DIS dijet production in Background Field Approach

General formalism and methods

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[based on arXiv: 2603.08805]

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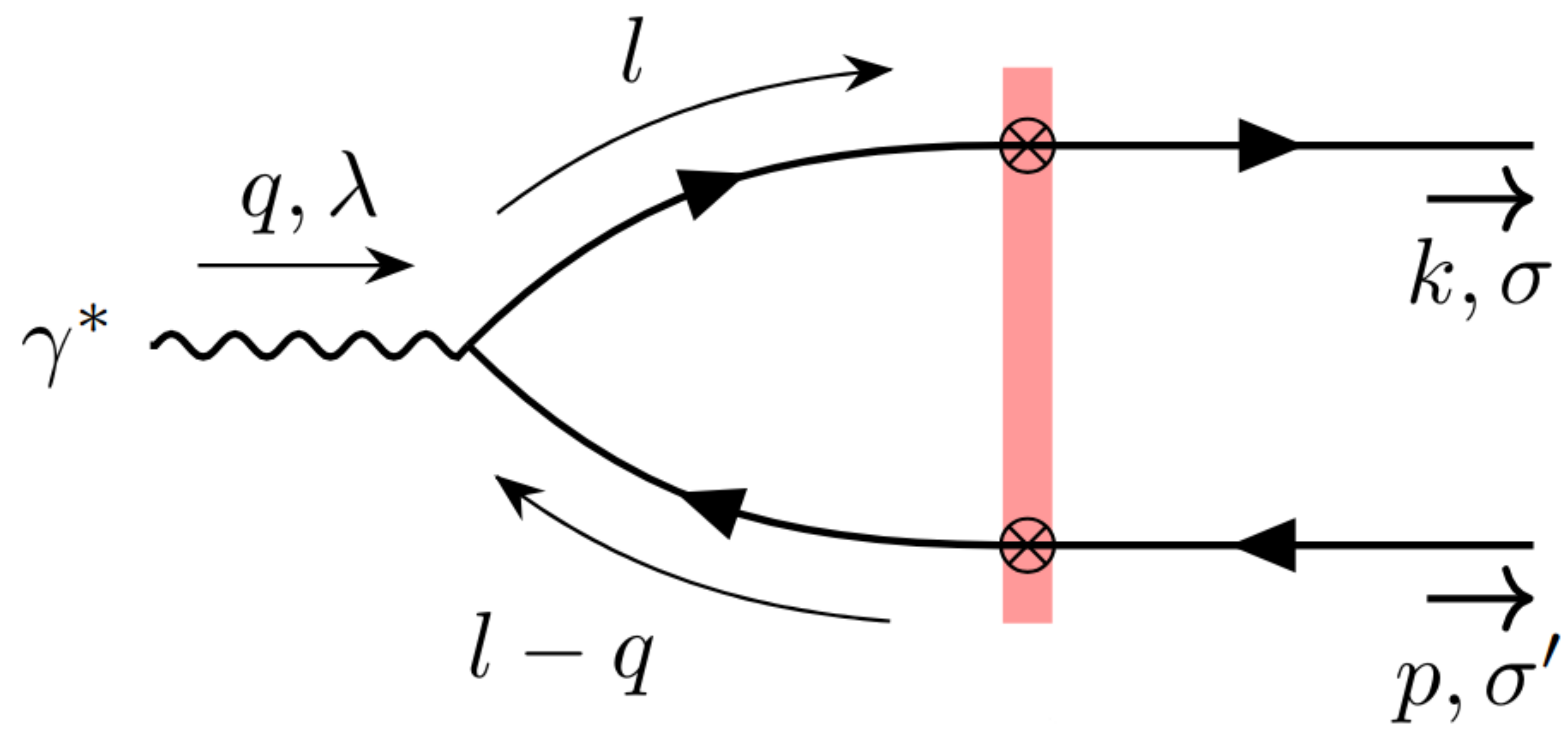
- Motivation
- Background field formalism
- Quark propagator
- Projection onto different kinematic regimes
- A generalised eikonal cross-section
- Conclusion

Dijet Production in DIS

- Saturation at EIC: What are emergent properties of dense system of gluons?

- Dijet production is the most intensively discussed process for EIC Physics.

- Small $x \to$ the dipole picture of DIS.

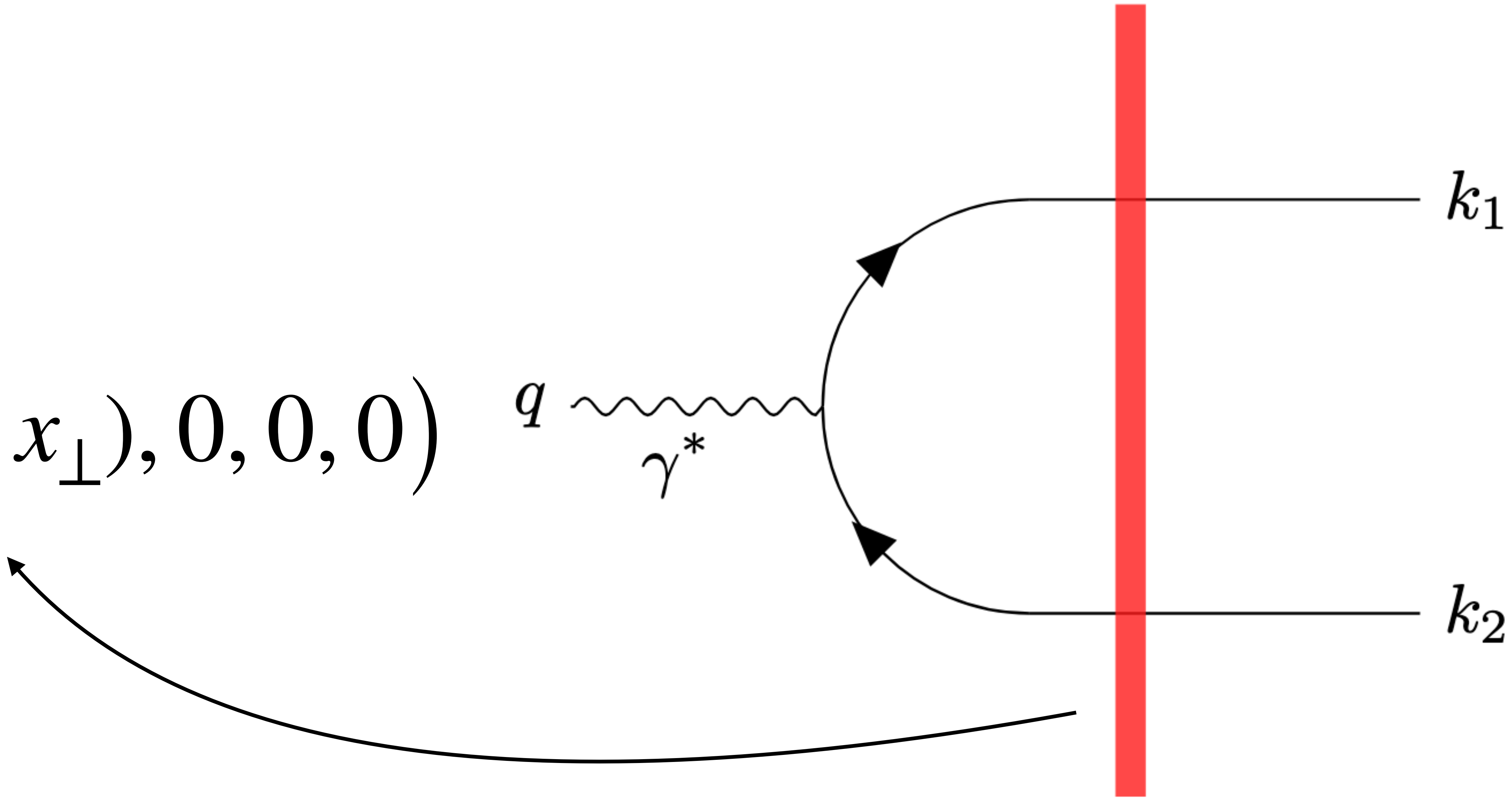


Color Glass Condensate

Conventional eikonal approximation

- "Wrong" light-cone gauge condition : $A_+ = 0$

$$A_\mu = \left(A_-(x^-, x_\perp), 0, 0, 0\right)$$



- Right-moving nucleus:

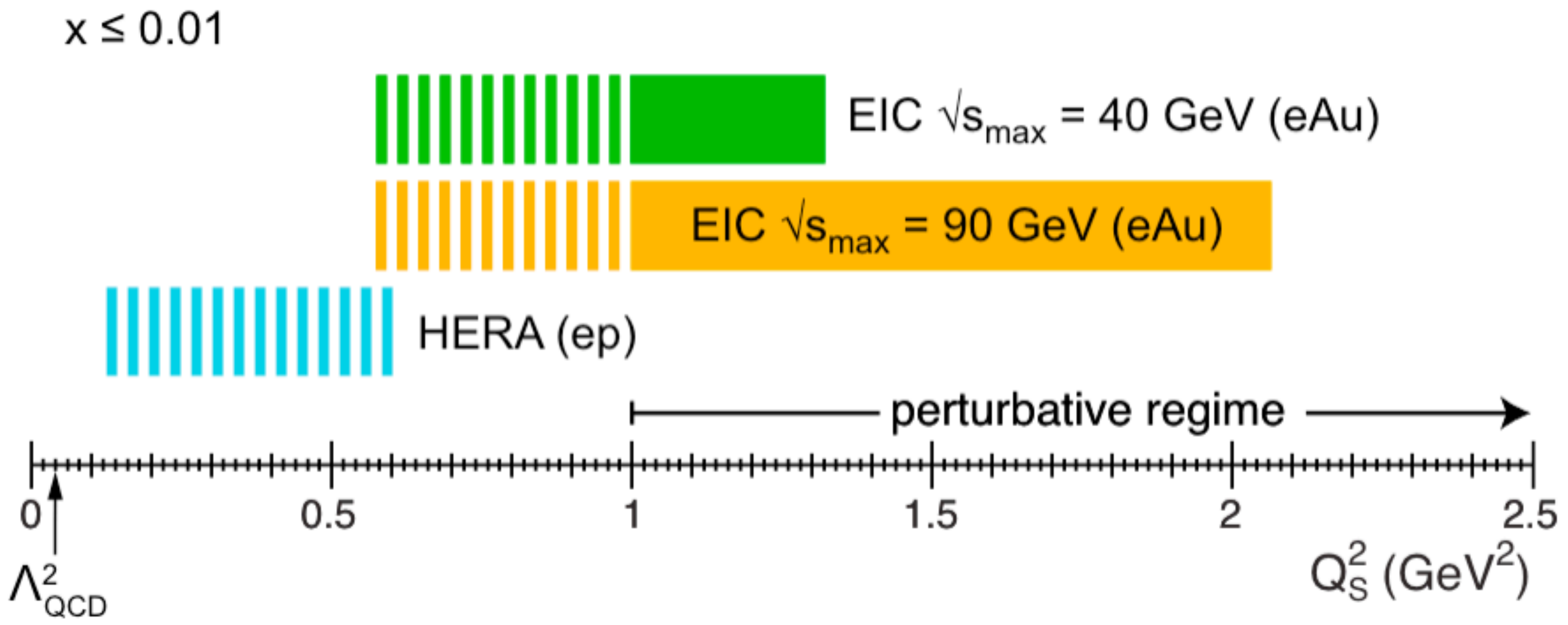
  $P^+ \gg P^-$

- Left moving electron:

  $q^+ \ll q^-$

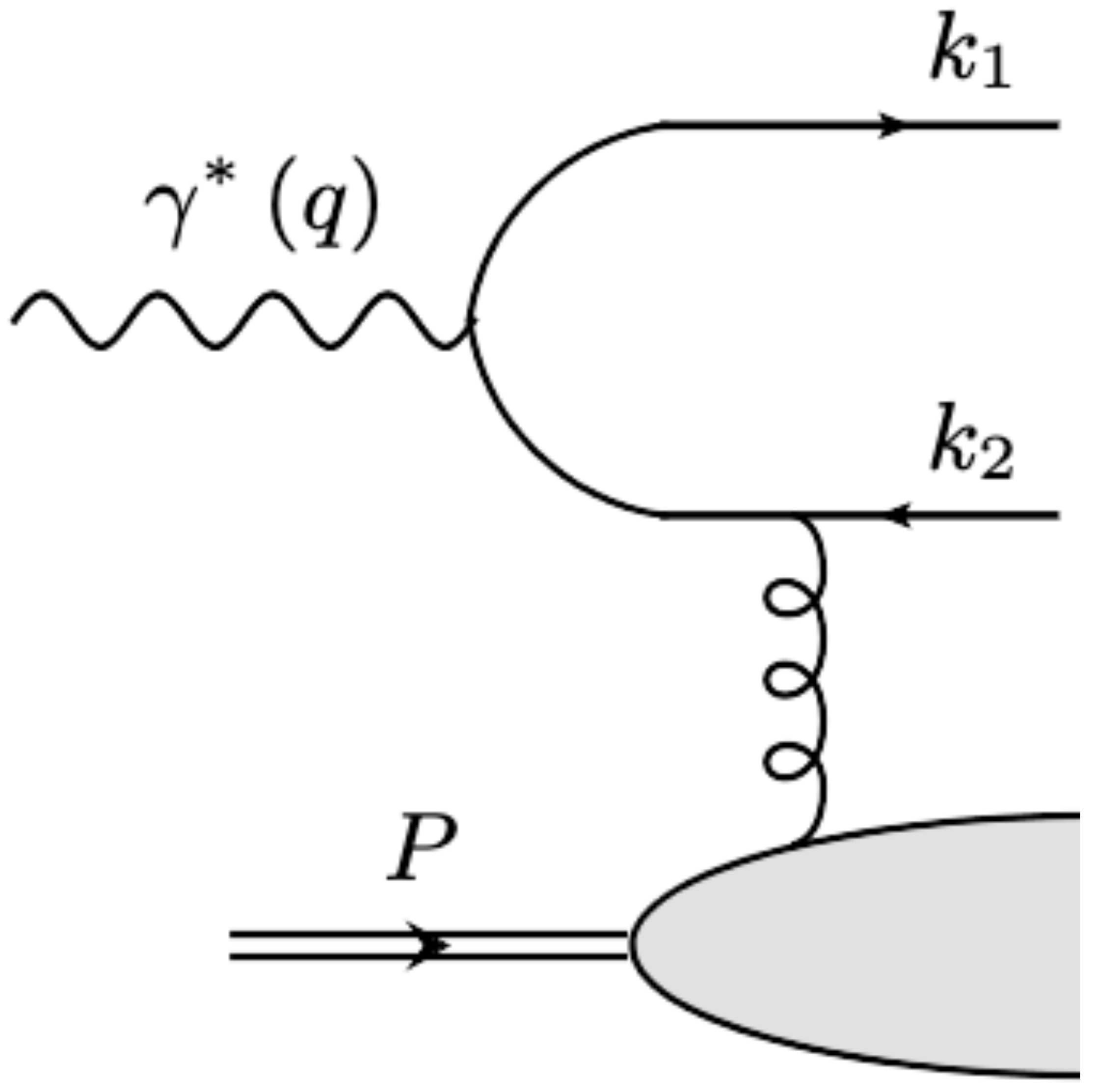
- Subeikonal corrections to the diet production cross-section.
  [Agostini, Altinoluk, Armesto, 2403.04603, Altinoluk, Beuf, Czayka, Tymowska, 2012.03886]

- A generalised eikonal prescription.



TMD factorisation

- Applicable at arbitrary-x.
- Back-to-back dijet configuration: $|\Delta_\perp| \ll |P_\perp|$
- Expansion parameter: $\frac{\Delta_\perp}{P_\perp} \ll 1$



$$\Delta_\perp = k_{1\perp} + k_{2\perp}$$

$$P_\perp = \zeta k_{1\perp} - \bar{\zeta} k_{2\perp}$$

- Cross-section factorises in terms of gluon TMD operators.

Why a new formalism?

Approach	Strength	Limitation
CGC	Small-x	Limited access to x-dependence
TMD	Arbitrary-x	Limited access to higher twist contribution

Our Approach: Gauge covariant propagator reproducing the result of these two approaches

- A general formalism for connecting these approaches

Background Field Method

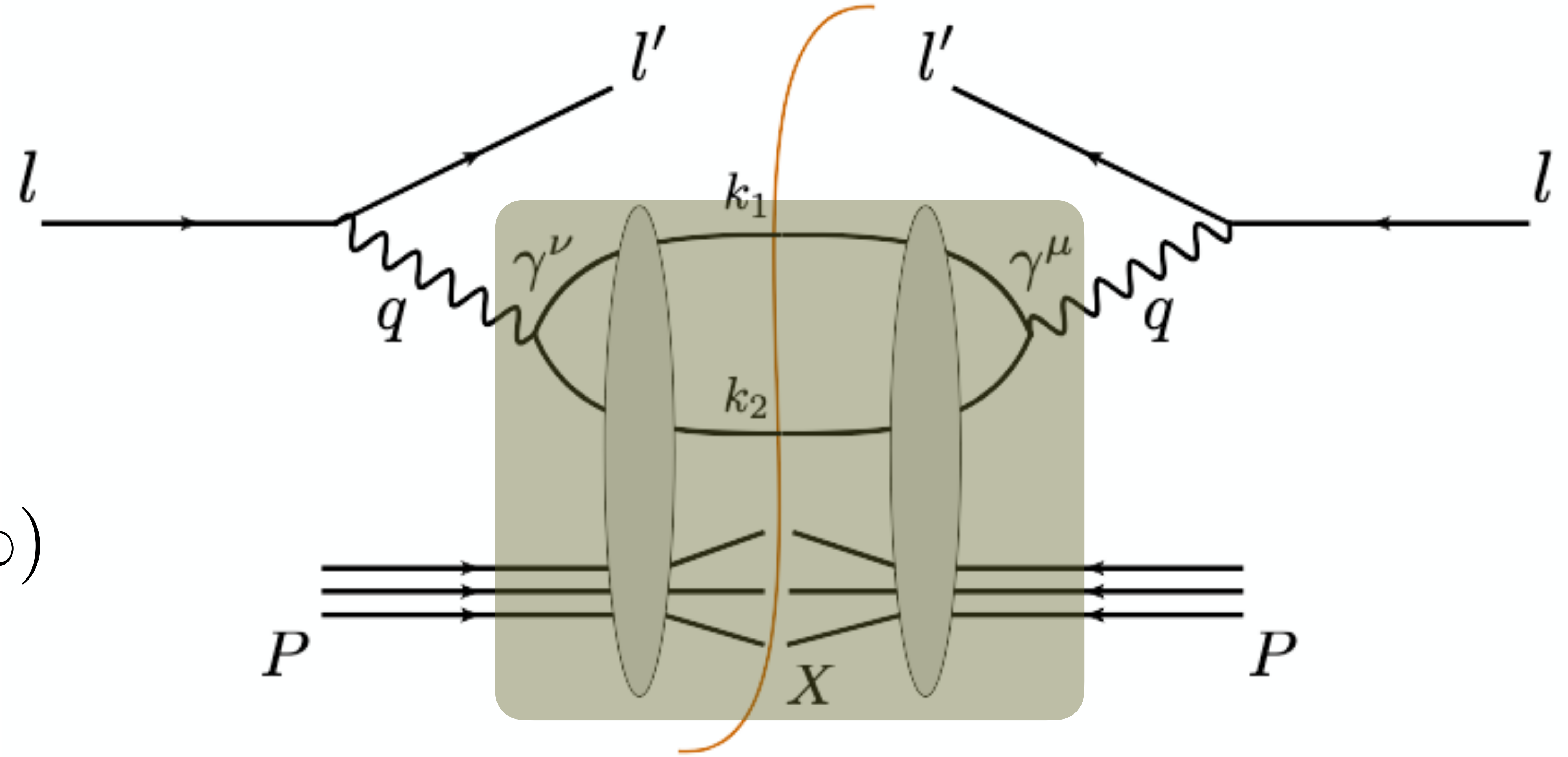
Our approach

- In the cross-section:

$$\mathcal{W}^{\mu\nu} = \sum_X \langle P, S|\tilde{T}\{\psi(x_2)j^\mu(x)\bar{\psi}(x_1)\}|X\rangle\langle X|T\{\psi(y_1)j^\nu(y)\bar{\psi}(y_2)\}|p, S\rangle$$

$$= \mathcal{N}^{-1}\int \mathcal{D}C \int \mathcal{D}\tilde{C}\ \Psi^*_{P,S}(\tilde{\vec{C}}(t_i))\,\tilde{\psi}(x_2)\,\tilde{j}^\mu(x)\,\tilde{\bar{\psi}}(x_1)$$

$$\times\, \psi(y_1)j^\nu(y)\,\bar{\psi}(y_2)\,\Psi_{P,S}(\vec{C}(t_i))\,e^{iS_{QCD}(C)-iS_{QCD}(\tilde{C})}$$

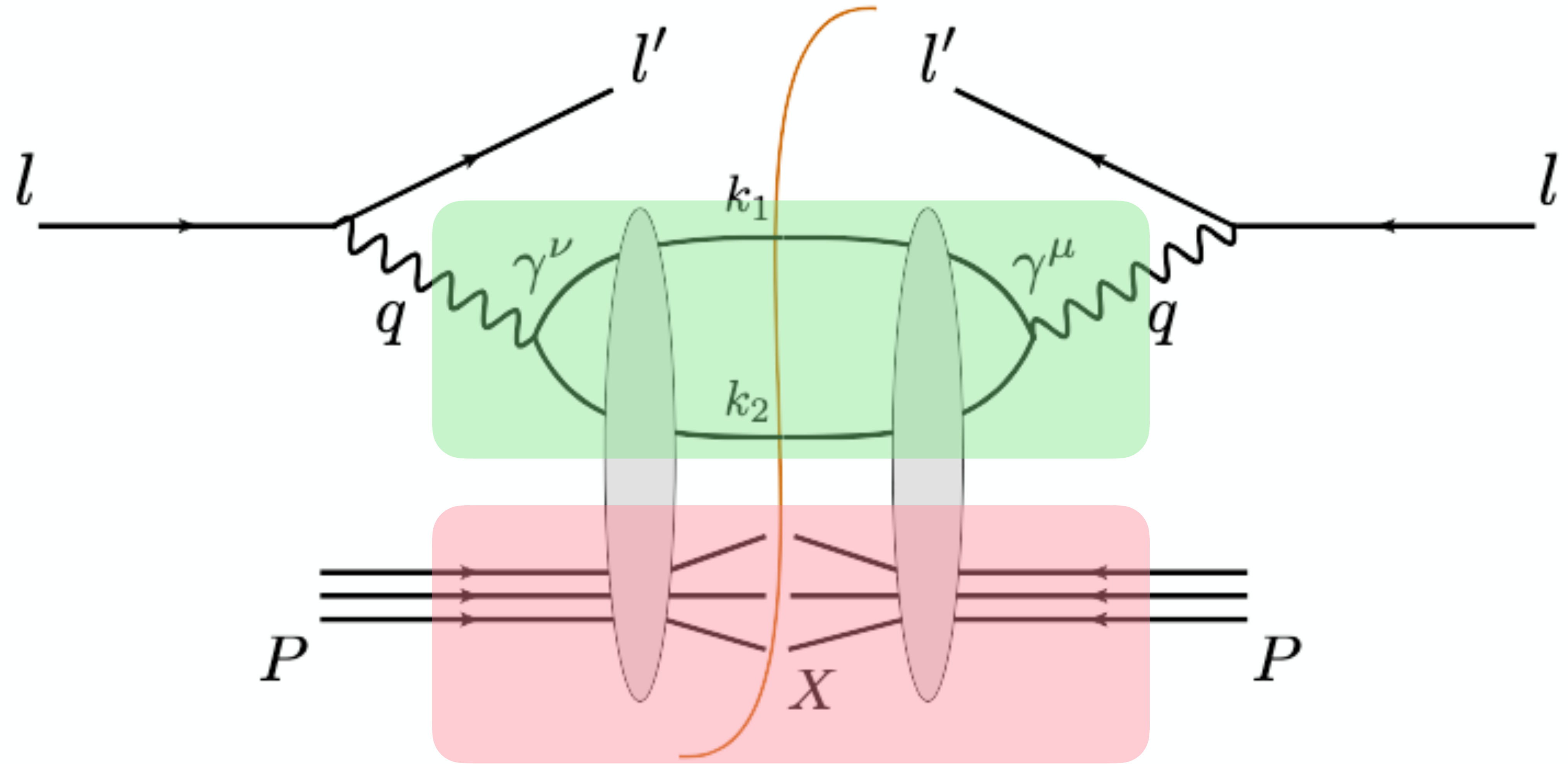


Boundary condition: $\tilde{C}(\vec{x}, t = \infty) = C(\vec{x}, t = \infty)$

- Fields can be decomposed into perturbative and non-perturbative parts.

$$C = A + B$$

Propagation of "quantum" particles A in background field B

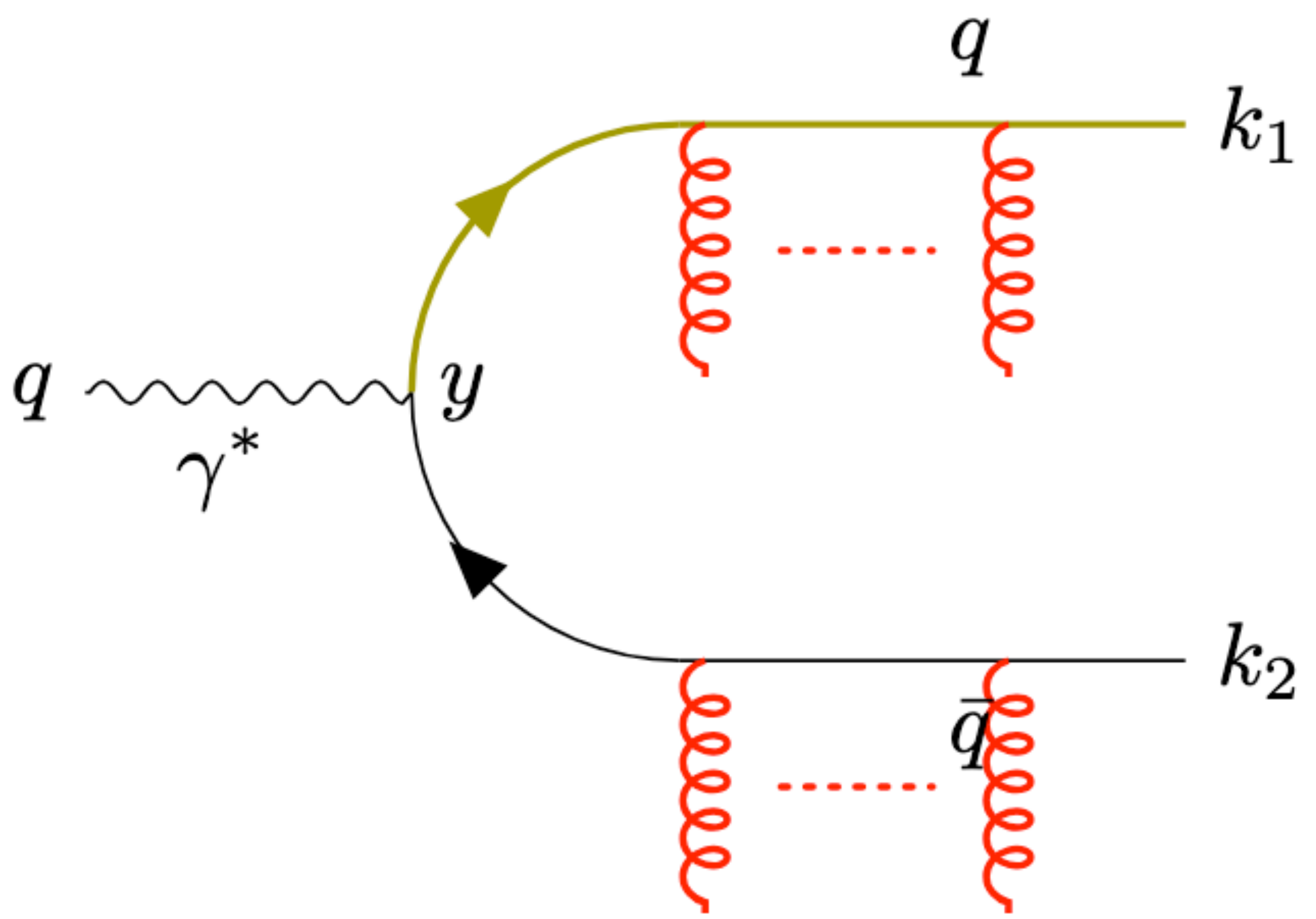


- Background QCD action: $S_{bg}(A, B) \equiv S_{QCD}(A+B) - S_{QCD}(B)$

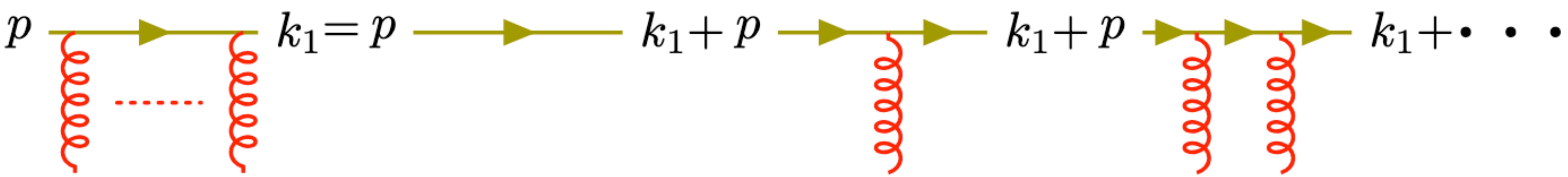
Quark Propagator

- The amplitude:

$$i\mathcal{M} = -ie \int d^4y\, \bar{u}(k_1)\not{k}_1 \underbrace{(k_1|\not{P}\frac{i}{\not{P}^2 + i\epsilon}|y)}_{\text{Quark propagator in the background field}}\gamma^\nu (y|\frac{i}{\not{P}^2 + i\epsilon}\not{P}| - k_2)\not{k}_2 v(k_2)\epsilon_\nu(q)e^{-iq.y} - (B = 0)$$



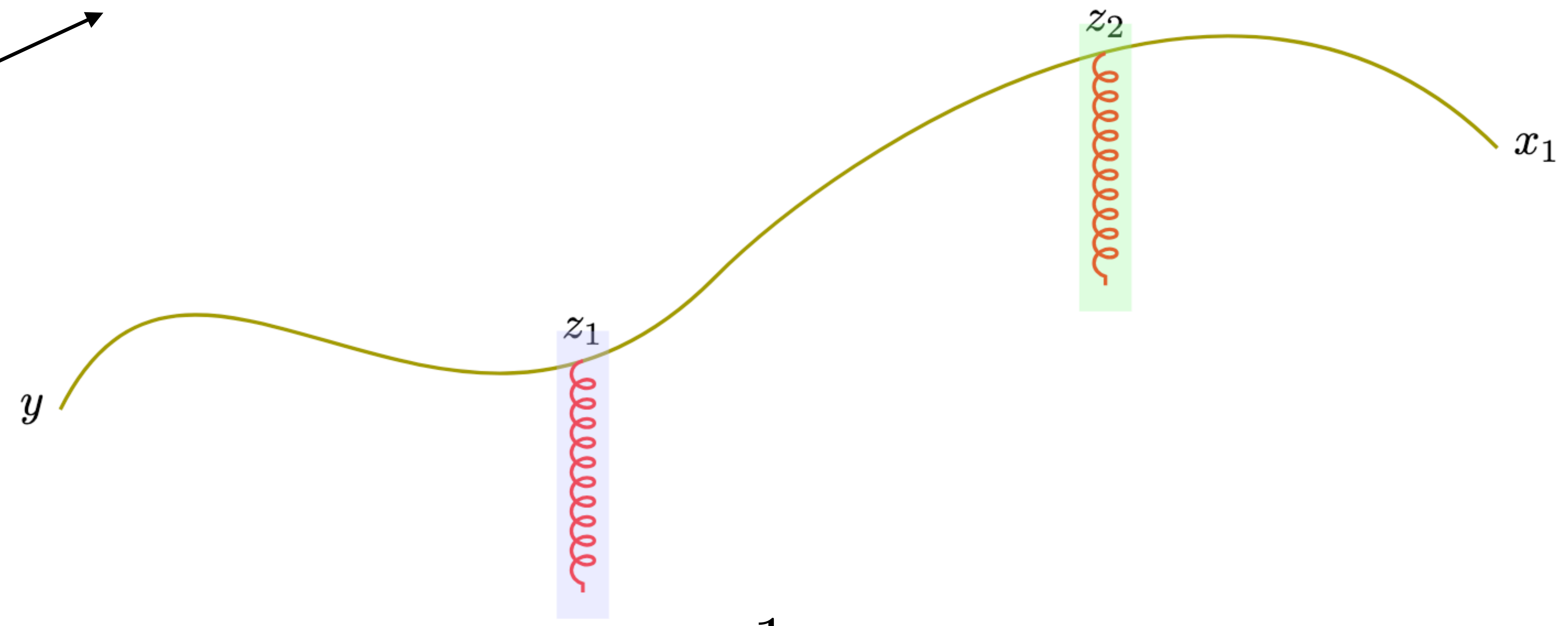
- Covariant momentum: $P_\mu = p_\mu + g B_\mu$



- The quark leg in background field:

$$\bar{u}(k_1)\not{k}_1(k_1|\not{P}\frac{i}{\not{P}^2+i\epsilon}|y) = \bar{u}(k_1)k_1^2(k_1|\frac{i}{P^2+\frac{g}{2}\sigma^{\mu\nu}F_{\mu\nu}}|y)$$

Pauli vertex: The fermionic factor in the propagator.

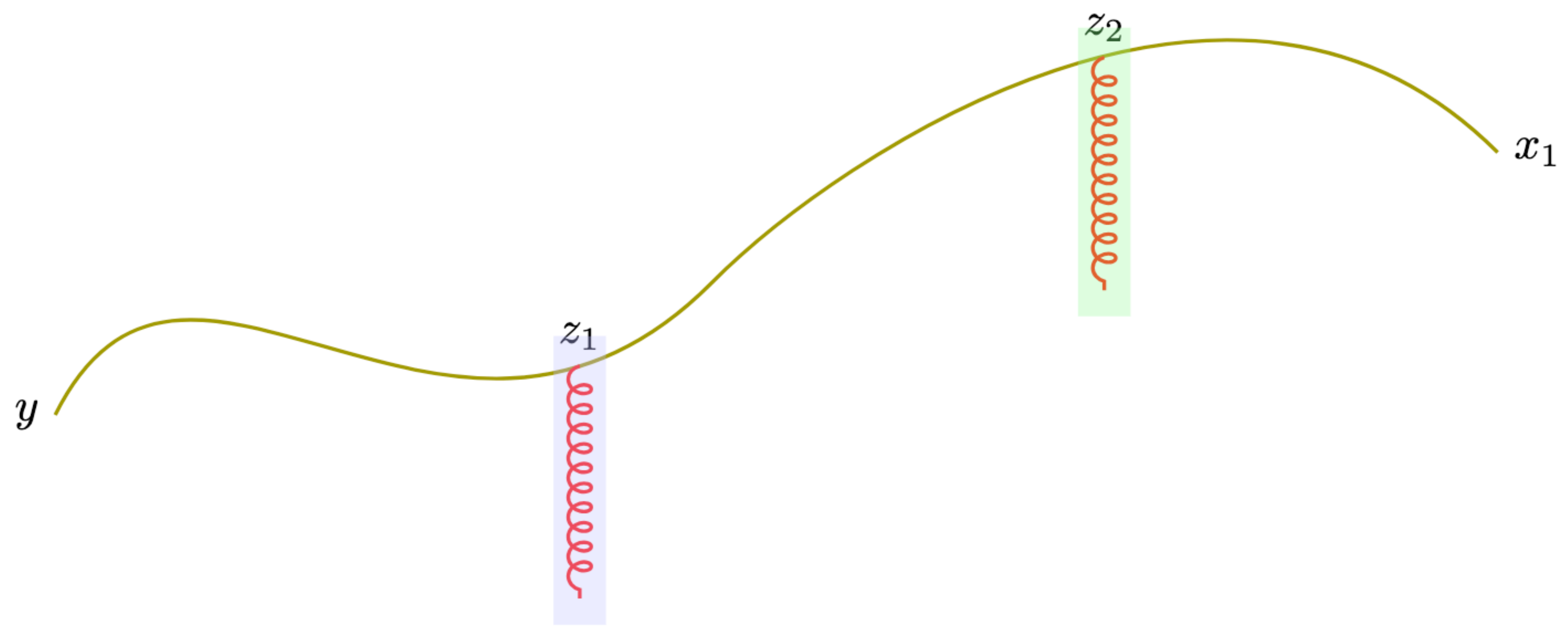


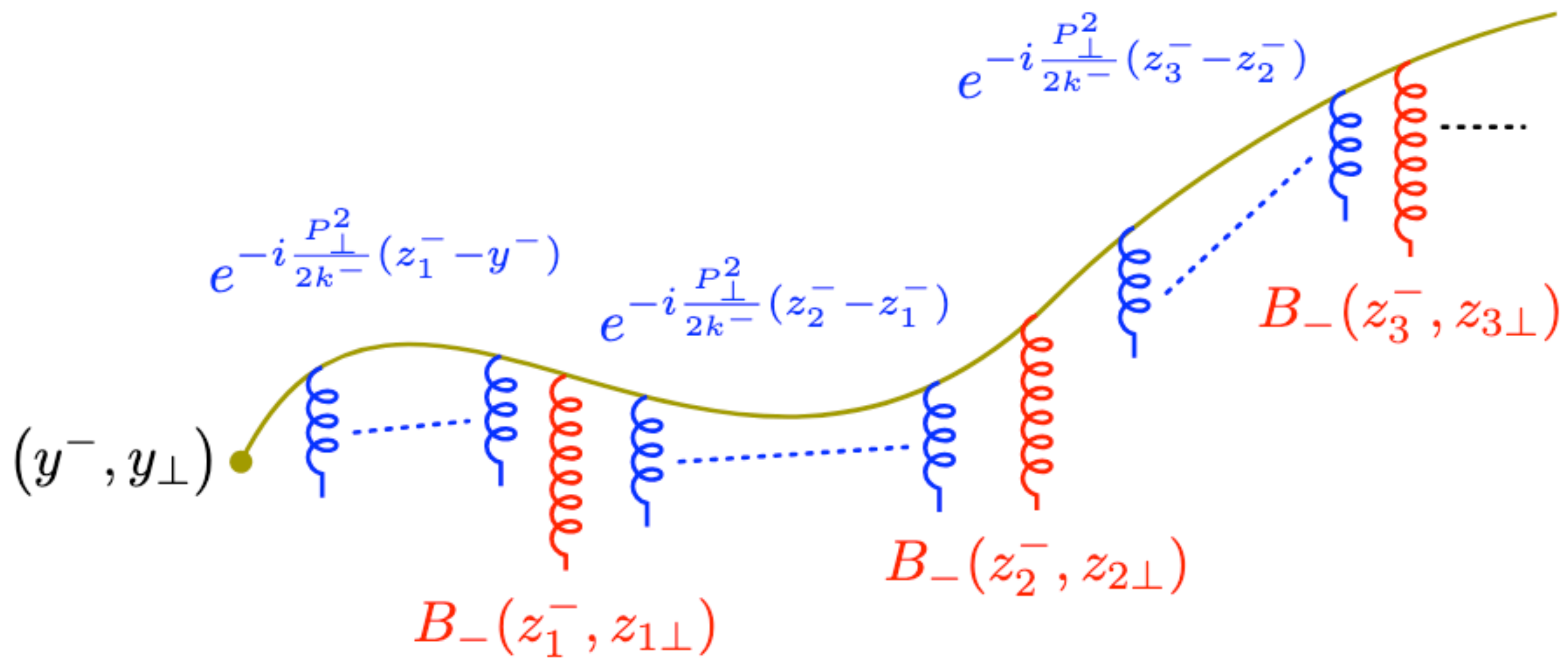
$$k_1^2(k_1|\frac{1}{P^2+\frac{g}{2}\sigma^{\mu\nu}F_{\mu\nu}}|y) = k_1^2(k_1|\frac{1}{p^2} - \frac{1}{p^2}\left(g\{p^\mu, A_\mu\} + g^2A^\mu A_\mu + \frac{g}{2}\sigma^{\mu\nu}F_{\mu\nu}\right)\frac{1}{p^2}$$

$$+\frac{1}{p^2}\left(g\{p^\mu, A_\mu\} + g^2A^\mu A_\mu + \frac{g}{2}\sigma^{\mu\nu}F_{\mu\nu}\right)\frac{1}{p^2}\left(g\{p^\mu, A_\mu\} + g^2A^\mu A_\mu + \frac{g}{2}\sigma^{\mu\nu}F_{\mu\nu}\right)\frac{1}{p^2} - \ldots|y)$$

$$k_1^2 (k_1| \frac{1}{P^2 + \frac{g}{2}\sigma^{\mu\nu}F_{\mu\nu}} |y) = k_1^2 (k_1| \frac{1}{p^2} - \frac{1}{p^2}\left(g\{p^\mu, A_\mu\} + g^2 A^\mu A_\mu + \frac{g}{2}\sigma^{\mu\nu}F_{\mu\nu}\right)\frac{1}{p^2}$$

$$+ \frac{1}{p^2}\left(g\{p^\mu, A_\mu\} + g^2 A^\mu A_\mu + \frac{g}{2}\sigma^{\mu\nu}F_{\mu\nu}\right)\frac{1}{p^2}\left(g\{p^\mu, A_\mu\} + g^2 A^\mu A_\mu + \frac{g}{2}\sigma^{\mu\nu}F_{\mu\nu}\right)\frac{1}{p^2} - \dots |y)$$





$$\lim_{k^2\to 0} k^2 (k| \frac{1}{P^2 + \frac{g}{2}\sigma^{\mu\nu}F_{\mu\nu} + i\epsilon} |y) = \lim_{k^2\to 0} (k_\perp|\theta(k^-)\mathcal{P}\exp\left\{i\int_{y^-}^{\infty} dz^-\left(e^{i\frac{P_\perp^2(z^-)}{2p^-}z^-}\left(iD_- + \frac{g}{4p^-}\sigma^{\mu\nu}F_{\mu\nu}\right)\right.\right.$$

$$\left.\left.\times\, e^{-i\frac{P_\perp^2(z^-)}{2p^-}z^-} - \frac{P_\perp^2(z^-)}{2p^-}\right)\right\} e^{i\frac{P_\perp^2(y^-)}{2k^-}y^-} |y_\perp) e^{ik^-y^+}$$

Boundary condition:

$$\lim_{z^-\to\infty} B_\mu(z^-) = 0$$

- Gauge-covariant propagator: Starting point for our approach
- What is the role of those transverse exponentials?

Bare Propagator

- Vacuum propagator:

$$(x|\frac{1}{p^2+i\epsilon}|y)\Big|_{x^->y^-} = \frac{-i}{2\pi}\int_0^\infty \frac{dp^-}{2p^-} e^{-ip^-(x^+-y^+)}(x_\perp| \; e^{-i\frac{p_\perp^2}{2p^-}(x^--y^-)} \; |y_\perp)$$

Dictates the shift in transverse position from $y_\perp$ to $x_\perp$

- Without transverse propagator:

$$(x_\perp|y_\perp) = \delta^{(2)}(x_\perp - y_\perp)$$

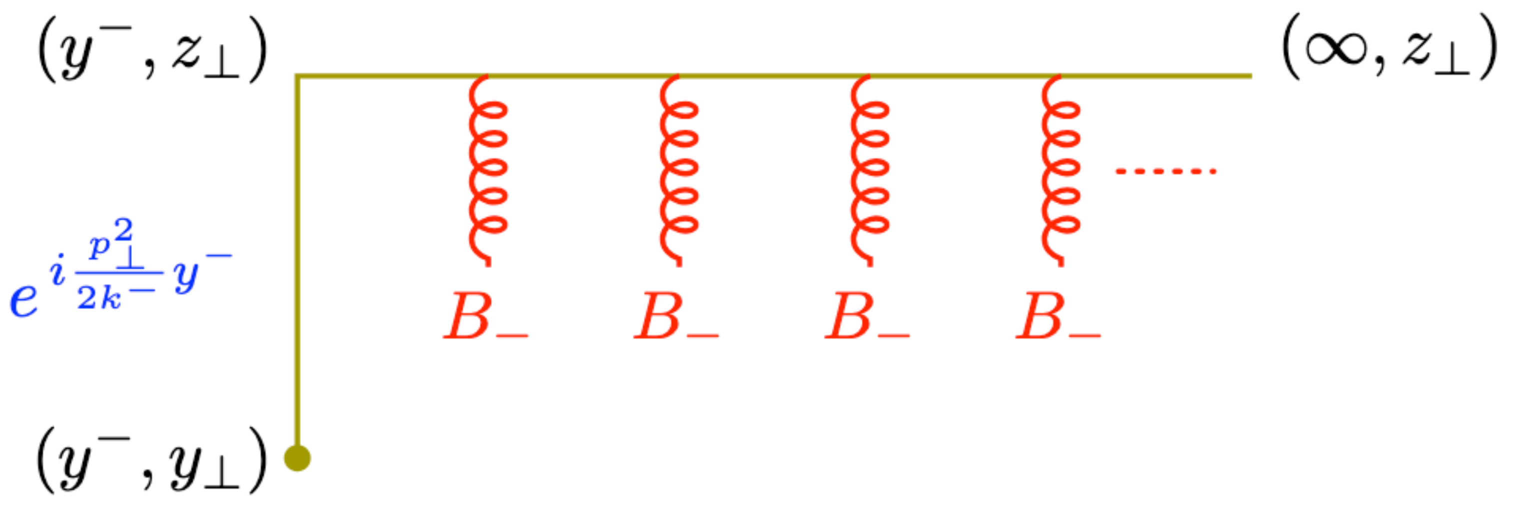
- Scalar propagator:

$B_i = 0$ & neglecting the exponentials with z^-

$$\lim_{k^2\to 0} k^2(k|\frac{1}{P^2+i\epsilon}|y)\Big|_{B_i=0;k^->0}$$

$$\approx \lim_{k^2\to 0}\int d^2z_\perp e^{-ik_\perp z_\perp}U(\infty,y^-;z_\perp)$$

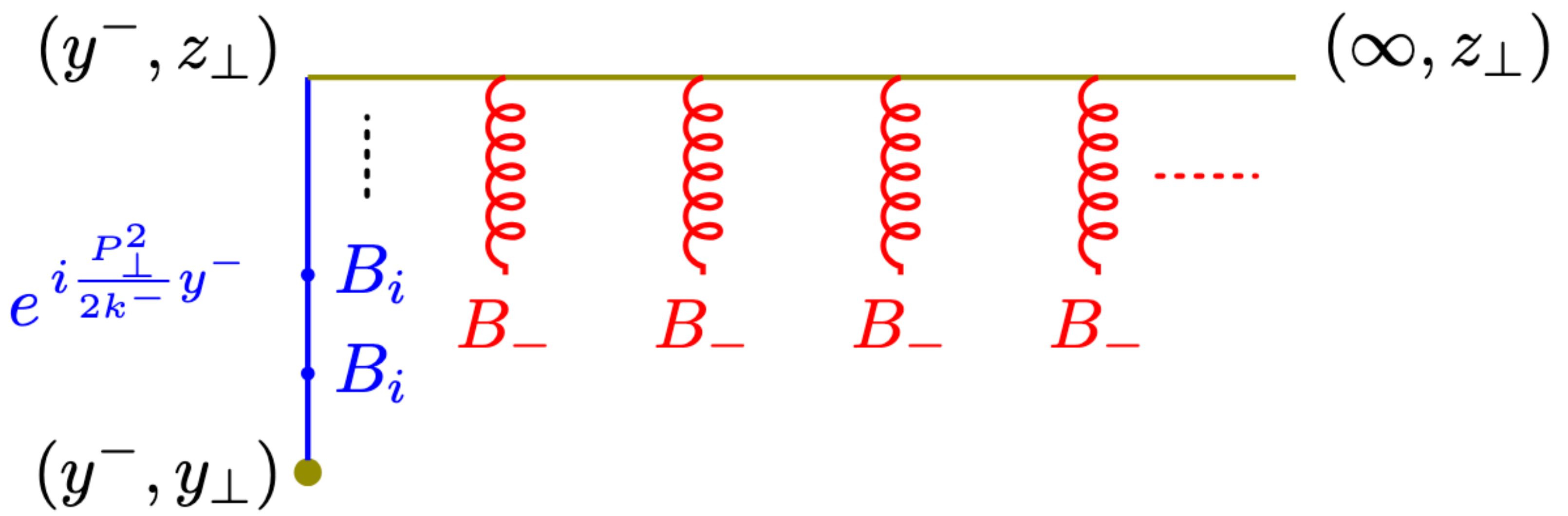
$$\times\,(z_\perp|e^{i\frac{p_\perp^2}{2k^-}y^-}|y_\perp)e^{ik^-y^+}$$

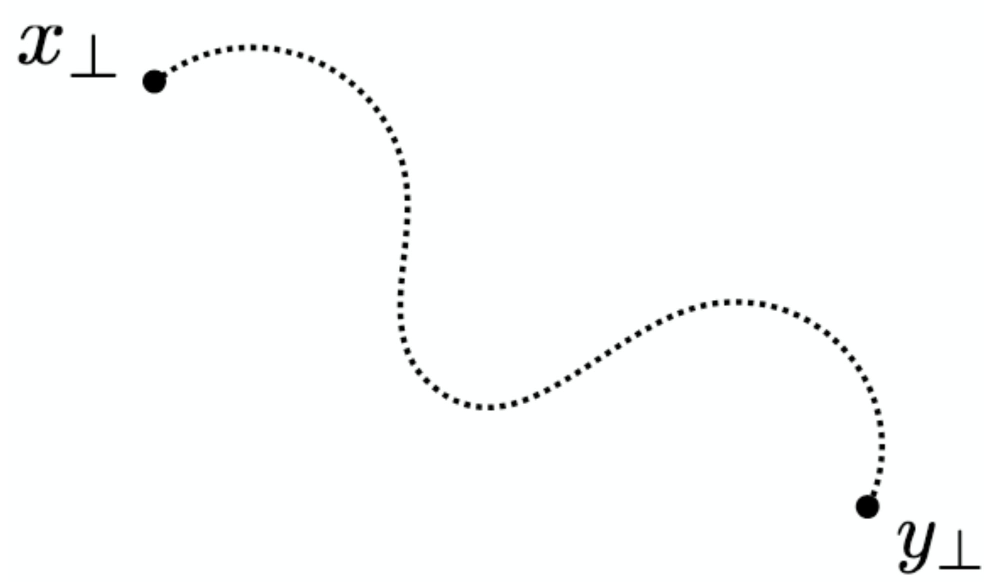


Transverse Gauge Link

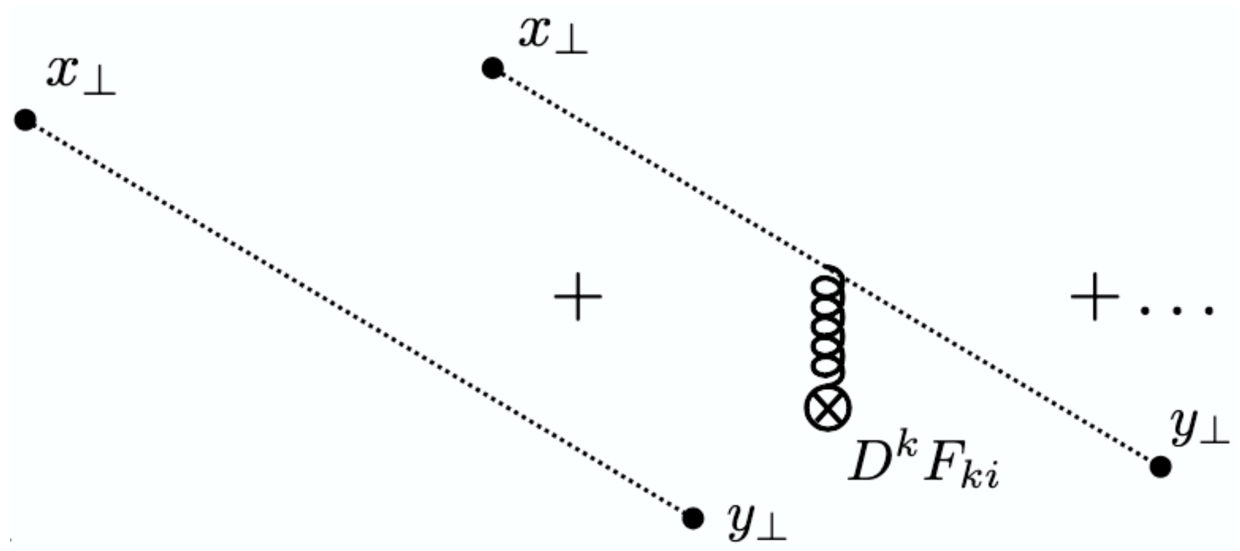
- For non-vanishing B_i :

$$(x_\perp|e^{i\frac{P_\perp^2}{2k^-}z^-}|y_\perp)$$





Linear Parametrization



$$(x_\perp|e^{isP_\perp^2}|y_\perp) = (x_\perp|e^{isp_\perp^2}|y_\perp)\Big\{U(x_\perp, y_\perp) + s\int_0^1 du\ u\bar{u}\ (x-y)^k U(x_\perp, \xi_u) D_m F_{mk}(\xi_u) U(\xi_u, y_\perp)$$

$$+ 2is\int_0^1 du\ \bar{u}\int_0^u dv\ v\ (x-y)^k (x-y)^m U(x_\perp, \xi_u) F_{ks}(\xi_u) U(\xi_u, \xi_v) F_{ms}(\xi_v) U(\xi_v, y_\perp) + \ldots\Big\}$$

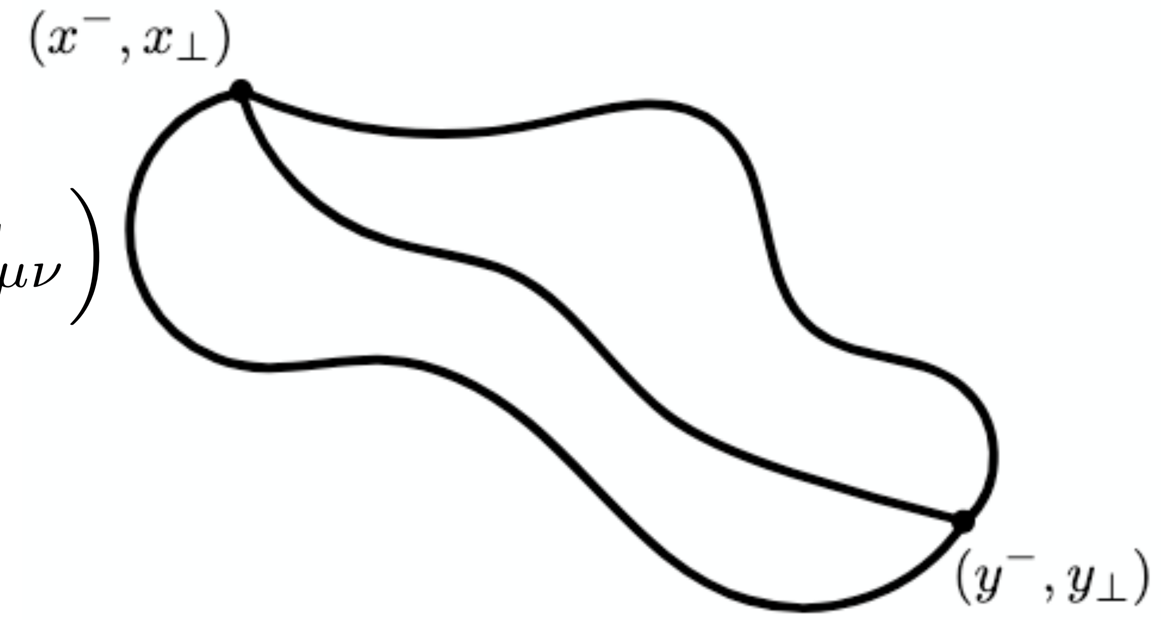
[Balitsky, Braun, Nucl. Phys. B, 311:541–584, 1989]

- Not just a transverse Wilson line!

Contour Projections

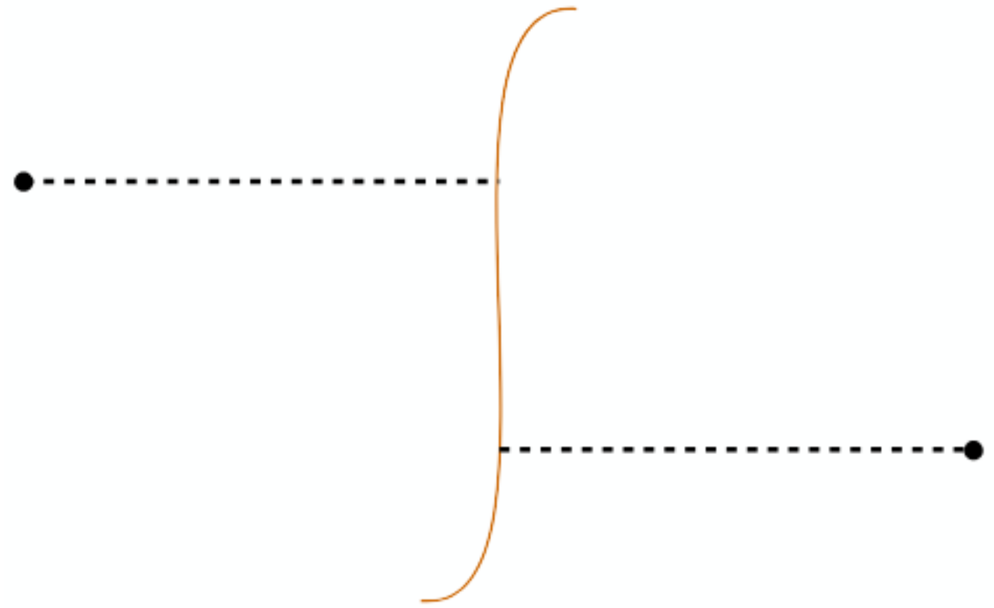
Dictionary for switching different kinematic regimes

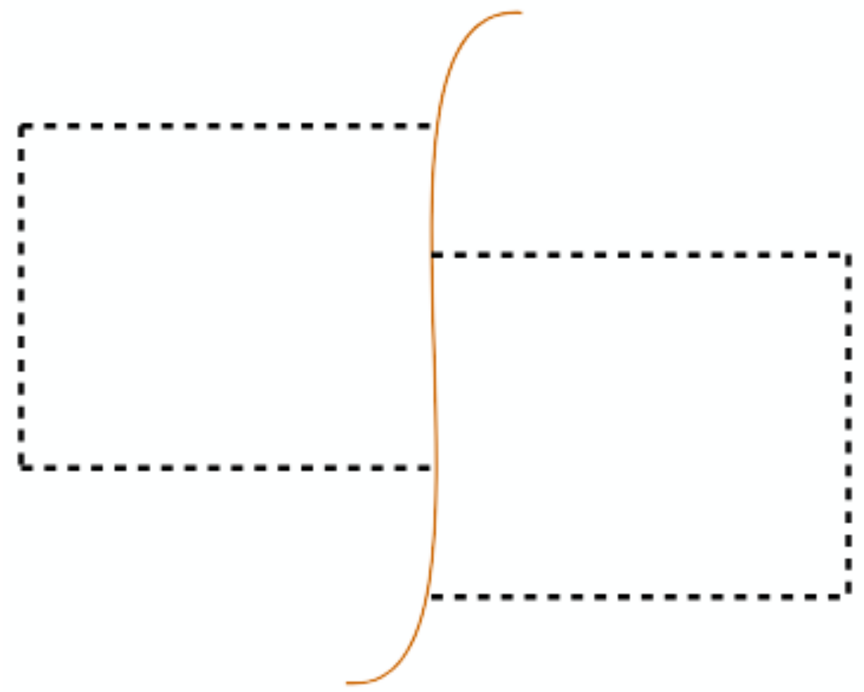
- The path ordered exponential describes propagation of the quantum partons along arbitrary path.



$$\lim_{k^2\to 0} k^2 (k|\frac{1}{P^2+\frac{g}{2}\sigma^{\mu\nu}F_{\mu\nu}+i\epsilon}|y) = \lim_{k^2\to 0}(k_\perp|\theta(k^-)\mathcal{P}\exp\left\{i\int_{y^-}^{\infty} dz^- \left(e^{i\frac{P_\perp^2(z^-)}{2p^-}z^-}\left(iD_- + \frac{g}{4p^-}\sigma^{\mu\nu}F_{\mu\nu}\right)\right.\right.$$
$$\left.\left.\times\, e^{-i\frac{P_\perp^2(z^-)}{2p^-}z^-} - \frac{P_\perp^2(z^-)}{2p^-}\right)\right\} e^{i\frac{P_\perp^2(y^-)}{2k^-}y^-}|y_\perp)e^{ik^-y^+}$$

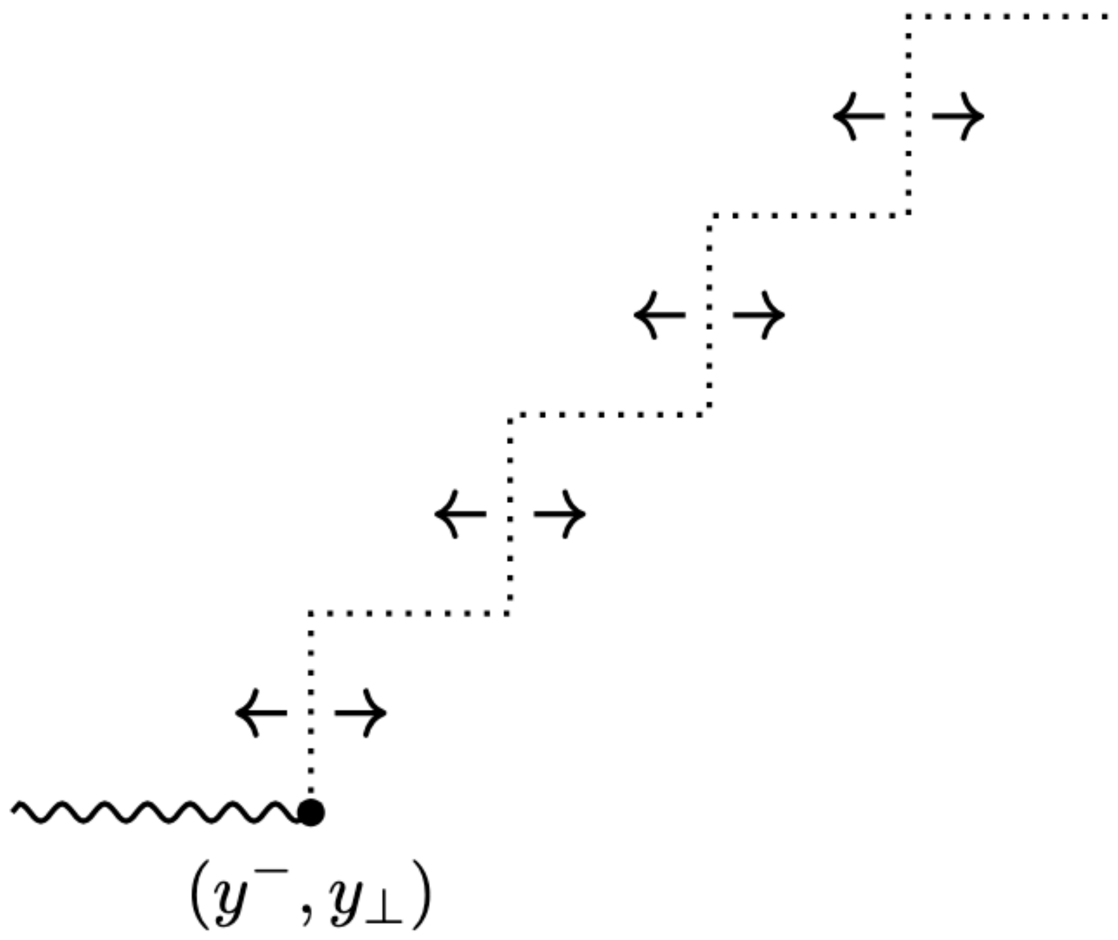
- In order to study different kinematic regimes we need to expand this exponential onto certain paths.





Procedure of the Contour Expansion

- A 2-step process.
  1. Expansion of the exponential.
  2. Commuting the background field operators to the desired transverse position.



$$e^{-i\frac{P_\perp^2(x^-)}{2p^-}x^-}\mathcal{P}\exp\left\{i\int_{y^-}^{x^-}dz^-\left(e^{i\frac{P_\perp^2(z^-)}{2p^-}z^-}\left(iD_- + \frac{g}{4p^-}\sigma^{\mu\nu}F_{\mu\nu}\right)e^{-i\frac{P_\perp^2(z^-)}{2p^-}z^-} - \frac{P_\perp^2(z^-)}{2p^-}\right)\right\}e^{i\frac{P_\perp^2(y^-)}{2p^-}y^-}$$

$$= e^{-i\frac{P_\perp^2(x^-)}{2p^-}(x^- - \xi^-)}\mathcal{P}\exp\left\{i\int_{y^-}^{x^-}dz^-\left(e^{i\frac{P_\perp^2(z^-)}{2p^-}(z^- - \xi^-)}\left(iD_- + \frac{g}{4p^-}\sigma^{\mu\nu}F_{\mu\nu}\right)e^{-i\frac{P_\perp^2(z^-)}{2p^-}(z^- - \xi^-)} - \frac{P_\perp^2(z^-)}{2p^-}\right)\right\}$$

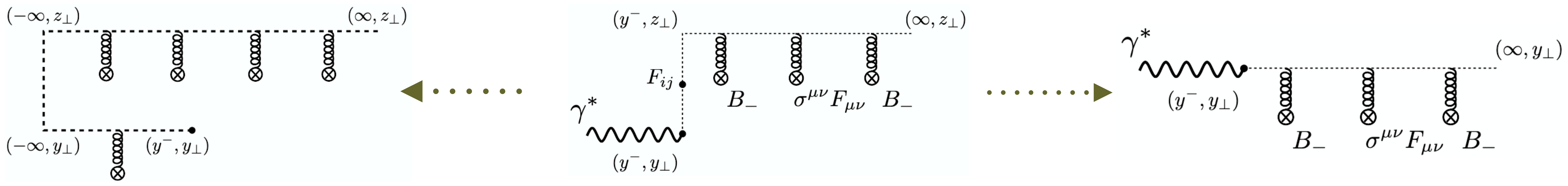
$$\times\, e^{i\frac{P_\perp^2(y^-)}{2p^-}(y^- - \xi^-)}$$

- Propagator remains gauge covariant after shifting transverse exponential.

- The transverse exponential at a finite z^- implies infinite number of operator insertions.

$$(x_\perp|e^{isP_\perp^2}|y_\perp) = (x_\perp|e^{isp_\perp^2}|y_\perp)\Big\{U(x_\perp,y_\perp) + s\int_0^1 du\ u\bar{u}\ (x-y)^k U(x_\perp,\xi_u)D_m F_{mk}(\xi_u)U(\xi_u,y_\perp)$$
$$+ 2is\int_0^1 du\ \bar{u}\int_0^u dv\ v\ (x-y)^k(x-y)^m U(x_\perp,\xi_u)F_{ks}(\xi_u)U(\xi_u,\xi_v)F_{ms}(\xi_v)U(\xi_v,y_\perp) + \dots\Big\}$$

- The fields are pure gauge at $z^- = \pm\infty$. $\Longrightarrow$ Shift the transverse phase at $\pm\infty$.



- What is the correct way of expansion?

Background Field

Power-counting in the high energy limit

- For a nucleus boosted along x^+ direction with boost parameter λ:

$$B_-(x^+, x^-, x_\perp) \sim \lambda \tilde{B}_-(\lambda^{-1}x^+, \lambda x^-, x_\perp);$$
$$B_i(x^+, x^-, x_\perp) \sim \tilde{B}_i(\lambda^{-1}x^+, \lambda x^-, x_\perp);$$
$$B_+(x^+, x^-, x_\perp) \sim \lambda^{-1} \tilde{B}_-(\lambda^{-1}x^+, \lambda x^-, x_\perp)$$

$$\Longrightarrow$$

$$F_{-i}(x^+, x^-, x_\perp) \sim \lambda \tilde{F}_{-i}(\lambda^{-1}x^+, \lambda x^-, x_\perp) \quad ;$$
$$F_{-+}(x^+, x^-, x_\perp) \sim \tilde{F}_{-+}(\lambda^{-1}x^+, \lambda x^-, x_\perp);$$
$$F_{ij}(x^+, x^-, x_\perp) \sim \tilde{F}_{ij}(\lambda^{-1}x^+, \lambda x^-, x_\perp);$$
$$F_{+i}(x^+, x^-, x_\perp) \sim \lambda^{-1} \tilde{F}_{+i}(\lambda^{-1}x^+, \lambda x^-, x_\perp)$$

- An example: Implementing the power-counting

$$\int \underbrace{dz^-}_{\sim\lambda^{-1}} \frac{1}{2k_1^-} \sigma^{-i} \underbrace{F_{-i}}_{\sim\lambda} \sim \lambda^0$$

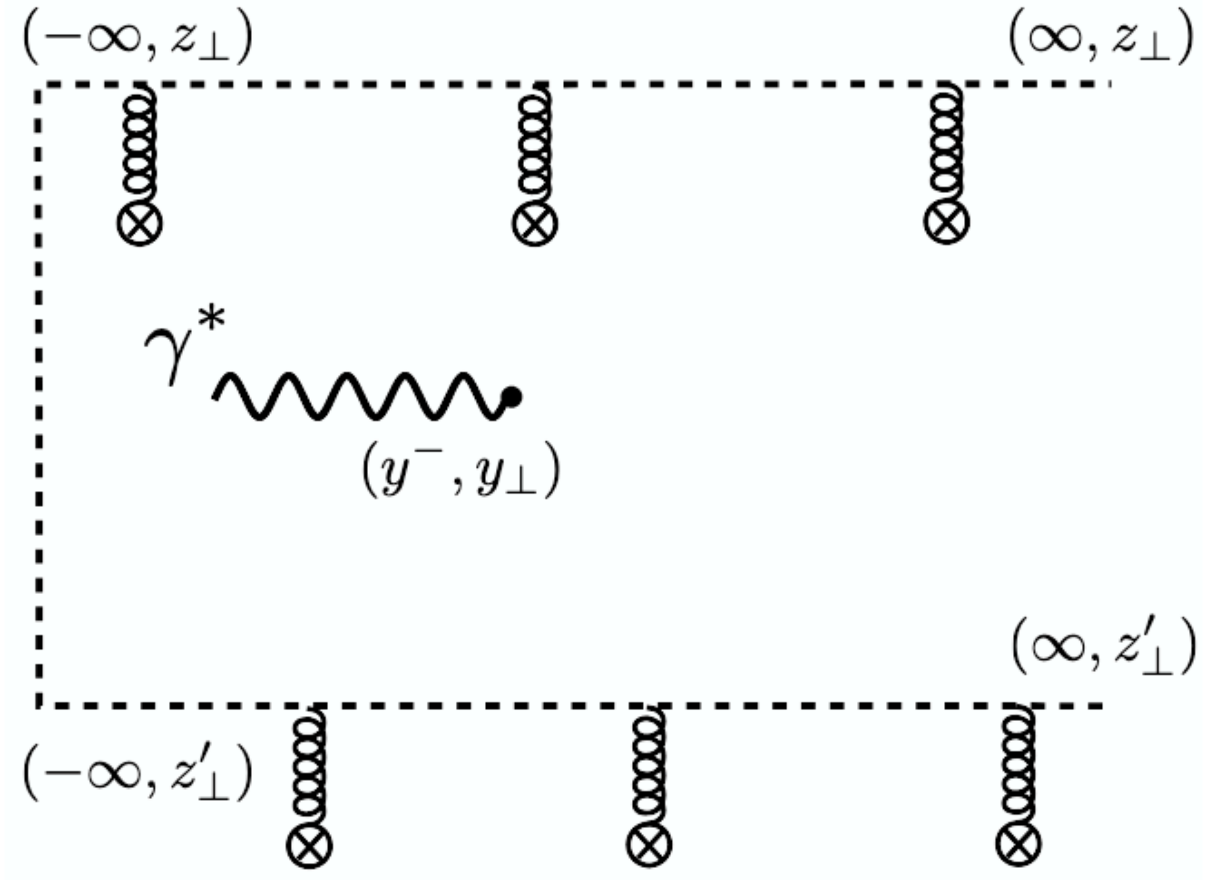
$$\underset{\sim\lambda^0}{U(x^-, y^-)} = 1 + i \int_{y^-}^{x^-} dz^- A_-(z^-) + i^2 \int_{y^-}^{x^-} dz_1^- A_-(z_1^-) \int_{y^-}^{z_1^-} dz_2^- A_-(z_2^-) + \ldots$$

- The quark propagator:

$$\lim_{k^2\to 0} k^2 (k|\frac{1}{P^2+\frac{g}{2}\sigma^{\mu\nu}F_{\mu\nu}+i\epsilon}|y) = \lim_{k^2\to 0}\int d^2z_\perp\Big(\theta(y^-)e^{i\frac{k_{1\perp}^2}{2k_1^-}y^-}e^{-ik_{1\perp}y_\perp}U(\infty,y^-;y_\perp)+\theta(-y^-)\int d^2z_\perp e^{-ik_{1\perp}}$$

$$+\frac{ig}{2p^-}\int_{-\infty}^{\infty}dz^-U(\infty,z^-;z_\perp)\sigma^{-k}F_{-k}(z^-,z_\perp)U(z^-,-\infty;z_\perp)\Big)(z_\perp|e^{i\frac{P_\perp^2(-\infty)}{2k_1^-}y^-}|y_\perp)U(-\infty,y^-;y_\perp)\Big)e^{ik^-y^+}$$

  - The first term in the expansion of the z^- exponential remains.
  - $dz^-F_{-i}\sim\lambda^0$



- The Eikonal amplitude:

$$iM^\nu_{s_1,s_2}(q,k_1,k_2)\Big|_{\text{eik.}}=\delta(q^--k_1^--k_2^-)2z\bar{z}q^-\int d^2z_\perp e^{-ik_{1\perp}z_\perp}\int d^2z'_\perp e^{-ik_{2\perp}z'_\perp}$$

$$\times\,\bar{u}_{s_1}(k_1)\Big\{\Big(-i\gamma^\nu+\frac{k_{1m}}{2k_1^-}\sigma^{-m}\gamma^\nu-\frac{k_{2m}}{2k_2^-}\gamma^\nu\sigma^{-m}\Big)K_0(\sqrt{z\bar{z}Q^2}|z_\perp-z'_\perp|)-i\sqrt{z\bar{z}Q^2}\Big(\frac{\sigma^{-m}\gamma^\nu}{2k_1^-}$$

$$+\frac{\gamma^\nu\sigma^{-m}}{2k_2^-}\Big)\frac{z_m-z'_m}{|z_\perp-z'_\perp|}K_1(\sqrt{z\bar{z}Q^2}|z_\perp-z'_\perp|)\Big\}v_{s_2}(k_2)\Big(U(\infty,-\infty;z_\perp)\quad U(z_\perp,z'_\perp;-\infty)\quad U(-\infty,\infty;z'_\perp)-1\Big)$$

Generalised eikonal cross-section

$$d\sigma_{00}^{\gamma^* p}\Big|_{\rm eik.} = \sum_f \frac{2\pi e_f^2 \alpha_{\rm EM} x}{V_3 Q^2}\left(\frac{Q}{q^-}\right)^2 \frac{dk_1^- d^2k_{1\perp}}{(2\pi)^3 2k_1^-}\Big|_{k_1^2=0} \frac{dk_2^- d^2k_{2\perp}}{(2\pi)^3 2k_2^-}\Big|_{k_2^2=0} \delta(q^- - k_1^- - k_2^-) 32 z^3 \bar{z}^3 (q^-)^4$$
$$\times \int d^2\omega_\perp e^{ik_{1\perp}\omega_\perp} \int d^2\omega'_\perp e^{ik_{2\perp}\omega'_\perp} K_0(\sqrt{z\bar{z}Q^2}|\omega_\perp - \omega'_\perp|) \int d^2z_\perp e^{-ik_{1\perp}z_\perp} \int d^2z'_\perp e^{-ik_{2\perp}z'_\perp}$$
$$\times K_0(\sqrt{z\bar{z}Q^2}|z_\perp - z'_\perp|)\mathcal{U}(z_\perp, z'_\perp; \omega_\perp, \omega'_\perp)$$

$$d\sigma_{\lambda=\pm1\lambda'=\pm1}^{\gamma^* p}\Big|_{\rm eik.} = \sum_f \frac{2\pi e_f^2 \alpha_{\rm EM} x}{V_3} \frac{dk_1^- d^2k_{1\perp}}{(2\pi)^3 2k_1^-}\Big|_{k_1^2=0} \frac{dk_2^- d^2k_{2\perp}}{(2\pi)^3 2k_2^-}\Big|_{k_2^2=0}$$
$$\times \delta(q^- - k_1^- - k_2^-) 8 (q^-)^2 z^2 \bar{z}^2 \int d^2\omega_\perp e^{ik_{1\perp}\omega_\perp} \int d^2\omega'_\perp e^{ik_{2\perp}\omega'_\perp} K_1(\sqrt{z\bar{z}Q^2}|\omega_\perp - \omega'_\perp|)$$
$$\times \int d^2z_\perp e^{-ik_{1\perp}z_\perp} \int d^2z'_\perp e^{-ik_{2\perp}z'_\perp} K_1(\sqrt{z\bar{z}Q^2}|z_\perp - z'_\perp|)\Big(\delta_{\lambda\lambda'}(-1+2z\bar{z})(g^{km} + i\epsilon^{km})$$
$$\times \frac{\omega_k - \omega'_k}{|\omega_\perp - \omega'_\perp|}\frac{z_m - z'_m}{|z_\perp - z'_\perp|} + 2z\bar{z}\delta_{\lambda(-\lambda')} e^{i\lambda\phi}\Big)\mathcal{U}(z_\perp, z'_\perp; \omega_\perp, \omega'_\perp)$$

$$\mathcal{U}(z_\perp, z'_\perp; \omega_\perp, \omega'_\perp) \equiv \langle P, S|{\rm tr_c}\Big(U(\infty, -\infty; \omega'_\perp)U(\omega'_\perp, \omega_\perp; -\infty)U(-\infty, \infty; \omega_\perp) - 1\Big)U(\omega_\perp, z_\perp; \infty)$$
$$\times \Big(U(\infty, -\infty; z_\perp)U(z_\perp, z'_\perp; -\infty)U(-\infty, \infty; z'_\perp) - 1\Big)U(z'_\perp, \omega'_\perp; \infty)|P, S\rangle$$

CGC cross-section

$$d\sigma_{00}^{\gamma^* p}\Big|_{\rm eik.}^{\rm CGC} = \sum_f \frac{2\pi e_f^2 \alpha_{\rm EM} x}{V_3 Q^2}\left(\frac{Q}{q^-}\right)^2 \frac{dk_1^- d^2k_{1\perp}}{(2\pi)^3 2k_1^-}\Big|_{k_1^2=0} \frac{dk_2^- d^2k_{2\perp}}{(2\pi)^3 2k_2^-}\Big|_{k_2^2=0} \delta(q^- - k_1^- - k_2^-) 32 z^3 \bar{z}^3 (q^-)^4$$
$$\times \int d^2\omega_\perp e^{ik_{1\perp}\omega_\perp} \int d^2\omega'_\perp e^{ik_{2\perp}\omega'_\perp} K_0(\sqrt{z\bar{z}Q^2}|\omega_\perp - \omega'_\perp|) \int d^2z_\perp e^{-ik_{1\perp}z_\perp} \int d^2z'_\perp e^{-ik_{2\perp}z'_\perp}$$
$$\times K_0(\sqrt{z\bar{z}Q^2}|z_\perp - z'_\perp|)\mathcal{U}(z_\perp, z'_\perp; \omega_\perp, \omega'_\perp)\Big|_{\rm CGC}$$

$$d\sigma_{\lambda=\pm1\lambda'=\pm1}^{\gamma^* p}\Big|_{\rm eik.}^{\rm CGC} = \sum_f \frac{2\pi e_f^2 \alpha_{\rm EM} x}{V_3} \epsilon_s^{\lambda=\pm1*}\epsilon_l^{\lambda'=\pm1} \frac{dk_1^- d^2k_{1\perp}}{(2\pi)^3 2k_1^-}\Big|_{k_1^2=0} \frac{dk_2^- d^2k_{2\perp}}{(2\pi)^3 2k_2^-}\Big|_{k_2^2=0}$$
$$\times \delta(q^- - k_1^- - k_2^-) 8 (q^-)^2 z^2 \bar{z}^2 \int d^2\omega_\perp e^{ik_{1\perp}\omega_\perp} \int d^2\omega'_\perp e^{ik_{2\perp}\omega'_\perp} K_1(\sqrt{z\bar{z}Q^2}|\omega_\perp - \omega'_\perp|)$$
$$\times \int d^2z_\perp e^{-ik_{1\perp}z_\perp} \int d^2z'_\perp e^{-ik_{2\perp}z'_\perp} K_1(\sqrt{z\bar{z}Q^2}|z_\perp - z'_\perp|)\Big(\delta_{\lambda=\lambda'}(-1+2z\bar{z})(g^{km} + i\epsilon^{km})$$
$$\times \frac{\omega_k - \omega'_k}{|\omega_\perp - \omega'_\perp|}\frac{z_m - z'_m}{|z_\perp - z'_\perp|} + 2z\bar{z}\delta_{\lambda=-\lambda'} e^{i\lambda\phi}\Big)\mathcal{U}(z_\perp, z'_\perp; \omega_\perp, \omega'_\perp)\Big|_{\rm CGC}$$

$$\mathcal{U}(z_\perp, z'_\perp; \omega_\perp, \omega'_\perp)\Big|_{\rm CGC}$$
$$\equiv \langle P, S|{\rm tr}\Big(U(\infty, -\infty; \omega'_\perp)U(-\infty, \infty; \omega_\perp) - 1\Big)\Big(U(\infty, -\infty; z_\perp)U(-\infty, \infty; z'_\perp) - 1\Big)|P, S\rangle$$

Conclusion

- Path-ordered exponent expansion provides a universal gauge-covariant tool for background-field calculations.
- The formalism systematically generates the operator hierarchy relevant for any chosen kinematic limit.
- In small-x diet production, transverse background fields contribute already at eikonal order.
- Different kinematic regimes (TMDs and small-x) are connected through contour re-expansion, providing a direct matching framework.
- The method is general and can be extended to sub-eikonal, loop-level, and multi-parton observables.

Thank you!