Model-Independent Determination of Proton Structure Functions at the Electron-Ion Collider

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- Overview of structure functions results from HERA.
- Model independent extraction of F_2, F_L and xF_3 at EIC.
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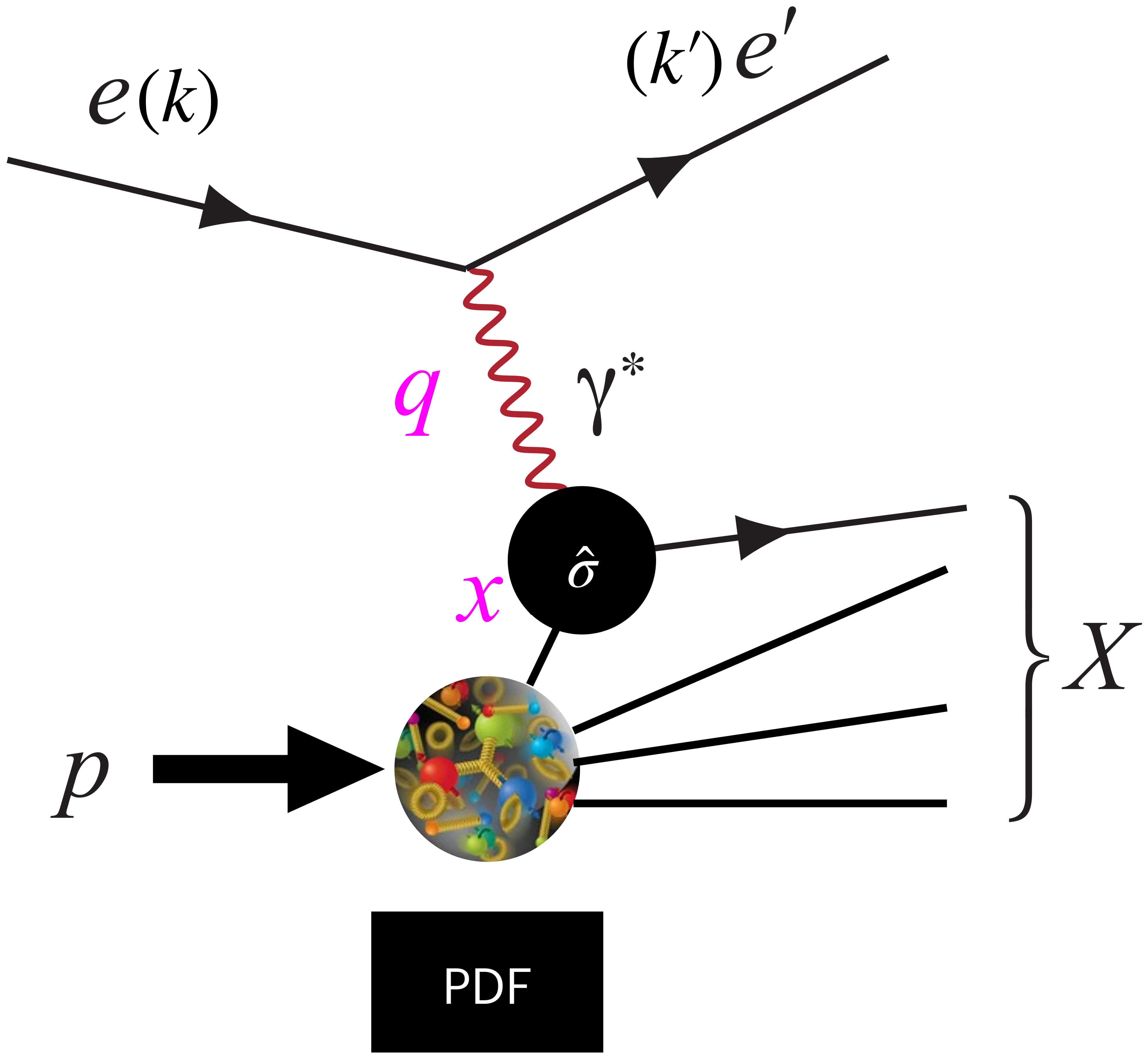
Overview of Deep Inelastic Scattering



The reduced cross-section for NC DIS for $Q^2 \ll M_Z^2$ is:

$$\sigma_r^{e^\pm p}(x, Q^2, y) = F_2\left(x, Q^2\right) - \frac{y^2}{Y_+} F_L\left(x, Q^2\right)$$

where x is the Bjorken scaling variable, Q^2 the virtuality, y the inelasticity and $Y_\pm = 1 \pm (1 - y)^2$.

As x and Q^2 are known, F_2 and F_L can be extracted as the free parameters of a linear fit.

This is known as the Rosenbluth-separation technique (*Phys. Rev. 79, 615*).

This method assumes $xF_3 = 0$, neglecting the exchange of Z bosons and the γZ interference.

Overview of Deep Inelastic Scattering

If xF_3 is not neglected, the full cross section formula for NC DIS is:

$$\sigma_r^{e^\pm p}(x, Q^2, y) = F_2\left(x, Q^2\right) \mp \frac{Y_-}{Y_+} xF_3\left(x, Q^2\right) - \frac{y^2}{Y_+} F_L\left(x, Q^2\right)$$

where:

$$F_2 = F_2^\gamma - \kappa_z \nu_e F_2^{\gamma Z} + \kappa_z^2(\nu_e^2 + a_e^2)F_2^Z$$

$$F_L = F_L^\gamma - \kappa_z \nu_e F_L^{\gamma Z} + \kappa_z^2(\nu_e^2 + a_e^2)F_L^Z$$

$$xF_3 = -\kappa_z a_2 xF_3^{\gamma Z} + \kappa_z^2 2\nu_e a_e xF_3^Z$$

with

$$\kappa_z(Q^2) = \frac{Q^2}{(Q^2 + M_Z^2)(4\sin^2\theta_\mathrm{w}\cos^2\theta_\mathrm{w})}$$

and ν_e and a_e are the vector and axial-vector weak couplings of the electron to the Z boson.

Electroweak effects are considerably suppressed for $Q^2 \ll M_Z^2$ but that does not mean we can fully neglect them!

Overview of Deep Inelastic Scattering

If xF_3 is not neglected, the full cross section formula for NC DIS is:

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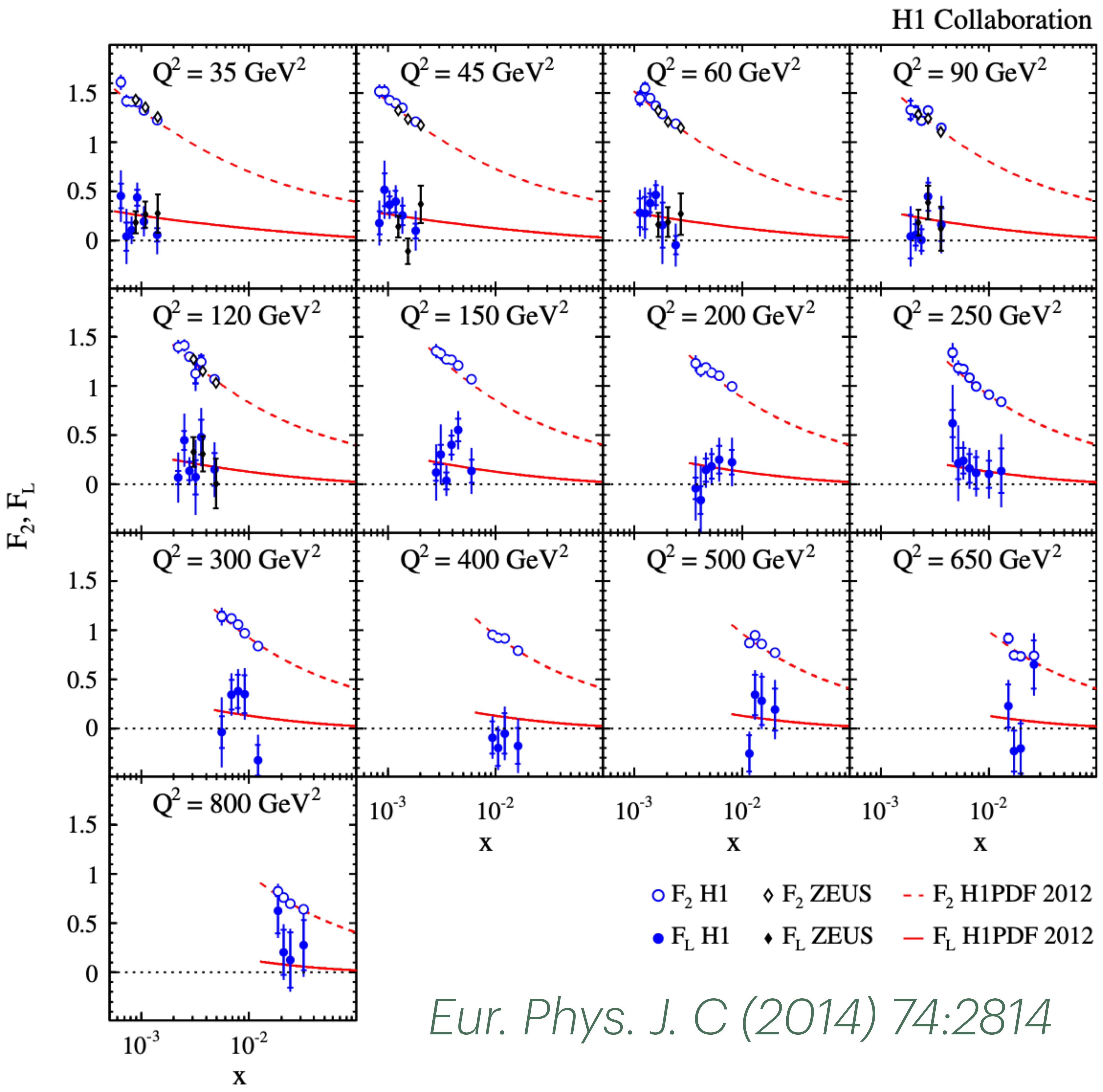
From this equation, it is possible to extract xF_3 using measurements from electron-proton and positron-proton events:

$$xF_3 = \frac{Y_+}{2Y_-}\left(\sigma_r^{e^-p} - \sigma_r^{e^+p}\right)$$

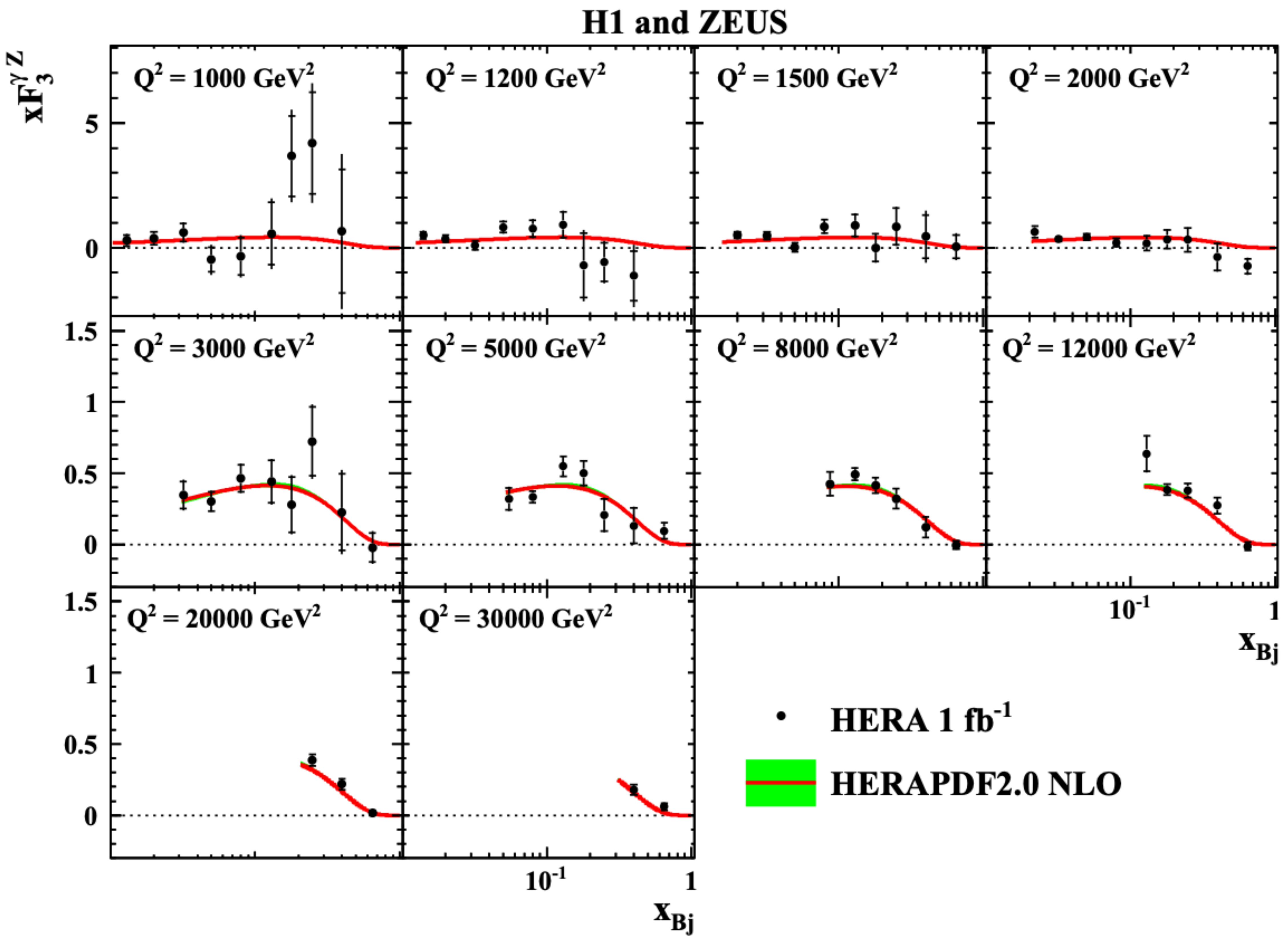
This was the only possible methodology that could be employed at HERA to extract measurements of xF_3.

However, there is a problem: one needs to have electrons and positrons available, but EIC will not have positrons and thus, this method can not be applied.

Results from HERA



Eur. Phys. J. C (2014) 74:2814



Eur. Phys. J. C (2015) 75:580

The simultaneous extraction of the 3 structure functions was not attempted at HERA.

EIC pseudo-data simulation

S5

e-beam energy (GeV)	p-beam energy (GeV)	$\sqrt{s}$ (GeV)	Integrated lumi (fb^{-1})
18	275	141	15.4
10	275	105	100
10	100	63	79.0
5	100	45	61.0
5	41	29	4.4

S17

E_e [GeV] \ E_p [GeV]	41	100	120	165	180	275
5	29	45	49	57	60	74
10	40	63	69	81	**85**	105
18	54	85	93	109	114	141

- Cross section generated with HERAPDF2.0 NNLO with xFitter.
- Gaussian smearing according to two possible scenarios:
    - Conservative scenario: 1.9 % of correlated systematics and 3.4 % of uncorrelated systematics $\Longrightarrow$ total uncertainty of 3.9 %.
    - Optimistic scenario: total uncertainty of 1 %.

Note that the uncertainty due to the normalisation between different beam energies is not considered.

MC replica method

Aiming to sample the distribution of possible outcomes for the values and uncertainties of the structure functions, 1000 replicas of each dataset are considered.

The averaging procedure, introduced in *PRD 105, 074006 (2022)*, is given by the following:

$$\bar{v} = \mathcal{S}_1 / N \qquad (\Delta v)^2 = \frac{\mathcal{S}_2 - \mathcal{S}_1^2 / N}{N - 1}$$

where:

$$\mathcal{S}_n = \sum_{i=1}^{N} v_i^n$$

and v_i stands for extracted value of the structure function in the i-th MC replica.

Potential bias in the extraction of F_L

As the cross section is a measurable observable, it must satisfy:

$$\Delta = \sigma_r^{\text{full}} - \sigma_r^{\text{Rosenbluth}} = 0$$

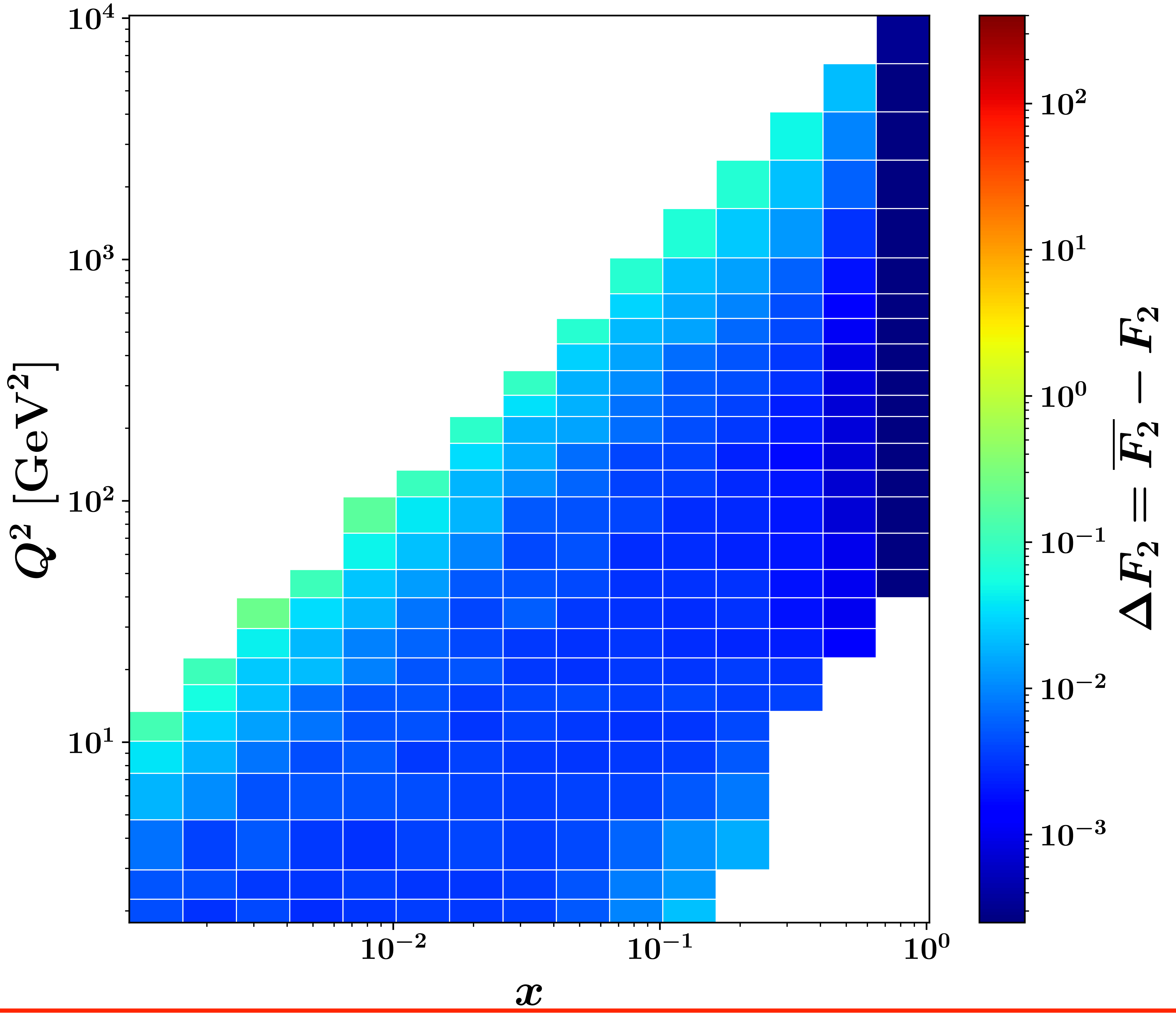
If we denote by $\overline{F}_2$ and $\overline{F}_L$ the structure functions obtained with the Rosenbluth method, one finds:

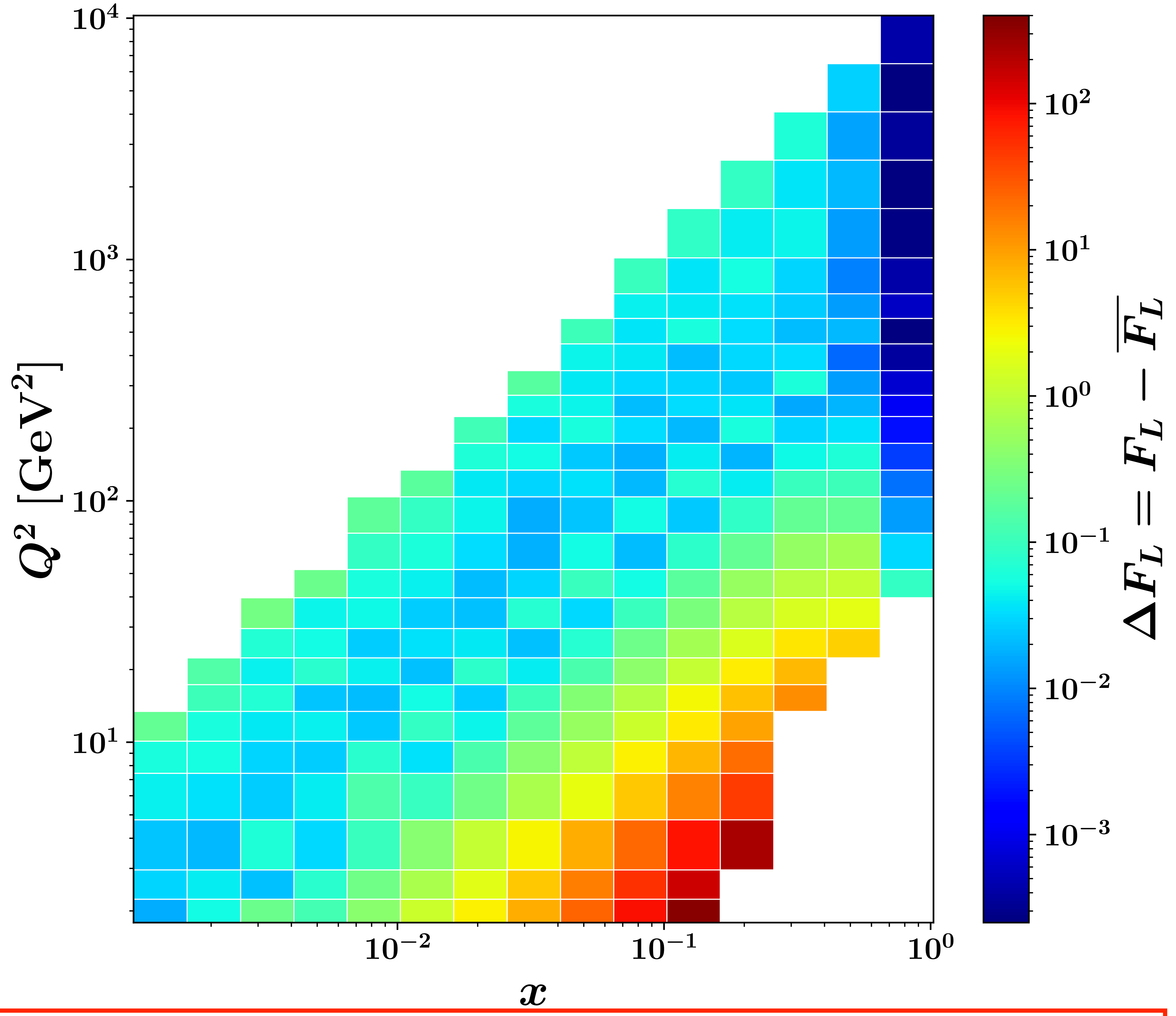
$$\Delta = F_2 - \overline{F}_2 + \frac{1}{Y_+}\left[(2y - y^2)xF_3 - y^2(F_L - \overline{F}_L)\right] = 0$$

As the difference $F_2 - \overline{F}_2$ can be neglected, the bias in F_L is:

$$F_L - \overline{F}_L \sim xF_3$$

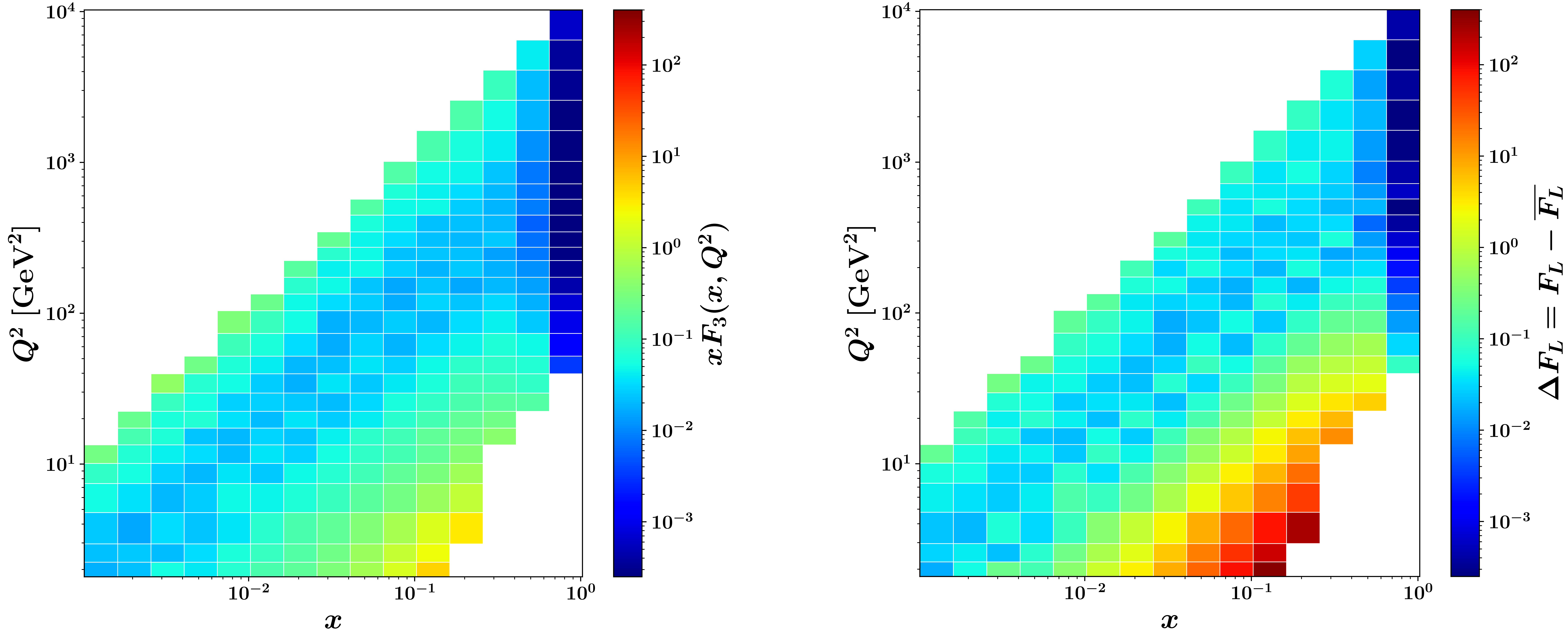
Potential bias in the extraction of F_L (S17 + optimistic)





The difference in F_2 due to the inclusion of xF_3 is negligible when compared to the difference in F_L

Potential bias in the extraction of F_L (S17 + optimistic)



Simultaneous extraction of F_2, F_L and xF_3

The extraction is done by fitting the structure functions according to the following relation with the reduced cross section:

$$\sigma_r^{e^\pm p}(x, Q^2, y) = F_2\left(x, Q^2\right) \mp \frac{Y_-}{Y_+} xF_3\left(x, Q^2\right) - \frac{y^2}{Y_+} F_L\left(x, Q^2\right)$$

As we are introducing an additional free parameter, we must be careful at the time of performing the fit. We propose the following fitting strategy:

- Obtain F_2 and F_L using the Rosenbluth method.

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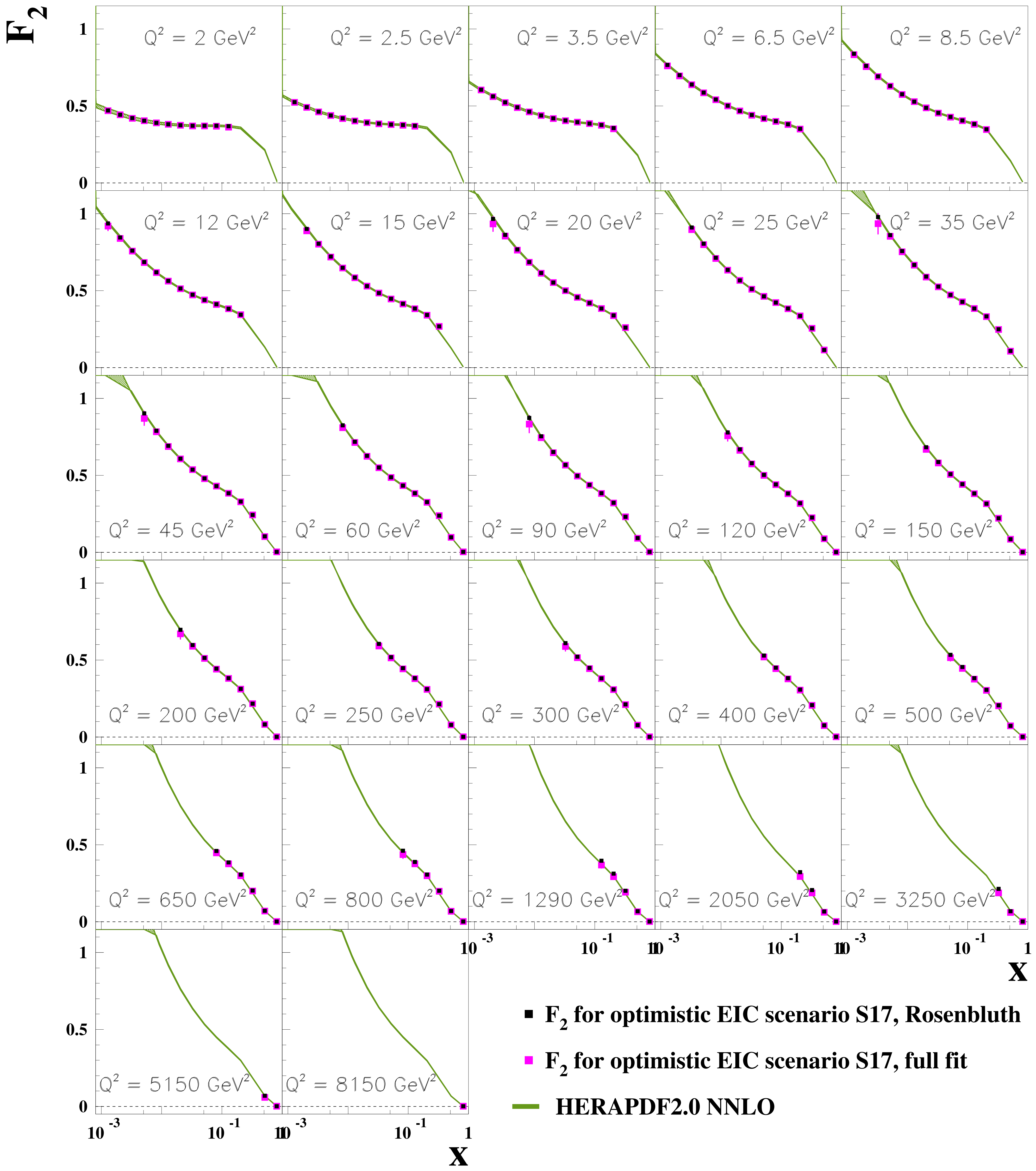
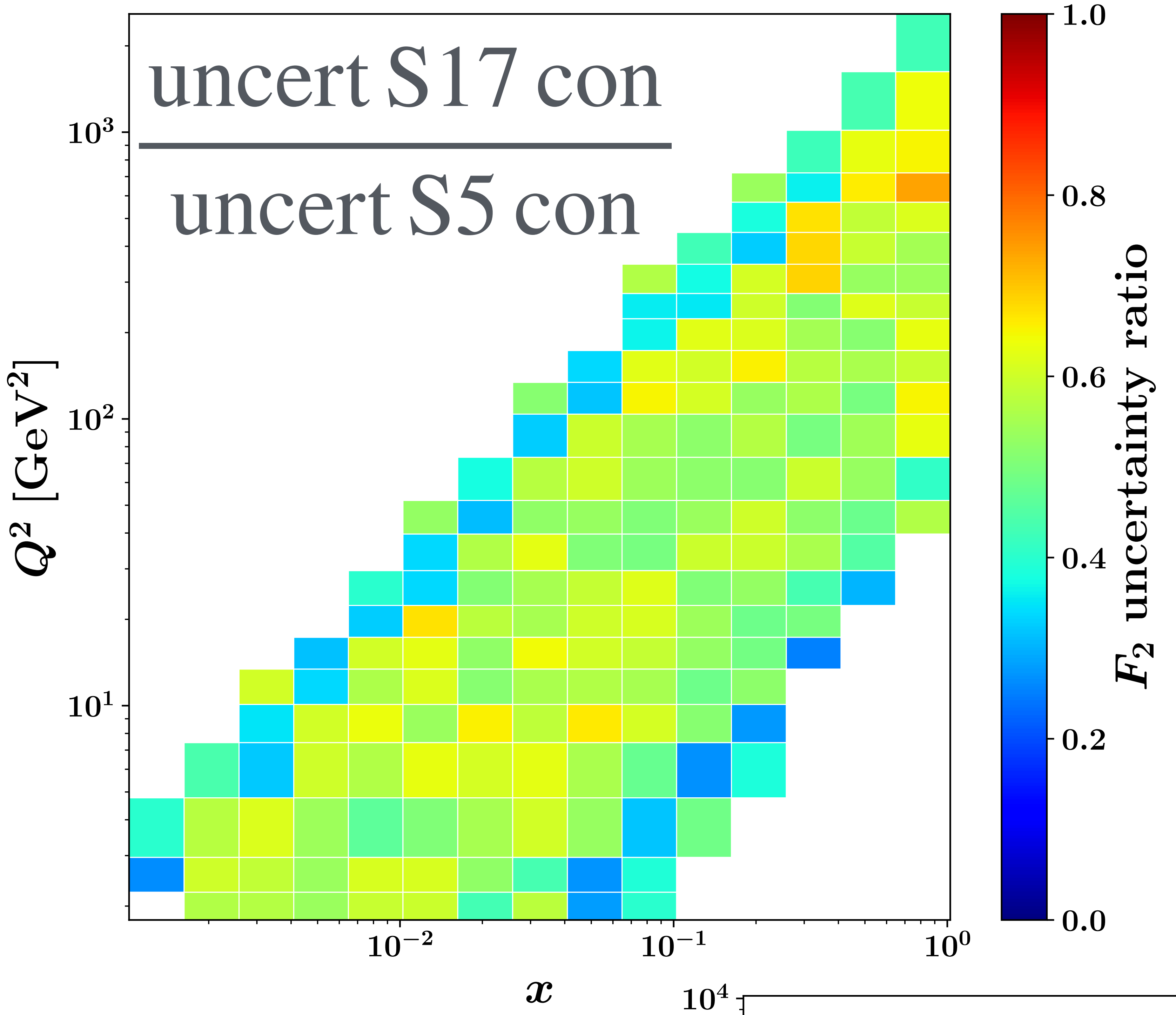
As we are introducing an additional free parameter, we must be careful at the time of performing the fit. We propose the following fitting strategy:

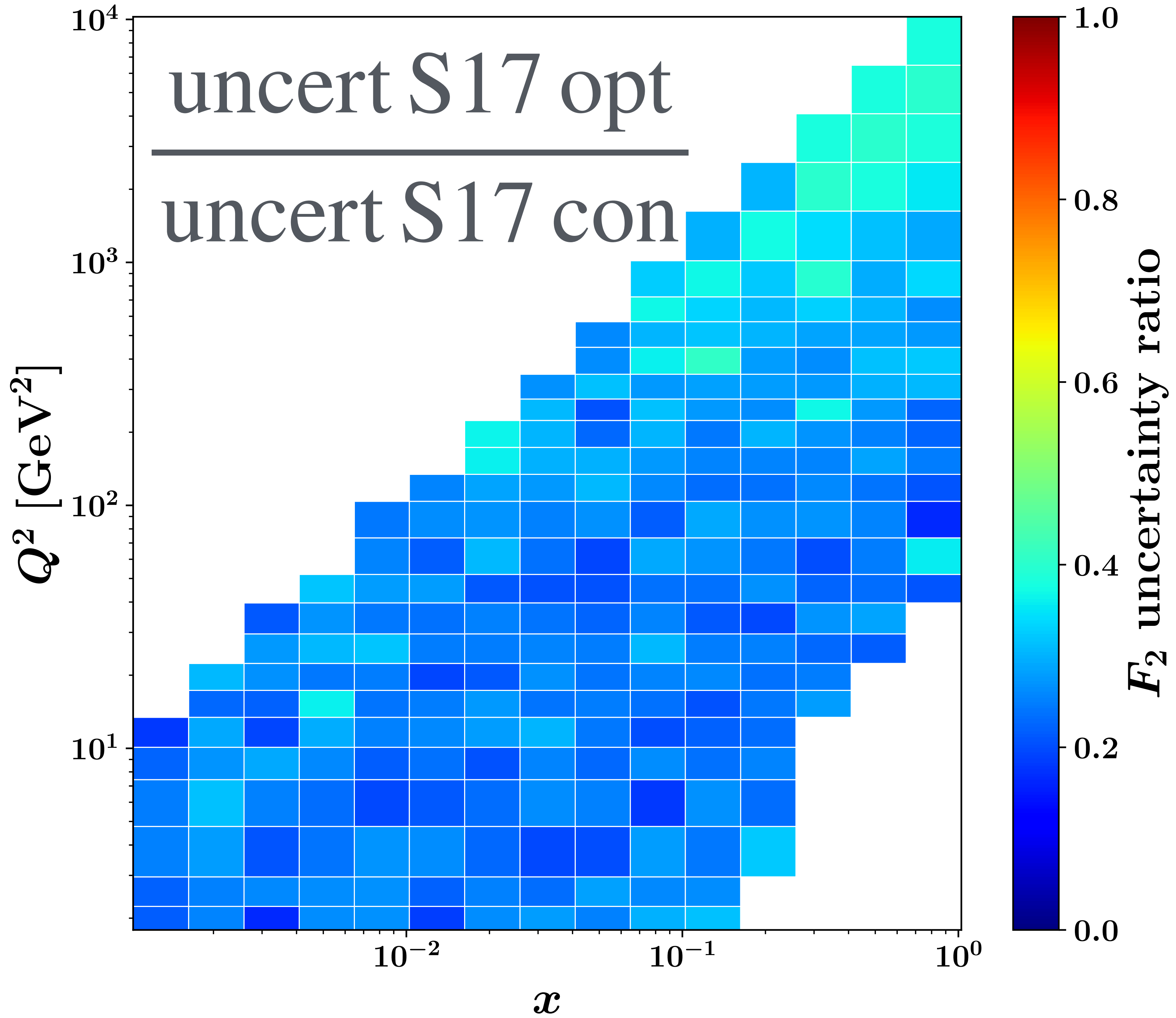
- Obtain F_2 and F_L using the Rosenbluth method.
- Use those values as initial guesses of the full fit in order to improve numerical stability and convergence.

Additionally, we impose the condition $xF_3 \geq 0$ during the fitting process to ensure physically meaningful results.

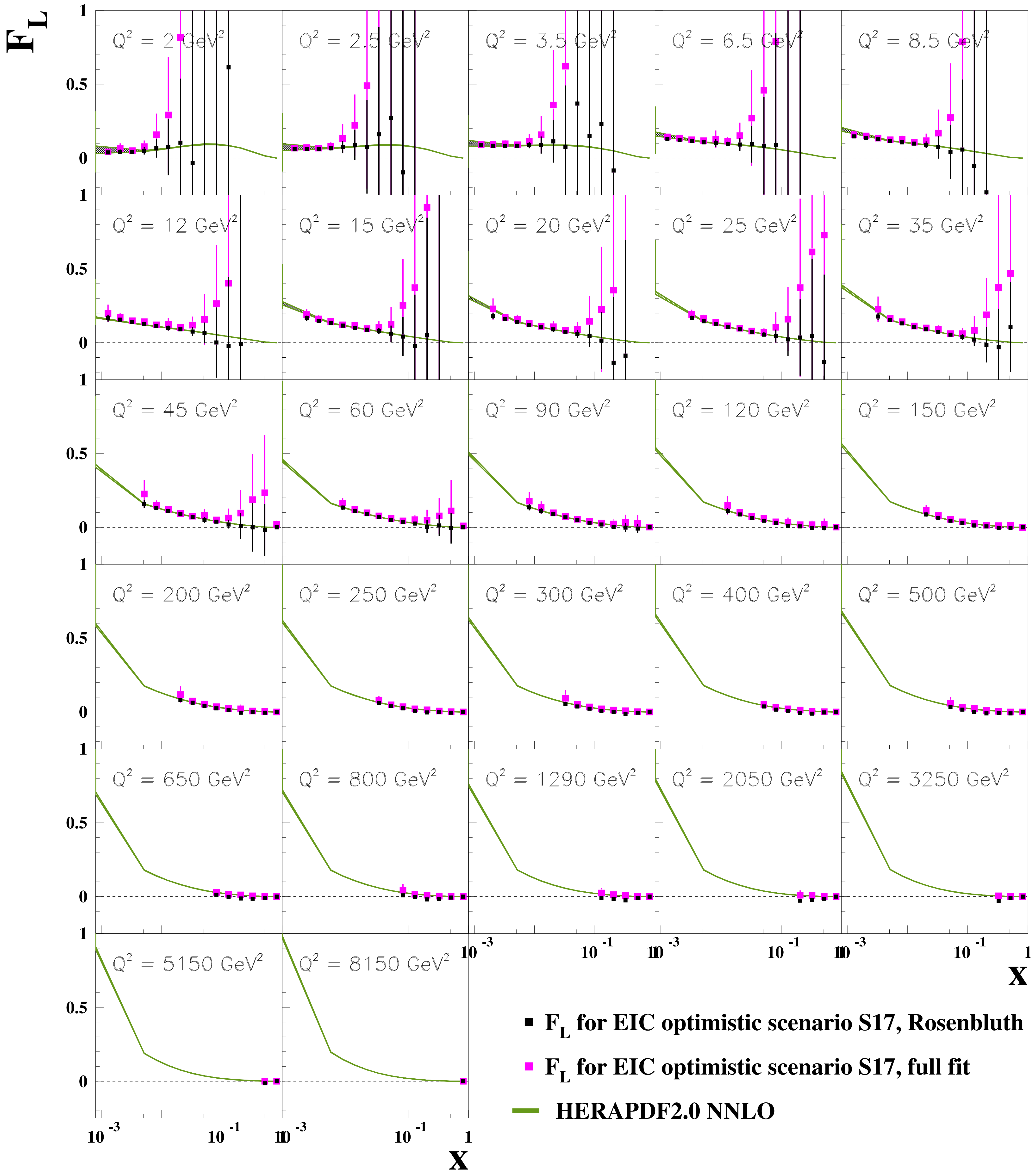
Results for F_2

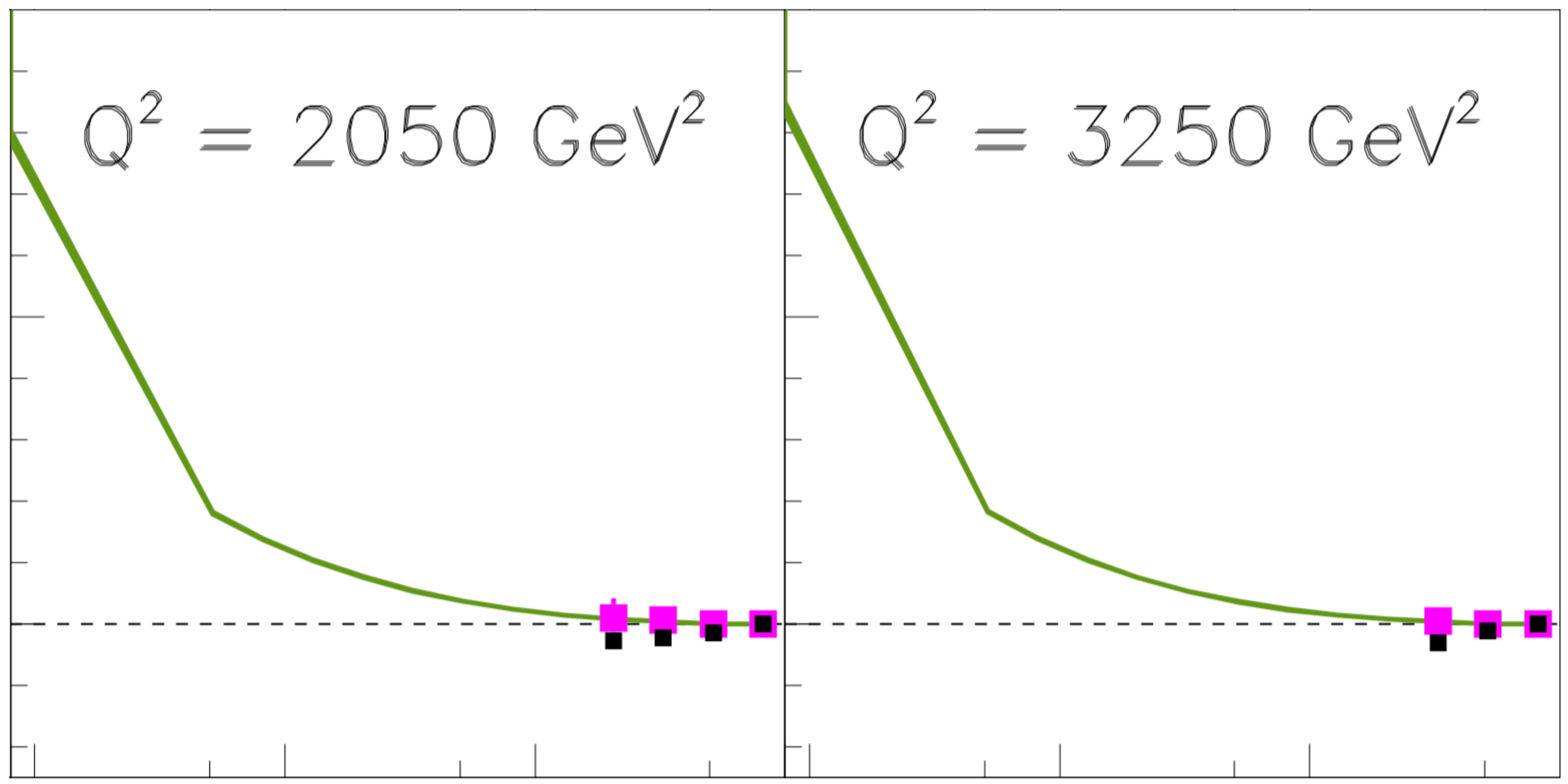






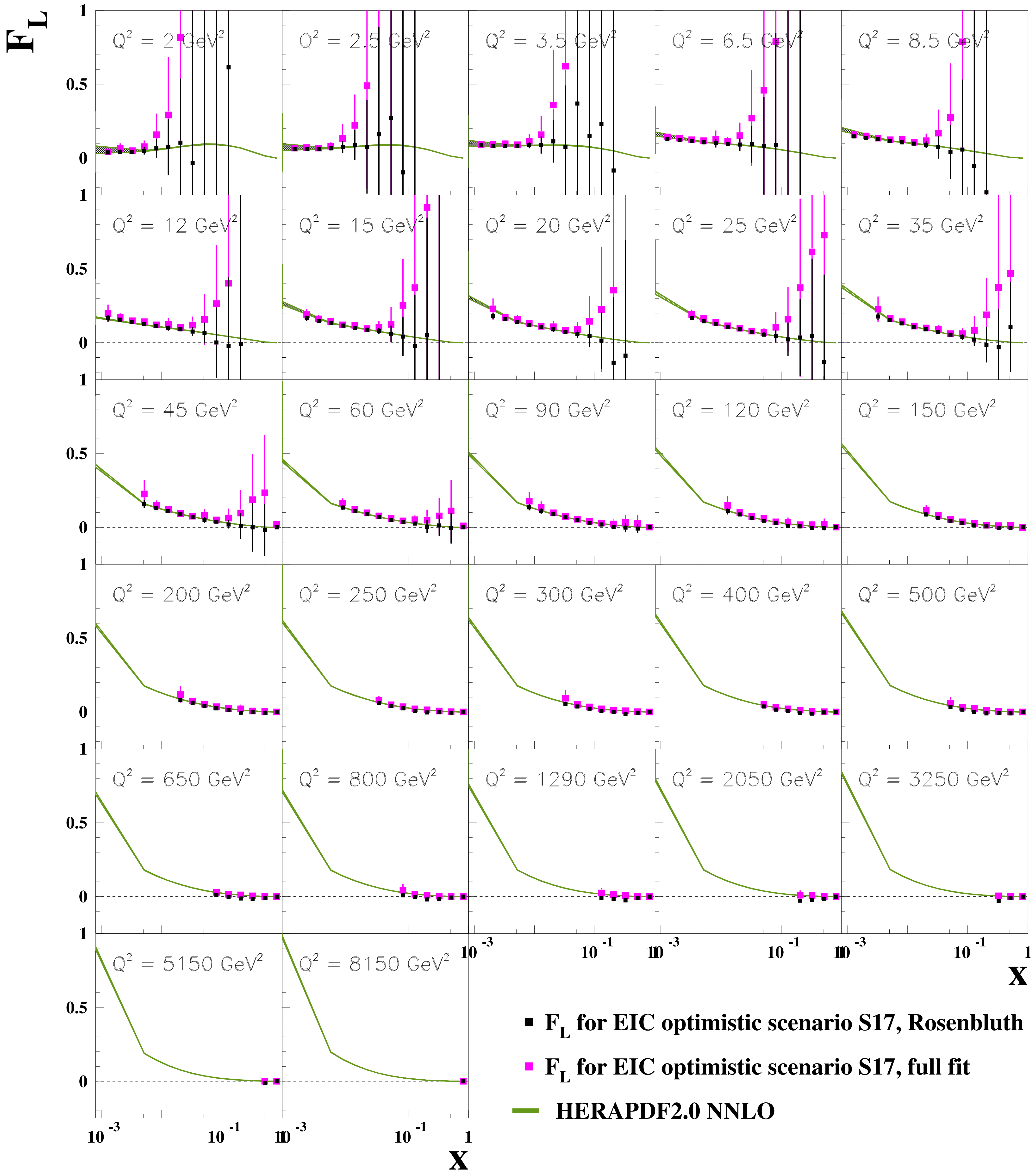
Results for F_L

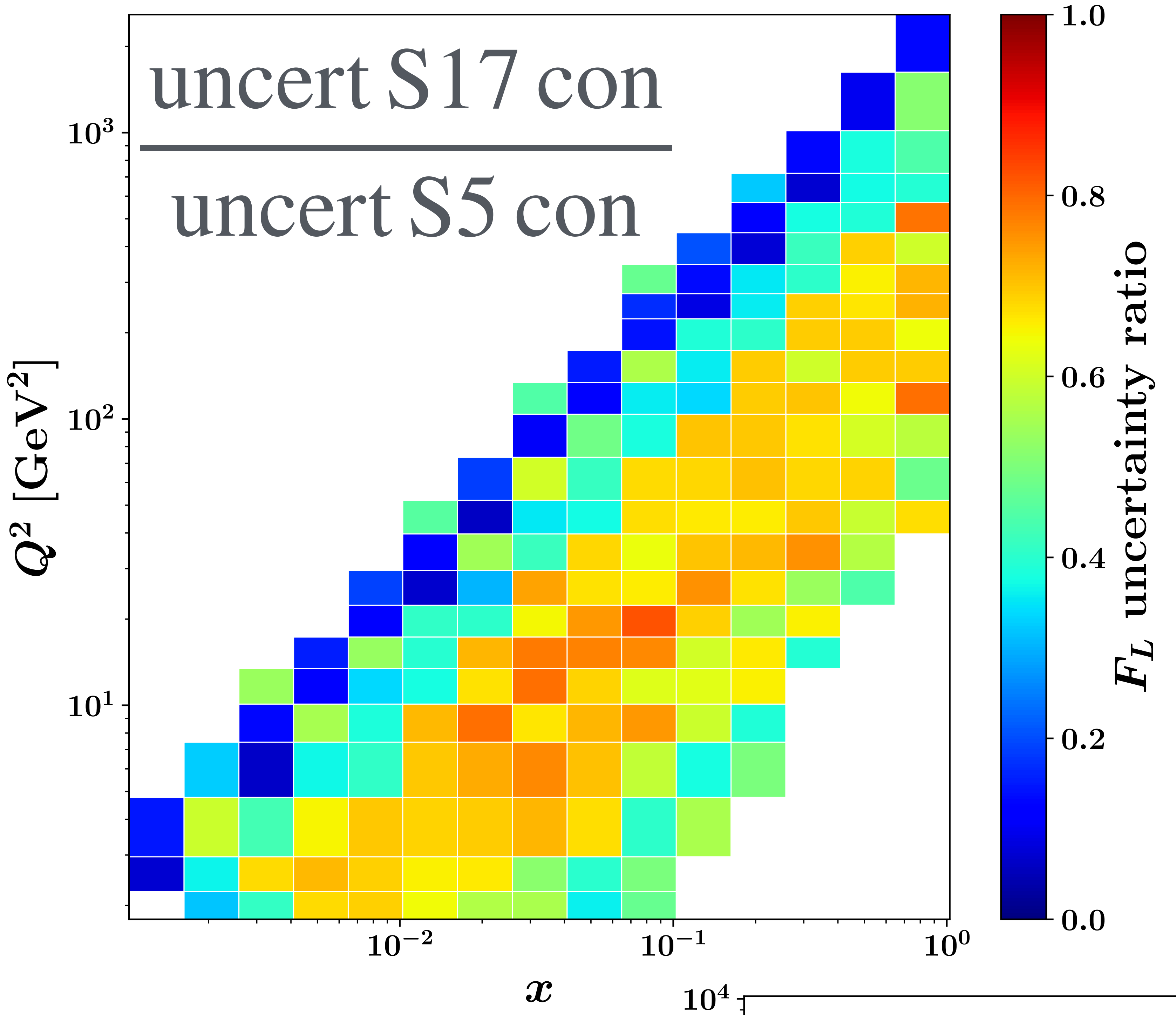


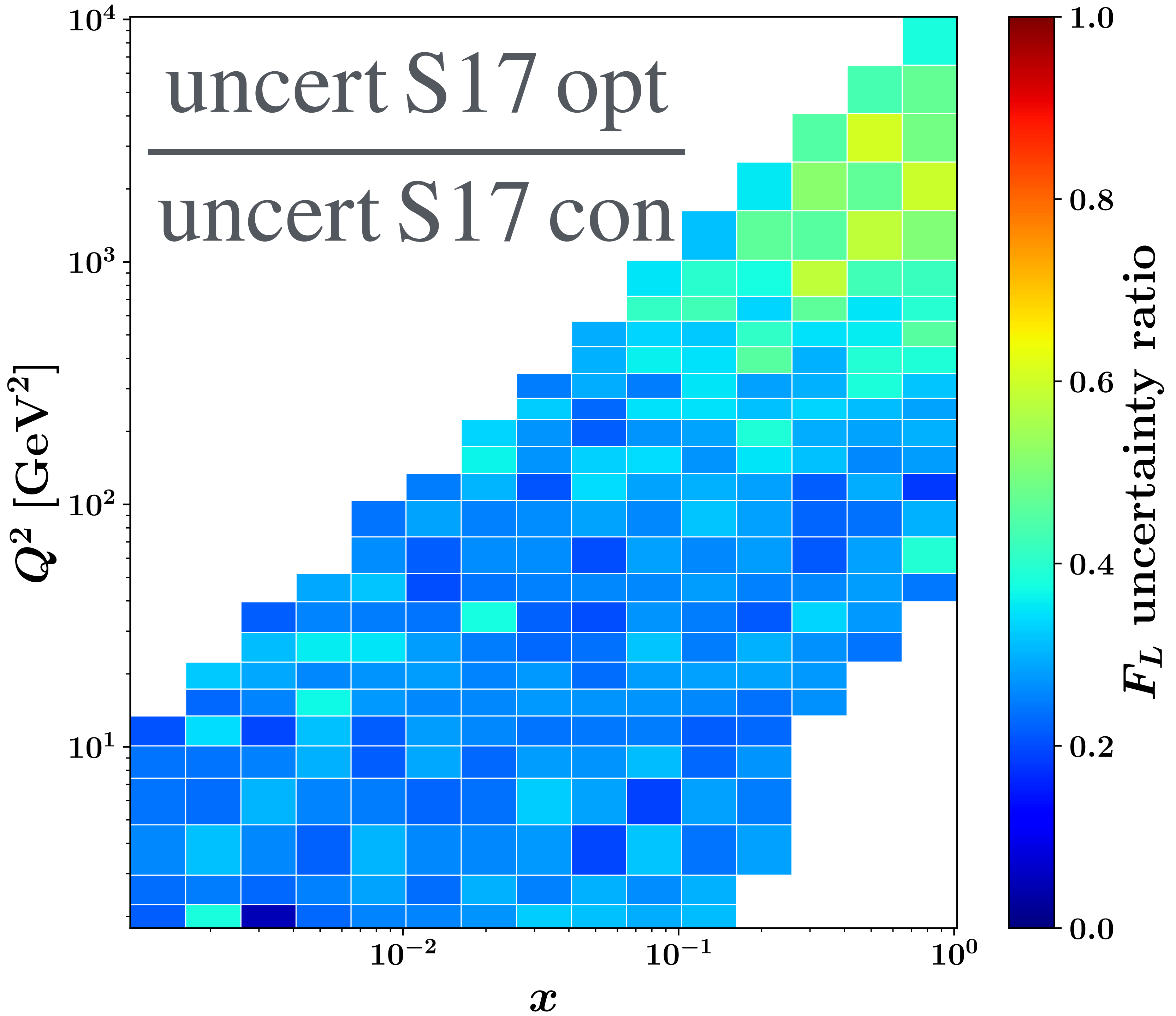


The Rosenbluth extraction yields negative values for F_L that are corrected by the inclusion of xF_3. In the optimistic uncertainty scenario and with enough c.o.m beam configurations, this difference might be observable.

Results for F_L







Results for xF_3 (conservative scenario)

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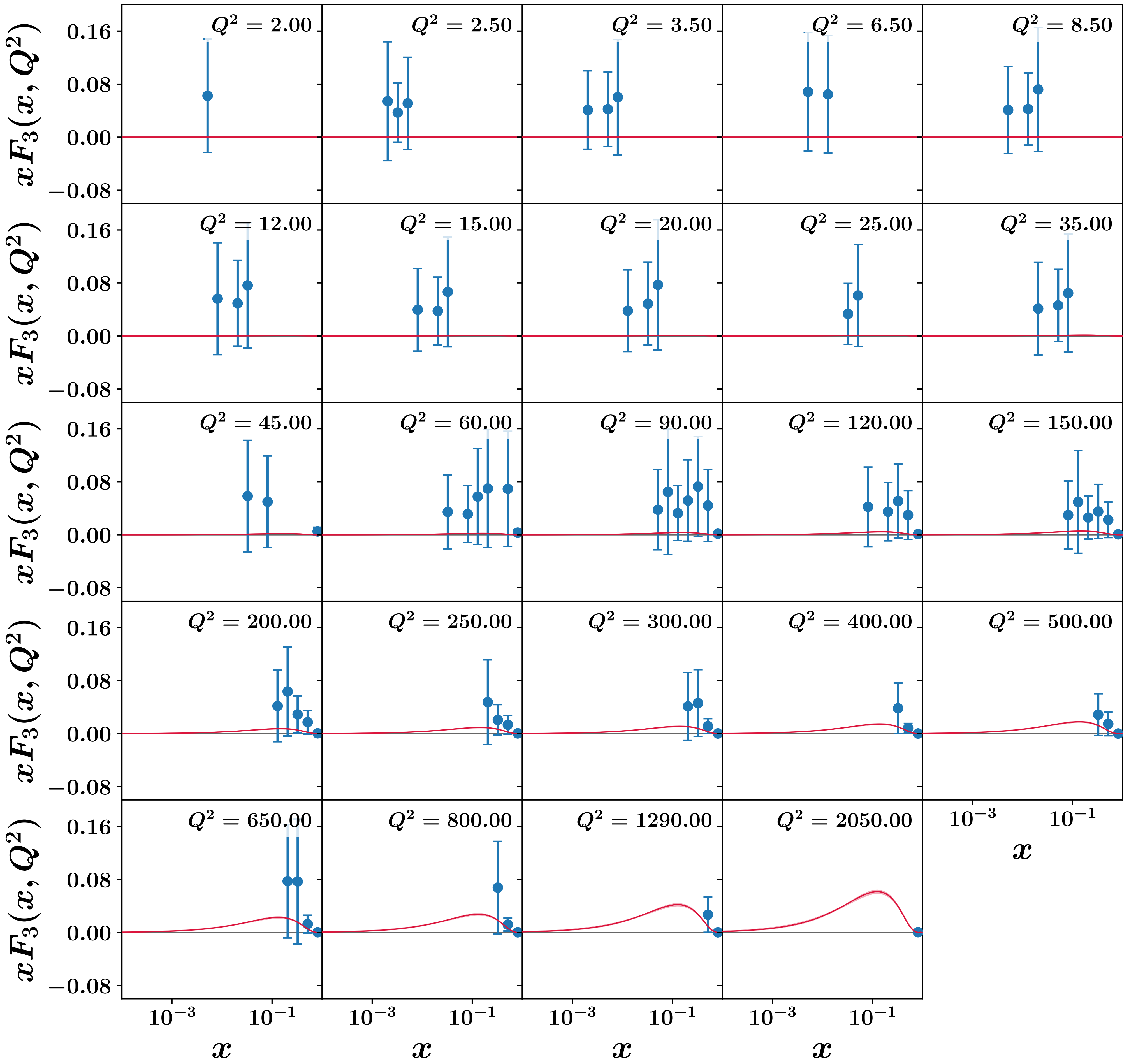
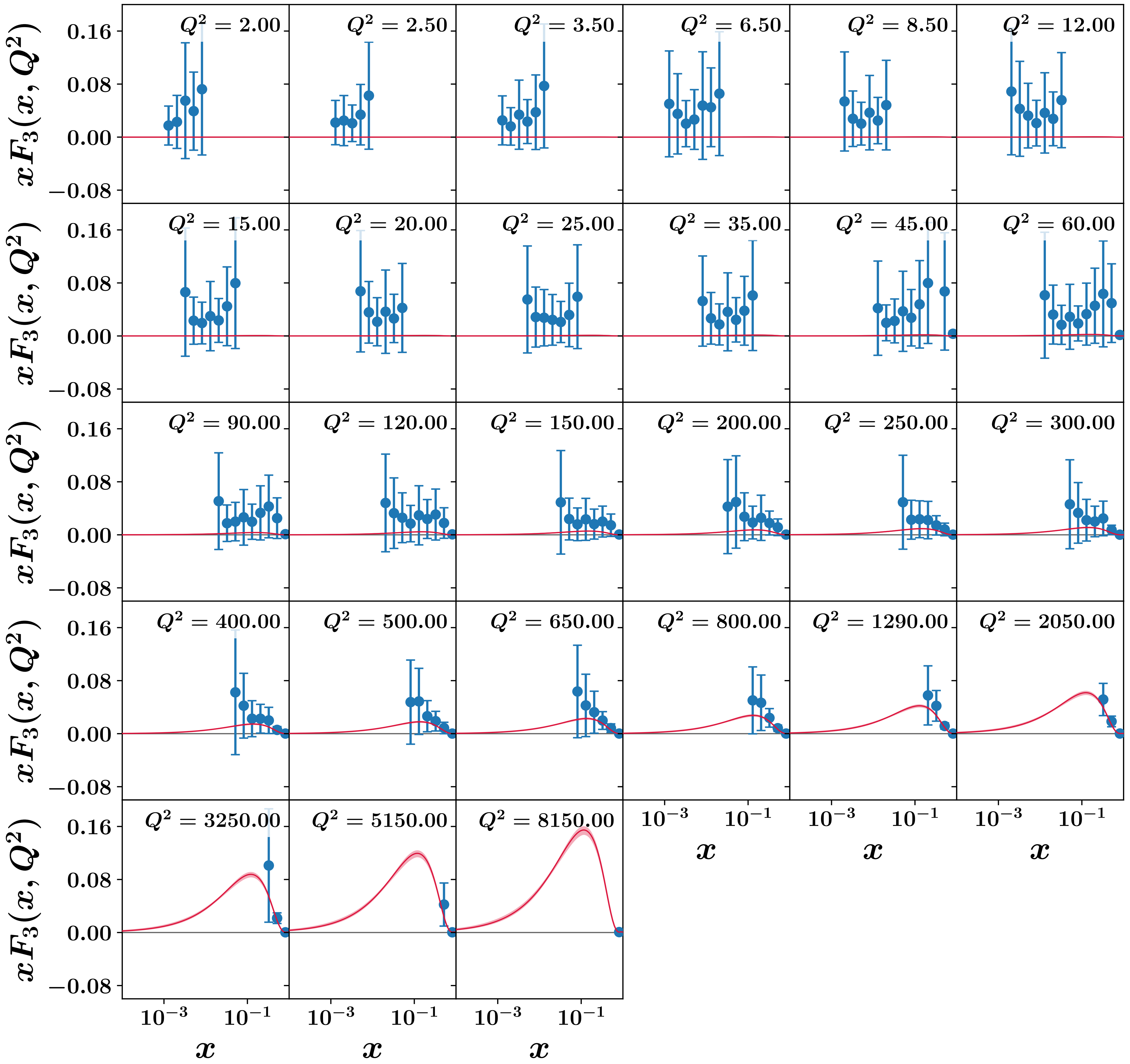
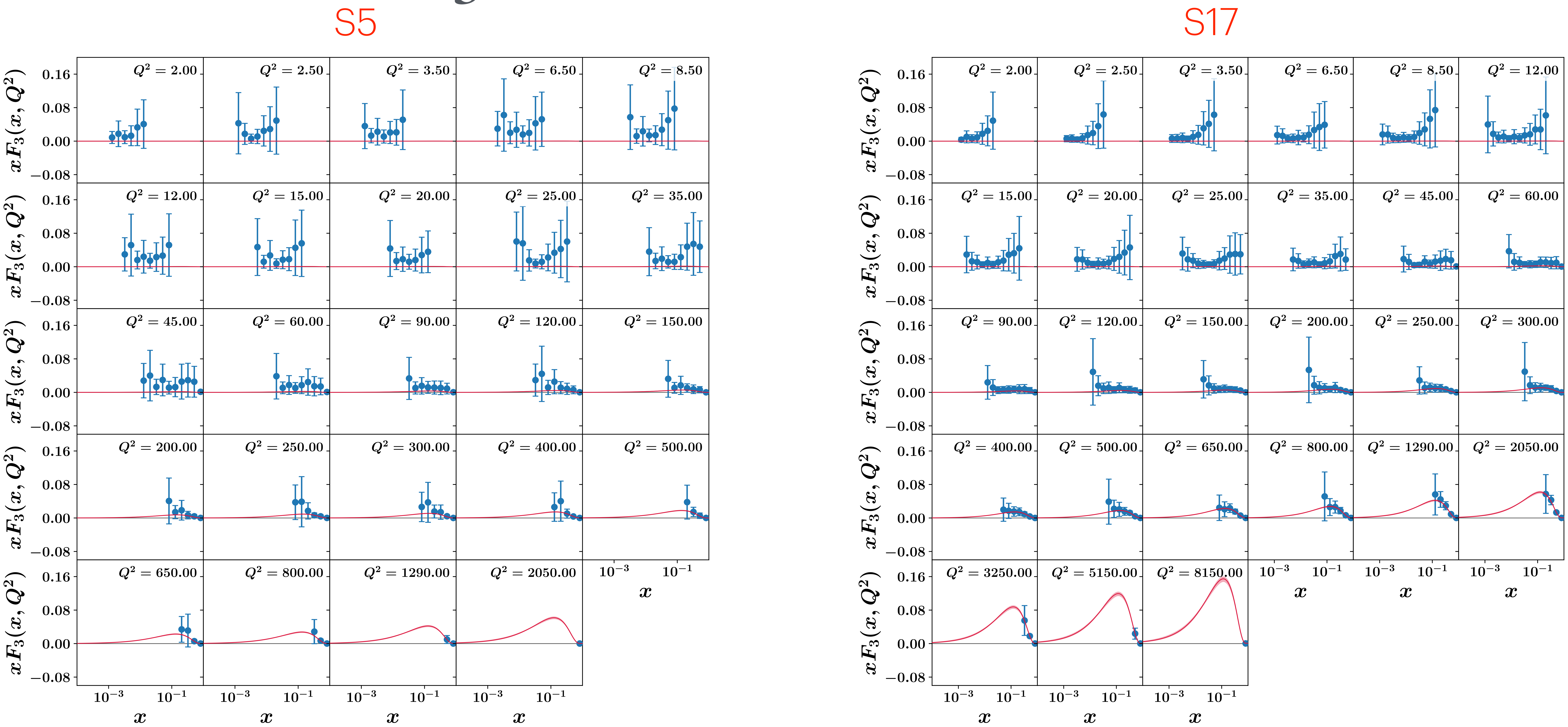


S17



Predictions to compare with are done with HERAPDF2.0 NNLO using the YADISM package (*Eur.Phys.J.C (2024) 84:697)* and points whose uncertainty is above 0.1 are not displayed for visual clarity.

Results for xF_3 (optimistic scenario)



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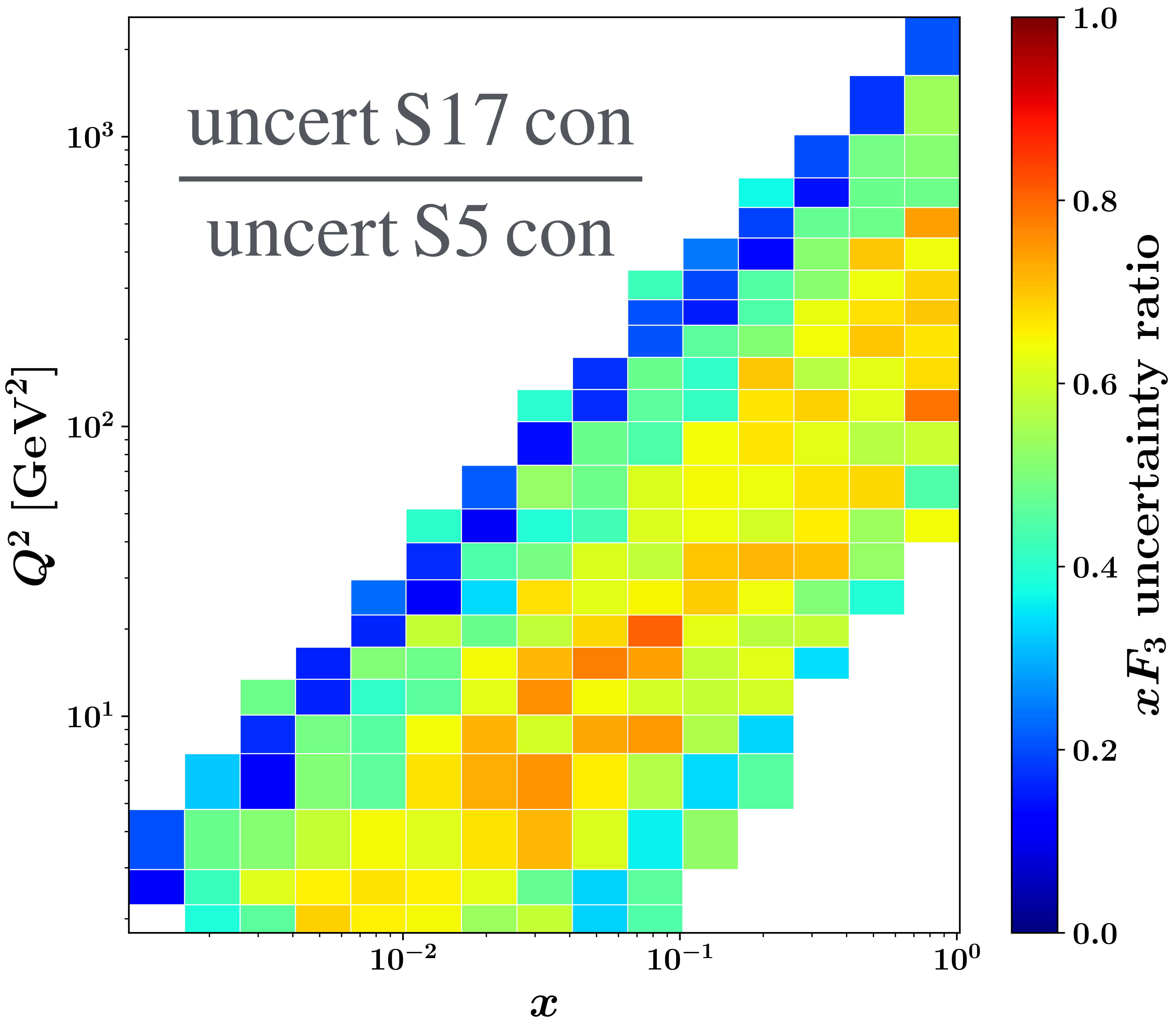
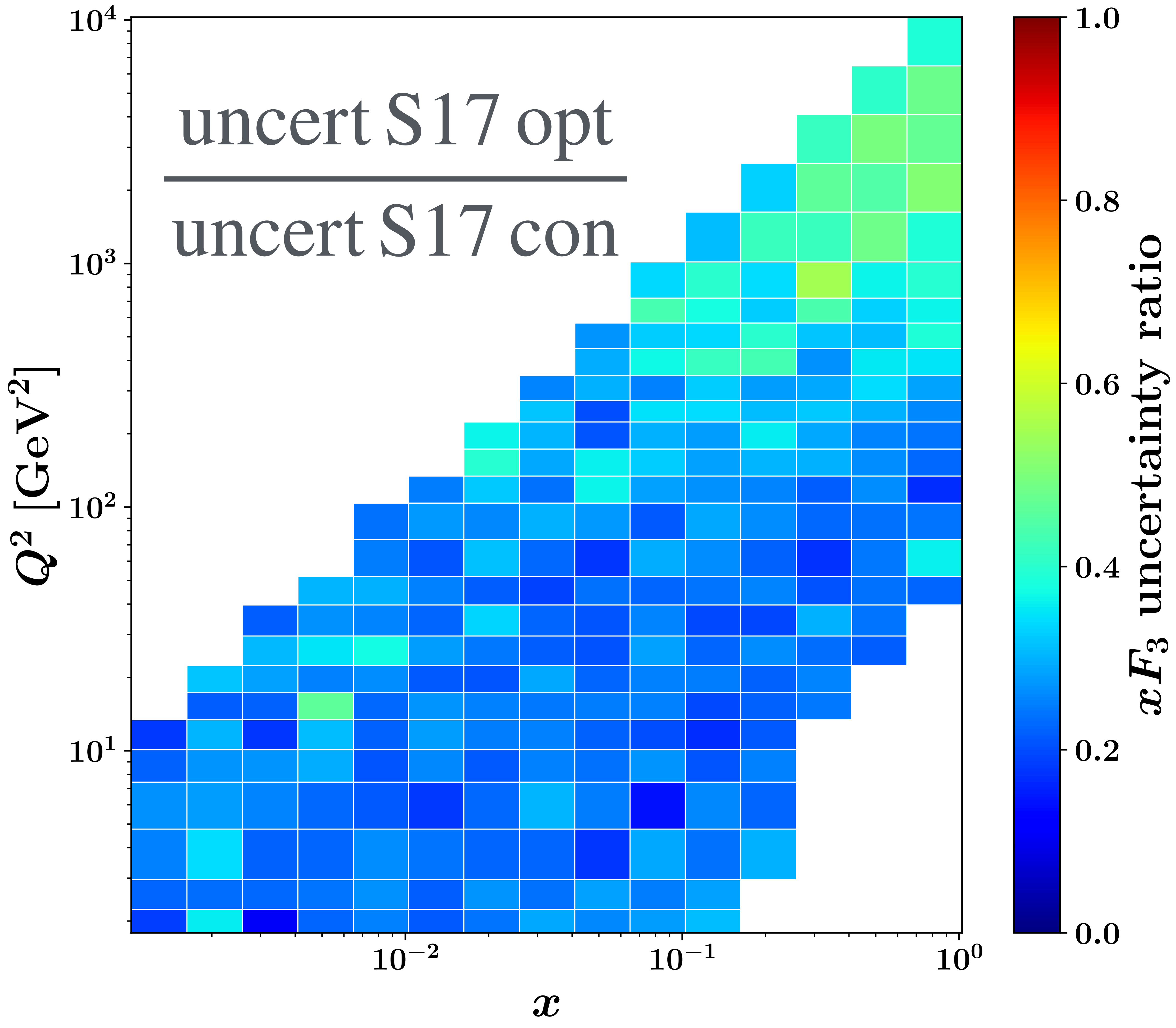
Results for xF_3

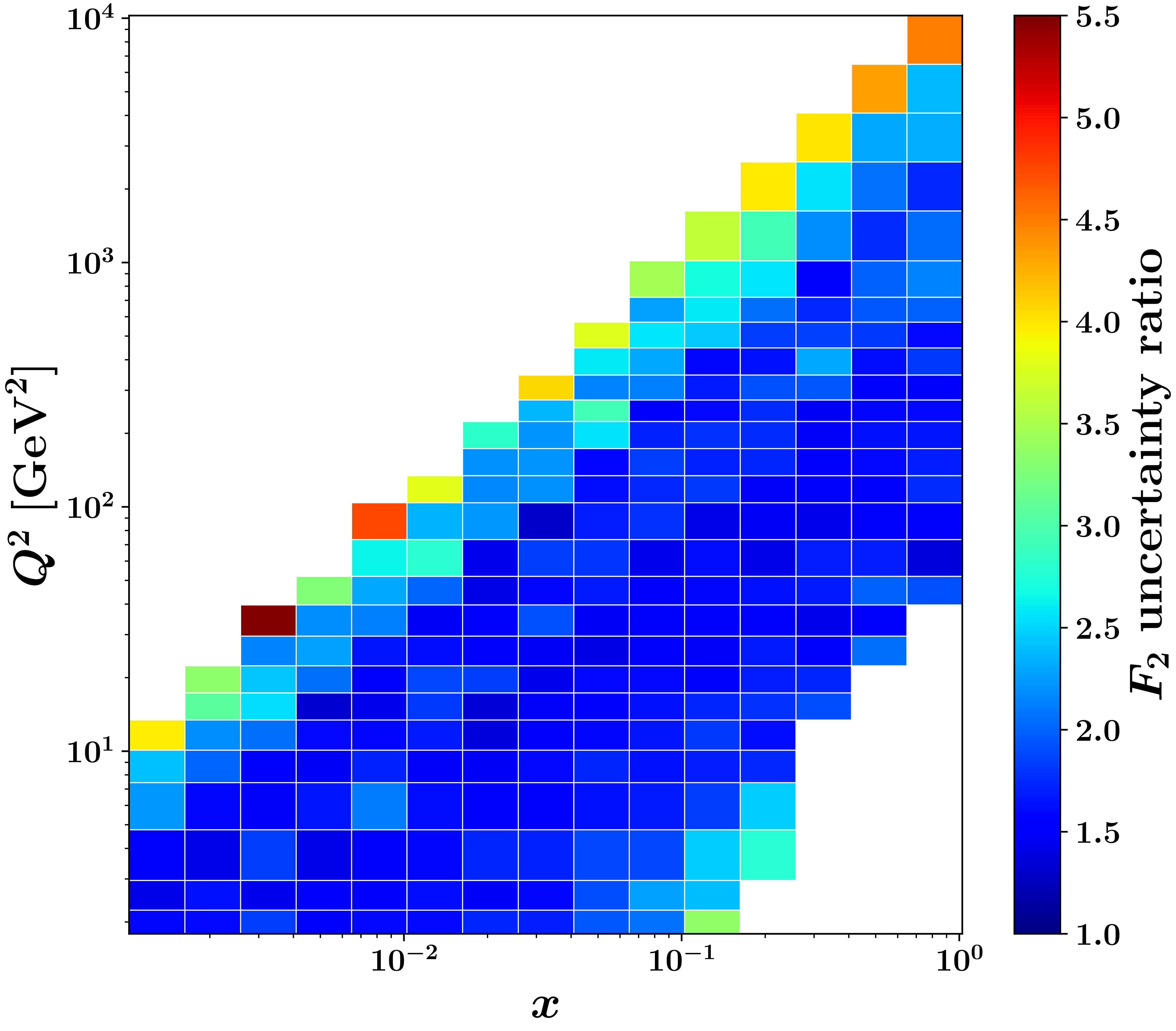


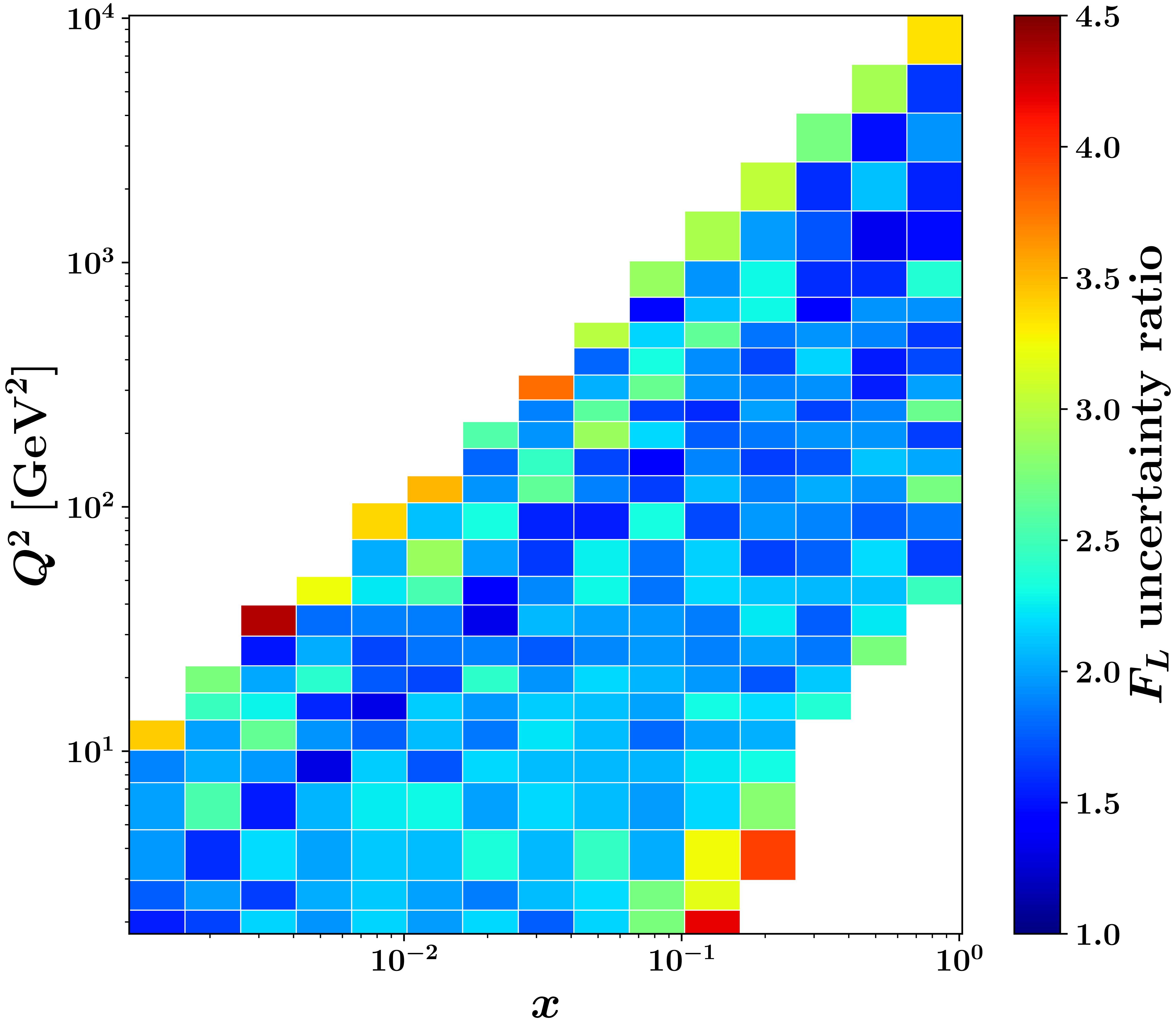


Uncertainties are considerably reduced in the optimistic scenario for the three structure functions.

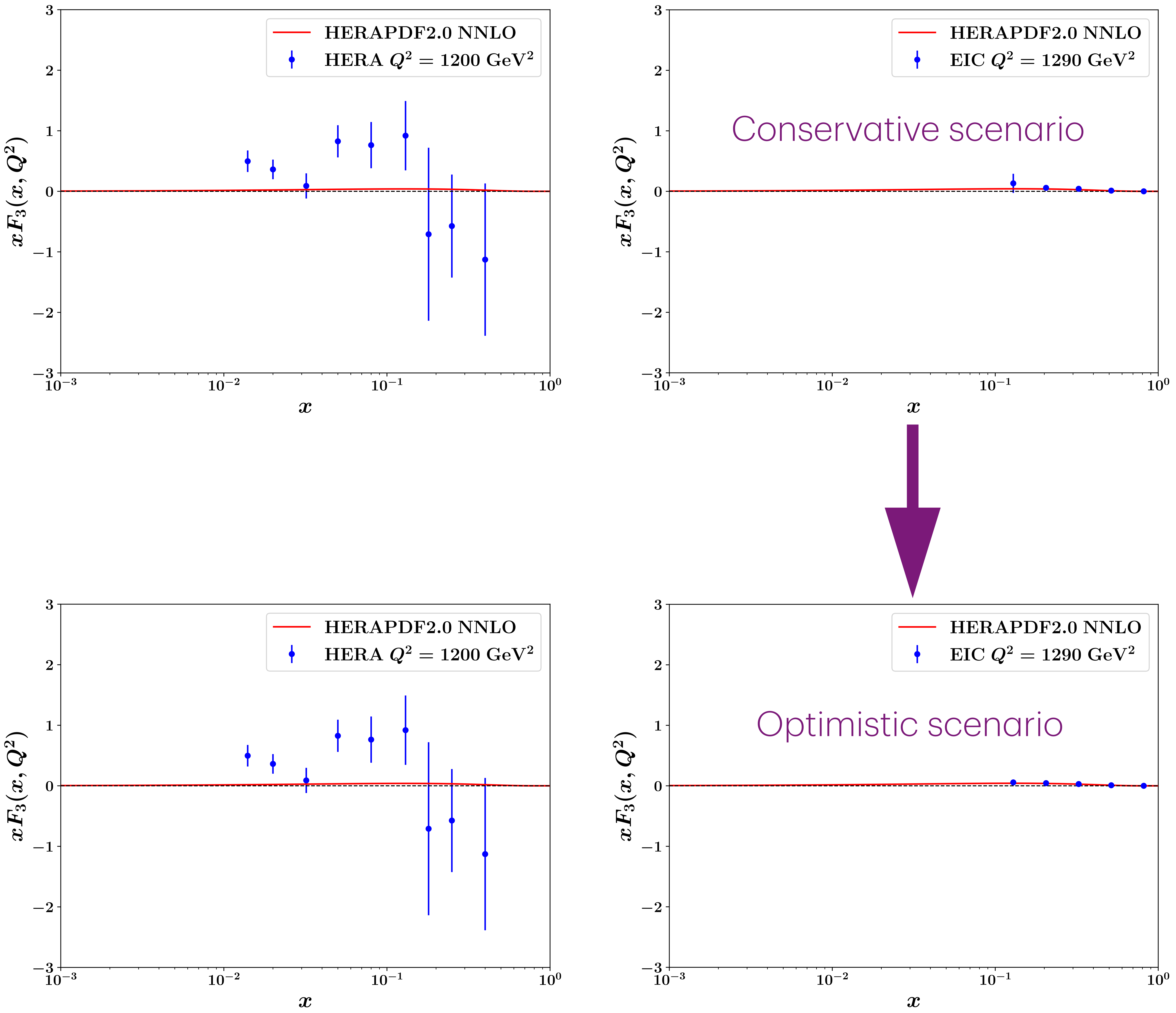
Comparison with Rosenbluth uncertainties (conservative scenario)

$$\frac{\text{uncert S17 full fit}}{\text{uncert S17 Rosenbluth}}$$





Comparison with HERA results



EIC will help to constrain xF_3 in an unexplored region of phase space

+

Unprecedented precision in the extraction of F_2 and F_L

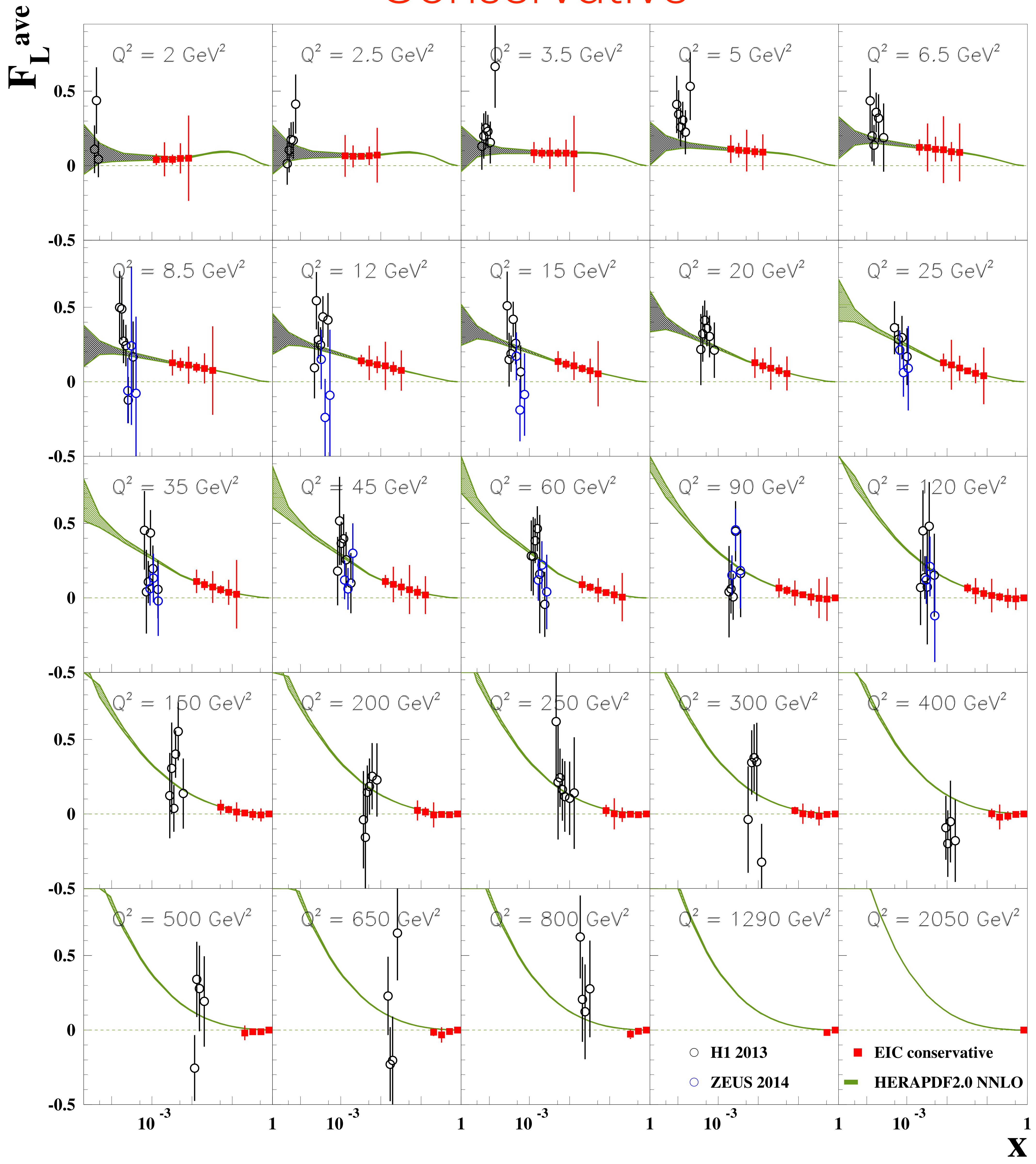
Summary and conclusions

- The three proton structure functions can be simultaneously obtained at the EIC.
- The Rosenbluth-type extraction introduces a potentially measurable bias in the longitudinal structure function, linked to the non-zero contribution of xF_3.
- The number of available beam energy configurations is crucial for these measurements.
- The precision of the extraction is directly linked to the uncertainties on the cross sections and is systematics dominated $\Longrightarrow$ getting close to the optimistic scenario is key for the obtention of the proton structure functions.

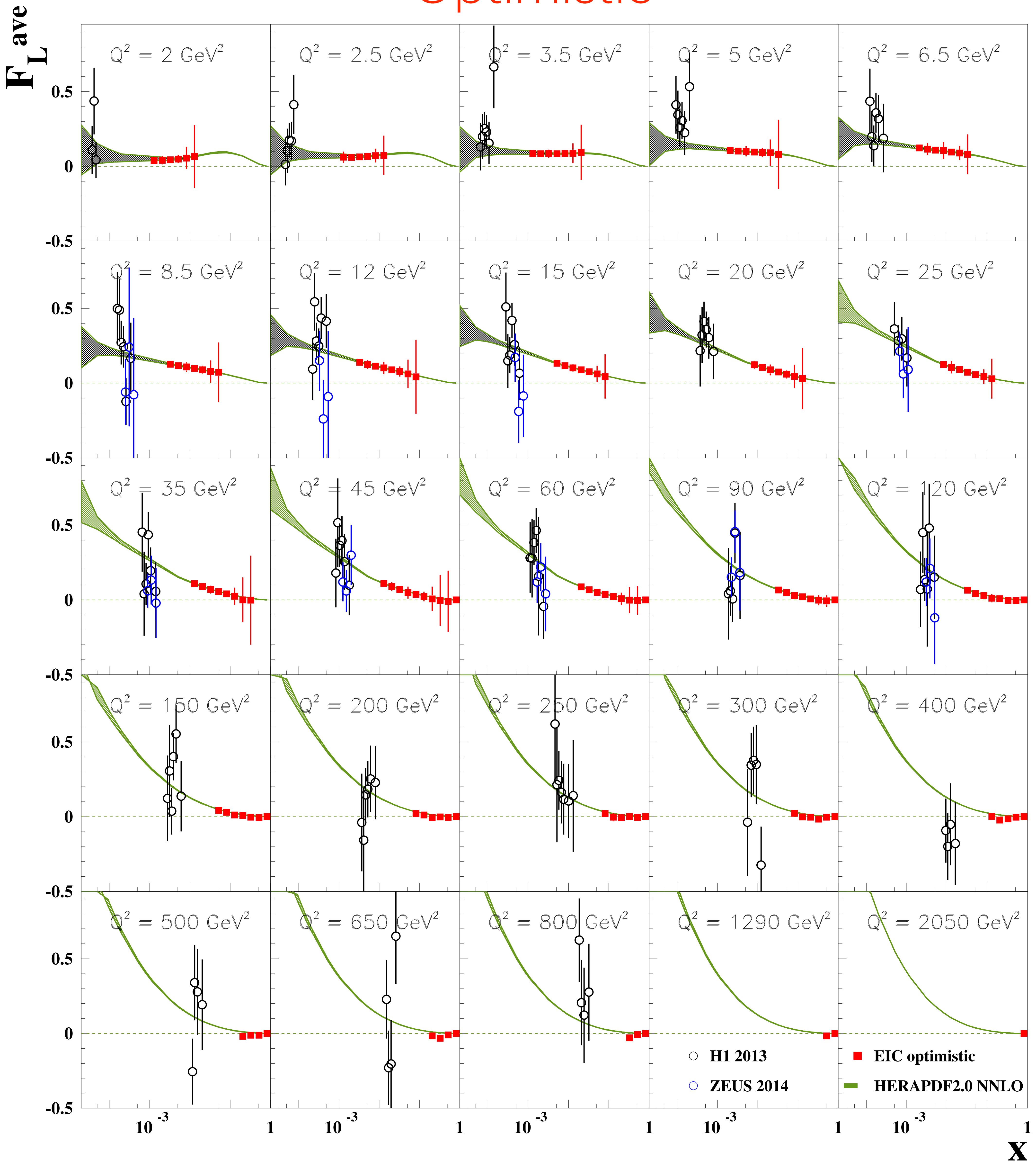
Backup

$F_L(x, Q^2)$ from Rosenbluth fit (conservative vs. optimistic)

Conservative



Optimistic



$F_L(x, Q^2)$ from full cross section fit (conservative vs. optimistic)