Brookhaven National Laboratory

Energy Correlators and 3D Hadron Structure

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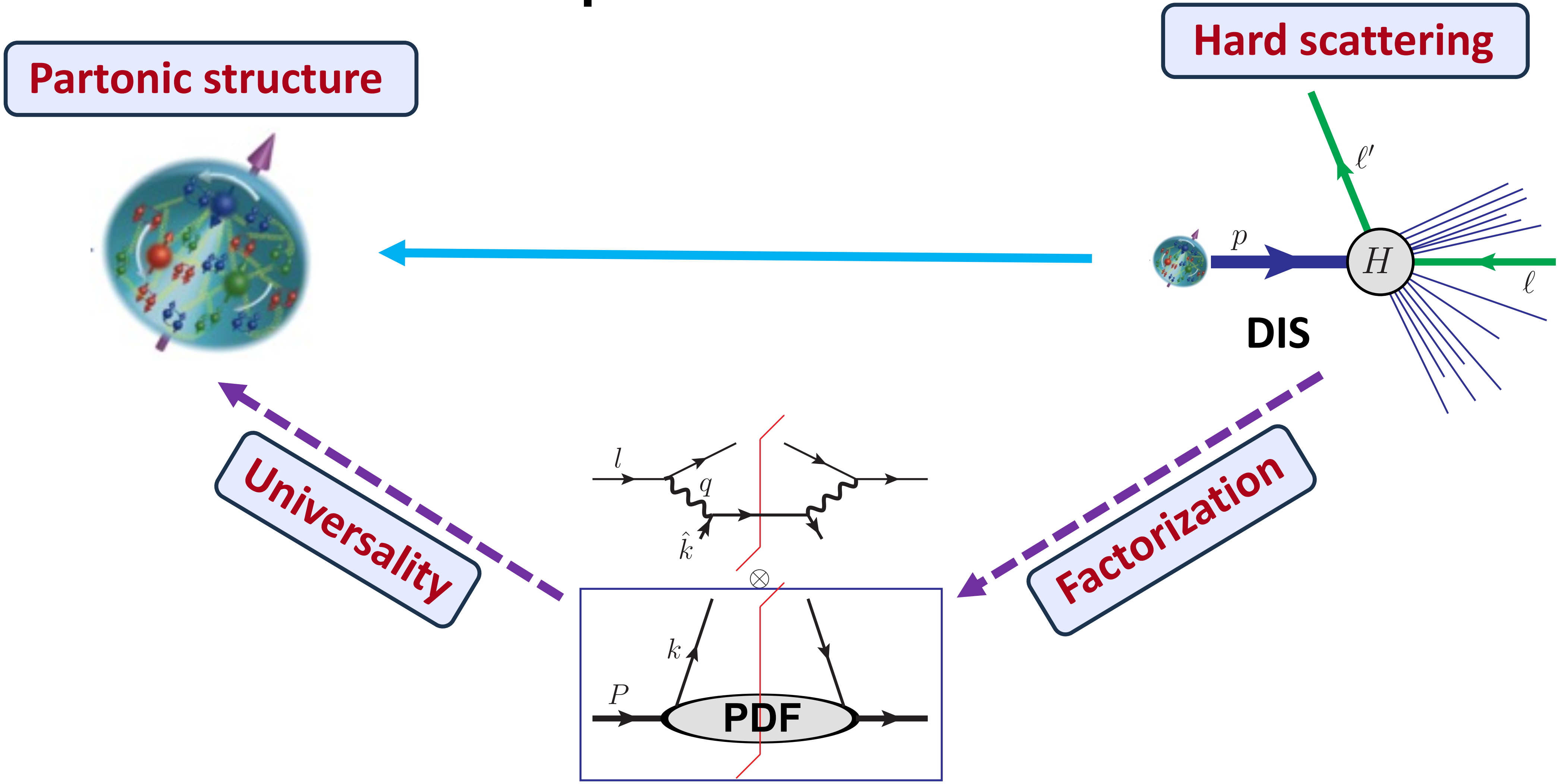
Based on JHEP 02 (2026) 244 [arXiv: 2509.18892]

In collaboration with Qing-Hong Cao, C.-P. Yuan, Shu-Tao Zhang, and Hua Xing Zhu

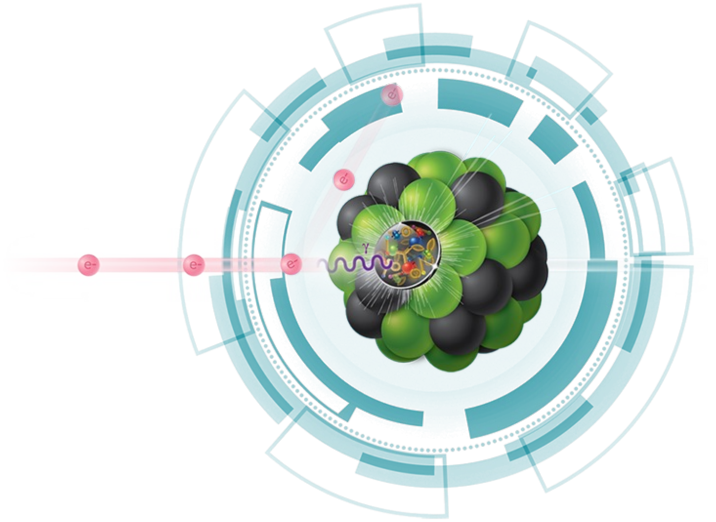
HET/BNL Lunch Time Talk, Mar 20, 2026

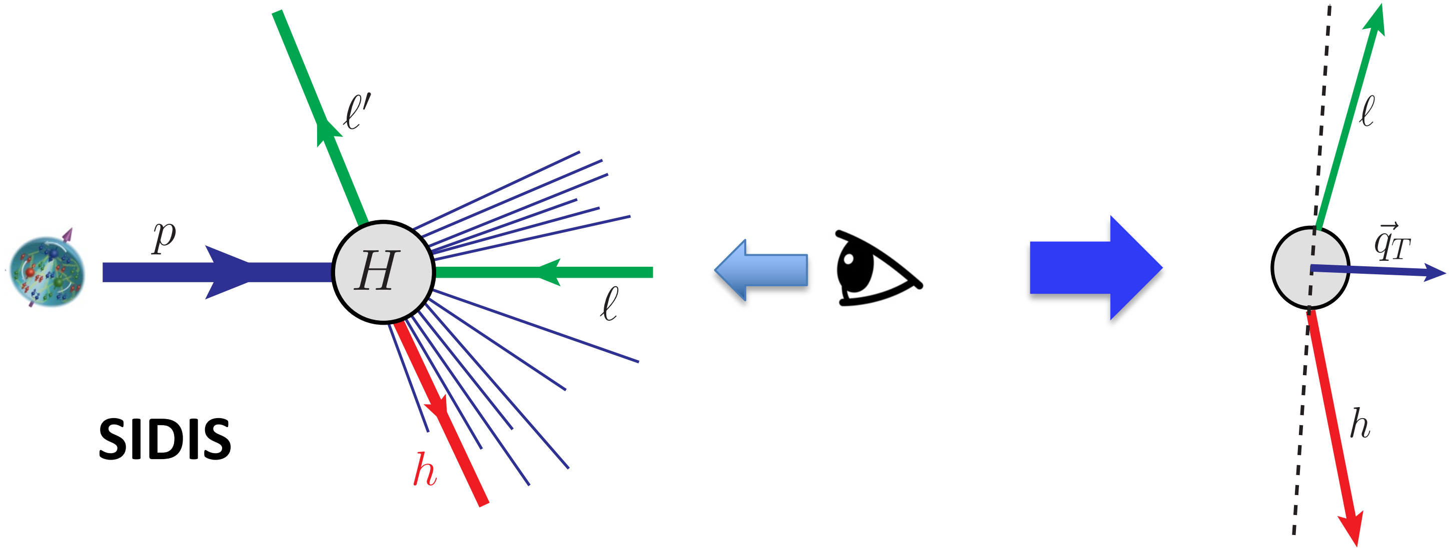
Hadron structure from experiments

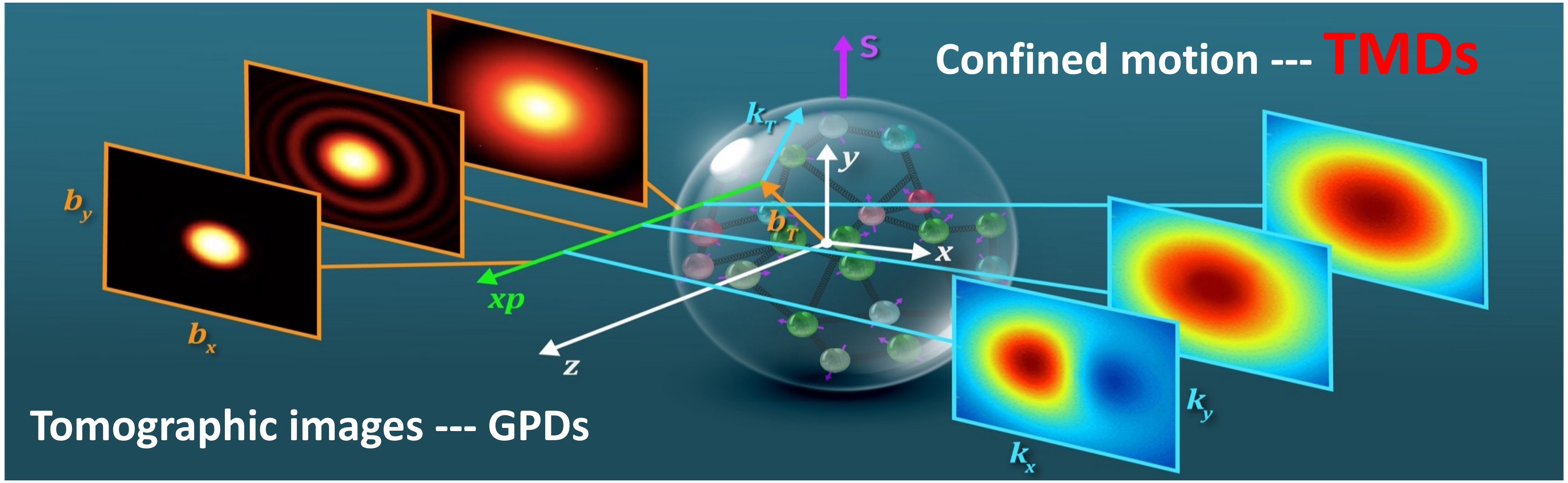


$$f_q(x) = \int_{-\infty}^{\infty} \frac{dy^-}{4\pi} e^{-ixP^+y^-} \langle P|\bar{\psi}(y^-)\gamma^+ W_n(y^-,0)\psi(0)|P\rangle$$

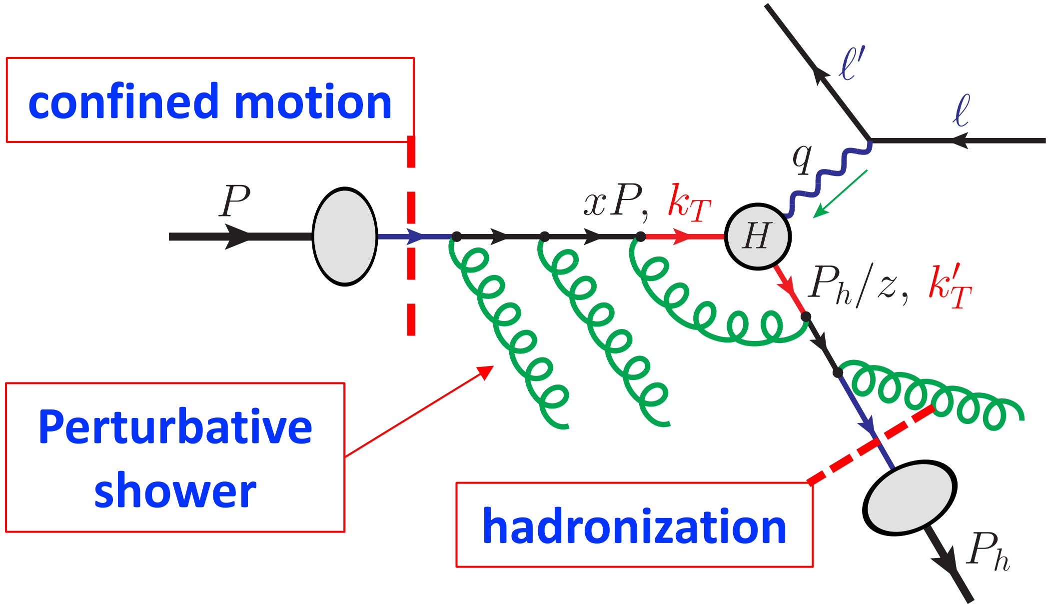
3D hadron structure at EIC







TMDs and challenges



- **Pairwise occurrences**

  Hard to disentangle various TMDs

- **Perturbative dominance**

  Confined dynamics diluted

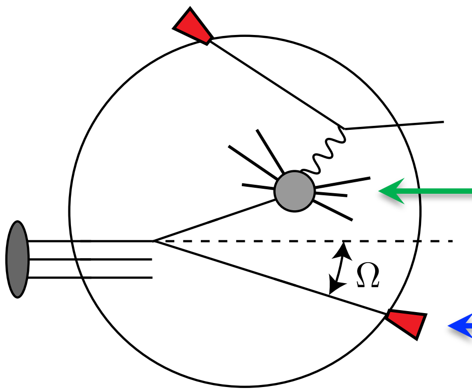
- **Scheme dependence**
  - Bare TMD have rapidity divergences
  - ***Intrinsic*** k_T ambiguously defined

$$f_q(x, \boldsymbol{k}_T) \stackrel{?}{=} \int \frac{dy^- \, d^2\boldsymbol{y}_T}{2(2\pi)^3} e^{-ixP^+y^- + i\boldsymbol{k}_T\cdot\boldsymbol{y}_T} \langle P|\bar{\psi}(0^+, y^-, \boldsymbol{y}_T)\gamma^+\psi(0)|P\rangle$$

Another probe of hadron structure

❑ Hadron scattering with extra energy flow measurement

Liu and Zhu, PRL 130 (2023), 091901



Hard parton scattering

Energy flow in the forward region along direction $\Omega(\theta,\phi)$

❑ Modified cross section

Inclusive DIS cross section

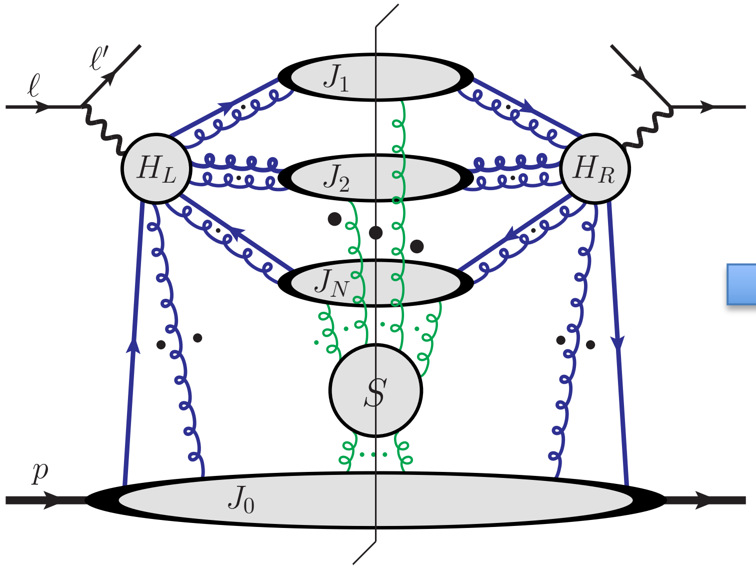
$$\frac{d\sigma}{d\Pi_\ell} = \sum_N \int d\Pi_N \frac{d\sigma}{d\Pi_\ell\, d\Pi_N}$$

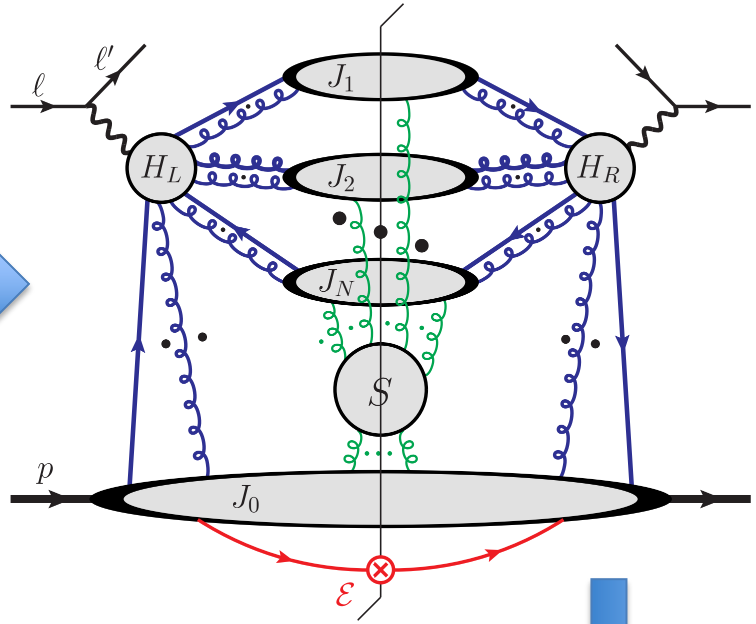
Energy flow weighted

$$\frac{d\Sigma}{d\Pi_\ell\, d\Omega} = \sum_N \int d\Pi_N \frac{d\sigma}{d\Pi_\ell\, d\Pi_N} \left[\sum_{i\in N} E_i\, \delta^{(2)}(\vec{\Omega} - \vec{\Omega}_i) \right]$$

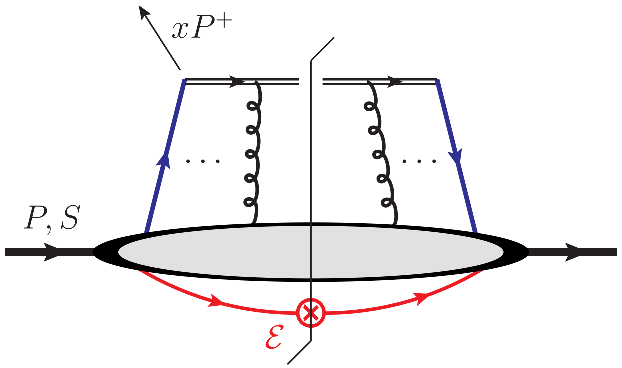
Factorization

$$\frac{d\Sigma}{d\Pi_\ell \, d\Omega} = \sum_N \int d\Pi_N \frac{d\sigma}{d\Pi_\ell \, d\Pi_N} \left[\sum_{i \in N} E_i \, \delta^{(2)}(\vec{\Omega} - \vec{\Omega}_i) \right]$$





- Energy weight suppresses soft lines
- For small θ, the energy correlator dominantly resides in the beam jet
- Bottom part factorized same as PDF



➢ Nucleon energy correlator (NEC)

$$F_q^{[\Gamma]}(x, \boldsymbol{n}; P, S) = \sum_X \int \frac{dy^-}{2\pi} e^{-ixP^+y^-} \left[\sum_{i \in X} E_i \, \delta^{(2)}(\boldsymbol{n} - \boldsymbol{n}_i) \right] \langle P, S | \bar{\psi}(y^-) W^\dagger(\infty, y^-) | X \rangle \, \Gamma \, \langle X | W(\infty, 0) \psi(0) | P, S \rangle$$

$$= \int \frac{dy^-}{2\pi} e^{-ixP^+y^-} \langle P, S | \bar{\psi}(y^-) W^\dagger(\infty, y^-) \, \hat{\mathcal{E}}(\boldsymbol{n}) \, \Gamma \, W(\infty, 0) \psi(0) | P, S \rangle$$

Energy flow operator

$$F_q^{[\Gamma]}(x, \boldsymbol{n}; P, S) = \int \frac{dy^-}{2\pi} e^{-ixP^+y^-} \langle P, S|\bar{\psi}(y^-)W^\dagger(\infty, y^-)\, \hat{\mathcal{E}}(\boldsymbol{n})\, \Gamma\, W(\infty, 0)\psi(0)|P, S\rangle$$

Energy-momentum tensor

$$\hat{\mathcal{E}}(\boldsymbol{n}) = \lim_{r\to\infty} \int_{-\infty}^{\infty} dt\, r^2\, n^i T^{i0}(t, r\boldsymbol{n})$$

records energy flow at celestial infinity --- detector put at infinity

$$\hat{\mathcal{E}}(\boldsymbol{n})|\{p_j\}; \mathrm{out}\rangle = \sum_{j=1}^{N} E_j\, \delta^{(2)}(\boldsymbol{n} - \boldsymbol{n}_j)|\{p_j\}; \mathrm{out}\rangle$$

Basham, et al., PRL 41 (1978) 1585;
Sveshnikov and Tkachov, PLB 382, 403 (1996);
Korchemsky and Sterman, NPB 555, 335 (1999);
… See the review by Moult and Zhu, 2506.09119.

- **Interpolates between hadron and jet**
  - **Infrared safe**
  - **Avoids jet algorithm**
- **Extending to n-point NEC**

$$F_{q,N}^{[\Gamma]}(x, \{\boldsymbol{n}_j\}; P, S) = \int \frac{dy^-}{2\pi} e^{-ixP^+y^-} \langle P, S|\bar{\psi}(y^-)W^\dagger(\infty, y^-)\, \hat{\mathcal{E}}(\boldsymbol{n}_1)\, \hat{\mathcal{E}}(\boldsymbol{n}_2) \cdots \hat{\mathcal{E}}(\boldsymbol{n}_N)\, \Gamma\, W(\infty, 0)\psi(0)|P, S\rangle$$

Spin structure of NEC

$$F_q^{[\Gamma]}(x, \boldsymbol{n}; P, S) = \int \frac{dy^-}{2\pi} e^{-ixP^+y^-} \langle P, S|\bar{\psi}(y^-)W^\dagger(\infty, y^-)\,\hat{\mathcal{E}}(\boldsymbol{n})\,\Gamma\, W(\infty, 0)\psi(0)|P, S\rangle$$

- **Unpolarized quark: $\boldsymbol{\Gamma = \gamma^+/2}$**

$$F^{[\gamma^+/2]}(x, \boldsymbol{n}; P, S) = f_1(x, \theta; P) + \frac{(\boldsymbol{s}_T \times \boldsymbol{n}_T)^z}{|\boldsymbol{n}_T|} f_{1T}^\perp(x, \theta; P)$$

Sivers-type NEC

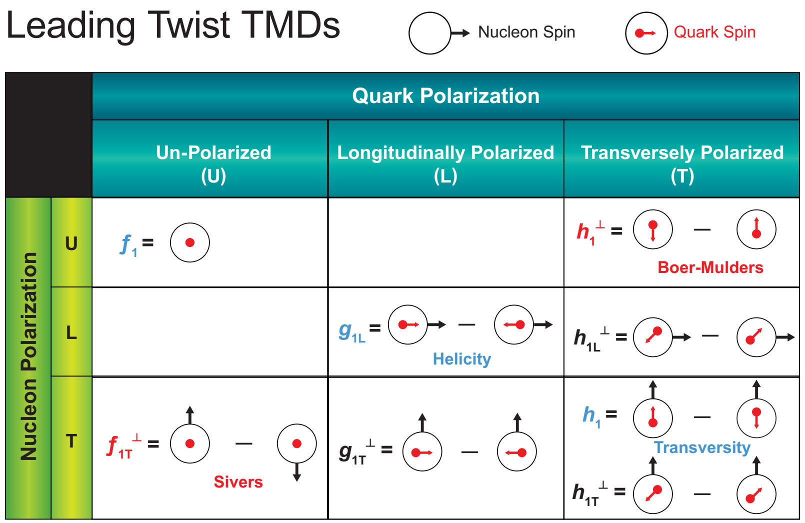
- **Longitudinally polarized quark: $\boldsymbol{\Gamma = \gamma^+\gamma_5/2}$**

$$F^{[\gamma^+\gamma_5/2]}(x, \boldsymbol{n}; P, S) = \lambda_N g_{1L}(x, \theta; P) + \frac{\boldsymbol{s}_T \cdot \boldsymbol{n}_T}{|\boldsymbol{n}_T|} g_{1T}^\perp(x, \theta; P)$$

- **Transversely polarized quark: $\boldsymbol{\Gamma = \gamma^+\gamma^\perp\gamma_5/2}$**

$$F^{[\gamma^+\gamma^i\gamma_5/2]}(x, \boldsymbol{n}; P, S) = s_T^i h_1(x, \theta; P) + \frac{(\hat{z} \times \boldsymbol{n}_T)^i}{|\boldsymbol{n}_T|} h_1^\perp(x, \theta; P)$$

$$+ \lambda_N \frac{n_T^i}{|\boldsymbol{n}_T|} h_{1L}^\perp(x, \theta; P) + \frac{\left(n_T^i n_T^j - \frac{1}{2}|n_T|^2\delta^{ij}\right) s_T^j}{|n_T|^2} h_{1T}^\perp(x, \theta; P)$$



Relation to TMD

Liu and Zhu, 2403.08874

Parton k_T is given by the opposite of remnant k_T

$$f_q^{[\Gamma]}(x, \boldsymbol{k}_T) = \sum_X \int \frac{dy^-}{2\pi} e^{-ixP^+y^-} \langle P, S|\bar{\psi}(y^-)W^\dagger(\infty, y^-)\, \delta^{(2)}(\boldsymbol{k}_T + \boldsymbol{k}_{X,T})\, |X\rangle\, \Gamma\, \langle X|W(\infty, 0)\psi(0)|P, S\rangle$$

Transverse energy flow

$$= \sum_X \int \frac{dy^-}{2\pi} e^{-ixP^+y^-} \langle P, S|\bar{\psi}(y^-)W^\dagger(\infty, y^-)\, \delta^{(2)}\left(\boldsymbol{k}_T + \int d^2\Omega\, \mathcal{E}(\boldsymbol{n})\, \boldsymbol{n}_T\right) |X\rangle\, \Gamma\, \langle X|W(\infty, 0)\psi(0)|P, S\rangle$$

$$\Rightarrow \int d^2\boldsymbol{k}_T\, (-k_T^{i_1})\, (-k_T^{i_2}) \cdots (-k_T^{i_N})\, f_q^{[\Gamma]}(x, \boldsymbol{k}_T) = \int d^2\Omega_1 \cdots d^2\Omega_N\, n_{1T}^{i_1} \cdots n_{NT}^{i_N}\, F_{q,N}^{[\Gamma]}(x, \{\boldsymbol{n}_i\}; P, S)$$

N-th k_T moment of TMD — Angular integral of N-point NEC

❑ Example for unpolarized quark at $N = 1$: Sivers TMD and Sivers-type NEC

$$\int \frac{dk_T^2}{2M} k_T^2\, f_{1T}^\perp(x, k_T^2) = -\int_0^\pi d\theta \sin^2\theta\, f_{1T}^\perp(x, \theta; P)$$

- θ plays the role of k_T
- More of conceptual use

Advantages of NEC

❑ Collinear factorization!

$$\frac{d\sigma^{\rm DIS}}{dx_B\, dQ^2} = C^{\rm DIS}(x) \otimes f(x) \quad \Rightarrow \quad \frac{d\Sigma^{\rm DIS}}{dx_B\, dQ^2\, d^2\Omega} = C^{\rm DIS}(x) \otimes F(x;\Omega)$$

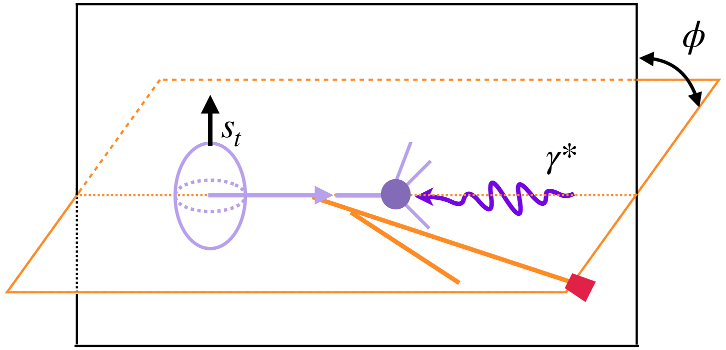
(1) No k_T convolution; (2) No rapidity divergences; (3) No Sudakov suppression

❑ As easily extracted as collinear PDF

Shares the same hard coefficient as the plain observables

E.g.: for single transverse target spin asymmetry, $F(x;\Omega) = f_1(x,\theta) + s_T \sin(\phi - \phi_S)\, f_{1T}^{\perp}(x,\theta)$

Spin structure + azimuthal modulation separate various NECs



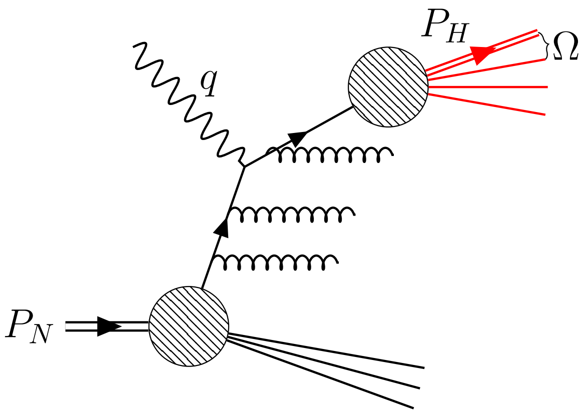
❑ Extracted at the same kinematic region

DIS / SIDIS + extra energy flow tagging

Extending to fragmentation region

Liu and Zhu, 2403.08874

❑ Tag an energy flow around a measured hadron



Fragmentation energy correlator (FEC)

$$D_{h/q}^{[\Gamma]}(z,\boldsymbol{n};p_h)=\frac{z}{2N_c}\sum_X\int\frac{dy^-}{2\pi}e^{ip_h^+y^-/z}\,\mathrm{Tr}\Big[\Gamma\,\langle 0|W(\infty,y^-)\psi(y^-)\times\mathcal{E}(\boldsymbol{n})|h,X\rangle\langle h,X|\bar{\psi}(0)W^\dagger(\infty,0)|0\rangle\Big]$$

❑ Two FECs for unpolarized hadron

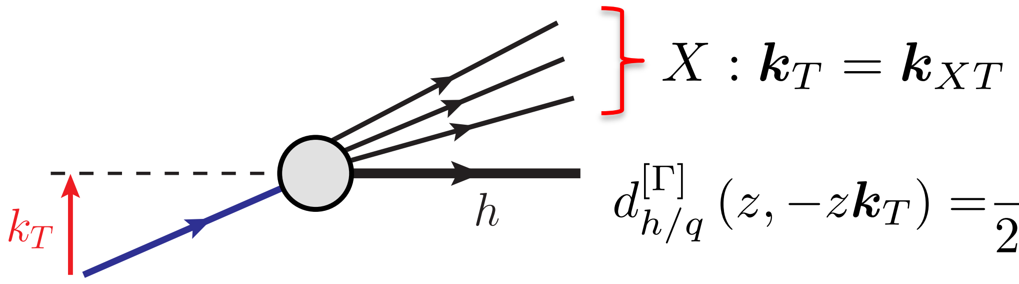
- **Unpolarized quark** $D_{h/q}^{[\gamma^+/2]}(z,\boldsymbol{n};p_h)=\mathcal{D}_{1,h/q}(z,\theta;p_h)$
- **Transversely polarized quark** $D_{h/q}^{[\gamma^+\gamma^i\gamma_5/2]}(z,\boldsymbol{n};p_h)=\frac{(\hat{z}\times\boldsymbol{n}_T)^i}{|\boldsymbol{n}_T|}\mathcal{H}_{1,h/q}^{\perp}(z,\theta;p_h)$

Collins-type FEC

- **Longitudinally polarized quark** $D_{h/q}^{[\gamma^+\gamma_5/2]}(z,\boldsymbol{n};p_h)=0$

Relation to TMD fragmentation function

❑ **In the "hadron frame", parton k_T equals the k_T of fragmented products**



$$d_{h/q}^{[\Gamma]}(z, -z\boldsymbol{k}_T) = \frac{1}{2zN_c} \sum_X \int \frac{dy^-}{2\pi} e^{ip_h^+ y^-/z} \, \mathrm{Tr}\Big[\Gamma \langle 0|W(\infty, y^-)\psi(y^-) \times \delta^{(2)}\Big(\boldsymbol{k}_T - \int d\Omega \, \mathcal{E}(\boldsymbol{n}) \, \boldsymbol{n}_T\Big) |h, X\rangle\langle h, X|\bar{\psi}(0)W^\dagger(\infty, 0)|0\rangle\Big]$$

$$z^2 \int d^2\boldsymbol{k}_T k_T^{i_1} \cdots k_T^{i_N} d_{h/q}^{[\Gamma]}(z, -z\boldsymbol{k}_T) = \int d\Omega_1 \cdots d\Omega_N n_{1T}^{i_1} \cdots n_{NT}^{i_N} D_{h/q,N}^{[\Gamma]}(z, \{\boldsymbol{n}_i\}; p_h)$$

❑ **For one-point FEC: Collins-type FEC**

$$\int_0^\pi d\theta \sin^2\theta \, \mathcal{H}_{1,h/q}^{\perp}(z, \theta; p_h) = -z^2 \int \frac{dk_T^2}{2m_h} k_T^2 \, H_{1,h/q}^{\perp}(z, zk_T)$$

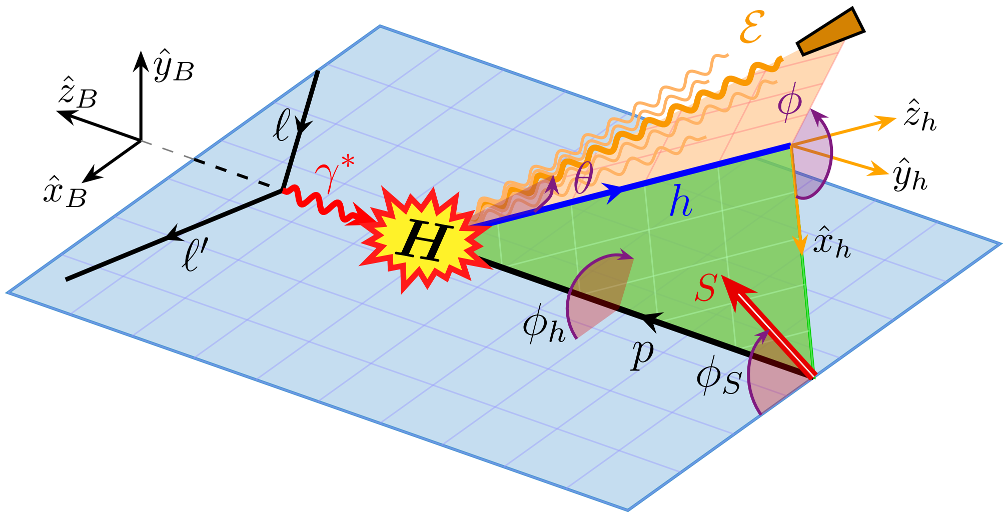
Collins-type FEC **Collins function**

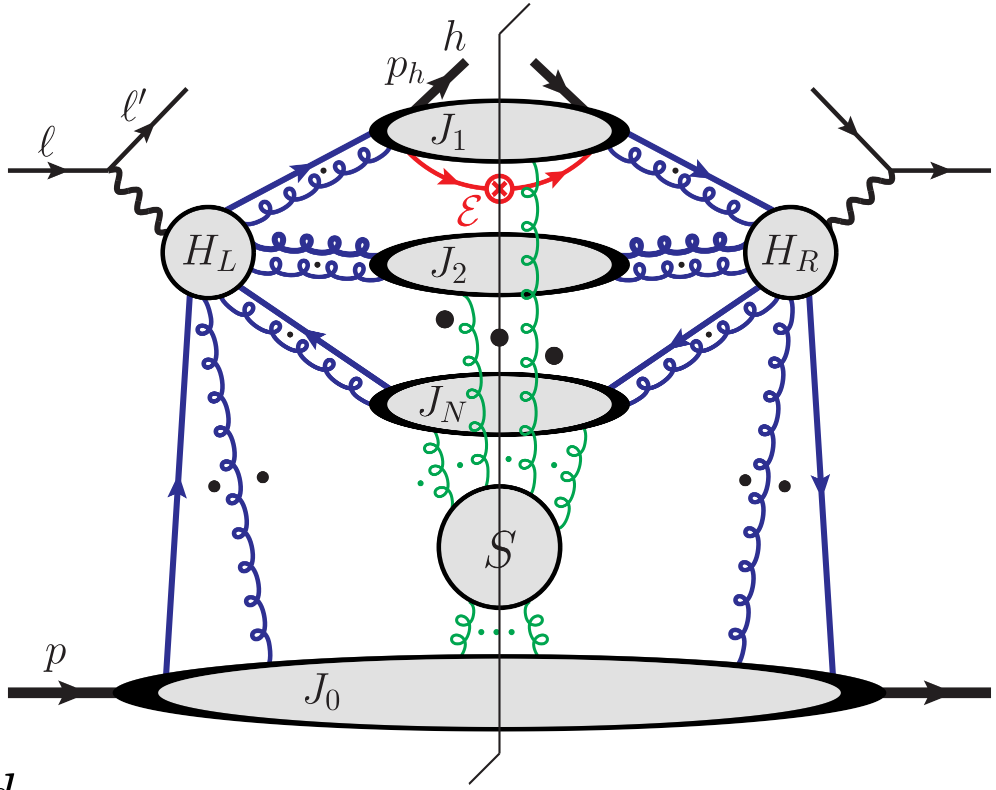
FEC from energy-weighted SIDIS cross section

Q.-H. Cao, ZY, C.-P. Yuan, S.-T. Zhang, and H. X. Zhu, JHEP 02 (2026) 244

❑ High-p_T hadron in Breit frame, with an energy flow measurement

$$d\sigma^{\rm SIDIS} \longrightarrow \frac{d\Sigma^{\rm SIDIS}}{d^2\Omega} = \int d\sigma^{\rm SIDIS} \times \sum_{i\in X} E_i\, \delta^{(2)}(\boldsymbol{n}-\boldsymbol{n}_i)$$





Factorization ➡

$$d\sigma^{\rm SIDIS} = f_{i/p} \otimes C_{ij} \otimes d_{h/j}$$

$$d\Sigma^{\rm SIDIS} = f_{i/p} \otimes C_{ij} \otimes D_{h/j}$$ (when θ is small)

Energy-weighted SIDIS at leading power (for quark channel)

$$\varepsilon = \frac{1-y}{1-y+y^2/2}$$

$$\left[\frac{y^2}{z(1-\varepsilon)}\frac{\alpha_e^2}{(8\pi^2 Q^2)^2}\right]^{-1}\frac{d\Sigma}{dx\,dQ^2\,d\phi_S\,dz\,dp_{hT}^2\,d\phi_h\,d\cos\theta\,d\phi}$$

$$= \mathcal{C}_{f,\mathcal{D}}\left[F_{UU,T}\right] + \varepsilon\,\mathcal{C}_{f,\mathcal{D}}\left[F_{UU,L}\right] + \sqrt{2\varepsilon(1+\varepsilon)}\,\mathcal{C}_{f,\mathcal{D}}\left[F_{UU}^{(1)}\right]\cos\phi_h + \varepsilon\,\mathcal{C}_{f,\mathcal{D}}\left[F_{UU}^{(2)}\right]\cos(2\phi_h)$$

$$+ P_e\sqrt{2\varepsilon(1-\varepsilon)}\,\mathcal{C}_{f,\mathcal{D}}\left[F_{LU}\right]\sin\phi_h + P_N\left\{\sqrt{2\varepsilon(1+\varepsilon)}\,\mathcal{C}_{g,\mathcal{D}}\left[F_{UL}^{(1)}\right]\sin\phi_h + \varepsilon\,\mathcal{C}_{g,\mathcal{D}}\left[F_{UL}^{(2)}\right]\sin(2\phi_h)\right\}$$

$$+ P_e P_N\left\{\sqrt{1-\varepsilon^2}\,\mathcal{C}_{g,\mathcal{D}}\left[F_{LL}\right] + \sqrt{2\varepsilon(1-\varepsilon)}\,\mathcal{C}_{g,\mathcal{D}}\left[F_{LL}^{(1)}\right]\cos\phi_h\right\}$$

$$+ s_T\left\{\left(-\mathcal{C}_{h,\mathcal{H}^\perp}\left[F_{UT,T}\right] - \varepsilon\,\mathcal{C}_{h,\mathcal{H}^\perp}\left[F_{UT,L}\right]\right)\sin(\phi-\phi_S+\phi_h)\right.$$

$$+ \sqrt{2\varepsilon(1+\varepsilon)}\left[\mathcal{C}_{h,\mathcal{H}^\perp}\left[F_{UT}^{(1+)}\right]\sin(\phi-\phi_S+2\phi_h) + \mathcal{C}_{h,\mathcal{H}^\perp}\left[F_{UT}^{(1-)}\right]\sin(\phi-\phi_S)\right]$$

$$\left. + \varepsilon\left[\mathcal{C}_{h,\mathcal{H}^\perp}\left[F_{UT}^{(2+)}\right]\sin(\phi-\phi_S+3\phi_h) + \mathcal{C}_{h,\mathcal{H}^\perp}\left[F_{UT}^{(2-)}\right]\sin(\phi-\phi_S-\phi_h)\right]\right\}$$

$$+ P_e s_T\left\{\sqrt{1-\varepsilon^2}\,\mathcal{C}_{h,\mathcal{H}^\perp}\left[F_{LT}\right]\cos(\phi-\phi_S+\phi_h)\right.$$

$$\left. + \sqrt{2\varepsilon(1-\varepsilon)}\left[\mathcal{C}_{h,\mathcal{H}^\perp}\left[F_{LT}^{(1+)}\right]\cos(\phi-\phi_S+2\phi_h) + \mathcal{C}_{h,\mathcal{H}^\perp}\left[F_{LT}^{(1-)}\right]\cos(\phi-\phi_S)\right]\right\}$$

Modified FEC definition with boost invariance

Q.-H. Cao, ZY, C.-P. Yuan, S.-T. Zhang, and H. X. Zhu, JHEP 02 (2026) 244

❑ Lack of Lorentz covariance

Explicit dependence on $\boldsymbol{E}$ and $\boldsymbol{\theta}$

- FEC explicitly depends on the energy of hadron h.
- Complicates comparison across different experiments, even the same experiment analyzed in different frames.
- Partial integration of $\boldsymbol{p}_{hT}$ is not applicable.

❑ Modified energy flow operator

$+/-$ components and rapidity η defined w.r.t. hadron h

$$\mathcal{E}_L(\eta,\phi)|X;\text{out}\rangle = \sum_{i\in X} p_i^+ \delta(\eta-\eta_i)\delta(\phi-\phi_i)|X;\text{out}\rangle$$

$$\mathcal{E}_L(\eta,\phi) = \frac{1+\tanh\eta}{\sqrt{2}\cosh^2\eta}\mathcal{E}(\theta,\phi)$$

Boost covariantly $U(\alpha)\,\mathcal{E}_L(\eta,\phi)\,U^{-1}(\alpha) = e^{-\alpha}\mathcal{E}_L(\eta+\alpha,\phi)$

❑ Boost-invariant FEC definition

$$\Lambda_F = m_h\, e^{y_h-\eta} = \sqrt{2}\,p_h^+ e^{-\eta}$$

$$D_{h/q}^{[\Gamma]}(z,\Lambda_F,\phi) = \frac{z}{2N_c}\sum_X \int \frac{dy^-}{2\pi p_h^+} e^{ip_h^+ y^-/z}\, \text{Tr}\Big[\Gamma\langle 0|\,W(\infty,y^-)\,\psi(y^-) \times \mathcal{E}_L(\eta,\phi)\,|h,X;\text{out}\rangle\langle h,X;\text{out}|\,\bar{\psi}(0)\,W^\dagger(\infty,0)\,|0\rangle\Big]$$

Energy-weighted SIDIS with modified definition

$$d\sigma \longrightarrow \frac{d\Sigma'}{d\eta d\phi} = \int d\sigma \times \sum_{i\in X} \frac{p_i^+}{p_h^+}\delta(\eta-\eta_i)\delta(\phi-\phi_i)$$

$$\begin{aligned}
&\left[\frac{y^2}{z(1-\varepsilon)}\frac{\alpha_e^2}{(8\pi^2Q^2)^2}\right]^{-1}\frac{d\Sigma'}{dx\,dQ^2\,d\phi_S\,dz\,dp_{hT}^2\,d\phi_h\,d\eta\,d\phi}\\
&= \mathcal{C}'_{f,\mathcal{D}}[F_{UU,T}] + \varepsilon\,\mathcal{C}'_{f,\mathcal{D}}[F_{UU,L}] + \sqrt{2\varepsilon(1+\varepsilon)}\,\mathcal{C}'_{f,\mathcal{D}}[F_{UU}^{(1)}]\cos\phi_h + \varepsilon\,\mathcal{C}'_{f,\mathcal{D}}[F_{UU}^{(2)}]\cos(2\phi_h)\\
&\quad + P_e\sqrt{2\varepsilon(1-\varepsilon)}\,\mathcal{C}'_{f,\mathcal{D}}[F_{LU}]\sin\phi_h + P_N\left\{\sqrt{2\varepsilon(1+\varepsilon)}\,\mathcal{C}'_{g,\mathcal{D}}[F_{UL}^{(1)}]\sin\phi_h + \varepsilon\,\mathcal{C}'_{g,\mathcal{D}}[F_{UL}^{(2)}]\sin(2\phi_h)\right\}\\
&\quad + P_eP_N\left\{\sqrt{1-\varepsilon^2}\,\mathcal{C}'_{g,\mathcal{D}}[F_{LL}] + \sqrt{2\varepsilon(1-\varepsilon)}\,\mathcal{C}'_{g,\mathcal{D}}[F_{LL}^{(1)}]\cos\phi_h\right\}\\
&\quad + s_T\Big\{\left(-\mathcal{C}'_{h,\mathcal{H}^\perp}[F_{UT,T}] - \varepsilon\,\mathcal{C}'_{h,\mathcal{H}^\perp}[F_{UT,L}]\right)\sin(\phi-\phi_S+\phi_h)\\
&\qquad + \sqrt{2\varepsilon(1+\varepsilon)}\left[\mathcal{C}'_{h,\mathcal{H}^\perp}[F_{UT}^{(1+)}]\sin(\phi-\phi_S+2\phi_h) + \mathcal{C}'_{h,\mathcal{H}^\perp}[F_{UT}^{(1-)}]\sin(\phi-\phi_S)\right]\\
&\qquad + \varepsilon\left[\mathcal{C}'_{h,\mathcal{H}^\perp}[F_{UT}^{(2+)}]\sin(\phi-\phi_S+3\phi_h) + \mathcal{C}'_{h,\mathcal{H}^\perp}[F_{UT}^{(2-)}]\sin(\phi-\phi_S-\phi_h)\right]\Big\}\\
&\quad + P_es_T\Big\{\sqrt{1-\varepsilon^2}\,\mathcal{C}'_{h,\mathcal{H}^\perp}[F_{LT}]\cos(\phi-\phi_S+\phi_h)\\
&\qquad + \sqrt{2\varepsilon(1-\varepsilon)}\left[\mathcal{C}'_{h,\mathcal{H}^\perp}[F_{LT}^{(1+)}]\cos(\phi-\phi_S+2\phi_h) + \mathcal{C}'_{h,\mathcal{H}^\perp}[F_{LT}^{(1-)}]\cos(\phi-\phi_S)\right]\Big\}
\end{aligned}$$

What if we integrate over $\vec{p}_{hT}$?

❑ Why?

- Most events reside in low-p_{hT} region.
- Simplify azimuthal analysis.

❑ How?

- Inclusive with p_{hT}
- But ϕ is defined on event-by-event basis
- Importantly, FEC convolutions need to be compatible with this integration!

$$\mathcal{C}'_{f,\mathcal{D}}[F] \equiv \int \frac{d\xi_1}{\xi_1} \int \frac{d\xi_2}{\xi_2^2} f_{q/p}(\xi_1)\, \mathcal{D}_{h/q}(\xi_2, \Lambda_F)\, F(Q^2, x/\xi_1, z/\xi_2, p_{hT}^2/\xi_2^2)$$

What if we integrate over $\vec{p}_{hT}$?

$$\left[\frac{y^2}{z(1-\varepsilon)}\frac{\alpha_e^2}{(8\pi^2Q^2)^2}\right]^{-1}\frac{d\Sigma'}{dx\,dQ^2\,d\phi_S\,dz\,dp_{hT}^2\,d\phi_h\,d\eta\,d\phi}$$

$$= \mathcal{C}'_{f,\mathcal{D}}\left[F_{UU,T}\right] + \varepsilon\,\mathcal{C}'_{f,\mathcal{D}}\left[F_{UU,L}\right] + \sqrt{2\varepsilon(1+\varepsilon)}\,\mathcal{C}'_{f,\mathcal{D}}\left[F_{UU}^{(1)}\right]\cos\phi_h + \varepsilon\,\mathcal{C}'_{f,\mathcal{D}}\left[F_{UU}^{(2)}\right]\cos(2\phi_h)$$

$$+ P_e\sqrt{2\varepsilon(1-\varepsilon)}\,\mathcal{C}'_{f,\mathcal{D}}\left[F_{LU}\right]\sin\phi_h + P_N\left\{\sqrt{2\varepsilon(1+\varepsilon)}\,\mathcal{C}'_{g,\mathcal{D}}\left[F_{UL}^{(1)}\right]\sin\phi_h + \varepsilon\,\mathcal{C}'_{g,\mathcal{D}}\left[F_{UL}^{(2)}\right]\sin(2\phi_h)\right\}$$

$$+ P_eP_N\left\{\sqrt{1-\varepsilon^2}\,\mathcal{C}'_{g,\mathcal{D}}\left[F_{LL}\right] + \sqrt{2\varepsilon(1-\varepsilon)}\,\mathcal{C}'_{g,\mathcal{D}}\left[F_{LL}^{(1)}\right]\cos\phi_h\right\}$$

$$+ s_T\left\{\left(-\mathcal{C}'_{h,\mathcal{H}^\perp}\left[F_{UT,T}\right] - \varepsilon\,\mathcal{C}'_{h,\mathcal{H}^\perp}\left[F_{UT,L}\right]\right)\sin(\phi-\phi_S+\phi_h)\right.$$

$$+ \sqrt{2\varepsilon(1+\varepsilon)}\left[\mathcal{C}'_{h,\mathcal{H}^\perp}\left[F_{UT}^{(1+)}\right]\sin(\phi-\phi_S+2\phi_h) + \mathcal{C}'_{h,\mathcal{H}^\perp}\left[F_{UT}^{(1-)}\right]\sin(\phi-\phi_S)\right]$$

$$\left.+ \varepsilon\left[\mathcal{C}'_{h,\mathcal{H}^\perp}\left[F_{UT}^{(2+)}\right]\sin(\phi-\phi_S+3\phi_h) + \mathcal{C}'_{h,\mathcal{H}^\perp}\left[F_{UT}^{(2-)}\right]\sin(\phi-\phi_S-\phi_h)\right]\right\}$$

$$+ P_es_T\left\{\sqrt{1-\varepsilon^2}\,\mathcal{C}'_{h,\mathcal{H}^\perp}\left[F_{LT}\right]\cos(\phi-\phi_S+\phi_h)\right.$$

$$\left.+ \sqrt{2\varepsilon(1-\varepsilon)}\left[\mathcal{C}'_{h,\mathcal{H}^\perp}\left[F_{LT}^{(1+)}\right]\cos(\phi-\phi_S+2\phi_h) + \mathcal{C}'_{h,\mathcal{H}^\perp}\left[F_{LT}^{(1-)}\right]\cos(\phi-\phi_S)\right]\right\}$$

0 0

Is that true?

Azimuthal singularity under $\vec{p}_{hT}$ integration

❑ $\int d^2\vec{p}_{hT}$ acts on hard coefficients

Separation $\int d^2\boldsymbol{p}_T f(\boldsymbol{p}_T) = \frac{1}{2}\int_0^\infty dp_T^2 \int_0^{2\pi} d\phi\, f(p_T^2, \phi)$ **is singular if $f(\boldsymbol{p}_T)$ has a singularity at $\boldsymbol{p}_T = 0$**

➡ **A vector-type plus distribution**

$$\int_0^{\Lambda^2} \frac{dp_T^2}{(p_T^2)^{1+\epsilon}} f(p_T^2, \phi) = \int_0^{\Lambda^2} dp_T^2 \left[\frac{1}{p_T^2}\right]_+ f(p_T^2, \phi) + f(0, \phi_0) \int_0^{\Lambda^2} \frac{dp_T^2}{(p_T^2)^{1+\epsilon}}$$

$\phi_0 = \pi$ in our convention

❑ The only nontrivial term

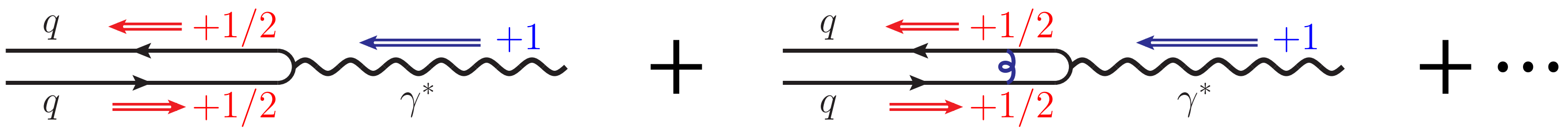
$\mathcal{C}'_{h,\mathcal{H}^\perp}\left[F_{UT}^{(2-)}\right] \sin(\phi - \phi_S - \phi_h)$ leaves a remaining $\sin(\phi - \phi_S)$ under $\vec{p}_{hT}$ integration

$\propto p_{hT}^{-2}$

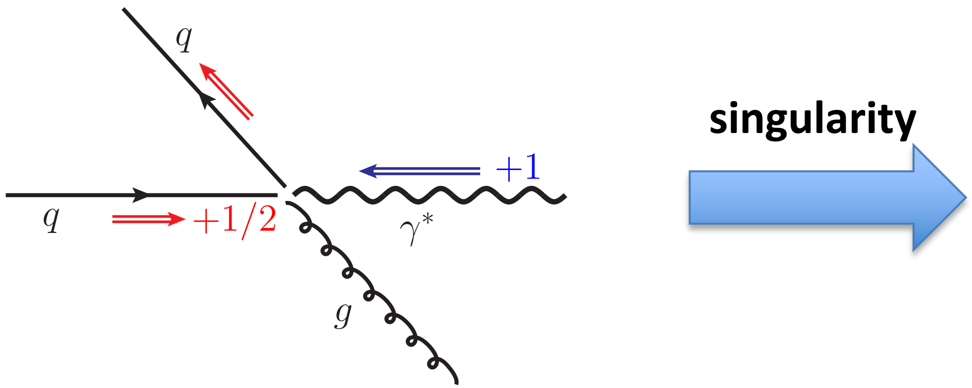
This also cancels the LO singular contribution with no gluon radiation!

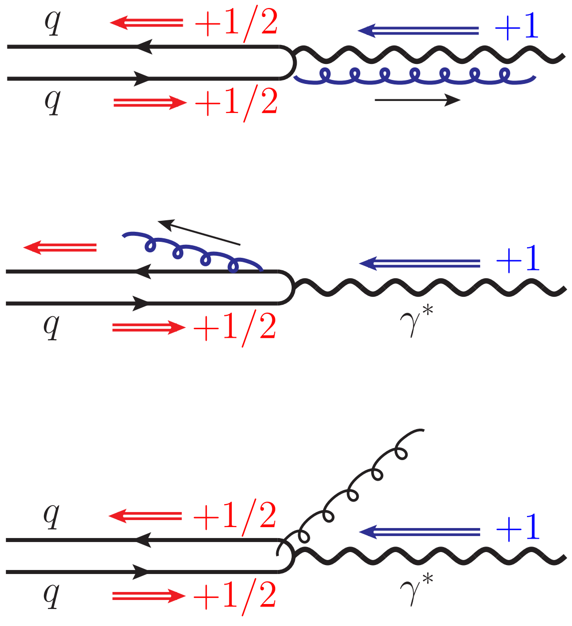
Physical understanding

❑ Without real radiation (starting at LO)



❑ With real radiation (starting from NLO)





Initial-state collinear

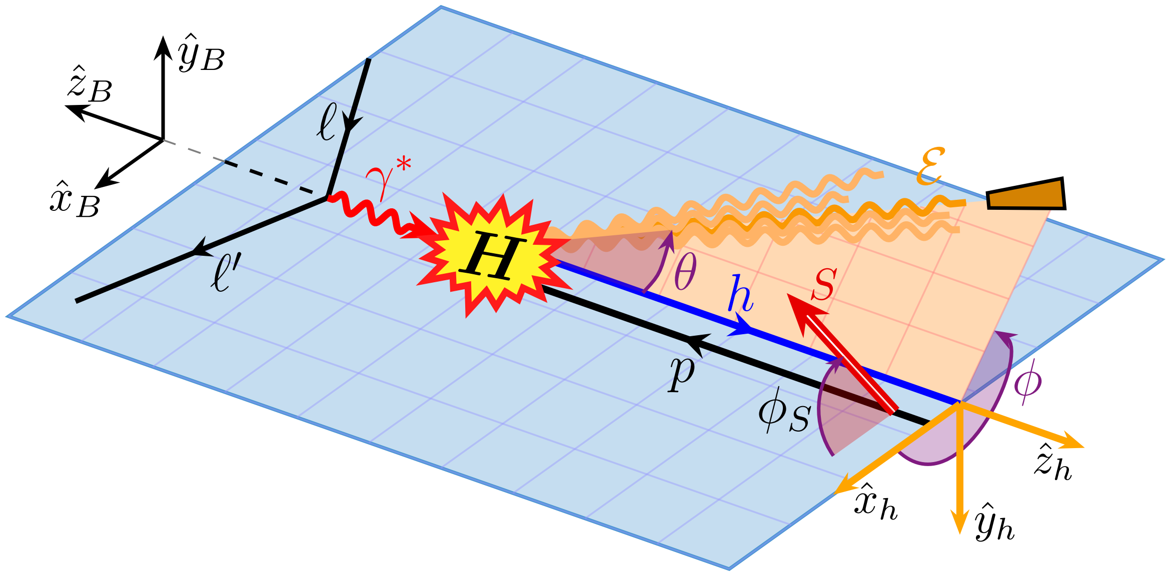
Final-state collinear

Soft

Singular configuration shares the same helicity structure as virtual diagrams!

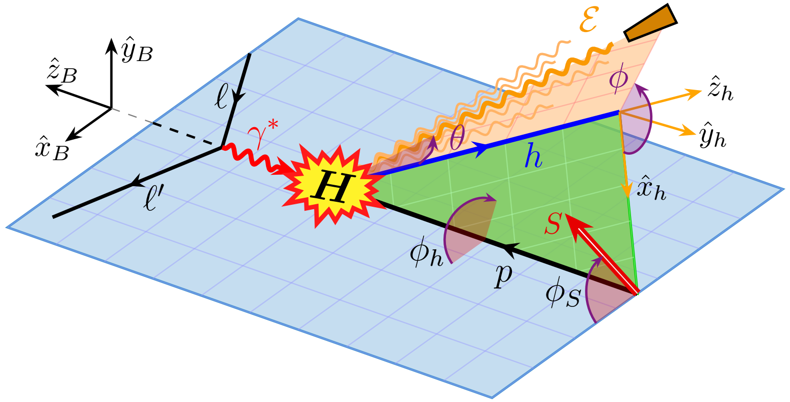
Energy-weighted SIDIS inclusive with $\vec{p}_{hT}$

$$\frac{d\Sigma'}{dx\, dQ^2\, d\phi_S\, dz\, d\eta\, d\phi} = \frac{y^2}{z\,Q^4}\frac{\alpha_e^2}{(1-\varepsilon)}\Big\{\mathcal{C}'_{f,\mathcal{D}}\big[f_{UU,T}\big] + \varepsilon\,\mathcal{C}'_{f,\mathcal{D}}\big[f_{UU,L}\big] + P_N P_e\sqrt{1-\varepsilon^2}\,\mathcal{C}'_{g,\mathcal{D}}\big[f_{LL}\big]$$
$$+ s_T\Big(\varepsilon\,\mathcal{C}'_{h,\mathcal{H}^\perp}\big[f_{UT}^{(2-)}\big] + \sqrt{2\varepsilon(1+\varepsilon)}\,\mathcal{C}'_{h,\mathcal{H}^\perp}\big[f_{UT}^{(1-)}\big]\Big)\sin(\phi-\phi_S)\Big\}$$

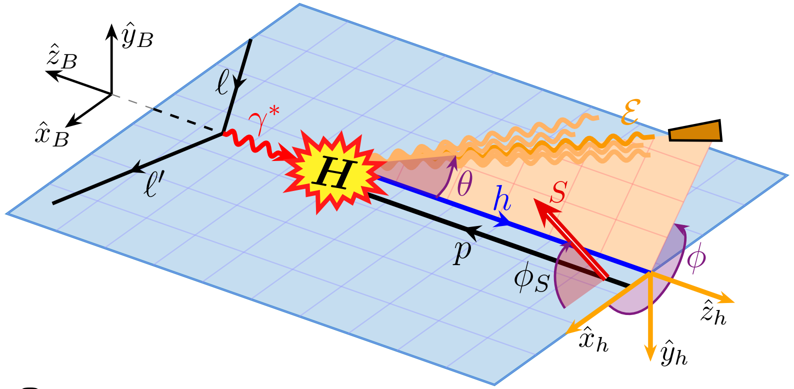


Summary

- ❑ **NEC/FEC helps to probe 3D hadron or fragmentation structure**



- ❑ **Collinear factorization**
- ❑ **Suitably studied at EIC**
  - **Applicable to both ep and eA**
  - **NEC and FEC can be studied individually or simultaneously**



- ❑ **Two kinds of observables**
  - **Large p_{hT}**
  - **Inclusive with p_{hT}**

Thank you!