# Leveraging the FCC-ee capabilities to study the invisible decay of a dark photon

*Brookhaven National Laboratory*
*Physics Colloquium*
*June 23rd, 2026*


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*Department of Physics*
*University of Hartford, CT*

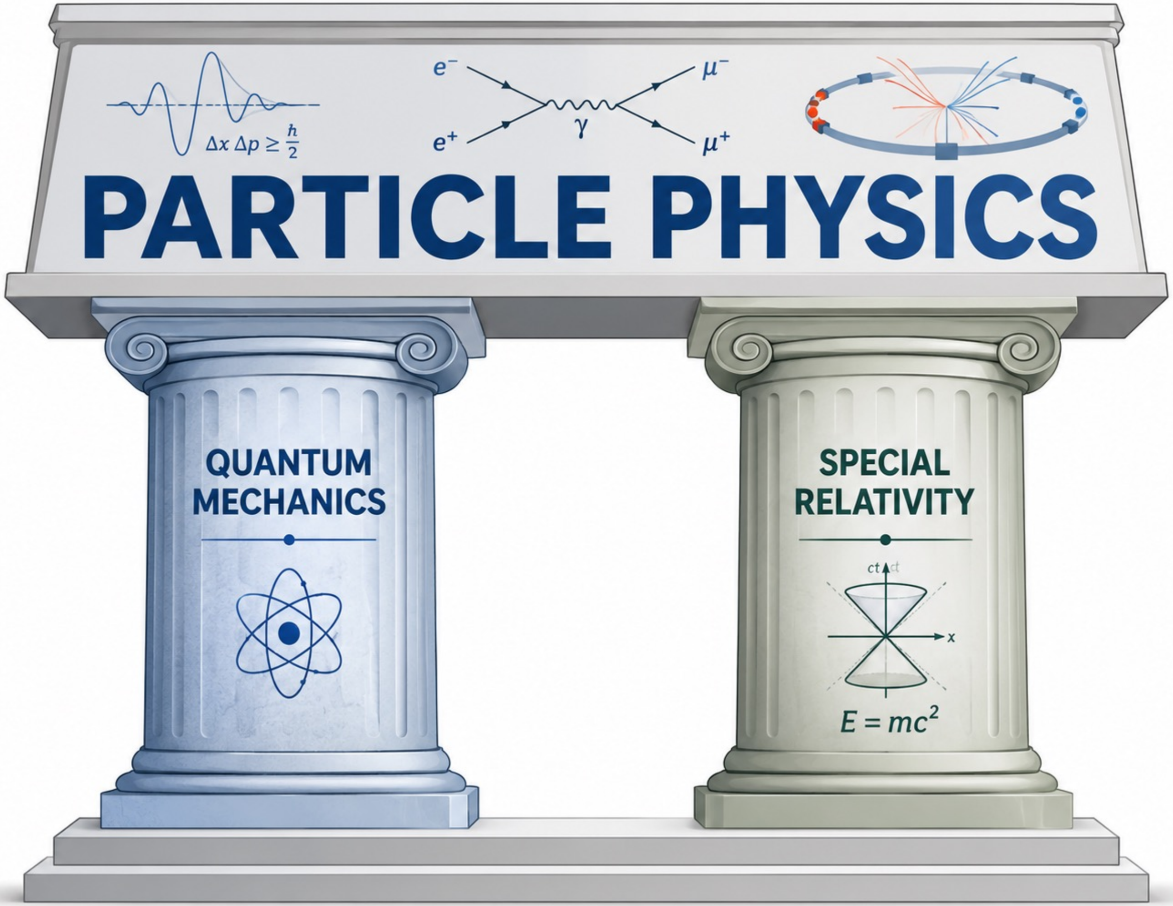

UNIVERSITY
OF HARTFORD

20th century revolutions:

*Quantum Mechanics*

*&*

*Special Relativity*



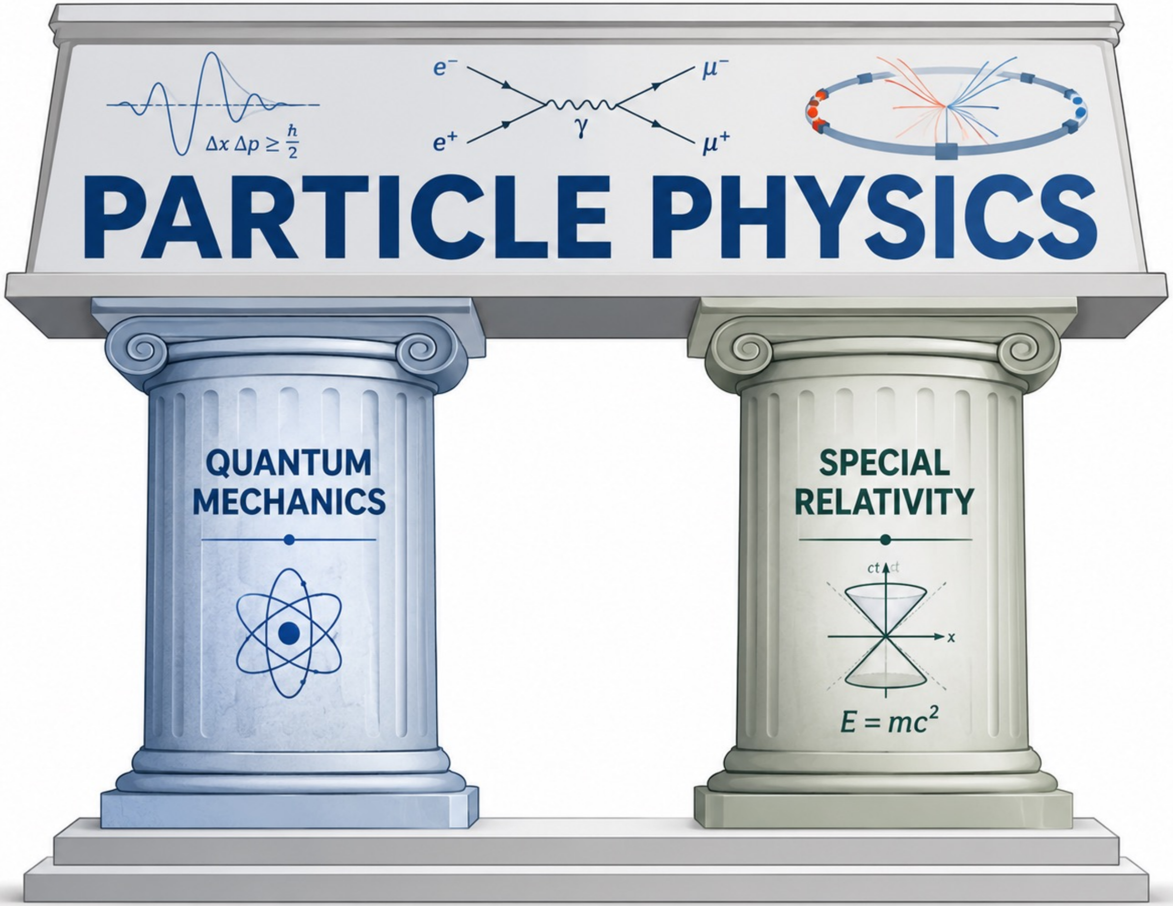
20th century revolutions:

*Quantum Mechanics*

*&*

*Special Relativity*

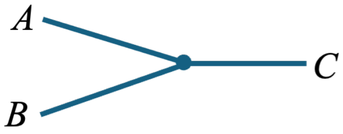
Quantum Field Theory
(QFT)



# What particles and interactions does QFT predict?

The marriage between QM and SR amazingly restricts the number of allowed particles and their interactions:

At "long" distances, particles interacting as




with spins:

$$0 \qquad \frac{1}{2} \qquad 1 \qquad \frac{3}{2} \qquad 2$$

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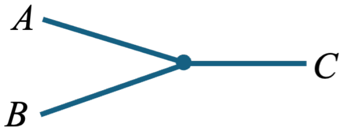


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**Fermions**

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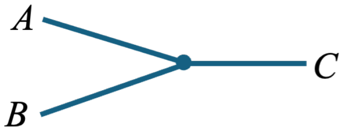


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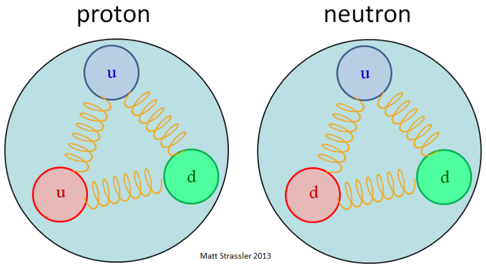
**Quarks** (make up protons and neutrons)

**Leptons** (e.g. electrons)



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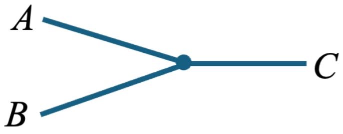


with spins:

$$\boxed{0} \quad \frac{1}{2} \quad \boxed{1} \quad \frac{3}{2} \quad \boxed{2}$$

**Bosons**

The force carriers

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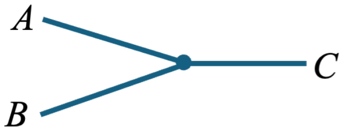


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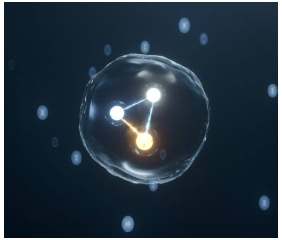
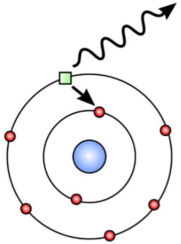
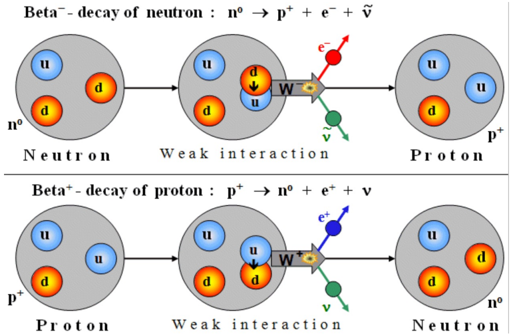




Massless: Photon, gluon

Massive: $W^{\pm}, Z$



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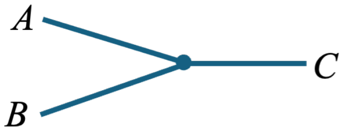


with spins:

[Peter Higgs (middle). Photo courtesy of CERN]

$\boxed{0}$ $\quad \frac{1}{2} \quad$ $\boxed{1}$ $\quad \frac{3}{2} \quad$ $\boxed{2}$

Higgs

(discovered in 2012)

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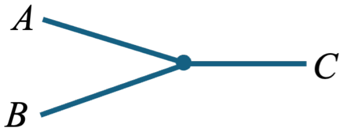


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Unique particle
(describes gravity)

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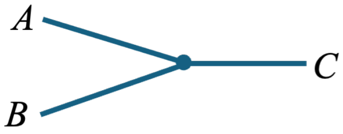


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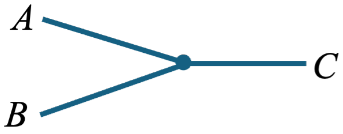


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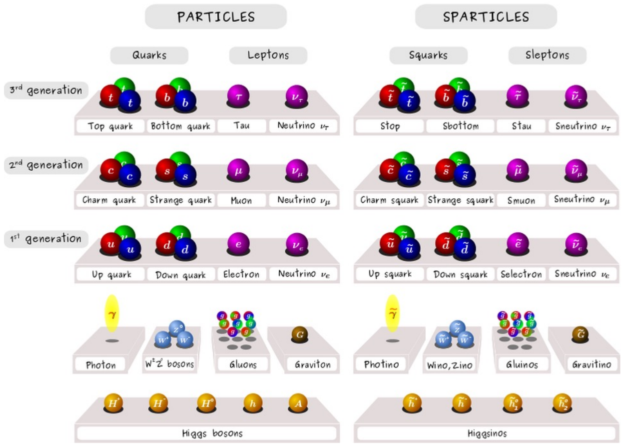
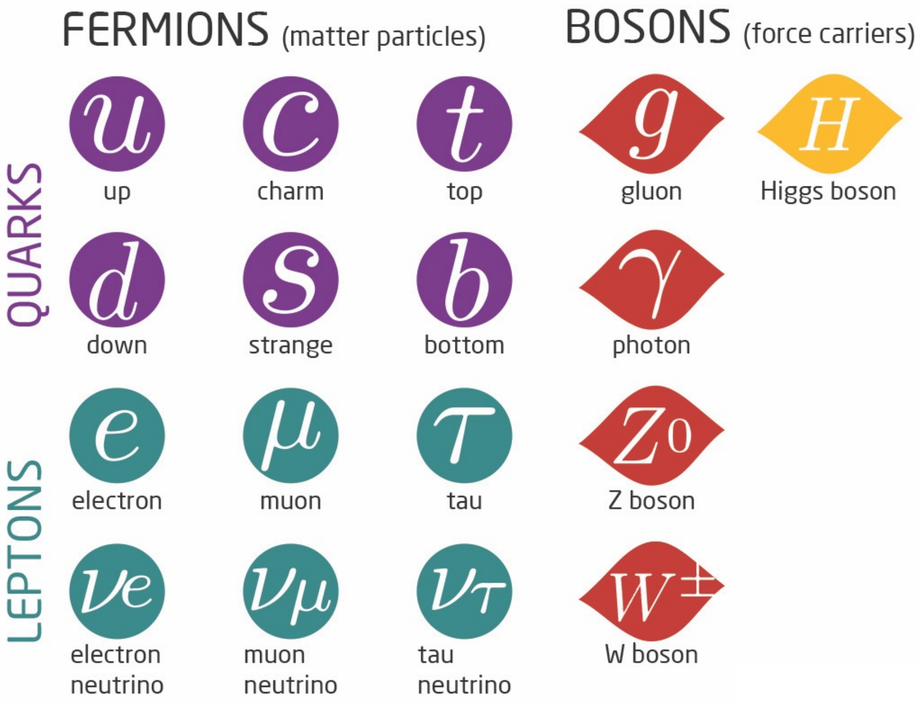



SUPERSYMMETRY

(A symmetry of spacetime)

# The Standard Model of Particle Physics




Amazingly successful but with many shortcomings!

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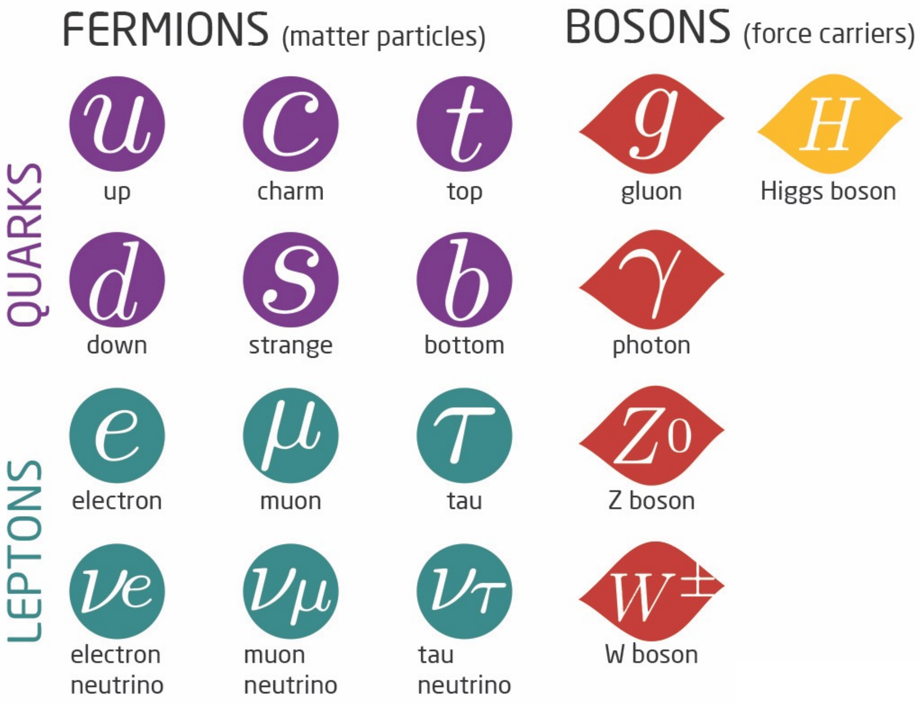


Amazingly successful but with many shortcomings!

## OPEN QUESTIONS

- No mechanism to explain neutrino masses
- No dark matter candidate
- No explanation for matter-antimatter asymmetry
- Hierarchy problem
- …

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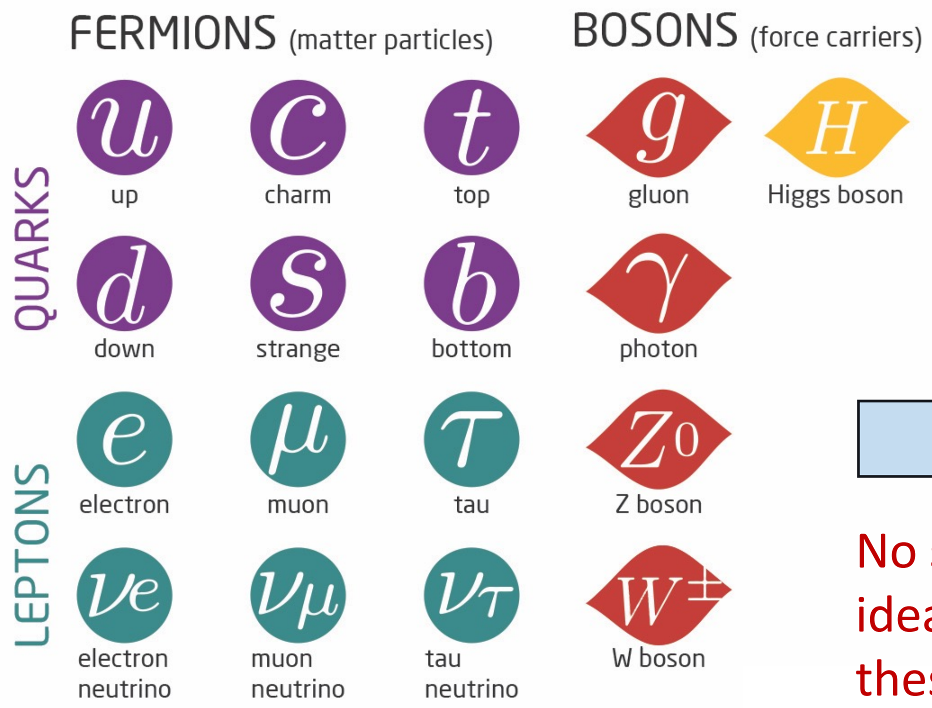
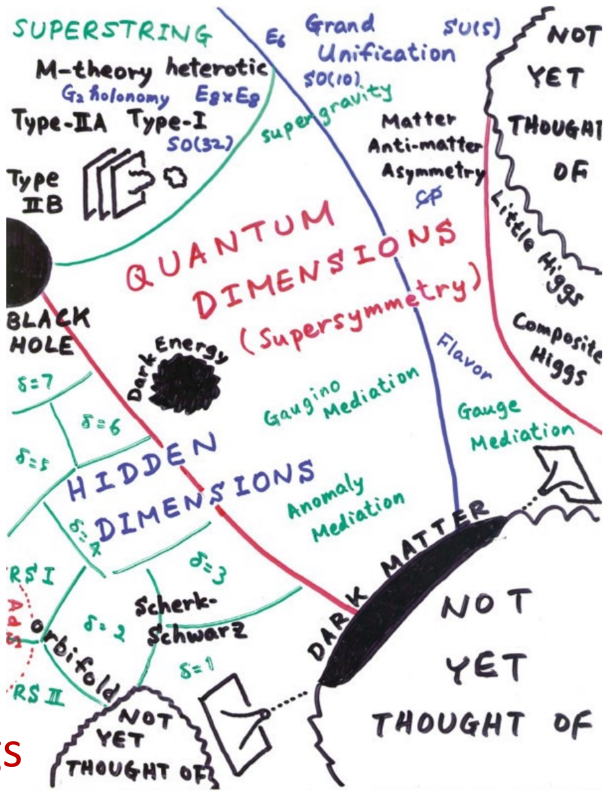



No shortage of ideas to explain these shortcomings




[Illustration by Hitoshi Murayama]

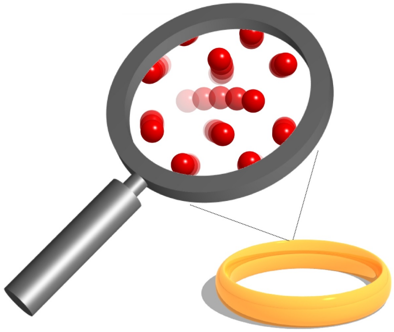
Amazingly successful but with many shortcomings!

Robert Millikan (1932)

Robert Millikan during his Nobel Prize lecture on May 23, 1924, said:

"The fact that Science walks forward on two feet, namely theory and experiment, is nowhere better illustrated than in the two fields for slight contributions to which you have done me the great honour of awarding the Nobel Prize in Physics for the year 1923. Sometimes it is one foot that is put forward first, sometimes the other, but continuous progress is only made by the use of both—by theorizing and then testing, or by finding new relations in the process of experimenting and then bringing the theoretical foot up and pushing it on beyond, and so on in unending alterations."

# How do we study the tiniest constituents of our universe?



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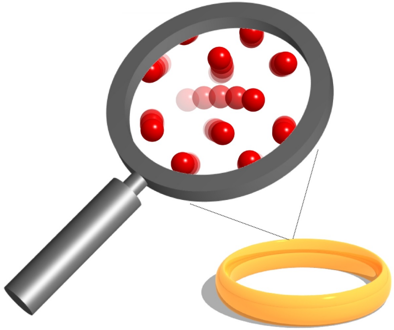

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*Energy*

*Wavelength (length scale)*

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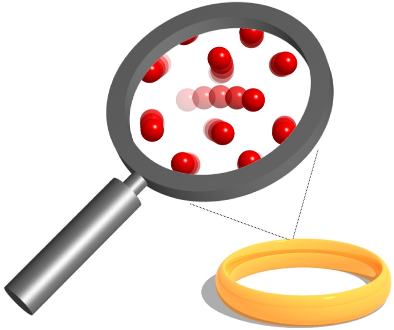

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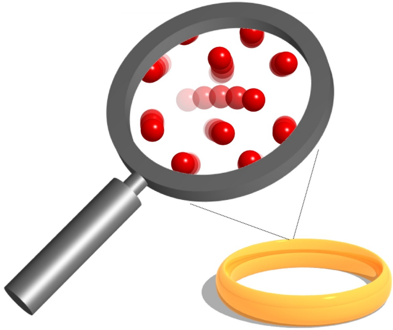

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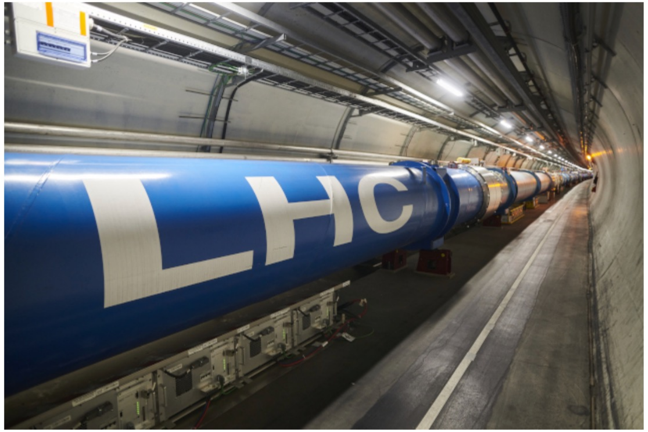
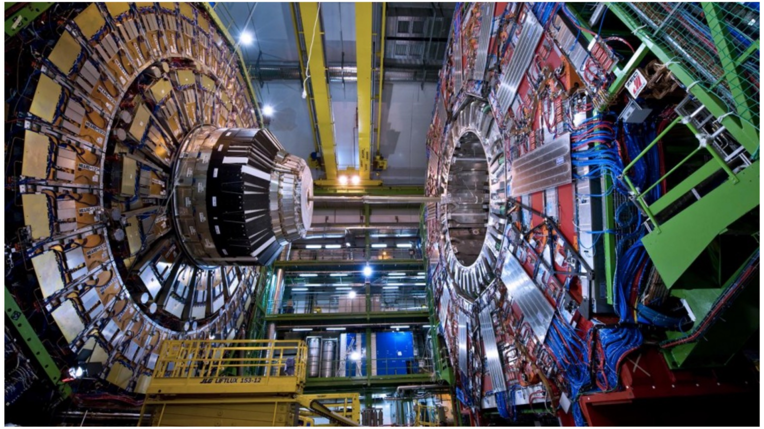
**In particular physics, our magnifying glass is a particle collider**

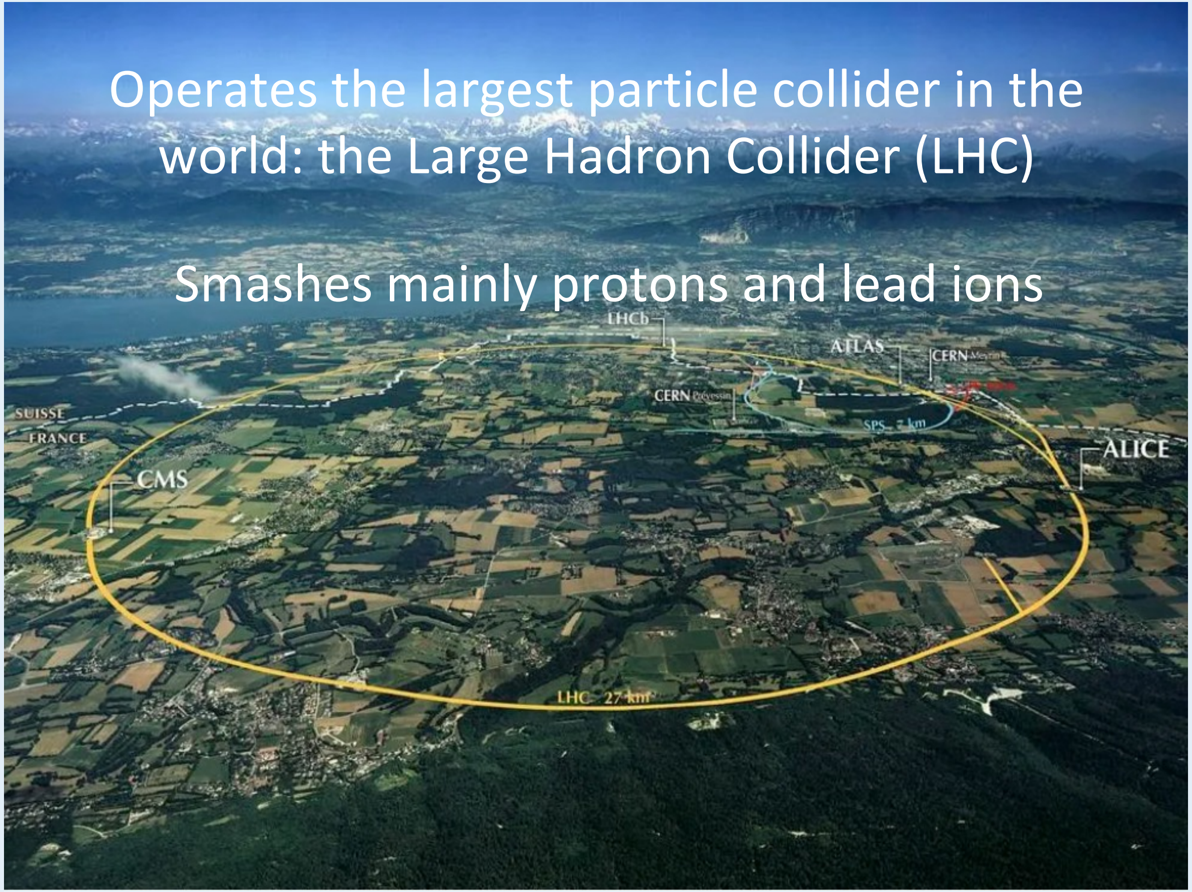


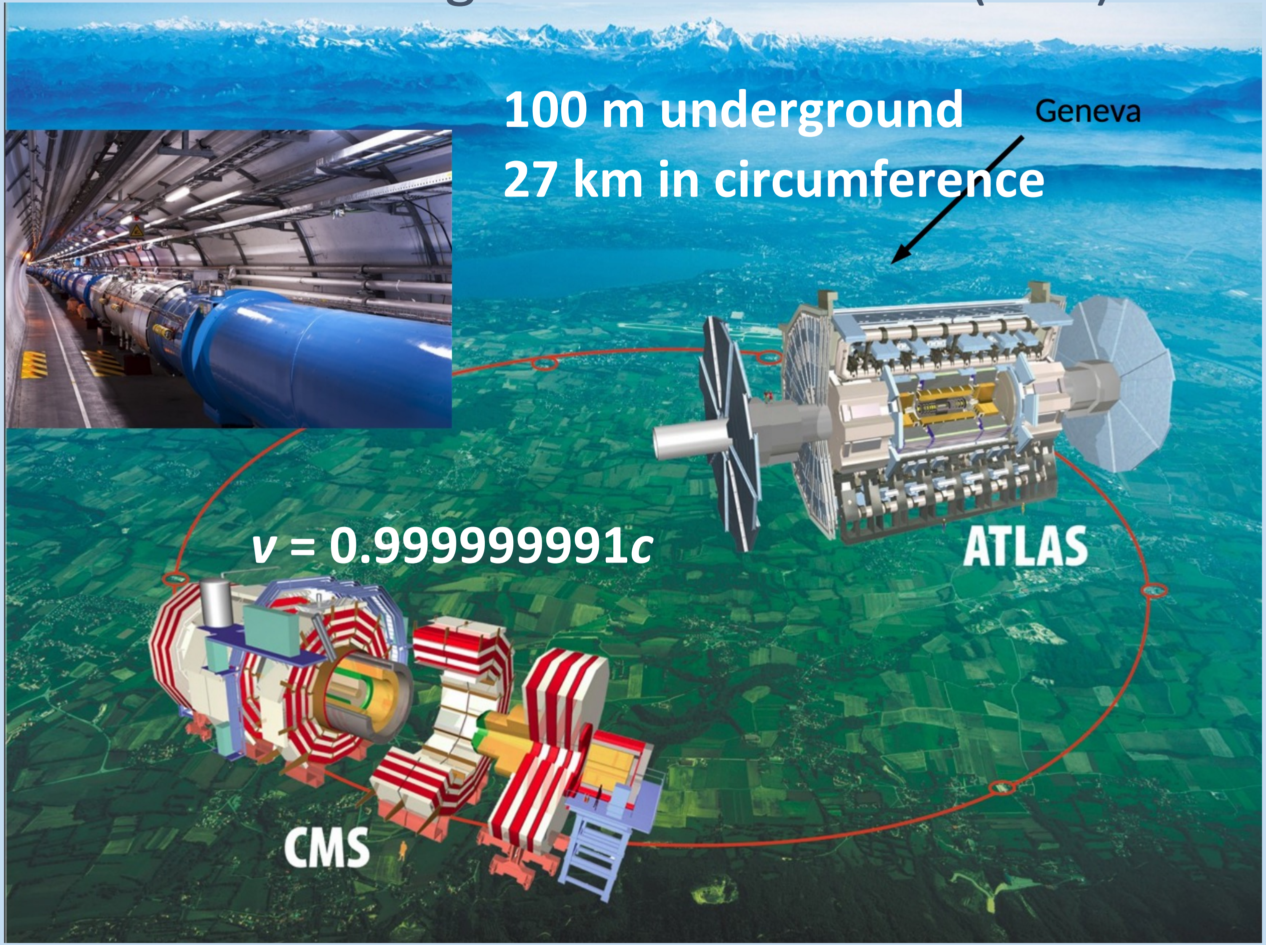
# CERN: European Center for Nuclear Research

Operates the largest particle collider in the world: the Large Hadron Collider (LHC)

Smashes mainly protons and lead ions



# The Large Hadron Collider (LHC)

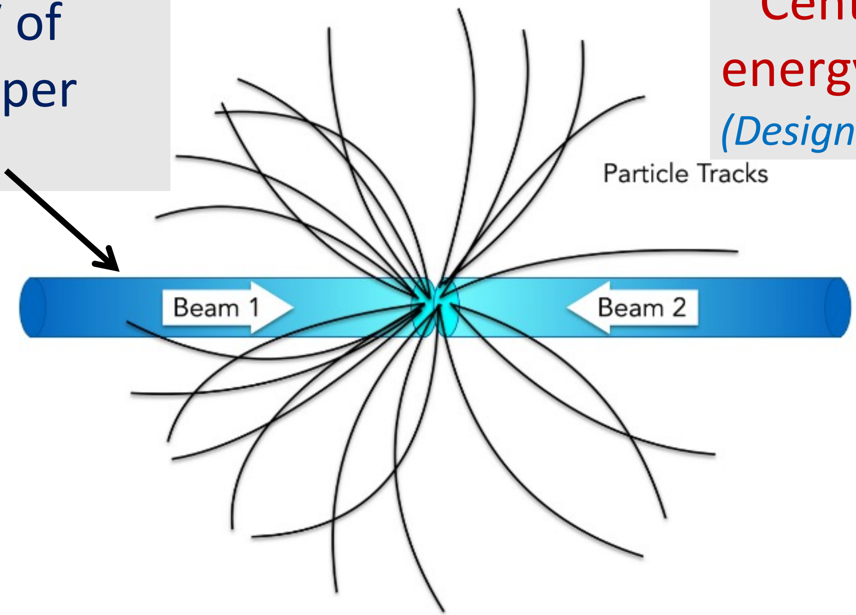


# LHC collision energy as of 2023

6.8 TeV of energy per beam

Center-of-mass energy of 13.6 TeV

*(Design energy: 14 TeV)*



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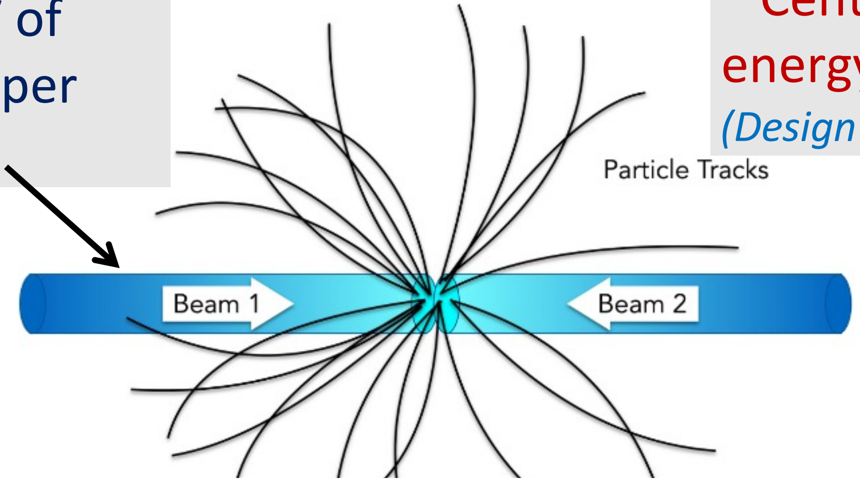
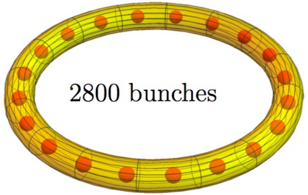


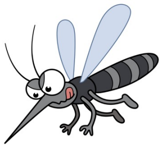

$10^{11}$ protons/bunch

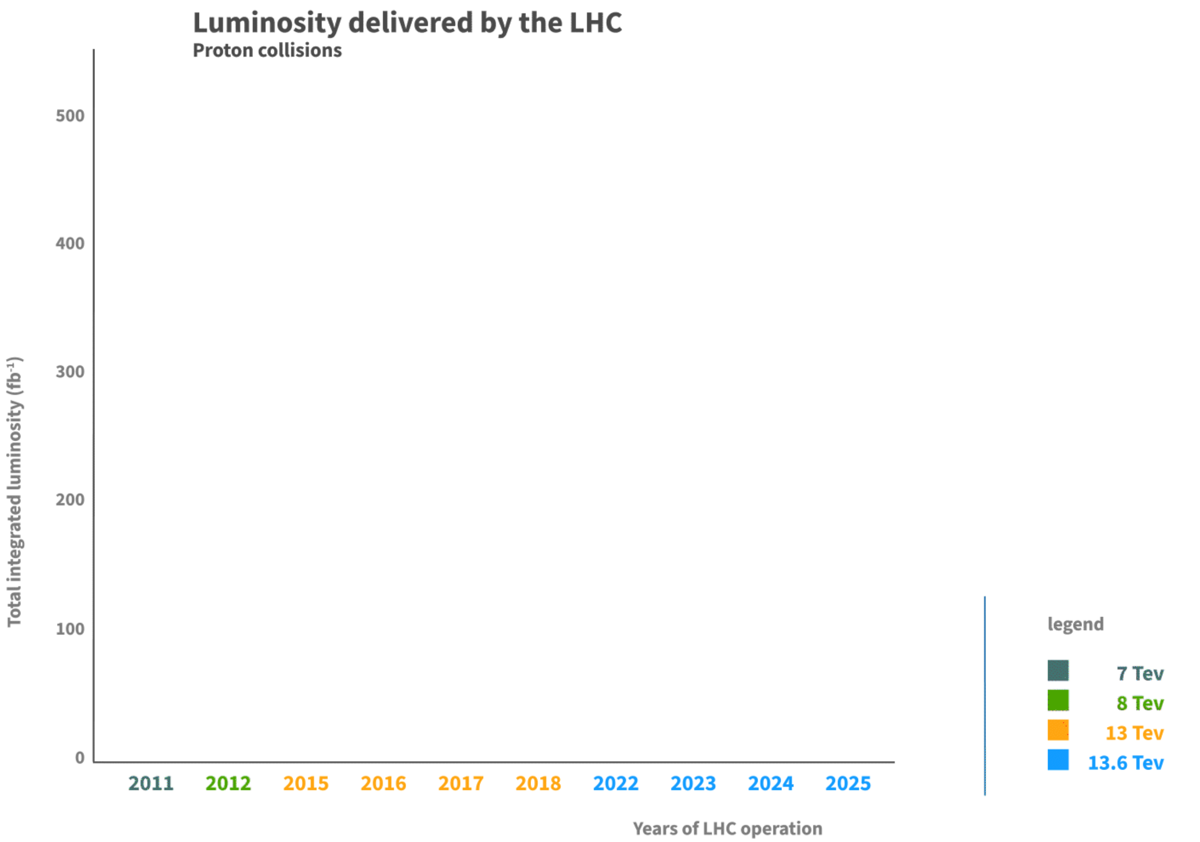



bunch spacing = 25 ns

$$1 \text{ TeV} = 10^{12} \text{ eV} = 0.1602\ \mu\text{J}$$

= energy of a flying mosquito



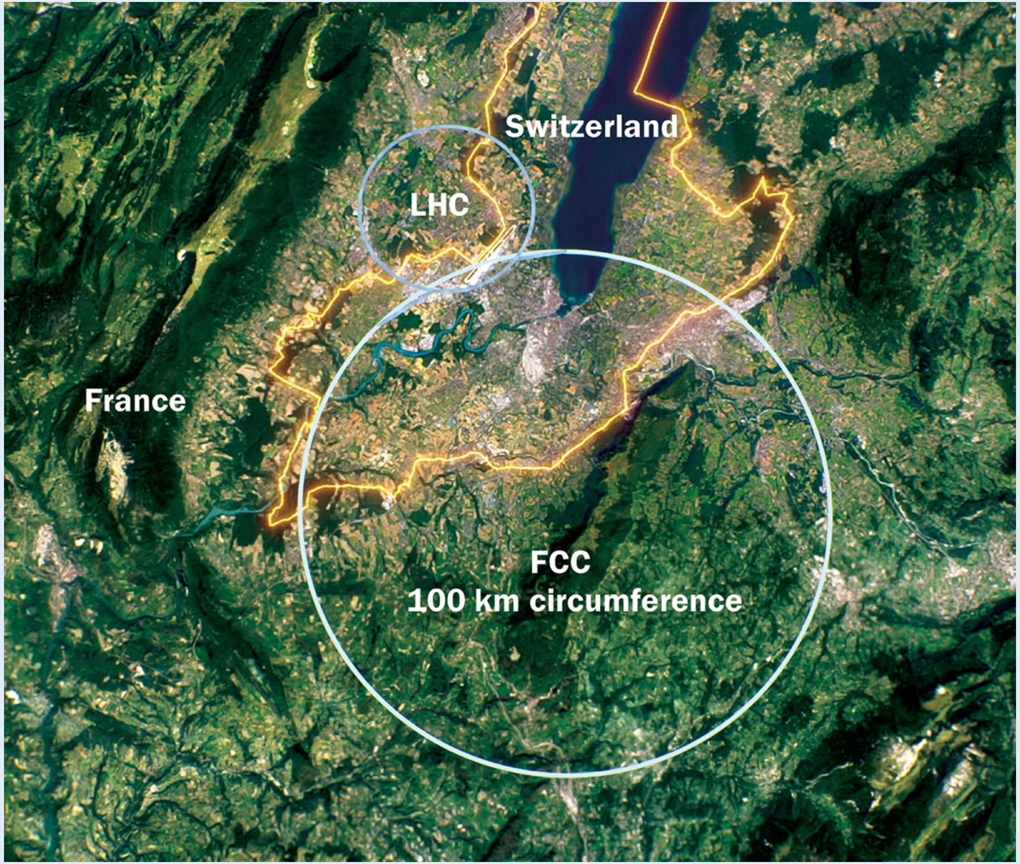



$fb^{-1}$ (time-integrated luminosity): cumulative rate of collisions over a given area

1 $fb^{-1} \approx$ 80 trillion p-p collisions

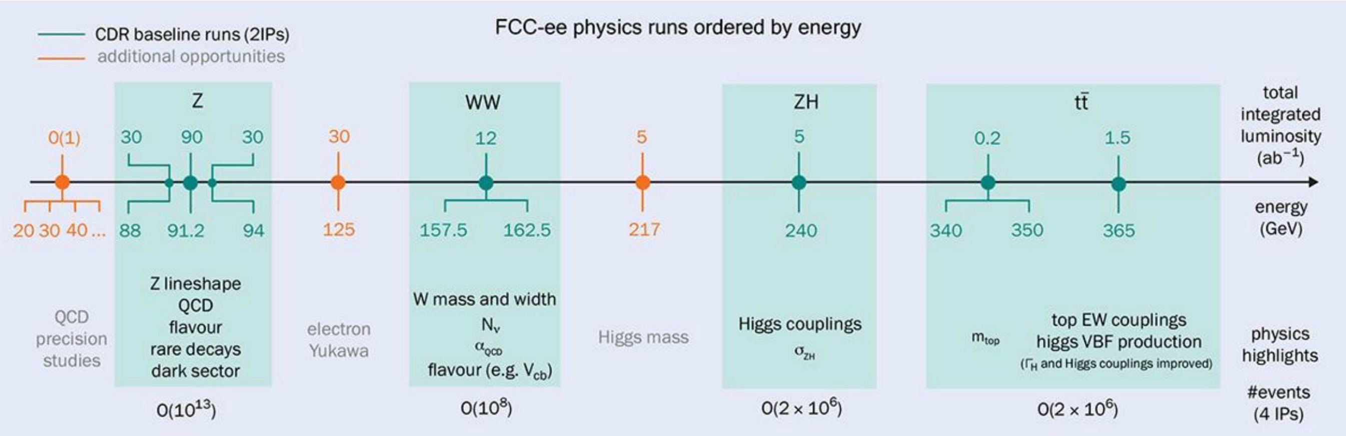
Animation: CERN

# Future Circular Collider (FCC-ee)



Slide content: courtesy of
T. Bose (SUSY 2026)

# FCC-ee timeline



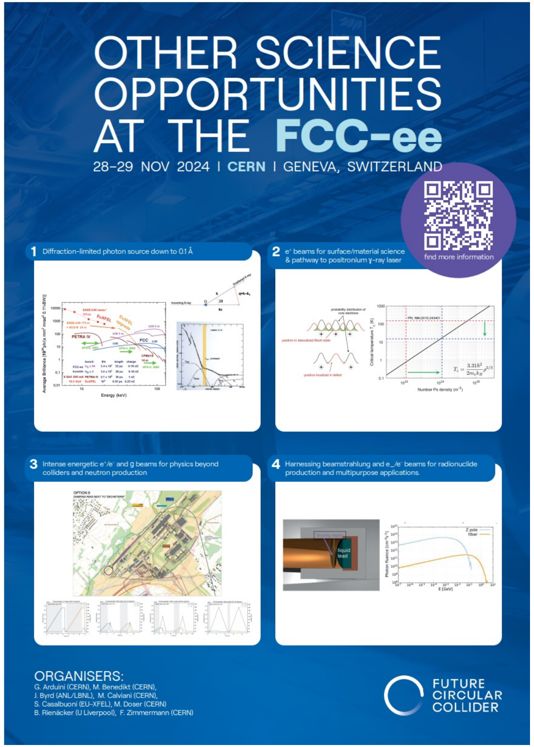

2026-2027: Preparatory engineering design, environmental impact studies

2028: CERN Member States formally vote to authorize and fund civil engineering

2033-2040: Heavy tunneling and civil engineering phase

~ 2045: First collisions at the FCC-ee

Complex funding landscape; Public consultation and debate process currently ongoing in France & Switzerland (May-Oct 2026)



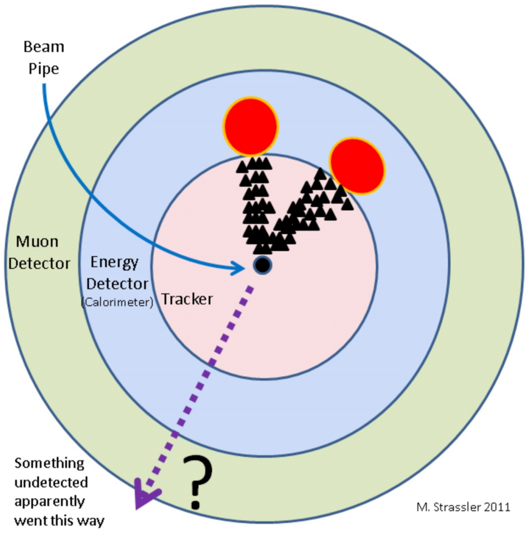

https://cds.cern.ch/record/2928809

On May 22, 2026, CERN Council voted on adopting the ESG recommendations.

# Analyzing an event at particle colliders

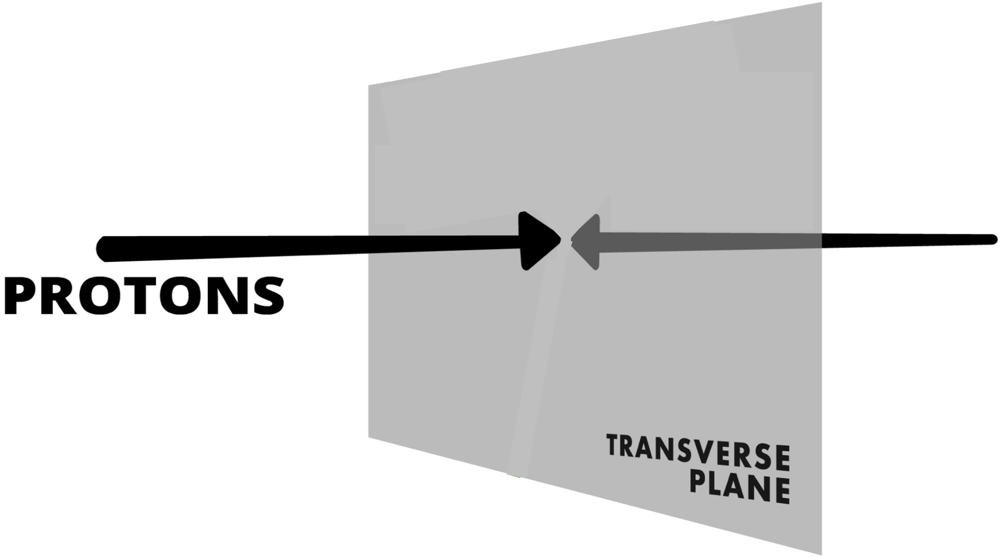

- Interesting physics events at colliders are reconstructed from final states.
- New particles are mostly very short-lived and cannot be detected directly.

# How to detect a (new) particle from its **invisible** decay products?




Look for missing energy!

# How to detect particles that don't want to be detected?



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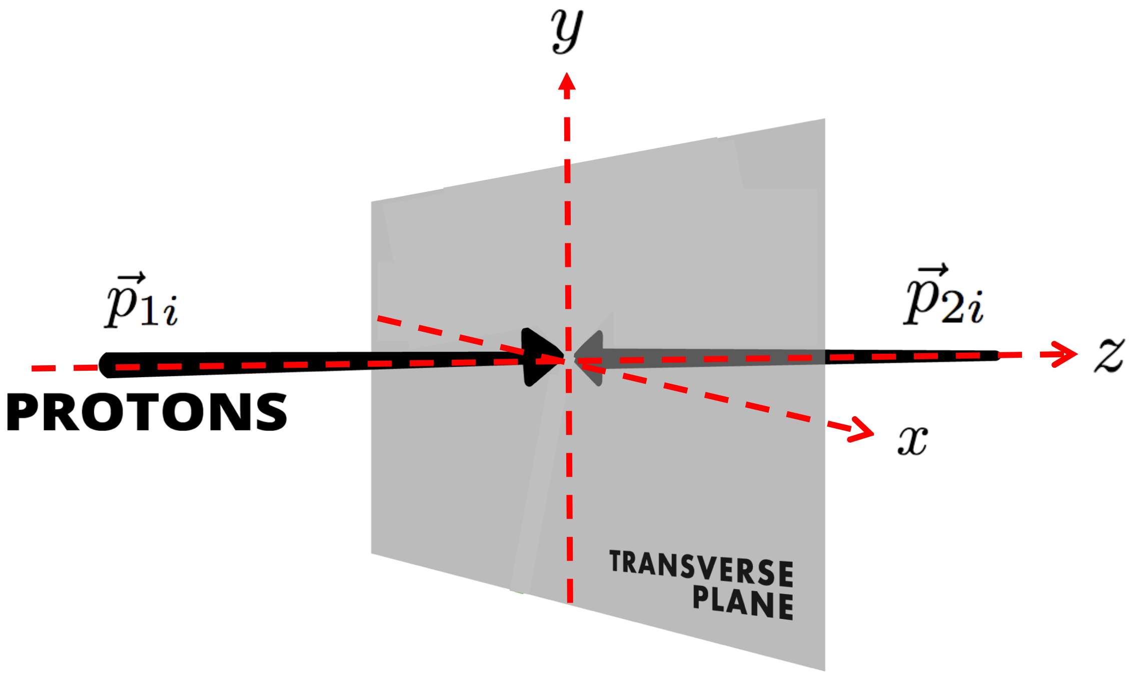

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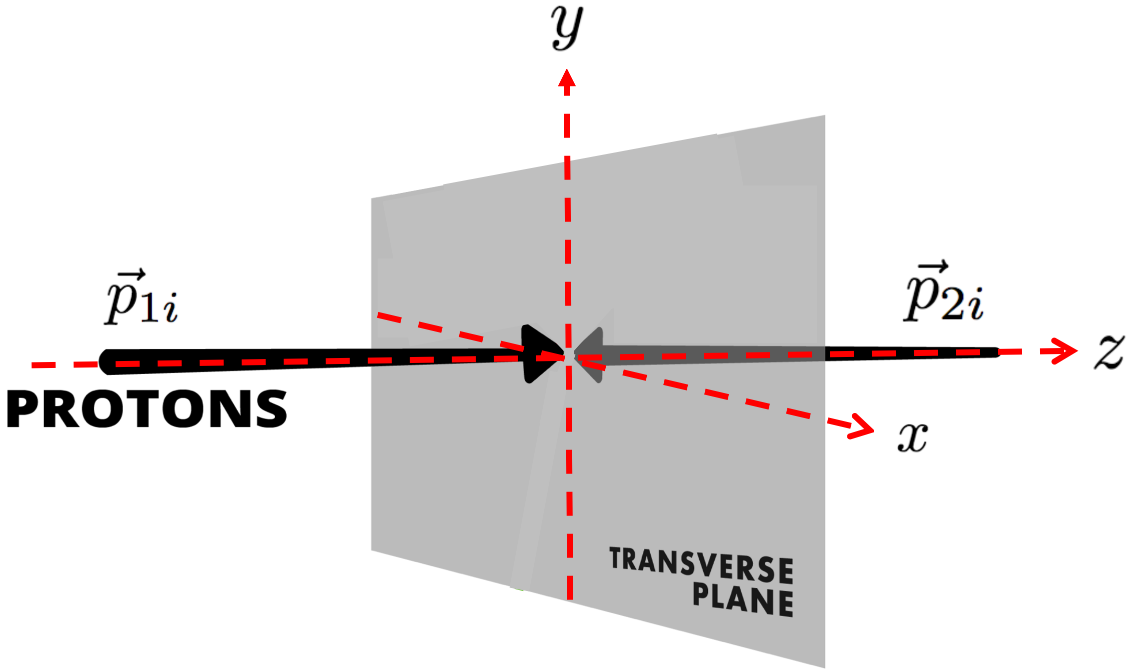


**Conservation of momentum:** $\vec{p}_{1i} + \vec{p}_{2i} = \vec{p}_{1f} + \vec{p}_{2f} + \vec{p}_{3f} + \cdots$

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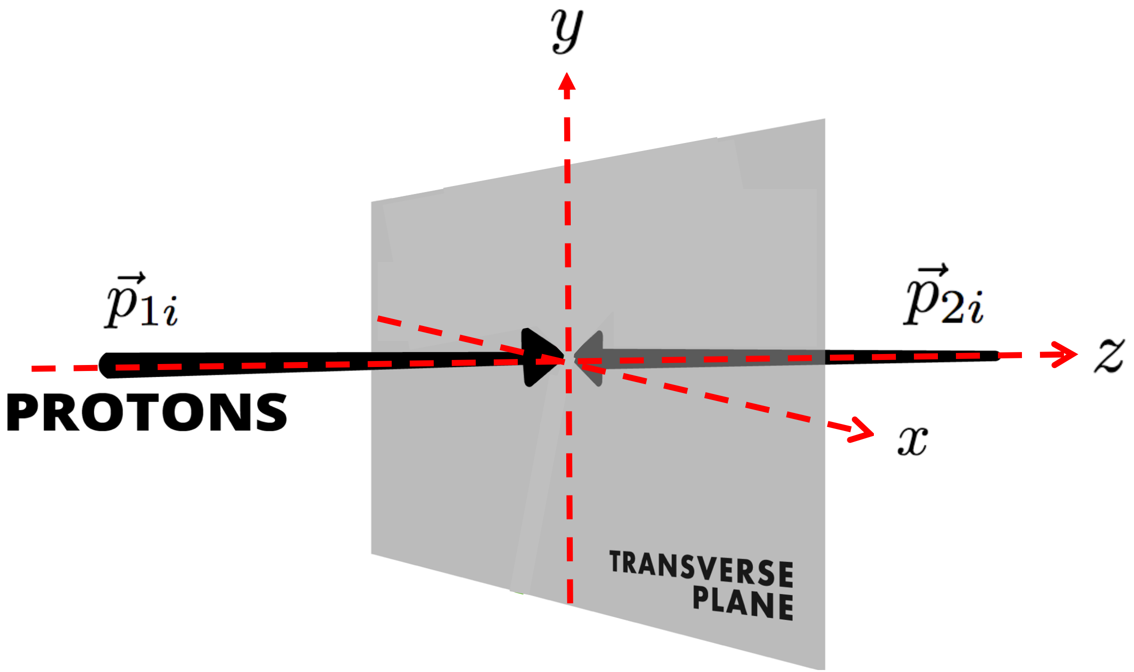


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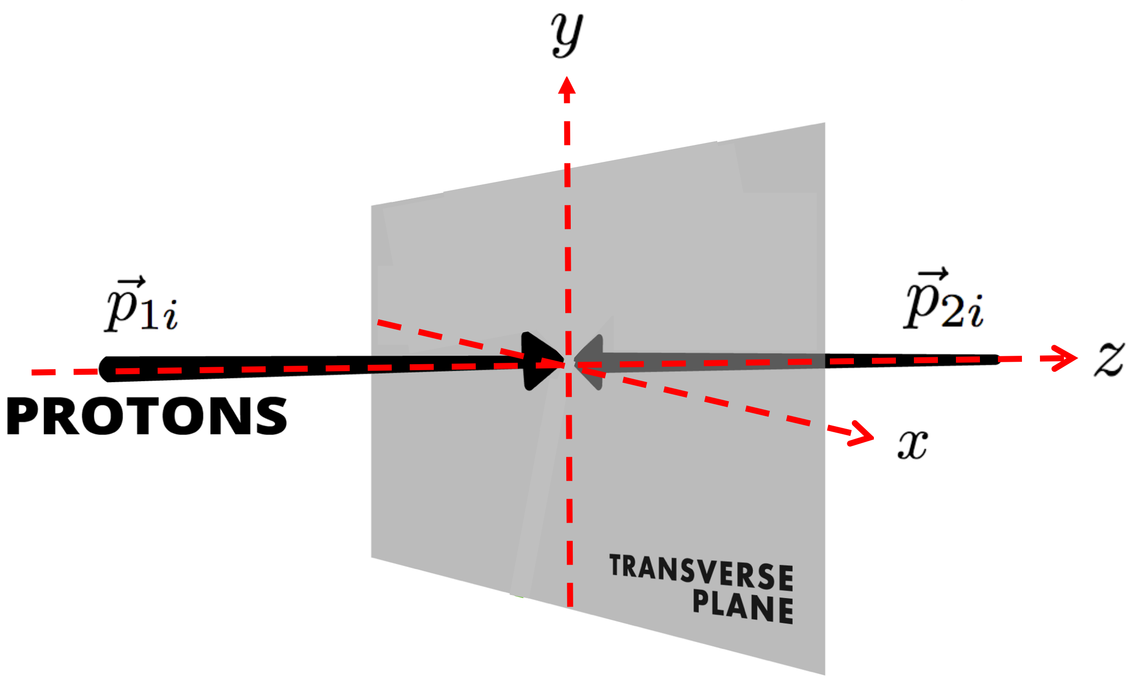
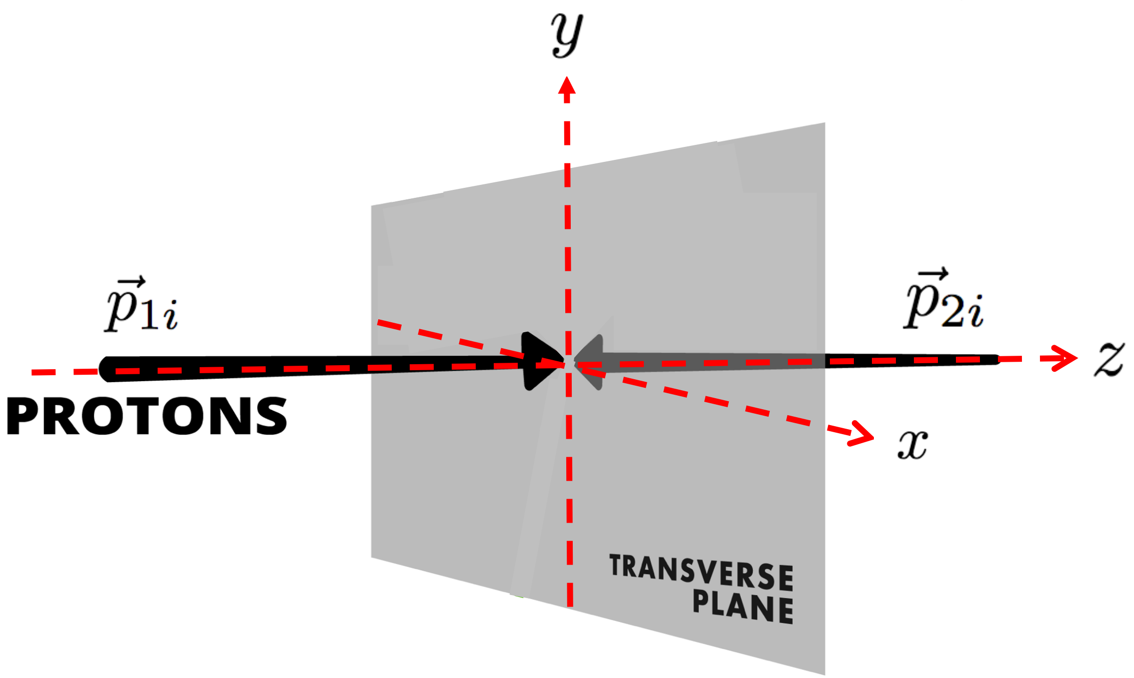



**Conservation of momentum:** $\vec{p}_{1i} + \vec{p}_{2i} = \sum_n \vec{p}_{n,f} = 0$ Center-of-mass frame

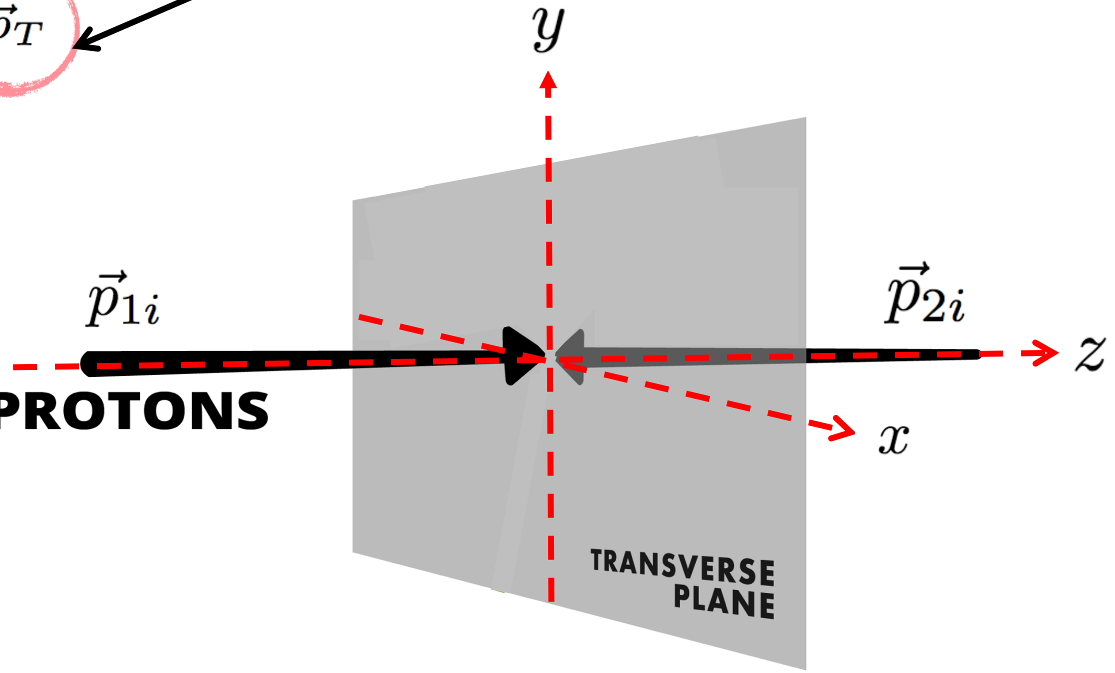
$$\vec{p} = \vec{p}_x + \vec{p}_y + \vec{p}_z$$

# Momentum decomposition



$$\vec{p} = \underbrace{\vec{p}_x + \vec{p}_y}_{\vec{p}_T} + \vec{p}_z$$

## Transverse momentum



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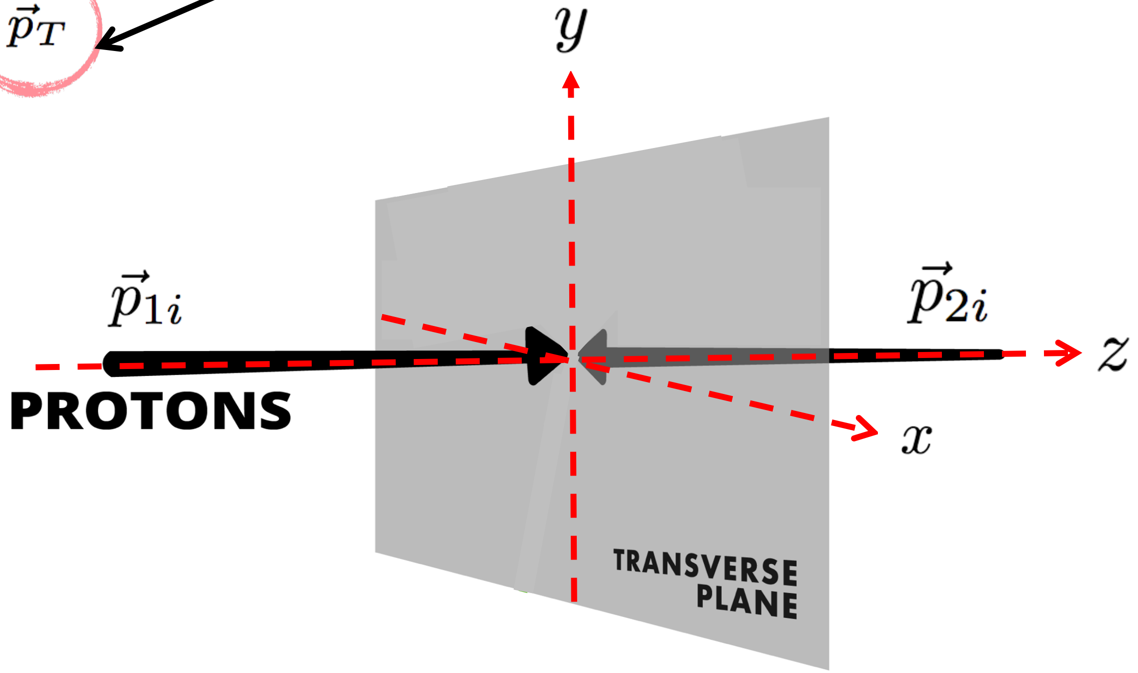
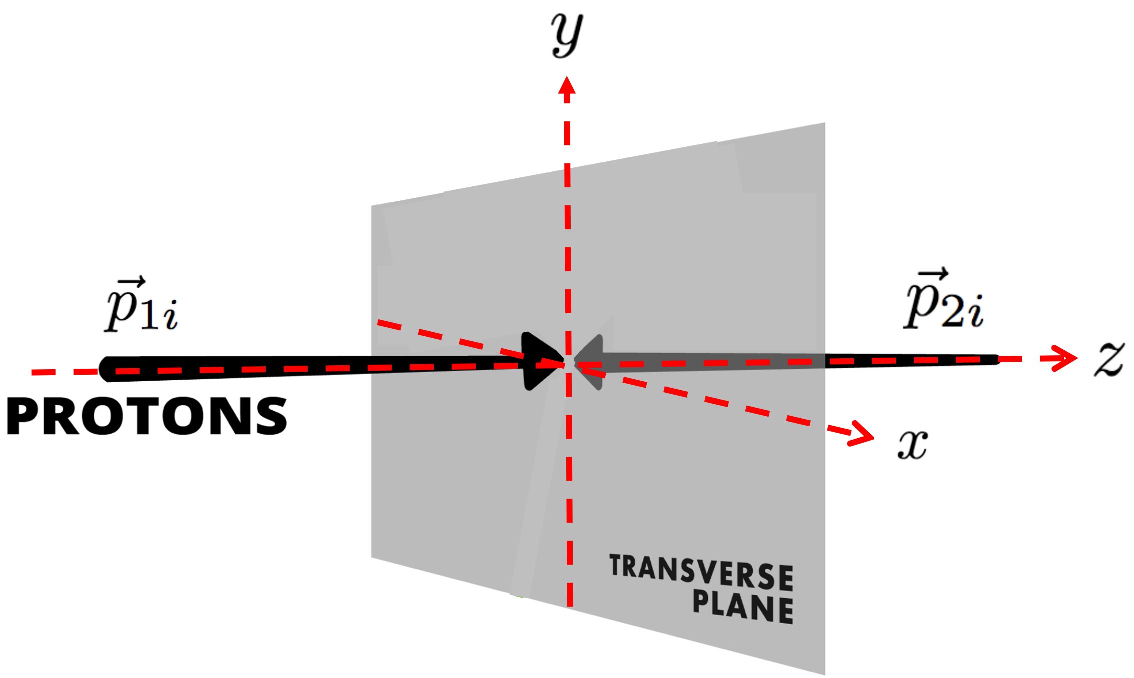


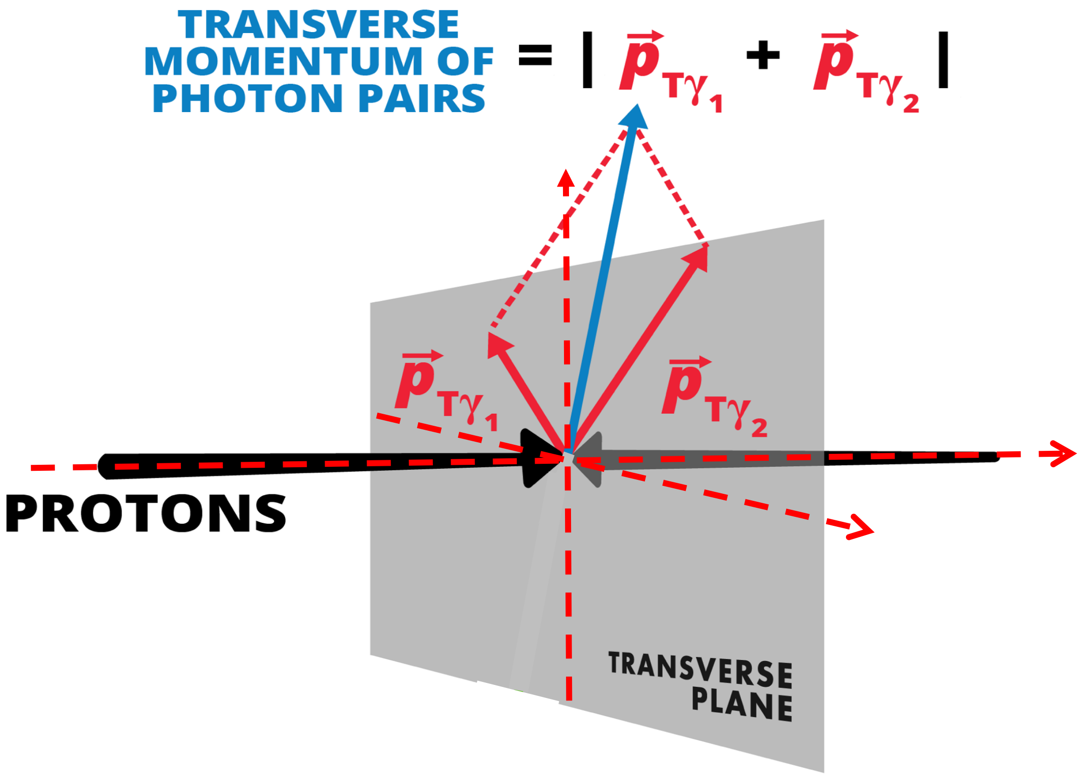

**Initial momenta entirely on the *z*-axis** → $\boxed{\vec{p}_{Ti} = \vec{p}_{Tf} = 0}$

# Consider *three particles* in the final state




**Conservation of momentum:** assuming three particles in the final state:

$$(\vec{p}_{T\,1,f} + \vec{p}_{T\,2,f}) + \vec{p}_{T\,3,f} = 0$$

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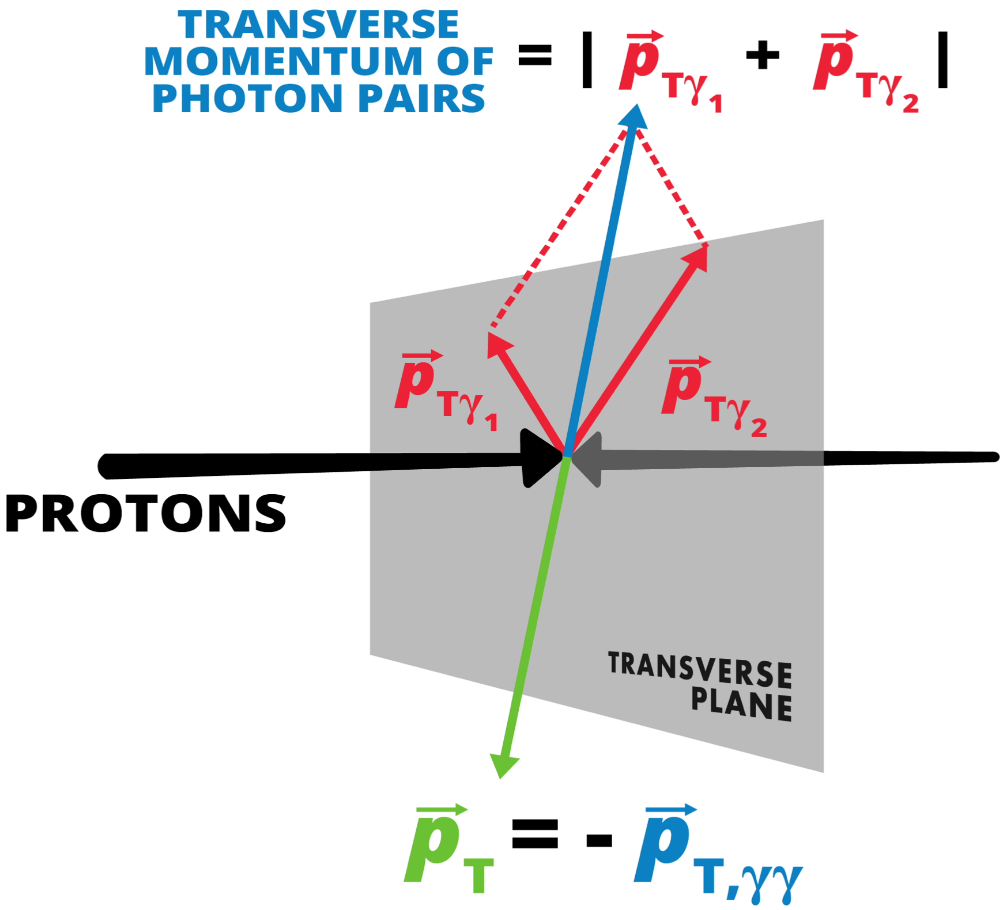
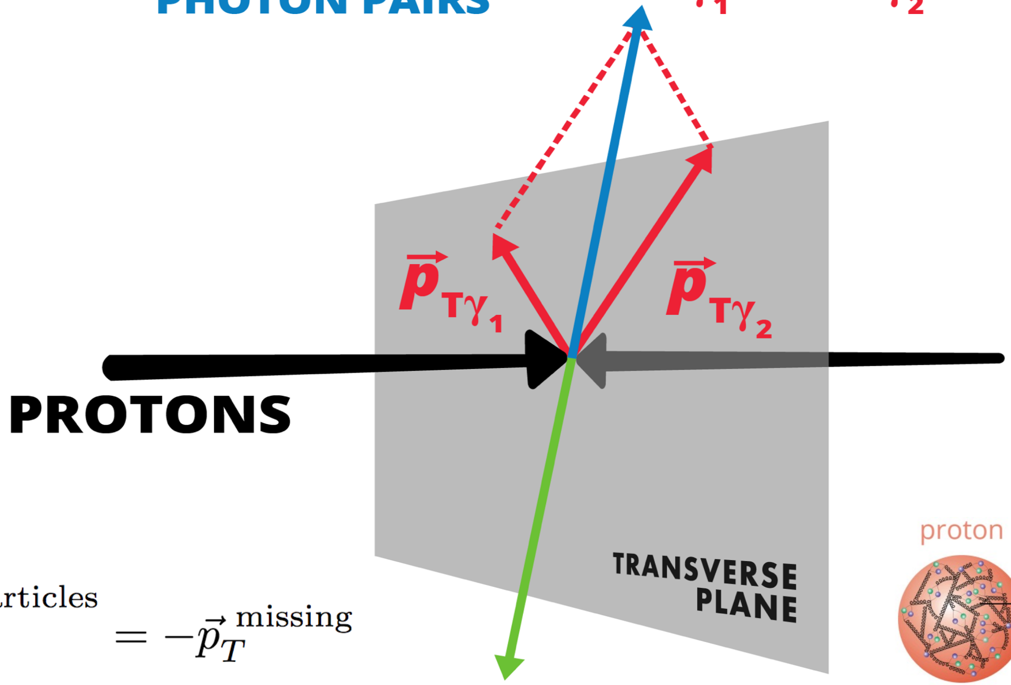
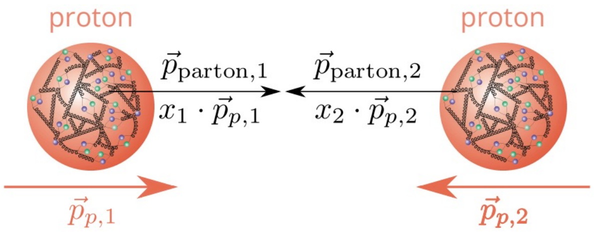

TRANSVERSE MOMENTUM OF PHOTON PAIRS $= |\vec{p}_{T\gamma_1} + \vec{p}_{T\gamma_2}|$
$\vec{p}_{T\gamma_1}$
$\vec{p}_{T\gamma_2}$
PROTONS
TRANSVERSE PLANE
$\vec{p}_T = -\vec{p}_{T,\gamma\gamma}$

**TRANSVERSE MOMENTUM OF PHOTON PAIRS** $= | \vec{p}_{T\gamma_1} + \vec{p}_{T\gamma_2} |$

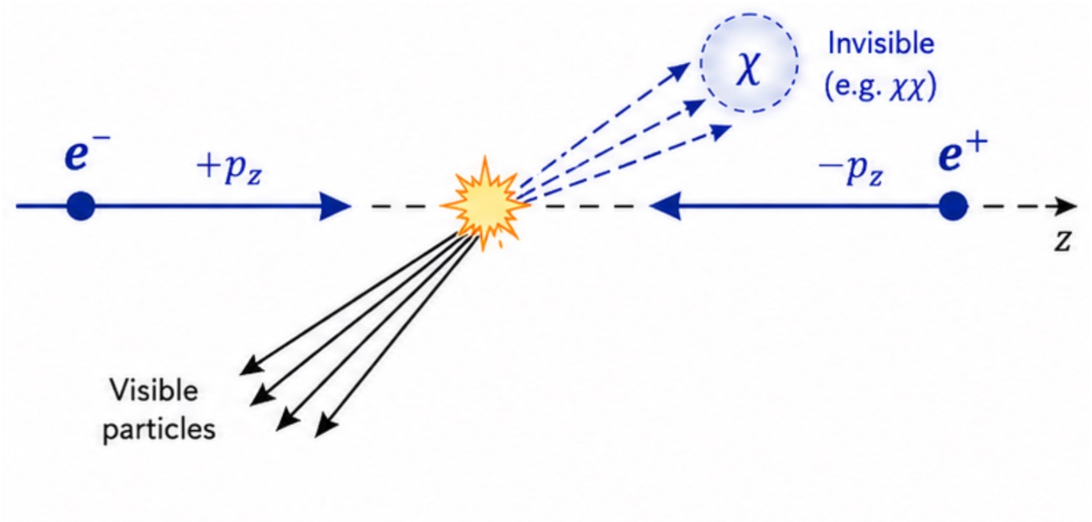



$$\left(\sum \vec{p}_T\right)^{\text{detected particles}} = -\vec{p}_T^{\,\text{missing}}$$

$$\vec{p}_T = -\,\vec{p}_{T,\gamma\gamma}$$



# Reconstruction of invisible final states at FCC-ee



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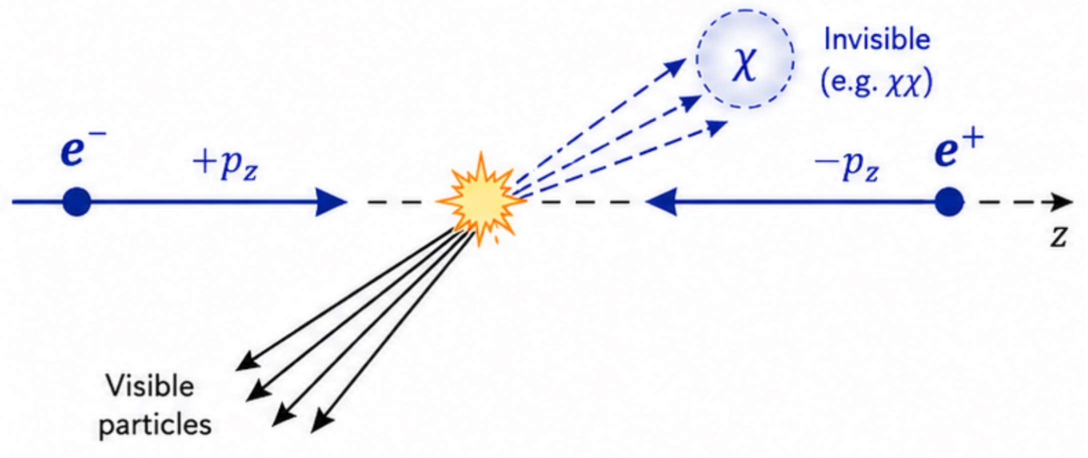



- Total initial four-momentum is **known**
- Momentum conservation enables **full reconstruction** of invisible system

$$p_{\text{miss}} = p_{e^-} + p_{e^+} - \sum p_{\text{vis}}$$

and

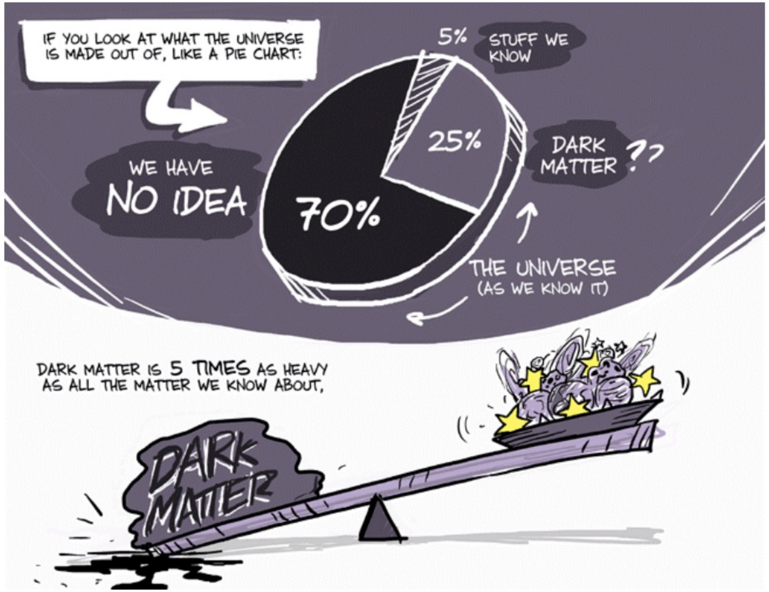
$$\begin{aligned} m_{\text{inv}}^2 &= p_{\text{miss}}^2 \\ &= (p_{e^-} + p_{e^+} - p_{\text{vis}})^2 \end{aligned}$$

# The Dark Sector of the universe


**Sandbox Studio for Symmetry Magazine**

https://www.symmetrymagazine.org/article/voyage-into-the-dark-sector

# The energy budget of the Universe




[Jorge Cham: *Dark Matters - A Tales from the Road Comic*]

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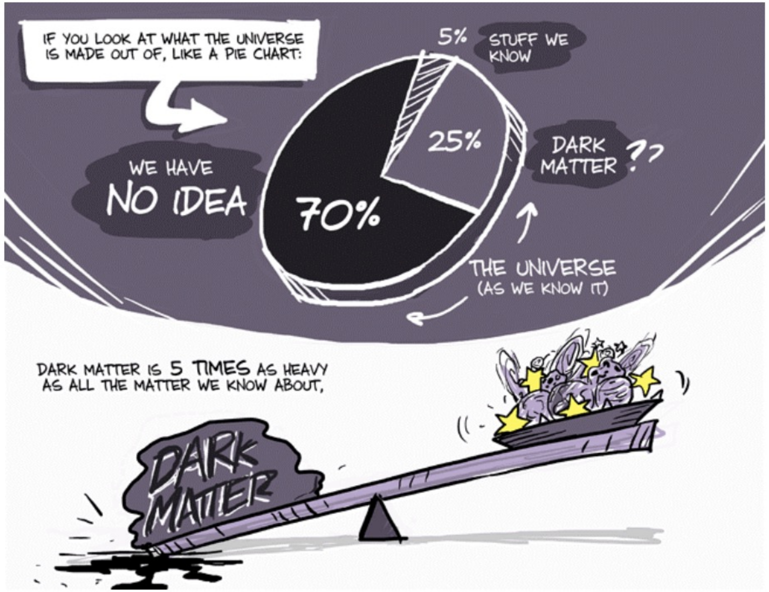


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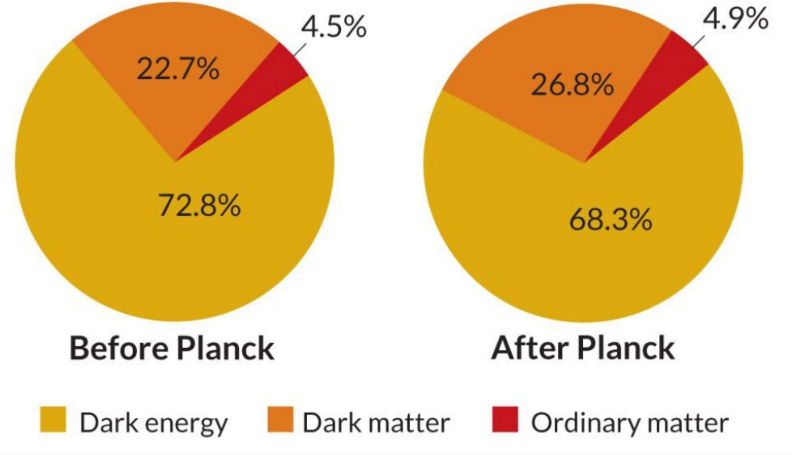


The Planck Satellite experiment (2009-2013) has heralded the era of precision cosmology

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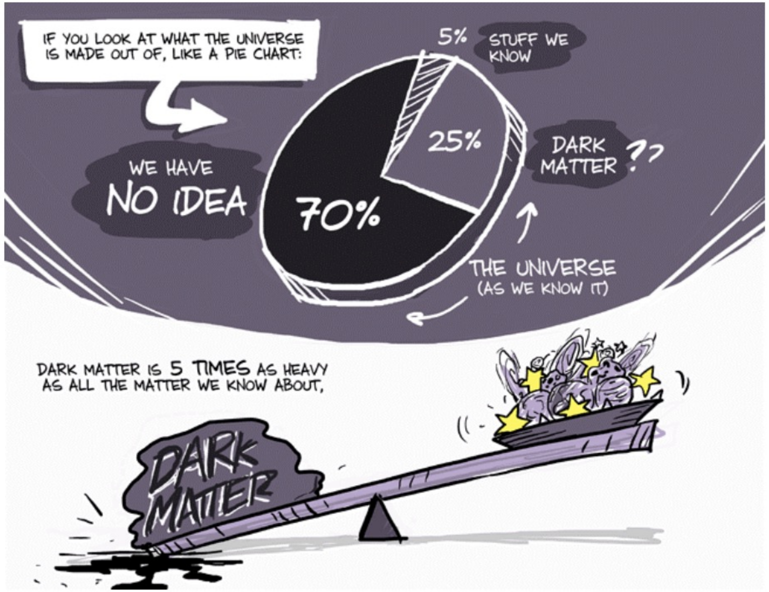


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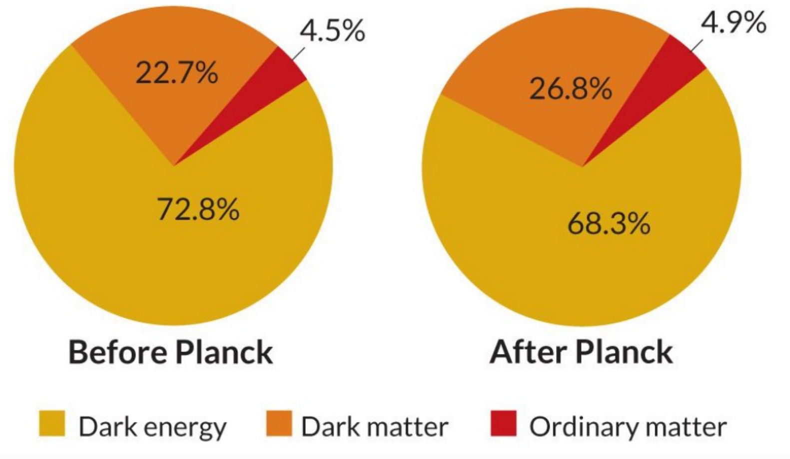


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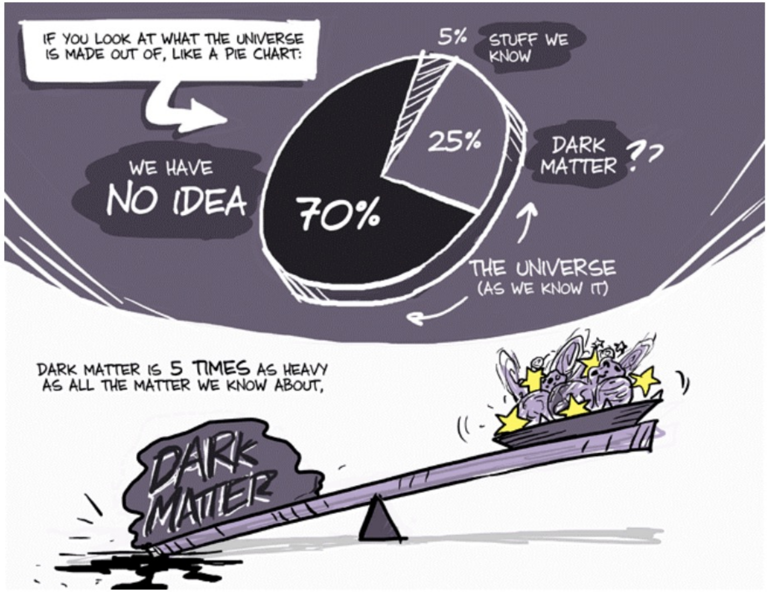


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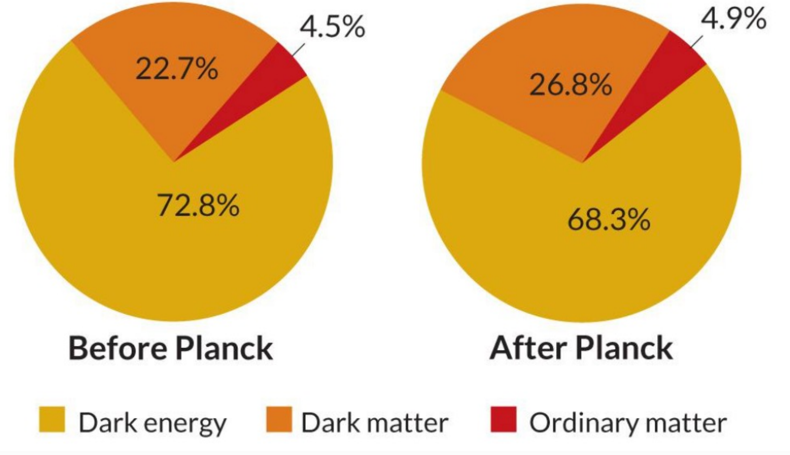


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**Dark matter** is believed to have seeded structure formation in the early universe. The imprints of which are left in the **Cosmic Microwave Background** (CMB) radiation

**Dark energy** does not seem to dilute as the universe expands. It drives the **accelerated expansion** of the universe

# A new vector boson as a portal to the dark sector

Visible sector

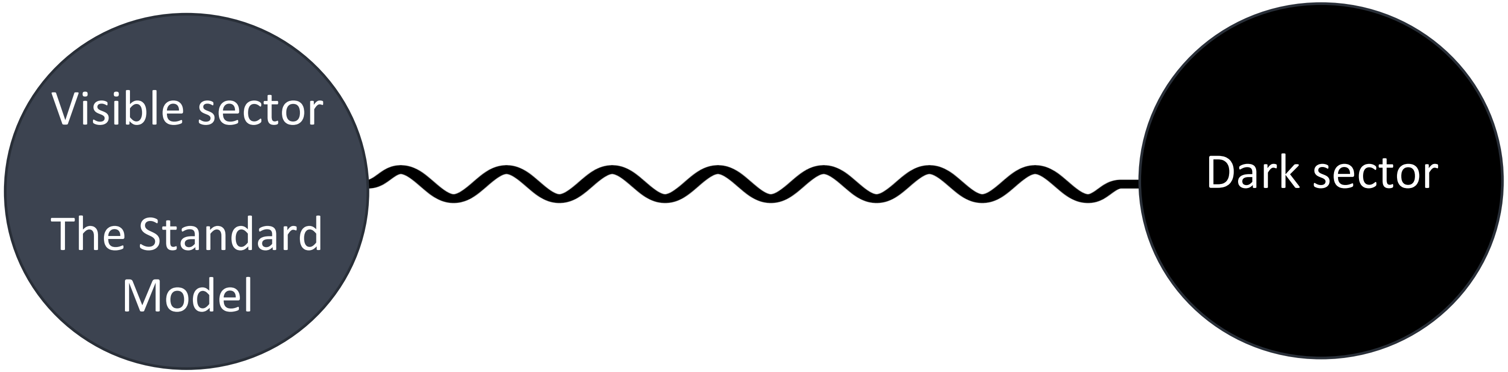
The Standard Model

Dark sector

$$\underbrace{SU(3)_C \times SU(2)_L \times U(1)_Y}_{}$$

Standard Model
Gauge group

# A new vector boson as a portal to the dark sector




$$\underbrace{SU(3)_C \times SU(2)_L \times U(1)_Y}_{\text{Standard Model Gauge group}} \longrightarrow SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_X$$

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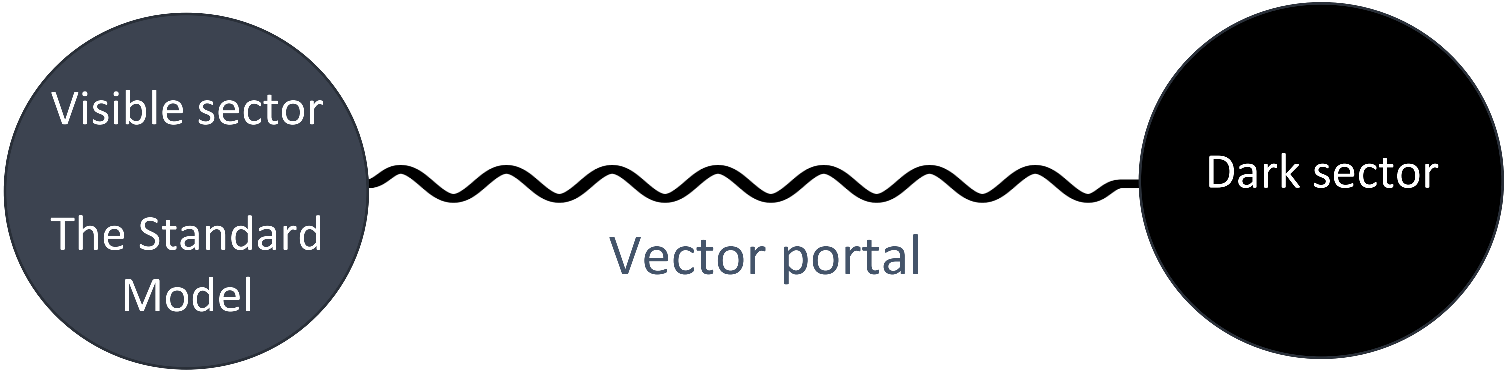


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Gives a new vector boson

**Hidden:** SM particles are not charged under $U(1)_X$

# The many realizations of the dark photon

Electroweak sector of the Standard Model $SU(2)_L \times U(1)_Y$

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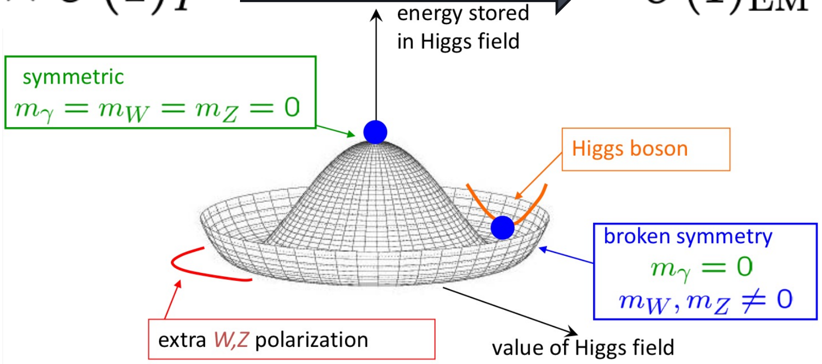


[Illustration: Mark Neubauer]

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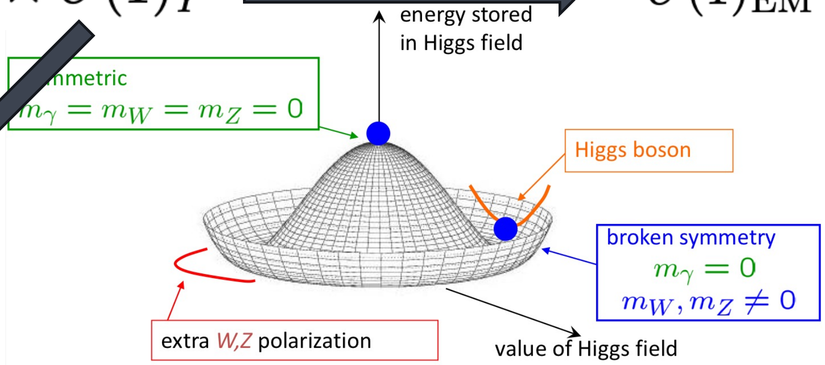


[Illustration: Mark Neubauer]

Processes **above** EWSB scale (~ 250 GeV):

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"Dark photon" couples to hypercharge

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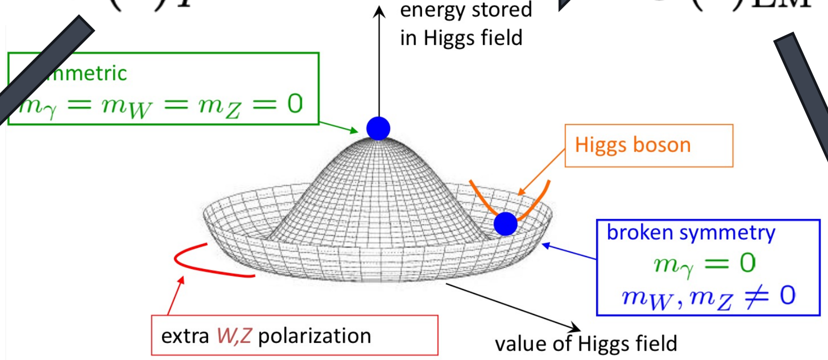


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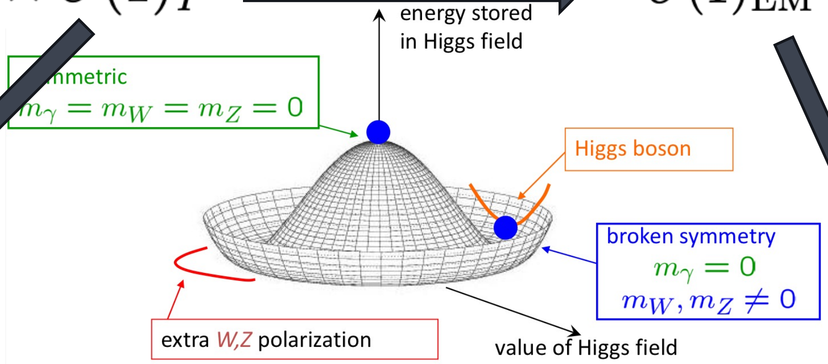


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**Two types of mixing:**

- Kinetic mixing $\quad \frac{\delta}{2} F_{\mu\nu} F'^{\mu\nu}$
- Mass mixing $\quad m_A \, m'_A A_\mu A'^\mu$

# Massive dark photon / dark Z

**Kinetic mixing** between $U(1)_Y$ and $U(1)_X$ [Holdom, PLB **166**, 196 (1986)]

$$\mathcal{L} \supset -\frac{1}{4}B_{\mu\nu}B^{\mu\nu} - \frac{1}{4}C_{\mu\nu}C^{\mu\nu} - \frac{\delta}{2}C_{\mu\nu}B^{\mu\nu}$$

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$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$$

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Hypercharge field strength

New gauge field strength

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" **Dark *Z*** " model: [Davoudiasl, Lee, Marciano (2012)]

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**Acquiring mass:**

- EWSB (new scalar sector)
- Stueckelberg mechanism

[Feldman, Liu, Nath, PRD **75** (2007)]

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$$\mathcal{L} = \mathcal{L}_{\rm SM} + \Delta\mathcal{L},$$

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The SM fields <u>do not</u> carry U(1)$_X$ quantum numbers, and the fields in the hidden sector do not carry quantum numbers of the SM gauge group.

# The basis

This allows one to work in the diagonal basis, denoted by $E$

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$$V = (K\mathcal{O}\mathcal{R})E$$

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Three mass eigenstates:

- massive dark $Z$
- massive $Z$ boson
- massless photon

**Dark $Z$ inherits the properties of the $Z$ boson, including axial coupling!**

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**NOTE:** Rotating to diagonal basis imparts a **millicharge** to dark matter

$$\boxed{\epsilon = \frac{M_2}{M_1}} \longrightarrow \boxed{\epsilon_D = \frac{g_X Q_X}{g_Y}\epsilon}$$

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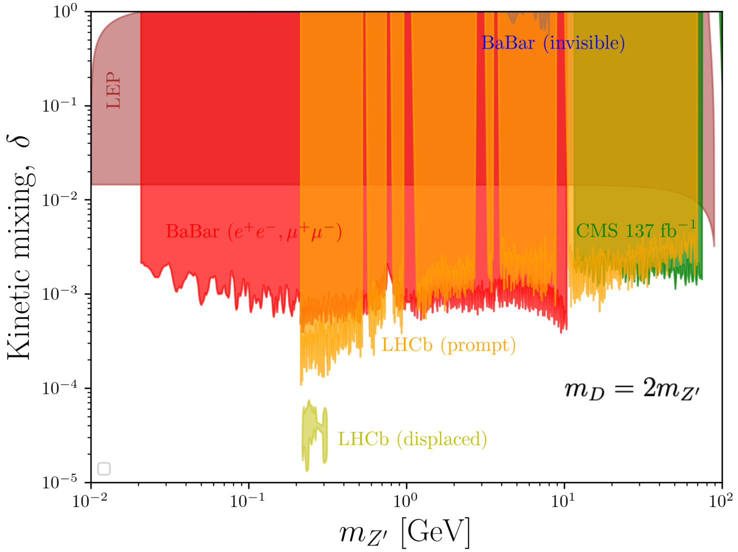
Neglect for this study

# Current constraints on StSM

Dark $Z$ decaying into **visible final states** (SM fermions)

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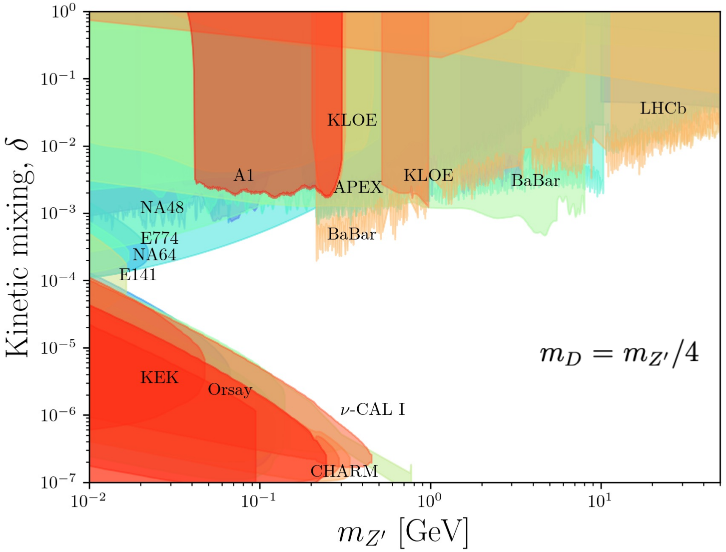
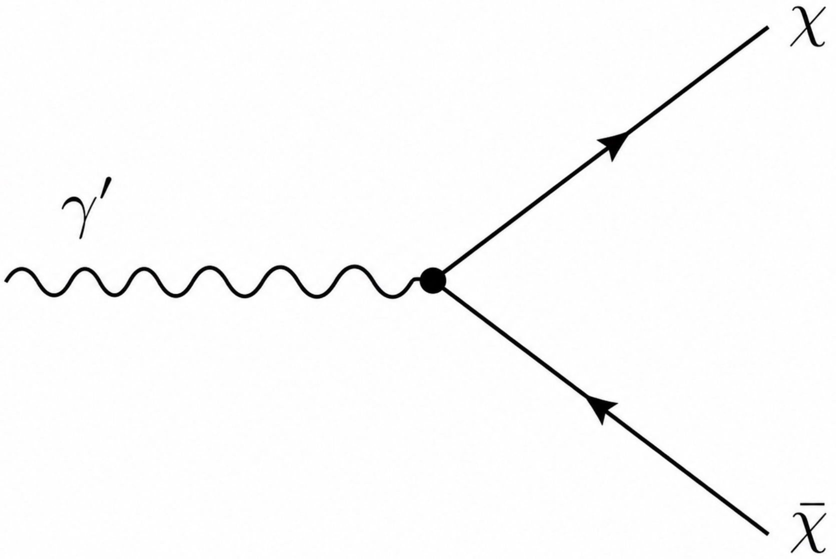



Agnostic to DM constraints

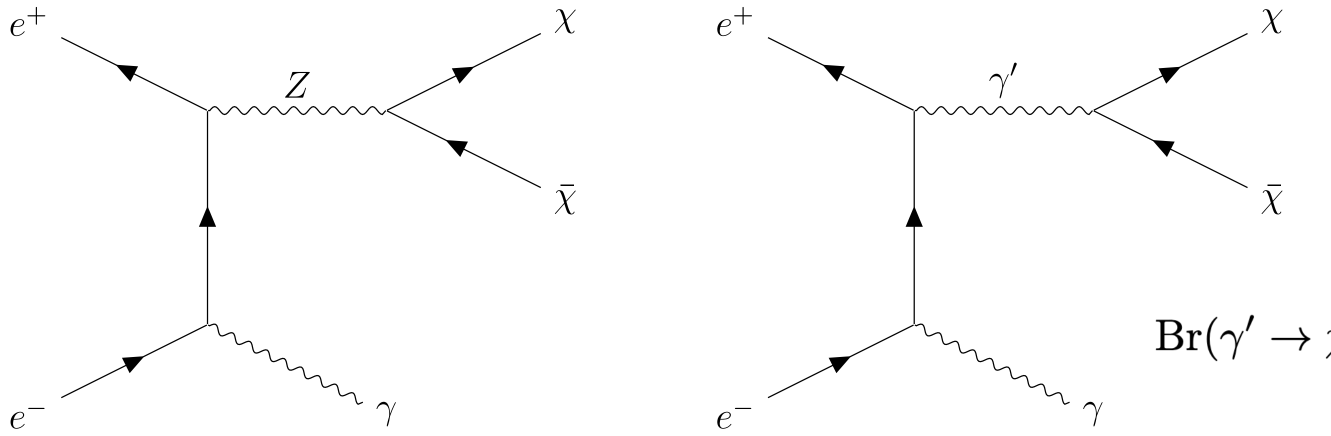
Generated using DarkCast
[P. Ilten *et al*, JHEP **06**, 004 (2018)]

**AA**, M.M. Altakach, M. Klasen, P. Nath and Z.Y. Wang, JHEP **03**, 182 (2023)

# The **invisible** decay of the dark Z at FCC-ee



# The signal: Invisible decay of a Stueckelberg dark photon / dark Z (StSM model)




$\mathrm{Br}(\gamma' \to \chi\bar{\chi}) \approx 1$

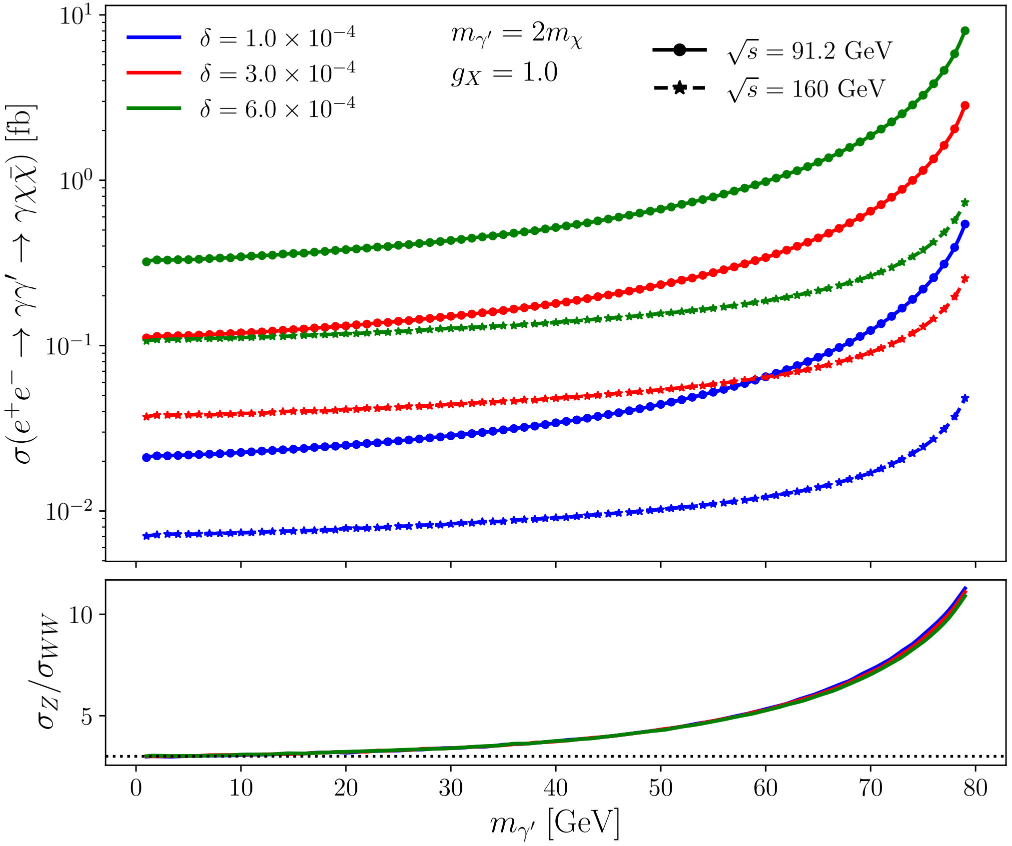
**Signal** process: Mono-photon $e^-e^+ \to \gamma\gamma' \to \gamma\chi\bar{\chi}$ (with ISR)

**Background** processes: Irreducible (dominant) $\nu\bar{\nu}\gamma$

Reducible (but large) $e^-e^+\gamma$ (misidentification of leptons)

……

# Cross-section vs. dark photon mass



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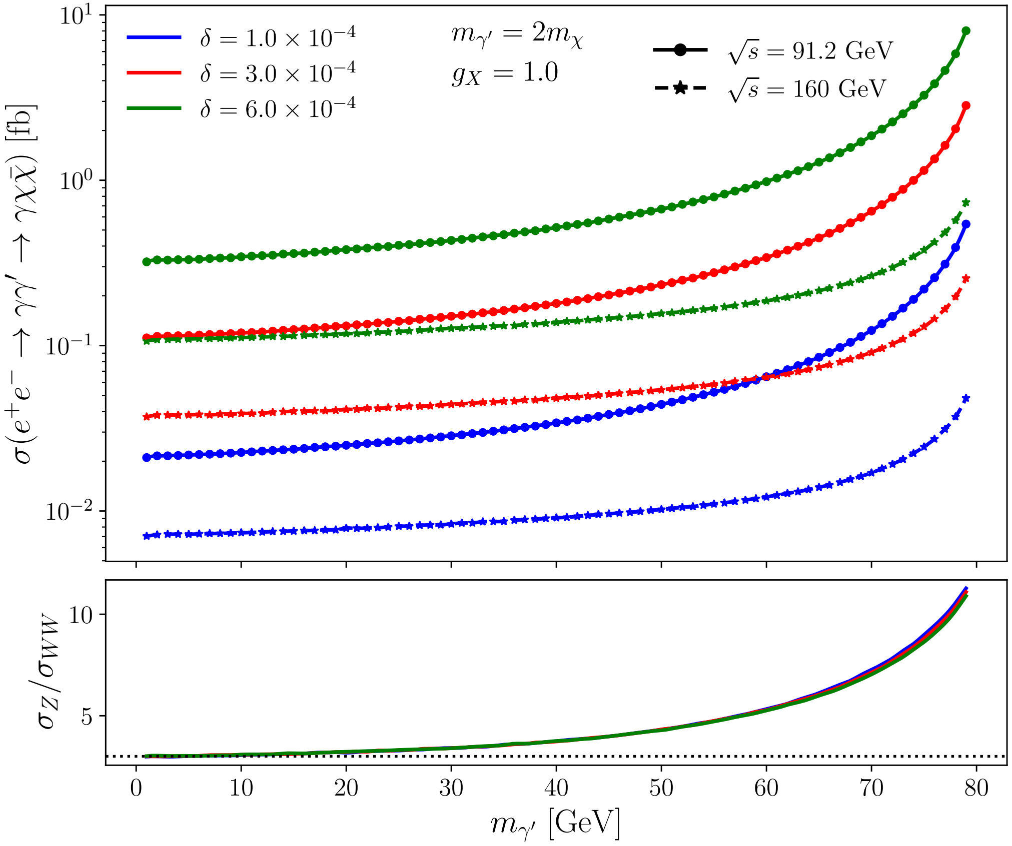


$$\sigma(s) \sim \frac{\delta^2\alpha^2}{s}\left(1 - \frac{m_{\gamma'}^2}{s}\right)^{-1}$$

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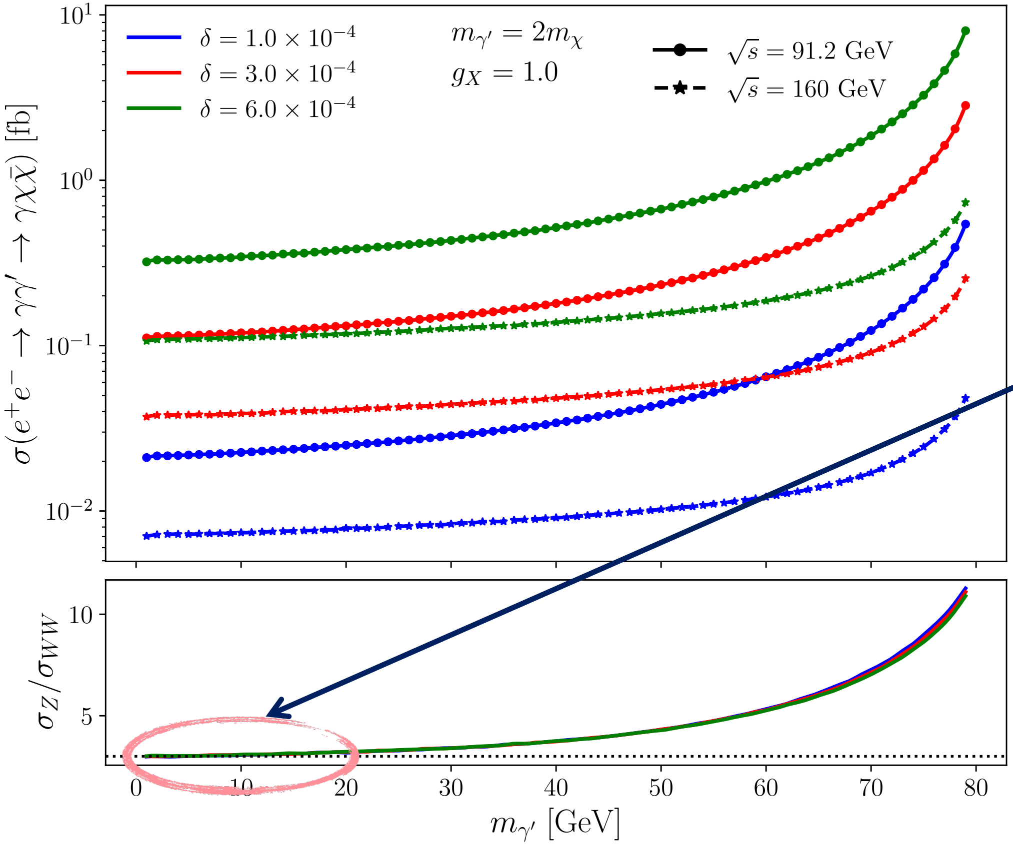


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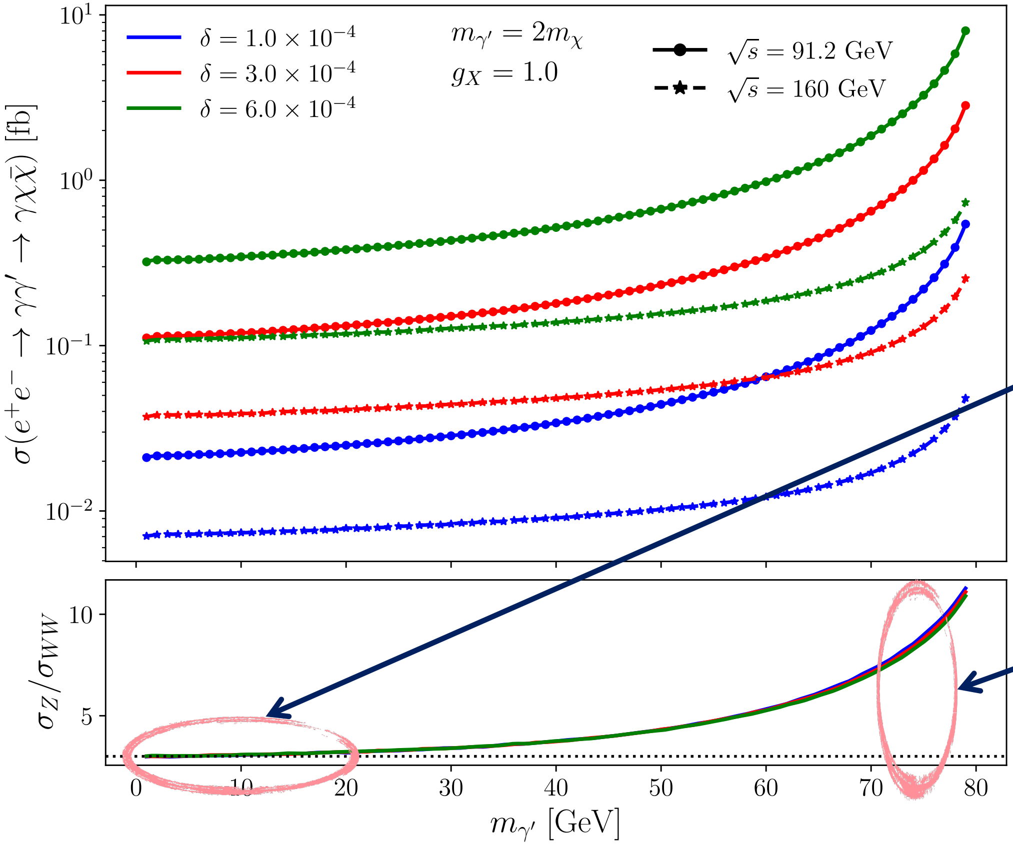


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$$\frac{\sigma_Z}{\sigma_{WW}} \sim \frac{160^2(1-75^2/160^2)}{91^2(1-75^2/91^2)} \sim 7$$

# Signal and Background Monte Carlo samples

**Each signal sample: 100,000 events x 2 = 200 K (train + test)**

**Background** processes: Irreducible (dominant) $\nu\bar{\nu}\gamma$

**70 Million events x 2 = 140 Million events (train + test)**

Reducible (but large) $e^{-}e^{+}\gamma$ (misidentification of leptons)

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# Preselection criteria

**Truth-level cuts:** $p_T^\gamma > 5.0\ \text{GeV}$ and $|\eta| < 2.6$

G. Polesello, [arXiv:2511.23337 [hep-ph]]

**Preselection criteria:** Electron veto, Muon veto, At least one photon in an event

# Reconstruction of invisible mass

**Conservation of 4-momentum:**

$$p_{\text{invisible}} = p_{\text{initial}} - p_{\gamma}$$

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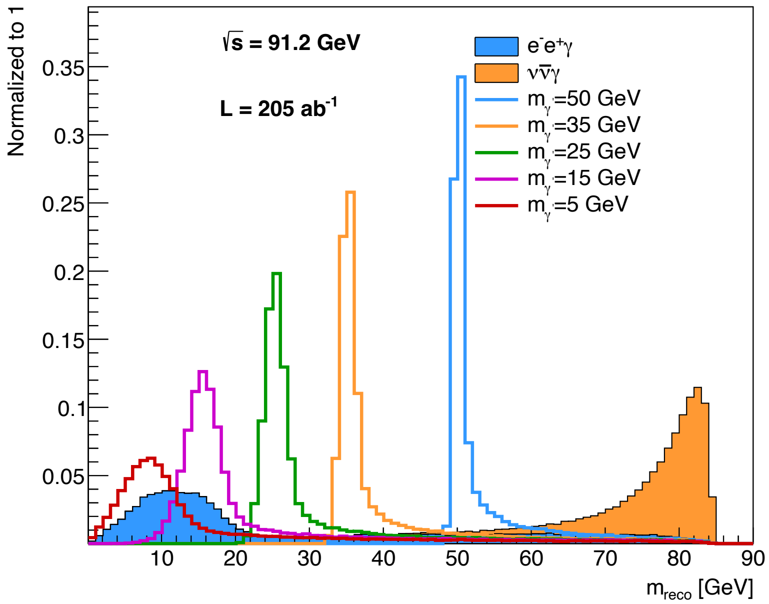


# Reconstruction of invisible mass

**Conservation of 4-momentum:**

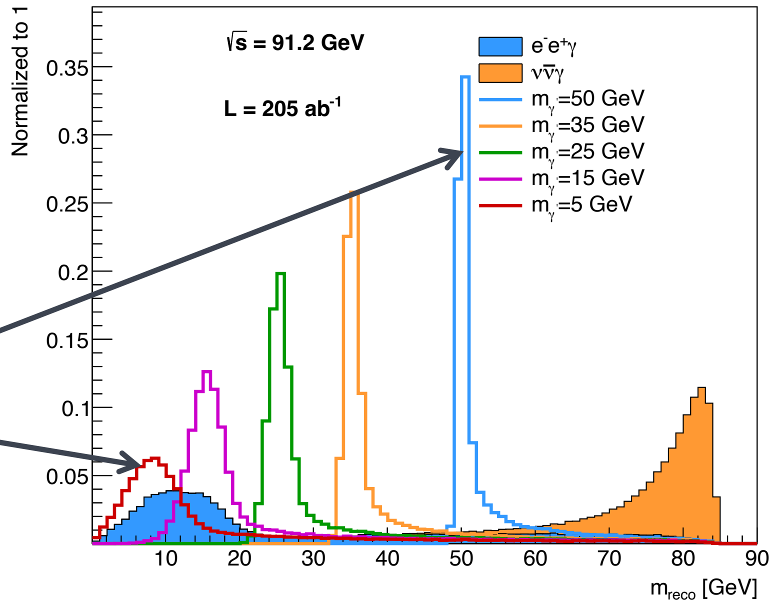
$$p_{\text{invisible}} = p_{\text{initial}} - p_\gamma$$

$$m_{\text{recoil}} = \sqrt{p^2_{\text{invisible}}}$$

Loop over all photons in an event and select the one that gives a recoil mass closest to the true mass

Poor (very good) reconstruction efficiency for lower (higher) masses

$$m^2_{\text{reco}} = s - 2\sqrt{s}E_\gamma \quad \Rightarrow \quad \delta m_{\text{reco}} \sim \frac{\sqrt{s}}{m_{\text{reco}}}\delta E_\gamma$$



# Boosted Decision Tree (BDT) training

- We train a BDT using the kinematic variables:

$$E_\gamma \,, \qquad p_T^\gamma \,, \qquad p_T^{\mathrm{miss}} \,, \qquad |\cos\theta_\gamma| = \frac{|p_z^\gamma|}{|\vec{p}_\gamma|} \,, \qquad \chi^2_{\mathrm{recoil}} = \frac{E_\gamma - E_{\mathrm{rec}}}{\sigma_E} \,.$$

$$E_{\mathrm{rec}} = \frac{s - m^2_{\gamma'}}{2\sqrt{s}} \quad \textbf{and} \quad \frac{\sigma_E}{E} = \frac{0.03}{\sqrt{E}} \oplus 0.005 \oplus \frac{0.002}{E} \,.$$

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Baseline ECAL resolution
in IDEA detector

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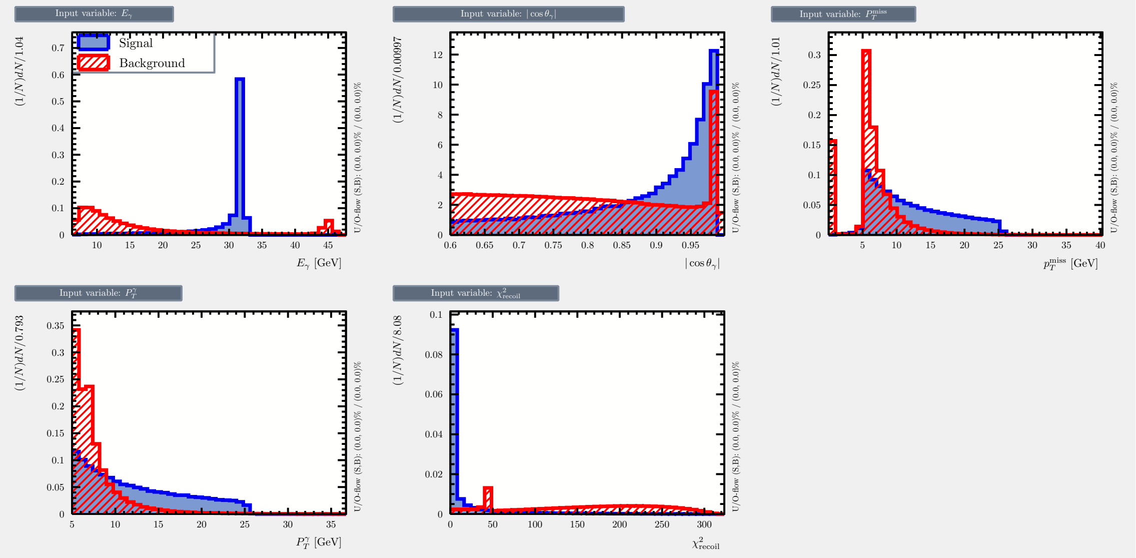

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- Training done using **BDT** on every signal and background events

- Additional cuts: **BDT cut optimized**, and $\chi^2_{\rm recoil} > 30$

$$Z_A = \sqrt{2\left[(S+B)\ln\left(1+\frac{S}{B}\right) - S\right]} \qquad |\cos\theta_\gamma| > 0.60$$

# Variation of detector configuration

Haider Abidi *et al*, PRD **112**, 052002 (2025)

1. Baseline configuration (current IDEA detector)
2. 30% better energy resolution (ECAL only)
3. 30% worse energy resolution (ECAL only)
4. Two times more granular
5. Two times less granular (not yet implemented)
6. Ideal VXD+Calo

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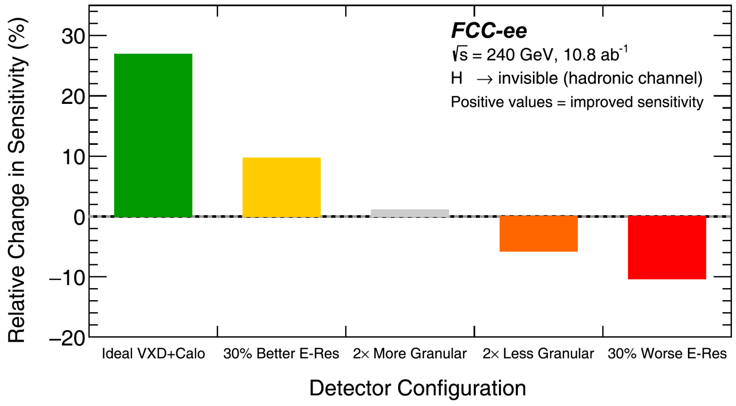
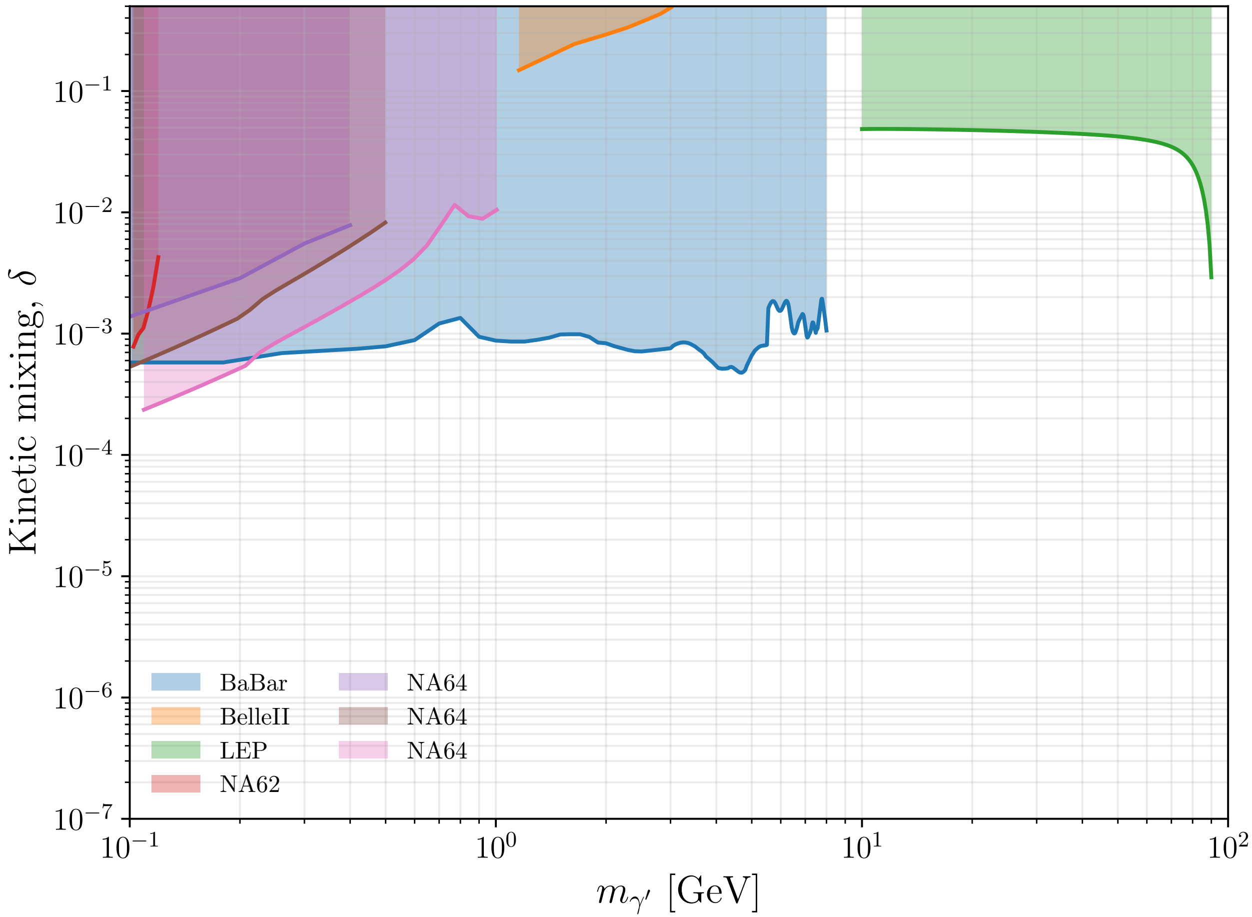


# The 95% CL upper limits on kinetic mixing in StSM

[Preliminary]



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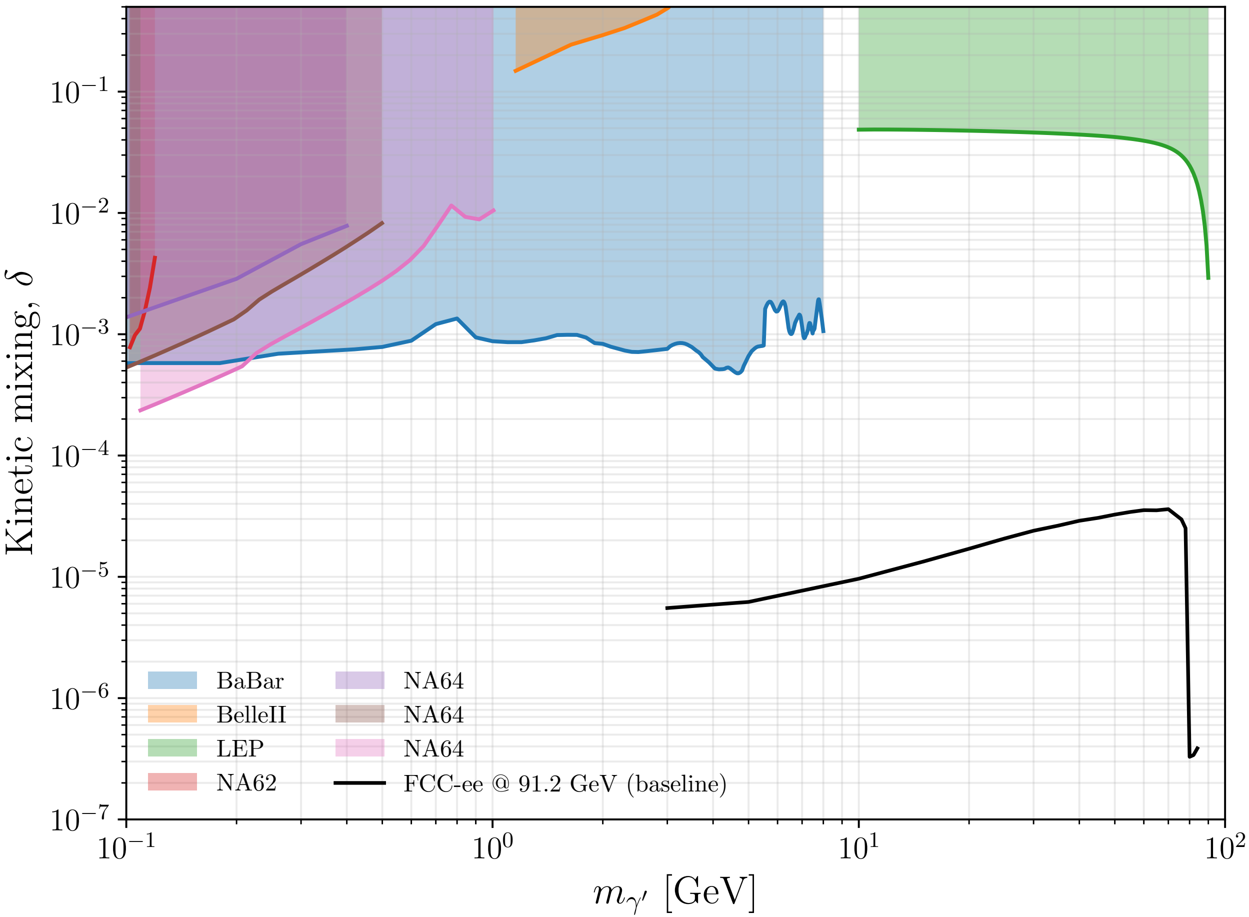

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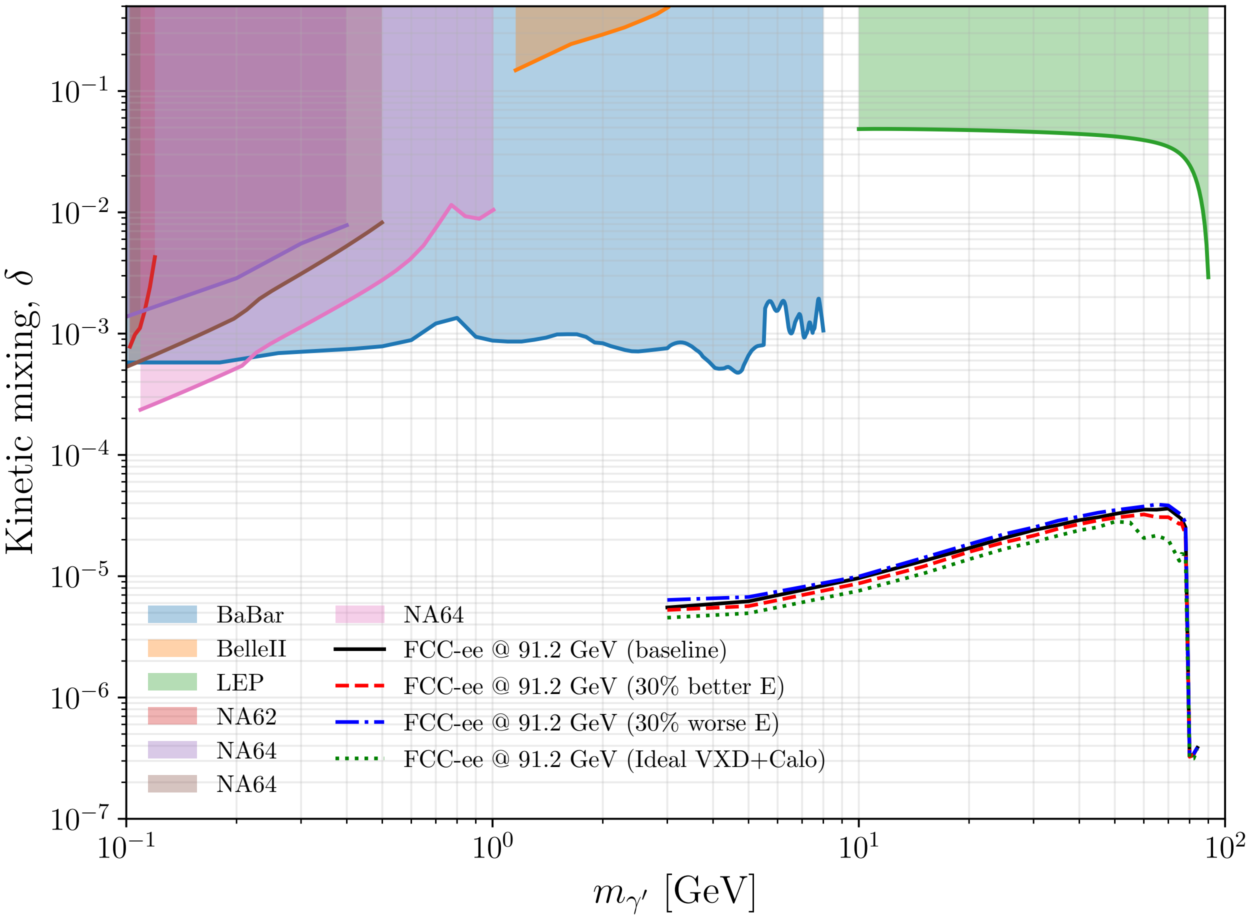

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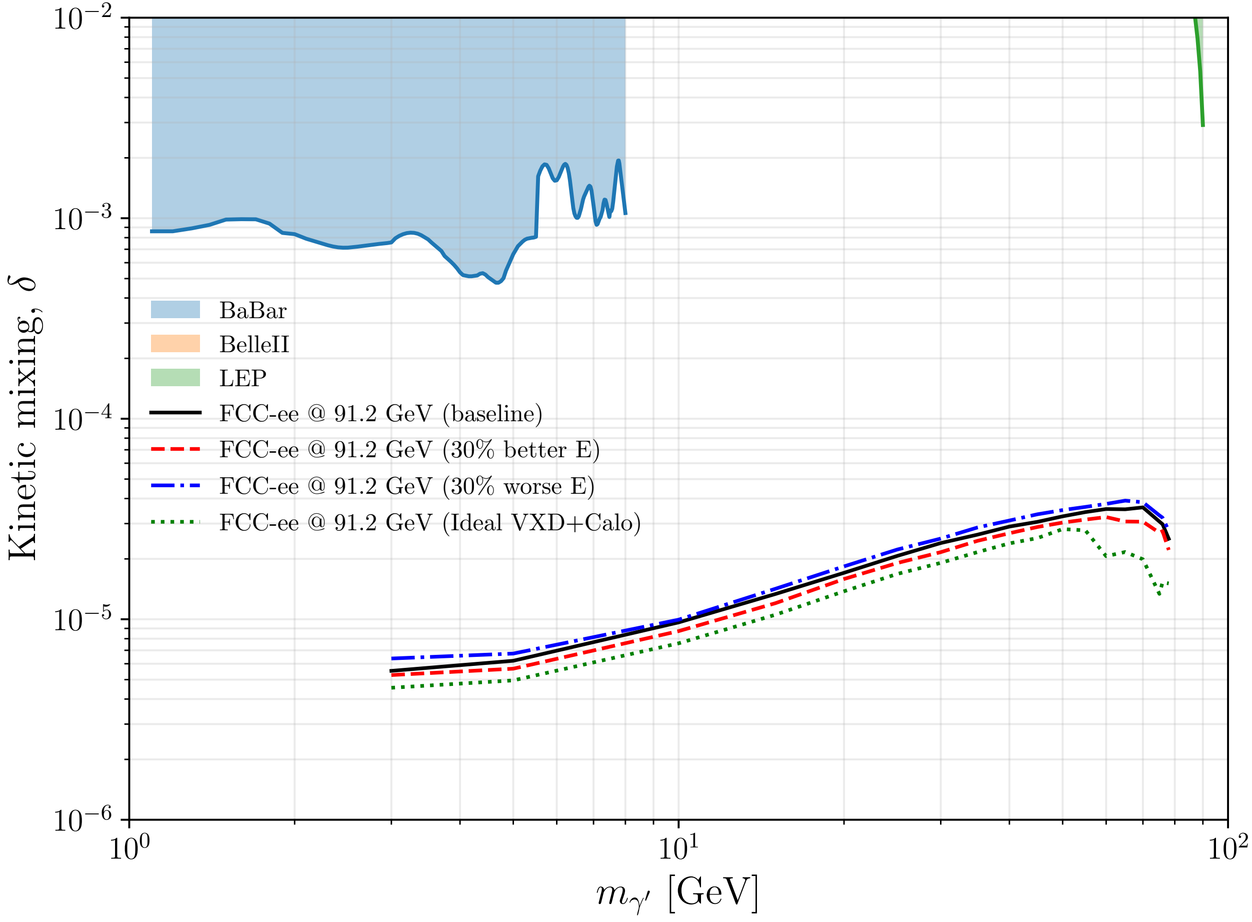
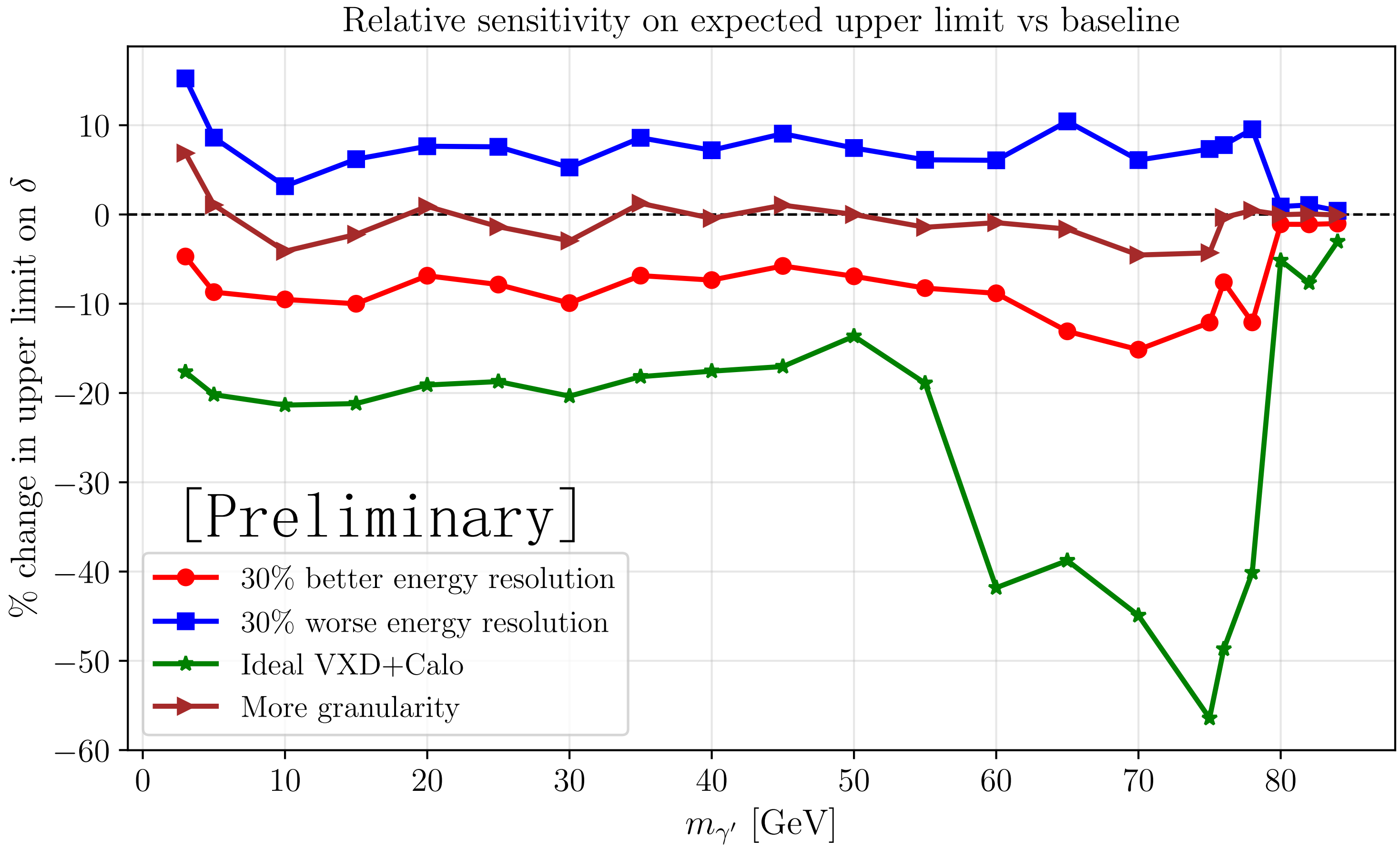


# Effects of variations in detector configuration



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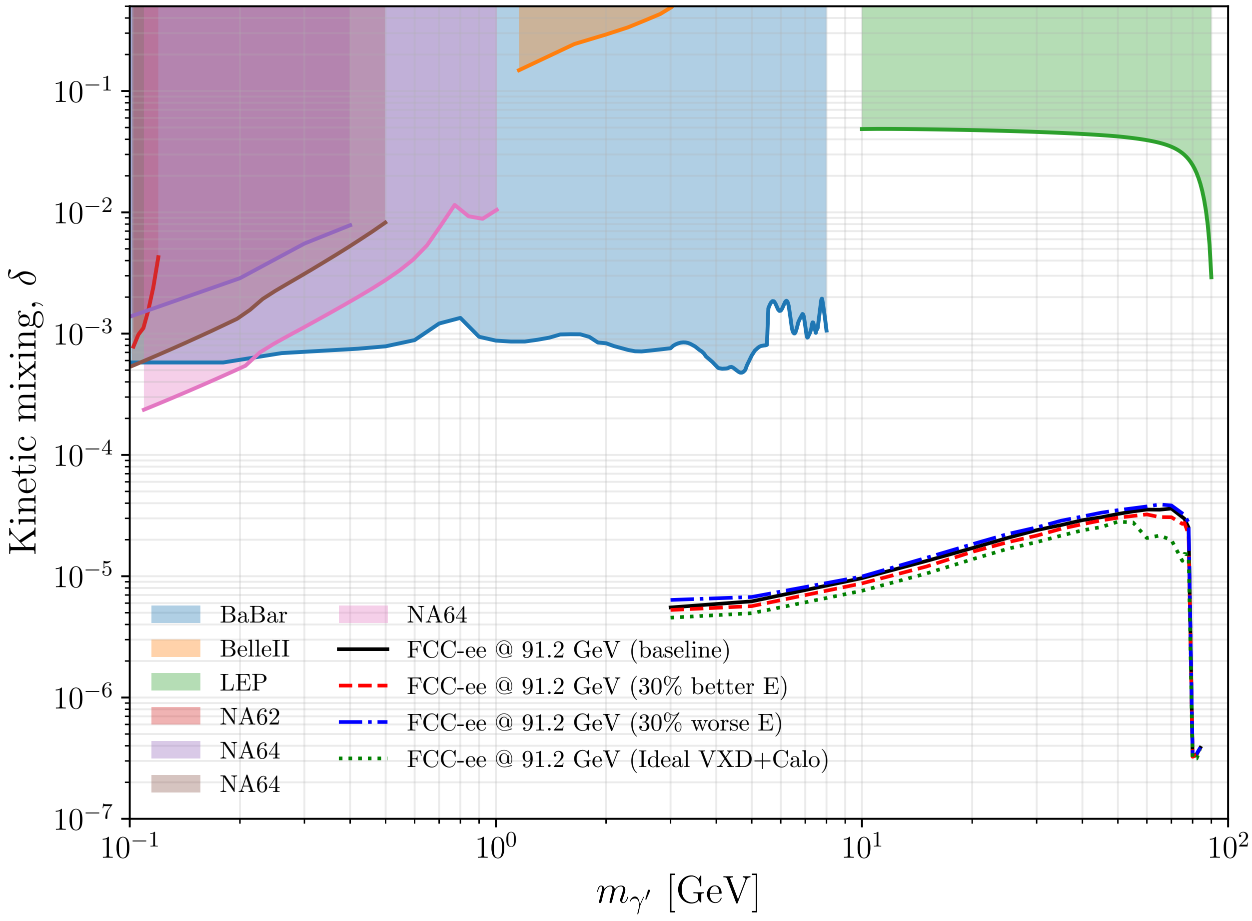

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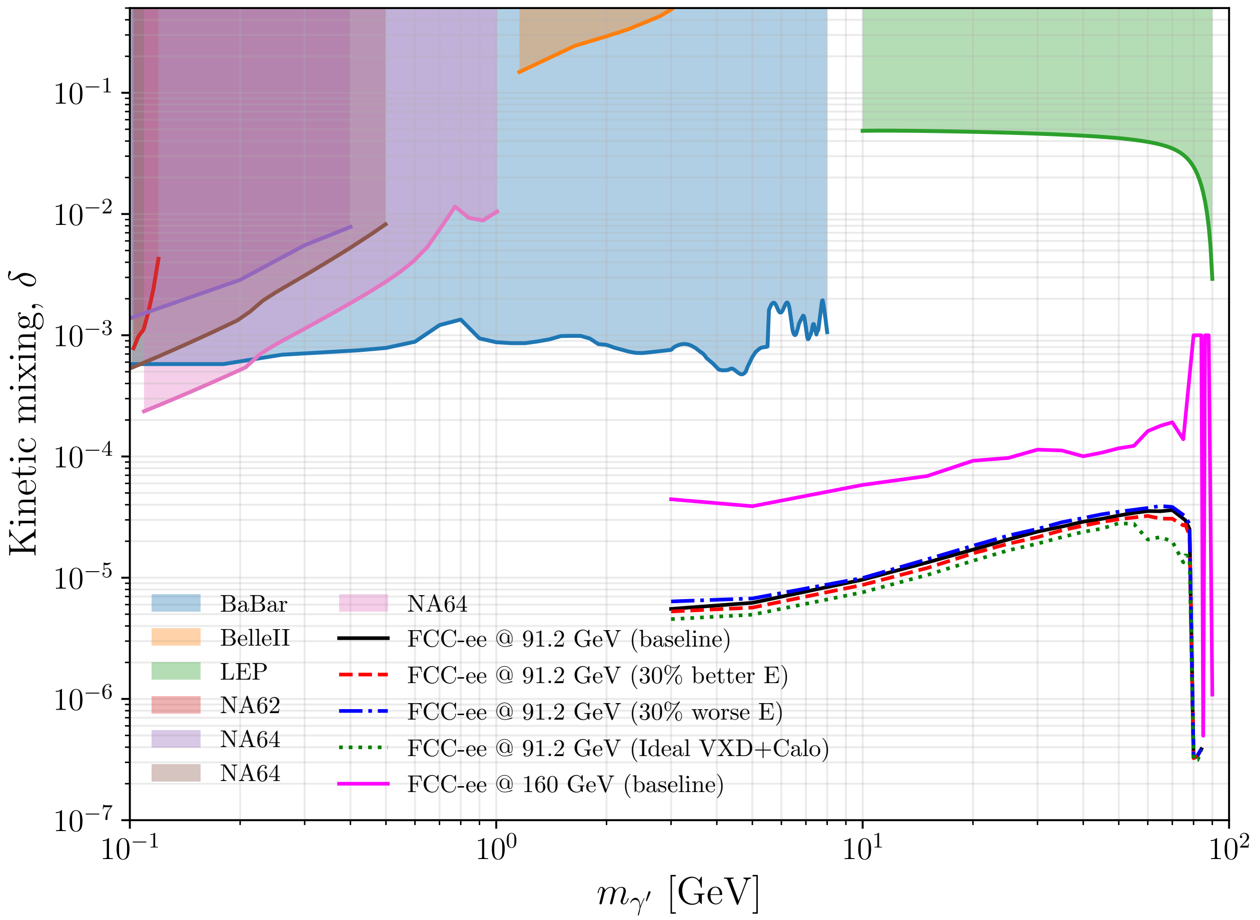

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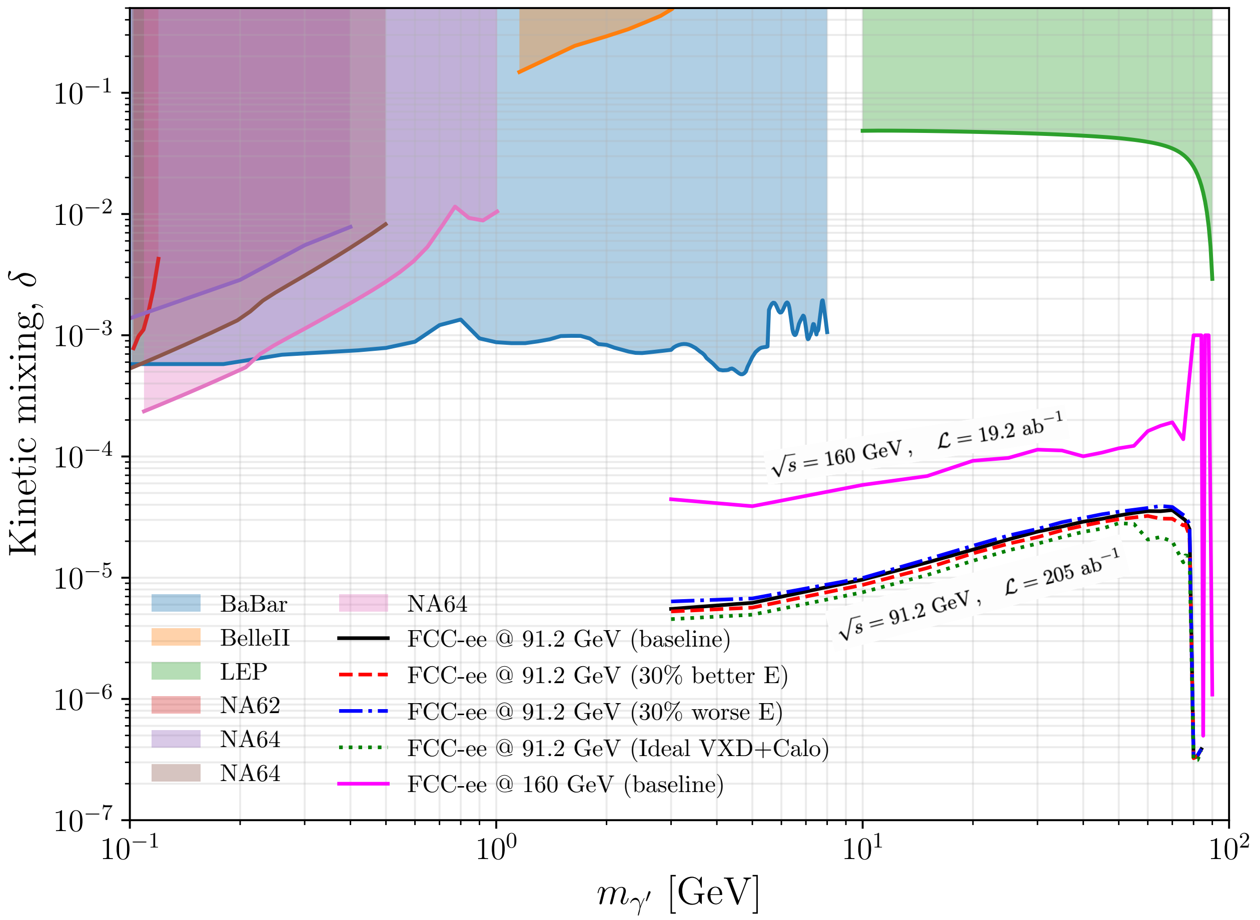

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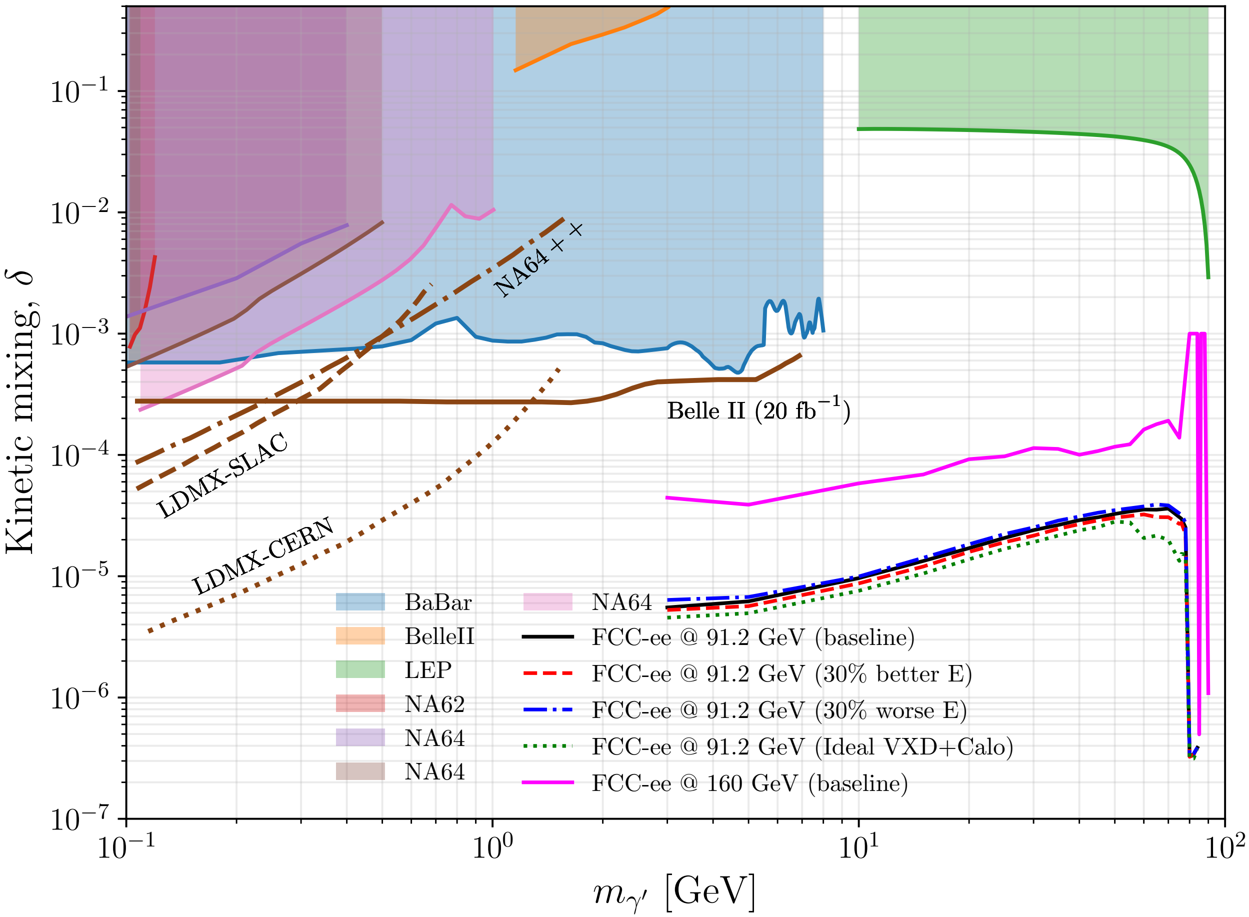

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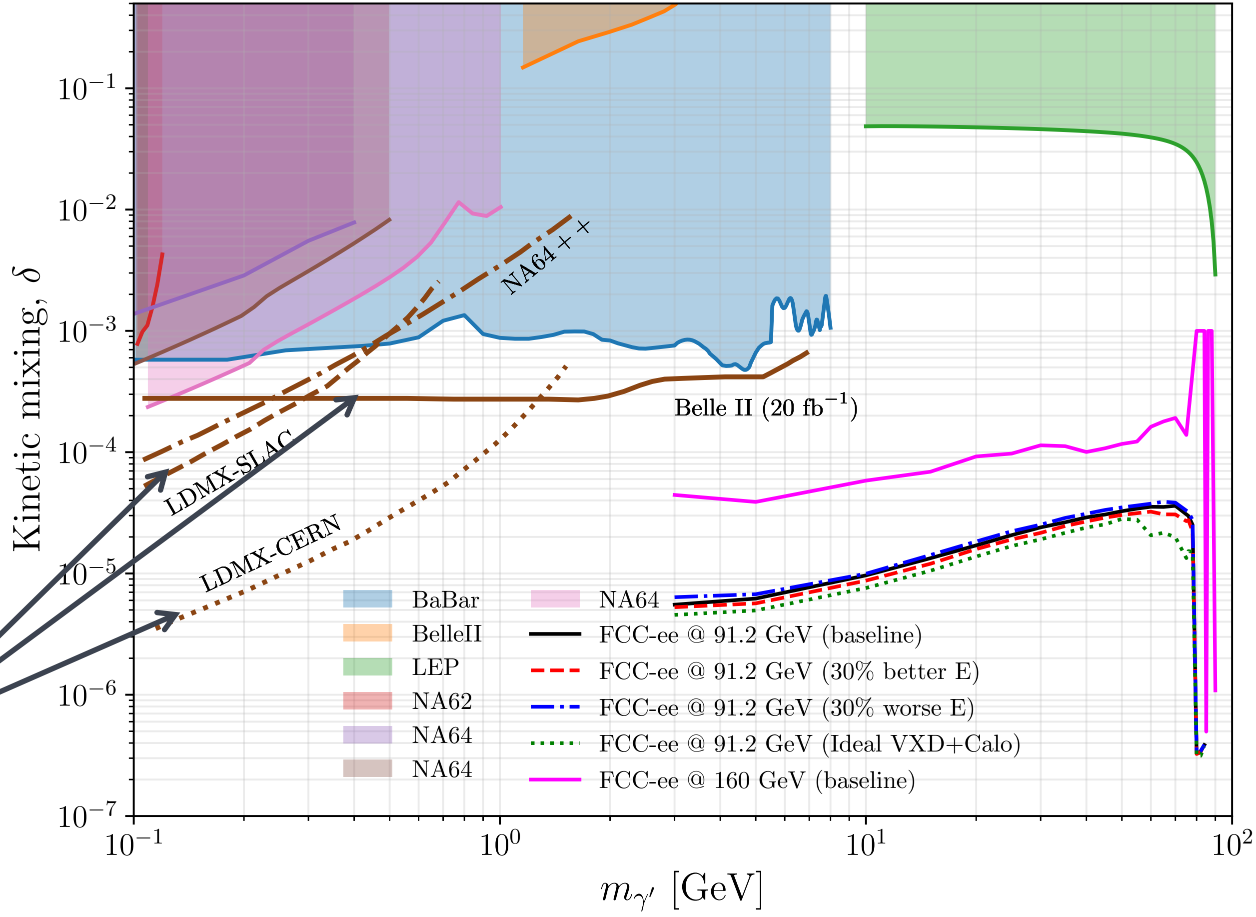


Other future sensitivities

Adapted from Fig. 3.6 (The Physics of the Dark Photon), *Marco Fabbrichesi, Emidio Gabrielli, Gaia Lanfranchi*

# Conclusion

- The FCC-ee provides a clean environment unlike the LHC
- Invisible final states can be reconstructed efficiently at FCC-ee
- StSM gives a dark photon / dark *Z* within a well-motivated theoretical framework
- Strong limits can be placed on the dark *Z* in the 3 GeV – 90 GeV mass range with very weak couplings
- Detector configurations:
  - ~10% enhancement in limits for 30% increase in ECAL resolution
  - ~10% degradation in limits for 30% decrease in ECAL resolution
  - No significant effect on limits from changes in granularity

# Got more time?

# Massless dark photon

# Massless dark photon

Lagrangian describing kinetic mixing between $U(1)_{\rm EM}$ and **unbroken** $U(1)_X$

$$\mathcal{L} \supset -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{4}F'_{\mu\nu}F'^{\mu\nu} + \frac{\delta}{2}F^{\mu\nu}F'_{\mu\nu} - eA_\mu J^\mu - e'A'_\mu J'^\mu$$

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Kinetic mixing

SM photon

Visible current

# Massless dark photon

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Dark photon

Dark current

# Massless dark photon

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Rotate to the diagonal basis: $A'^\mu \to A'^\mu + \delta A^\mu$

$$\mathcal{L} \supset -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{4}F'_{\mu\nu}F'^{\mu\nu} - A_\mu(eJ^\mu + \delta\, e' J'^\mu) - e'A'_\mu J'^\mu$$

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SM photon couples to both
visible and dark currents

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SM photon couples to both visible and dark currents

Dark photon only couples to dark current

# Massless dark photon

Lagrangian describing kinetic mixing between $U(1)_{\text{EM}}$ and **unbroken** $U(1)_X$

$$\mathcal{L} \supset -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{4}F'_{\mu\nu}F'^{\mu\nu} + \frac{\delta}{2}F^{\mu\nu}F'_{\mu\nu} - eA_\mu J^\mu - e'A'_\mu J'^\mu$$

Rotate to the diagonal basis: $A'^\mu \to A'^\mu + \delta A^\mu$

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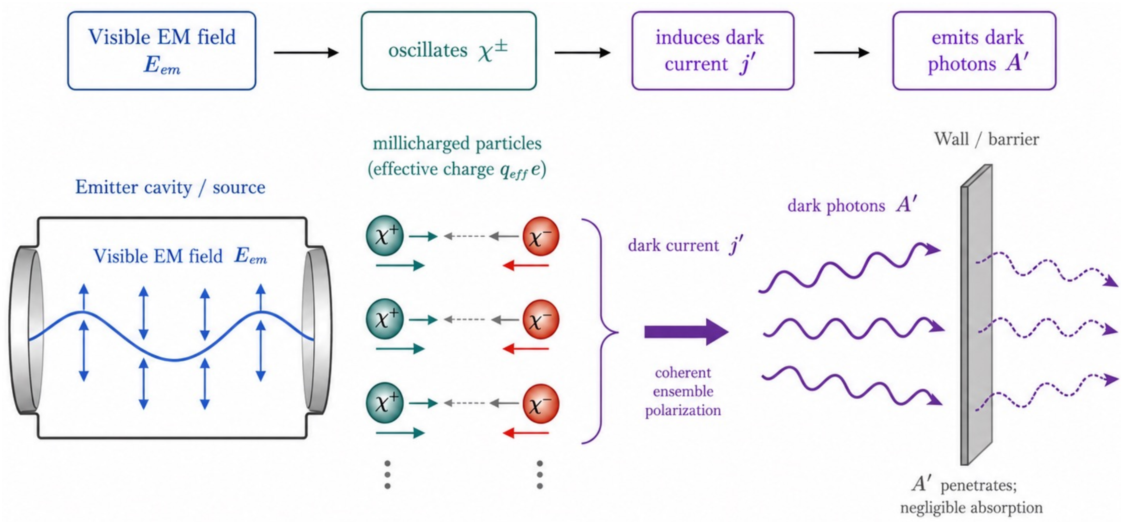
**Massless dark photons DO NOT couple to the electromagnetic current!**

# Massless dark photon

Lagrangian describing kinetic mixing between $U(1)_{\mathrm{EM}}$ and **unbroken** $U(1)_X$

$$\mathcal{L} \supset -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{4}F'_{\mu\nu}F'^{\mu\nu} - A_\mu(eJ^\mu + \delta\, e' J'^\mu) - e' A'_\mu J'^\mu$$



# BACK UP SLIDES

# Stueckelberg U(1) extension of the Standard Model

**This mechanism allows the additional gauge boson to gain mass with out extending the scalar sector**

A prototype Stueckelberg Lagrangian is

B. Kors and P. Nath, JHEP **07**, 069 (2005)

$$\mathcal{L} = -\frac{1}{4}\mathcal{F}_{\mu\nu}\mathcal{F}^{\mu\nu} - \frac{1}{2}(mA_\mu + \partial_\mu\sigma)(mA^\mu + \partial^\mu\sigma)$$

It is **gauge invariant** if $\delta A_\mu = \partial_\mu \epsilon \ , \quad \delta\sigma = -m\epsilon$

Add *gauge fixing term* $\mathcal{L}_{\rm gf} = -\frac{1}{2\xi}\left(\partial_\mu A^\mu + \xi m\sigma\right)^2$

Total Lagrangian becomes

$$\mathcal{L} + \mathcal{L}_{\rm int} + \mathcal{L}_{\rm gf} = -\frac{1}{4}\mathcal{F}_{\mu\nu}\mathcal{F}^{\mu\nu} - \frac{m^2}{2}A_\mu A^\mu - \frac{1}{2\xi}(\partial_\mu A^\mu)^2 - \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma - \xi\frac{m^2}{2}\sigma^2$$

Mass terms decoupled

# Stueckelberg U(1) extension of the Standard Model

*(with kinetic and mass mixings)*

The **extended electroweak sector** has the following Lagrangian

A A, P. Nath and Z. Y. Wang, JHEP **12**, 148 (2021)

AA, M.M. Altakach, M. Klasen, P. Nath and Z.Y. Wang, JHEP **03**, 182 (2023)

$$\mathcal{L} = \mathcal{L}_{\rm SM} + \Delta\mathcal{L},$$

$$\begin{aligned}\Delta\mathcal{L} = &-\frac{1}{4}C_{\mu\nu}C^{\mu\nu} + i\bar{D}\gamma^\mu\partial_\mu D - m_D\bar{D}D \\ &-\frac{\delta}{2}C_{\mu\nu}B^{\mu\nu} - \frac{1}{2}(\partial_\mu\sigma + M_1C_\mu + M_2B_\mu)^2 \\ &+ g_X Q_X\bar{D}\gamma^\mu DC_\mu.\end{aligned}$$

Terms represent **non-diagonal kinetic terms**
and
**non-diagonal mass terms**

# Let's diagonalize them!

In the basis $V^T = (C, B, A^3)$, the *non-diagonal kinetic term* is

$$\mathcal{K} = \begin{bmatrix} 1 & \delta & 0 \\ \delta & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

and the *non-diagonal mass term* is

$$M_{\mathrm{St}}^2 = \begin{bmatrix} M_1^2 & M_1 M_2 & 0 \\ M_1 M_2 & M_2^2 + \frac{1}{4}v^2 g_Y^2 & -\frac{1}{4}v^2 g_2 g_Y \\ 0 & -\frac{1}{4}v^2 g_2 g_Y & \frac{1}{4}v^2 g_2^2 \end{bmatrix}$$

A **simultaneous diagonalization** of the kinetic energy and of the mass matrix can be obtained by a transformation

$$K^T M_{\mathrm{St}}^2 K$$

*GL(3) transformation that diagonalizes the kinetic term*

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A **simultaneous diagonalization** of the kinetic energy and of the mass matrix can be obtained by a transformation

$$\mathcal{O}^T (K^T M^2_{\rm St} K) \mathcal{O}$$

*Followed by an orthogonal transformation because it is nice!*

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A **simultaneous diagonalization** of the kinetic energy and of the mass matrix can be obtained by a transformation

$$M_D = \mathcal{R}^T (\mathcal{O}^T K^T M_{\rm St}^2 K \mathcal{O}) \mathcal{R}$$

*Finally, a rotation matrix to seal the deal!*

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A **simultaneous diagonalization** of the kinetic energy and of the mass matrix can be obtained by a transformation

$$\begin{aligned} M_D &= \mathcal{R}^T \mathcal{O}^T K^T M_{\rm St}^2 K \mathcal{O} \mathcal{R} \\ &= {\rm diag}(m_{\gamma'}^2, m_Z^2, 0) \end{aligned}$$

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In the basis $V^T = (C, B, A^3)$, the *non-diagonal kinetic term* is

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Three mass eigenstates:

- massive dark photon
- massive $Z$ boson
- massless photon

# The couplings in StSM

The modified SM interactions are:

$$\Delta\mathcal{L}'_{\rm SM} = \frac{g_2}{2\cos\theta}\bar{\psi}_f\gamma^\mu\Big[(v_f - \gamma_5 a_f)Z_\mu\Big]\psi_f + e\bar{\psi}_f\gamma^\mu Q_f A^\gamma_\mu\psi_f$$

$$v_f = \cos\psi[(1 + s_\delta\tan\psi\sin\theta)T_{3f} - 2\sin^2\theta(1 + s_\delta\csc\theta\tan\psi)Q_f],$$
$$a_f = \cos\psi(1 + s_\delta\tan\psi\sin\theta)T_{3f}.$$

The new interactions are:

$$\Delta\mathcal{L}_{\rm int} = \bar{D}\gamma^\mu(g_{Z'}Z'_\mu + g_Z Z_\mu + g_\gamma A^\gamma_\mu)D + \frac{g_2}{2\cos\theta}\bar{\psi}_f\gamma^\mu(v'_f - \gamma_5 a'_f)Z'_\mu\psi_f.$$

$$v'_f = -\cos\psi[(\tan\psi - s_\delta\sin\theta)T_{3f} - 2\sin^2\theta(-s_\delta\csc\theta + \tan\psi)Q_f],$$
$$a'_f = -\cos\psi(\tan\psi - s_\delta\sin\theta)T_{3f}.$$

$$\tan\phi = -\sinh\delta,\quad \tan\theta = \frac{g_Y}{g_2}\cosh\delta\cos\phi,\ \tan 2\psi = \frac{2m_Z^2\sin\theta\tan\phi}{m_{Z'}^2 - m_Z^2 + (m_{Z'}^2 + m_Z^2 - m_W^2)\tan^2\phi}$$

# Construction of the 95% CL upper limit on kinetic mixing

- For each signal mass hypothesis, we construct a binned likelihood in the reconstructed invisible mass

- We then extract a 95% CL upper limit on the kinetic mixing

- The likelihood is a product of independent Poisson terms over $m_{\mathrm{inv}}$ bins:

$$\mathcal{L}(\delta) = \prod_{i=1}^{N_{\mathrm{bins}}} \mathrm{Pois}(n_i \mid \lambda_i(\delta)) \,, \qquad \lambda_i(\delta) = B_i + S_i(\delta),$$

where $n_i$ are **observed** (Asimov) counts, $B_i$ are **background expectations** and $S_i$ is the **signal expectation** in bin $i$

# Construction of the 95% CL upper limit on kinetic mixing

- The signal yield is modeled as

$$S_i(\delta) \approx \alpha_i\,\delta + \beta_i\,\delta^2, \qquad S_i(0) = 0.$$

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Accounts for interference with SM

Pure BSM

# Construction of the 95% CL upper limit on kinetic mixing

- The signal yield is modeled as

$$S_i(\delta) \approx \alpha_i\,\delta + \beta_i\,\delta^2, \qquad S_i(0) = 0.$$

- To compute expected limits, we use a **background-only Asimov** dataset,

$$n_i = B_i.$$

- This produces the median expected limit under the background-only hypothesis

# Construction of the 95% CL upper limit on kinetic mixing

- **For each mass point:**

1. Group available couplings

2. Determine BDT cuts that optimizes $Z_A$

$$Z_A = \sqrt{2\left[(S+B)\ln\left(1+\frac{S}{B}\right)-S\right]}$$

3. Build $m_{\text{inv}}$ background and signal histograms

- Fit $\alpha_i$ and $\beta_i$ for each bin using a least-squares fit per bin

# Construction of the 95% CL upper limit on kinetic mixing

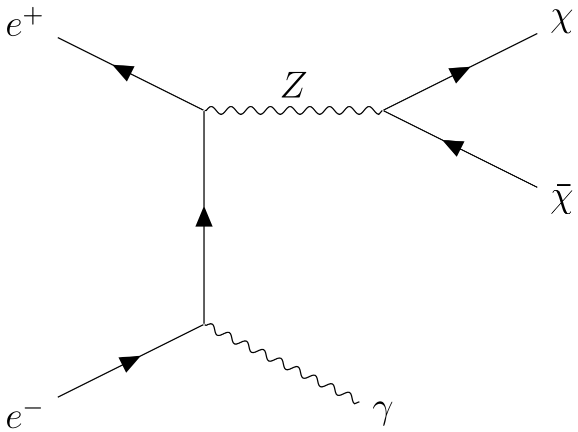
- **For each bin *i* :**

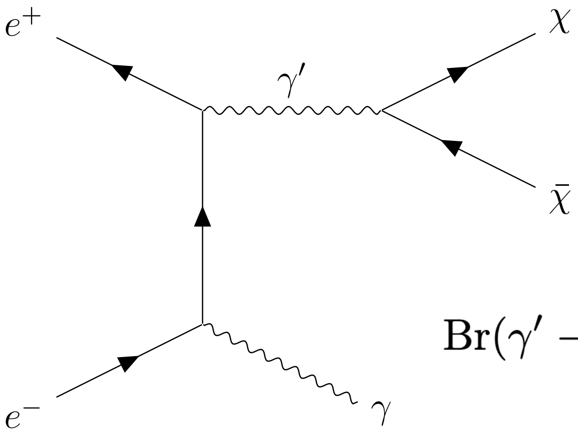
1. Create observable $n_i$
2. Get $B_i$, $\alpha_i$ and $\beta_i$
3. Define the Poisson mean (**expected**): $\lambda_i(g) = B_i + \alpha_i\,\delta + \beta_i\,\delta^2$
4. Create the term $P(n_i \mid \lambda_i)$ and add it to the product likelihood
5. The resulting likelihood

$$\mathcal{L}(\log\delta) = \prod_{i=1}^{N_{\text{bins}}} \text{Pois}\Big(n_i \mid \lambda_i\Big(\delta = 10^{\log\delta}\Big)\Big)$$

6. We then find the test statistic $q(\delta) = -2\ln\lambda(\delta) = -2\ln\dfrac{\mathcal{L}(\delta)}{\mathcal{L}(\hat{\delta})}$

# The signal: Invisible decay of a Stueckelberg dark photon




$$\mathrm{Br}(\gamma' \to \chi\bar{\chi}) \approx 1$$

FeynRules $\xrightarrow{\text{UFO}}$ MG5 → PY8 → Delphes

**FeynRules**: Model implementation

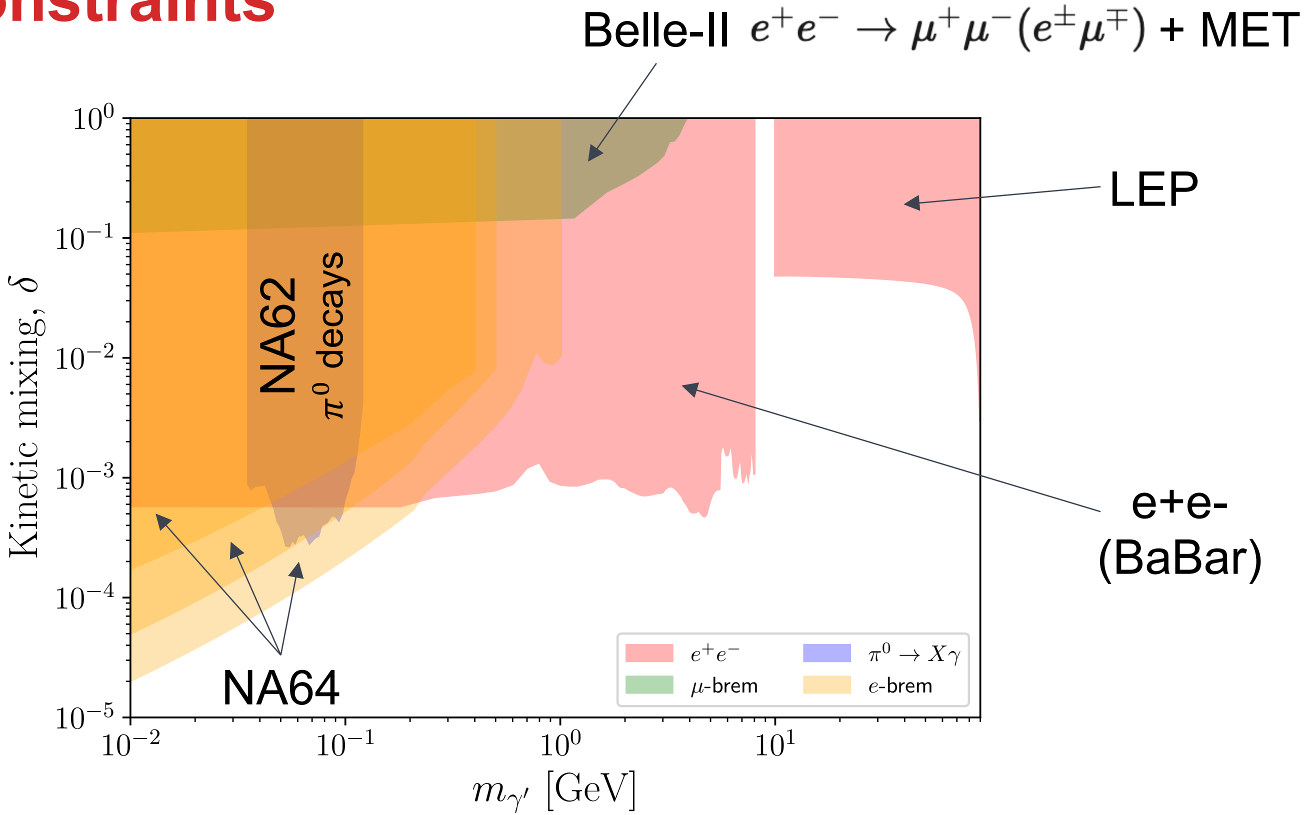
**MG5**: Monte Carlo Simulation of e+e- collisions (*Including ISR*)

**PY8**: Showering / hadronization

**Delphes**: IDEA detector simulation

# Current constraints

$$\mathrm{Br}(\gamma' \to \chi\bar{\chi}) \approx 1$$



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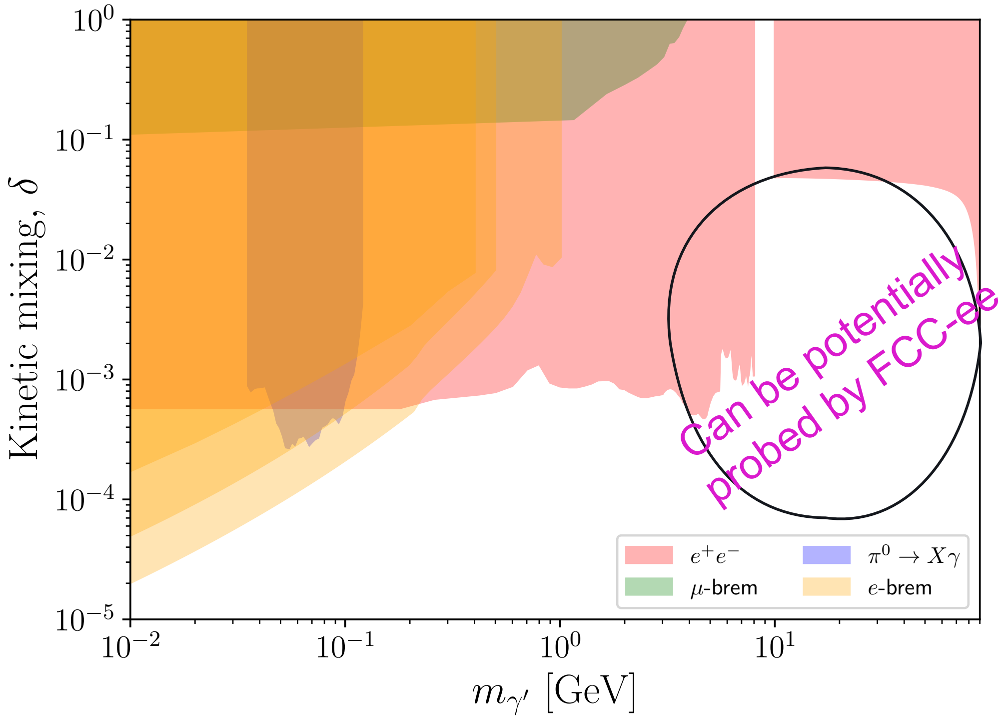
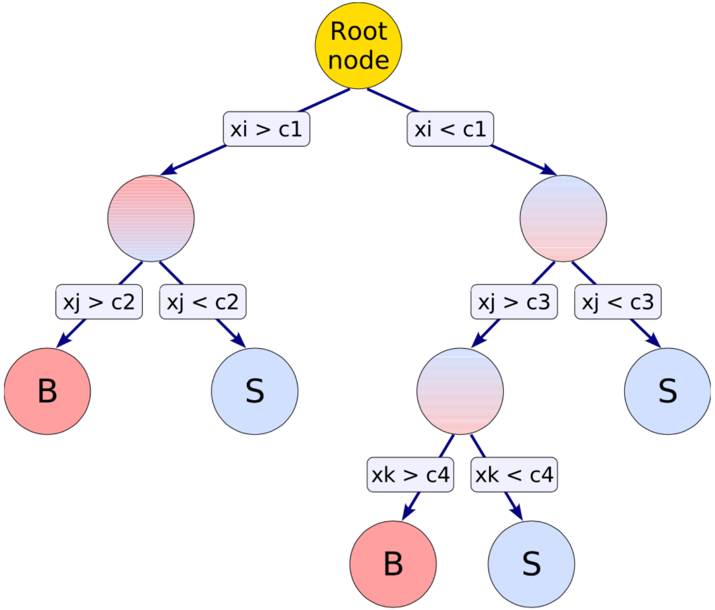
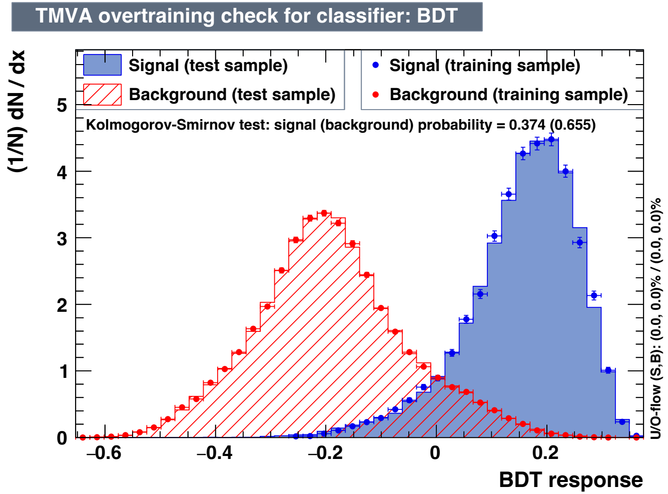
$$\mathrm{Br}(\gamma' \to \chi\bar{\chi}) \approx 1$$



# MACHINE LEARNING TECHNIQUES

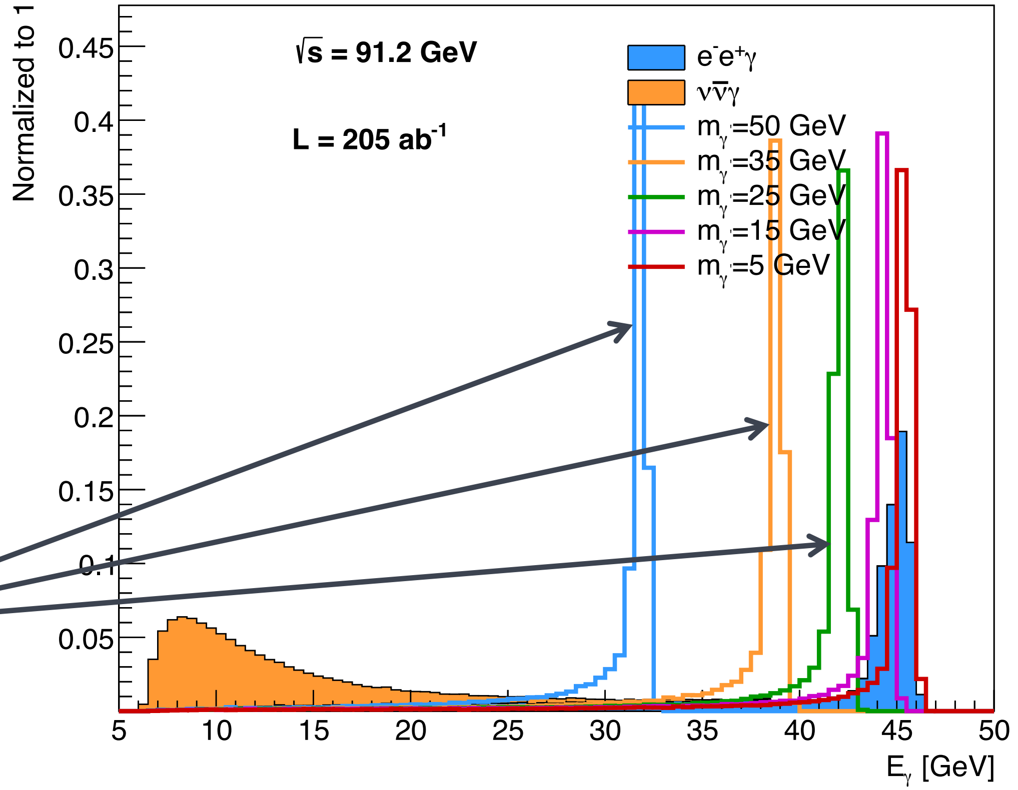
## Boosted Decision Tree (BDT)
## Deep Neural Network, …



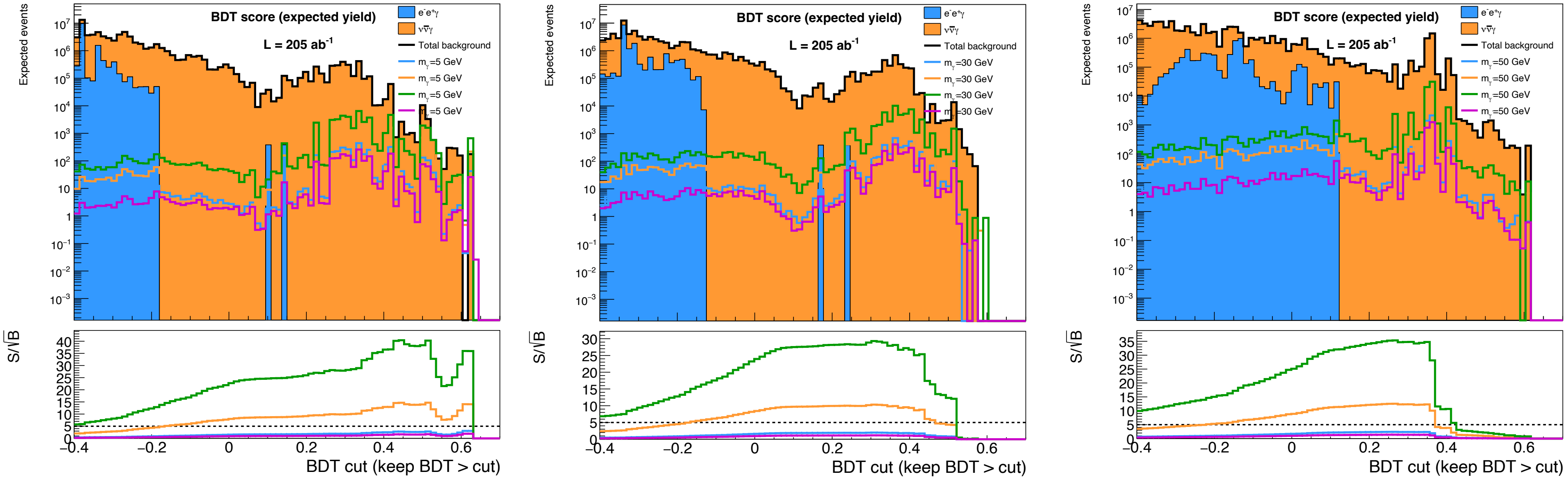



# Photon recoil energy

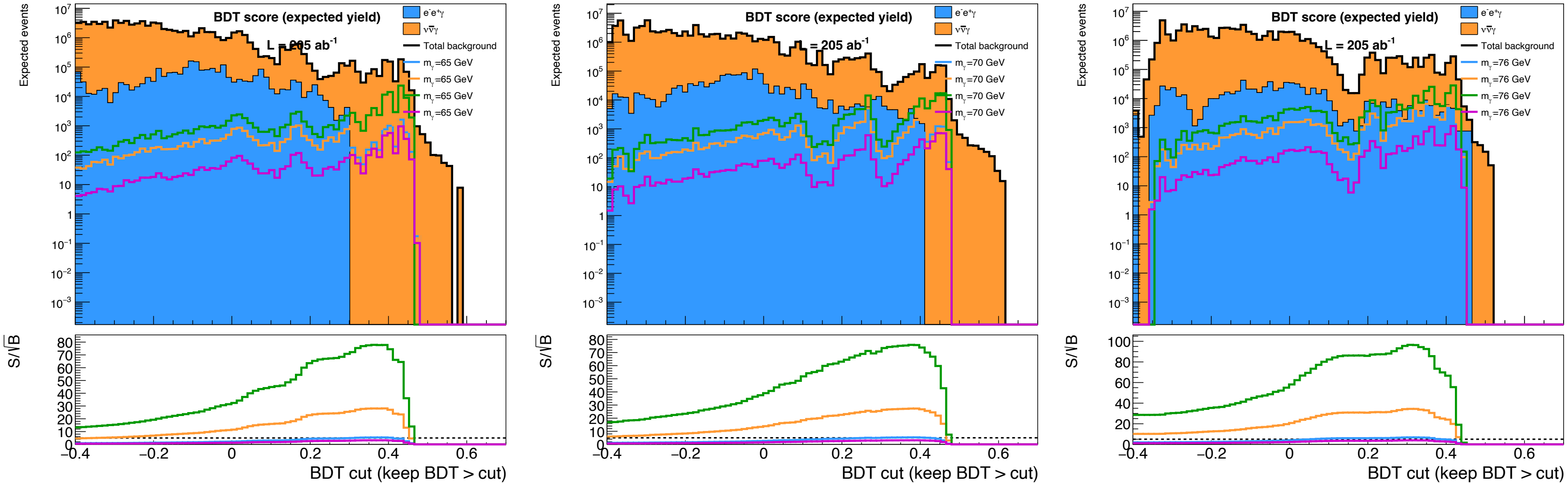
$$E_{\mathrm{rec}} = \frac{s - m_{\gamma'}^2}{2\sqrt{s}}$$



# BDT score distribution

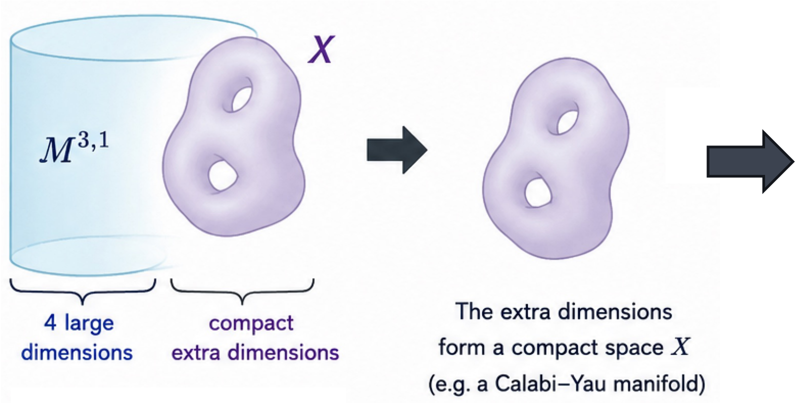


# BDT score distribution



# The origin of the U(1) gauge groups

Hidden U(1) gauge groups are ubiquitous in string and supergravity theories




Gauge fields propagating in these curled up dimensions manifest as massive or massless vector bosons in 4D

$$U(N) \to SU(N) \times U(1)$$

Not charged under the SM gauge group

Mixing of gauge fields

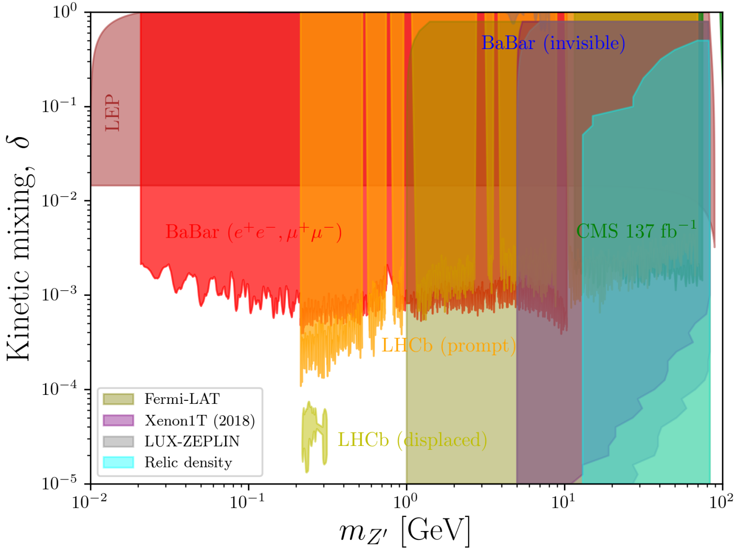
The Lagrangian: $\mathcal{L} \supset -\frac{1}{4}F_{a\mu\nu}F_a^{\mu\nu} - \frac{1}{4}F_{b\mu\nu}F_a^{\mu\nu} + \frac{\delta}{2}F_{a\mu\nu}F_b^{\mu\nu}$

- The dark photon can have a range of masses: a few MeV to several TeV
- Can be directly and indirectly produced at the LHC

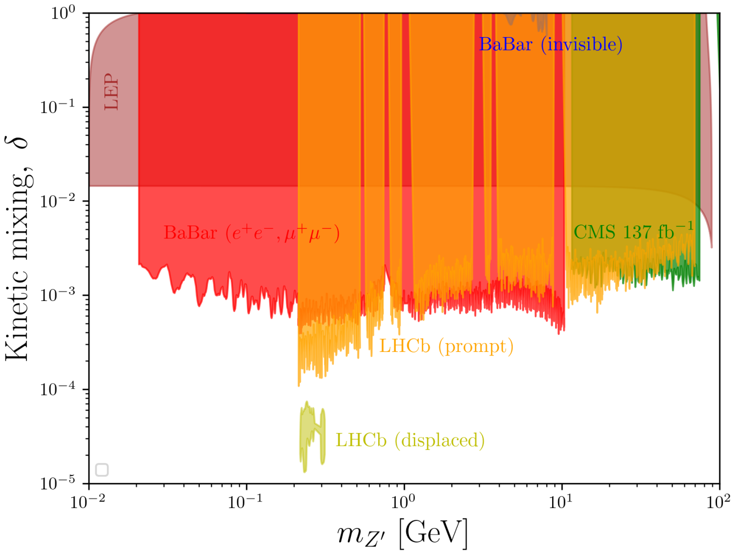
# Current constraints on StSM

Dark photon decaying into **visible final states** (SM fermions)

With DM constraints




Agnostic to DM constraints

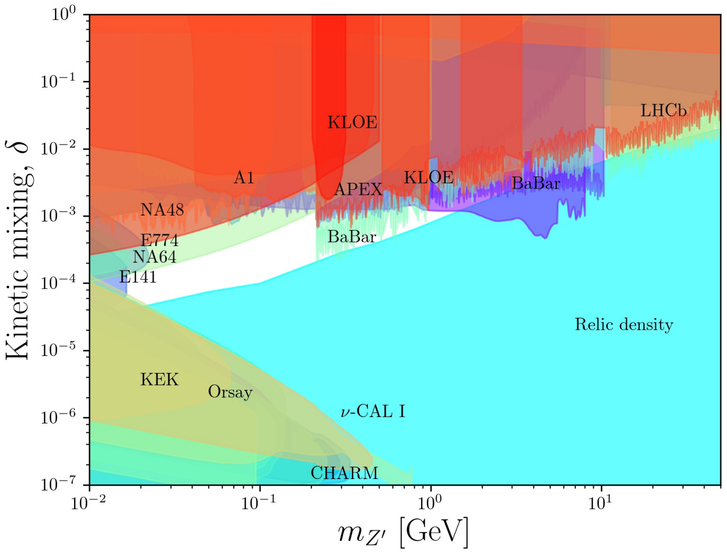



**AA**, M.M. Altakach, M. Klasen, P. Nath and Z.Y. Wang, JHEP **03**, 182 (2023)

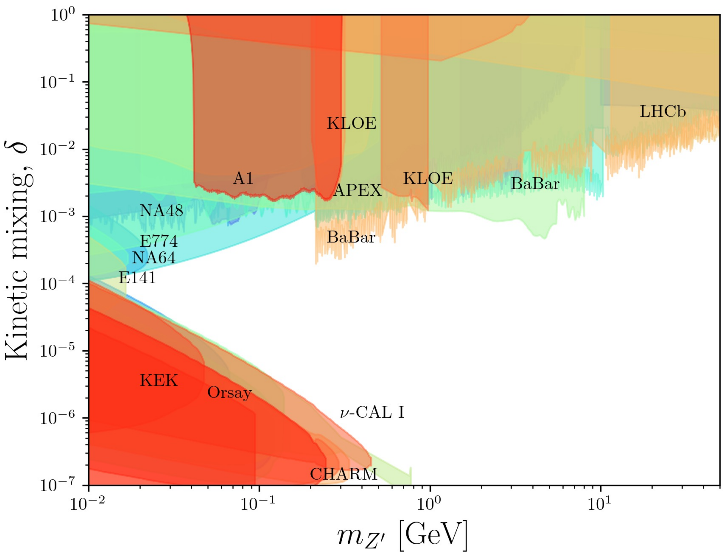
# Current constraints on StSM

Dark photon decaying into **visible and invisible final states** (SM fermions)

With DM constraints




Agnostic to DM constraints




**AA**, M.M. Altakach, M. Klasen, P. Nath and Z.Y. Wang, JHEP **03**, 182 (2023)

## Stueckelberg mass terms

The Stueckelberg mass terms

$$\frac{1}{2}(\partial_\mu \sigma + m_1 C_\mu + m_2 B_\mu)^2$$

are invariant under the $U(1)_X \times U(1)_Y$ gauge transformations. [5]

---

$U(1)_Y$ gauge transformation:

$$\delta_Y B_\mu = \partial_\mu \lambda_Y, \quad \delta_Y C_\mu = 0, \quad \delta_Y \sigma = -m_2 \lambda_Y.$$

---

$U(1)_X$ gauge transformation:

$$\delta_X B_\mu = 0, \quad \delta_X C_\mu = \partial_\mu \lambda_X, \quad \delta_X \sigma = -m_1 \lambda_X.$$

[5] The Stueckelberg mechanism can be viewed as the $U(1)$ Higgs mechanism with the Higgs boson mass taken to be infinity.

Earlier iteration
(February 2026)

# Preselection, BDT training

- **Generator-level cuts:** $p_T^\gamma > 5.0$ GeV and $|\eta| < 2.6$ G. Polesello, [arXiv:2511.23337 [hep-ph]]
- **Preselection criteria:** Electron veto, Muon veto
  At least one photon in an event
- Reconstruction of invisible mass: $p_{\text{invisible}} = p_{\text{initial}} - p_\gamma$
  $$m_{\text{recoil}} = \sqrt{p^2_{\text{invisible}}}$$
  Loop over all photons in an event and select the one that gives a recoil mass closest to the true mass
- We train a BDT using the kinematic variables:
  $$E_T^{\text{miss}}, \quad \frac{m_{\text{recoil}}}{p_T^\gamma}, \quad \Delta R(\vec{p}_{\text{recoil}}, \vec{p}_\gamma), \quad p_T^\gamma \text{ (leading)}$$
- Training done using **one BDT** on all signal and background events

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- We train a BDT using the kinematic variables:

Ranks highest in discriminatory power

$$E_T^{\text{miss}}, \quad \frac{m_{\text{recoil}}}{p_T^\gamma}, \quad \Delta R(\vec{p}_{\text{recoil}}, \vec{p}_\gamma), \quad p_T^\gamma \text{ (leading)}$$

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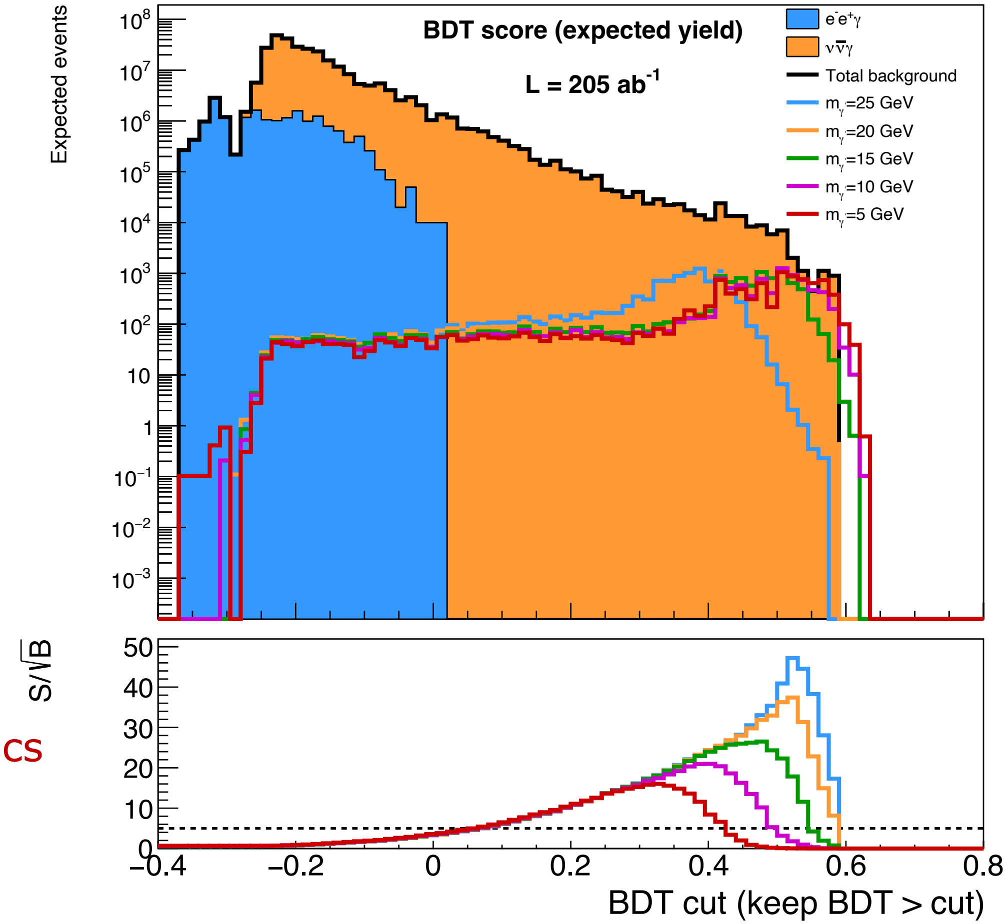
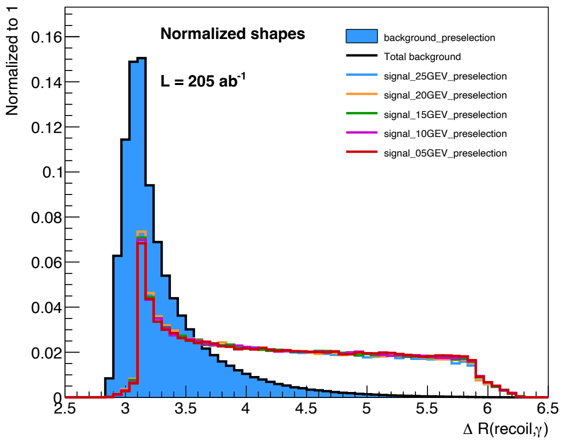
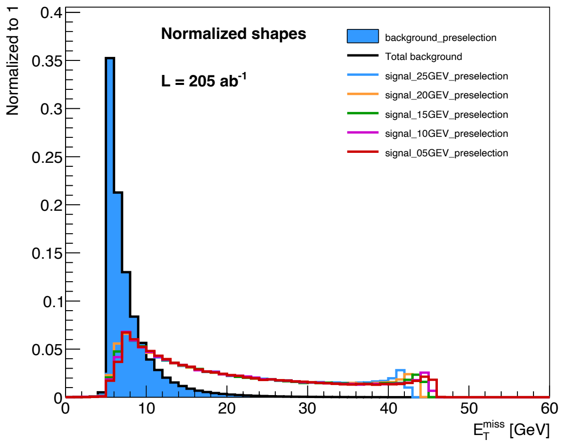
# Results (BDT score)

No systematics included




Normalized shapes
L = 205 ab$^{-1}$
background_preselection
Total background
signal_25GEV_preselection
signal_20GEV_preselection
signal_15GEV_preselection
signal_10GEV_preselection
signal_05GEV_preselection
Normalized to 1
Δ R(recoil,γ)


Normalized shapes
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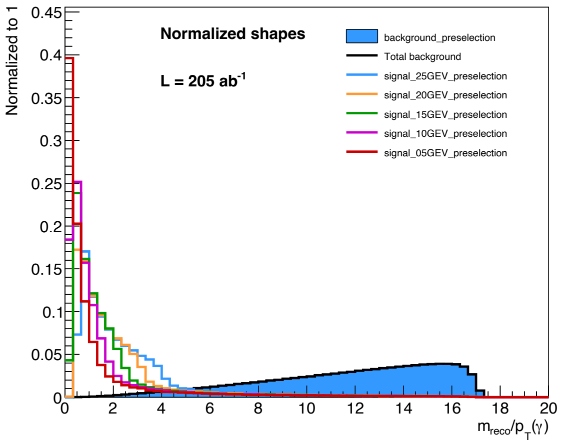
Total background
signal_25GEV_preselection
signal_20GEV_preselection
signal_15GEV_preselection
signal_10GEV_preselection
signal_05GEV_preselection
Normalized to 1
$E_T^{miss}$ [GeV]


Normalized shapes
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signal_25GEV_preselection
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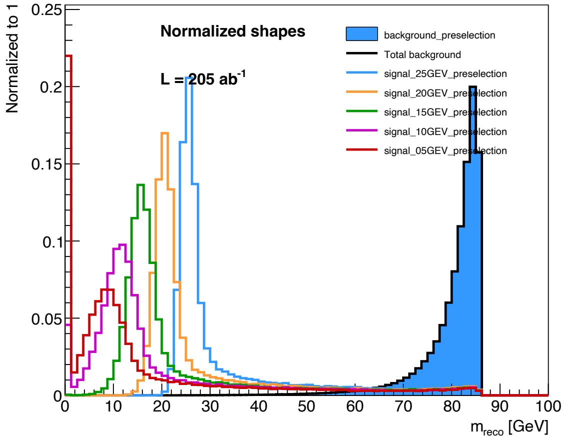
signal_10GEV_preselection
signal_05GEV_preselection
Normalized to 1
$m_{reco}/p_T(\gamma)$


Normalized shapes
L = 205 ab$^{-1}$
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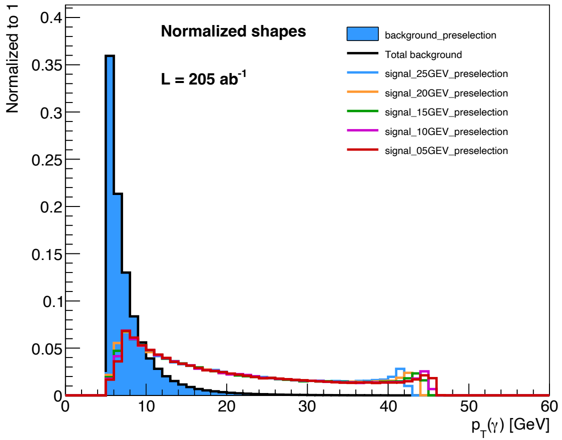
Total background
signal_25GEV_preselection
signal_20GEV_preselection
signal_15GEV_preselection
signal_10GEV_preselection
signal_05GEV_preselection
Normalized to 1
$m_{reco}$ [GeV]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signal_25GEV_preselection
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signal_10GEV_preselection
signal_05GEV_preselection
Normalized to 1
$p_T(\gamma)$ [GeV]