Uncertainties in Jet Bins

Frank Tackmann

Massachusetts Institute of Technology

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Perturbative Structure of Jet Cross Sections

$$\sigma_{\mathrm{total}} = \underbrace{\int_0^{p_T^{\mathrm{cut}}} \mathrm{d}p_T \frac{\mathrm{d}\sigma}{\mathrm{d}p_T}}_{\sigma_0(p_T^{\mathrm{cut}})} + \underbrace{\int_{p_T^{\mathrm{cut}}}^{\infty} \mathrm{d}p_T \frac{\mathrm{d}\sigma}{\mathrm{d}p_T}}_{\sigma_{\geq 1}(p_T^{\mathrm{cut}})}$$

$$\sigma_{\mathrm{total}} = 1 + \alpha_s + \alpha_s^2 + \cdots$$

$$\sigma_{\geq 1}(p_T^{\mathrm{cut}}) = \alpha_s(L^2 + L) + \alpha_s^2(L^4 + L^3 + L^2 + L) + \cdots$$

$$\begin{aligned} \sigma_0(p_T^{\mathrm{cut}}) &= \sigma_{\mathrm{total}} - \sigma_{\geq 1}(p_T^{\mathrm{cut}}) \\ &= \left[1 + \alpha_s + \alpha_s^2 + \cdots\right] - \left[\alpha_s(L^2 + L) + \alpha_s^2(L^4 + \cdots) + \cdots\right] \end{aligned}$$

where $L^2 = 2\ln^2(p_T^{\mathrm{cut}}/m_H)$ or $L^2 = \ln^2(\mathcal{T}^{\mathrm{cut}}/m_H)$

- Perturbative series in σ_{total} and $\sigma_{\geq 1}(p_T^{\mathrm{cut}})$ have different structures and are unrelated
- Apparent small uncertainties in $\sigma_0(p_T^{\mathrm{cut}})$ arise from cancellation between two series with large corrections

Division Into Jet Bins

To first approximation, one should treat perturbative series in σ_{total} and $\sigma_{\geq 1}$ as independent with uncorrelated perturbative uncertainties, similarly for $\sigma_{\geq 1}$ and $\sigma_{\geq 2}$

- First consider *inclusive* jet cross sections

$$\sigma_{\text{total}},\ \sigma_{\geq 1},\ \sigma_{\geq 2} \quad \Rightarrow \quad C = \begin{pmatrix} \Delta^2_{\text{total}} & 0 & 0 \\ 0 & \Delta^2_{\geq 1} & 0 \\ 0 & 0 & \Delta^2_{\geq 2} \end{pmatrix}$$

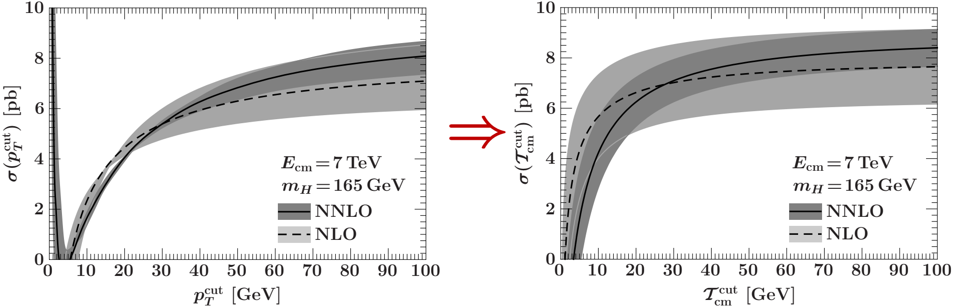
- Then transform to *exclusive* jet cross sections

$$\sigma_0 = \sigma_{\text{total}} - \sigma_{\geq 1}\,, \qquad \sigma_1 = \sigma_{\geq 1} - \sigma_{\geq 2}\,, \qquad \sigma_{\geq 2}$$

$$\Rightarrow \quad C = \begin{pmatrix} \Delta^2_{\text{total}} + \Delta^2_{\geq 1} & -\Delta^2_{\geq 1} & 0 \\ -\Delta^2_{\geq 1} & \Delta^2_{\geq 1} + \Delta^2_{\geq 2} & -\Delta^2_{\geq 2} \\ 0 & -\Delta^2_{\geq 1} & \Delta^2_{\geq 2} \end{pmatrix}$$

Fixed-Order Scale Uncertainties

Using naive scale variation for σ_0 $\Rightarrow$ Using above procedure for σ_0



New procedure

- Uncertainties reproduce naive scale variation at large cut values
- Larger uncertainties at small cut values

  $\longrightarrow$ Take into account presence of large logarithmic corrections

How to get $\Delta\sigma_{\geq i}$ in Practice

- Simplest: Use relative uncertainties from fixed-order codes

$$\Delta\sigma_{\geq i} = \left[\frac{\Delta\sigma_{\geq i}}{\sigma_{\geq i}}\right]_{\mathrm{FO}} \times [f_{\geq i}]_{\mathrm{MC}}\, \sigma_{\mathrm{total}}$$

  - for $\Delta\sigma_{\geq 1}$: HNNLO, FEHiP, MCFM should be identical
  - for $\Delta\sigma_{\geq 2}$: HNNLO, FEHiP give LO, can use MCFM for NLO

- Reweight to new HqT including all its uncertainties
  - propagates its uncertainties into $\sigma_{\geq 1}$
  - for $\Delta\sigma_{\geq 2}$: use HNNLO/FEHiP or MCFM

Example Numbers

Using HNNLO (Jianming's numbers) to get $\{\Delta\sigma_{\text{total}}, \Delta\sigma_{\geq 1}, \Delta\sigma_{\geq 2}\}$

cut	$\frac{\Delta\sigma_{\text{total}}}{\sigma_{\text{total}}}$	$\frac{\Delta\sigma_{\geq 1}}{\sigma_{\geq 1}}$	$\frac{\Delta\sigma_{\geq 2}}{\sigma_{\geq 2}}$	$\frac{\Delta\sigma_0}{\sigma_0}$	$\frac{\Delta\sigma_1}{\sigma_1}$
$p_T^{\text{cut}} = 30\,\text{GeV},\ \eta^{\text{cut}} = 3$	10%	21%	45%	17%	29%

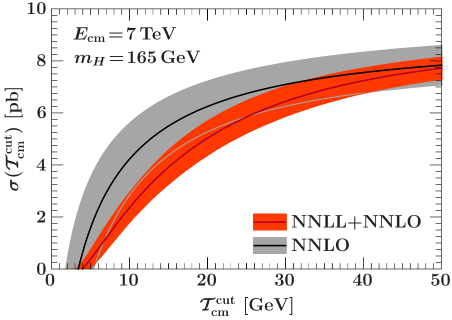
Correlations for $\{\sigma_0, \sigma_1, \sigma_{\geq 2}\}$

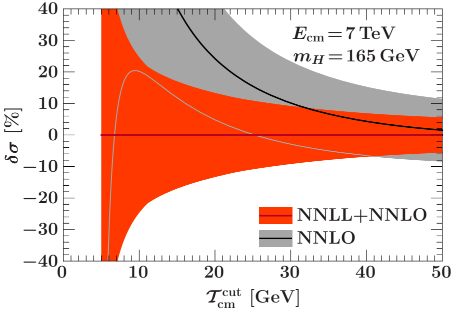
$$\begin{pmatrix} 1 & -0.49 & 0 \\ -0.49 & 1 & -0.42 \\ 0 & -0.42 & 1 \end{pmatrix}$$

Correlations for $\{\sigma_{\text{total}}, f_0, f_1, f_2\}$

$$\begin{pmatrix} 1 & 0.44 & -0.33 & -0.22 \\ 0.44 & 1 & -0.92 & -0.10 \\ -0.33 & -0.92 & 1 & -0.31 \\ -0.22 & -0.10 & -0.31 & 1 \end{pmatrix}$$

Comparison to Resummation





cut	order	$\frac{\Delta\sigma_{\rm total}}{\sigma_{\rm total}}$	$\frac{\Delta\sigma_{\geq 1}}{\sigma_{\geq 1}}$	$\frac{\Delta\sigma_0}{\sigma_0}$
$\mathcal{T}_{\rm cm}^{\rm cut} = 20\,$GeV	NNLO	8.5%	28%	16%
$\mathcal{T}_{\rm cm}^{\rm cut} = 20\,$GeV	NNLL+NNLO	5.2%	21%	13%

- NNLO uncertainties now consistent with those from resummation

$\Rightarrow$ Resummation can improve uncertainties (as one would expect)