Bootstrapping QCD Amplitudes

Based on 2510.20565 & 2602.02783

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Sérgio Carrôlo | Loopfest 2026

MAX-PLANCK-INSTITUT FÜR PHYSIK

— Outline —

Why? I. Current Frontier of QCD Amplitudes

What? II. The Skeleton of QCD Amplitudes

$$\mathcal{A}_n^{(L)} = \sum_{\omega} C_{ij}^{(\omega)} R_i^{(\omega)} I_j^{(\omega)}$$

III. Rational Prefactor Problem

How? IV. Setup and Simplifications
1. Planar
2. Maximal weight *
3. Symbol

Ingredients
- V. Function Space
- VI Leading Singularities = On-Shell Diagrams

Method – VII Bootstrap v2.0: Prescriptive Unitarity Edition

VIII Structure of the Results

Next? IX Summary and Outlook

— Current Frontier of QCD Amplitudes —

* Scattering amplitudes @ NNLO and beyond are needed to match experimental precision.

* The current frontier is set at

= Massless 5 parton Scattering @ NNLO =

(i) Feynman Integrals (2015 → 2018)

[Gehrmann et al '15 ; Chicherin et al '18 ; Abreu et al '18 ...]

(ii) Scattering amplitudes (2020 - 2021)

[Abreu et al '20 ; Abreu et al '21 ; Badger et al '21 ...]

(iii) $d\sigma$ $pp \to 3\gamma$; $pp \to 2\gamma + j$; $pp \to 3j$ (2020 - 2021)

[Kallweit et al '20 ; Chawdhry et al '21 ; Czakon et al '21 ...]

— Current Frontier of QCD Amplitudes —

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Our Goal : m=0 6 parton Scattering @ NNLO

— The Skeleton of QCD Amplitudes —

$$\mathcal{A}_n^{(L)} = \boxed{\sum_{\omega=0}^{2L}} \sum_{i,j} \boxed{C_{ij}^{(\omega)}}\, \boxed{R_i^{(\omega)}}\, \boxed{\mathcal{I}_j^{(\omega)}}$$

- $\sum_{\omega=0}^{2L}$ → Graded transcendental function : ω - weight
- $C_{ij}^{(\omega)}$ → Coefficients : rational numbers
- $R_i^{(\omega)}$ → (Rational) Kinematic prefactors
  - ↓ Challenge: Proliferation in number and size (⚠ jumpscare)
- $\mathcal{I}_j^{(\omega)}$ → Basis of Transcendental functions
  - ↓ Complicated multi-variate transcendental functions

— Rational Prefactor Problem —

Usual way:

Ansatz

↓

Compute R_i numerically
[From unitarity]

↓

Solve linear System for n_i, d_i

$$R_i = \frac{N_i}{D_i} \longrightarrow \begin{aligned} N_i &= \sum n_{i,\vec{\alpha}} \left(\prod S_{ij}^{\alpha_{ij}} \right) \\ D_i &= \sum d_{i,\vec{\alpha}} \left(\prod S_{ij}^{\alpha_{ij}} \right) \end{aligned}$$

Problems:

- Runtimes
- # unknowns

5-gluon amplitudes

Helicity remainder	ansatz size
$R^{(2),(2,0)}_{+++--}$	21,910
$R^{(2),(2,0)}_{++-+-}$	54,148
$R^{(2),(1,0)}_{+++--}$	163,635
$R^{(2),(1,0)}_{+-++-}$	241,156
$R^{(2),(1,0)}_{--+++}$	82,180
$R^{(2),(1,1)}_{+++--}$	21,910
$R^{(2),(1,1)}_{++-+-}$	54,148
$R^{(2),(0,1)}_{+++--}$	118,880
$R^{(2),(0,1)}_{+-++-}$	209,018
$R^{(2),(0,1)}_{--+++}$	76,845
$R^{(2),(-1,1)}_{+++--}$	5,320
$R^{(2),(-1,1)}_{++-+-}$	9,384

[De Laurentis, Ita, Klinkert, Sotnikov '23]

— Rational Prefactor Problem —

In the end, they look like (Split MHV : $--+++$)

```
{R[1] -> -5/2,

R[2] -> -1/(4 s12^3 (s12 - s34 - s45)^3) (6 s12^5 s15 + 15 s12^4 s15^2 + 10 s12^3 s15^3 + 6 s12^5 s23 + 15 s12^4 s23^2 + 10 s12^3 s23^3 - 12 s12^4 s15 s34 - 15 s12^3 s15^2 s34 - 18 s12^4 s23 s34 - 45 s12^3 s23^2 s34 - 30 s12^2 s23^3 s34 + 6 s12^3 s15 s34^2 + 18 s12^3 s23 s34^2 +
  45 s12^2 s23^2 s34^2 + 30 s12 s23^3 s34^2 - 6 s12^2 s23 s34^3 - 15 s12 s23^2 s34^3 - 10 s23^3 s34^3 - 18 s12^4 s15 s45 - 45 s12^3 s15^2 s45 - 30 s12^2 s15^3 s45 - 12 s12^4 s23 s45 - 15 s12^3 s23^2 s45 + 6 s12^4 s34 s45 + 54 s12^3 s15 s34 s45 +
  60 s12^2 s15^2 s34 s45 + 54 s12^3 s23 s34 s45 + 60 s12^2 s15 s23 s34 s45 + 30 s12 s15^2 s23 s34 s45 + 60 s12^2 s23^2 s34 s45 + 30 s12 s15 s23^2 s34 s45 - 12 s12^3 s34^2 s45 - 36 s12^2 s15 s34^2 s45 - 72 s12^2 s23 s34^2 s45 -
  60 s12 s15 s23 s34^2 s45 - 75 s12 s23^2 s34^2 s45 - 30 s15 s23^2 s34^2 s45 + 6 s12^2 s34^3 s45 + 30 s12 s23 s34^3 s45 + 30 s23^2 s34^3 s45 + 18 s12^3 s15 s45^2 + 45 s12^2 s15^2 s45^2 + 30 s12 s15^3 s45^2 + 6 s12^3 s23 s45^2 - 12 s12^3 s34 s45^2 -
  72 s12^2 s15 s34 s45^2 - 75 s12 s15^2 s34 s45^2 - 36 s12^2 s23 s34 s45^2 - 60 s12 s15 s23 s34 s45^2 - 30 s15^2 s23 s34 s45^2 + 27 s12^2 s34^2 s45^2 + 60 s12 s15 s34^2 s45^2 + 60 s12 s23 s34^2 s45^2 + 60 s15 s23 s34^2 s45^2 -
  15 s12 s34^3 s45^2 - 30 s23 s34^3 s45^2 - 6 s12^2 s15 s45^3 - 15 s12 s15^2 s45^3 - 10 s15^3 s45^3 + 6 s12^2 s34 s45^3 + 30 s12 s15 s34 s45^3 + 30 s15^2 s34 s45^3 - 15 s12 s34^2 s45^3 - 30 s15 s34^2 s45^3 + 10 s34^3 s45^3 - 6 s12^4 tr5 -
  15 s12^3 s15 tr5 - 10 s12^2 s15^2 tr5 - 15 s12^3 s23 tr5 - 10 s12^2 s15 s23 tr5 - 10 s12^2 s23^2 tr5 + 12 s12^3 s34 tr5 + 15 s12^2 s15 s34 tr5 + 30 s12^2 s23 s34 tr5 + 10 s12 s15 s23 s34 tr5 + 20 s12 s23^2 s34 tr5 -
  6 s12^2 s34^2 tr5 - 15 s12 s23 s34^2 tr5 - 10 s23^2 s34^2 tr5 + 12 s12^3 s45 tr5 + 30 s12^2 s15 s45 tr5 + 20 s12 s15^2 s45 tr5 + 15 s12^2 s23 s45 tr5 + 10 s12 s15 s23 s45 tr5 - 27 s12^2 s34 s45 tr5 - 35 s12 s15 s34 s45 tr5 -
  35 s12 s23 s34 s45 tr5 - 20 s15 s23 s34 s45 tr5 + 15 s12 s34^2 s45 tr5 + 20 s23 s34^2 s45 tr5 - 6 s12^2 s45^2 tr5 - 15 s12 s15 s45^2 tr5 - 10 s15^2 s45^2 tr5 + 15 s12 s34 s45^2 tr5 + 20 s15 s34 s45^2 tr5 - 10 s34^2 s45^2 tr5),

R[4] -> 1/(6 s12^3 s34^3) (10 s12^3 s15^3 - 30 s12^3 s15^2 s23 + 30 s12^3 s15 s23^2 - 10 s12^3 s23^3 - 15 s12^3 s15^2 s34 + 30 s12^3 s15 s23 s34 - 15 s12^3 s23^2 s34 + 6 s12^3 s15 s34^2 - 6 s12^3 s23 s34^2 + 3 s12^3 s34^3 - 6 s12^2 s23 s34^3 -
  15 s12 s23^2 s34^3 - 10 s23^3 s34^3 - 30 s12^2 s15^3 s45 + 60 s12^2 s15^2 s23 s45 - 30 s12^2 s15 s23^2 s45 + 60 s12^2 s15^2 s34 s45 - 60 s12^2 s15 s23 s34 s45 + 30 s12 s15^2 s23 s34 s45 - 30 s12 s15 s23^2 s34 s45 -
  36 s12^2 s15 s34^2 s45 - 60 s12 s15 s23 s34^2 s45 - 30 s15 s23^2 s34^2 s45 + 6 s12^2 s34^3 s45 + 30 s12 s23 s34^3 s45 + 30 s23^2 s34^3 s45 + 30 s12 s15^3 s45^2 - 30 s12 s15^2 s23 s45^2 - 75 s12 s15^2 s34 s45^2 +
  30 s12 s15 s23 s34 s45^2 - 30 s15^2 s23 s34 s45^2 + 60 s12 s15 s34^2 s45^2 + 60 s15 s23 s34^2 s45^2 - 15 s12 s34^3 s45^2 - 30 s23 s34^3 s45^2 - 10 s15^3 s45^3 + 30 s15^2 s34 s45^3 - 30 s15 s34^2 s45^3 + 10 s34^3 s45^3 - 10 s12^2 s15^2 tr5 +
  20 s12^2 s15 s23 tr5 - 10 s12^2 s23^2 tr5 + 15 s12^2 s15 s34 tr5 - 15 s12^2 s23 s34 tr5 + 10 s12 s15 s23 s34 tr5 - 10 s12 s23^2 s34 tr5 - 6 s12^2 s34^2 tr5 - 15 s12 s23 s34^2 tr5 - 10 s23^2 s34^2 tr5 + 20 s12 s15^2 s45 tr5 -
  20 s12 s15 s23 s45 tr5 - 35 s12 s15 s34 s45 tr5 + 10 s12 s23 s34 s45 tr5 - 20 s15 s23 s34 s45 tr5 + 15 s12 s34^2 s45 tr5 + 20 s23 s34^2 s45 tr5 - 10 s15^2 s45^2 tr5 + 20 s15 s34 s45^2 tr5 - 10 s34^2 s45^2 tr5),

R[5] -> -1/(s12^3 s45^3) (10 s12^3 s15^3 - 30 s12^3 s15^2 s23 + 30 s12^3 s15 s23^2 - 10 s12^3 s23^3 + 30 s12^2 s15^2 s23 s34 - 60 s12^2 s15 s23^2 s34 + 30 s12^2 s23^3 s34 + 30 s12 s15 s23^2 s34^2 - 30 s12 s23^3 s34^2 + 10 s23^3 s34^3 +
  15 s12^3 s15^2 s45 - 30 s12^3 s15 s23 s45 + 15 s12^3 s23^2 s45 + 60 s12^2 s15 s23 s34 s45 + 30 s12 s15^2 s23 s34 s45 - 60 s12^2 s23^2 s34 s45 - 30 s12 s15 s23^2 s34 s45 - 30 s12 s15 s23 s34^2 s45 + 75 s12 s23^2 s34^2 s45 +
  30 s15 s23^2 s34^2 s45 - 30 s23^2 s34^3 s45 + 6 s12^3 s15 s45^2 - 6 s12^3 s23 s45^2 + 36 s12^2 s23 s34 s45^2 + 60 s12 s15 s23 s34 s45^2 + 30 s15^2 s23 s34 s45^2 - 60 s12 s23 s34^2 s45^2 - 60 s15 s23 s34^2 s45^2 + 30 s23 s34^3 s45^2 +
  6 s12^2 s15 s45^3 + 15 s12 s15^2 s45^3 + 10 s15^3 s45^3 - 6 s12^2 s34 s45^3 - 30 s12 s15 s34 s45^3 - 30 s15^2 s34 s45^3 + 15 s12 s34^2 s45^3 + 30 s15 s34^2 s45^3 - 10 s34^3 s45^3 + 10 s12^2 s15^2 tr5 - 20 s12^2 s15 s23 tr5 +
  10 s12^2 s23^2 tr5 + 20 s12 s15 s23 s34 tr5 - 20 s12 s23^2 s34 tr5 + 10 s23^2 s34^2 tr5 + 15 s12^2 s15 s45 tr5 + 10 s12 s15^2 s45 tr5 - 15 s12^2 s23 s45 tr5 - 10 s12 s15 s23 s45 tr5 - 10 s12 s15 s34 s45 tr5 +
  35 s12 s23 s34 s45 tr5 + 20 s15 s23 s34 s45 tr5 - 20 s23 s34^2 s45 tr5 + 6 s12^2 s45^2 tr5 + 15 s12 s15 s45^2 tr5 + 10 s15^2 s45^2 tr5 - 15 s12 s34 s45^2 tr5 - 20 s15 s34 s45^2 tr5 + 10 s34^2 s45^2 tr5)}
```

↳ From 5-Parton N^2LO amps [Abreu, Dorman, Febres Cordero, Ita, Page, Sotnikov '19]

Hope for 6pt N^2LO ?

→ Need better understanding of their <u>structure</u> ←

Foreshadowing: CAN COMPUTE THEM USING ON-SHELL DIAGRAMS!

— Setup and Simplifications —

6-particle QCD amplitudes @ 2 loops in the

planar limit with Large number of flavours

$$\begin{cases} \text{Colours} & N_c \to \infty \\ \text{flavours} & N_f \to \infty \end{cases} \quad \text{with} \quad \begin{cases} \lambda = g_{YM}^2 N_c & \text{fixed} \\ N_f / N_c & \text{finite} \end{cases}$$

Example:

$$\mathcal{A}(1^\pm \ldots 6^\pm) \sim \sum_\sigma \mathrm{Tr}[T^{a_{\sigma_1}} T^{a_{\sigma_2}} \ldots T^{a_{\sigma_6}}] \, A(\sigma(1) \ldots \sigma(6)) + \ldots$$

$$A_6^{(2)} = A_6^{(2),0} + \frac{N_f}{N_c} A_6^{(2),1} + \left(\frac{N_f}{N_c}\right)^2 A_6^{(2),2}$$

— Setup and Simplifications —

6-particle QCD amplitudes @ 2 loops in the

(1) **planar limit with Large number of flavours**

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(2) **Maximal Transcendental weight**

$$A_6^{(2)} \supset \sum_{i,j} c_{ij} R_i I_j$$ I_j : Transc. functions of $w = 4$

— Maximal Transcendental Weight —

In this approximation we capture

$$A_6^{(2)} \supset \sum_{i,j} C_{ij}^{(4)} R_i^{(4)} \mathcal{I}_j^{(\omega=4)}$$

How to think of $R_i^{(4)}$? [Henn, Torres Bobadilla '21]

$A^{(2)} = \int \omega^{(2)}$ Integrand for the Amplitude !

8 integrations

→ $\omega^{(2)}$ is decomposed into $\begin{cases} \mathcal{I}_i^{(4)} = \sum_j b_{ij} \overbrace{d\log(\alpha_{ij}^{1})}^{\text{Simple poles}} \wedge \ldots \wedge d\log(\alpha_{ij}^{8}) \\ + \\ \text{Double poles in } \ell \end{cases}$

Maximal weight Projection = Forget about double poles

$$\mathcal{P}(\omega^{(2)}) = \sum c_i \mathcal{I}_i^{(4)}$$

⇒ Prefactors are related to the maximal (8) residues of the integrand

— Maximal Transcendental Weight —
[4 = 4-2ε ?]

How to think of $R_i^{(4)}$? [Henn, Torres Bobadilla '21]

$$A^{(2)} = \int \omega^{(2)}$$ Integrand for the Amplitude?

Should uplift to $D = 4-2\varepsilon$ + Deal with divergences

→ R_i would get contributions from pure D-dim information
<u>But</u> appropriate IR subtraction removes beyond 4D effects (weight = 4)

Hard Function:

$$H^{(2)} = A^{(2)} - I^{(1)} A^{(1)} - I^{(2)} A^{(0)}$$

$$I^{(1)} = -\frac{1}{\varepsilon^2} \sum_i \left(\frac{-S_{i\,i+1}}{\mu^2} \right)^{-\varepsilon} \quad \text{and} \quad I^{(2)} = -\frac{1}{2} \left(I^{(1)} \right)^2$$

+

Successful bootstrap confirms completeness of R_i (Hindsight)

— Setup and Simplifications —

6-particle QCD amplitudes @ 2 loops in the

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$$A_6^{(2)} = A_6^{(2),0} + \frac{N_f}{N_c} A_6^{(2),1} + \left(\frac{N_f}{N_c}\right)^2 A_6^{(2),2}$$

(2) Maximal Transcendental weight

$$A_6^{(2)} \supset \sum_{i,j} C_{ij} R_i I_j \quad : \text{Transc. functions of } \omega = 4$$

(3) Symbol level

$$\mathcal{S}(A_6^{(2)}) = \sum C_{ij} R_i \, \mathcal{S}(I_j) \quad : \quad \mathcal{S} \sim \alpha_1 \otimes \alpha_2 \otimes \alpha_3 \otimes \alpha_4$$

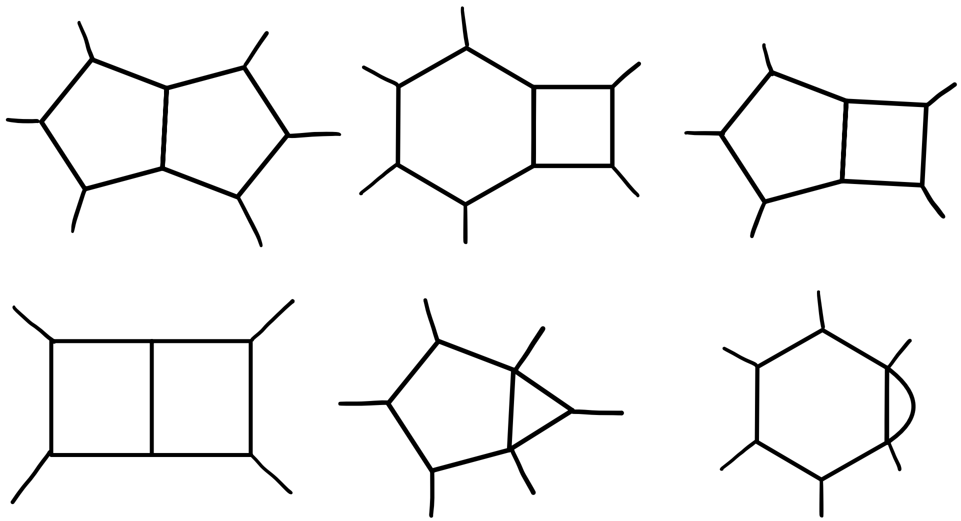
[Goncharov, Spradlin, Vergu, Volovich '10]

— Hexagon Functions @ 2-Loops —

[Abreu, Monni, Page, Usovitsch '24 ; Henn, Matijašić, Miczajka, Peraro, Xu, Zhang '25]

Features :

- 2 loop , $m=0$, 6-particle processes
- Answers are iterated dlog integrals with 167 4 dimensional letter alphabet



For our purposes :

Transcendental weight	1	2	3	4
# All symbols	9	62	319	945
# Two-loop six-point symbols	9	62	266	639
# Two-loop five-point one-mass symbols	9	59	263	594
# One-loop squared symbols	9	59	221	428
# Genuine two-loop six-point symbols	0	0	3	45

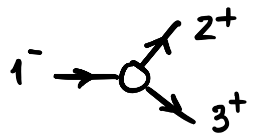
→ Function Space is bigger than pure 2-loops because of IR Subtraction !

$$H^{(2)} = A^{(2)} \underbrace{- I^{(1)} A^{(1)} - I^{(2)} A^{(0)}}_{\text{Need these too !}}$$

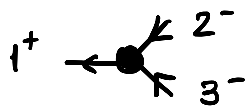
— Leading Singularities = On-shell Diagrams —

Pre-factors $R_i \in \text{Span}\{\text{maximal cuts}\}$

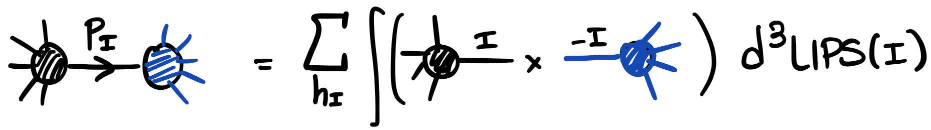
maximal Cuts = "Glue" trees [Arkani-Hamed, Bourjaily, Cachazo, Goncharov, Postnikov, Trnka '12]

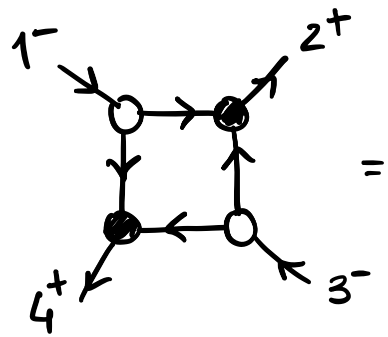


$$= \frac{[23]^4}{[12][23][31]} \delta^4(p)$$



$$= \frac{\langle 23\rangle^4}{\langle 12\rangle\langle 23\rangle\langle 31\rangle} \delta^4(p)$$

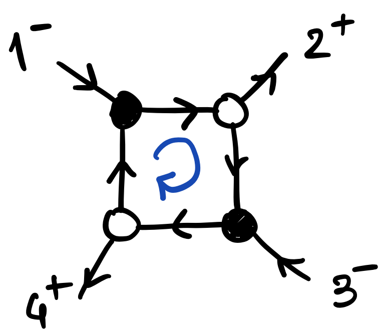




$$= \frac{\langle 13\rangle^4}{\langle 12\rangle\langle 23\rangle\langle 34\rangle\langle 41\rangle}$$

¡Parke-Taylor!
(Tree-level amp)

But also ... Diagrams with helicity cycles!



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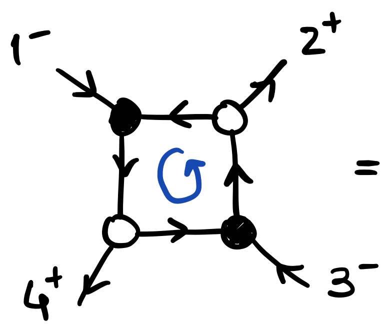
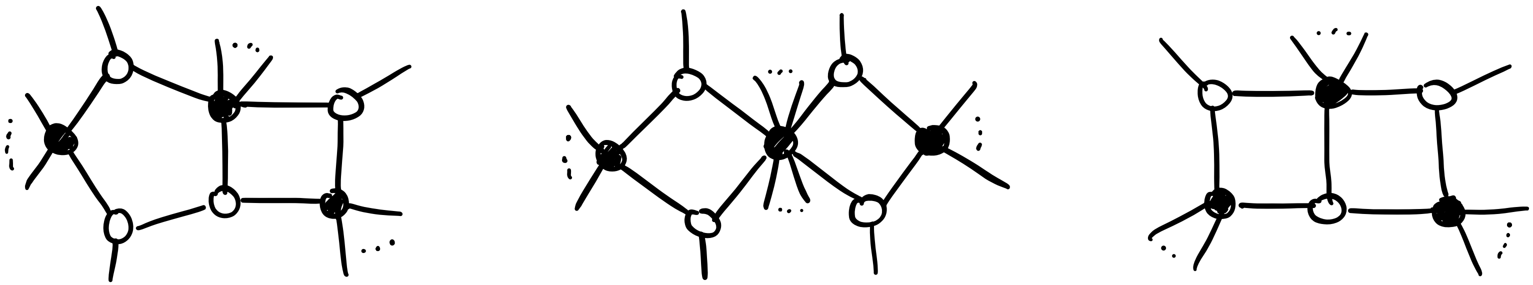


$$= \frac{\langle 13\rangle^4}{\langle 12\rangle\langle 23\rangle\langle 34\rangle\langle 41\rangle}\left[\left(\frac{\langle 12\rangle\langle 34\rangle}{\langle 13\rangle\langle 24\rangle}\right)^4 + \left(\frac{\langle 14\rangle\langle 23\rangle}{\langle 13\rangle\langle 24\rangle}\right)^4\right]$$

¡New pre-factor @ 1 loop!

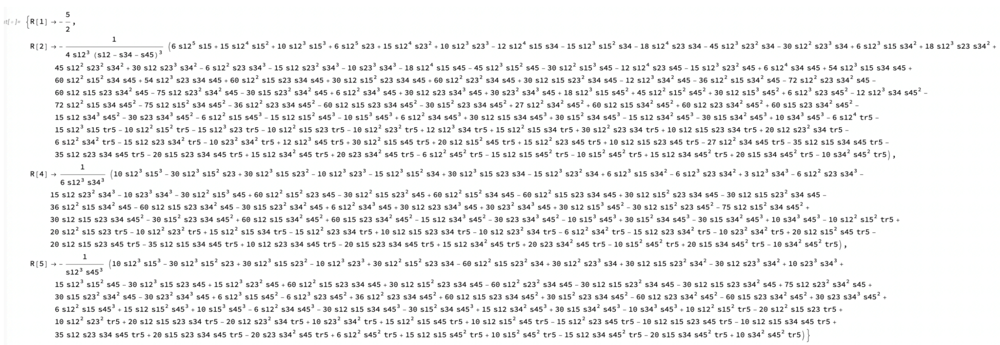
— Leading Singularities at two-loops —

For MHV amplitudes, we only need 3 topologies



All Conformal invariant $\sum_{i=1}^{n} \frac{\partial^2}{\partial \lambda_i \partial \tilde{\lambda}_i}$ [Witten '03; Henn, Power, Zoia '19]

Major Simplification!



$$\left\{1, -\frac{(\langle 12\rangle\langle 34\rangle - 2\langle 13\rangle\langle 24\rangle)\left(-10\langle 12\rangle\langle 13\rangle\langle 24\rangle\langle 34\rangle + \langle 12\rangle^2\langle 34\rangle^2 + 10\langle 13\rangle^2\langle 24\rangle^2\right)}{\langle 12\rangle^3\langle 34\rangle^3},\right.$$
$$-\frac{(\langle 12\rangle\langle 35\rangle - 2\langle 13\rangle\langle 25\rangle)\left(-10\langle 12\rangle\langle 13\rangle\langle 25\rangle\langle 35\rangle + \langle 12\rangle^2\langle 35\rangle^2 + 10\langle 13\rangle^2\langle 25\rangle^2\right)}{\langle 12\rangle^3\langle 35\rangle^3},$$
$$\left.-\frac{(\langle 12\rangle\langle 45\rangle - 2\langle 14\rangle\langle 25\rangle)\left(-10\langle 12\rangle\langle 14\rangle\langle 25\rangle\langle 45\rangle + \langle 12\rangle^2\langle 45\rangle^2 + 10\langle 14\rangle^2\langle 25\rangle^2\right)}{\langle 12\rangle^3\langle 45\rangle^3}\right\}$$

Conformal invariance

— The Bootstrap v2.0: Prescriptive Unitarity Edition —

→ Usual Bootstrap approach is Naive:

$$H_6^{(2)} = \sum_{i,j} C_{ij} \, R_i \, \mathcal{I}_j$$

C_{ij} – All R_i are Paired w/ All $\mathcal{I}_j$

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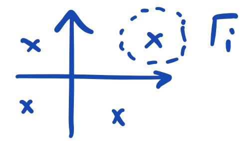
→ Prescriptive Unitarity Partially Solves this! [Bourjaily, Hermann, Trnka '17]

(1) 2-loop Leading Singularities $\hat{R}_i = \oint_{\Gamma_i} \omega^{(2)}$



(2) Prescriptive Basis G_i is Diagonal in Γ_i

(3) Improved Ansatz:

$$H_6^{(2)} = \sum_{L=0,1} C_{ij}\, R_i^{(L)}\, \mathcal{I}_j + \sum R_i^{(2)}\, G_i$$

— The Bootstrap v2.0 : Prescriptive Unitarity Edition —

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$$H_6^{(2)} = \sum_{i,j} C_{ij}\, R_i\, \mathcal{I}_j$$

C_{ij} – All R_i are Paired w/ All $\mathcal{I}_j$

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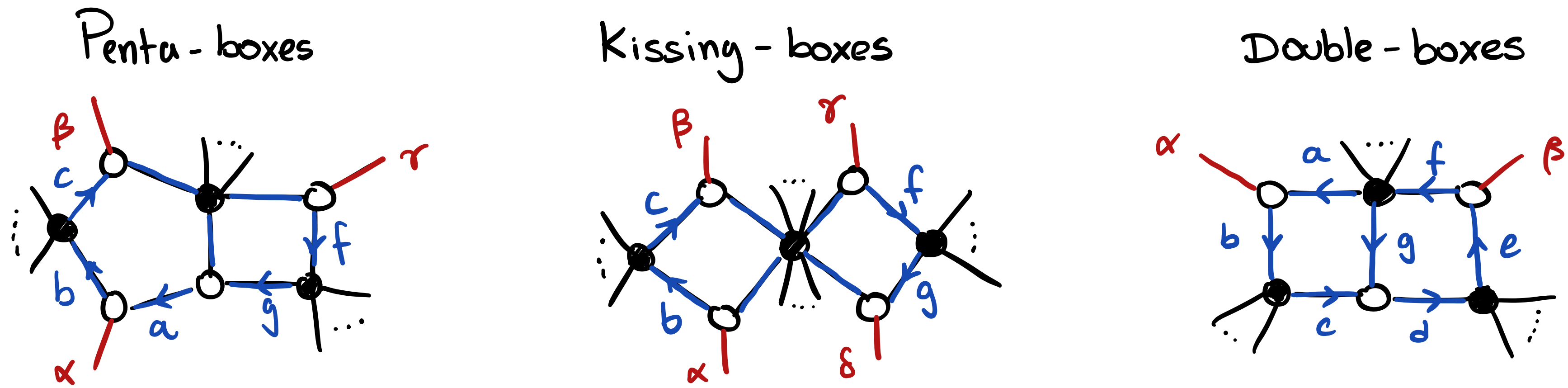
Improved Ansatz

$$H_6^{(2)} = \sum_{L=0,1} C_{ij}\, R_i^{(L)}\, \mathcal{I}_j + \sum R_i^{(2)}\, G_i$$

2-Loop MHV Basis G_i : [Bourjaily, Hermann, Langer, McLeod, Trnka '19]



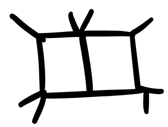
— Structure of the Results —

Prescriptive Unitarity gives a good organizing principle

But it's not enough! Still need to bootstrap the remaining C_{ij}

The final result looks like:

$$
\begin{cases}
H(--++++) = PT\, f_0 + \sum R^{db} G^{db} \\
H(-+-+++) = PT\, g_0 + \sum R_i^{(1)} g_i + \sum R^{Pb} G^{Pb} + \sum R^{db} G^{db} \\
H(-++-++) = PT\, h_0 + \sum R_i^{(1)} h_i + \sum R^{kb} G^{kb} + \sum R^{Pb} G^{Pb} + \sum R^{db} G^{db}
\end{cases}
$$

In $\mathcal{N}=4$ SYM, no poles @ ∞ $\Rightarrow$ No 

Indeed:

$$
H^{(2)}_{YM} \xrightarrow[R^{db} \to 0]{R^{\alpha \neq db} \to 1} H^{(2)}_{\mathcal{N}=4}
$$

¡Generalization of the maximal transcendentality Principle!

— Summary and Outlook —

Take home messages:

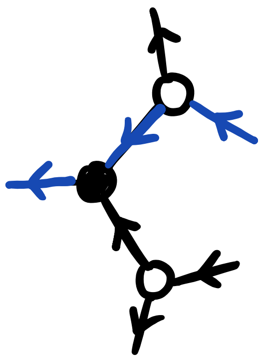
1. Not a usual Symbol bootstrap because we have the function Space!

2. Rational prefactors @ maximal weight = On-shell diagrams!

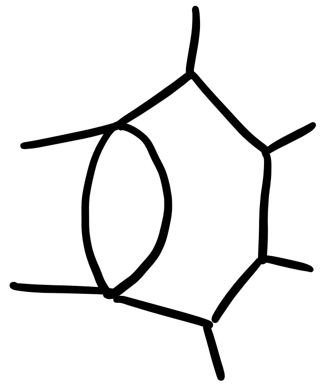
3. Prescriptive unitarity dramatically Simplifies our result
   + Highlights a general structure

Future:

- Other helicity Sectors? NMHV?
- Lower weight prefactors? Rational terms?
- Uplift of Symbol → function level? Nice positive properties?
- MHV for all multiplicity?



THANK YOU!



Back-up Slides

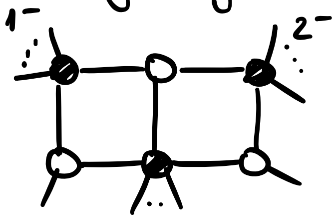
— Function Space Bootstrap —

Working example: $1^- 2^- 3^+ 4^+ 5^+ 6^+$ in pure Yang-Mills

Recall $\quad \mathcal{S}(\mathcal{A}) = \sum_{i,j} C_{ij} \times R_i \times I_j$

C_{ij} → **Unknowns**
$\# = 2412$
(Naive Counting $\sim 7k$)

R_i → 7 different DOUBLE-BOX Leading Singularities



I_j → 945 hexagon functions from Differential Eqs

↓

Fix them by imposing Physical Constraints

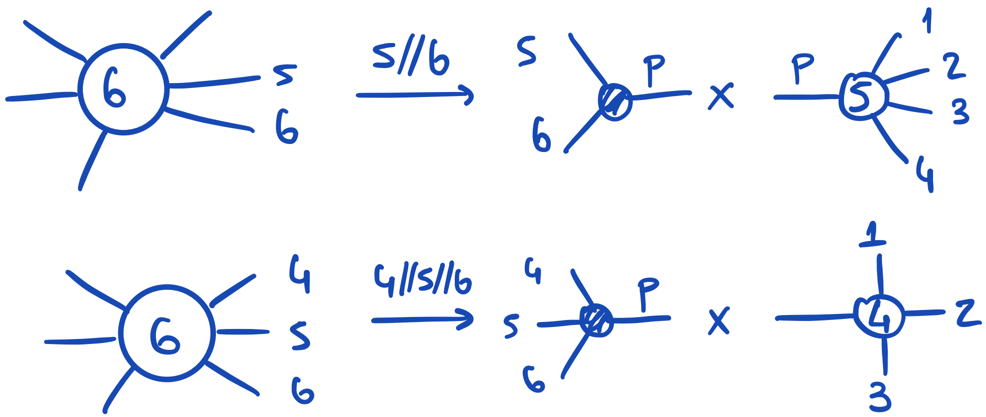


Table I. Two-loop bootstrap constraints for $\mathcal{H}^{(2)}_{\rm YM}$

Condition on $\mathcal{H}^{(2)}_{\rm YM}$	No. of constraints
dimensionless symbols G	996
no spurious/high-order poles	
$\langle 36 \rangle = 0$	333
$\langle 35 \rangle = 0$	614
$\langle 34 \rangle = 0$	629
$\langle 45 \rangle = 0$	343
collinear limit $p_5 \vert\vert p_6$	
$--++++$	1785
$+--+++$	1307
$++--++$	1785
$+++--+$	1646
$-++++-$	1646
triple collinear limit $p_4 \vert\vert p_5 \vert\vert p_6$	
$--++++$	1836
$++--++$	724
Total	2412

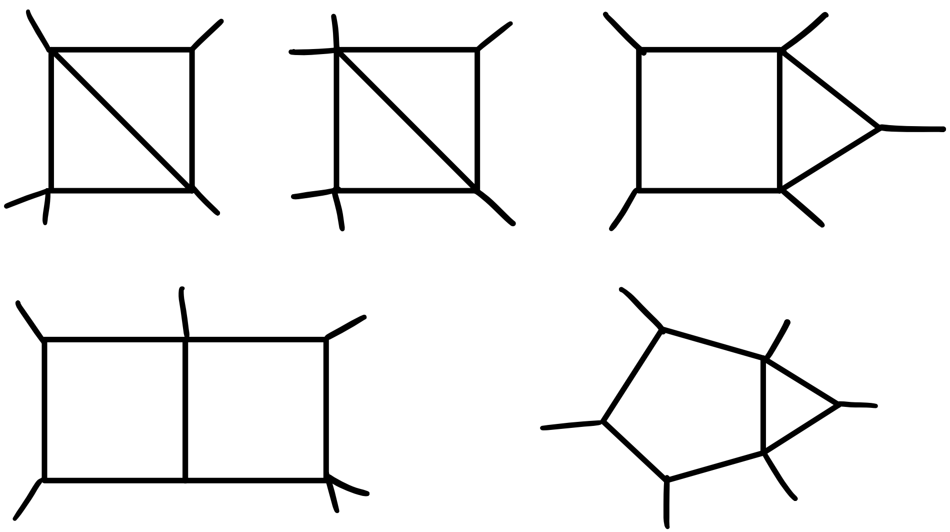
¡ UNIQUELY FIXED !

— Singularities of Bootstrapped Result —

$\mathcal{I}_j$ have a 167 letter alphabet $\longrightarrow$ $H_6^{(2)}(--++++)$ has 137 letters

Among 137 we have 48 rational letters coming from two-loop topologies

Come from Landau analysis of



Conjecture: It's the complete 6-pt 2 loop MHV alphabet for QCD

Cluster algebraic interpretation:

[Bossinger, Li '24]

[Pokraka, Spradlin, Volovich, Weng '25]

[Bossinger, Drummond, Glew, Gürdoğan, Wright '25]

[Aliaj, Dian, Papathanasiou '26]