On-shell amplitudes and EFT

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$$\mathcal{L}_{\mathrm{EFT}} = \mathcal{L}_{\mathrm{SM}} + \sum_i \frac{1}{\Lambda^{d_i - 4}} c_i \mathcal{O}_i$$

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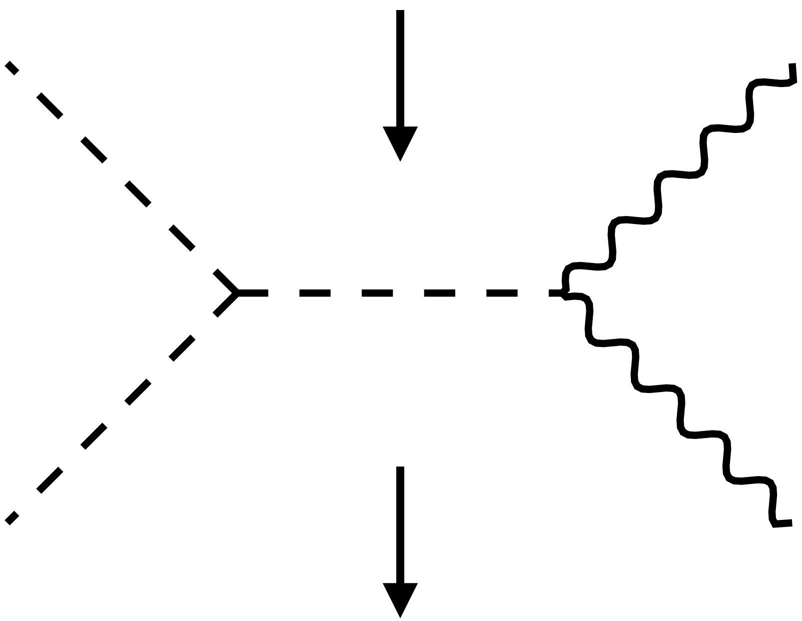
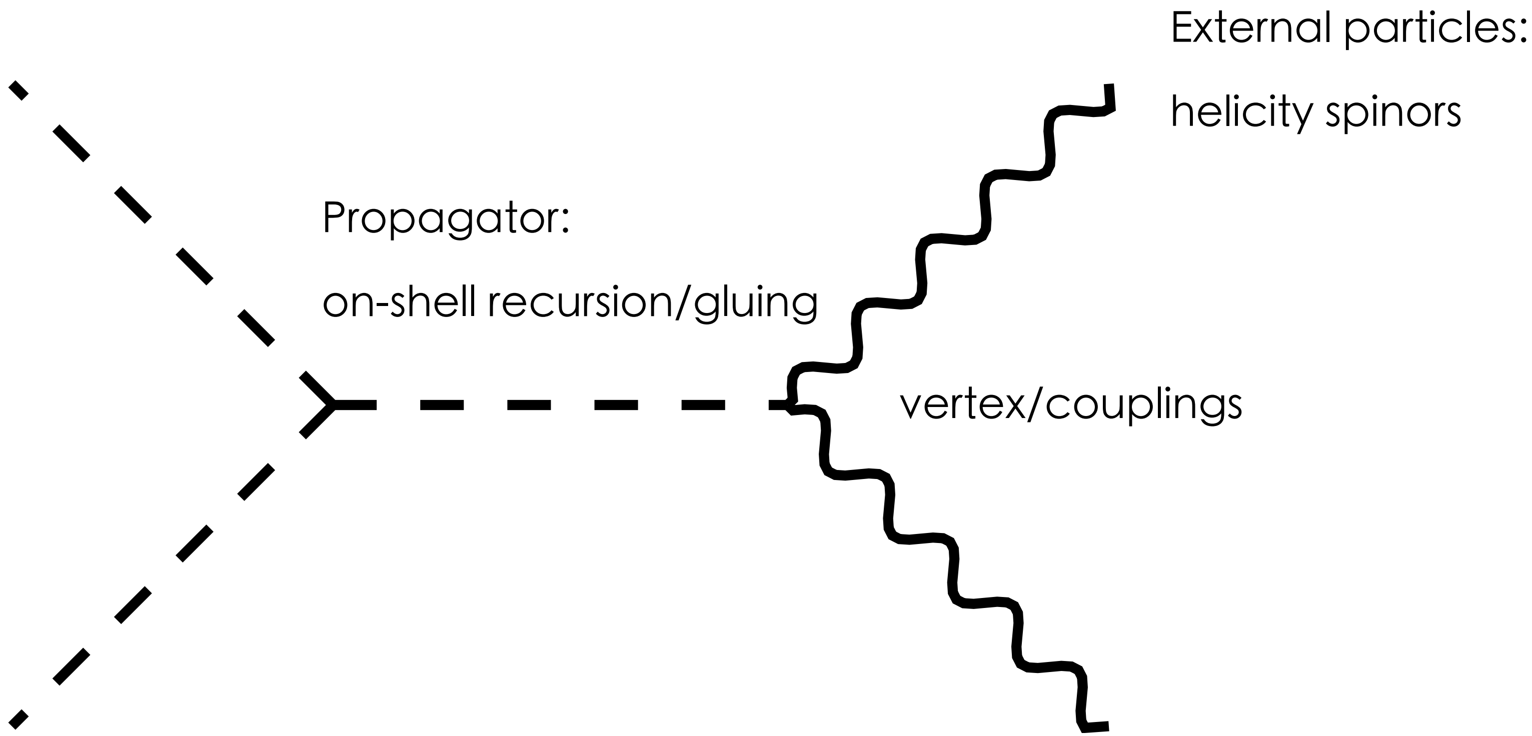


$$\mathcal{A}_{\mathrm{EFT}} = \mathcal{A}_{\mathrm{SM}} + \sum_i \frac{1}{\Lambda^{d_i-4}} c_i \mathcal{A}_i$$

- Amplitude is related to the physical observables, building from external physical states.



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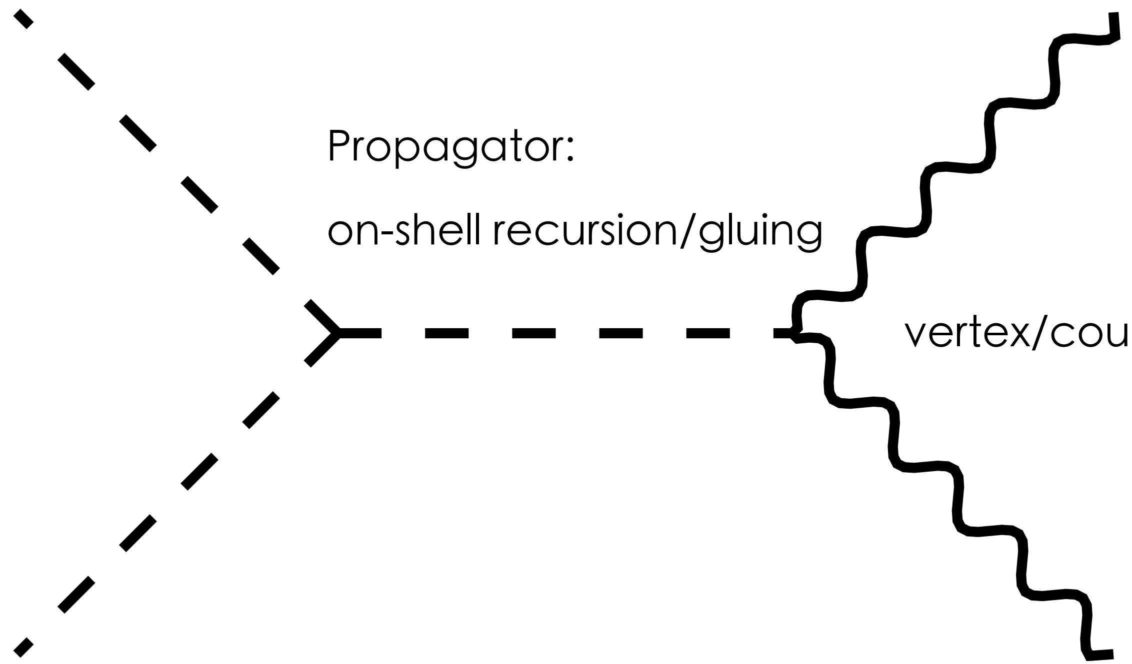


- What independent amplitudes are allowed by Lorentz invariance, little-group covariance, locality, and symmetry?

Massless particle

$$p_i^2 = \det(p^i_\mu \sigma^\mu_{\alpha\dot{\alpha}}) = 0 \xrightarrow{\text{rank-1}} p_{\alpha\dot{\alpha}} \equiv p^i_\mu \sigma^\mu_{\alpha\dot{\alpha}} = i\rangle_\alpha [i_{\dot{\alpha}}$$

- The momentum is invariant under the transformation.

$$h = -\frac{1}{2} \qquad i\rangle \to e^{+i\phi/2} i\rangle, \quad i] \to e^{-i\phi/2} i] \qquad h = \frac{1}{2}$$

- The little-group U(1) transformation can be understood as a rotation around the direction of motion.
- A spin-s massless particle can be built from 2s helicity spinors
- The spinor structure must have the correct little-group weight for the external particles.

Massless 3-point amplitudes

- The combination of little group scaling and locality uniquely fixes the massless 3-pt amplitudes structure. In terms of complex momenta,

$$\mathcal{M}_3(h_1, h_2, h_3) \propto \begin{cases} [12]^{+h_1+h_2-h_3}\,[23]^{-h_1+h_2+h_3}\,[13]^{+h_1-h_2+h_3}\,, & \text{for } h_1+h_2+h_3>0 \\ \langle 12\rangle^{-h_1-h_2+h_3}\langle 23\rangle^{+h_1-h_2-h_3}\langle 13\rangle^{-h_1+h_2-h_3}\,, & \text{for } h_1+h_2+h_3<0 \end{cases}$$

One example: massless fermion + massless photon $\mathcal{A}_3(1_f^+\ 2_{\bar{f}}^-\ 3_\gamma^+) = e\dfrac{[13]^2}{[12]}$

Massive particle

[Arkani-Hamed, Huang, Huang, 1709.04891]

- For massive particles, the little group becomes SU(2): $i^J] \equiv \boldsymbol{i}], \quad i^J\rangle \equiv \boldsymbol{i}\rangle$

$$p_i \cdot p_i = \det(p^i_\mu \sigma^\mu_{\alpha\dot\alpha}) = m^2 \longrightarrow p_{\alpha\dot\alpha} = i^J\rangle_\alpha [i_{J\dot\alpha} = i^1\rangle_\alpha [i_{1\dot\alpha} + i^2\rangle_\alpha [i_{2\dot\alpha}$$

$$p^\mu = (E,0,0,P) \longrightarrow p^\mu = k^\mu + q^\mu$$

$$k^\mu = (\frac{E+P}{2}, 0, 0, \frac{E+P}{2})$$

$$q^\mu = (\frac{E-P}{2}, 0, 0, -\frac{E-P}{2})$$

$$k_{\alpha\dot\alpha} = i^2\rangle_\alpha [i_{2\dot\alpha} \qquad q_{\alpha\dot\alpha} = i^1\rangle_\alpha [i_{1\dot\alpha}$$

$$i^2\rangle, i^1] \equiv k\rangle, k] \propto \sqrt{E} \qquad i^1\rangle, i^2] \equiv q\rangle, q] \propto m/\sqrt{E}$$

- For spin-s particle, need 2s symmetrized spinors, the spinors associated with massive particles are bolded, to imply symmetrization over little-group indices $V_i = \boldsymbol{i})^{\{I}\boldsymbol{i})^{J\}}$

$$V_i^+ = \boldsymbol{i})^1\boldsymbol{i})^1, \quad V_i^- = \boldsymbol{i})^2\boldsymbol{i})^2, \quad V_i^0 = \frac{\boldsymbol{i})^1\boldsymbol{i})^2 + \boldsymbol{i})^2\boldsymbol{i})^1}{\sqrt{2}}$$

Massive 3-point amplitudes

- Helicity is not a good quantum number, instead, each particle is a spin-s SU(2) multiplet.

- The number of independent structures can be obtained by irreducible representations in the addition of the three spins.

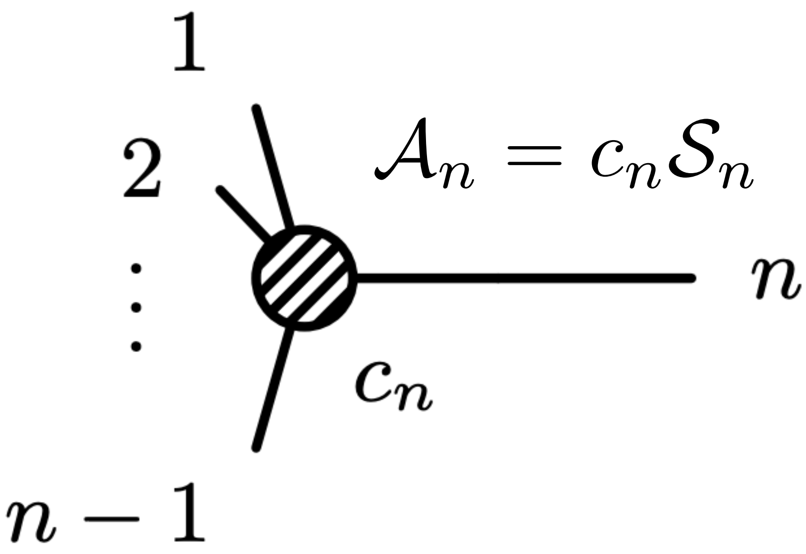
$$N_3(s_1,s_2,s_3)=\sum_{s=s_3-s_2}^{s_3+s_2}\sum_{m=|s-s_1|}^{s+s_1}1=(2s_1+1)(2s_2+1)-p(1+p),\quad p=\max(0,s_1+s_2-s_3)$$

- All EW massive 3-pt amplitudes are given in [Durieux, Kitahara, Shadmi, Weiss, 1909.10551]

One example $\psi^c\psi Z$

$$\mathcal{M}(\mathbf{1}_{\psi^c},\mathbf{2}_{\psi},\mathbf{3}_Z)=\frac{c^{RRR}_{\psi^c\psi Z}}{\bar{\Lambda}}[\mathbf{13}][\mathbf{23}]+\frac{c^{LR0}_{\psi^c\psi Z}}{m_Z}\langle\mathbf{13}\rangle[\mathbf{23}]+\frac{c^{RL0}_{\psi^c\psi Z}}{m_Z}[\mathbf{13}]\langle\mathbf{23}\rangle+\frac{c^{LLL}_{\psi^c\psi Z}}{\bar{\Lambda}}\langle\mathbf{13}\rangle\langle\mathbf{23}\rangle$$

Dimension



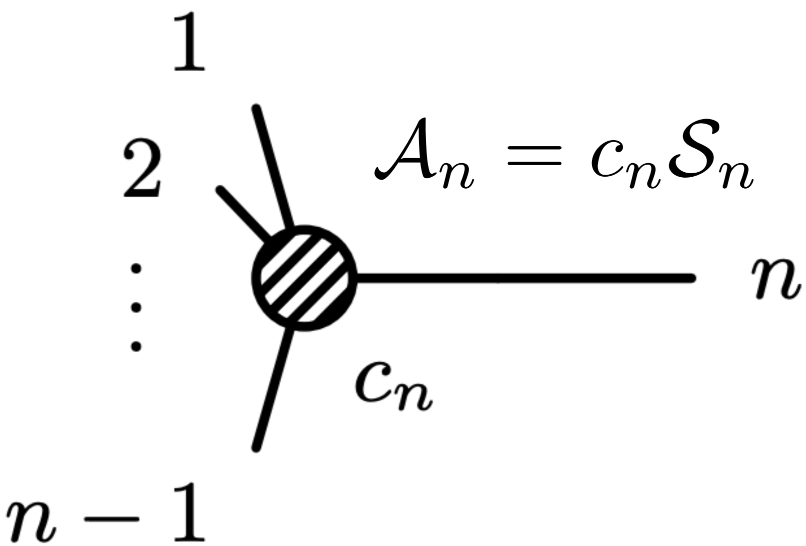
$$\mathcal{A}_n = c_n \mathcal{S}_n$$

Dimension of an n-pt amplitude

$$d[\mathcal{A}_n] = 4 - n$$

$$d[\mathcal{A}_n] = d[c_n] + d[\mathcal{S}_n]$$

Dimension



Dimension of an n-pt amplitude

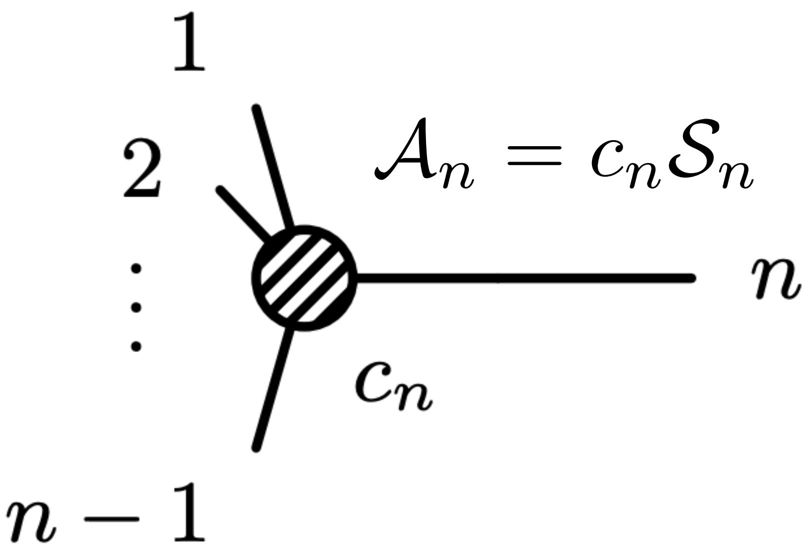
$$d[\mathcal{A}_n] = 4 - n$$

$$d[\mathcal{A}_n] = d[c_n] + d[\mathcal{S}_n]$$

Dimension of the operator

$$d[\mathcal{O}] = 4 - d[c_n] = n + d[\mathcal{S}_n]$$

Dimension



Dimension of an n-pt amplitude

$$d[\mathcal{A}_n] = 4 - n$$

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Dimension of the operator

$$d[\mathcal{O}] = 4 - d[c_n] = n + d[\mathcal{S}_n]$$

The minimum spinor dimension $\longrightarrow$ The minimal operator dimension

For non-factorizable amplitude $d[\mathcal{S}_n] = \sum_h |h_i| + n_{p,\text{ins.}}$

$V_1^+ V_2^+ V_3^- V_4^-$	$[12]^2 \langle 34 \rangle^2$	$d[\mathcal{O}] = 4 + 4 = 8$
$f_1^+ f_2^+ f_3^+ f_4^- f_5^- f_6^-$	$[12]\langle 56 \rangle [3p4\rangle$	$d[\mathcal{O}] = 6 + 3 + 1 = 10$

Non-factorizable contact terms: general structures

Little-group index

Mandelstam invariants

$$\sum_n S_n^{\{I\}} L_n \left[\frac{s_{ij}}{\Lambda^2} \right]$$

Spinor structures

Polynomials

Carrying little-group weights and polarizations

Only depends on scattering angle

Four-point massless amplitudes

[Durieux, Machado, 1912.08827]

Determined by 3-point amplitudes

↑

$$\mathcal{A}_4 = \mathcal{A}_4^{\text{fac}} + \mathcal{A}_4^{\text{non-fac.}}$$

Satisfy little-group scaling and locality.

Use Schouten identity

$$\langle ij\rangle\langle kl\rangle + \langle ik\rangle\langle lj\rangle + \langle il\rangle\langle jk\rangle = 0,$$

and momentum conservation to remove redundancies.

mult.	min. dim.	helicity conf.	spinor structures
3-pt	dim-3	sss	constant
	dim-4	f^+f^+s	$[12]$
	dim-5	v^+v^+s	$[12]^2$
		$v^+f^+f^+$	$[12][13]$
	dim-6	$v^+v^+v^+$	$[12][13][23]$
4-pt	dim-4	$ssss$	constant; s_{ij}; $s_{ij}s_{kl}$
	dim-5	f^+f^+ss	$[12](s_{ij})$
	dim-6	v^+v^+ss	$[12]^2(s_{ij})$
		$v^+f^+f^+s$	$[12][13](s_{ij})$
		$f^+f^+f^+f^+$	$[12][34](s_{ij})$, $[13][24](s_{ij})$
		$f^+f^+f^-f^-$	$[12]\langle 34\rangle(s_{ij})$
		f^+f^-ss	$[1(3-4)2\rangle(s_{ij})$
	dim-7	$v^+v^+v^+s$	$[12][13][23]$
		$v^+v^+f^+f^+$	$[12]^2[34]$, $[12]([14][23]+[13][24])$
		$v^+v^+f^-f^-$	$[12]^2\langle 34\rangle$
		$v^+f^+f^-s$	$[12][123\rangle$
		v^+sss	$[1231]$
		$f^+f^+f^+f^-$	$[12][3(1-2)4\rangle$
	dim-8	$v^+v^+v^+v^+$	$[12]^2[34]^2$, $[13]^2[24]^2$, $[14]^2[23]^2$
		$v^+v^+v^-v^-$	$[12]^2\langle 34\rangle^2$
		$v^+v^+f^+f^-$	$[12]^2[3(1-2)4\rangle$
		$v^+v^-f^+f^-$	$[13]\langle 24\rangle[1(3-4)2\rangle$
		v^+v^-ss	$[1(3-4)2\rangle^2$
		$v^+f^-f^-s$	$[1231]\langle 23\rangle$
	dim-9	$v^+v^+v^-s$	$[12]^2\langle 3123\rangle$
		$v^+v^-f^+f^+$	$[34][1(3-4)2\rangle^2$
	dim-10	$v^+v^+v^+v^-$	$[12]^2[3(1-2)4\rangle^2$

Four-point massive amplitudes

[Durieux, Kitahara, Machado, Shadmi, Weiss, 2008.09652]

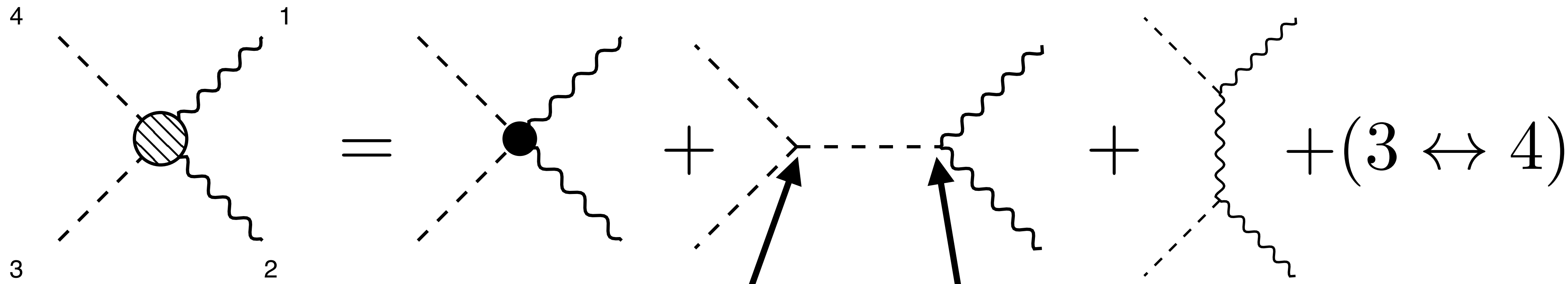
$$\mathcal{A}_4 = \mathcal{A}_4^{\text{fac}} + \mathcal{A}_4^{\text{non-fac.}}$$

↓

$$N_{\geq 4}(s_i) = \prod_i (2s_i + 1)$$

spins	n_{SCT}	n_{s}	hel. cat.	spinor structures	n_{perm}	$\min\{d_{\text{op}}\}$
$ssss$	1	1	(0000)	constant	1	4
$vsss$	$4 \to 3$	3	(0000)	$[\mathbf{121}\rangle, [\mathbf{131}\rangle$	1	5
			(+000)	$[\mathbf{1231}] \to [\mathbf{1231}] - \langle\mathbf{1231}\rangle$	$\not{2} \to 1$	7
$ffss$	4	4	(++00)	$[\mathbf{12}]$	2	5
			(+−00)	$[\mathbf{132}\rangle$	2	6
$vvss$	$10 \to 9$	9	(0000)	$[\mathbf{12}]\langle\mathbf{12}\rangle, [\mathbf{131}\rangle[\mathbf{232}\rangle$	1	4,6
			(+000)	$[\mathbf{12}][\mathbf{132}\rangle$	4	6
			(++00)	$[\mathbf{12}]^2$	2	6
			(+−00)	$[\mathbf{132}\rangle^2 \to [\mathbf{132}\rangle^2 - \langle\mathbf{132}]^2$	$\not{2} \to 1$	8
$ffvs$	$14 \to 12$	12	(++00)	$[\mathbf{12}]\{[\mathbf{313}\rangle, [\mathbf{323}\rangle\}$	2	6
			(+−00)	$[\mathbf{13}]\langle\mathbf{23}\rangle$	2	5
			(+++0)	$[\mathbf{13}][\mathbf{23}]$	2	6
			(++−0)	$[\mathbf{12}]\langle\mathbf{3123}\rangle \to \varnothing$	$\not{2} \to 0$	8
			(+−+0)	$[\mathbf{13}][\mathbf{312}\rangle$	4	7
$ffff$	18	16	(++++)	$[\mathbf{12}][\mathbf{34}], [\mathbf{13}][\mathbf{24}]$	2	6
			(++−−)	$[\mathbf{12}]\langle\mathbf{34}\rangle$	6	6
			(+++−)	$[\mathbf{12}][\mathbf{324}\rangle$	8	7
$vvvs$	$35 \to 27$	27	(0000)	$[\mathbf{12}][\mathbf{343}\rangle\langle\mathbf{12}\rangle, [\mathbf{13}][\mathbf{242}\rangle\langle\mathbf{13}\rangle, [\mathbf{23}][\mathbf{141}\rangle\langle\mathbf{23}\rangle$	1	5
			(+000)	$[\mathbf{12}][\mathbf{13}]\langle\mathbf{23}\rangle$	6	5
			(++00)	$[\mathbf{12}]^2\{[\mathbf{313}\rangle, [\mathbf{323}\rangle\}$	6	7
			(+−00)	$[\mathbf{13}][\mathbf{132}\rangle\langle\mathbf{23}\rangle$	$6 \to 4$	7
			(+++0)	$[\mathbf{12}][\mathbf{13}][\mathbf{23}]$	2	7
			(++−0)	$[\mathbf{12}]^2\langle\mathbf{3123}\rangle \to \varnothing$	$\not{6} \to 0$	9
$vvff$	$46 \to 38$	36	(00++)	$\langle\mathbf{12}\rangle \times \{[\mathbf{12}][\mathbf{34}], [\mathbf{13}][\mathbf{24}]\}$	2	5
			(00+−)	$\langle\mathbf{14}\rangle\langle\mathbf{231}][\mathbf{23}], \langle\mathbf{24}\rangle\langle\mathbf{132}][\mathbf{13}]$	2	6
			(0−++)	$\langle\mathbf{12}\rangle[\mathbf{34}]\langle\mathbf{241}] \to \langle\mathbf{12}\rangle[\mathbf{34}](\langle\mathbf{241}]/m_1 - \langle\mathbf{142}]/m_2)$	$\not{4} \to 2$	7
			(0+++)	$\langle\mathbf{132}] \times \{[\mathbf{12}][\mathbf{34}], [\mathbf{13}][\mathbf{24}]\}$	4	7
			(0++−)	$\langle\mathbf{14}\rangle[\mathbf{12}][\mathbf{23}]$	8	6
			(+++−)	$[\mathbf{12}]^2[\mathbf{314}\rangle$	4	8
			(++++)	$[\mathbf{12}] \times \{[\mathbf{12}][\mathbf{34}], [\mathbf{13}][\mathbf{24}]\}$	2	7
			(−+++)	$\langle\mathbf{1231}\rangle[\mathbf{23}][\mathbf{24}] \to \varnothing$	$\not{4} \to 0$	9
			(++−−)	$[\mathbf{12}]^2\langle\mathbf{34}\rangle$	2	7
			(+−−+)	$[\mathbf{14}][\mathbf{132}\rangle\langle\mathbf{23}\rangle \to [\mathbf{14}][\mathbf{132}\rangle\langle\mathbf{23}\rangle - [\mathbf{24}][\mathbf{231}\rangle\langle\mathbf{13}\rangle$	$\not{4} \to 2$	8
$vvvv$	$116 \to 85$	81	(0000)	$\{[\mathbf{12}][\mathbf{34}], [\mathbf{13}][\mathbf{24}]\} \times \{\langle\mathbf{12}\rangle\langle\mathbf{34}\rangle, \langle\mathbf{13}\rangle\langle\mathbf{24}\rangle\}$	1	4
			(+000)	$\{[\mathbf{12}][\mathbf{34}], [\mathbf{13}][\mathbf{24}]\} \times [\mathbf{142}\rangle\langle\mathbf{34}\rangle \to \cdots$	$\not{8} \to 6$	6
			(++00)	$\{[\mathbf{12}][\mathbf{34}], [\mathbf{13}][\mathbf{24}]\} \times [\mathbf{12}]\langle\mathbf{34}\rangle$	12	6
			(+−00)	$[\mathbf{13}][\mathbf{14}]\langle\mathbf{23}\rangle\langle\mathbf{24}\rangle$	12	6
			(+++0)	$\{[\mathbf{12}][\mathbf{34}], [\mathbf{13}][\mathbf{24}]\} \times [\mathbf{23}][\mathbf{134}\rangle$	8	8
			(++−0)	$[\mathbf{12}]^2\langle\mathbf{34}\rangle\langle\mathbf{324}] \to [\mathbf{12}]^2\langle\mathbf{34}\rangle(\langle\mathbf{324}]/m_4 - \langle\mathbf{423}]/m_3) \to \cdots$	$\not{24} \to \not{12} \to 5$	8
			(++++)	$[\mathbf{12}]^2[\mathbf{34}]^2, [\mathbf{12}][\mathbf{13}][\mathbf{24}][\mathbf{34}], [\mathbf{13}]^2[\mathbf{24}]^2$	2	8
			(+++−)	$[\mathbf{12}][\mathbf{13}][\mathbf{23}]\langle\mathbf{4124}\rangle \to \varnothing$	$\not{8} \to 0$	10
			(++−−)	$[\mathbf{12}]^2\langle\mathbf{34}\rangle^2$	6	8

Bottom-up construction to WWhh

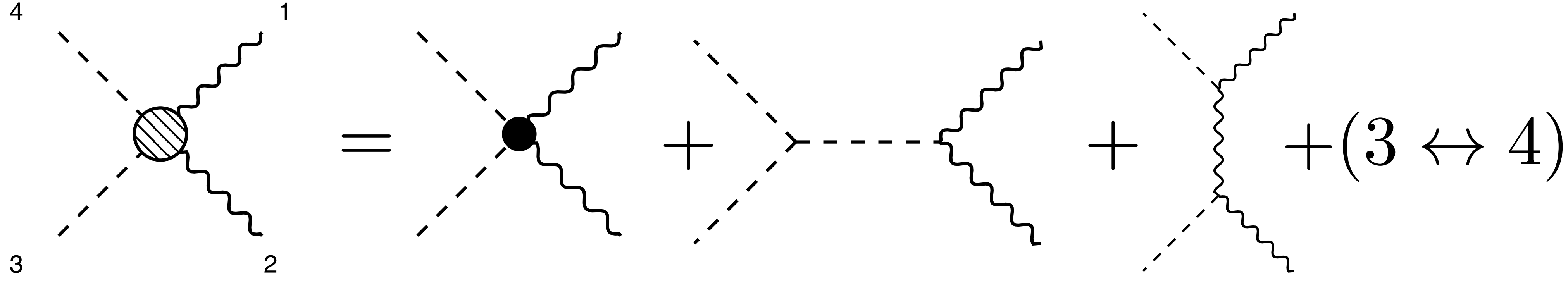


Three-point terms $\quad m_h C_{hhh}, \quad C^{00}_{WWh} \frac{[\mathbf{12}]\langle\mathbf{12}\rangle}{m_W}$

Four-point contact terms $\quad C^{00,\mathrm{fac}}_{WWhh} \frac{[\mathbf{12}]\langle\mathbf{12}\rangle}{m_W^2} + C^{00}_{WWhh} \frac{[\mathbf{12}]\langle\mathbf{12}\rangle}{\Lambda^2}$

→ determined by factorizable part

Bottom-up construction to WWhh



renormalizable | nonrenormalizable | s-channel

$$\mathcal{M}(W^+, W^-, h, h) = c^{00,\text{fac}}_{WWhh}\frac{[\mathbf{12}]\langle\mathbf{12}\rangle}{m_W^2} + c^{00}_{WWhh}\frac{[\mathbf{12}]\langle\mathbf{12}\rangle}{\Lambda^2} - \frac{m_h m_W c^{00}_{WWh} c_{hhh}}{s_{12} - m_h^2}\frac{[\mathbf{12}]\langle\mathbf{12}\rangle}{m_W^2}$$

$$+ \frac{{c^{00}_{WWh}}^2}{s_{13} - m_W^2}\left(\frac{\langle\mathbf{131}]\langle\mathbf{242}]}{2m_W^2} - [\mathbf{12}]\langle\mathbf{12}\rangle\right) + \frac{{c^{00}_{WWh}}^2}{s_{14} - m_W^2}\left(\frac{\langle\mathbf{141}]\langle\mathbf{232}]}{2m_W^2} - [\mathbf{12}]\langle\mathbf{12}\rangle\right)$$

t-channel | u-channel

WWhh: gauge invariance

Take high-energy limit for the transverse W $[\mathbf{12}]\langle\mathbf{12}\rangle \to [1_k 2_q]\langle 1_q 2_k\rangle$

$$i_q) = \frac{m_W}{(i_k \xi_i)} \xi_i) \,,\ \xi_i) \not\propto i_k) \quad ; \quad (i_k i_q) = m_W$$

$$\mathcal{M}(W^+, W^-, h, h) \to c^{00,fac}_{WWhh} \frac{[1\xi_2]\langle\xi_1 2\rangle}{[2\xi_2]\langle 1\xi_1\rangle} + \frac{{c^{00}_{WWh}}^2}{2}\left(\frac{[13\xi_1\rangle[\xi_2 42\rangle}{s_{13}[2\xi_2]\langle 1\xi_1\rangle} + \frac{[14\xi_1\rangle[\xi_2 32\rangle}{s_{14}[2\xi_2]\langle 1\xi_1\rangle}\right)$$

Take $(\xi_1, \xi_2) = (2, 1)$ $\quad \mathcal{M}(W^+, W^-, h, h) \to -\frac{{c^{00}_{WWh}}^2}{2}\frac{\langle 23\rangle\langle 24\rangle}{\langle 13\rangle\langle 14\rangle}$

or $(\xi_1, \xi_2) = (3, 3)$ $\quad \mathcal{M}(W^+, W^-, h, h) \to c^{00,\text{fac}}_{WWhh}\frac{\langle 23\rangle\langle 24\rangle}{\langle 13\rangle\langle 14\rangle}$

Gauge invariance requires $\quad c^{00,\text{fac}}_{WWhh} + \frac{{c^{00}_{WWh}}^2}{2} = 0$

WWhh: perturbative unitarity

Take high-energy limit for the longitudinal W

$$\mathcal{M}(W^+, W^-, h, h) \supset -\left(c^{00,\text{fac}}_{WWhh} + \frac{{c^{00}_{WWh}}^2}{2}\right)\frac{s_{12}}{2m_W^2}$$

Due to the gauge invariance $c^{00,\text{fac}}_{WWhh} + \frac{{c^{00}_{WWh}}^2}{2} = 0$

Gauge invariance guarantee perturbative unitarity

WWhh: non-factorizable contact terms

[Durieux, Kitahara, Machado, Shadmi, Weiss, 2008.09652]

spins	$n_{\rm SCT}$	n_s	hel. cat.	spinor structures	$n_{\rm perm}$	$\min\{d_{\rm op}\}$
$vvss$	$10 \to 9$	9	(0000)	$[\mathbf{12}]\langle\mathbf{12}\rangle, [\mathbf{131}\rangle[\mathbf{232}\rangle$	1	4,6
			(+000)	$[\mathbf{12}][\mathbf{132}\rangle$	4	6
			(++00)	$[\mathbf{12}]^2$	2	6
			(+−00)	$[\mathbf{132}\rangle^2 \to [\mathbf{132}\rangle^2 - \langle\mathbf{132}]^2$	$\not{2} \to 1$	8

$[\mathbf{132}\rangle^2 + \langle\mathbf{132}]^2$

is not independent

W^+W^-hh at E^2

(0000)	$[\mathbf{12}]\langle\mathbf{12}\rangle$
$(++00)$	$[\mathbf{12}]^2$
$(--00)$	$\langle\mathbf{12}\rangle^2$

Match to D6 SMEFT

$$C^{00}_{WWhh} = 4(C_{H\Box} - \frac{1}{4}C_{HD})$$

$$C^{\pm\pm}_{WWhh} = 4(C_{HW} \pm iC_{H\tilde{W}})$$

WWhh: non-factorizable contact terms

[Durieux, Kitahara, Machado, Shadmi, Weiss, 2008.09652]

spins	n_{SCT}	n_{s}	hel. cat.	spinor structures	n_{perm}	$\min\{d_{\mathrm{op}}\}$
$vvss$	$10 \to 9$	9	(0000)	$[\mathbf{12}]\langle\mathbf{12}\rangle, [\mathbf{131}\rangle[\mathbf{232}\rangle$	1	4,6
			(+000)	$[\mathbf{12}][\mathbf{132}\rangle$	4	6
			(++00)	$[\mathbf{12}]^2$	2	6
			(+−00)	$[\mathbf{132}\rangle^2 \to [\mathbf{132}\rangle^2 - \langle\mathbf{132}]^2$	$\not{2} \to 1$	8

$[\mathbf{132}\rangle^2 + \langle\mathbf{132}]^2$

is not independent

W^+W^-hh at E^3, E^4

(0000)	$[\mathbf{131}\rangle[\mathbf{232}\rangle + [\mathbf{141}\rangle[\mathbf{242}\rangle, \tilde{s}_{12}[\mathbf{12}]\langle\mathbf{12}\rangle$	2
$(++00)$	$\tilde{s}_{12}[\mathbf{12}]^2$	2
$(+-00)$	$[\mathbf{1(3-4)2}\rangle^2$	2

WWhh: non-factorizable contact terms

[Durieux, Kitahara, Machado, Shadmi, Weiss, 2008.09652]

spins	n_{SCT}	n_{s}	hel. cat.	spinor structures	n_{perm}	$\min\{d_{\mathrm{op}}\}$
$vvss$	$10 \to 9$	9	(0000)	$[\mathbf{12}]\langle\mathbf{12}\rangle,\ [\mathbf{131}\rangle[\mathbf{232}\rangle$	1	4,6
			(+000)	$[\mathbf{12}][\mathbf{132}\rangle$	4	6
			(++00)	$[\mathbf{12}]^2$	2	6
			(+−00)	$[\mathbf{132}\rangle^2 \to [\mathbf{132}\rangle^2 - \langle\mathbf{132}]^2$	$\not{2} \to 1$	8

$$[\mathbf{132}\rangle^2 + \langle\mathbf{132}]^2$$

is not independent

$$(+000) \qquad [\mathbf{12}][\mathbf{1}(\mathbf{3}-\mathbf{4})\mathbf{2}\rangle(\tilde{s}_{13} - \tilde{s}_{14}) \text{ at } E^5$$

Collider phenomenology

- In the 1990s, Lance Dixon and Yael Shadmi studied the tr(G^3) operator and found that its interference with the SM QCD amplitude vanishes for $gg \to gg$ processes. This orthogonality is lifted in $gg \to ggg$ processes [Dixon, Shadmi, hep-ph/9312363].

- Inspired by this work, we study the tr(W^3)operator in the process:

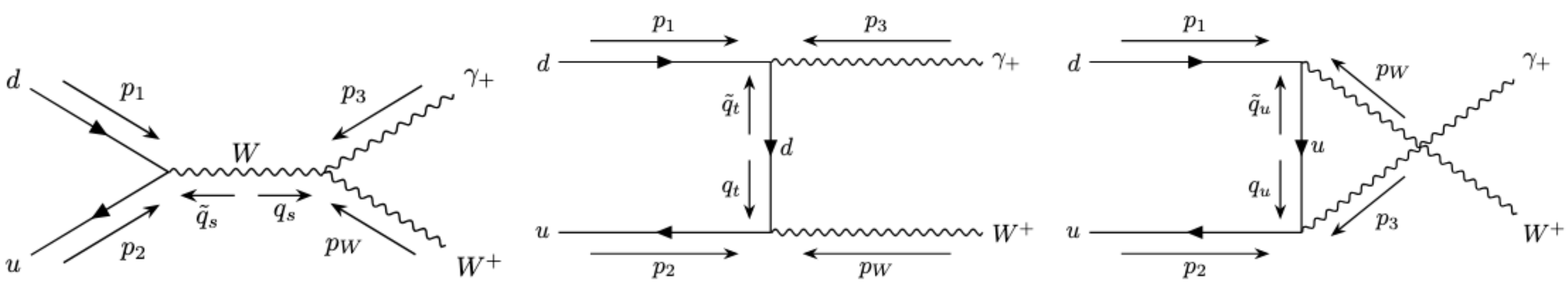
$$q\bar{q}' \to \gamma W,\, W \to \ell\nu$$

Collider phenomenology: Wγ production in pp collisions

The relevant production amplitude is $\mathcal{A}_4(d\bar{u}\gamma W^+)$

Here, we take all the particles as incoming

At SM



$$\mathcal{A}^{\rm SM}_{4,+}(d\bar{u}\gamma W) = -\frac{\sqrt{2}eg(Q_u s_{13} + Q_d s_{23})[13][23]}{(s_{12} - m_W^2)s_{13}s_{23}}\langle 1\mathbf{P_w}\rangle^2,$$

$$\mathcal{A}^{\rm SM}_{4,-}(d\bar{u}\gamma W) = -\frac{\sqrt{2}eg(Q_u s_{13} + Q_d s_{23})\langle 13\rangle\langle 23\rangle}{(s_{12} - m_W^2)s_{13}s_{23}}[2\mathbf{P_w}]^2,$$

$$\mathcal{A}^{\rm SM}_{4,++} = \mathcal{A}^{\rm SM}_{4,+}(1,1) = \frac{ieg}{3\sqrt{2}}\frac{m_W^2}{(s_{12} - m_W^2)}(3\cos\theta + 1)\cot\left(\frac{\theta}{2}\right),$$

$$\mathcal{A}^{\rm SM}_{4,--} = \mathcal{A}^{\rm SM}_{4,-}(2,2) = \frac{ieg}{3\sqrt{2}}\frac{m_W^2}{(s_{12} - m_W^2)}(3\cos\theta + 1)\tan\left(\frac{\theta}{2}\right),$$

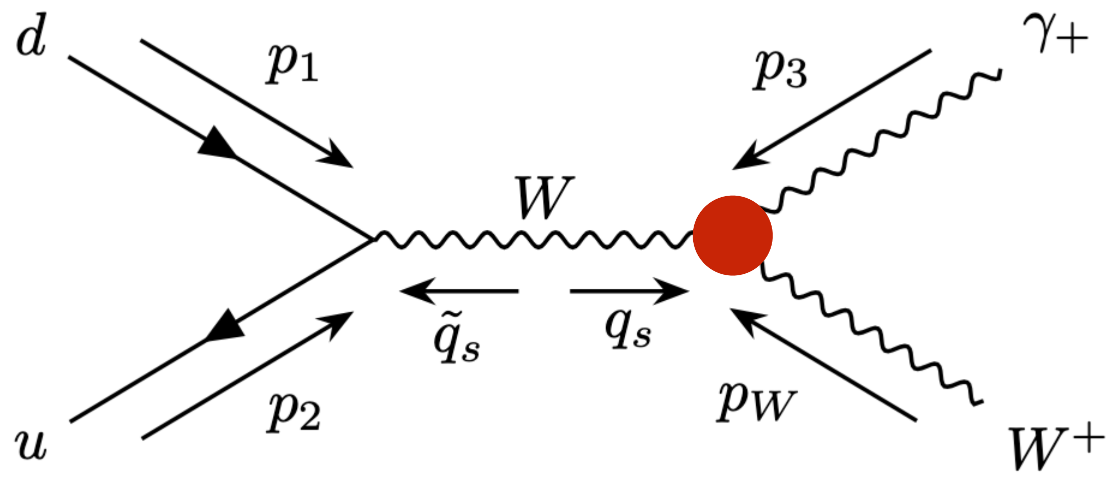
$$\mathcal{A}^{\rm SM}_{4,+0} = \frac{\mathcal{A}^{\rm SM}_{4,+}(1,2) + \mathcal{A}^{\rm SM}_{4,+}(2,1)}{\sqrt{2}} = \frac{ieg}{3}\frac{m_W\sqrt{s_{12}}}{(s_{12} - m_W^2)}(3\cos\theta + 1),$$

$$\mathcal{A}^{\rm SM}_{4,-0} = \frac{\mathcal{A}^{\rm SM}_{4,-}(1,2) + \mathcal{A}^{\rm SM}_{4,-}(2,1)}{\sqrt{2}} = \mathcal{A}^{\rm SM}_{4,+0},$$

$$\mathcal{A}^{\rm SM}_{4,+-} = \mathcal{A}^{\rm SM}_{4,+}(2,2) = \frac{ieg}{3\sqrt{2}}\frac{s_{12}}{(s_{12} - m_W^2)}(3\cos\theta + 1)\tan\left(\frac{\theta}{2}\right),$$

$$\mathcal{A}^{\rm SM}_{4,-+} = \mathcal{A}^{\rm SM}_{4,-}(1,1) = \frac{ieg}{3\sqrt{2}}\frac{s_{12}}{(s_{12} - m_W^2)}(3\cos\theta + 1)\cot\left(\frac{\theta}{2}\right),$$

Collider phenomenology: 4-point (D6 SMEFT)



$$\mathcal{O}_W = W^3 \text{ and } \mathcal{O}_{\tilde{W}} = \tilde{W}W^2$$

$$c_{6,\pm} = C_W \pm i C_{\tilde{W}}$$

$$\mathcal{A}^{\rm EFT}_{3,+}(WW\gamma) = -3\sqrt{2}s_W \frac{c_{6,+}}{\Lambda^2}[\mathbf{12}][\mathbf{1}3][\mathbf{2}3],$$

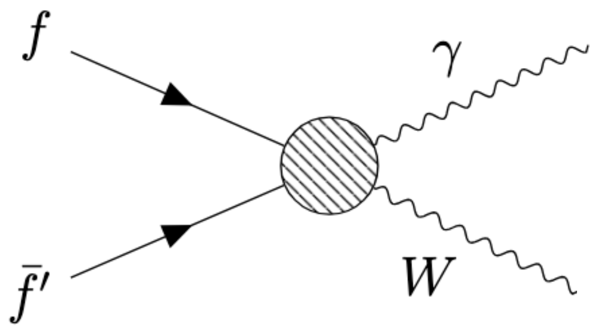
$$\mathcal{A}^{\rm EFT}_{3,-}(WW\gamma) = -3\sqrt{2}s_W \frac{c_{6,-}}{\Lambda^2}\langle\mathbf{12}\rangle\langle\mathbf{1}3\rangle\langle\mathbf{2}3\rangle$$

Gluing

$$\mathcal{A}^{\rm EFT_6}_{4,+}(d\bar{u}\gamma W) = 3\sqrt{2}g s_W \frac{c_{6,+}}{\Lambda^2}\frac{[2\mathbf{P_w}][3\mathbf{P_w}]\langle 12\rangle[23]}{s_{12}-m_W^2},$$

$$\mathcal{A}^{\rm EFT_6}_{4,-}(d\bar{u}\gamma W) = 3\sqrt{2}g s_W \frac{c_{6,-}}{\Lambda^2}\frac{\langle 1\mathbf{P_w}\rangle\langle 3\mathbf{P_w}\rangle[12]\langle 13\rangle}{s_{12}-m_W^2}.$$

Collider phenomenology: 4-point (D8 SMEFT)



- At dimension-8, we consider two operators

$$\mathcal{A}_{4,+}^{\mathrm{EFT}_8}(d\bar{u}\gamma W) = \frac{c_{8,+}}{\Lambda^4}[3\mathbf{P_w}]^2[2(\bar{\mathbf{P}}_{\mathbf{w}} - \bar{3})1\rangle,$$
$$\mathcal{A}_{4,-}^{\mathrm{EFT}_8}(d\bar{u}\gamma W) = \frac{c_{8,-}}{\Lambda^4}\langle 3\mathbf{P_w}\rangle^2[2(\bar{\mathbf{P}}_{\mathbf{w}} - \bar{3})1\rangle.$$

Collider phenomenology: angular distributions

The leading EFT contributions are from the terms with same photon and W helicities

$$\mathcal{A}^{\mathrm{EFT}}_{4,++} = i\,6\sqrt{2} g s_W\, c_{6,+} \left(\frac{E_{\mathrm{CM}}}{\Lambda}\right)^2 \sin\theta + i\,16\, c_{8,+} \left(\frac{E_{\mathrm{CM}}}{\Lambda}\right)^4 \sin\theta,$$

$$\mathcal{A}^{\mathrm{EFT}}_{4,--} = -i\,6\sqrt{2} g s_W\, c_{6,-} \left(\frac{E_{\mathrm{CM}}}{\Lambda}\right)^2 \sin\theta + i\,16\, c_{8,-} \left(\frac{E_{\mathrm{CM}}}{\Lambda}\right)^4 \sin\theta,$$

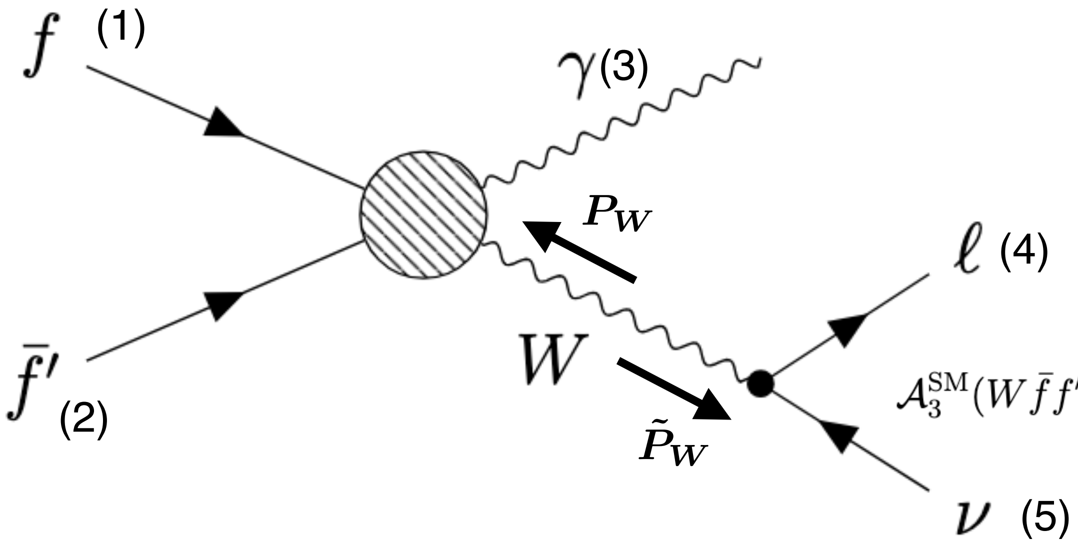
$$\mathcal{A}^{\mathrm{EFT}}_{4,+0} = i\,6 g s_W\, c_{6,+} \left(\frac{m_W E_{\mathrm{CM}}}{\Lambda^2}\right) \sin^2\left(\frac{\theta}{2}\right),$$

$$\mathcal{A}^{\mathrm{EFT}}_{4,-0} = i\,6 g s_W\, c_{6,-} \left(\frac{m_W E_{\mathrm{CM}}}{\Lambda^2}\right) \sin^2\left(\frac{\theta}{2}\right),$$

$$\mathcal{A}^{\mathrm{EFT}}_{4,+-} = \mathcal{A}^{\mathrm{EFT}}_{4,-+} = 0,$$

The leading interference comes from opposite W helicities $|\Delta h_W| = 2$

Collider phenomenology: 5-point

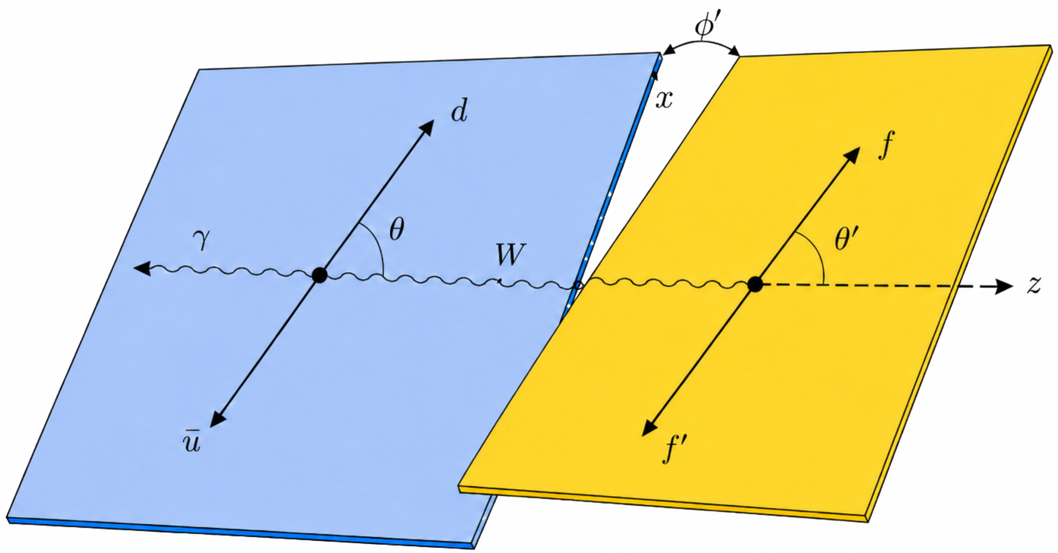


$$\mathcal{A}_3^{\rm SM}(W\bar{f}f') = g\frac{\langle 4\tilde{\mathbf{P}}_{\mathbf{w}}\rangle[5\tilde{\mathbf{P}}_{\mathbf{w}}]}{m_W}$$

The 5-pt amplitudes can be obtained by gluing the 4-pt and 3-pt ones

$$-\frac{1}{P_W^2-m_W^2}\left[\sum_{I,J,\tilde{I},\tilde{J}=1,2}\frac{\mathcal{A}_4^{IJ}(P_W)+\mathcal{A}_4^{JI}(P_W)}{2}\,\epsilon_{I\tilde{I}}\,\epsilon_{J\tilde{J}}\,\frac{\mathcal{A}_3^{\tilde{I}\tilde{J}}(\tilde{P}_W)+\mathcal{A}_3^{\tilde{J}\tilde{I}}(\tilde{P}_W)}{2}\right]_{P_W^2=m_W^2}$$

Collider phenomenology: 5-point



$$|\mathcal{A}^{\mathrm{SM}}_{5,\pm}|^2_{\mathrm{LO}} = \frac{\pi}{18} e^2 g^4 \frac{m_W}{\Gamma_W} (3\cos\theta + 1)^2 \tan^{\pm 2}\left(\frac{\theta}{2}\right) \left(\frac{1 \pm \cos\theta'}{2}\right)^2 ,$$

$$|\mathcal{A}^{\mathrm{SM}}_{5,\pm}|^2_{\mathrm{NLO}} = -\frac{\pi}{18} e^2 g^4 \left(\frac{m_W}{E_{\mathrm{CM}}}\right) \frac{m_W}{\Gamma_W} (3\cos\theta + 1)^2 \tan^{\pm 1}\left(\frac{\theta}{2}\right) \sin\theta' \left(\frac{1 \pm \cos\theta'}{2}\right)^2 \sin\phi' .$$

$$|\mathcal{A}^{\mathrm{int}_6}_{5,\pm}|^2_{\mathrm{LO}} = 2\pi e g^4 s_W c_{6,\pm} \left(\frac{E_{\mathrm{CM}}}{\Lambda}\right)^2 \frac{m_W}{\Gamma_W} (3\cos\theta + 1) \left(\frac{1 \mp \cos\theta}{2}\right) \times \sin^2\theta' \{\mp \mathrm{Re}[c_{6,\pm}] \cos(2\phi') + \mathrm{Im}[c_{6,\pm}] \sin(2\phi')\},$$

Collider phenomenology: 5-point

- Proton contains both d and ubar

$$\frac{d\sigma}{d\tau d\cos\theta d\cos\theta' d\phi'} = \frac{1}{(64\pi^2)^2\pi s}\sum_{h=\pm}\left\{F_d|\mathcal{A}_{5,h}|^2(\theta,0) + F_{\bar{u}}|\mathcal{A}_{5,h}|^2(\pi-\theta,\pi)\right\}$$

$$F_d(x_{\min},\tau) \equiv \int_{x_{\min}}^1 \frac{dx}{x} f_{1,d}(x) f_{2,\bar{u}}(\tau/x),$$

$$F_{\bar{u}}(x_{\min},\tau) \equiv \int_{x_{\min}}^1 \frac{dx}{x} f_{1,\bar{u}}(x) f_{2,d}(\tau/x),$$

- We define the forward and backward distributions

$$\int_0^y d\cos\theta \frac{d\sigma}{d\tau d\cos\theta d\cos\theta' d\phi'} \equiv \frac{1}{(64\pi^2)^2 s}\mathcal{F}(x_{\min},\tau,\theta',\phi',y),$$

$$\int_{-y}^0 d\cos\theta \frac{d\sigma}{d\tau d\cos\theta d\cos\theta' d\phi'} \equiv \frac{1}{(64\pi^2)^2 s}\mathcal{B}(x_{\min},\tau,\theta',\phi',y).$$

Collider phenomenology: 5-point

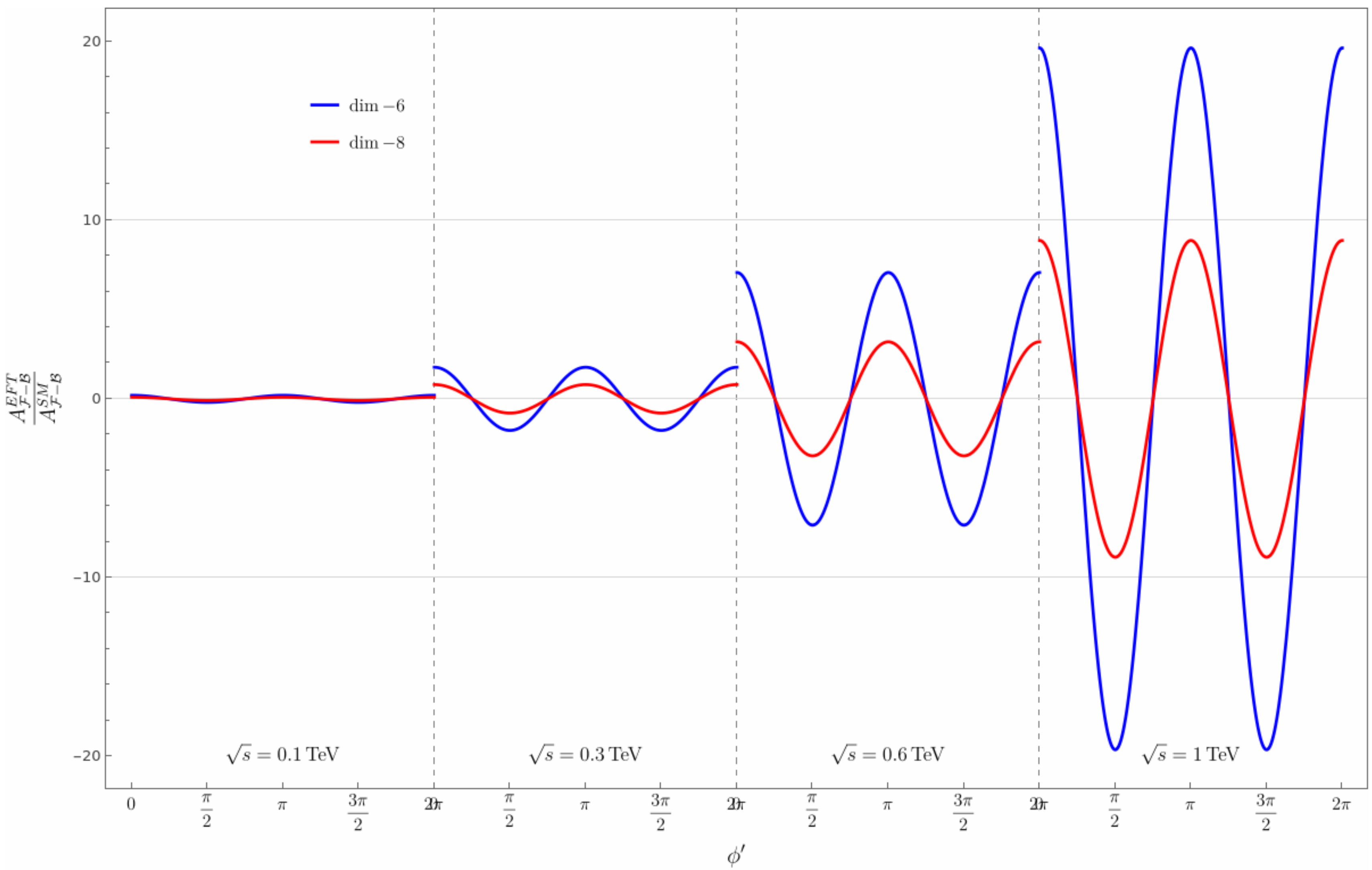
$$\mathcal{F} \pm \mathcal{B} = f \left(\sum_n [a^{\pm}_{n\phi'} \cos(n\phi') + b^{\pm}_{n\phi'} \sin(n\phi')] + c^{\pm} \right) F_{\pm}.$$

$$F_{\pm}(x_{\min}, \tau) \equiv F_d(x_{\min}, \tau) \pm F_{\bar{u}}(x_{\min}, \tau).$$

Coefficient	SM LO	SM NLO	SM NNLO	dim-6 LO	dim-8 LO
f	$\frac{e^2 g^4}{36} \frac{m_W}{\Gamma_W}$	$\frac{e^2 g^4}{18} \frac{m_W^2}{\Gamma_W E_{\text{CM}}}$	$\frac{e^2 g^4}{72} \frac{m_W^3}{\Gamma_W E_{\text{CM}}^2}$	$2\pi e g^4 s_W \frac{m_W}{\Gamma_W} \left(\frac{E_{\text{CM}}}{\Lambda}\right)^2$	$-\frac{8\sqrt{2} e g^3}{3} \frac{m_W}{\Gamma_W} \left(\frac{E_{\text{CM}}}{\Lambda}\right)^4$
$a^-_{2\phi'}$	0	0	$-3y^2 (1 - \cos(2\theta'))$	$-\frac{y^2}{2} (1 - \cos(2\theta'))$ $\times \text{Re}[c_{6,+} - 2c_{6,-}]$	$\frac{y^2}{2} (1 - \cos(2\theta'))$ $\times (\text{Re}[c_{8,+}] + 2\text{Re}[c_{8,-}])$
$b^-_{2\phi'}$	0	0	0	$\frac{y^2}{2} (1 - \cos(2\theta'))$ $\times \text{Im}[c_{6,+} + 2c_{6,-}]$	$-\frac{y^2}{2} (1 - \cos(2\theta'))$ $\times (\text{Im}[c_{8,+}] - 2\text{Im}[c_{8,-}])$
$a^-_{\phi'}$	0	0	0	0	0
$b^-_{\phi'}$	0	$-\left[(3y^2 + 7)S_y - 7\right] \sin(2\theta')$ $+ 12 (S_y - 1) \sin\theta'$	0	0	0
c^-	$(36y^2 + 40L_y) \cos\theta'$ $- (3y^2 + 6L_y) \cos(2\theta')$ $- (9y^2 + 18L_y)$	0	$4 (9y^2 + 10L_y) \cos\theta'$ $- 3 (3y^2 + 2L_y) \cos(2\theta')$ $- 3 (y^2 + 6L_y)$	0	0

Table 1: Coefficients in the $\mathcal{F} - \mathcal{B}$ combination.

Collider phenomenology: 5-point



Summary

- We derive the four-point contact terms of the standard model particles, keeping terms with up to quartic energy growth. It will complete the 4-point amplitudes together with the factorizable parts.

- The on-shell method is a powerful tool for obtaining angular observables in the process involving more than 4 external legs.

Thanks!

Backup

HEFT and SMEFT mapping

[HL, Ma, Shadmi, Waterbury, 2301.11349]

Massive $d=6$ amplitudes	SMEFT Wilson coefficients
$\mathcal{M}(W_L^+W_L^-hh) = C^{00}_{WWhh}\langle\mathbf{12}\rangle[\mathbf{12}]$	$C^{00}_{WWhh} = (c^{(+)}_{(H^\dagger H)^2} - 3c^{(-)}_{(H^\dagger H)^2})/2$
$\mathcal{M}(W_\pm^+W_\pm^-hh) = C^{\pm\pm}_{WWhh}(\mathbf{12})^2$	$C^{\pm\pm}_{WWhh} = 2c^{\pm\pm}_{WWHH}$
$\mathcal{M}(Z_LZ_Lhh) = C^{00}_{ZZhh}\langle\mathbf{12}\rangle[\mathbf{12}]$	$C^{00}_{ZZhh} = -2c^{(+)}_{(H^\dagger H)^2}$
$\mathcal{M}(Z_\pm Z_\pm hh) = C^{\pm\pm}_{ZZhh}(\mathbf{12})^2$	$C^{\pm\pm}_{ZZhh} = c_W^2 c^{\pm\pm}_{WWHH} + s_W^2 c^{\pm\pm}_{BBHH} + c_W s_W c^{\pm\pm}_{BWHH}$
$\mathcal{M}(g_\pm g_\pm hh) = C^{\pm\pm}_{gghh}(12)^2$	$C^{\pm\pm}_{gghh} = c^{\pm\pm}_{GGHH}$
$\mathcal{M}(\gamma_\pm\gamma_\pm hh) = C^{\pm\pm}_{\gamma\gamma hh}(12)^2$	$C^{\pm\pm}_{\gamma\gamma hh} = s_W^2 c^{\pm\pm}_{WWHH} + c_W^2 c^{\pm\pm}_{BBHH} - c_W s_W c^{\pm\pm}_{BWHH}$
$\mathcal{M}(\gamma_\pm Zhh) = C^{\pm}_{\gamma Zhh}(\mathbf{12})^2$	$C^{\pm}_{\gamma Zhh} = s_W c_W c^{\pm\pm}_{WWHH} - s_W c_W c^{\pm\pm}_{BBHH} + \frac{1}{2}(s_W^2 - c_W^2)c^{\pm\pm}_{BWHH}$
$\mathcal{M}(hhhh) = C_{hhhh}$	$C_{hhhh} = -3c_{(H^\dagger H)^2} + 45\, v^2 c_{(H^\dagger H)^3}$
$\mathcal{M}(f^c_\pm f_\pm hh) = C^{\pm\pm}_{ffhh}(\mathbf{12})$	$C^{\pm\pm}_{ffhh} = 3c^{\pm\pm}_{\Psi\psi HHH}v/(2\sqrt{2})$
$\mathcal{M}(f^c_+ f'_- W_L h) = C^{+-0}_{ffWh}[\mathbf{13}]\langle\mathbf{23}\rangle$	$C^{+-0}_{ffWh} = (c^{+-,(+)}_{\Psi\Psi HH} - c^{+-,(-)}_{\Psi\Psi HH})/2$
$\mathcal{M}(f^c_- f'_+ W_L h) = C^{-+0}_{ffWh}\langle\mathbf{13}\rangle[\mathbf{23}]$	$C^{-+0}_{ffWh} = c^{-+}_{\psi_R\psi'_R HH}$
$\mathcal{M}(f^c_\pm f'_\pm W_\pm h) = C^{\pm\pm\pm}_{ffWh}(\mathbf{13})(\mathbf{23})$	$C^{\pm\pm\pm}_{ffWh} = c^{\pm\pm\pm}_{\Psi\psi WH}/2$
$\mathcal{M}(f^c_+ f_- Z_L h) = C^{+-0}_{ffZh}[\mathbf{13}]\langle\mathbf{23}\rangle$	$C^{+-0}_{e_Le_LZh} = -i\sqrt{2}c^{+-,(+)}_{\Psi\Psi HH}$, $C^{+-0}_{\nu_L\nu_LZh} = -i(c^{+-,(+)}_{\Psi\Psi HH} + c^{+-,(-)}_{\Psi\Psi HH})/\sqrt{2}$
$\mathcal{M}(f^c_- f_+ Z_L h) = C^{-+0}_{ffZh}\langle\mathbf{13}\rangle[\mathbf{23}]$	$C^{-+0,\mathrm{CT}}_{ffZh} = -i\sqrt{2}c^{-+}_{\psi\psi HH}$
$\mathcal{M}(f^c_\pm f_\pm Z_\pm h) = C^{\pm\pm\pm}_{ffZh}(\mathbf{13})(\mathbf{23})$	$C^{\pm\pm\pm}_{ffZh} = -(s_W c^{\pm\pm\pm}_{\Psi\psi BH} + c_W c^{\pm\pm\pm}_{\Psi\psi WH})/\sqrt{2}$
$\mathcal{M}(f^c_\pm f_\pm \gamma_\pm h) = C^{\pm\pm\pm}_{ff\gamma h}(\mathbf{13})(\mathbf{23})$	$C^{\pm\pm\pm}_{ff\gamma h} = (-s_W c^{\pm\pm\pm}_{\Psi\psi WH} + c_W c^{\pm\pm\pm}_{\Psi\psi BH})/\sqrt{2}$
$\mathcal{M}(q^c_\pm q_\pm g^A_\pm h) = C^{\pm\pm\pm}_{qqgh}\lambda^A(\mathbf{13})(\mathbf{23})$	$C^{\pm\pm\pm}_{qqgh} = c^{\pm\pm\pm}_{\Psi\psi GH}/\sqrt{2}$

- In ±± helicity of VVhh, 10 HEFT parameters and 8 SMEFT parameters

Schouten identity

$$\varepsilon^{\alpha\beta}\varepsilon^{\gamma\delta} - \varepsilon^{\alpha\gamma}\varepsilon^{\beta\delta} + \varepsilon^{\alpha\delta}\varepsilon^{\beta\gamma} = 0\,,$$

$$\varepsilon_{\dot\alpha\dot\beta}\varepsilon_{\dot\gamma\dot\delta} - \varepsilon_{\dot\alpha\dot\gamma}\varepsilon_{\dot\beta\dot\delta} + \varepsilon_{\dot\alpha\dot\delta}\varepsilon_{\dot\beta\dot\gamma} = 0\,,$$

$$\begin{aligned} 0 &= \left(\varepsilon^{\alpha\beta}\varepsilon^{\gamma\delta} - \varepsilon^{\alpha\gamma}\varepsilon^{\beta\delta} + \varepsilon^{\alpha\delta}\varepsilon^{\beta\gamma}\right)\lambda^I_{1,\gamma}\lambda^J_{2,\delta} \\ &= -\varepsilon^{\alpha\beta}(\lambda^I_1\lambda^J_2) - \lambda^{I,\alpha}_1\lambda^{J,\beta}_2 + \lambda^{I,\beta}_1\lambda^{J,\alpha}_2\,, \end{aligned}$$

$$\langle \mathbf{1}^I \mathbf{2}^J\rangle\delta^\beta_\alpha = |\mathbf{2}^J\rangle_\alpha\langle\mathbf{1}^I|^\beta - |\mathbf{1}^I\rangle_\alpha\langle\mathbf{2}^J|^\beta\,.$$

Massless dim-6 SMEFT

- Massless SMEFT contact term [Ma, Shu, Xiao '19]
- Each kinematic structure in the physical amplitudes can be associated with a specific operator in the Warsaw basis

[HL, Ma, Shadmi, Waterbury '23]

Amplitude	Contact term	Warsaw basis operator	Coefficient
$\mathcal{A}(H_i^c H_j^c H_k^c H^l H^m H^n)$	$T^{+\,lmn}_{ijk}$	$\mathcal{O}_H/6$	$c_{(H^\dagger H)^3}$
$\mathcal{A}(H_i^c H_j^c H^k H^l)$	$s_{12} T^{+\,kl}_{ij}$	$\mathcal{O}_{HD}/2 + \mathcal{O}_{H\Box}/4$	$c^{(+)}_{(H^\dagger H)^2}$
$\mathcal{A}(H_i^c H_j^c H^k H^l)$	$(s_{13} - s_{23}) T^{-\,kl}_{ij}$	$\mathcal{O}_{HD}/2 - \mathcal{O}_{H\Box}/4$	$c^{(-)}_{(H^\dagger H)^2}$
$\mathcal{A}(B^\pm B^\pm H_i^c H^j)$	$(12)^2 \delta_i^j$	$(\mathcal{O}_{HB} \pm i\mathcal{O}_{H\tilde{B}})/2$	$c^{\pm\pm}_{BBHH}$
$\mathcal{A}(B^\pm W^{I\pm} H_i^c H^j)$	$(12)^2 (\sigma^I)_i^j$	$\mathcal{O}_{HWB} \pm i\mathcal{O}_{H\tilde{W}B}$	$c^{\pm\pm}_{BWHH}$
$\mathcal{A}(W^{I+} W^{J+} H_i^c H^j)$	$(12)^2 \delta^{IJ} \delta_i^j$	$(\mathcal{O}_{HW} \pm i\mathcal{O}_{H\tilde{W}})/2$	$c^{\pm\pm}_{WWHH}$
$\mathcal{A}(g^{A\pm} g^{B\pm} H_i^c H^j)$	$(12)^2 \delta^{AB} \delta_i^j$	$(\mathcal{O}_{HG} \pm i\mathcal{O}_{H\tilde{G}})/2$	$c^{\pm\pm}_{GGHH}$
$\mathcal{A}(L_i^c e H_j^c H^k H^l)$	$[12] T^{+\,kl}_{ij}$	$\mathcal{O}_{eH}/2$	c^{++}_{LeHHH}
$\mathcal{A}(Q^c_{a,i} d^b H_j^c H^k H^l)$	$[12] T^{+\,kl}_{ij} \delta_a^b$	$\mathcal{O}_{dH}/2$	c^{++}_{QdHHH}
$\mathcal{A}(Q^c_{a,i} u^b H_j^c H_k^c H^l)$	$[12] \varepsilon_{im} T^{+\,ml}_{jk} \delta_a^b$	$\mathcal{O}_{uH}/2$	c^{++}_{QuHHH}
$\mathcal{A}(e^c e H_i^c H^j)$	$\langle 142] \delta_i^j$	$\mathcal{O}_{He}/2$	c^{-+}_{eeHH}
$\mathcal{A}(u_a^c u^b H_i^c H^j)$	$\langle 142] \delta_i^j \delta_a^b$	$\mathcal{O}_{Hu}/2$	c^{-+}_{uuHH}
$\mathcal{A}(d_a^c d^b H_i^c H^j)$	$\langle 142] \delta_i^j \delta_a^b$	$\mathcal{O}_{Hd}/2$	c^{-+}_{ddHH}
$\mathcal{A}(u_a^c d^b H^i H^j)$	$\langle 142] \epsilon^{ij} \delta_a^b$	$\mathcal{O}_{Hud}/2$	c^{-+}_{udHH}
$\mathcal{A}(L_i^c L^j H_k^c H^l)$	$[142\rangle T^{+\,jl}_{ik}$	$\left(\mathcal{O}^{(1)}_{HL} + \mathcal{O}^{(3)}_{HL}\right)/8$	$c^{+-,(+)}_{LLHH}$
$\mathcal{A}(L_i^c L^j H_k^c H^l)$	$[142\rangle T^{-\,jl}_{ik}$	$\left(\mathcal{O}^{(1)}_{HL} - \mathcal{O}^{(3)}_{HL}\right)/8$	$c^{+-,(-)}_{LLHH}$
$\mathcal{A}(Q^c_{a,i} Q^{b,j} H_k^c H^l)$	$[142\rangle T^{+\,jl}_{ik} \delta_a^b$	$\left(3\mathcal{O}^{(1)}_{HQ} + \mathcal{O}^{(3)}_{HQ}\right)/8$	$c^{+-,(+)}_{QQHH}$
$\mathcal{A}(Q^c_{a,i} Q^{b,j} H_k^c H^l)$	$[142\rangle T^{-\,jl}_{ik} \delta_a^b$	$(\mathcal{O}^{(1)}_{HQ} - \mathcal{O}^{(3)}_{HQ})/8$	$c^{+-,(-)}_{QQHH}$
$\mathcal{A}(L_i^c e B^+ H^j)$	$[13][23] \delta_i^j$	$-i\mathcal{O}_{eB}/(2\sqrt{2})$	c^{+++}_{LeBH}
$\mathcal{A}(Q^c_{a,i} d^b B^+ H^j)$	$[13][23] \delta_i^j \delta_a^b$	$-i\mathcal{O}_{dB}/(2\sqrt{2})$	c^{+++}_{QdBH}
$\mathcal{A}(Q^c_{a,i} u^b B^+ H_j^c)$	$[13][23] \epsilon_{ij} \delta_a^b$	$-i\mathcal{O}_{uB}/(2\sqrt{2})$	c^{+++}_{QuBH}
$\mathcal{A}(L_i^c e W^{I+} H^j)$	$[13][23] (\sigma^I)_i^j$	$-i\mathcal{O}_{eW}/(2\sqrt{2})$	c^{+++}_{LeWH}
$\mathcal{A}(Q^c_{a,i} d^b W^{I+} H^j)$	$[13][23] (\sigma^I)_i^j \delta_a^b$	$-i\mathcal{O}_{dW}/(2\sqrt{2})$	c^{+++}_{QdWH}
$\mathcal{A}(Q^c_{a,i} u^b W^{I+} H_j^c)$	$[13][23] (\sigma^I)_{ik} \epsilon^k_j \delta_a^b$	$-i\mathcal{O}_{uW}/(2\sqrt{2})$	c^{+++}_{QuWH}
$\mathcal{A}(Q^c_{a,i} d^b g^{A+} H^j)$	$[13][23] \delta_i^j (\lambda^A)_a^b$	$-i\mathcal{O}_{dG}/(2\sqrt{2})$	c^{+++}_{QdGH}
$\mathcal{A}(Q^c_{a,i} u^b g^{A+} H_j^c)$	$[13][23] \epsilon_{ij} (\lambda^A)_a^b$	$-i\mathcal{O}_{uG}/(2\sqrt{2})$	c^{+++}_{QuGH}
$\mathcal{A}(W^{I\pm} W^{J\pm} W^{K\pm})$	$(12)(23)(31)\epsilon^{IJK}$	$(\mathcal{O}_W \pm i\mathcal{O}_{\tilde{W}})/6$	$c^{\pm\pm\pm}_{WWW}$
$\mathcal{A}(g^{A\pm} g^{B\pm} g^{C\pm})$	$(12)(23)(31) f^{ABC}$	$(\mathcal{O}_G \pm i\mathcal{O}_{\tilde{G}})/6$	$c^{\pm\pm\pm}_{GGG}$

WWhh

$$T^{\pm\,kl}_{ij} \equiv \frac{1}{2}(\delta^k_i \delta^l_j \pm \delta^k_j \delta^l_i)$$

Amplitude	Contact term	Warsaw basis operator	Coefficient
$\mathcal{A}(H^c_i H^c_j H^c_k H^l H^m H^n)$	$T^{+\,lmn}_{ijk}$	$\mathcal{O}_H/6$	$c_{(H^\dagger H)^3}$
$\mathcal{A}(H^c_i H^c_j H^k H^l)$	$s_{12} T^{+\,kl}_{ij}$	$\mathcal{O}_{HD}/2 + \mathcal{O}_{H\Box}/4$	$c^{(+)}_{(H^\dagger H)^2}$
$\mathcal{A}(H^c_i H^c_j H^k H^l)$	$(s_{13} - s_{23}) T^{-\,kl}_{ij}$	$\mathcal{O}_{HD}/2 - \mathcal{O}_{H\Box}/4$	$c^{(-)}_{(H^\dagger H)^2}$

$$\mathcal{A}(H^c_i H^c_j H^k H^l) = \frac{c_+}{\Lambda^2} s_{12} T^{+\,kl}_{ij} + \frac{c_-}{\Lambda^2}(s_{13} - s_{23}) T^{-\,kl}_{ij}$$

$$[\mathbf{12}]\langle\mathbf{12}\rangle \equiv \frac{1}{2}[1^{\{I_1}2^{\{J_1}]\langle 1^{I_2\}}2^{J_2\}}\rangle \to \frac{1}{2}[12]\langle 12\rangle = -\frac{1}{2}s_{12}$$

$$\boldsymbol{\mathcal{M}}(W^+_L W^-_L hh) = \frac{c_+ - 3c_-}{2}\frac{[\mathbf{12}]\langle\mathbf{12}\rangle}{\Lambda^2}$$

$$\Rightarrow \quad c^{00}_{WWhh} = \frac{c_+ - 3c_-}{2} = 4(C_{H\Box} - \frac{1}{4}C_{HD})$$

HEFT and SMEFT mapping

- In ±± helicity of VVhh, 10 HEFT parameters and 8 SMEFT parameters

Massive $d = 6$ amplitudes	SMEFT Wilson coefficients
$\mathcal{M}(W_L^+ W_L^- hh) = C^{00}_{WWhh} \langle \mathbf{12} \rangle [\mathbf{12}]$	$C^{00}_{WWhh} = (c^{(+)}_{(H^\dagger H)^2} - 3c^{(-)}_{(H^\dagger H)^2})/2$
$\mathcal{M}(W_\pm^+ W_\pm^- hh) = C^{\pm\pm}_{WWhh} (\mathbf{12})^2$	$C^{\pm\pm}_{WWhh} = 2c^{\pm\pm}_{WWHH}$
$\mathcal{M}(Z_L Z_L hh) = C^{00}_{ZZhh} \langle \mathbf{12} \rangle [\mathbf{12}]$	$C^{00}_{ZZhh} = -2c^{(+)}_{(H^\dagger H)^2}$
$\mathcal{M}(Z_\pm Z_\pm hh) = C^{\pm\pm}_{ZZhh} (\mathbf{12})^2$	$C^{\pm\pm}_{ZZhh} = c_W^2 c^{\pm\pm}_{WWHH} + s_W^2 c^{\pm\pm}_{BBHH} + c_W s_W c^{\pm\pm}_{BWHH}$
$\mathcal{M}(g_\pm g_\pm hh) = C^{\pm\pm}_{gghh} (12)^2$	$C^{\pm\pm}_{gghh} = c^{\pm\pm}_{GGHH}$
$\mathcal{M}(\gamma_\pm \gamma_\pm hh) = C^{\pm\pm}_{\gamma\gamma hh} (12)^2$	$C^{\pm\pm}_{\gamma\gamma hh} = s_W^2 c^{\pm\pm}_{WWHH} + c_W^2 c^{\pm\pm}_{BBHH} - c_W s_W c^{\pm\pm}_{BWHH}$
$\mathcal{M}(\gamma_\pm Z hh) = C^{\pm}_{\gamma Zhh} (\mathbf{12})^2$	$C^{\pm}_{\gamma Zhh} = s_W c_W c^{\pm\pm}_{WWHH} - s_W c_W c^{\pm\pm}_{BBHH} + \frac{1}{2}(s_W^2 - c_W^2) c^{\pm\pm}_{BWHH}$
$\mathcal{M}(hhhh) = C_{hhhh}$	$C_{hhhh} = -3c_{(H^\dagger H)^2} + 45\, v^2 c_{(H^\dagger H)^3}$
$\mathcal{M}(f^c_\pm f_\pm hh) = C^{\pm\pm}_{ffhh} (\mathbf{12})$	$C^{\pm\pm}_{ffhh} = 3c^{\pm\pm}_{\Psi\psi HHH} v/(2\sqrt{2})$
$\mathcal{M}(f^c_+ f'_- W_L h) = C^{+-0}_{ffWh} [\mathbf{13}] \langle \mathbf{23} \rangle$	$C^{+-0}_{ffWh} = (c^{+-,(+)}_{\Psi\Psi HH} - c^{+-,(-)}_{\Psi\Psi HH})/2$
$\mathcal{M}(f^c_- f'_+ W_L h) = C^{-+0}_{ffWh} \langle \mathbf{13} \rangle [\mathbf{23}]$	$C^{-+0}_{ffWh} = c^{-+}_{\psi_R \psi'_R HH}$
$\mathcal{M}(f^c_\pm f'_\pm W_\pm h) = C^{\pm\pm\pm}_{ffWh} (\mathbf{13})(\mathbf{23})$	$C^{\pm\pm\pm}_{ffWh} = c^{\pm\pm\pm}_{\Psi\psi WH}/2$
$\mathcal{M}(f^c_+ f_- Z_L h) = C^{+-0}_{ffZh} [\mathbf{13}] \langle \mathbf{23} \rangle$	$C^{+-0}_{e_L e_L Zh} = -i\sqrt{2} c^{+-,(+)}_{\Psi\Psi HH}$, $C^{+-0}_{\nu_L \nu_L Zh} = -i(c^{+-,(+)}_{\Psi\Psi HH} + c^{+-,(-)}_{\Psi\Psi HH})/\sqrt{2}$
$\mathcal{M}(f^c_- f_+ Z_L h) = C^{-+0}_{ffZh} \langle \mathbf{13} \rangle [\mathbf{23}]$	$C^{-+0,\mathrm{CT}}_{ffZh} = -i\sqrt{2} c^{-+}_{\psi\psi HH}$
$\mathcal{M}(f^c_\pm f_\pm Z_\pm h) = C^{\pm\pm\pm}_{ffZh} (\mathbf{13})(\mathbf{23})$	$C^{\pm\pm\pm}_{ffZh} = -(s_W c^{\pm\pm\pm}_{\Psi\psi BH} + c_W c^{\pm\pm\pm}_{\Psi\psi WH})/\sqrt{2}$
$\mathcal{M}(f^c_\pm f_\pm \gamma_\pm h) = C^{\pm\pm\pm}_{ff\gamma h} (\mathbf{13})(\mathbf{23})$	$C^{\pm\pm\pm}_{ff\gamma h} = (-s_W c^{\pm\pm\pm}_{\Psi\psi WH} + c_W c^{\pm\pm\pm}_{\Psi\psi BH})/\sqrt{2}$
$\mathcal{M}(q^c_\pm q_\pm g^A_\pm h) = C^{\pm\pm\pm}_{qqgh} \lambda^A (\mathbf{13})(\mathbf{23})$	$C^{\pm\pm\pm}_{qqgh} = c^{\pm\pm\pm}_{\Psi\psi GH}/\sqrt{2}$

[HL, Ma, Shadmi, Waterbury ’23]