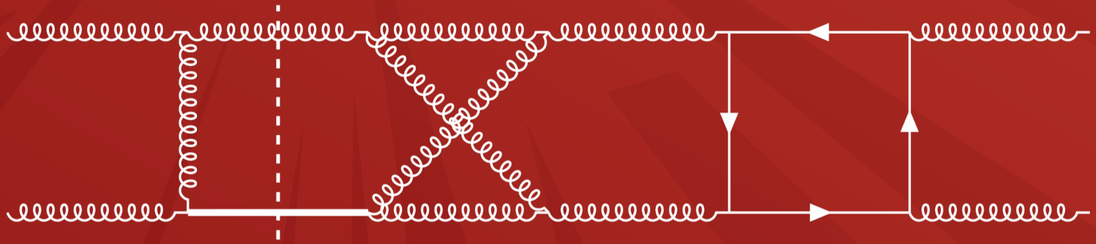
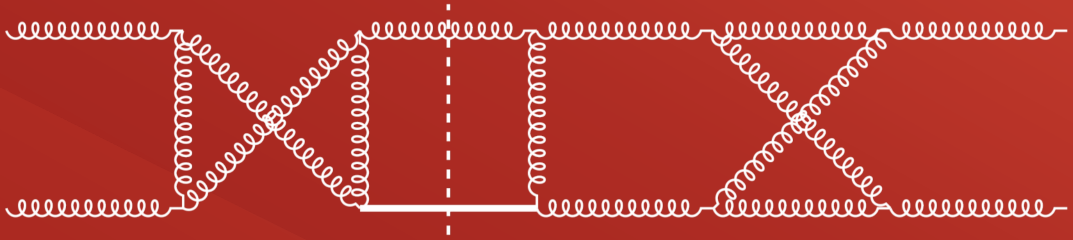
Taking the First Steps Toward the Higgs Production at N^{4}LO in QCD: the Single-real Ingredient

Adi Suresh & Bernhard Mistlberger

LoopFest XXIV



arXiv:2606.xxxxx



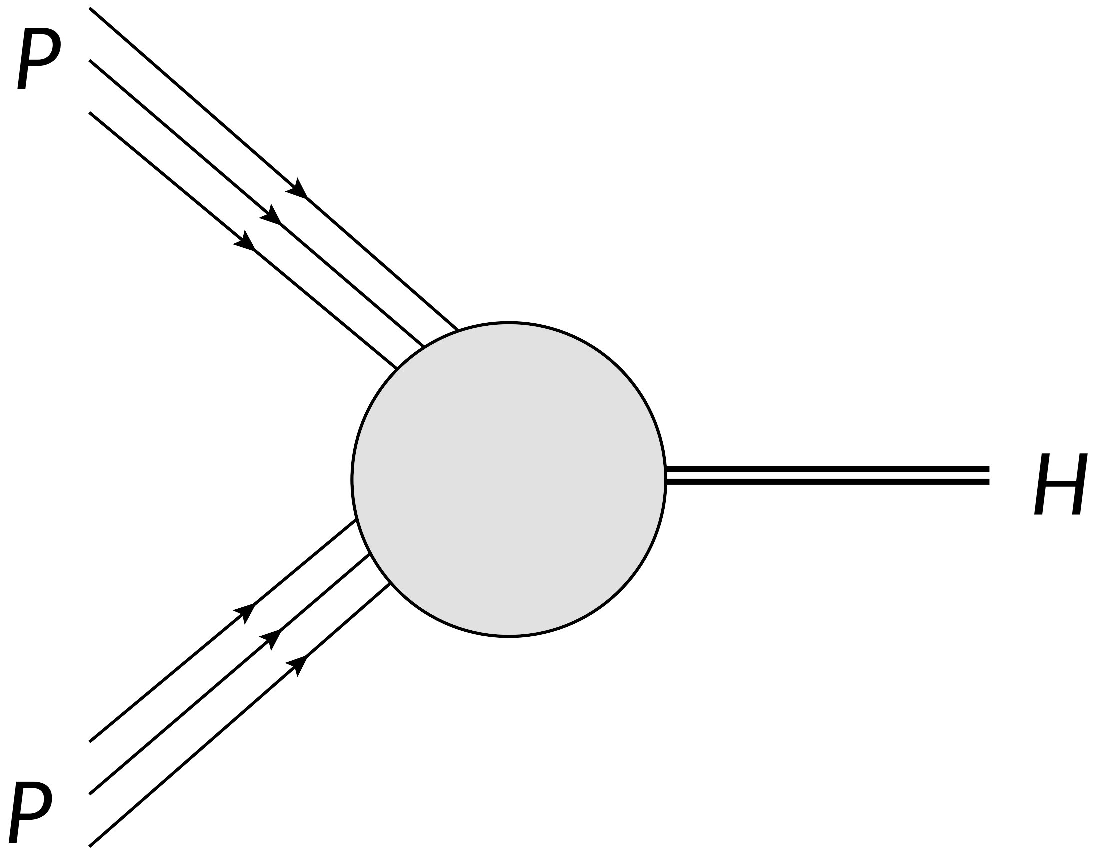
arXiv:2504.10574

SLAC NATIONAL ACCELERATOR LABORATORY

Stanford University

U.S. DEPARTMENT of ENERGY

How many Higgs bosons are produced at the LHC?



And how many did we expect…?

- i.e. what is the production rate?
- Signal strength:

$$\mu = \sigma_{\text{obs}}/\sigma_{\text{SM}}$$

Observed cross section

Standard Model calculation

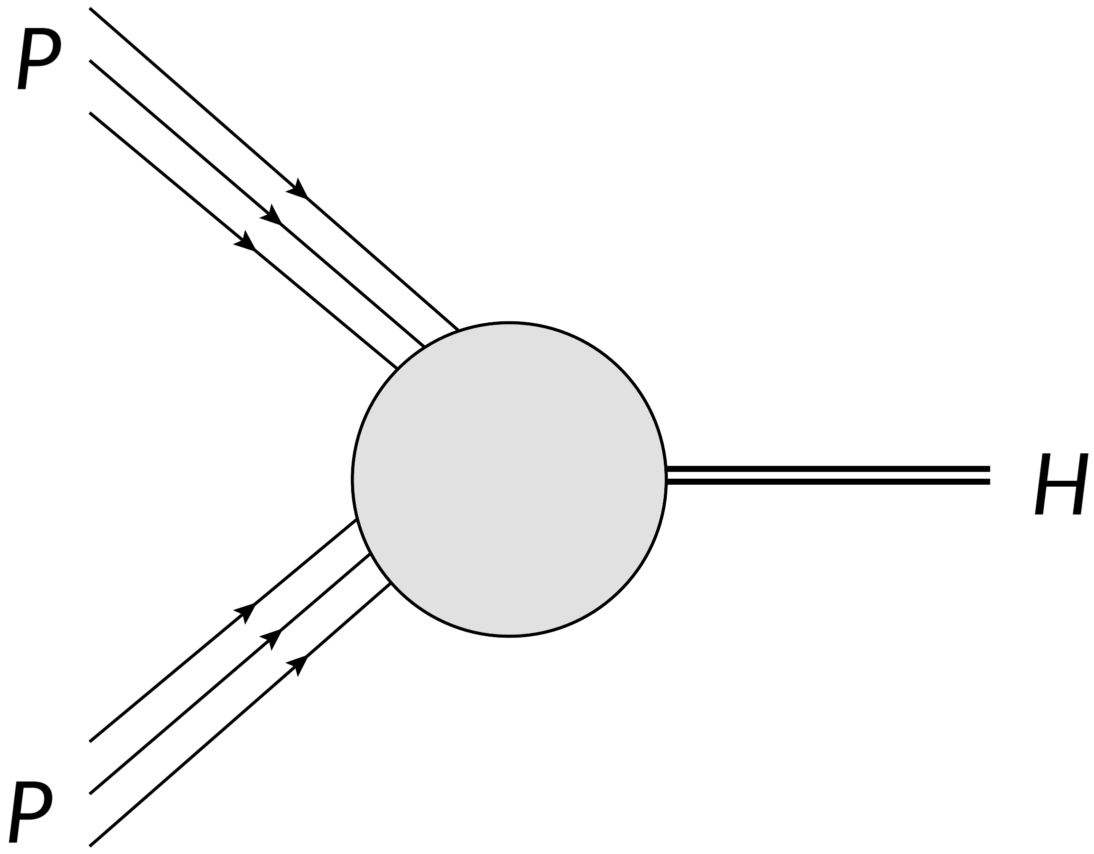
- ATLAS Run 2 (from *Nature* **607**, 52–59 (2022)):

$$\mu = 1.05 \pm 0.06$$

$$= 1.05 \pm \boxed{0.03(\text{stat.}) \pm 0.03(\text{exp.})}$$

$$\boxed{\pm 0.04(\text{sig. th.}) \pm 0.02(\text{bkg. th})}$$

How many Higgs bosons are produced at the LHC?



And how many did we expect...?

- i.e. what is the production rate?
- Signal strength:

$$\mu = \sigma_{\text{obs}}/\sigma_{\text{SM}}$$

Observed cross section → σ_{obs}

Standard Model calculation → σ_{SM}

- **ATLAS Run 2 Update** (from ATLAS-CONF-2025-006):

$$\begin{aligned}\mu &= 1.023^{+0.056}_{-0.053}\\ &= 1.023\boxed{\pm 0.028(\text{stat.})\ {}^{+0.026}_{-0.025}(\text{exp.})}\\ &\qquad\boxed{{}^{+0.039}_{-0.036}(\text{sig. th.}) \pm 0.012(\text{bkg. th})}\end{aligned}$$

In the foreseeable future:

Era	(Projected) Integrated Luminosity (fb^{-1})
Run 2	140
Run 3 (2026)	300+
HL-LHC (2040)	**3000+**

We will be collecting a lot more data!

- Along with improvements in experimental uncertainty, **much better statistics!**
- The LHC will deliver measurements at **the percent level!**
- We will need to get more precise on theory side…

Current Theoretical Uncertainty for Gluon-fusion Higgs

Contribution* of the QCD perturbative expansion at each order (at $13\ \mathrm{TeV}$):

*[Anastasiou, Duhr, Falko, Furlan, Gehrmann, Herzog, Lazopoulos, Mistlberger; arXiv:1602.00695]

$$
\begin{aligned}
\hat{\sigma} = \hat{\boldsymbol{\sigma}}_{\mathbf{LO}} \\
+\hat{\sigma}_{\mathrm{NLO}} \\
+\hat{\sigma}_{\mathrm{NNLO}} \\
+\hat{\sigma}_{\mathrm{N3LO}} \\
+\cdots
\end{aligned}
$$

Current Theoretical Uncertainty for Gluon-fusion Higgs

Correction* of the QCD perturbative expansion at each order (at 13 TeV):

*[Anastasiou, Duhr, Falko, Furlan, Gehrmann, Herzog, Lazopoulos, Mistlberger; arXiv:1602.00695]

$$\hat{\sigma} = \hat{\sigma}_{\mathrm{LO}} + \hat{\boldsymbol{\sigma}}_{\mathbf{NLO}}^{**} + \hat{\sigma}_{\mathrm{NNLO}} + \hat{\sigma}_{\mathrm{N3LO}} + \cdots$$

$$\boxed{\boldsymbol{O}(\mathbf{100\%})}$$

**[Dawson; Nucl.Phys.B 359 (1991), 283-300]

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$$\boxed{O(100\%)}$$

$$\boxed{\boldsymbol{O(25\%)}}$$

**[Dawson; Nucl.Phys.B 359 (1991), 283-300]
***[Harlander, Kilgore; arXiv:0201206], [Anastasiou, Melnikov; arXiv: 0207004]

Current Theoretical Uncertainty for Gluon-fusion Higgs

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$$\hat{\sigma} = \hat{\sigma}_{\text{LO}}$$
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$$+\hat{\sigma}_{\text{NNLO}}^{***}$$
$$+\boldsymbol{\hat{\sigma}_{\text{N3LO}}}^{*}$$
$$+\cdots$$

$$\boxed{O(100\%)}$$
$$\boxed{O(25\%)}$$
$$\boxed{O(5\%)}$$

**[Dawson; Nucl.Phys.B 359 (1991), 283-300]
***[Harlander, Kilgore; arXiv:0201206],
[Anastasiou, Melnikov; arXiv: 0207004]

Current Theoretical Uncertainty for Gluon-fusion Higgs

Uncertainty* due solely truncating the QCD perturbative expansion (at 13 TeV):

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$$\begin{aligned}\hat{\sigma} = \hat{\sigma}_{\mathrm{LO}} \\ +\hat{\sigma}_{\mathrm{NLO}} \\ +\hat{\sigma}_{\mathrm{NNLO}} \\ +\hat{\sigma}_{\mathrm{N3LO}} \\ +\xcancel{\cdots}\end{aligned}$$

- State of the art sufficient for current experimental needs
- But **N4LO** will take time to compute – **start now!**

$$\boxed{\sim 2.5\%}$$

Current Theoretical Uncertainty for Gluon-fusion Higgs

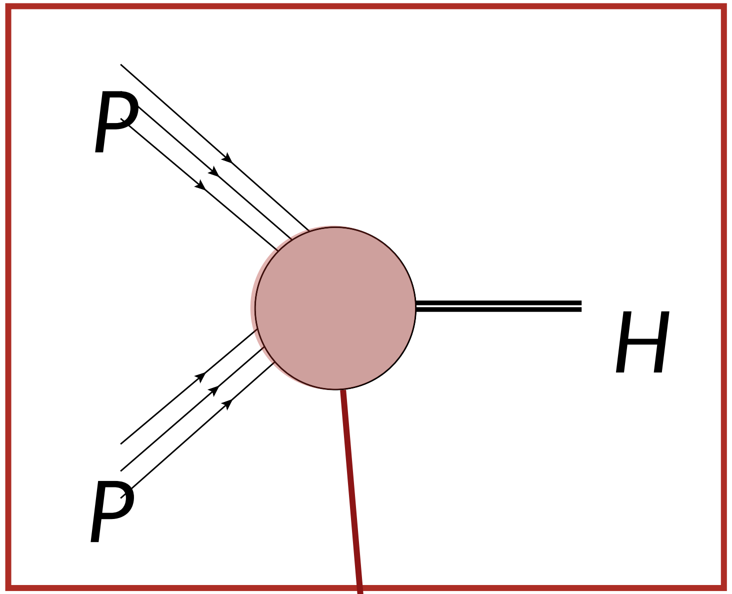
Uncertainty due solely truncating the QCD perturbative expansion?

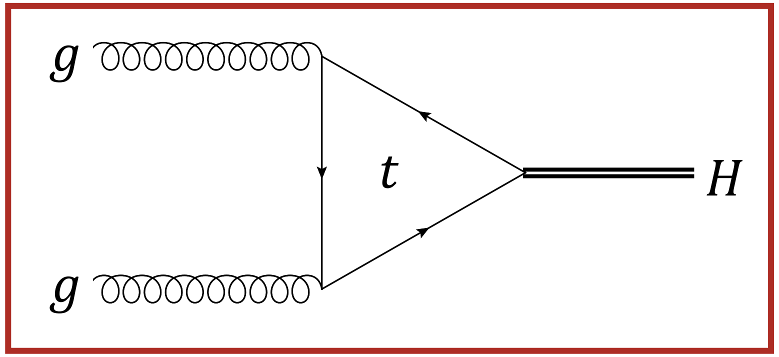
$$\begin{aligned}\hat{\sigma} = \hat{\sigma}_{\mathrm{LO}} \\ +\hat{\sigma}_{\mathrm{NLO}} \\ +\hat{\sigma}_{\mathrm{NNLO}} \\ +\hat{\sigma}_{\mathrm{N3LO}} \\ +\boldsymbol{\hat{\sigma}}_{\mathbf{N4LO}}\end{aligned}$$

- State of the art sufficient for current experimental needs
- But **N4LO** will take **years** to compute – **start now!**

$\boxed{\textbf{Aim for} \sim 1\%}$

How are Higgs bosons produced at the LHC?





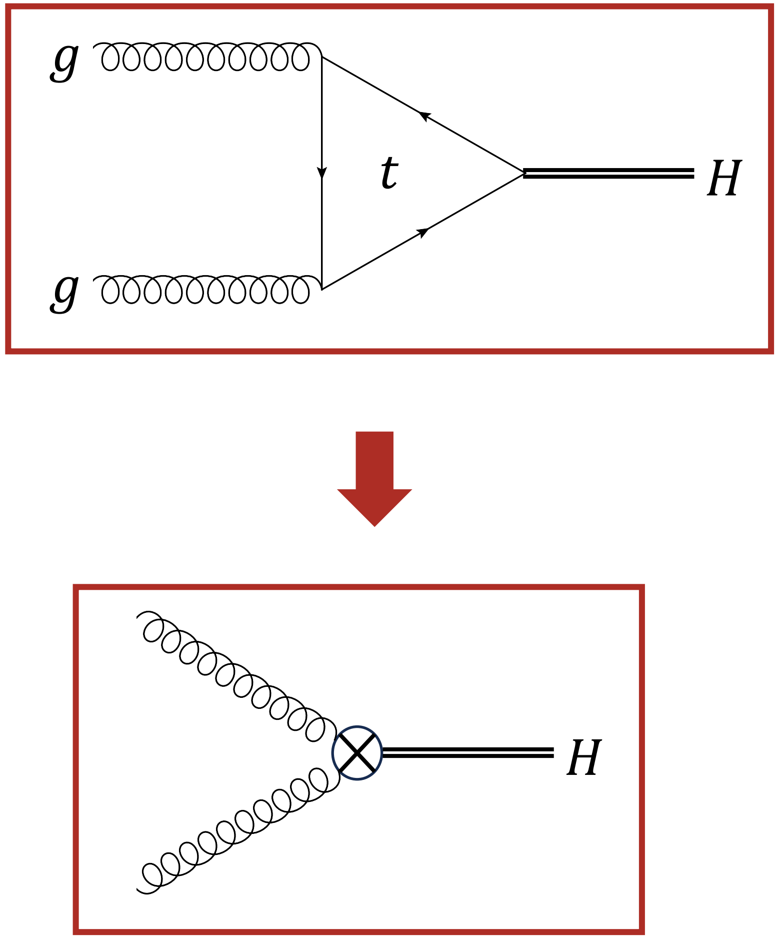
Hadronic process (long-range, nonperturbative)

Factorization model:

$$\sigma \sim \int dx\, dy\, f(x)\, f(y)\, \hat{\sigma}$$

Partonic process (short-range, perturbative!)

How are Higgs bosons produced at the LHC partonically?



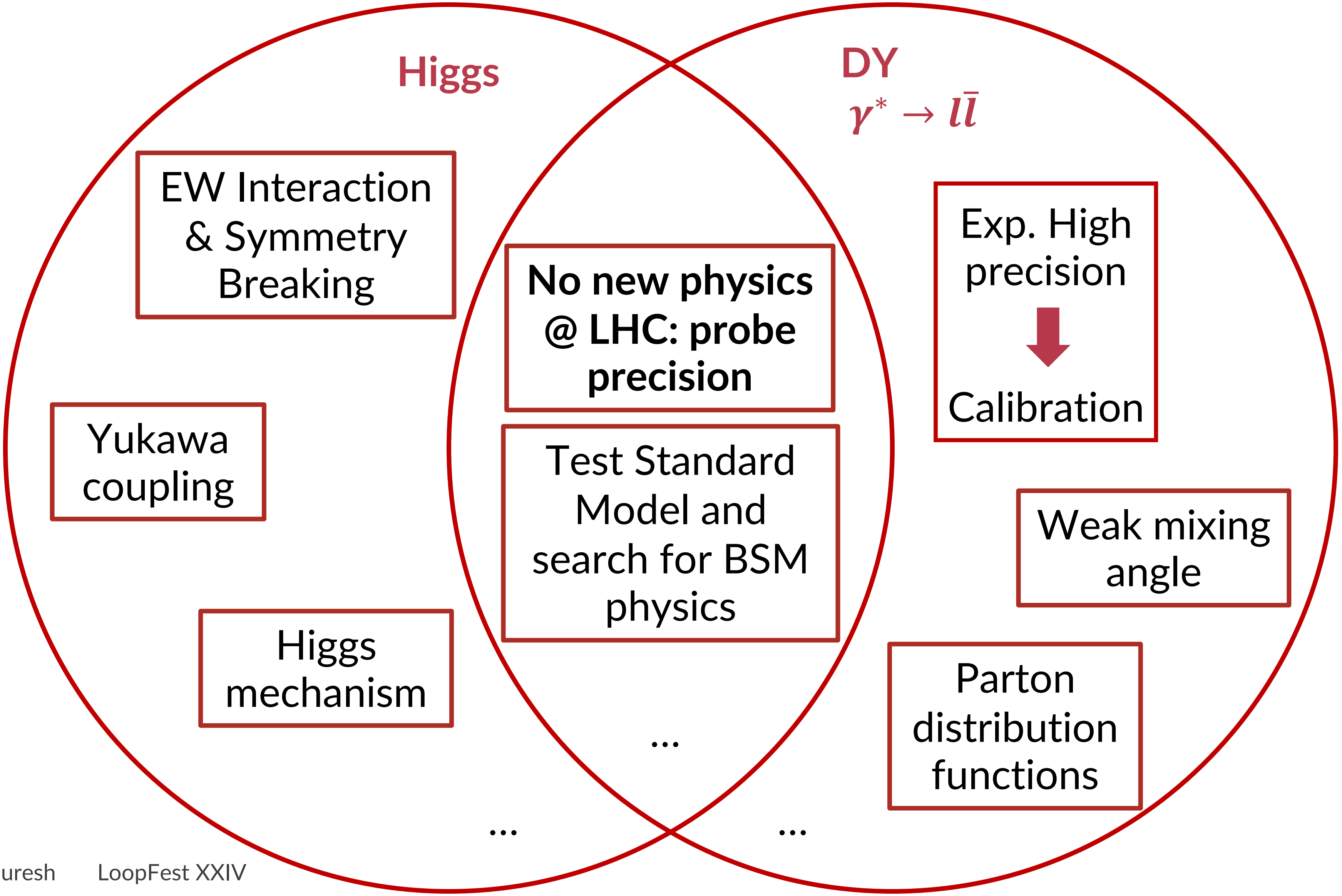
About 90% are produced via gluon fusion!

- $gg \to H$ via top quark loop
- We can simplify:

  take limit $m_{\text{top}} \to \infty$

  - i.e. leading term in m_H/m_{top} expansion
  - Higgs looks like it directly couples to gluons

Why probe Higgs and Drell-Yan Production?



Higgs

DY

$\gamma^* \to l\bar{l}$

EW Interaction & Symmetry Breaking

Yukawa coupling

Higgs mechanism

…

No new physics @ LHC: probe precision

Test Standard Model and search for BSM physics

…

Exp. High precision ⬇ Calibration

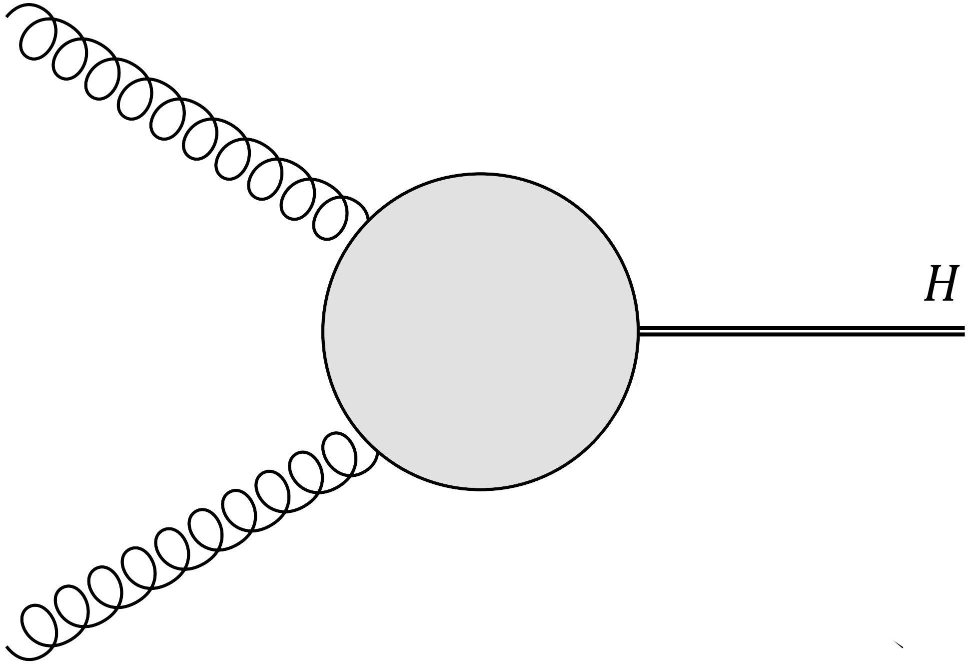
Weak mixing angle

Parton distribution functions

…

How do we compute how many Higgs bosons are produced?

To correctly compute an **inclusive** cross section:



- i.e. probability of Higgs produced

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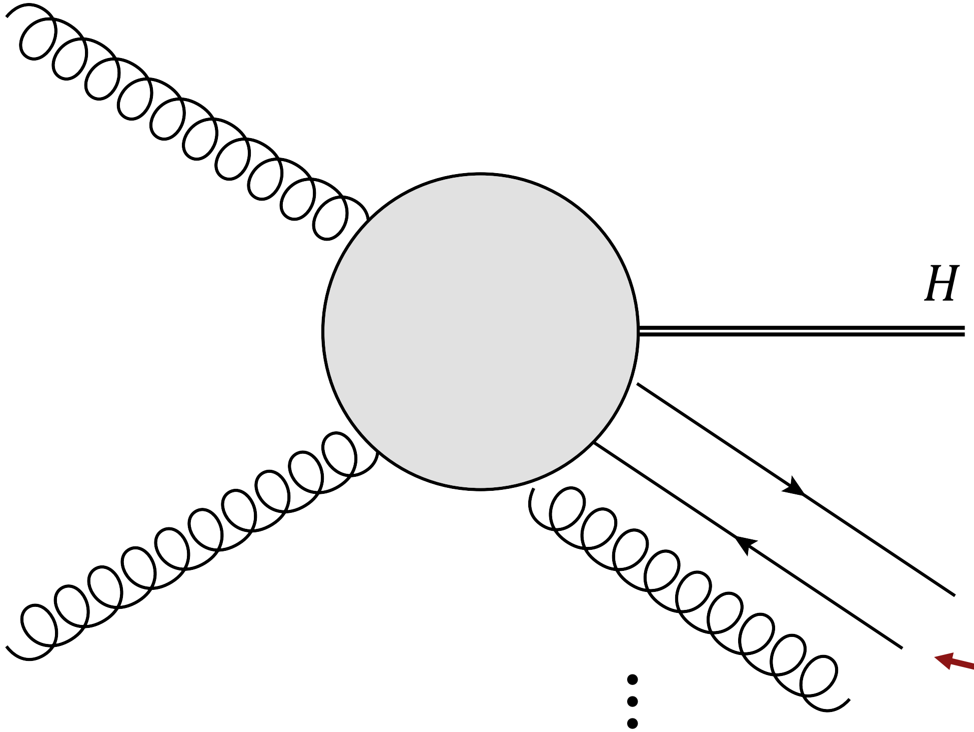
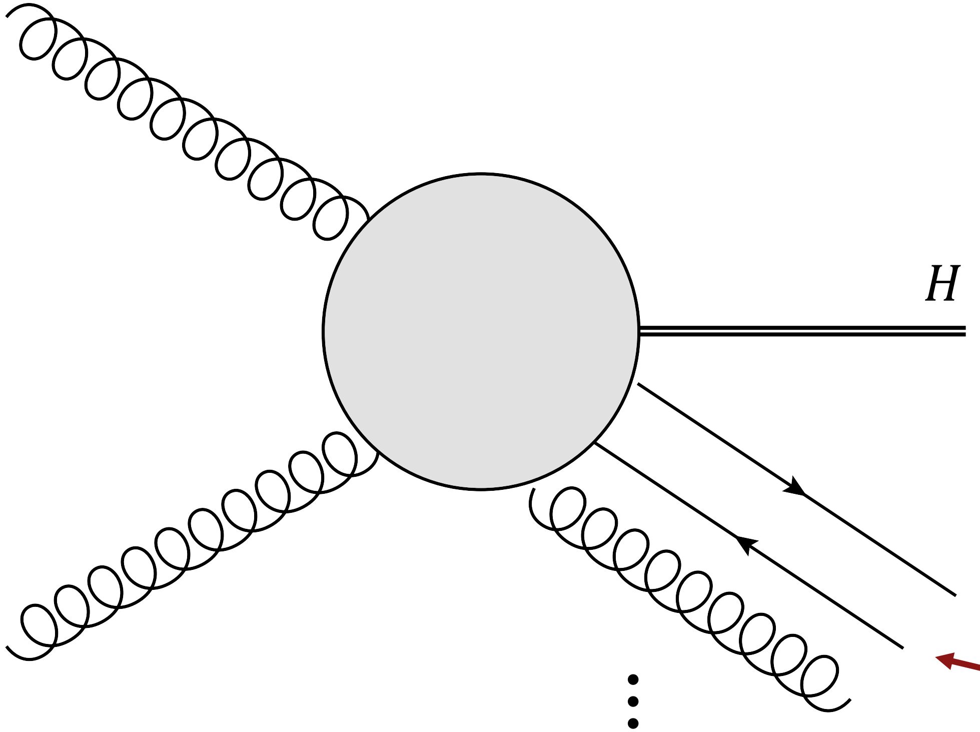


- i.e. probability of Higgs produced
- KLN theorem: Perturbative corrections must include:
  - loops &
  - **Radiation**

How do we compute how many Higgs bosons are produced?

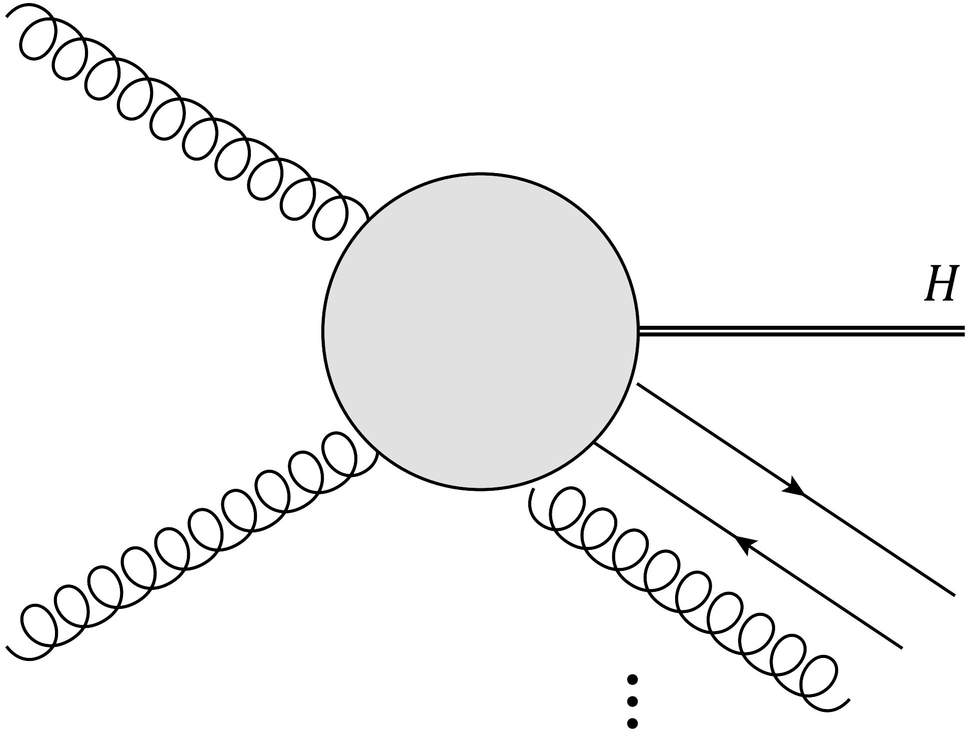
To correctly compute an **inclusive** cross section:



- i.e. probability of ***Higgs + X*** *produced*
- KLN theorem: Perturbative corrections must include:
  - loops &
  - **Radiation**

Building blocks at N^{4}LO:

To correctly compute an **inclusive** cross section:



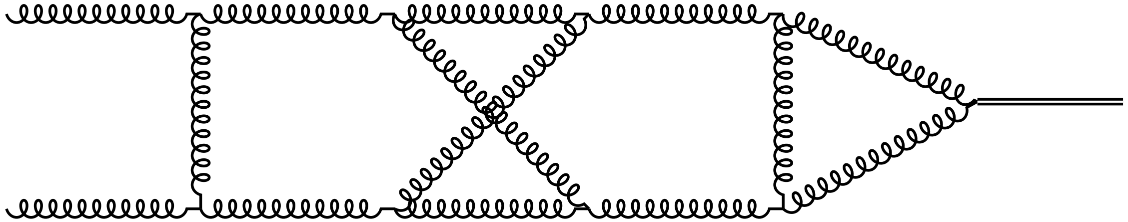
- Include up to 4 loops and 4 additional partons: $l + n = 4$
- Each piece is well-defined, gauge invariant
- Natural way to partition the computation

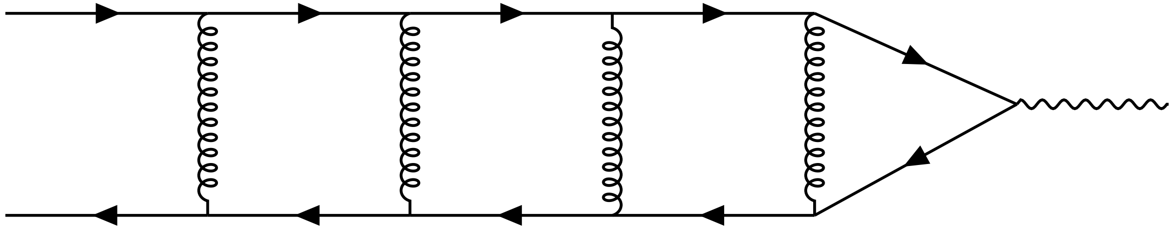
Fully virtual correction at N^{4}LO

Four loop amplitudes, “VVVV” contribution:

- Higgs and Drell-Yan first known approximately (2020) *
- Full computation (2022) **

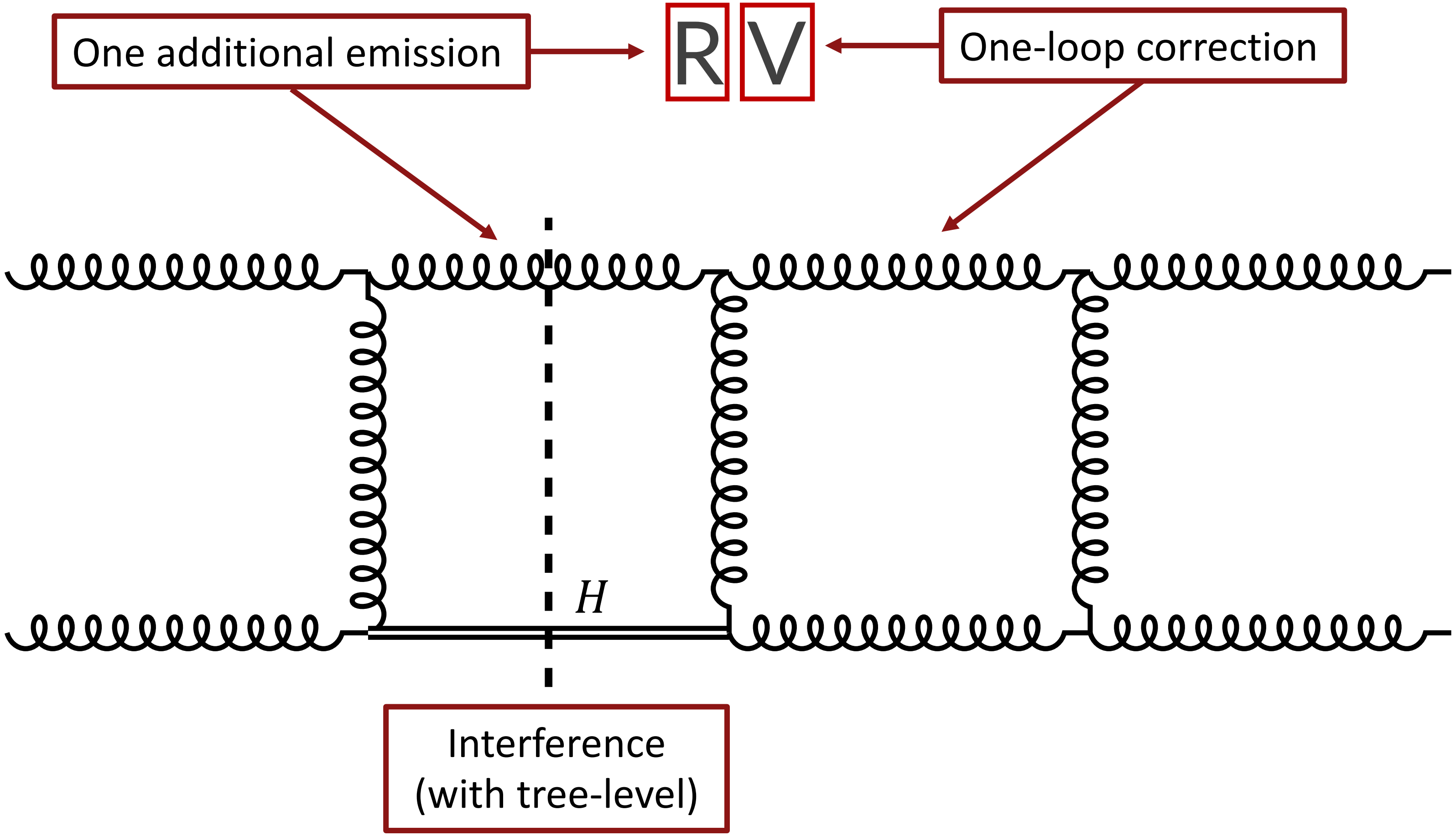
*[Das, Moch, Vogt; arXiv: 2004.00563]
**[Lee, von Manteuffel, Schabinger, Smirnov, Smirnov, Steinhauser; arXiv: 2202.04660]





What are the single-real ingredients? At NNLO:

First, let's look at "RV", NNLO correction for $gg \to H + X$.



One additional emission

RV

One-loop correction

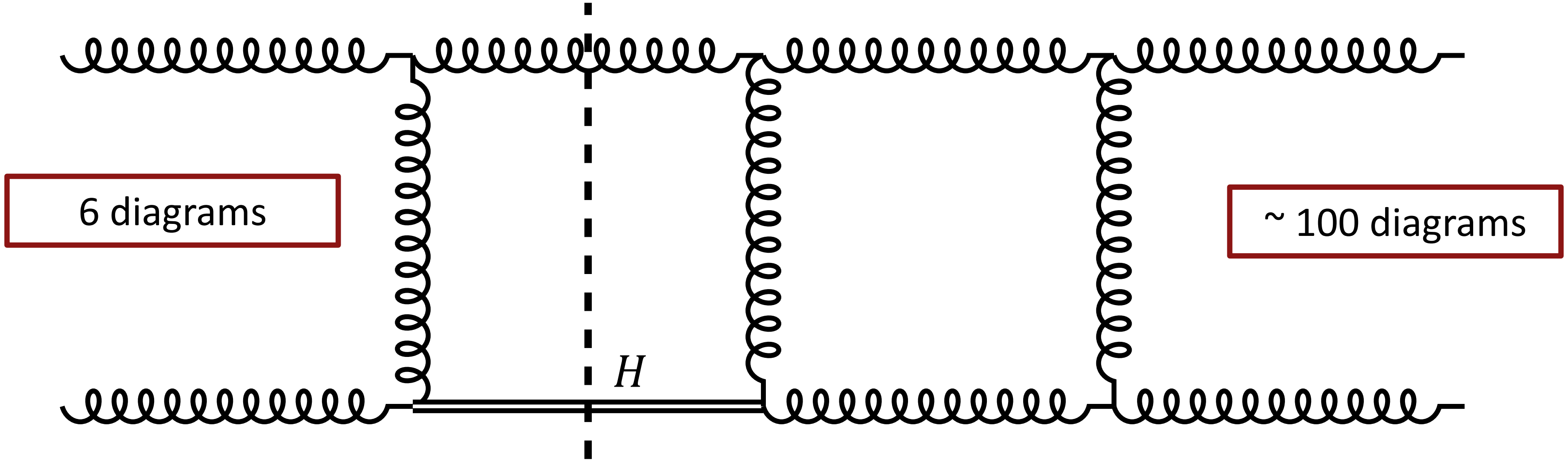
H

Interference
(with tree-level)

What are the single-real ingredients? At NNLO:

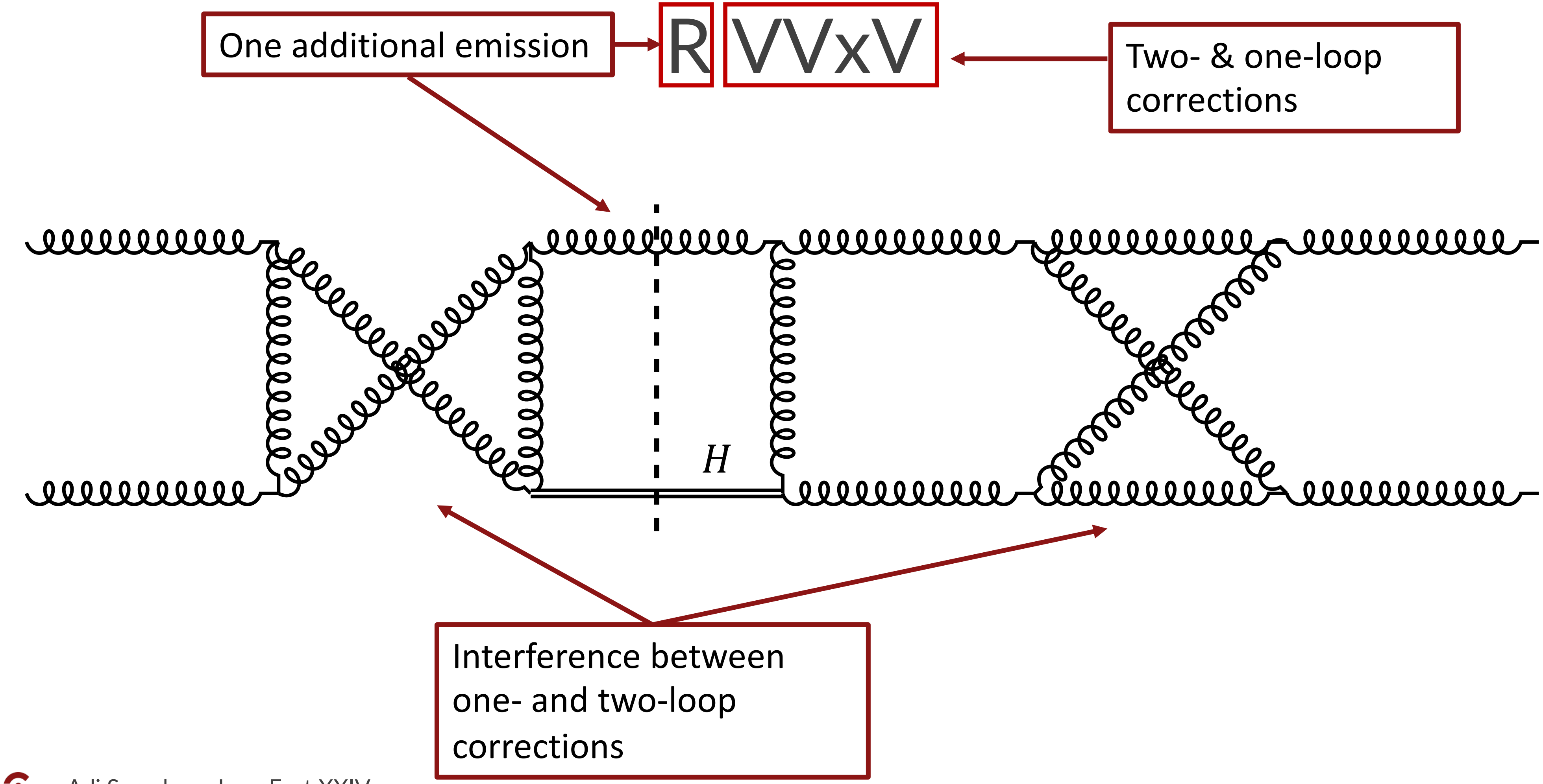
First, let’s look at ”RV”, an NNLO correction for $gg \to H + X$.

RV



What are the single-real ingredients? At N^{4}LO:

Now RVVxV: ”single-real, double-virtual cross single-virtual”.



One additional emission

R VVxV

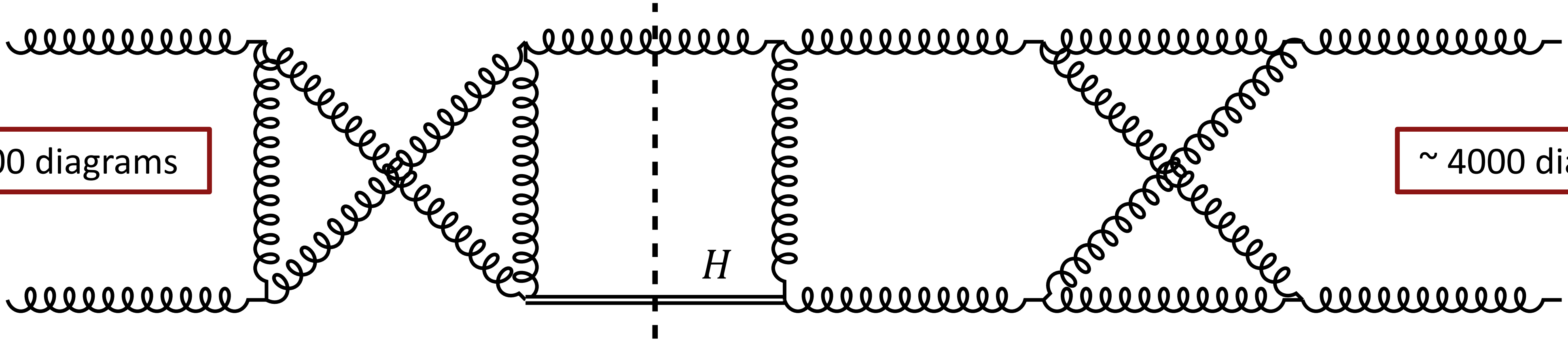
Two- & one-loop corrections

Interference between one- and two-loop corrections

What are the single-real ingredients? At N^{4}LO:

Now RVVxV: ”single-real, double-virtual cross single-virtual”.

RVVxV



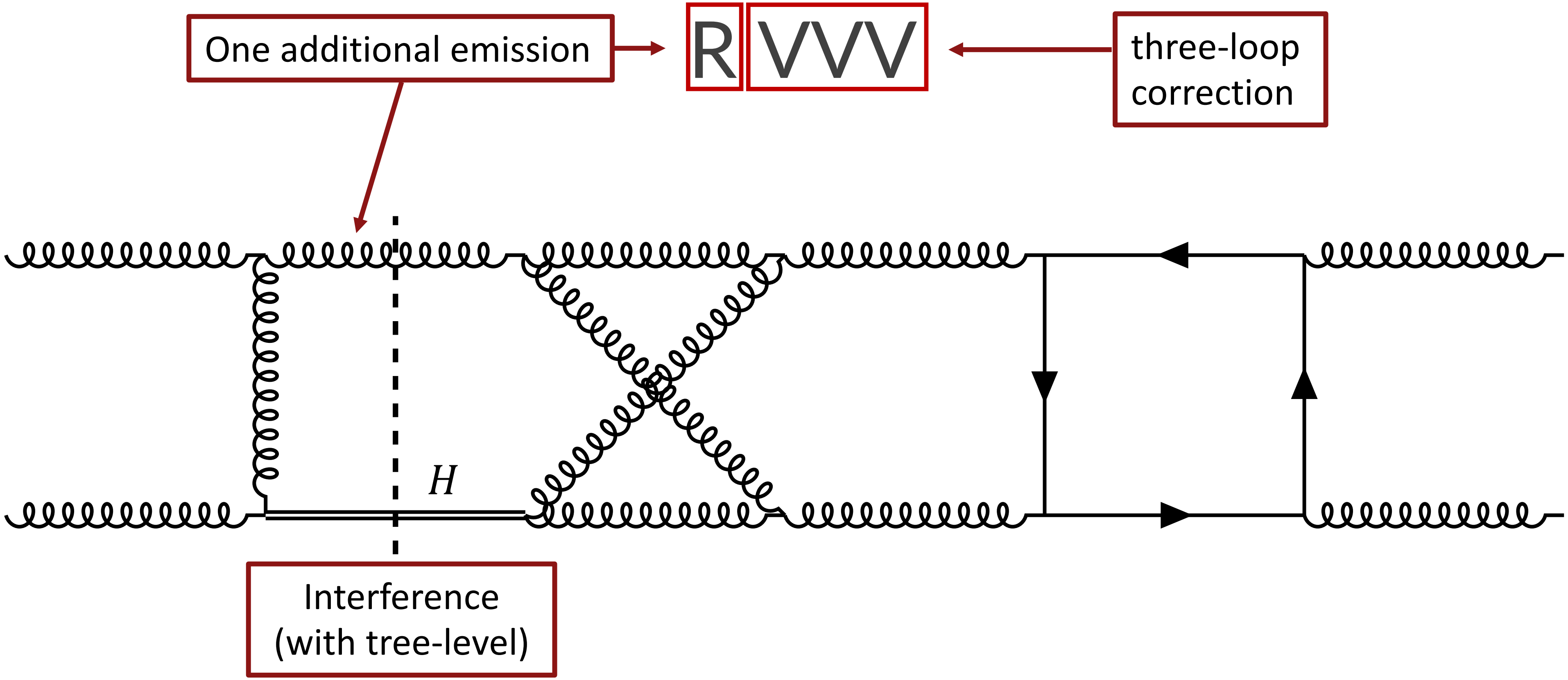
~ 100 diagrams

~ 4000 diagrams

Complexity grows fast!

What are the single-real ingredients? At N^{4}LO:

Now RVVV: ”single-real, triple-virtual”.


One additional emission

R VVV

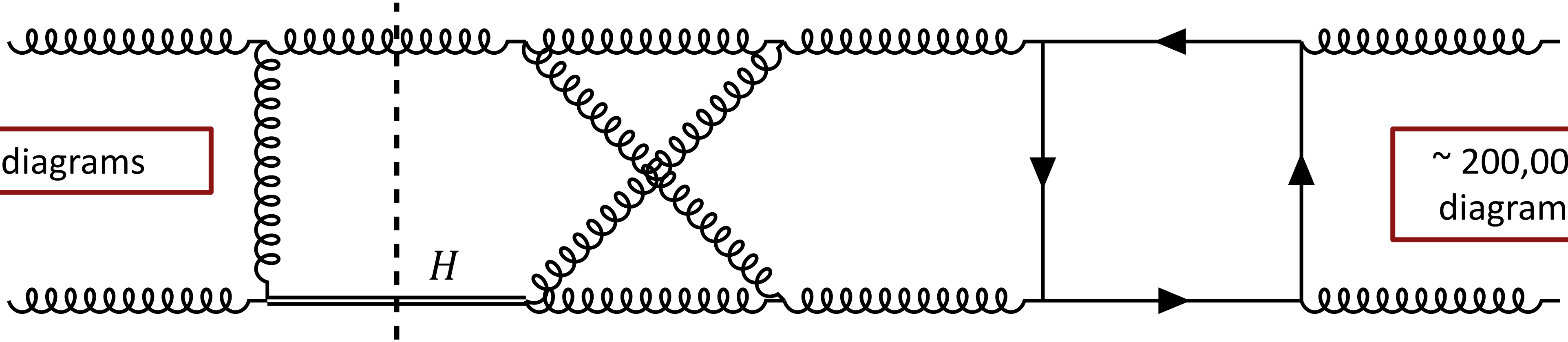
three-loop correction

Interference (with tree-level)

What are the single-real ingredients? At N^{4}LO:

Now RVVxV: ”single-real, double-virtual cross single-virtual”.

RVVV

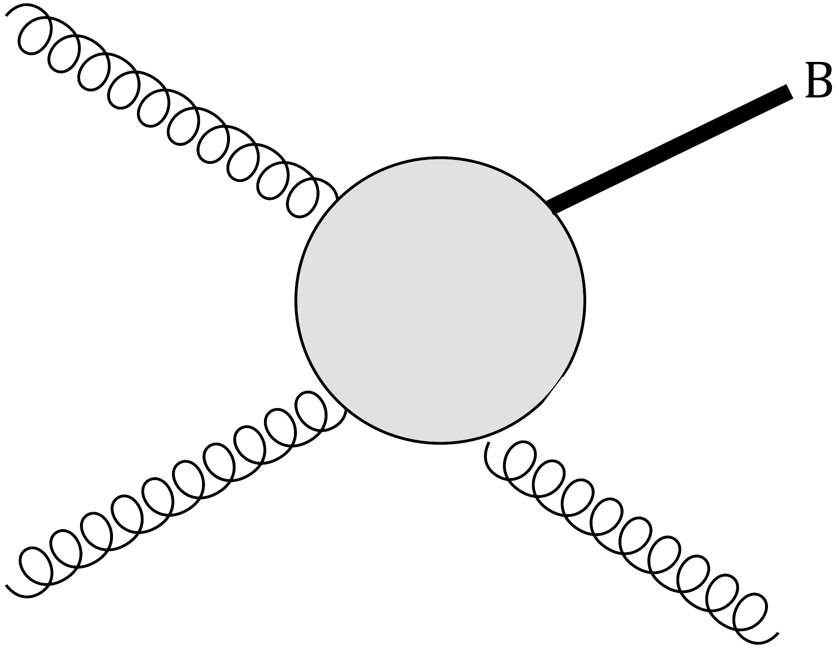


6 diagrams

~ 200,000 diagrams

The single-real ingredient as a probe

Only one additional emission is “easy”:

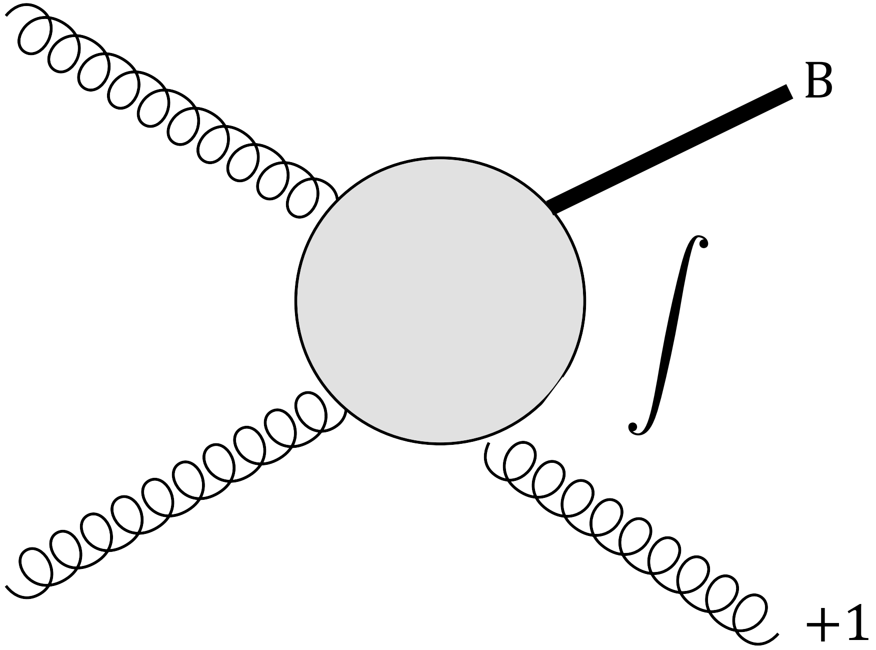


- Simplest (non-trivial) phase space
  - Can do **direct** phase space integration

Phase Space Integration

We only care about Higgs / DY production → Integrate over d.o.f. of final state

$$\hat{\sigma} \sim \boxed{\int d\,\Phi_{B+1}} A^{(1)} A^{*\,(2)}$$



Momentum conservation & rotational symmetry

⬇

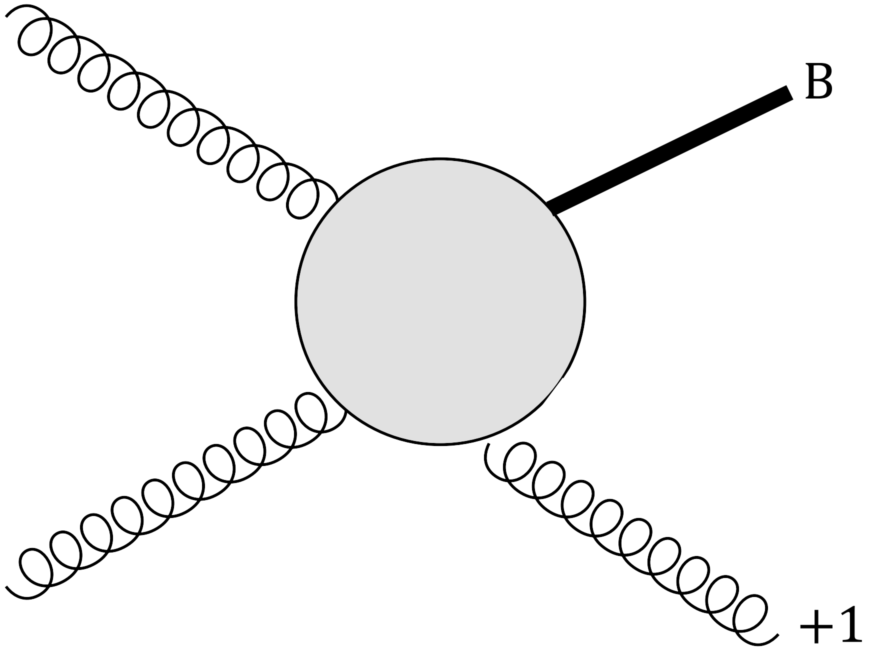
Only **one** actual d.o.f.:

Angle w.r.t. beamline:

$$\boldsymbol{\lambda} \in [\mathbf{0}, \mathbf{1}]$$

The single-real ingredient as a probe

Only one additional emission is "easy":

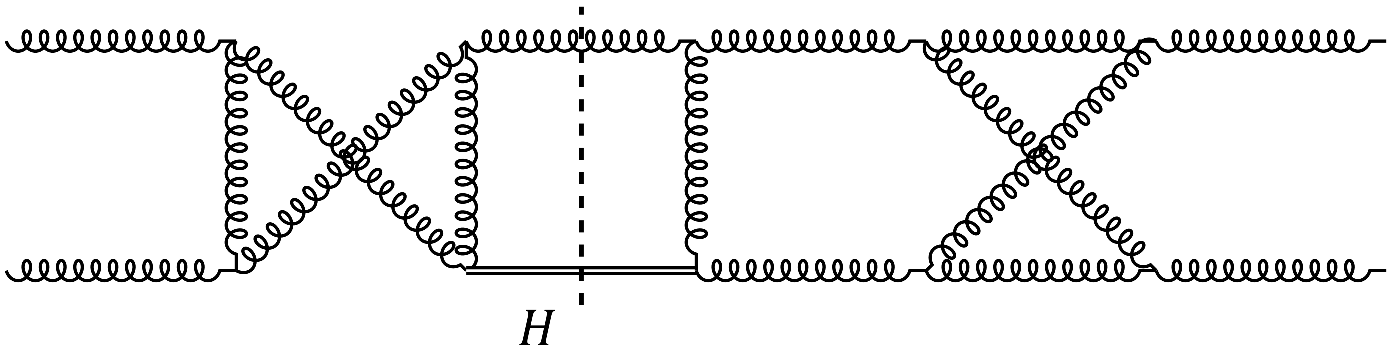


- Simplest (non-trivial) phase space
  - Can do **direct** phase space integration
  - Higher multiplicity involves the reverse unitarity approach (more loops)
- Probe complexity at N^{4}LO:
  - Function space
  - Color
  - Computational tools

Production Channels

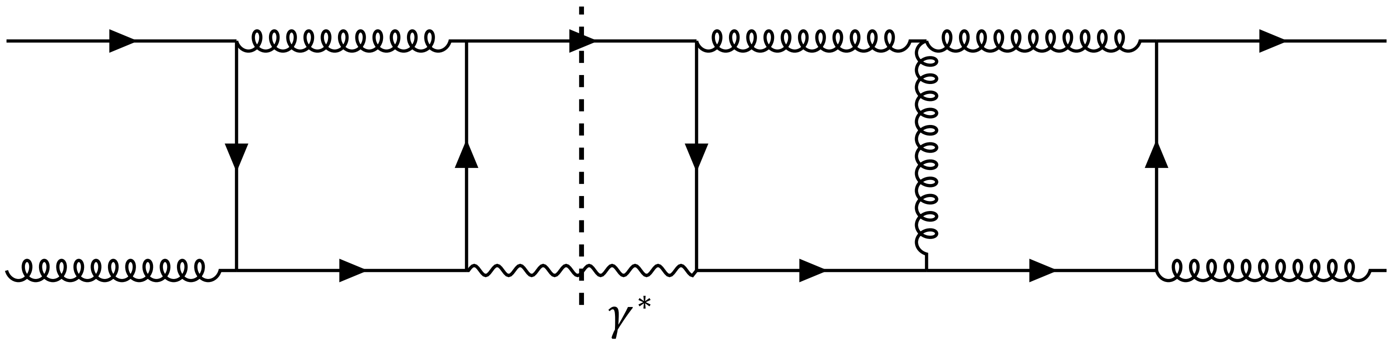
Higgs Production

- $g + g \to H + g$
- $q + \bar{q} \to H + g$
- $q + g \to H + q$



Drell-Yan Production

- $g + g \to \gamma^* + g$
  - Starts at 1-loop order
    - Only RVVxV
- $q + \bar{q} \to \gamma^* + g$
- $q + g \to \gamma^* + q$



RVVxV Contribution

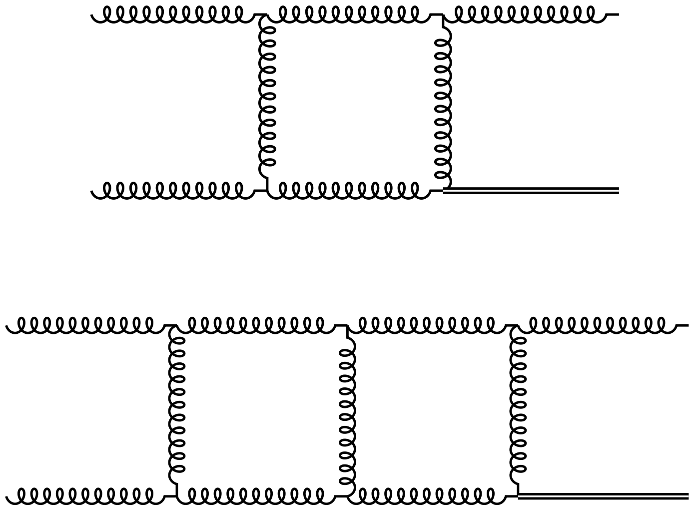
(Re)computing one- and two**-loop amplitudes

Generate all Feynman diagrams

Project onto gauge-invariant Lorentz basis

All particle live in $\mathrm{d} = 4 - 2\epsilon$ dimensions (conventional dimensional regularization)

Project onto basis of color tensors



Compute Loop integrals

- Use IBPs to obtain basis of “master integrals”
- Evaluate with method of differential equations
- In terms of generalized polylogarithms $G(\dots)$ up to transcendental weight eight

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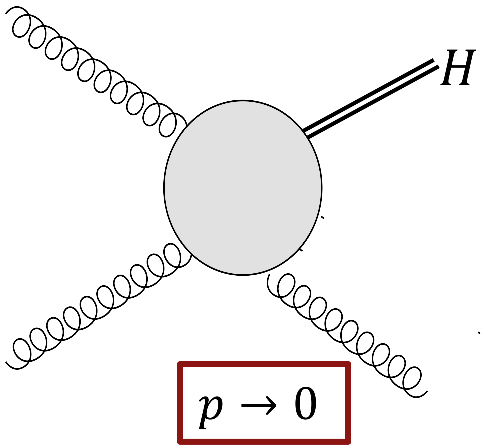
** [Gehrmann, Jakubčík, Mella, Syrrakos, Tancredi; arXiv:2301.10849]

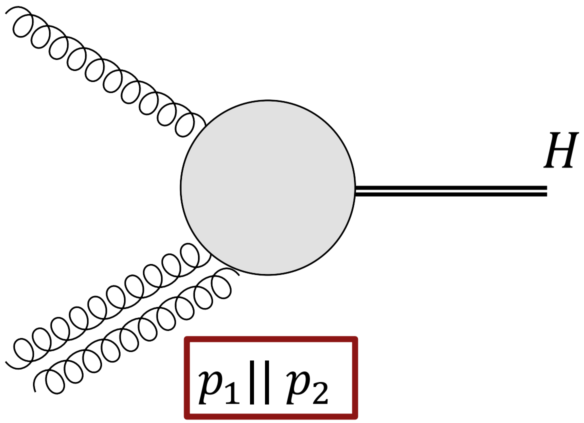
[Gehrmann, Jakubčík, Mella, Syrrakos, Tancredi; arXiv:2306.10170]

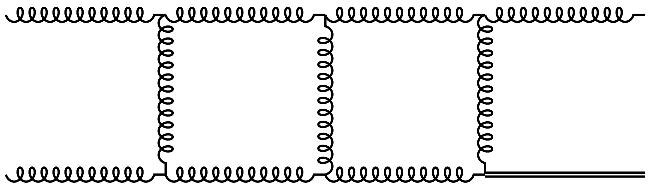
Checks on Amplitudes

Amplitudes factorize in:

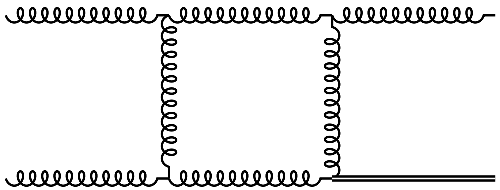
- Soft ($p \to 0$) limits
- Collinear ($p_1 || p_2$) limits
- Poles in ϵ ~ lower loop amplitudes







$$\sim \frac{1}{\epsilon^n}$$



(Re)computing one- and two**-loop amplitudes

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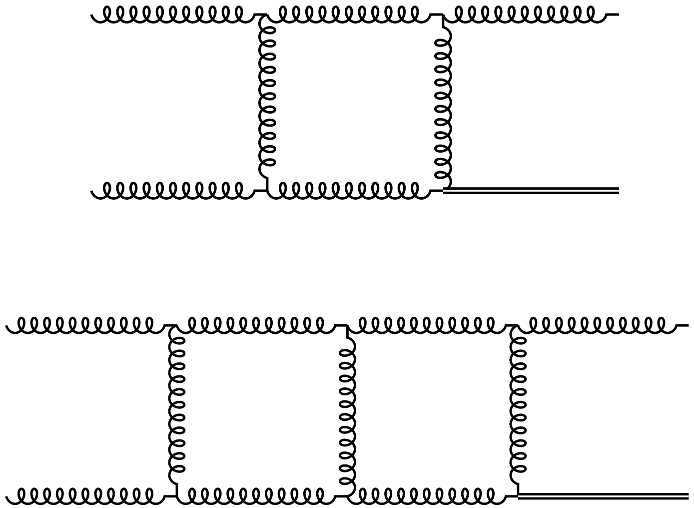
Project onto basis of color tensors

Debut Mathematica color algebra package: CIFAR



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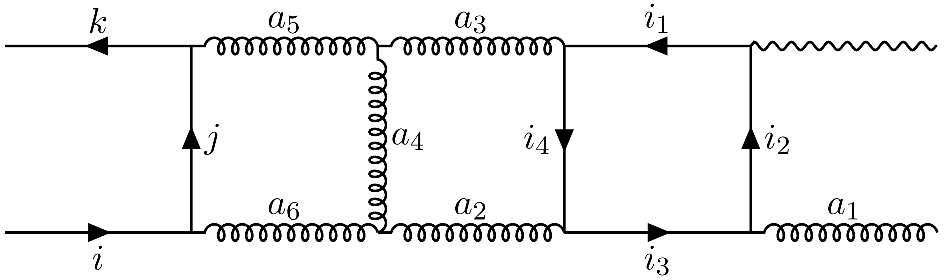
**[Gehrmann, Jakubčík, Mella, Syrrakos, Tancredi; arXiv:2301.10849]

[Gehrmann, Jakubčík, Mella, Syrrakos, Tancredi; arXiv:2306.10170]

Computing Color Factors in Gauge Theories

QCD gauge group → “color” factors in amplitude

- Gauge group is $\mathrm{SU}(n_c)$, with n_c number of colors
  - Factors reduce to polynomials in n_c
- Can instead consider **generalized gauge group**
  - Factors reduce to color group (“Casimir”) invariants C



Proj. $t^{a_1}_{ik}$

$$\delta_{i_1 i_2} t^{a_1}_{i_2 i_3} t^{a_2}_{i_3 i_4} t^{a_3}_{i_4 i_1} t^{a_1}_{ik} t^{a_5}_{kj} t^{a_6}_{ji} f^{a_3 a_4 a_5} f^{a_2 a_6 a_4}$$

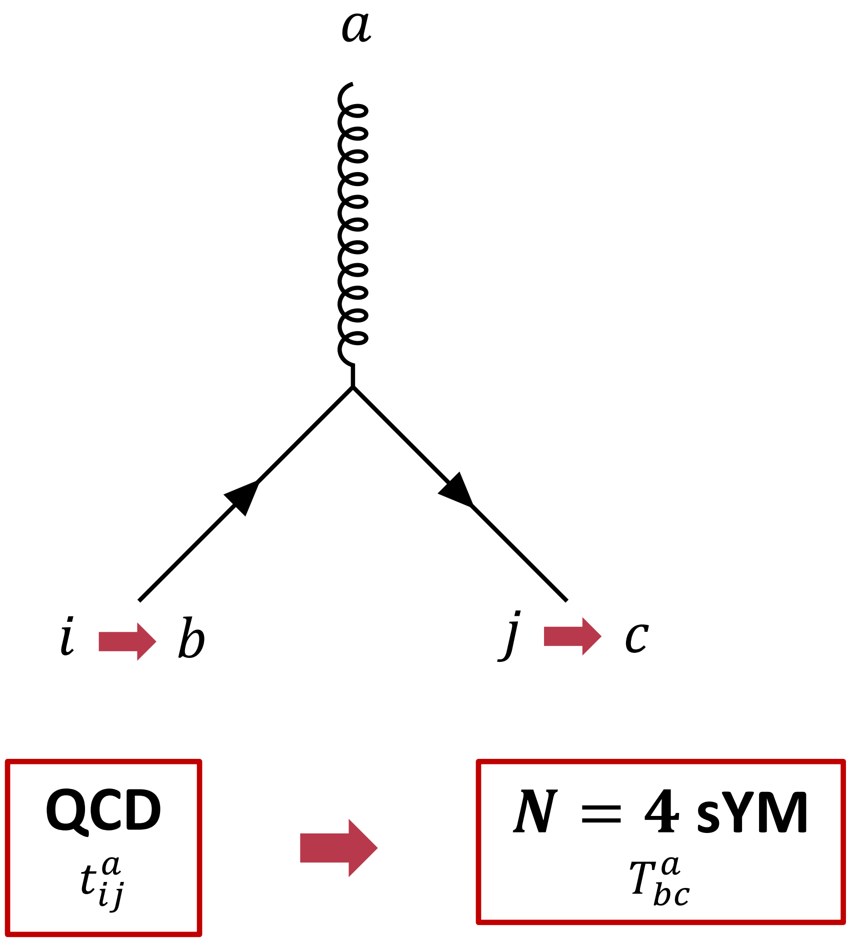
$$-\frac{1}{2} C_A C_3^{FF} - \frac{1}{8} T_F^2 D_A C_A^2$$

$$-\frac{1}{8} + \frac{3}{16} n_c^2 - \frac{1}{16} n_c^4$$

Computing Color Factors in Gauge Theories

QCD gauge group → “color” factors in amplitude

- Ubiquitous step in amplitude calculations
- Generalized gauge group → allows for comparison across theories
  - e.g. Set $C_F = C_A$ for $N = 4$ sYM (“Principle of maximal transcendentality”)
- Need an efficient and reliable computational pipeline



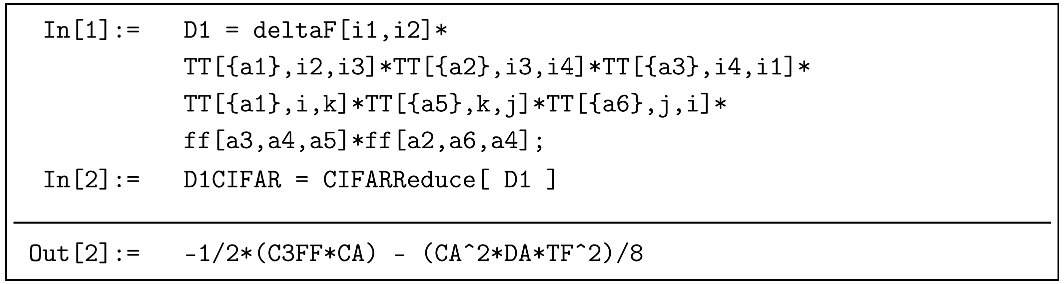
QCD
t_{ij}^{a}

$N = 4$ sYM
T_{bc}^{a}

CIFAR: A Color Algebra Package

QCD gauge group → “color” factors in amplitude

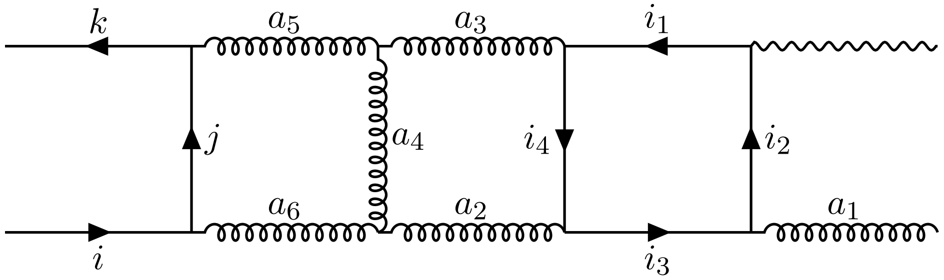
- CIFAR can do this computation for us!



```
In[1]:=    D1 = deltaF[i1,i2]*
           TT[{a1},i2,i3]*TT[{a2},i3,i4]*TT[{a3},i4,i1]*
           TT[{a1},i,k]*TT[{a5},k,j]*TT[{a6},j,i]*
           ff[a3,a4,a5]*ff[a2,a6,a4];
In[2]:=    D1CIFAR = CIFARReduce[ D1 ]

Out[2]:=   -1/2*(C3FF*CA) - (CA^2*DA*TF^2)/8
```

- Quartic Casimirs: Ready for N^{4}LO
- Available at: github.com/adisurtya/CIFAR



Proj. $t^{a_1}_{ik}$

$$\delta_{i_1 i_2} t^{a_1}_{i_2 i_3} t^{a_2}_{i_3 i_4} t^{a_3}_{i_4 i_1} t^{a_1}_{ik} t^{a_5}_{kj} t^{a_6}_{ji} f^{a_3 a_4 a_5} f^{a_2 a_6 a_4}$$

$$-\frac{1}{2} C_A C_3^{FF} - \frac{1}{8} T_F^2 D_A C_A^2$$

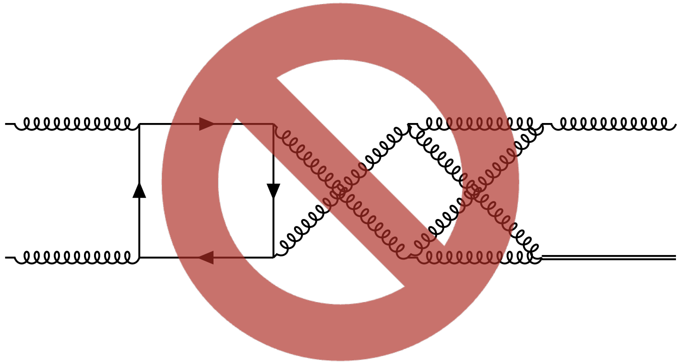
$$-\frac{1}{8} + \frac{3}{16} n_c^2 - \frac{1}{16} n_c^4$$

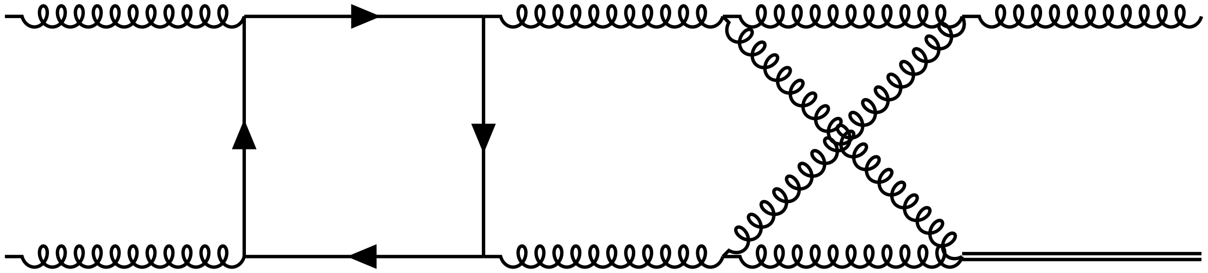
RVVV Contribution

Three-loop amplitudes

State-of-the-art: Leading color amplitudes

- Only leading power in $n_C{}^{\alpha} n_f{}^{\beta}$
- Drell-Yan known*
- Higgs recently computed**





e.g. $\{n_C^6, n_C^5 n_f, n_C^4 n_f^2, n_C^3 n_f^3\}$ for $gg \to Hg$

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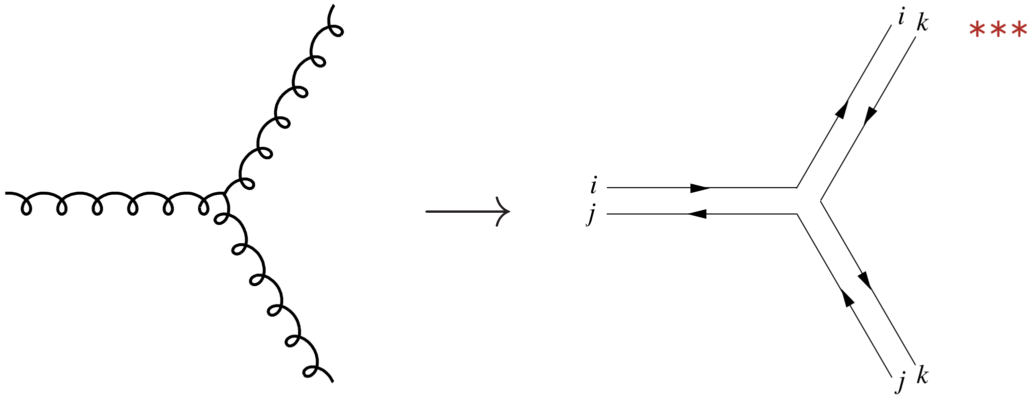
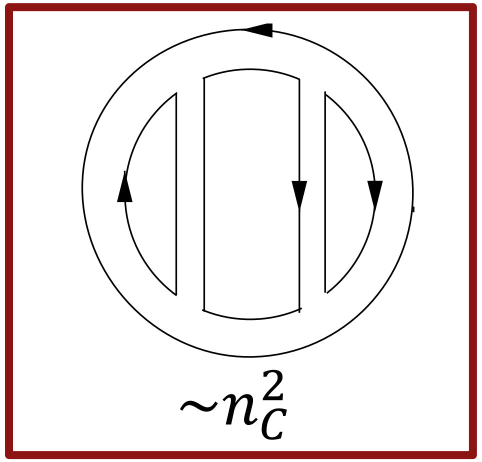
*[Gehrmann, Jakubčík, Mella, Syrrakos, Tancredi; arXiv:2307.15405]
**[Chen, Guan, Mistlberger; arXiv: 2504.06490]

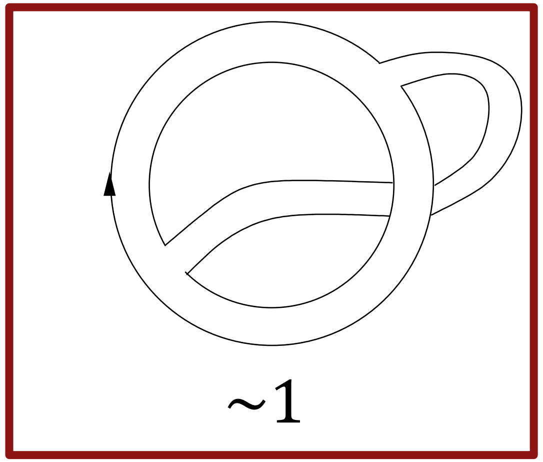
Generalized leading-color limit

Take $n_C \to \infty$

- Only planar color diagrams contribute
- Much easier to compute
  - E.g. Γ_{cusp} known to all orders in planar N=4 sYM* vs. N^4LO for non-planar **
- Subleading color suppressed by $\frac{1}{n_C^2} \sim \frac{1}{10}$.
  - For XS @ N^4LO, $\frac{1}{10}$ of 1%?







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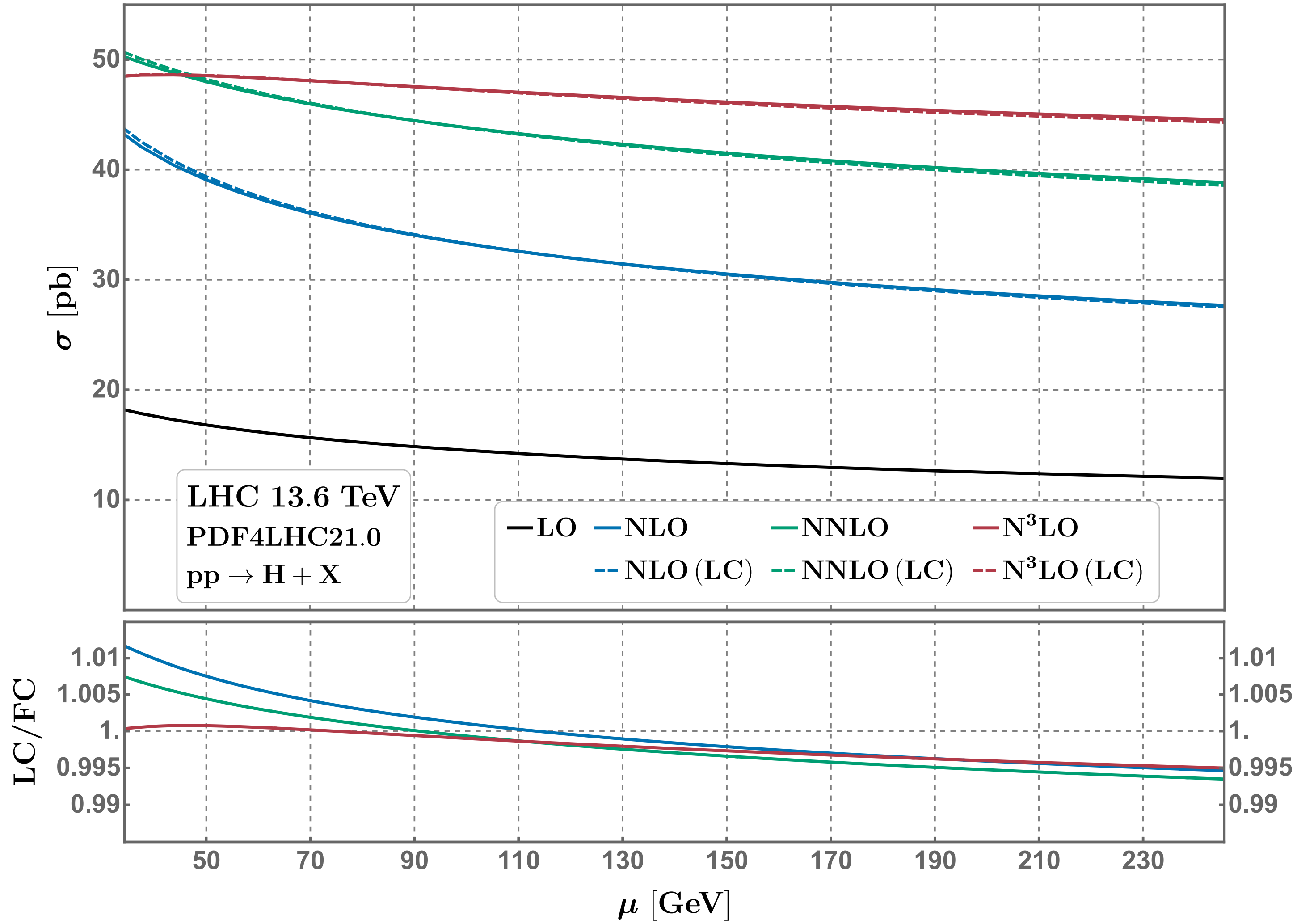
*[Beisert, Eden, Staudacher; arXiv:0610251]
**[Boels, Huber, Yang; arXiv:1705.03444]
***[davidtong.org/teaching/gauge-theory/]

Can we do **phenomenology** with only leading color?

Hadronic Cross Section up to N^{3}LO: Leading vs. Full Color



Leading color appears to capture the cross section at the <0.5% level!

Should hold for N^{4}LO.

RVVV Computation: Integration strategy for Higgs Channels

Problem: $\int_0^1 r(\lambda)G(a_1, \ldots, a_n, \lambda)\, d\lambda$

hard to evaluate when $a_i = \sqrt{\alpha + \beta\bar{z} + \gamma\bar{z}^2\,}\}$

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⬇

Partial Integration Idea: Evaluate $\int_0^1 r(\lambda) M_j(\lambda)\, d\lambda$!

Canonical Master Integral

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Partial Integration Idea: Evaluate $\int_0^1 r(\lambda) M_j(\lambda)\, d\lambda$!

$$= R\, M \Big|_0^1 - \int_0^1 R\, \partial_\lambda M_i \, d\lambda$$

$R(\lambda) = \int r(\lambda) d\lambda$

RVVV Computation: Integration strategy for Higgs Channels

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Know from differential equations!

$\partial_\lambda M_i = \epsilon A_{ij} M_j$

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↓

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$$= R\, M \,\Big|_0^1 - \int_0^1 R\, \partial_\lambda M_i \, d\lambda$$

↓

Next Iteration: $\epsilon \int_0^1 A_{ij} R(\lambda) M_j(\lambda)\, d\lambda$

RVVxV: After Performing Phase Space Integration

Results

- Laurant expansion in dimensional regulator, ϵ
- With $\bar{z} = 1 - \frac{m_B^2}{s}$, contains:
  - Rational functions
  - Multiple Polylogs: $G(\dots, \bar{z})$
  - Distributions: $\delta(\bar{z})$, $\left[\frac{\log^k \bar{z}}{\bar{z}}\right]_+$
    - (soft singularities)

Checks

- Recomputed all lower-order single-real (R…) interference contributions
- Soft-singular parts for:
  - $g + g \to H + g$
  - $q + \bar{q} \to \gamma^* + g$
  - Applied single-emission soft current to $g + g \to H$ and $q + \bar{q} \to \gamma^*$ to find agreement

RVVV (LC): After Performing Phase Space Integration

Results

- With $\bar{z} = 1 - \frac{m_B^2}{s}$, contains:
  - Everything in RVVxV **and**
  - More general class of iterated integrals:
    - New (not $d\log$) letters: $\frac{1}{\bar{z}\sqrt{1-\bar{z}}}, \frac{1}{\sqrt{3+\bar{z}}\sqrt{1-\bar{z}}}$
    - Entered at RRV and RRR for N^{3}LO [***]

Checks

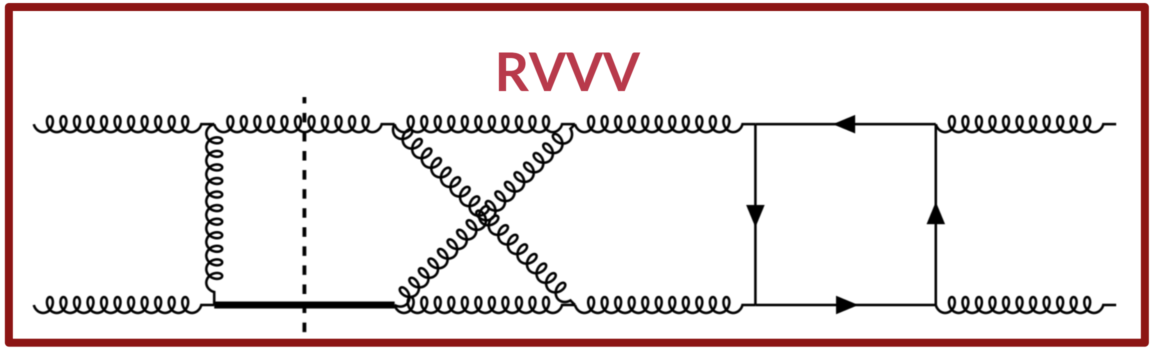
- Regulated IR singularities with known results:
  - Three-loop single-emission soft current[*]
  - Three-loop splitting amplitudes[**]
- Computed threshold expansion in $\bar{z}$ before and after phase-space integration

[*] [Herzog, Ma, Mistlberger, Suresh; arXiv:2309.07884]
[**] [Guan, Herzog, Ma, Mistlberger, Suresh; arXiv:2408.03019]
[***] [Mistlberger; arXiv:1802.00833]

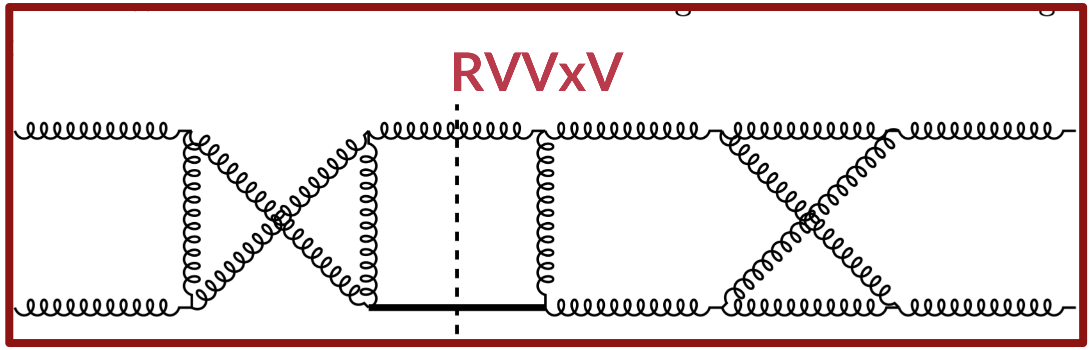
Conclusion and Next Steps

Unlock Full Phenomenological Potential of LHC & Higgs Physics: Need N^{4}LO Precision

- Two key steps in computing the production cross section at N^{4}LO:



RVVV



RVVxV

- Understanding complexity and building up technology (e.g. CIFAR) to tackle further N^{4}LO calculations
- How to compute other ingredients RRVV, RRV^2, …
  - Leading color? threshold expansion? …

Backup slides

Thank you!

Regulation of Soft Singularity

Expand soft singularity:

$$\left(\frac{1}{\bar{z}}\right)^{1+a\epsilon} = -\frac{1}{a\epsilon}\delta(\bar{z}) + \sum_{k=0}^{\infty}\frac{(-a\epsilon)^k}{k!}\left[\frac{\log^k \bar{z}}{\bar{z}}\right]_+ ,$$

Regulation of Soft Singularity

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Additional ϵ pole at soft limit $\bar{z} \to 0$

Regulation of Soft Singularity

Expand soft singularity:

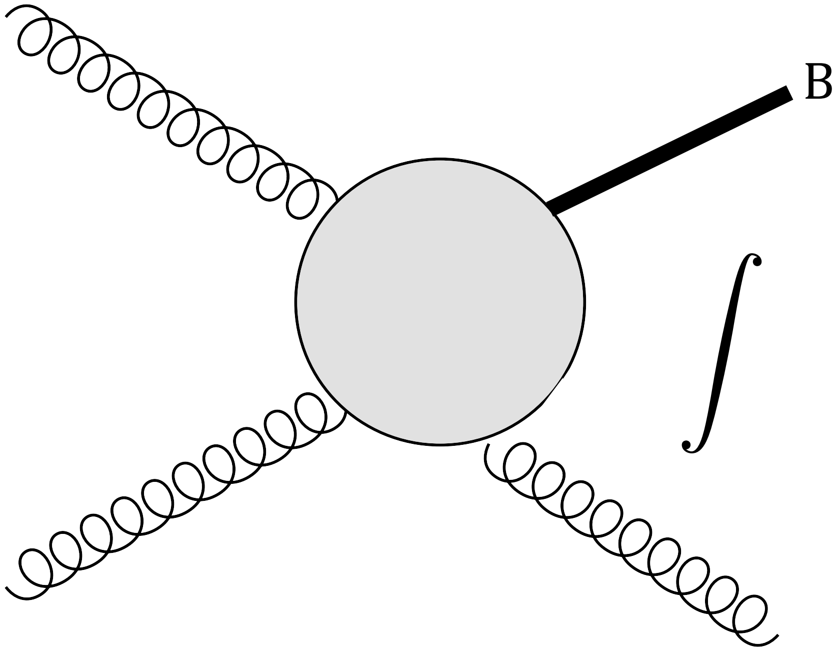
$$\left(\frac{1}{\bar{z}}\right)^{1+a\epsilon} = -\frac{1}{a\epsilon}\delta(\bar{z}) + \sum_{k=0}^{\infty}\frac{(-a\epsilon)^k}{k!}\left[\frac{\log^k \bar{z}}{\bar{z}}\right]_+ ,$$

Plus distribution:

$$\int_0^1 d\bar{z}\left[\frac{\log^k \bar{z}}{\bar{z}}\right]_+ f(\bar{z}) \equiv \int_0^1 \frac{\log^k \bar{z}}{\bar{z}}\big(f(\bar{z}) - f(0)\big).$$

Phase Space Integration

What do these integrals typically look like?

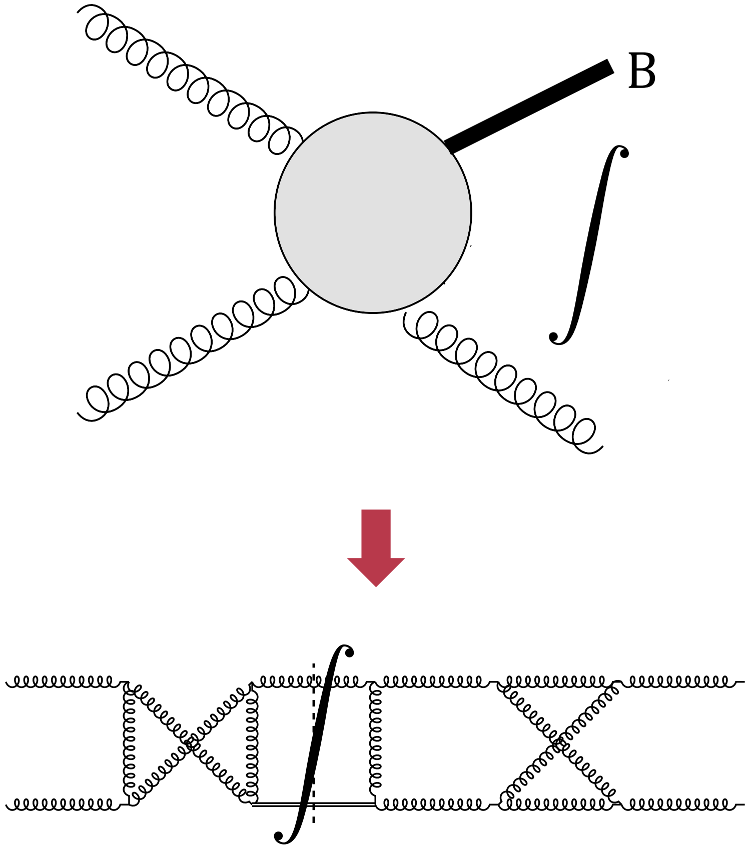


$$\hat{\sigma} \sim \int d\lambda \, \frac{\log\lambda}{\lambda - 1/\bar{z}} = \log\lambda \log\left(1 - \boxed{\bar{z}}\lambda\right) + \mathrm{Li}_2(\bar{z}\lambda)$$

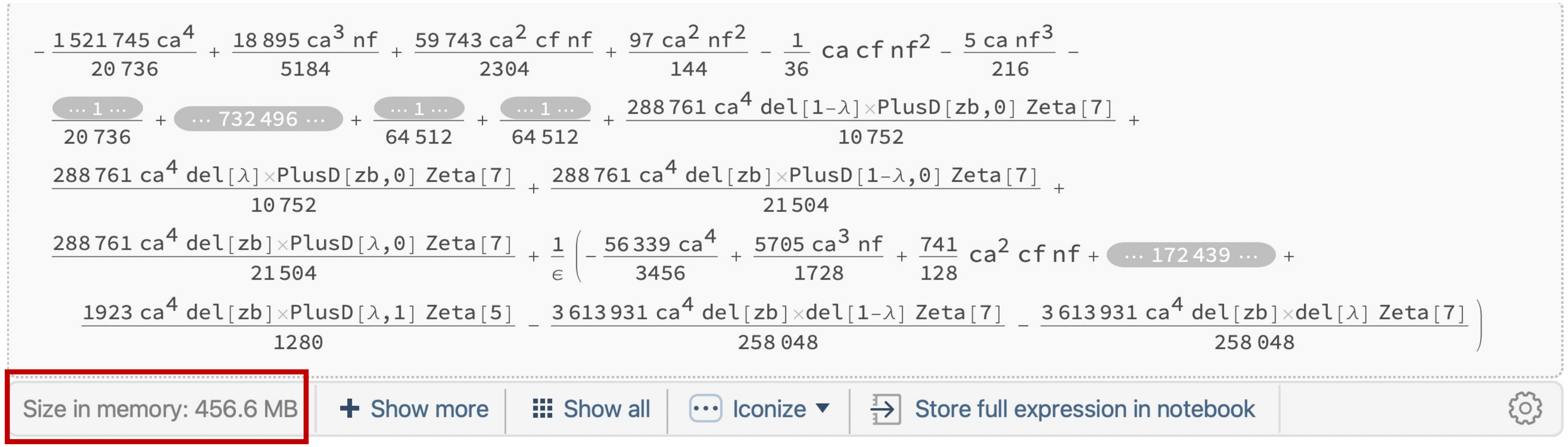
Parametrizes energy of scattering

Phase Space Integration

What do these integrals typically look like **in practice**?



$gg \to Hg$ integrand:



$$-\frac{1521745\,\mathrm{ca}^4}{20736}+\frac{18895\,\mathrm{ca}^3\,\mathrm{nf}}{5184}+\frac{59743\,\mathrm{ca}^2\,\mathrm{cf}\,\mathrm{nf}}{2304}+\frac{97\,\mathrm{ca}^2\,\mathrm{nf}^2}{144}-\frac{1}{36}\,\mathrm{ca}\,\mathrm{cf}\,\mathrm{nf}^2-\frac{5\,\mathrm{ca}\,\mathrm{nf}^3}{216}-$$
$$\frac{\cdots 1 \cdots}{20736}+\cdots 732496 \cdots+\frac{\cdots 1 \cdots}{64512}+\frac{\cdots 1 \cdots}{64512}+\frac{288761\,\mathrm{ca}^4\,\mathtt{del}[1-\lambda]\times\mathtt{PlusD}[\mathtt{zb},0]\,\mathtt{Zeta}[7]}{10752}+$$
$$\frac{288761\,\mathrm{ca}^4\,\mathtt{del}[\lambda]\times\mathtt{PlusD}[\mathtt{zb},0]\,\mathtt{Zeta}[7]}{10752}+\frac{288761\,\mathrm{ca}^4\,\mathtt{del}[\mathtt{zb}]\times\mathtt{PlusD}[1-\lambda,0]\,\mathtt{Zeta}[7]}{21504}+$$
$$\frac{288761\,\mathrm{ca}^4\,\mathtt{del}[\mathtt{zb}]\times\mathtt{PlusD}[\lambda,0]\,\mathtt{Zeta}[7]}{21504}+\frac{1}{\epsilon}\Bigg(-\frac{56339\,\mathrm{ca}^4}{3456}+\frac{5705\,\mathrm{ca}^3\,\mathrm{nf}}{1728}+\frac{741}{128}\,\mathrm{ca}^2\,\mathrm{cf}\,\mathrm{nf}+\cdots 172439 \cdots+$$
$$\frac{1923\,\mathrm{ca}^4\,\mathtt{del}[\mathtt{zb}]\times\mathtt{PlusD}[\lambda,1]\,\mathtt{Zeta}[5]}{1280}-\frac{3613931\,\mathrm{ca}^4\,\mathtt{del}[\mathtt{zb}]\times\mathtt{del}[1-\lambda]\,\mathtt{Zeta}[7]}{258048}-\frac{3613931\,\mathrm{ca}^4\,\mathtt{del}[\mathtt{zb}]\times\mathtt{del}[\lambda]\,\mathtt{Zeta}[7]}{258048}\Bigg)$$

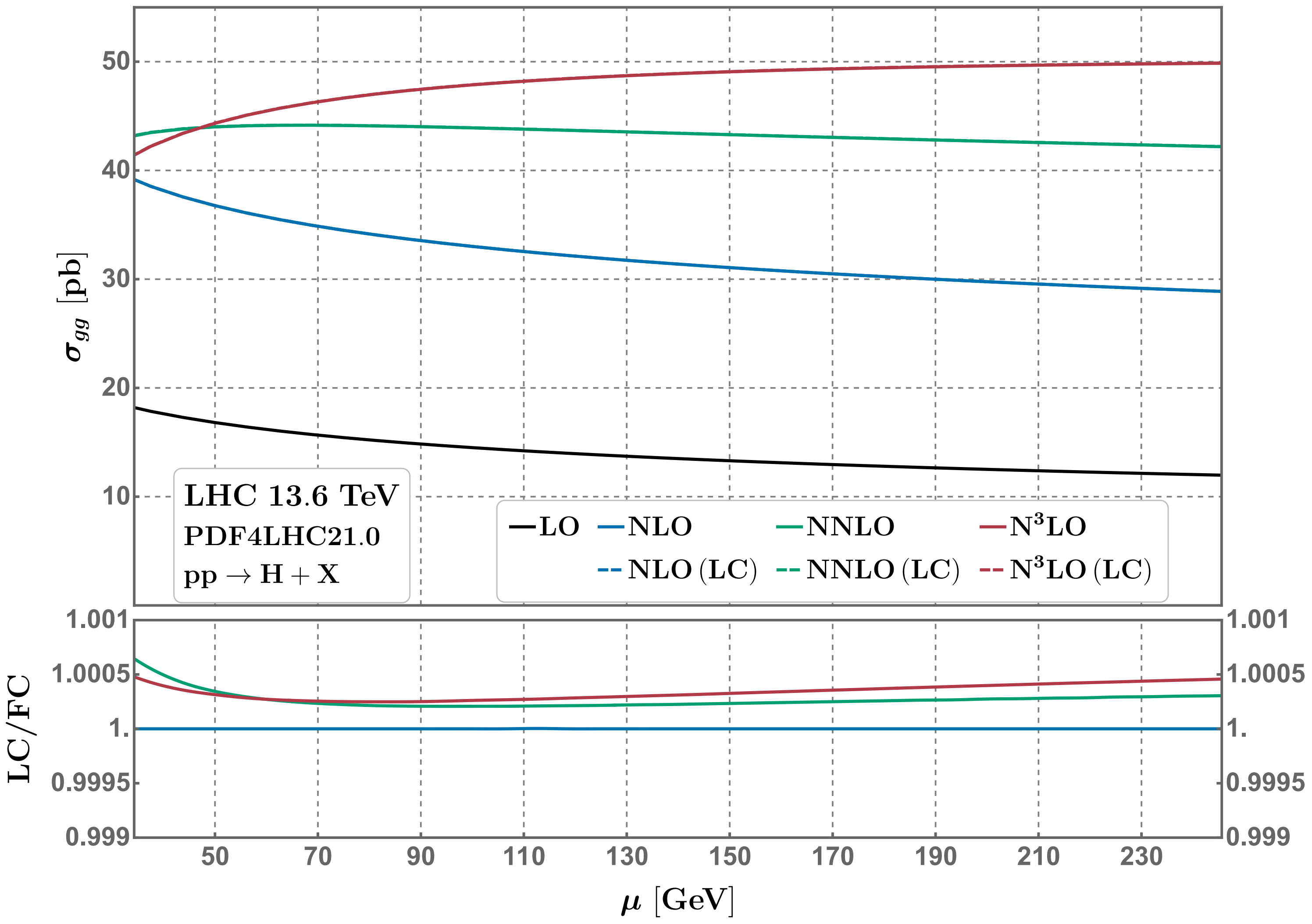
Huge and complicated! Contains various polylogs, distributions, renormalization factors…

Integration software based on PolyLogTools*

*[Claude Duhr, Falko Dulat; arXiv:1904.07279]

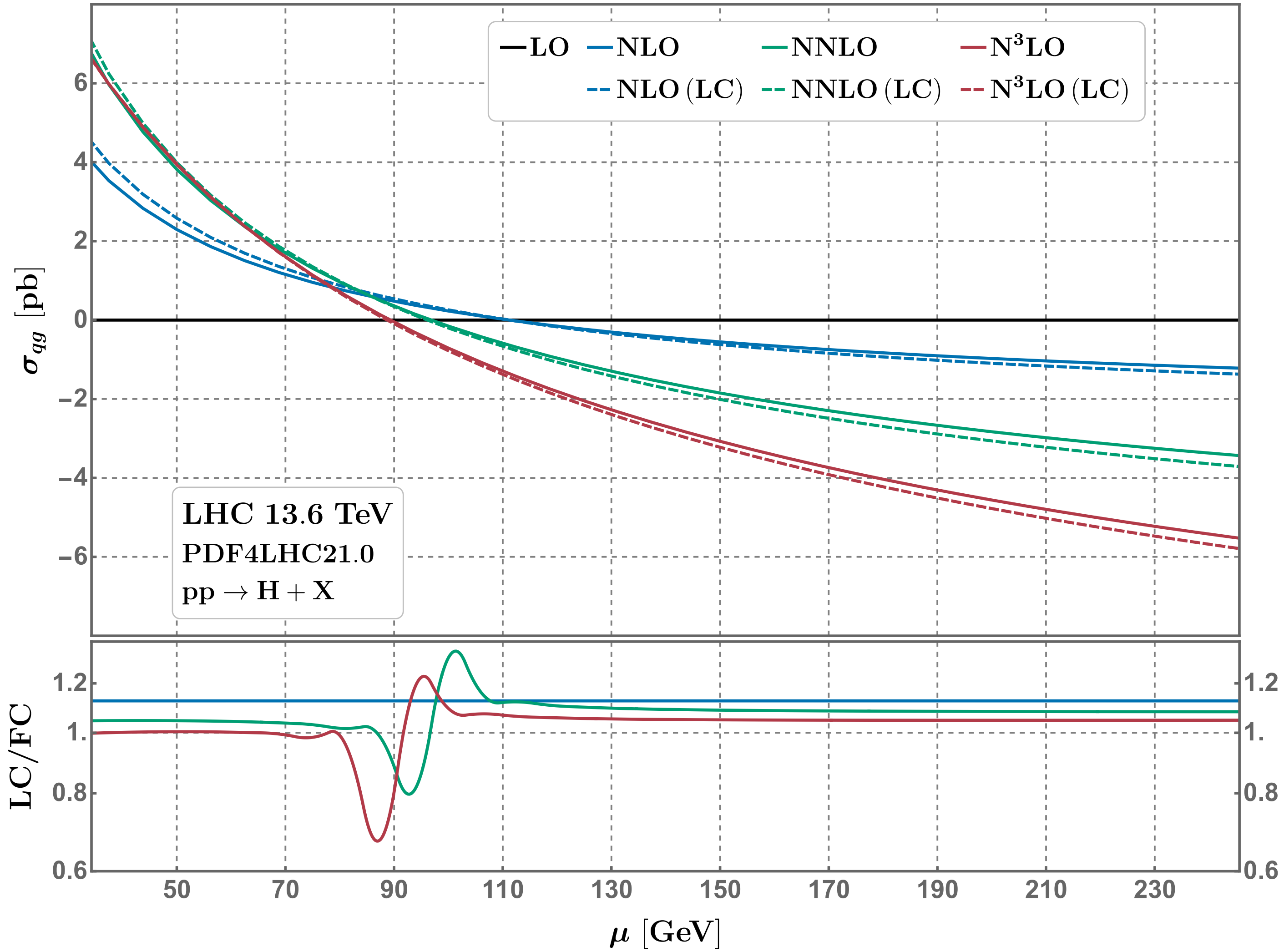
More Leading Color Phenomenology

$gg \to Hg$



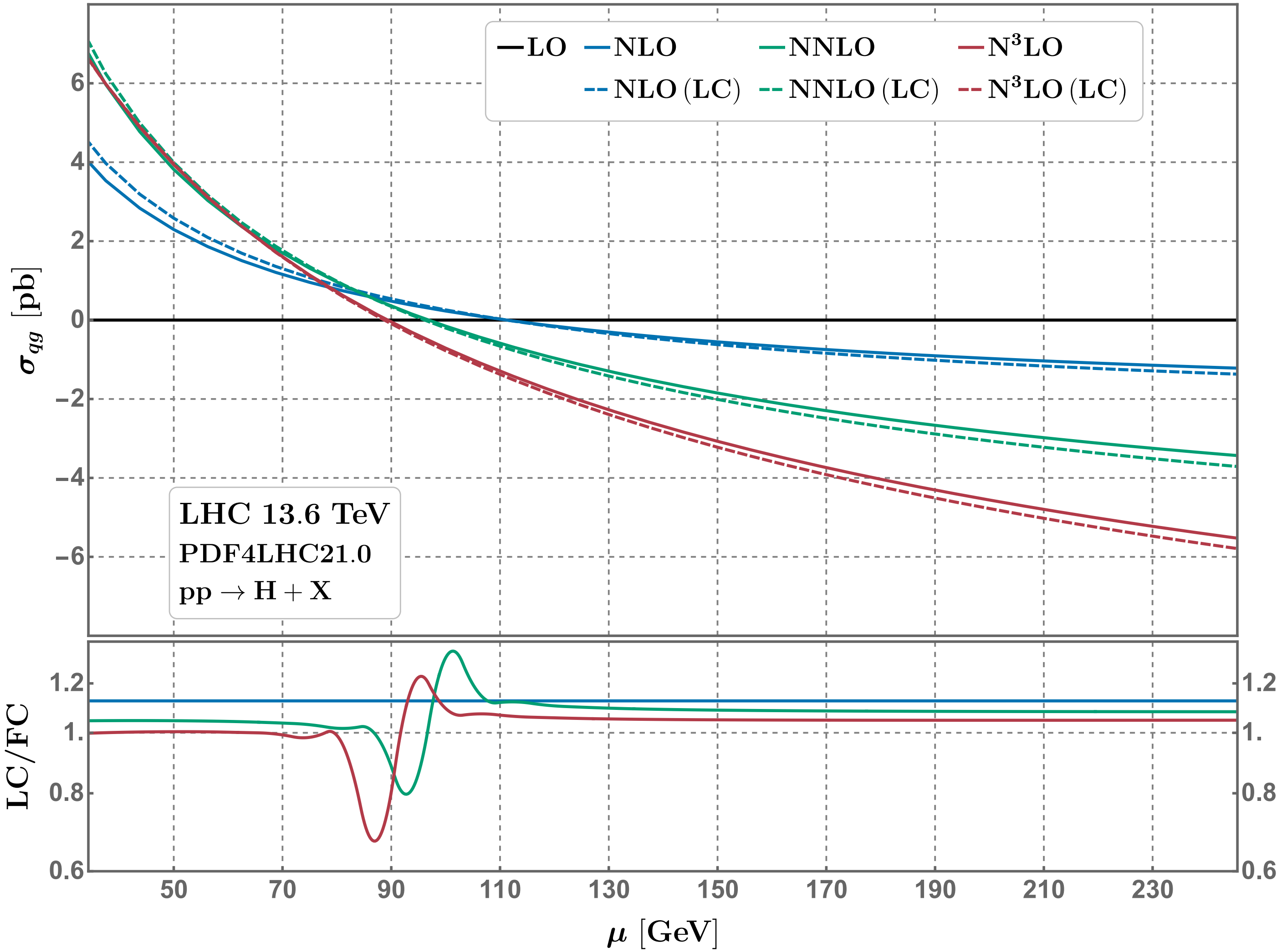
More Leading Color Phenomenology

$q\bar{q} \to Hg$



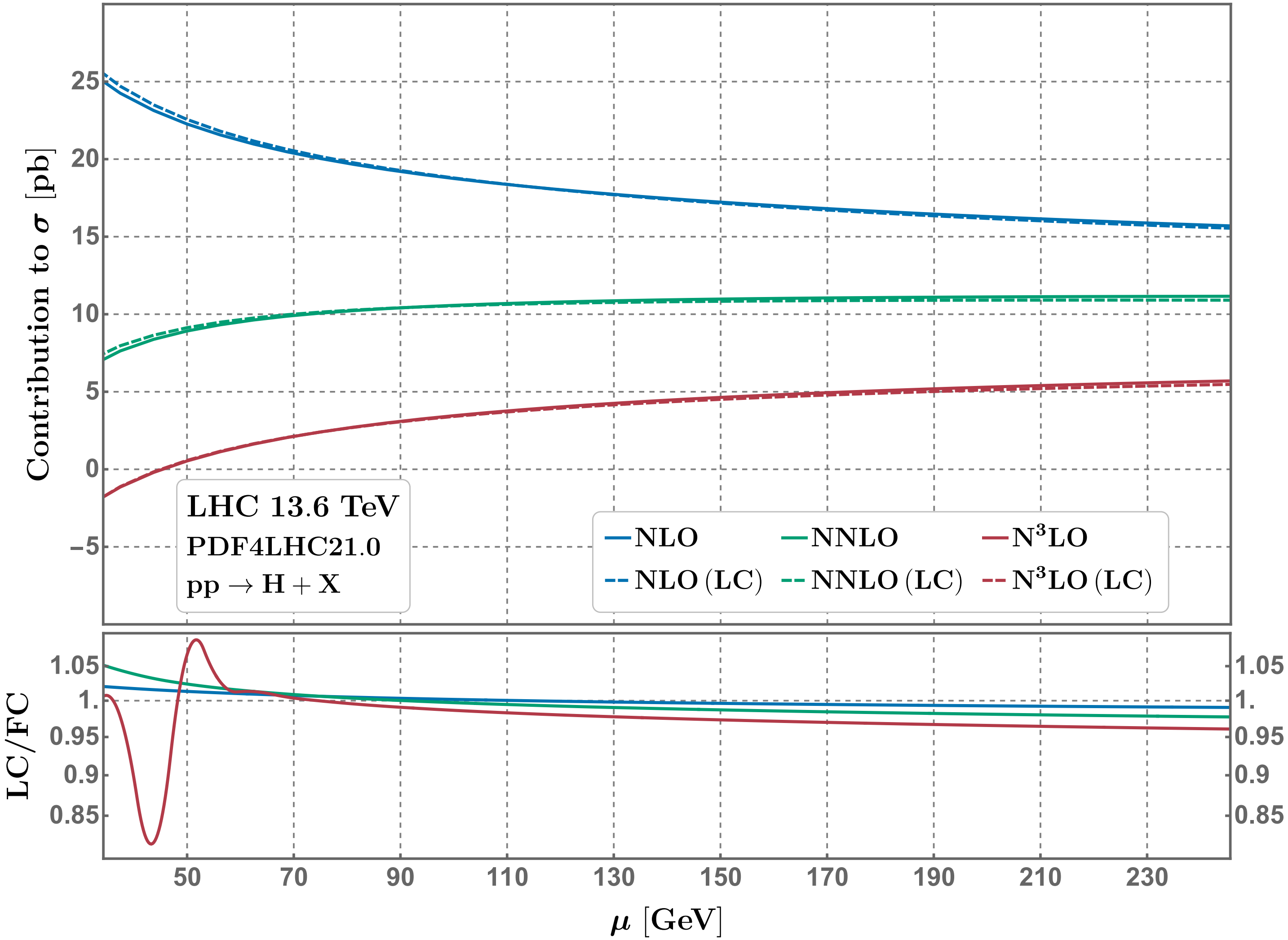
More Leading Color Phenomenology

$qg \to Hq$



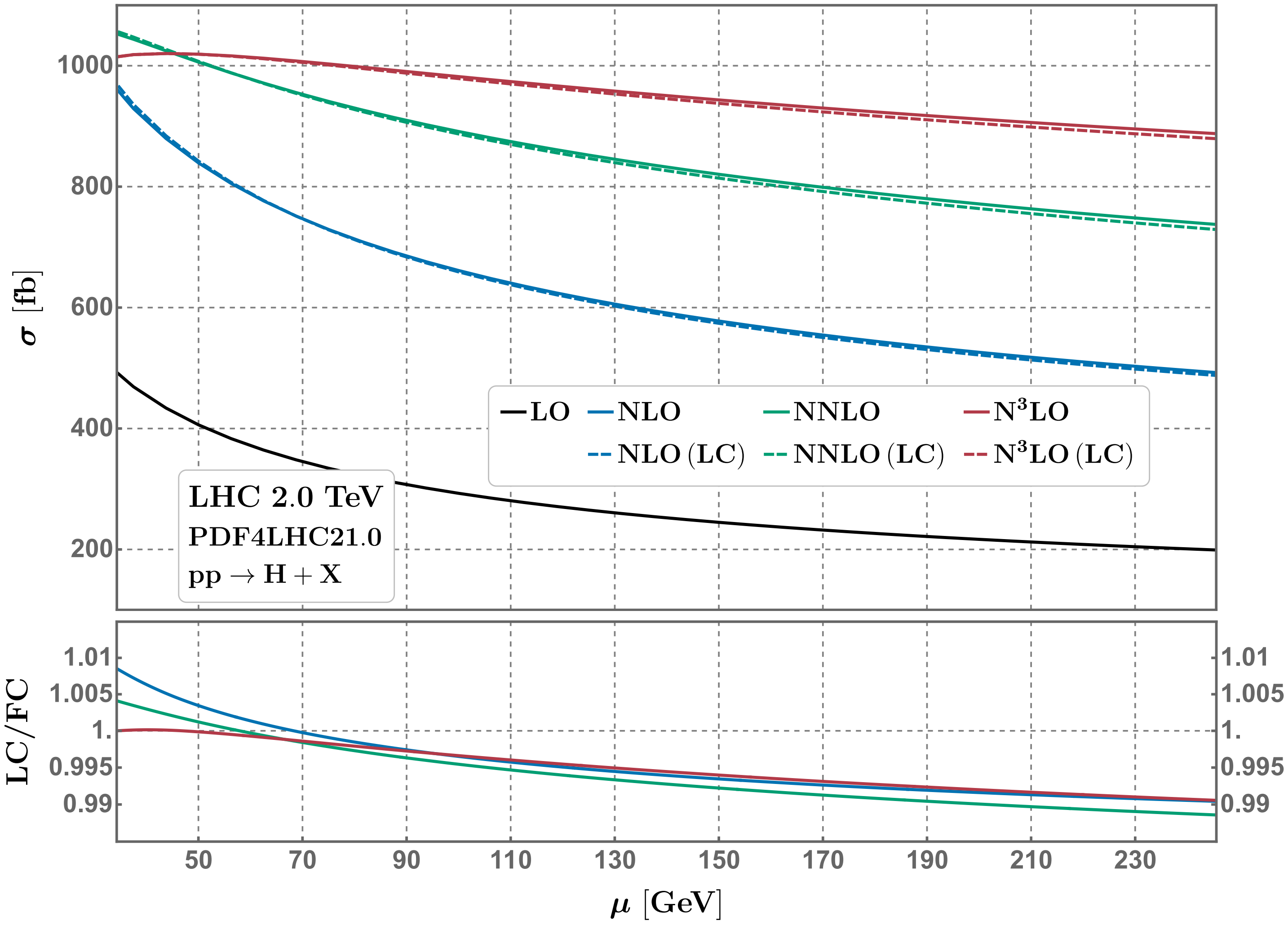
More Leading Color Phenomenology

Contribution to cross section at each order:



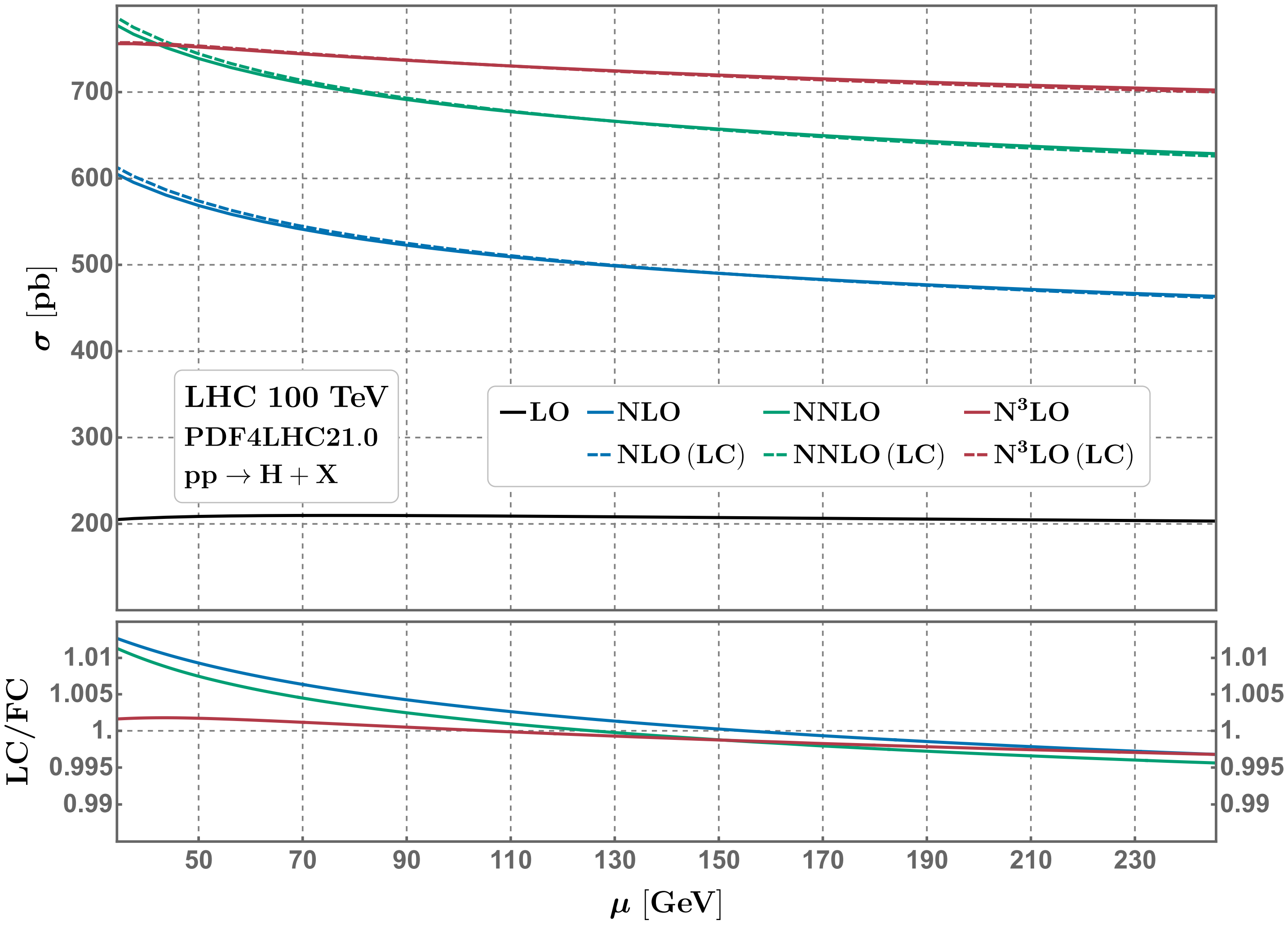
More Leading Color Phenomenology

Cross section at $\sqrt{s} = 2$ TeV



More Leading Color Phenomenology

Cross section at $\sqrt{s} = 100$ TeV



Integration strategy for RVVV Higgs Channels

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⬇

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Next Iteration: $\epsilon \int_0^1 A_{ij} R(\lambda) M_j(\lambda)\, d\lambda$