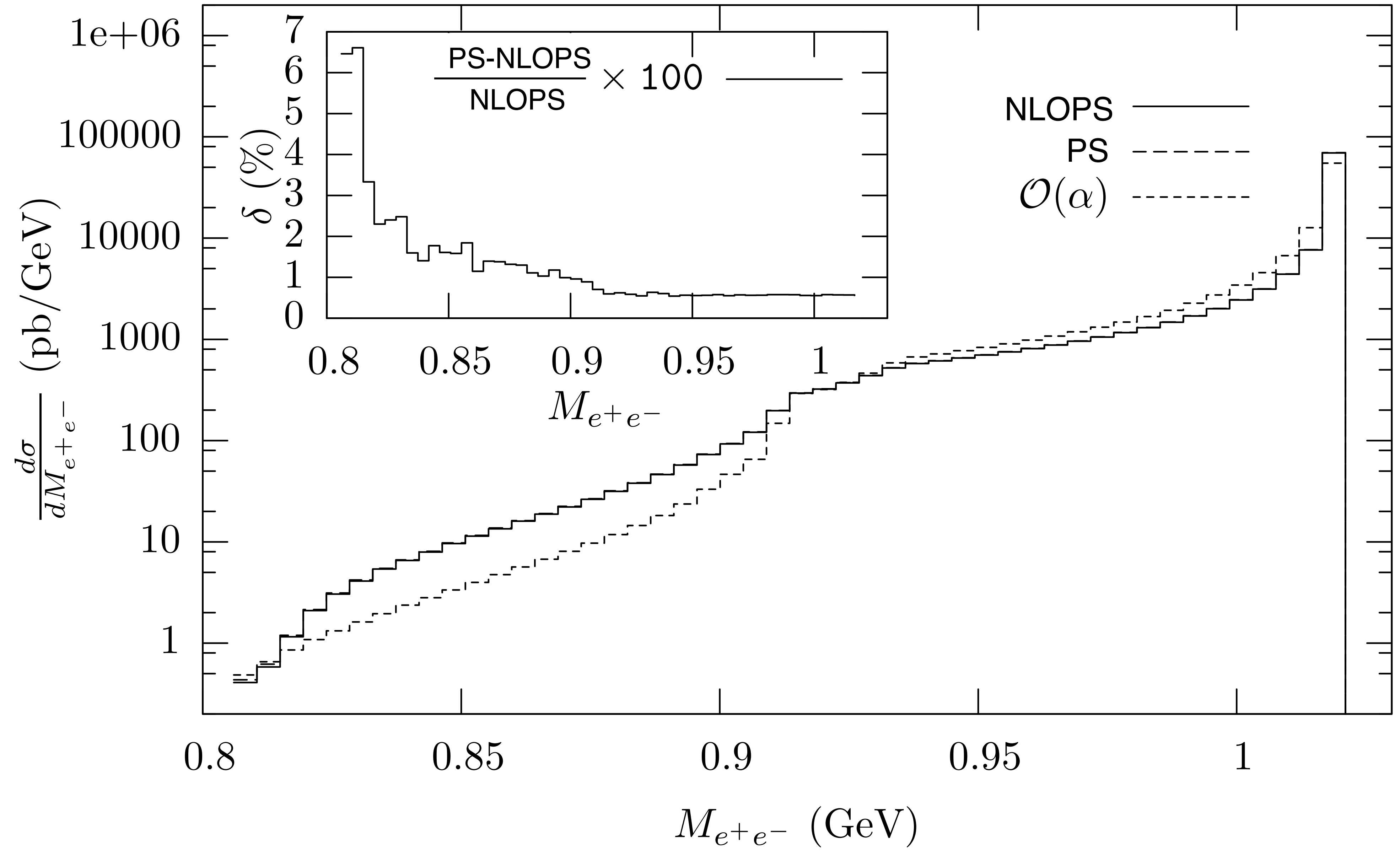
Theoretical precision

The size of radiative corrections has been studied in typical flavour factories setups

ϕ-factories
A $\sqrt{s} = 1.02$ GeV, $E_{min} = 0.408$ GeV, $20° < \theta_\pm < 160°$, $\xi_{max} = 10°$
B $\sqrt{s} = 1.02$ GeV, $E_{min} = 0.408$ GeV, $55° < \theta_\pm < 125°$, $\xi_{max} = 10°$

B-factories
C $\sqrt{s} = 10$ GeV, $E_{min} = 4$ GeV, $20° < \theta_\pm < 160°$, $\xi_{max} = 10°$
D $\sqrt{s} = 10$ GeV, $E_{min} = 4$ GeV, $55° < \theta_\pm < 125°$, $\xi_{max} = 10°$

Correction vs Setup	A	B	C	D
$\delta_\alpha = \frac{\sigma_{\rm NLO} - \sigma_{\rm LO}}{\sigma_{\rm LO}}$	-11.61	-14.72	-16.03	-19.57
$\delta_{\rm HO} = \frac{\sigma_{\rm NLOPS} - \sigma_{\rm NLO}}{\sigma_{\rm LO}}$	0.39	0.82	0.73	1.44
$\delta_{\rm HO}^{\rm PS} = \frac{\sigma_{\rm PS}^\infty - \sigma_{\rm PS}^\alpha}{\sigma_{\rm LO}}$	0.35	0.74	0.68	1.34
$\delta_\alpha^{\rm non-log} = \frac{\sigma_{\rm NLO} - \sigma_{\rm PS}^\alpha}{\sigma_{\rm LO}}$	-0.34	-0.57	-0.34	-0.56
$\delta_\infty^{\rm non-log} = \frac{\sigma_{\rm NLOPS} - \sigma_{\rm PS}}{\sigma_{\rm LO}}$	-0.30	-0.49	-0.29	-0.46



$\delta_{\rm HO}^{\rm PS} \simeq \delta_{\rm HO}$

The higher orders are added to the NLO

$\delta_\infty^{\rm non-log} \simeq \delta_\alpha^{\rm non-log}$

The missing NLO finite corrections are included

Tuned comparisons

At the Φ and τ–charm factories

setup	BabaYaga@NLO	BHWIDE	MCGPJ	$\delta(\%)$
$\sqrt{s} = 1.02$ GeV, $20^\circ \leq \vartheta_\mp \leq 160^\circ$	6086.6(1)	6086.3(2)	—	0.005
$\sqrt{s} = 1.02$ GeV, $55^\circ \leq \vartheta_\mp \leq 125^\circ$	455.85(1)	455.73(1)	—	0.030
$\sqrt{s} = 3.5$ GeV, $\lvert\vartheta_+ + \vartheta_- - \pi\rvert \leq 0.25$ rad	35.20(2)	—	35.181(5)	0.050

By BabaYaga group, Ping Wang and A. Sibidanov

At BaBar

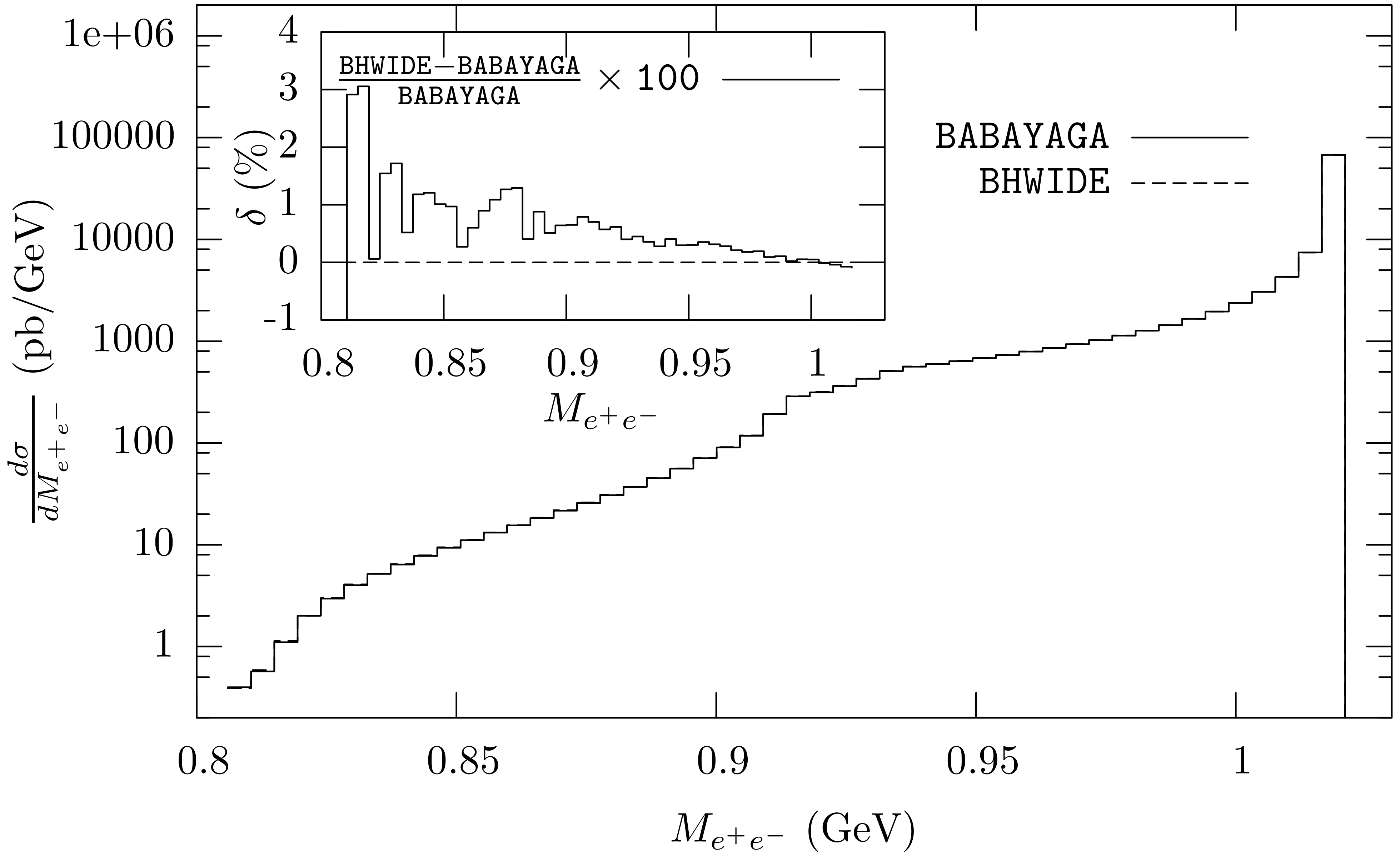
angular acceptance cuts	BabaYaga@NLO	BHWIDE	$\delta(\%)$
$15^\circ \div 165^\circ$	119.5(1)	119.53(8)	0.025
$40^\circ \div 140^\circ$	11.67(3)	11.660(8)	0.086
$50^\circ \div 130^\circ$	6.31(3)	6.289(4)	0.332
$60^\circ \div 120^\circ$	3.554(6)	3.549(3)	0.141

By A. Hafner and A. Denig

Tests shows an agreement at $0.1\ \%$ level, which is the theoretical accuracy of BabaYaga@NLO

At KLOE

S. Actis et al. Eur. Phys. J. C 66 (2010) 585

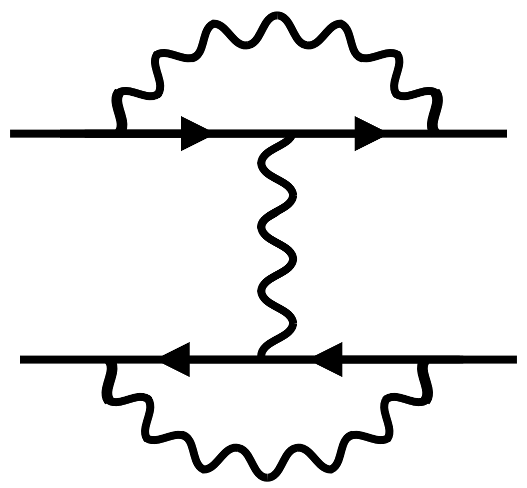


Theoretical error

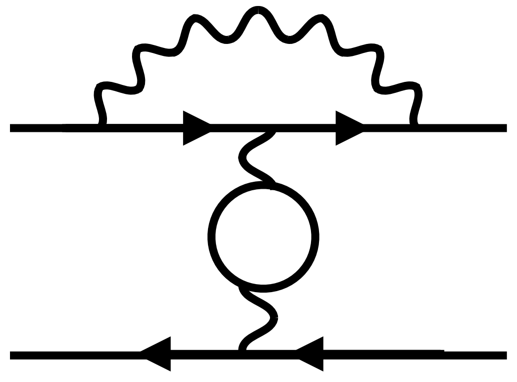
To quantify the theoretical error of BabaYaga, we can compare the exact NNLO versus the PS matched expanded

$$\sigma^{\alpha^2} = \sigma^{\alpha^2}_{\rm SV} + \sigma^{\alpha^2}_{\rm SV,H} + \sigma^{\alpha^2}_{\rm HH} \quad \text{vs BabaYaga@NLO } \mathcal{O}(\alpha^2)$$

$\sigma^{\alpha^2}_{\rm SV}$

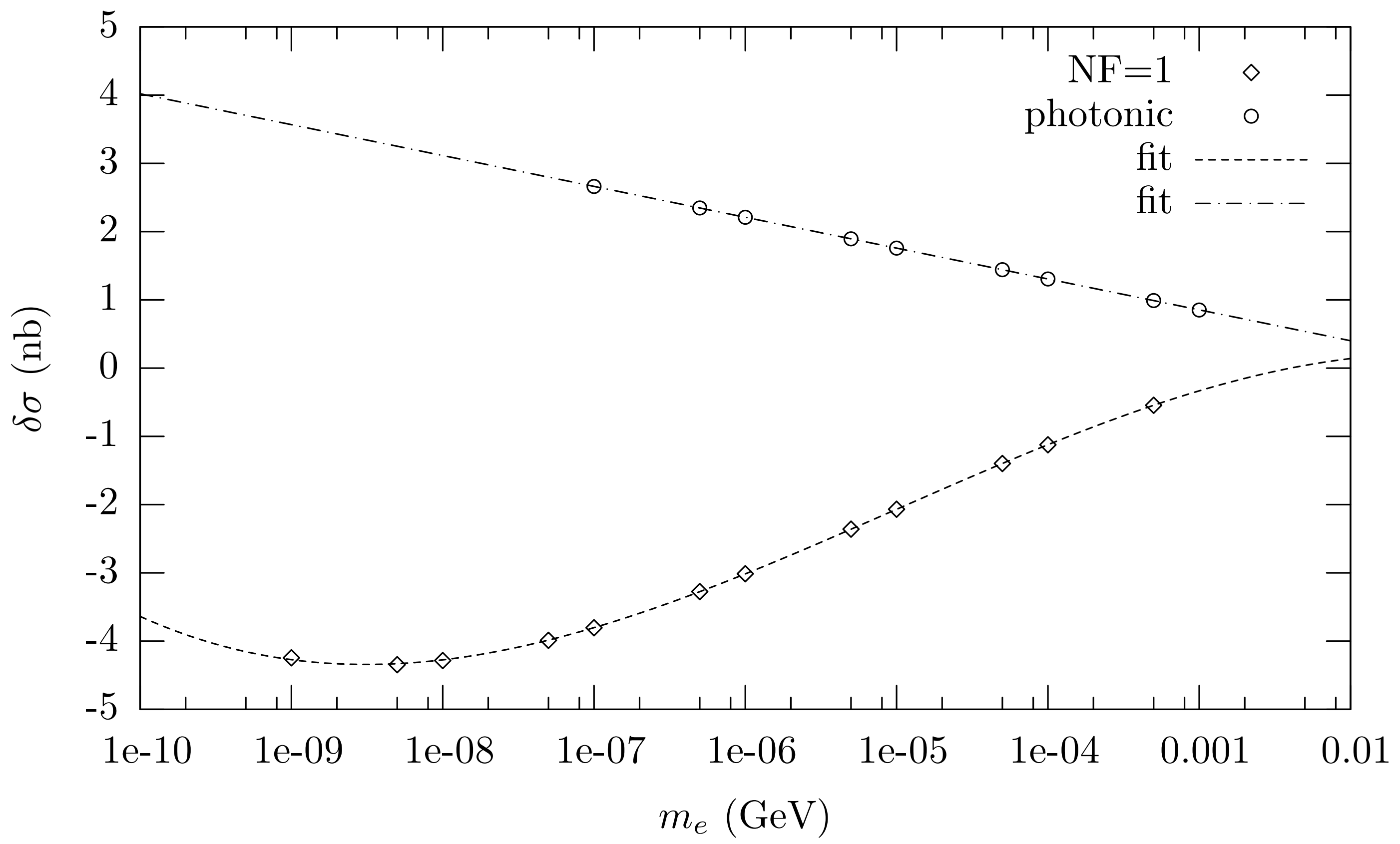


Photonic



Fermionic $N_f = 1$

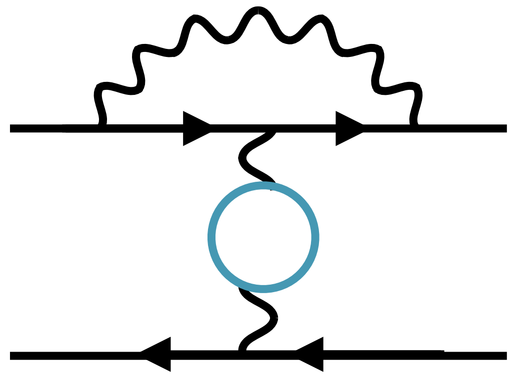
A. Penin, PRL 95 (2005) 010408
Nucl. Phys. B734 (2006) 185



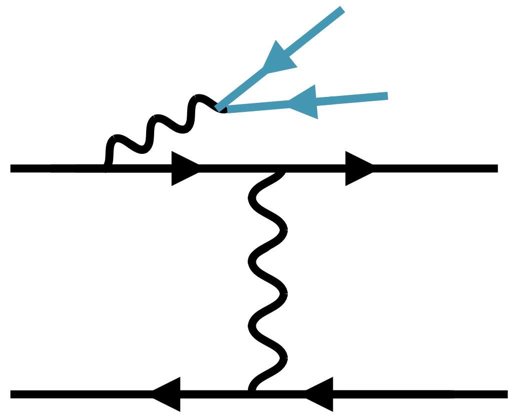
$$\frac{\delta\sigma(\text{Photonic})}{\sigma_{\rm LO}} \propto \alpha^2 L$$

$$\sigma^{\alpha^2}_{\rm SV}(\text{Penin}) - \sigma^{\alpha^2}_{\rm SV}(\text{BY@NLO}) < 0.02\,\%\,\sigma_{\rm LO}$$

$\sigma^{\alpha^2}_{\rm HH}$



Photonic $f \neq e$



Real pairs

	$\sqrt{s}$		$\sigma_{\rm BY}$	$S_{e^+e^-}$ [‰]	S_{lep} [‰]	S_{had} [‰]	S_{tot} [‰]
KLOE	1.020	NNLO		-3.935(4)	-4.472(4)	1.02(2)	-3.45(2)
		BB@NLO	455.71	-3.445(2)	-4.001(2)	0.876(5)	-3.126(5)
BES	3.650	NNLO		-1.469(9)	-1.913(9)	-1.3(1)	-3.2(1)
		BB@NLO	116.41	-1.521(4)	-1.971(4)	-1.071(4)	-3.042(5)
BaBar	10.56	NNLO		-1.48(2)	-2.17(2)	-1.69(8)	-3.86(8)
		BB@NLO	5.195	-1.40(1)	-2.09(1)	-1.49(1)	-3.58(2)
Belle	10.58	NNLO		-4.93(2)	-6.84(2)	-4.1(1)	-10.9(1)
		BB@NLO	5.501	-4.42(1)	-6.38(1)	-3.86(1)	-10.24(2)

Carloni, Czyż, Gluza, Gunia, Montagna, Nicrosini, Piccinini, Riemann et al., JHEP 1107 (2011) 126

$$\delta\sigma_{\rm pairs} \sim 10^{-4}$$

The pion form factor

The Pion FF is a key quantity to determine the $(g-2)_\mu$

$$a_\mu^{\text{HLO}} = \frac{\alpha}{\pi^2}\int_{4m_\pi^2}^{\infty}\frac{\text{d}s}{s}K(s)\left(\frac{\alpha(s)}{3}\frac{\sigma(e^+e^-\to\text{hadrons})}{\sigma(e^+e^-\to\mu^+\mu^-)}\right)$$

$$\simeq \frac{\alpha}{\pi^2}\int\frac{\text{d}s}{s}K(s)\beta_\pi^2\,|F_\pi(s)|^2 f(s)$$

In scan experiments, the PionFF is given by the ratio

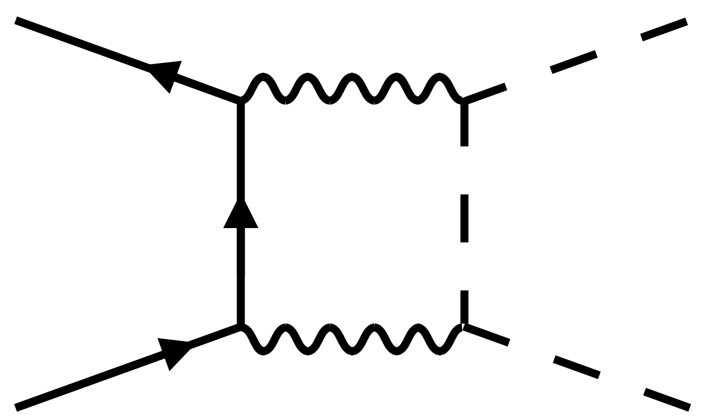
$$|F_\pi|^2 = \left(\frac{N_{\pi^+\pi^-}}{N_{e^+e^-}} - \Delta^{\text{bg}}\right)\cdot\frac{\sigma^0_{e^+e^-}\cdot(1+\delta_{e^+e^-})\cdot\varepsilon_{e^+e^-}}{\sigma^0_{\pi^+\pi^-}\cdot(1+\delta_{\pi^+\pi^-})\cdot\varepsilon_{\pi^+\pi^-}}$$

$\delta_{\pi^+\pi^-}$ Radiative corrections are estimated via MC

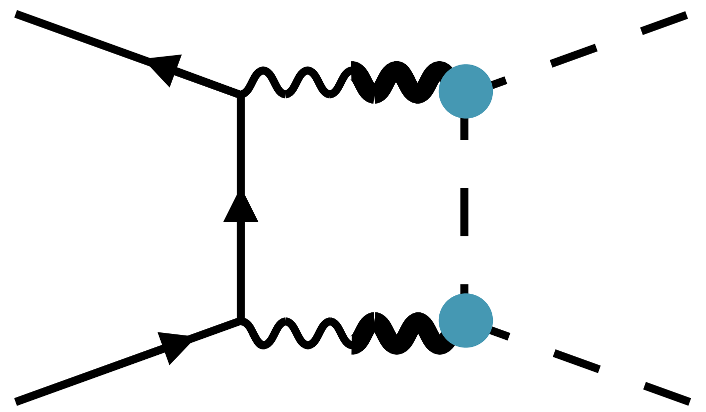
$\epsilon_{\pi^+\pi^-}$ Also the acceptance has a MC dependence via the charge asymmetry

$$A_{FB}^{\text{NLO}} = A_{FB}^{\text{LO}} + \frac{\alpha}{\pi}A_{FB}^{\alpha} = 0 + \frac{\alpha}{\pi}\left(\frac{\sigma_B^{\text{odd}} - \sigma_F^{\text{odd}}}{\sigma^{\text{NLO}}}\right)$$

The FF has to be modelled due its non-perturbative nature

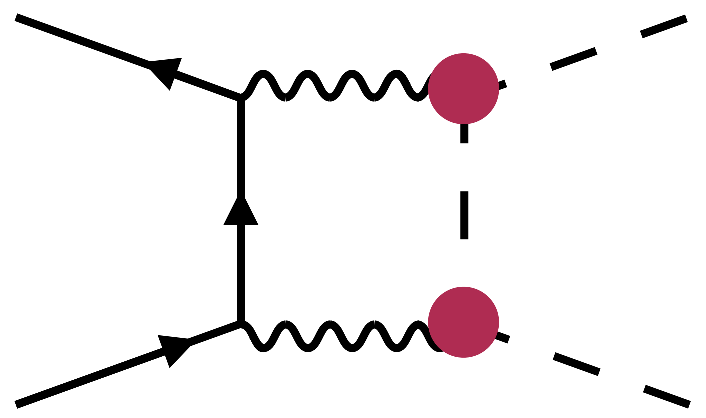


Factorised sQED — The FF is attached to virtual amplitude as to cancel infrared divergences



GVMD — A model based on the mixing of the photon with light resonances

$$F_\pi^{\text{BW}}(q^2) = \sum_{v=1}^{n_r} F_{\pi,v}^{\text{BW}}(q^2) = \frac{1}{c_t}\sum_{v=1}^{n_r} c_v\frac{\Lambda_v^2}{\Lambda_v^2 - q^2}$$



FsQED — Relies on the analytic properties of the FF, written via a dispersion relation

$$F_\pi(q^2) = 1 + \frac{q^2}{\pi}\int_{4m_\pi^2}^{\infty}\frac{\text{d}s'}{s'}\frac{\text{Im}F_\pi(s')}{s'-q^2-i\epsilon'}$$

Contact Interactions

"Electroweak Measurements in Electron–Positron Collisions at W-Boson-Pair Energies at LEP."
Physics Reports, vol. 532, no. 4, Nov. 2013, pp. 119–244. arXiv:1302.3415

In LEP analyses, contact interactions were parameterised as

$$\mathscr{L}_{\text{eff}} = \frac{g^2}{2\Lambda^2} \sum_{i,j=L,R} \eta_{ij} \left(\bar{e}_i \gamma_\mu e_i\right) \left(\bar{e}_j \gamma^\mu e_j\right)$$

$$\frac{g^2}{4\pi} = 1$$

Model	Λ^-_{ee} (TeV)	Λ^+_{ee} (TeV)
$LL^\pm$	8.0	8.7
$RR^\pm$	7.9	8.6
$VV^\pm$	15.3	20.6
$AA^\pm$	14.0	10.1
$LR^\pm$	8.5	11.9
$RL^\pm$	8.5	11.9
$V0^\pm$	11.2	12.4
$A0^\pm$	11.8	17.0
$A1^\pm$	4.0	3.9

Bound for NP scale

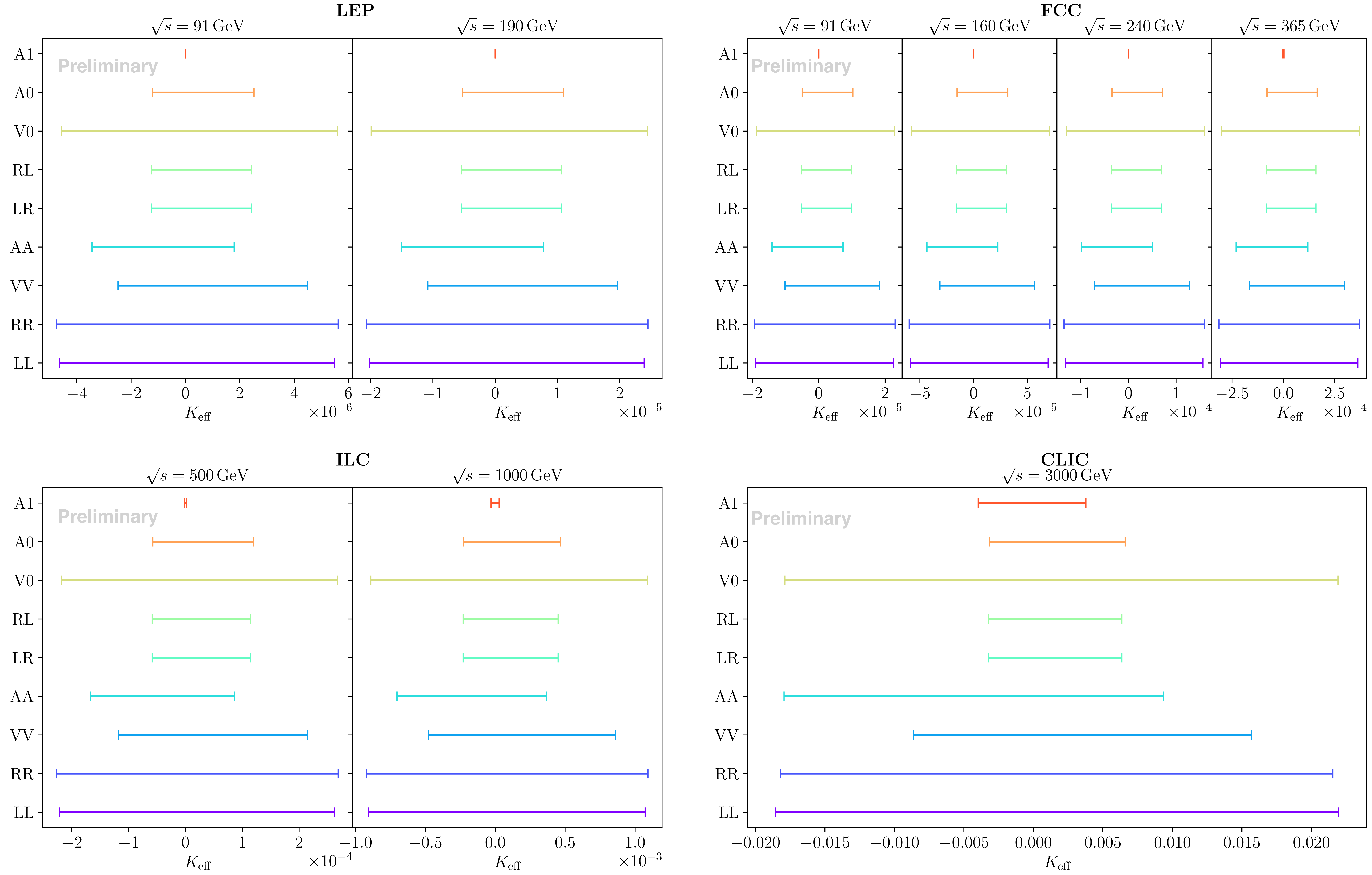
Λ_+ **Positive interference**

Λ_- **Negative interference**

$$\mathscr{M}(t)^\dagger_\gamma \mathscr{M}_{\text{LL/RR}} = -32\pi\alpha \frac{(1+\cos\theta)^2}{(1-\cos\theta)} s$$

$$\mathscr{M}(t)^\dagger_\gamma \mathscr{M}_{\text{RL/LR}} = -64\pi\alpha \frac{s}{(1-\cos\theta)}$$

Contact Interactions: results



Electroweak Sector in SMEFT

Using $\{\alpha, M_Z, G_\mu\}$ scheme the SMEFT Lagrangian in the EW sector can be written as

$$\mathscr{L}^{\rm NC}_{\rm SMEFT} = -\sqrt{4\pi\alpha}\,(\bar{e}\gamma^\mu e)\,A_\mu + \frac{\sqrt{4\pi\alpha}}{s_w c_W}\left\{\ \bar{e}_L\left(\hat{g}^Z_L + \frac{\Delta g^{Ze}_L}{\Lambda^2}\right)\gamma^\mu e_L + \bar{e}_R\left(\hat{g}^Z_R + \frac{\Delta g^{Ze}_R}{\Lambda^2}\right)\gamma^\mu e_R\right\} Z_\mu$$

Corrections to SM input parameters

I. Brivio arXiv:2012.11343

$$g = g_{\rm SM} + \Delta g$$

$$G_\mu = \frac{1}{\sqrt{2}v_T^2}(1 + \frac{1}{\sqrt{2}G_\mu}\left(C^{(3)11}_{Hl} + C^{(3)22}_{Hl} - C^{1221}_{ll}\right)$$

$$\alpha_{\rm em} = \frac{1}{4\pi}\frac{g_W^2 g_1^2}{g_W^2 + g_1^2}(1 + \Delta\alpha_{\rm em})$$

Coupling of Z boson to fermions

Linear combinations of WCs in Warsaw basis

$$\Delta g^{Z,e}_L = -\frac{1}{2}C^{(3)}_{\phi l} - \frac{1}{2}C_{\phi l} + f\left(-\frac{1}{2}, -1\right)$$

$$\Delta g^{Z,e}_R = -\frac{1}{2}C_{\phi e} + f\,(0, -1)$$

For SILH Basis arXiv:1610.07922

$$f(T^3, Q) = -Q\frac{s_w c_w}{c_w^2 - s_w^2}C_{\phi WB} + \left(\frac{1}{4}C_{ll,1221} - \frac{1}{2}C^{(3)}_{\phi l,11} - \frac{1}{2}C^{(3)}_{\phi l,22}\right)\left(T^3 + Q\frac{s_w^2}{c_w^2 - s_w^2}\right)$$

Fit methodology

$$\Delta A_{ab} = \sqrt{\sum_{k=a,b} \left(\frac{\partial A_{ab}}{\partial N_k}\right)^2 \Delta_k^2} = 2\sqrt{\frac{N_a N_b}{(N_a+N_b)^3}}\,.$$

The number of expected events is calculated with 6 months of effective run

Run al picco della Z — FCC-ee

$$\mathcal{L}_{\rm FCC} \;=\; 1.4 \times 10^{36}\,{\rm cm}^{-2}\,{\rm s}^{-1}$$

We obtain an error

$$\Delta A^0_{{\rm FB},\alpha} \lesssim 2\times 10^{-5}$$

We generate gaussian samples centred about the SM with statistical error

$$g\big(A^{\rm SM}_{\rm FB}, \Delta A^0_{\rm FB}\big)_\alpha$$

We solve the system MC and obtain the errors

$$\sum_{i\in 4f} \frac{C_i}{\Lambda^2_{\rm NP}} \left[\frac{(\sigma_{\rm F}-\sigma_{\rm B})^{(6)}_i}{(\sigma_{\rm F}-\sigma_{\rm B})_{\rm SM}} - \frac{(\sigma_{\rm F}+\sigma_{\rm B})^{(6)}_i}{(\sigma_{\rm F}+\sigma_{\rm B})_{\rm SM}}\right]_\alpha = \frac{\Delta A^0_{{\rm FB},\alpha}}{A^0_{{\rm FB},\alpha}},$$

Run a 250 GeV — ILC

$$\mathcal{L}_{\rm ILC} \;=\; 1.35 \times 10^{34}\,{\rm cm}^{-2}\,{\rm s}^{-1}$$

We consider predictions in bins of $\cos\theta = 0.02$

$$\mathbf{A}_\alpha \;\equiv\; \left(A^0_{\rm pol} - A^{\rm th}_{\rm pol}(\vec{C})\right)_\alpha$$

The gaussian likelihood is given as

$$L(\vec{C}) = \mathcal{N}\exp\left\{-\frac{1}{2}\mathbf{A}^T(\vec{C})\,W^{-1}\,\mathbf{A}(\vec{C})\right\}$$

Ellipses are obtained from the region $\chi^2(\vec{C}) \le 1$

The approach is equivalent to

$$\chi^2(\vec{C}) = \frac{1}{\Lambda^4_{\rm NP}} \sum_{i,j}\sum_{\alpha,\beta} C_i\, \kappa^{(6)}_{i,\alpha}\, W^{-1}_{\alpha\beta}\, \kappa^{(6)}_{j,\beta}\, C_j$$