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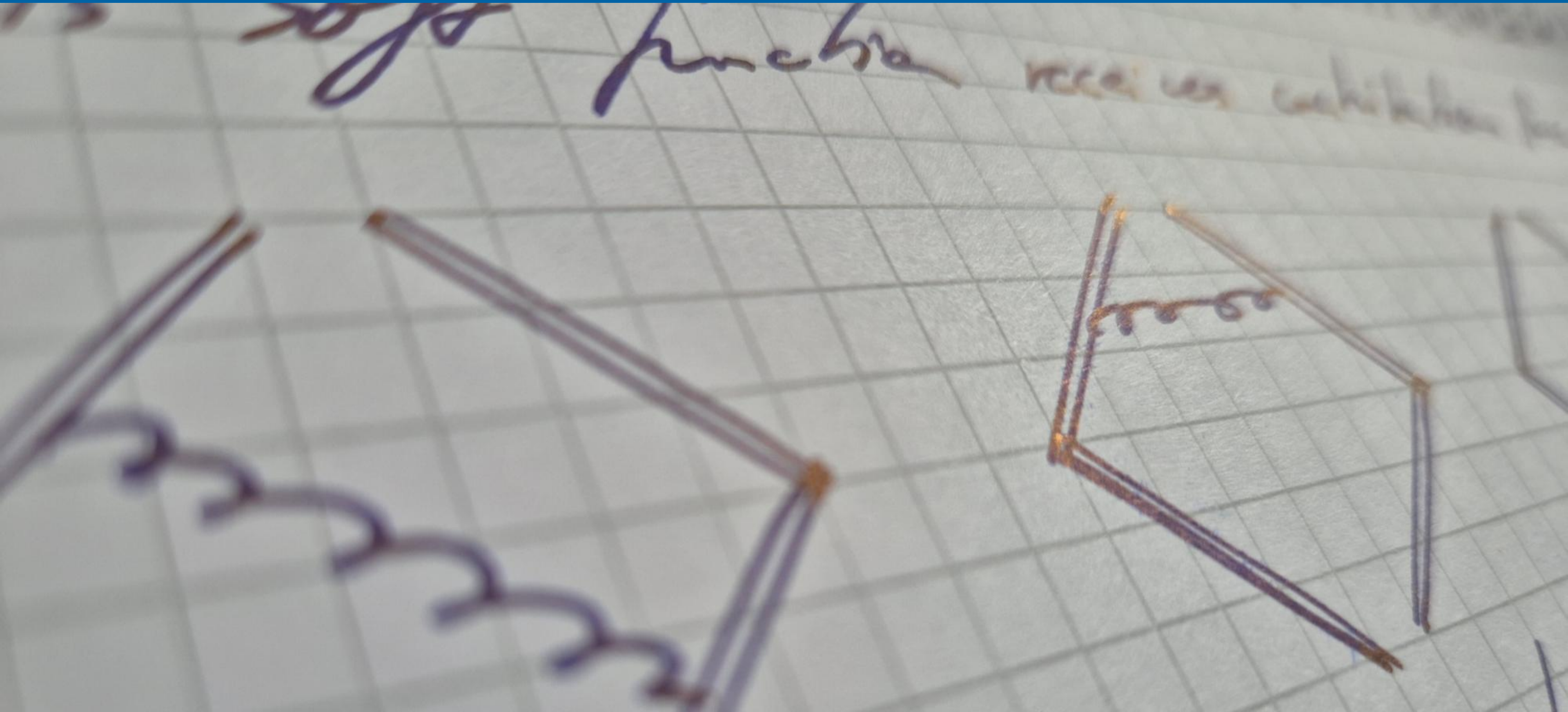
Automated calculation of non-global soft functions

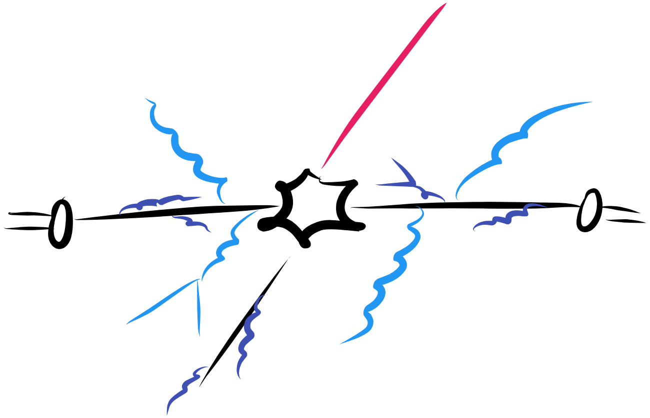
Rudi Rahn

Loopfest 2026, BNL, 27 May 2026



Agenda

- Soft functions
- Automation
- Non-globality



(this is a technical talk, but I'll try to explain everything as conceptual as possible)

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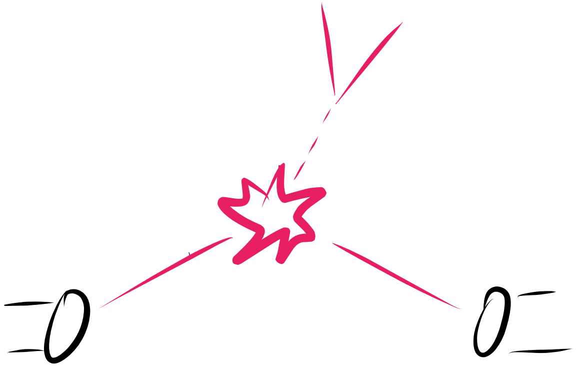
Factorisation

- Calculations in collider physics *factorise* (sometimes)

$$\sigma_{AB\to X} = f_a^A \otimes \hat{\sigma}_{ab\to X} \otimes f_b^B$$

Parton distributions: Λ_{QCD}

Partonic cross section: $\sqrt{\hat{s}}$



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Factorisation

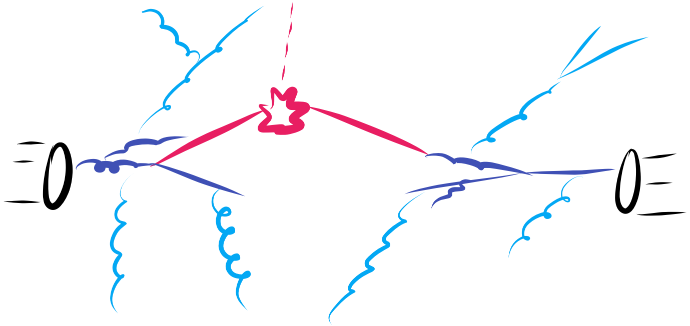
- Calculations in collider physics *factorise* (sometimes)

- With multiple scales (e.g. weak boson at low p_T):

$$\sigma_{AB\to X} \sim f_a^A \otimes \mathcal{I}_{ai}(p_T) \otimes H_{ij}^X(Q) \otimes \mathcal{I}_{bj}(p_T) \otimes f_b^B \otimes S(p_T)$$

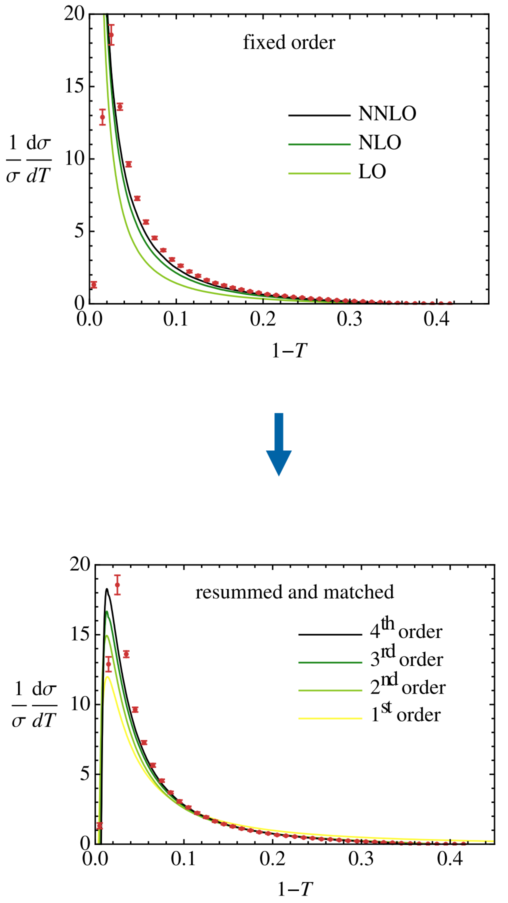
With parton distributions,
beam functions,
hard function, and

Soft function



Why factorise?

- *Divide and conquer*
- Reuse ingredients
- Resum large logarithms



[Becher, Schwartz, ’08]

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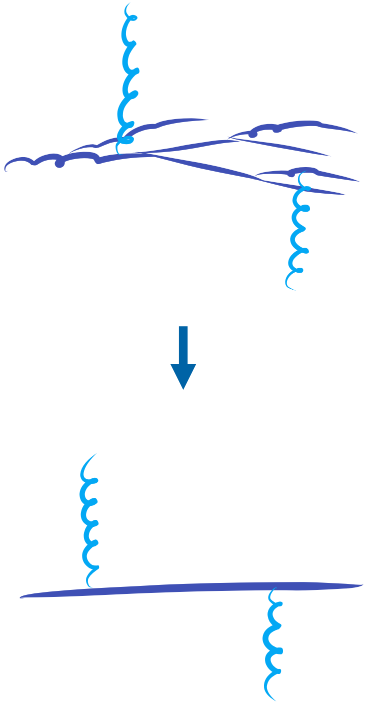
Soft functions

- Correlator of Wilson lines

$$W(x, x+sn) = \mathbf{P}\exp\left(ig\int_0^s \mathrm{d}s'\, n\cdot A(x+s'n)\right)$$

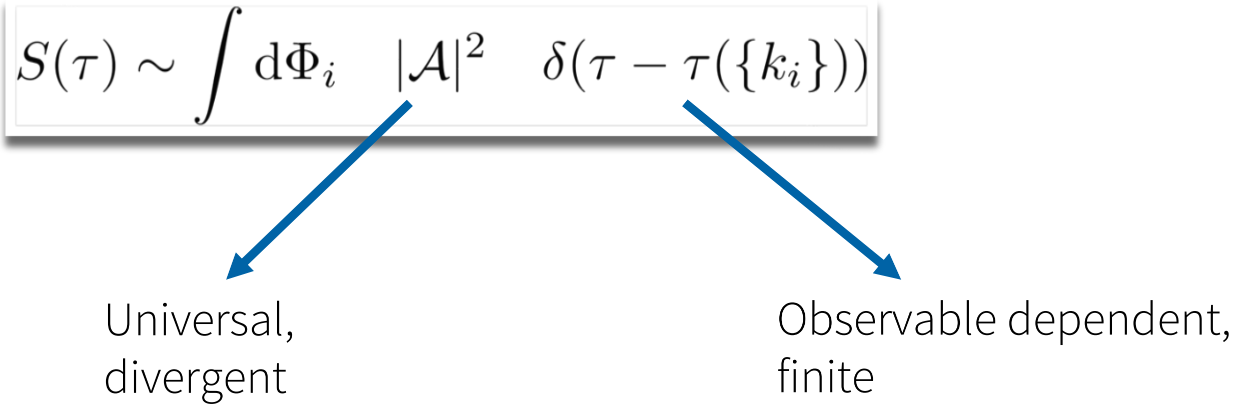
- Observable dependent

- Divergent ➞ analytic calculation



Soft functions – a closer look

- Matrix element and measurement



$$S(\tau) \sim \int \mathrm{d}\Phi_i \quad |\mathcal{A}|^2 \quad \delta(\tau - \tau(\{k_i\}))$$

Universal,
divergent

Observable dependent,
finite

- Extract divergences *before* integration

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The NLO case

- Eikonal matrix element for one emission

$$|\mathcal{A}^2| \sim \frac{1}{k_+ k_-}$$

- Use transverse momentum and rapidity

$$\mathrm{d}^d k \, |\mathcal{A}^2| \sim k_T^{-1-2\epsilon} \, y^{-1}$$

$$k_T = \sqrt{k_+ k_-} \qquad y = \frac{k_+}{k_-}$$

The NLO case

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$$k_T = \sqrt{k_+ k_-} \qquad y = \frac{k_+}{k_-}$$

- How does an observable τ behave near these divergences?

$$\boxed{\tau(k) = k_T^m \, y^{\frac{n}{2}} \, f(y, \theta_i)}$$

Mass dimension

Read off

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The NLO case - Automation

- Integrate k_T :

Soft divergence

Collinear divergence

$$S(\tau) \sim \tau^{-1-2\epsilon} \int \mathrm{d}y \int \mathrm{d}\theta_i \, y^{-1+n\epsilon} f^{2\epsilon}(y, \theta_i)$$

- Expand:

$$S(\tau) \sim \tau^{-1-2\epsilon} \times \left(\frac{I_1}{\epsilon} + I_2 + I_3\epsilon + \dots \right)$$

- Profit!

Examples (NLO)

Observable	n	$f(y,t)$
Thrust	1	$\min(1, \frac{1}{y})$
C-Parameter	1	$\frac{1}{1+y}$
Broadening	0	$\frac{1}{2}$
B-axis angularities	$B-1$	$(1+y)^{1-\frac{B}{2}}$
DY @ threshold	-1	$1+y$
p_T resummation	0	$2\lvert 1-2t\rvert$
1-jettiness	1	$\min(1, \frac{1}{y}, \frac{n_{jk}}{n_{ij}} + \frac{n_{ik}}{y\, n_{ij}} + \sqrt{\frac{2}{y\, n_{ij}}} n_k \cdot e_x (1-2t))$
Soft drop jet mass	$-1-\beta$	$(1+y)^{1+\frac{\beta}{2}}$

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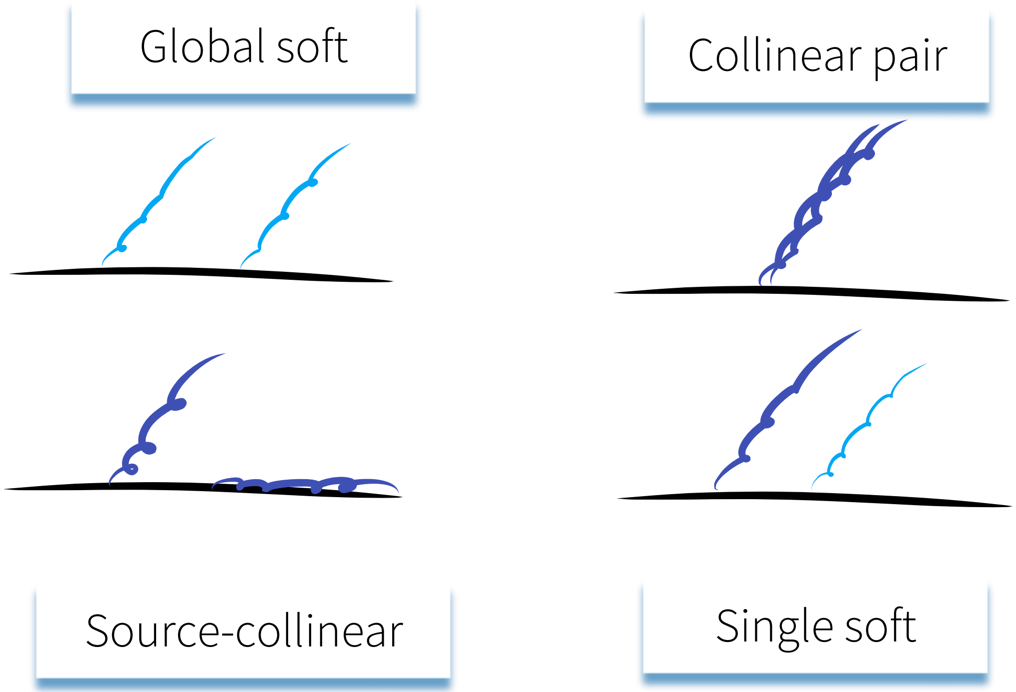
The NNLO case

- Four divergences



Global soft

Collinear pair

Source-collinear

Single soft

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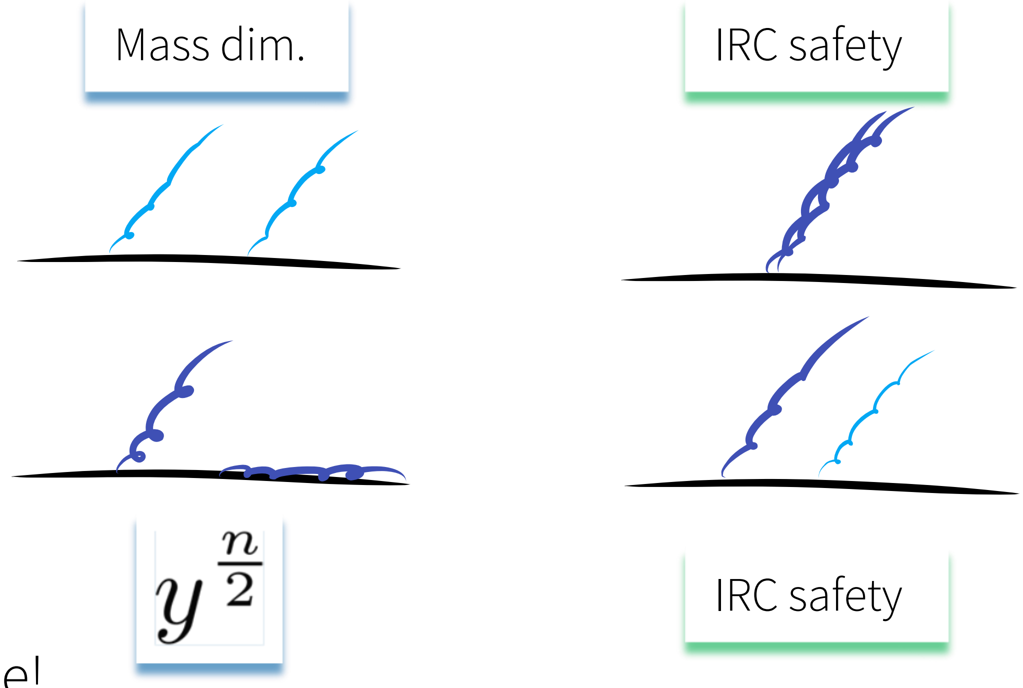
The NNLO case

- Four divergences



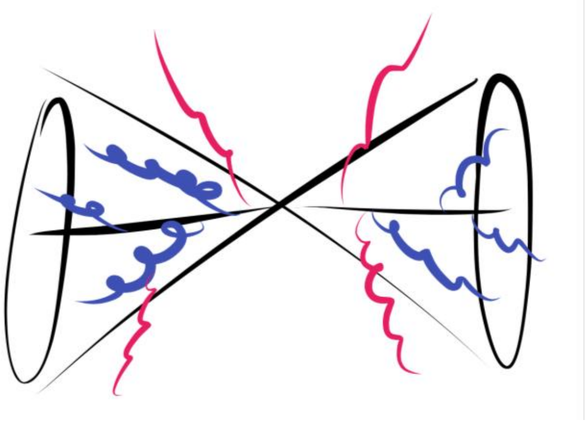
- IRC safety to the rescue!

$$\tau(k,l) = p_T^m \, y^{\frac{n}{2}} \, F(y,a,b,\theta_i)$$

[Bell, RR, Talbert, ’18]

Non-globality [Dasgupta, Salam, '01]

- Gap between jets
  - Blind vs. measurement region



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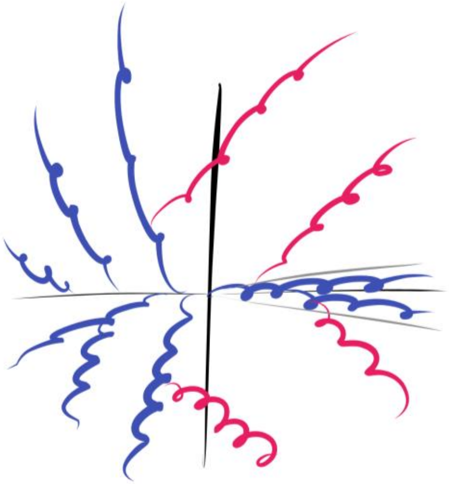
Non-globality [Dasgupta, Salam, '01]



- Gap between jets
  - Blind vs. measurement region
- Light jet mass? [Becher, Pecjak, Shao, '16]
  - Hemisphere soft function? $M_R \ll M_L$
    [Kelley, Schabinger, Schwartz, Zhu, '11]

$$S_{\text{hemi}}(k_L, k_R) = g(\epsilon)\,\delta(k_L) k_R^{-1-2\epsilon} + h(\epsilon, k_L, k_R) + g(\epsilon)\, k_L^{-1-2\epsilon}\delta(k_R)$$

Non-globality

- So far: implicit assumption of global observable

$$\int_{\mathbb{R}^d} \mathrm{d}^d k\, \delta^+(k^2)\; |\mathcal{A}(k)|^2\, \delta\big(\tau - \tau(k)\big)$$

- Non-global: restricted phase space, *or* piecewise definition (jets!)

- Required from factorisation: multiple Wilson lines

[Becher, Neubert, Rothen, Shao, ‘16]

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- Non-global: restricted phase space, *or* piecewise definition (jets!)

- Required from factorisation: multiple Wilson lines ✓

[Becher, Neubert, Rothen, Shao, '15/16]

[Bell, Dehnadi, Mohrmann, RR, '23]

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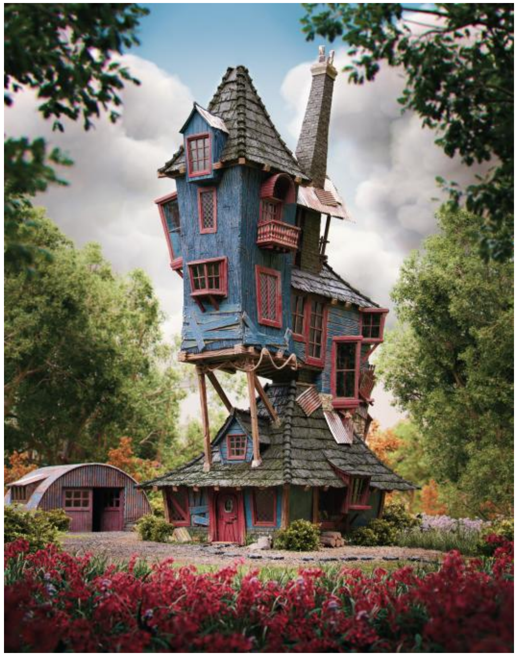
Some modifications

- Measurement on softer emissions
- Numerical instabilities
- Different mode contributions
- More integration variables



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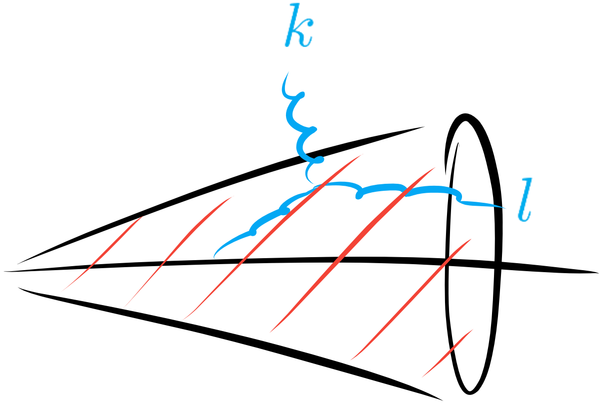
Non-globality – naïve expectations

- Simply add a step function constraint?

$$\int \mathrm{d}^d k\, \mathrm{d}^d l\, \theta_{\mathrm{accept}}(k,l)\, |\mathcal{A}(k,l)|^2\, \delta\big(\tau - \tau(k,l)\big)$$



- IRC safety for in-out radiation is ambiguous

- Modify the function definition:

$$\tau(k,l) = p_T^m\, b^{n_b}\, y^{\frac{n}{2}}\, F(y,a,b,\theta_i)$$

$$b = \frac{k_T}{l_T}$$

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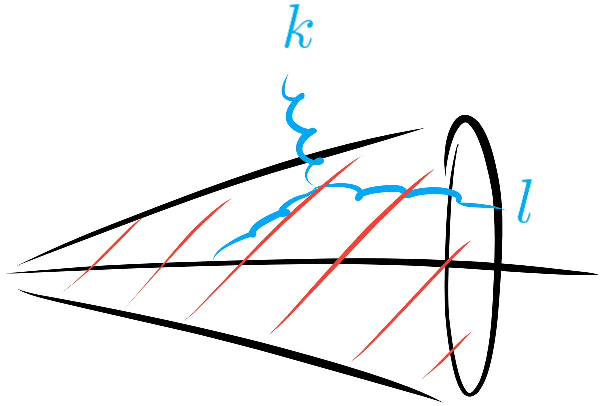

- IRC safety for in-out radiation is ambiguous

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$$b = \frac{k_T}{l_T}$$

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Non-globality – soft divergence

- Interesting pattern of appearance in the master formula:

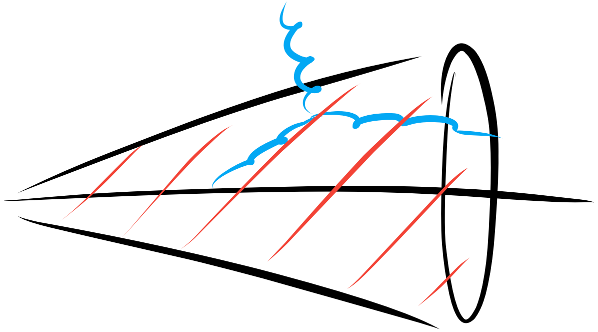
$$\int \mathrm{d}b\, b^{-1-2\epsilon}\left(b^{n_b} F(y,a,b,\theta_i)\right)^{4\epsilon} = \int \mathrm{d}b\, b^{-1+(4n_b-2)\epsilon}\, F(y,a,b,\theta_i)^{4\epsilon}$$

- Two hierarchies: softer particle measured: $n_b = 1$
  harder particle measured: $n_b = 0$

- Soft divergence cancels!
- Persists to different mass dimensions, and rapidity regularisation.

Non-globality – numerical problems

- Divergences partially cut off



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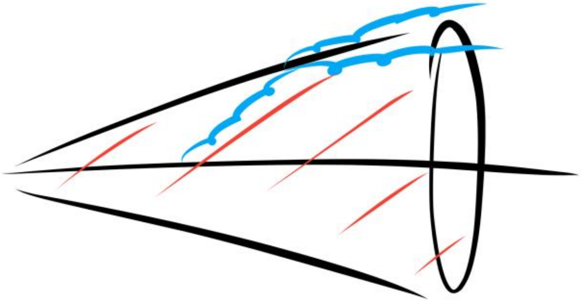
Non-globality – numerical problems

- Divergences partially cut off



- Numerically very unstable (though analytically fine):

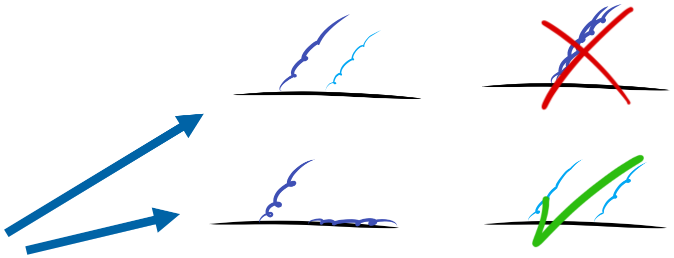
$$\int_0^1 \mathrm{d}u \, \mathrm{d}x_i \, \frac{1}{u} \theta(u - x_i) = - \int_0^1 \mathrm{d}x_i \, \ln x_i = 1$$

- Requires observable-dependent remedy…

Correlations and regions

- So far: in-out region treated like gaps between jets

- What if both emissions are measured? On different modes?
  - Product of coft and jet function in resummation context
  - But in a fixed order Method-of-Region expansion?



- Do these limits still commute? Apparently yes…

What else can we generalise?

- Mass dimension?
  Fairly easy, changes some exponents ✓

- Regulators?
  [Chiu, Jain, Neill, Rothstein, '11]
  Variations of phase space, eta, pure rapidity regulator $\left(\nu^n R(\{k_i\})\right)^\alpha$ ✓
  [Becher, Bell, '11] [Ebert, Moult, Stewart, Tackmann, Vita, Zhu, '19]

- Non-standard phase space suppression?
  Played with exponentials, … ✓

Some preliminary results

- Collinear-soft function for jet-based q_T -slicing

$$\tau(\{k_i\}) = \sum_i \theta_{\text{out}}(k_i)\vec{b}_T \cdot \vec{k}_{i,T} + \theta_{\text{in}}(k_i) b_T \cos\phi_b k_{i,x}$$

- Results for the one-in/one-out configuration:

$$S^{\text{in,out}}_{\text{global}} = (\mu b_T e^{\gamma_E})^{4\epsilon} R^{-4\epsilon} \left[C_F C_A \left(\frac{18.75(5)}{\epsilon} - 9.9(3) \right) + C_F T_F n_f \left(\frac{-7.425(7)}{\epsilon} + 9.39(4) \right) \right]$$

$$S^{\text{in,out}}_{\text{NGL,avg}} = (\mu b_T e^{\gamma_E})^{4\epsilon} R^{-2\epsilon} \left[C_F C_A \left(\frac{6.5800(8)}{\epsilon^2} + \frac{-17.742(5)}{\epsilon} + 45.03(3) \right) \right]$$

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Work in progress

- Hopefully soon™:

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Conclusion

- Non-global soft functions can be treated as a class
- Soft limit needs to be accommodated
- Some preliminary results are available
- Open questions: Wilson lines in d dimensions? Uncorrelated emissions? …

Thank you!

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Additional integration variables

- Generally: more divergent → more universal:

$$S_b^{(1)} = \overset{\text{universal}}{\overset{\downarrow}{\frac{\Gamma_0}{\epsilon^2}}} + \underset{\text{shared}}{\underset{\nearrow}{\frac{\gamma_0^S}{\epsilon}}} + \overset{\text{individual}}{\overset{\downarrow}{c_1}}$$

- Two sources: subtraction, expansion

$$x^{-1+\epsilon} = \frac{\delta(x)}{\epsilon} + \left[\frac{1}{x}\right]_+ + \ldots \qquad f^{2\epsilon}(y,t) = 1 + 2\epsilon \ln f(y,t) + \ldots$$

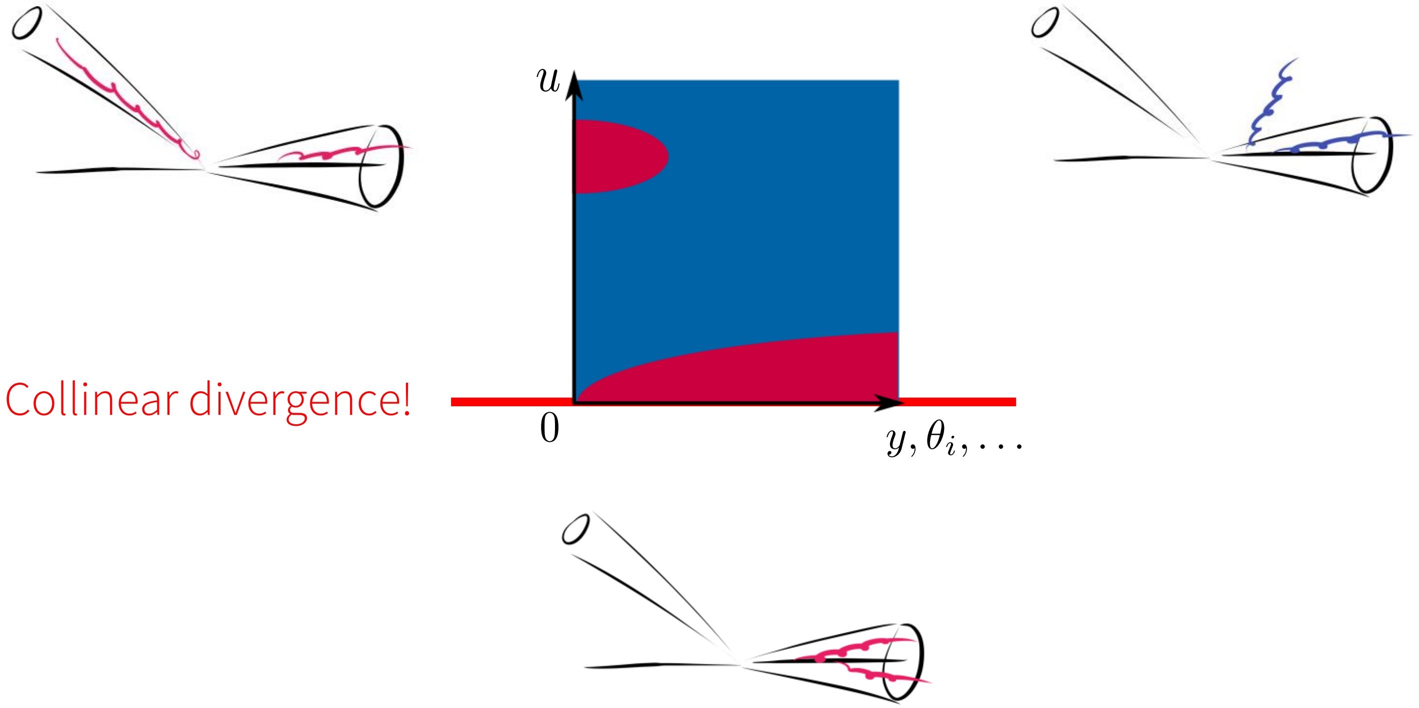
- For non-global observables:

$$\boxed{S(\tau) \sim \tau^{-1-2\epsilon} \int \mathrm{d}y \int \mathrm{d}\theta_i \, \theta_{\text{accept}}(y, \theta_i) \, y^{-1+n\epsilon} f^{2\epsilon}(y, \theta_i)}$$

- Consequence: One additional integration variable at NNLO

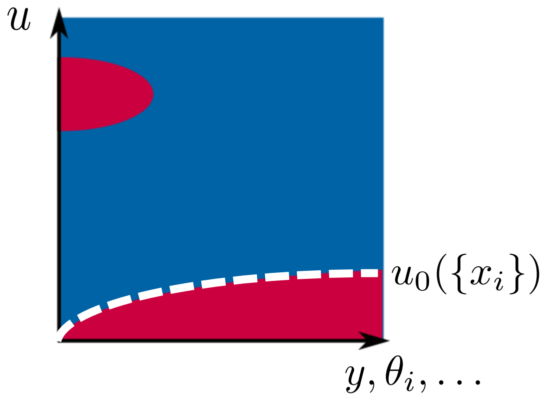
Improve the numerics

- Variable *u* encodes collinearity, look at in-out contribution


Collinear divergence!

Improve the numerics

- If we know where the boundary lies, we can rescale: $u = u_0^z$



$$\int_0^1 \mathrm{d}u\, \mathrm{d}x_i\, \frac{1}{u}\theta(u, u_0)$$
$$= \int_0^1 \mathrm{d}(u_0^z)\, \mathrm{d}x_i\, \frac{1}{u_0^z}\theta(u_0^z, u_0)$$
$$= \int_0^1 \mathrm{d}z\, \mathrm{d}x_i\, \ln u_0\, \theta(u_0^z, u_0)$$

- Unfortunately: needs user input

Regulator cancellation

- Regulator appears as $\left(\nu^n R(\{k_i\})\right)^\alpha$: e.g. for two emissions

$$\left(\frac{\nu^2}{(k_+ \pm k_-)(l_+ \pm l_-)}\right)^\alpha \qquad \left(\frac{\nu}{(k_+ + l_+) \pm (k_- + l_-)}\right)^\alpha \qquad \left(\frac{k_+ + l_+}{k_- + l_-}\right)^\alpha$$

- Regulator must cancel with collinear functions (insensitive to bulk)
- Dependence on bulk impossible in correlated emissions:

Forward limit →

$$\frac{\delta(y)}{\alpha} \cdot \left(1 + \alpha \ln R(y, \ldots) + \mathcal{O}(\alpha^2)\right)$$

- In uncorrelated emissions: cancellation with NLO^2