



Climbing the Spin Tower

Spinning Black Holes from fixed-Spin Theories

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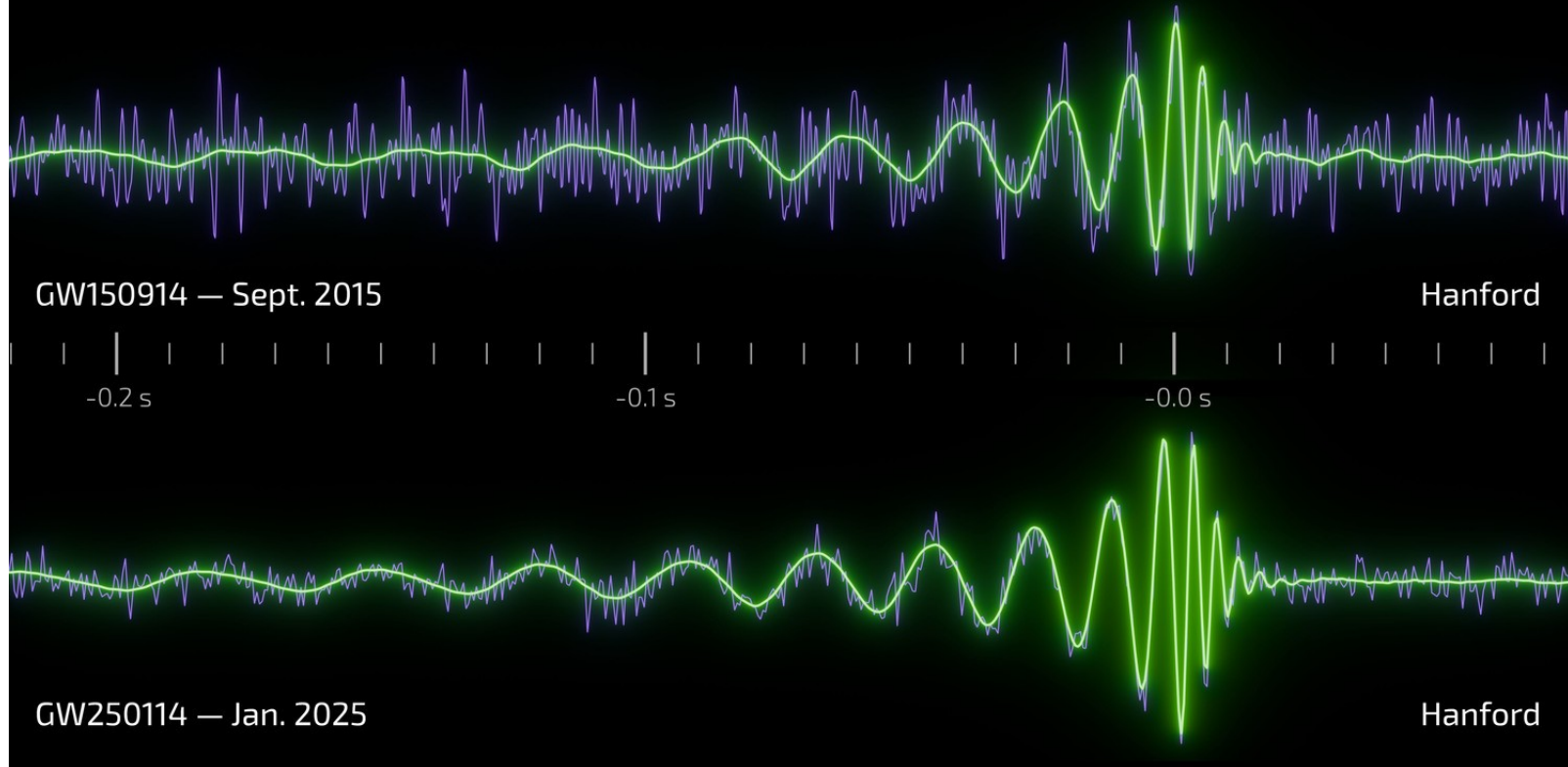


Introduction

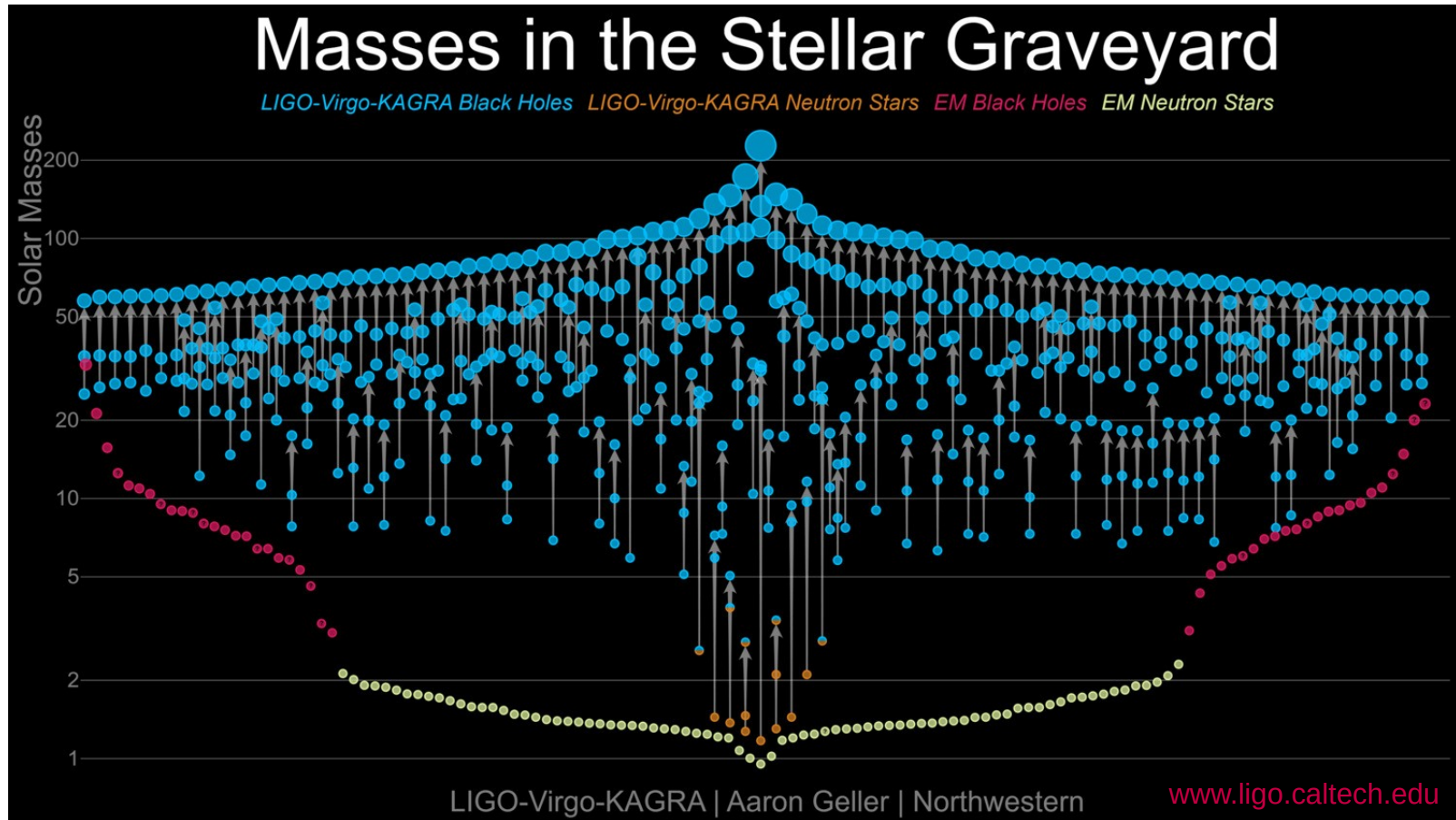
Gravitational Waves enter the Precision Era

10 Years Later: LIGO Hears Loud and Clear

www.ligo.caltech.edu



The Population of Binary Systems

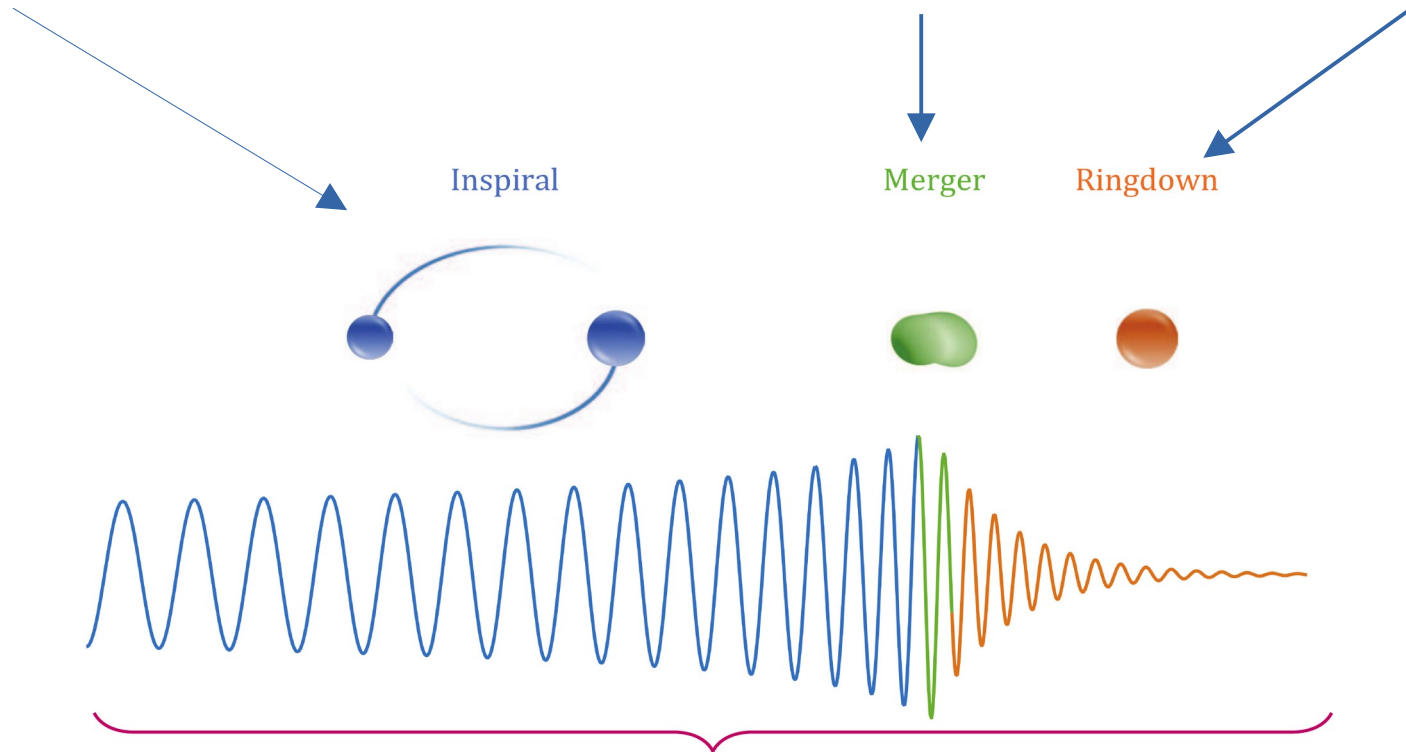


Stages of a Binary Merger

Perturbation Theory

Numerical GR

BHPT



Waveform

[Antelis, Moreno, EPJP 132 (2017) 1,10]

Future Gravitational Wave Observatories

Future ground based Observatories

- Advanced LIGO
- Einstein Telescope
- Cosmic Explorer

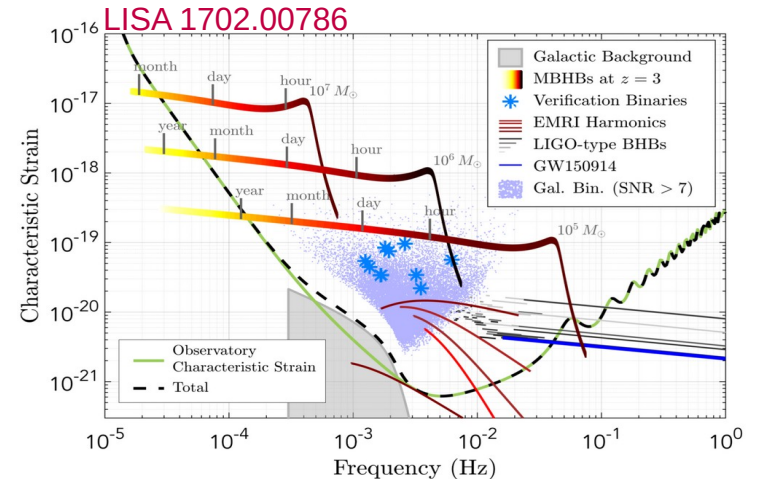
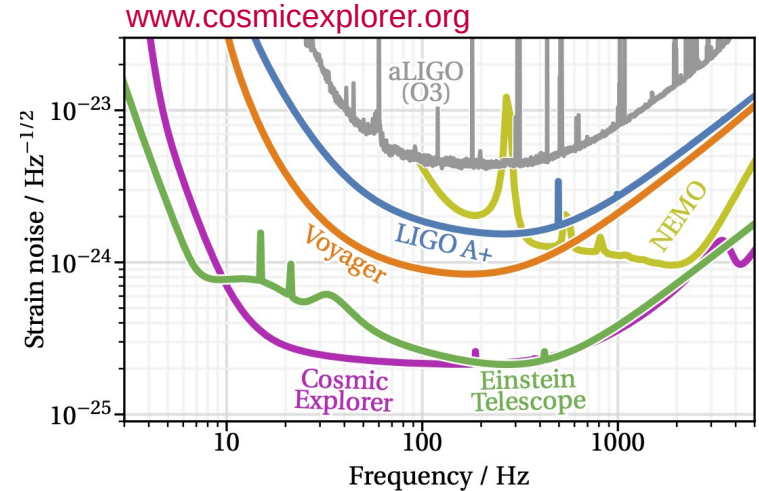
Future space based Observatories

- LISA
- TianQin
- Taiji

Physics goals:

- Tests of General Relativity
- Black Hole Formation
- Neutron Star Equation of State

Requires theoretical modeling of perturbative inspiral phase up to $\mathcal{O}(G^7)$



Non-Spinning Black Holes

General Relativity as a QFT

- The Action

$$S[\phi, g_{\mu\nu}] = \int d^4x \sqrt{-g} \left[-\frac{2}{\kappa^2} R + \frac{1}{2} \sum_{i=1}^2 (g^{\mu\nu} (\partial_\mu \phi_i)(\partial_\nu \phi_i) - m_i^2 \phi_i^2) \right]$$

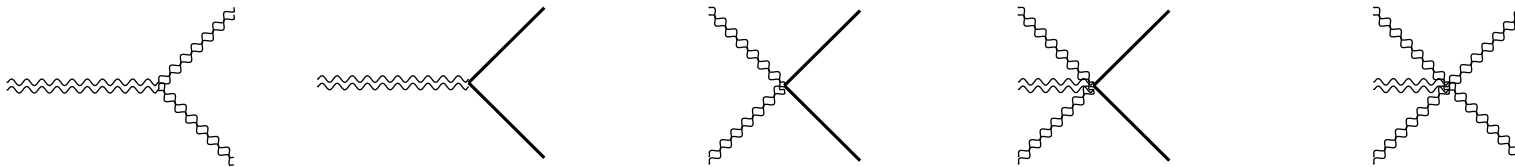
- Expand spacetime around flat space

$$g_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu} \quad \kappa = \sqrt{32\pi G}$$

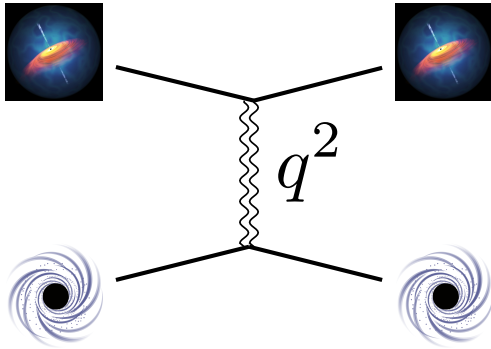
- General Relativity is **non-linear**

$$g^{\mu\nu} = \eta^{\mu\nu} - \kappa h^{\mu\nu} + \kappa^2 h^{\mu\lambda} h^\nu{}_\lambda + \dots$$

- Generates **infinite** many Feynman Rules



Quantum Field Theory at work - I



$$A^{\text{tree}} = \frac{4\pi G}{E_1 E_2} \frac{m_1^2 m_2^2 (1 - 2\sigma^2)}{q^2} + \dots$$

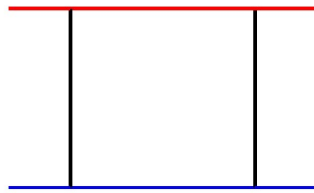
The classical potential is given by

$$\begin{aligned} V(r) &= \int \frac{d^3 q}{(2\pi)^3} e^{i\vec{q}\cdot\vec{r}} A^{\text{tree}} = -\frac{Gm_1 m_2}{r} \left(\frac{m_1 m_2}{E_1 E_2} (2\sigma^2 - 1) \right) \\ &= -\frac{Gm_1 m_2}{r} + \mathcal{O}(v^2) \end{aligned}$$

- $q \sim \hbar \rightarrow 0$ reveals the classical physics
- Amplitudes provide full **special relativistic** corrections
- Loop-Amplitudes provide higher-order $\mathcal{O}(G^n)$ corrections

Quantum Field Theory at work - II

$$p_1 = \bar{p}_1 - q/2 \quad p_4 = \bar{p}_1 + q/2$$



$$= \int \frac{d^4 \ell}{(2\pi)^4} \frac{1}{\ell^2 (\ell - q)^2 [(\ell + p_1)^2 - m_1^2] [(\ell - p_2)^2 - m_2^2]}$$

$$p_2 = \bar{p}_2 + q/2 \quad p_3 = \bar{p}_2 - q/2$$

$$= \frac{\hbar^4}{\hbar^4} \int \frac{d^4 \ell}{(2\pi)^4} \frac{1}{\ell^2 (\ell - q)^2} \left[\frac{1}{\hbar^2} \frac{1}{[2(\ell \cdot \bar{p}_1) + i\epsilon][-2(\ell \cdot \bar{p}_2) + i\epsilon]} + \mathcal{O}(\hbar) \right]$$

$$l \sim q \sim \hbar$$

High-Rank Tensor integrals can be dropped

$$\int \frac{d^4 \ell}{(2\pi)^4} \frac{\ell^\mu \ell^\nu \ell^\alpha \ell^\beta}{\ell^2 (\ell - q)^2 [(\ell + p_1)^2 - m_1^2] [(\ell - p_2)^2 - m_2^2]} \sim \mathcal{O} \left(\frac{\hbar^4}{\hbar^2} \right) = \mathcal{O}(\hbar^2)$$

[Parra-Martinez, Ruf, Zeng arXiv:2005.04236]

Spinning Black Holes

- Massive spinor helicity amplitudes (spin exponentiation) [Arkani-Hamed, Huang, Huang, '17 ...] High ranks for tensor integrals, difficult IBP reduction beyond one-loop
- Non-transverse higher-spin fields [Bern, Luna, Roiban, Shen, Zeng '20, Alavverdian, Bern, Kosmopoulos, Luna, Roiban, Scheopner, Teng '24] Needs separation between spin-magnitude changing effects and conservative dynamics; not yet achieved beyond one-loop
- **Finite-spin massive particles** [Holstein, Ross '07, Vaidya '14, Maybee, O'Connell, Vines '19, Damgaard, Haddad, Helset '19, Aoude, Haddad, Helset '20]
 - Straightforward option for > 1 -loop calculations
 - Spin- j accesses $\mathcal{O}(S^{2j})$ perturbative multipole moments
 - Plagued by **Casimir ambiguity**, resolved by spin-interpolation
- Beyond amplitudes: PMEFT with spin [Liu, Porto, Yang], WQFT [Jakobsen, Mogull, Plefka, Steinhoff '21, Haddad, Jakobsen, Mogull, Plefka '24], Generalized Wilson Lines [Bonocore, Kulesza, Pirsch '25]

State-of-the-art for Spin effects

- High orders for spin at 1-loop [Bern, Kosmopoulos, Luna, Roiban, Teng '22; Aoude, Haddad, Helset '22,'23; Bohnenblust, Cangemi, Johansson, Pichini '24]
- Gravitational Waveform at $\mathcal{O}(G^3 S^2)$ [Bohnenblust, Ita, MK, Schlenk '23,'25]
- Conservative + dissipative effects in WQFT at $\mathcal{O}(G^3 S^2)$ [Jakobsen, Mogull, '22]
- All-order-in-spin radiation reaction [Alessio, Di Vecchia, '22]
- Energy loss from PMEFT [Riva, Vernizzi, Wong '22]
- Three-loop in WQFT $\mathcal{O}(G^4 S^1)$ [Jakobsen, Mogull, Plefka, Sauer, Xu '23]
- Conservative dynamics at $\mathcal{O}(G^3 S^2)$ from amplitudes [Febres Cordero, MK, Lin, Ruf, Zeng '22; Akpinar, Febres Cordero, MK, Ruf, Zeng '24]
- Unprecedented results at $\mathcal{O}(G^3 S^4)$ [Akpinar, Febres Cordero, MK, Smirnov, Zeng '25]

Spin of a Spinning Black Hole?

- Soft expansion performed as in the case of non-spinning BHs
- Form-factor decomposition, e.g

$$\mathcal{A}(\phi V \rightarrow \phi V) = \sum_{n=1}^4 M_n \epsilon_\mu(p_1, \lambda_1) T_n^{\mu\nu} \epsilon_\nu^*(p_4, \lambda_4)$$

$$T_1 = (\epsilon_1 \cdot \epsilon_4^*)$$

$$T_2 = (\epsilon_1 \cdot q)(\epsilon_4^* \cdot q)$$

$$T_3 = q^2(\epsilon_1 \cdot \bar{p}_2)(\epsilon_4^* \cdot \bar{p}_2)$$

$$T_4 = (\epsilon_1 \cdot \bar{p}_2)(\epsilon_4^* \cdot q) - (\epsilon_1 \cdot q)(\epsilon_4^* \cdot \bar{p}_2)$$

- Spin dependence obtained from

$$\bar{\epsilon}^{\star\mu} \bar{\epsilon}^\nu = - \left(\eta^{\mu\nu} - \frac{\bar{p}_1'^{\mu} \bar{p}_1'^{\nu}}{m_1^2} \right) + \frac{i}{2m_1} \epsilon^{\mu\nu\alpha\beta} \bar{p}_{1\alpha}' S_\beta - \frac{1}{2} (S^\mu S^\nu + S^\nu S^\mu),$$

- Complicated expression at higher spin

Spin Universality and Ambiguity

- Fixed spin- j amplitudes have access to $\mathcal{O}(S^{2j})$ multipoles

$$\mathcal{A}(\phi_1\phi_2 \rightarrow \phi_1\phi_2) \sim \mathbb{1}, \mathcal{A}(\phi\psi \rightarrow \phi\psi) \sim \{\mathbb{1}, S \cdot l\},$$

$$\mathcal{A}(\phi V \rightarrow \phi V) \sim \{\mathbb{1}, S \cdot l, (q \cdot S)^2, (p \cdot S)^2\}$$

- **Spin Universality:** (conjecture!)
Classical multipole moments do not depend on the nature of the test particle

- For arbitrary spin there should be a fifth spin-operator

$$O_5 = q^2 S^2 \sim \frac{\hbar^2}{\hbar^2} = 1, \quad O_5^{\text{spin-}j} = q^2 S^2 = q^2 j(j+1)\mathbb{1}$$

- 1-Loop example: Classical physics lives at $1/|q|$ and quantum physics at $|q|$

$$\mathcal{A} = \frac{1}{|q|} [c_0\mathbb{1} + c_5(q^2 S^2) + \dots] = \frac{1}{|q|} [c_0 + c_5 q^2 j(j+1)] \mathbb{1}$$

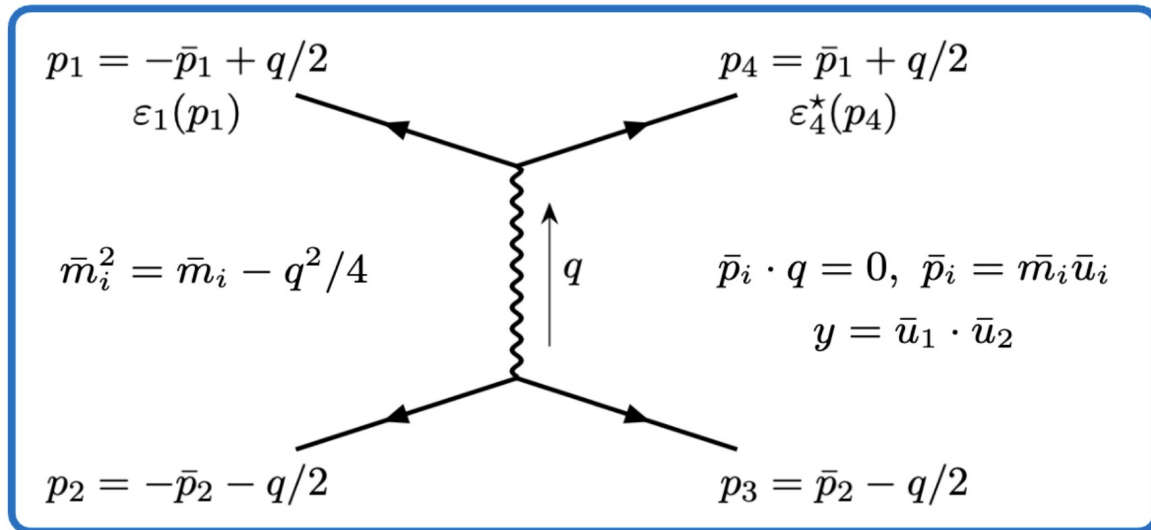
- Casimir operator indistinguishable from Quantum physics!

→ Spin Interpolation!

A Tower of Spin Theories

- Consider the following Scattering Processes

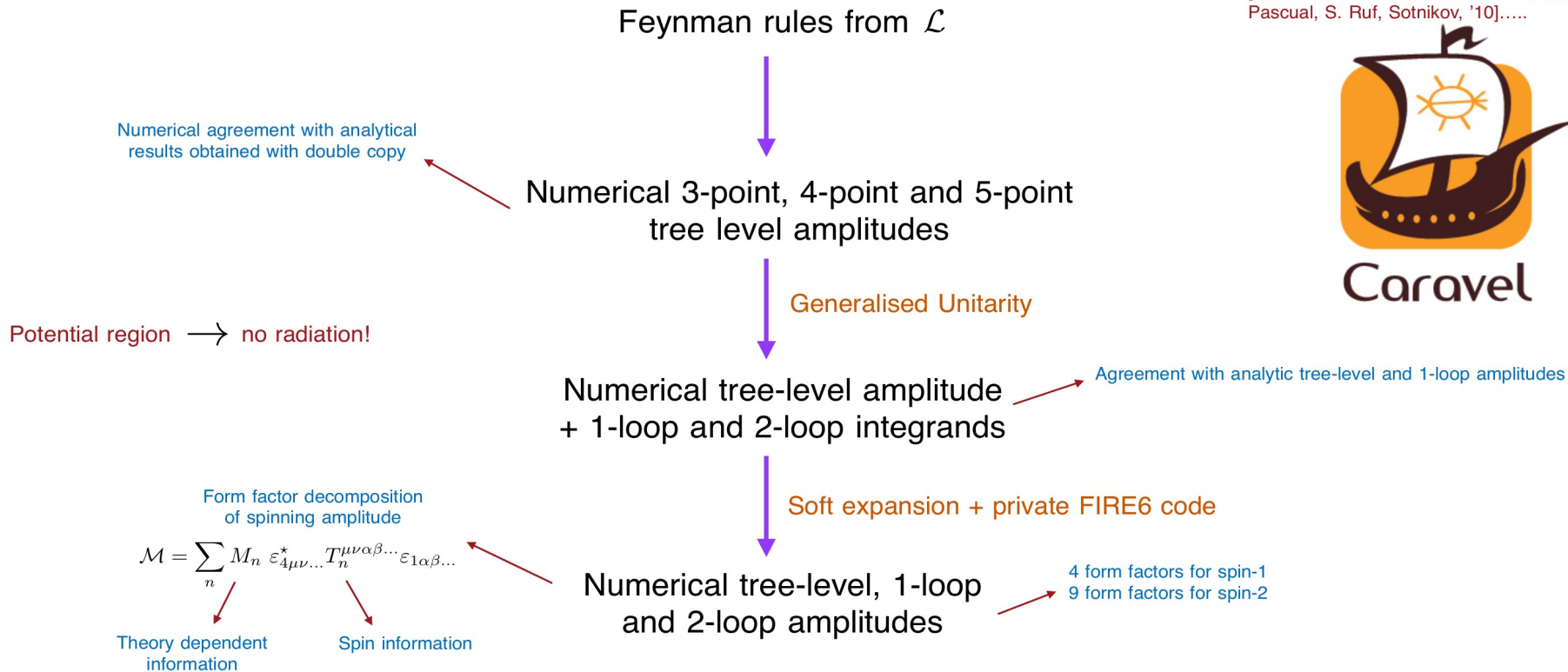
$$X(p_1, \epsilon_1) + \phi(p_2) \rightarrow \phi(p_3) + X(p_4, \epsilon_4) \quad X = \{\Phi, V^\mu, H^{\mu\nu}\}$$



$$\frac{\mathcal{L}}{\sqrt{-g}} = \frac{2R}{\kappa^2} + \frac{1}{2} g^{\mu\nu} (\partial_\mu \phi)(\partial_\nu \phi) - \frac{1}{2} m_2^2 \phi^2 + \mathcal{L}_{m_1}$$

A Tower of Spin Theories

[Abreu, Dormans, Febres Cordero, Ita, Kraus, Page, Pascual, S. Ruf, Sotnikov, '10].....



Spin in Quantum Field Theory

- Spin operator for massive spin-s boson

$$S_{ij}^\mu = \frac{(-1)^s}{2m} \epsilon^{\mu\lambda\alpha\beta} p_\lambda \epsilon_{iI(s)}^* (p) (\Sigma_{\alpha\beta})_{J(s)}^{I(s)} \epsilon_j^{J(s)} (p)$$

- With the Lorentz-Generators in the spin-s representation

$$(\Sigma_{\alpha\beta})_{J(s)}^{I(s)} = 2is \delta_{[\alpha}^{I_1} \eta_{\beta]} (J_1 \delta_{J_2}^{I_2} \cdots \delta_{J_s}^{I_s})$$

- The classical spin vector can be seen as the expectation value

$$S^\mu = \sum_{i,j} \omega_i^* S_{ij}^\mu \omega_j \quad \sum_i |\omega_i|^2 = 1$$

- Products of classical spin vectors → Symmetric products of spin operators

$$S^\mu S^\rho \cdots S^\sigma S^\nu = \sum_{i,k,\dots} \omega_i^* S_{ik}^{(\mu} S_{kj}^{\rho} \cdots S_{rm}^{\sigma} S_{mj}^{\nu)} \omega_j$$

Spin Interpolation - I

- Write Ansatz for the amplitude valid for generic spin representation

$$\{1, i(n \cdot S), (q \cdot S)^2, q^2 S^2, q^2 (\bar{p}_2 \cdot S)^2\} \quad n^\mu = \epsilon^{\mu\nu\alpha\beta} q_\nu \bar{p}_{1\alpha} \bar{p}_{2\beta}$$

- Ansatz for the finite part of the amplitude reads

$$\mathcal{M}_{fin}^{(L)} = \frac{\kappa^{2+2L}}{(-q^2)^{1-L/2+L\epsilon}} \sum_{n=0}^4 \sum_m \alpha^{(L,n,m)} \mathcal{O}^{(n,m)}$$

- Spin Universality
→ spin coefficients (**classical AND quantum**) are independent of spin representation!
- Spin structures $\mathcal{O}^{(n,m)}$ organized in terms of

$$\Omega_{\pm} = (q \cdot S)^2 \pm q^2 S^2$$

Spin Interpolation - II

- To fix the coefficients we **evaluate the Ansatz** for $s = 0, 1, 2$ and expand up to $\mathcal{O}(q^4)$
 - 14 classical coefficients, given by the leading q contribution of each structure
 - 6 quantum suppressed coefficients

- For each representation, project onto the appropriate **form factor basis**

$$\mathcal{A} = \sum_n M_n \epsilon_{4,\mu\nu,\dots}^* T_n^{\mu\nu\alpha\beta\dots} \epsilon_{1\alpha\beta\dots}$$

- Solve a linear system of equations by demanding equality with the corresponding spinning amplitude
 - 31 linear relations, of which **11 are linearly dependent**
 - 20 linearly independent relations fixes the $1+5+14=20$ spin coefficients

Obtained for the first time results at $\mathcal{O}(G^3 S^4)$

Spin-Shift Symmetry

- Spin-Shift Symmetry states the Amplitude is invariant under

$$S^\mu \rightarrow S^\mu + \xi q^\mu$$

One-loop conjecture
[Bern, Kosmopoulos, Luna, Roiban, Teng '22]
[Aoude, Haddad, Helset '22]

- The only possible way to organize Spin multipoles

$$c_+ \Omega_+ = c_+ [(q \cdot S)^2 + q^2 S^2] \Rightarrow c_+ = 0$$

$$c_- \Omega_- = c_- [(q \cdot S)^2 - q^2 S^2] \Rightarrow c_- \neq 0$$

- The two-loop quartic in spin amplitude organizes itself as

$$\alpha^{(2,n,m)} = \alpha_{\text{probe}}^{(2,n,m)} + m_1^3 m_2^3 \alpha_{\text{1SF}}^{(2,n,m)}$$

Spin-Shift symmetric

Spin-Shift Symmetry breaking

First time spin-shift symmetry observed up to $\mathcal{O}(G^3 S^4)$

The radial action and Observables

- Extract the radial action from the finite part of the Amplitude:

$$I_r = \int \frac{d^4 q}{(2\pi)^2} \delta(p_1 \cdot q) \delta(-p_2 \cdot q) e^{ib \cdot q} \mathcal{M}_{fin}$$

Agreement with [Gonzo, Shi, '24] in the limit of a spinless probe in a Kerr background to quartic order in spin

- Dirac Brackets to compute observables

Following [Gonzo, Shi '24]

$$\Delta \lambda^\mu = \sum_j \frac{1}{j!} \underbrace{\{I_r, \{I_r, \dots, \dots, \{I_r, \lambda^\mu\} \dots\}}_{j \text{ times}} \quad \text{for} \quad \lambda^\mu = \{u_1^\mu, u_2^\mu, a_1^\mu, a_2^\mu\}$$

$$a_i = S_i / m_i$$

$$\{b^\mu, u_i^\nu\} = -\text{sgn}_i \frac{(y^2 - 1)b^\mu b^\nu + l^\mu l^\nu}{m_i b^2 (y^2 - 1)}, \quad \{b^\mu, a_i^\nu\} = \text{sgn}_i \frac{u_i^\nu ((y^2 - 1)b^\mu (b \cdot a_i) + l^\mu (l \cdot a_i))}{m_i b^2 (y^2 - 1)}$$

$$\{b^\mu, b^\nu\} = \frac{(b^2 (y u_2^\mu - u_1^\mu) - l^\mu (a_1 \cdot u_2)) b^\nu - (\mu \leftrightarrow \nu)}{m_1 b^2 (y^2 - 1)} + (1 \leftrightarrow 2, b \rightarrow -b), \quad \{a_i^\mu, a_i^\nu\} = \frac{\epsilon^{\mu\nu\rho\sigma} u_i^\rho a_i^\sigma}{m_i}$$

Validation of our results

- Agreement with the aligned-spin scattering angle in **post-Newtonian results** [Bautista, Khalil, Sergola, Kavanagh, Vines, '24] through $\mathcal{O}(G^3 S^4)$
- Conservative observables from Dirac Brackets agree with our previous $\mathcal{O}(G^3 S^2)$ results from **KMOC** [Febres Cordero, MK, Lin, Ruf, Zeng '22] and with **WQFT** results [Jakobsen, Mogull '22]
- Using all-order-in-spin radiation reaction amplitudes [Alessio, Di Vecchia '22] and acting with Dirac Brackets, obtain **cancellation of high-energy divergences** in observables through $\mathcal{O}(G^3 S^4)$, previously observed through $\mathcal{O}(G^3 S^2)$ using WQFT [Jakobsen, Mogull '22]
- Strong and highly non-trivial checks provide confidence in our partly conjectural setup!

Conclusions & Outlook

Conclusions & Outlook

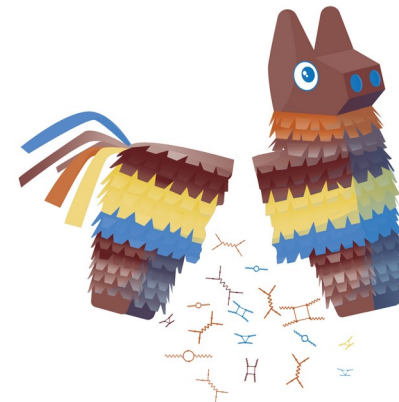
- First calculation of $\mathcal{O}(G^3 S^4)$ **conservative dynamics** of binary Kerr Black Hole using scattering amplitudes from two-loop numerical unitarity!

What is next?

- Dissipative effects?
 - Higher-loop orders?
 - Higher spin?
-
- What if we are interested in Neutron stars?
 - Higher-dimensional Operators
 - How to do Spin interpolation?
-
- What is the origin of the Spin-Shift symmetry?!

Outlook

- The last 3.5 years I spent forming a new research group in Mexico
- Research Topics: Amplitudes, Top-quark Pheno, Gravitational Waves
- Current Team: (1 Bachelor, 2 Master, 1 PhD, 2 Postdocs)



Particle Physics	Gravitational Waves
Nataly Almanza Avila 🇨🇴	Leonardo David Espinosa Roldán 🇲🇽
Luz Silvana Rodríguez Meléndez 🇲🇽	Allan Eduardo Alonzo Artiles 🇲🇽
Dr. Edier Mauricio Paredes Cruz 🇲🇽	Dr. César Moctezuma Mata Zamora 🇲🇽

- More national collaborators:
Dr. Fabiola Fortuna (UNAM) 🇲🇽
Dr. Jonathan Reyes Pérez (BUAP) 🇲🇽

**Watch out for new results
from Mexico!**