Abstract

The fermionic masses and mixings represent the majority of the free parameters of the SM. Decades of attempts to understand their relationships have not left us with a clear understanding. As the quark mass matrix is well measured, making useful predictions depends on the future measurements of neutrino parameters. I will show several broad classifications of neutrino mass model predictions. I will then put them in the context of different anticipated observables. First, I will discuss neutrinoless double beta decay, and focus on the "funnel" region where determining if neutrinos are Majorana becomes practically impossible. Second, I will discuss future oscillation and cosmology measurements and their impact in understanding the nature of flavor.

The Future of Lepton Flavor

Peter B. Denton

HET Group Lunch Discussion

July 17, 2026

2308.09737 PBD, J. Gehrlein
2606.05291 PBD, J. Gehrlein, H. Truelson

Brookhaven National Laboratory

Outline

1. Flavor predictions
2. Neutrinoless double beta decay and the funnel of death
3. Future neutrino measurements
4. Summary

Standard Model Parameters

In addition to the structure of the SM,
we must enumerate:

1. Gauge couplings (3)
2. Higgs vev and mass (2)
3. QCD θ parameter (1)
4. Quark masses and mixings (10)
5. Lepton masses and mixings: low energy (3+7+)

Standard Model Parameters

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Does not include:

1. General relativity
2. Dark matter
3. Dark energy
4. Inflation
5. Baryogenesis
6. Neutrino mass generation
7. Hierarchy?
8. Cosmological constant problem?
9. Strong CP problem?

Standard Model Parameters

One of the remarkable successes of the SM is collapsing many interactions to be governed by a small number of parameters

The majority of the free parameters remaining are the fermion masses and mixings

Can they also be collapsed?

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This is actually a Higgs talk?

Quarks

The quark mass matrix is relatively well measured:

Masses (MeV) and mixing:

m_u	2.16	1.9%
m_d	4.70	0.85%
m_s	92.9	0.43%
m_c	1272.9	0.21%
m_b	4185.9	0.081%
m_t	172 600	0.16%
A	0.826	2.1%
λ	0.2252	0.30%
$\bar{\rho}$	0.158	5.8%
$\bar{\eta}$	0.356	2.0%

PDG

Wolfenstein parameterization:

$$\lambda^2 = \frac{|V_{us}|^2}{|V_{ud}|^2 + |V_{us}|^2}$$

$$A^2\lambda^4 = \frac{|V_{cb}|^2}{|V_{ud}|^2 + |V_{us}|^2}$$

$$\bar{\rho} + i\bar{\eta} = -\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}$$

L. Wolfenstein PRL 1983

Good because mixing is very hierarchical

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Good because mixing is very hierarchical

No qualitative changes within uncertainties

Leptons

Usual parameterization:

$$U = \begin{pmatrix} 1 & & \\ & c_{23} & s_{23} \\ & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & & s_{13}e^{-i\delta} \\ & 1 & \\ -s_{13}e^{i\delta} & & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & \\ -s_{12} & c_{12} & \\ & & 1 \end{pmatrix} \begin{pmatrix} e^{i\alpha/2} & & \\ & e^{i\beta/2} & \\ & & 1 \end{pmatrix}$$

PDG

Good because easy extraction from measurements

PBD, R. Pestes 2006.09384

Leptons

Masses and mixing:

$m^2_{\nu_2} - m^2_{\nu_1}$	7.5×10^{-5} eV^2	1.3%
$\lvert m^2_{\nu_3} - m^2_{\nu_1} \rvert$	2.52×10^{-3} eV^2	1.0% GFs disagree $> 1\sigma$
$\sqrt{\sum_i \lvert U_{ei}\rvert^2 m^2_{\nu_i}}$	< 0.45 eV	90% CL KATRIN 2406.13516
$\sum_i m_{\nu_i}$	< 0.071 eV	95% CL DESI+CMB+ 2411.12022

Analyses disagree with each other,
osc data, and neutrinos

m_e	0.51 MeV	0.000000031%
m_μ	106 MeV	0.0000022%
m_τ	1776.9 MeV	0.0068%

s^2_{12}	0.309	2.2%
s^2_{13}	0.0225	0.30%
s^2_{23}	0.47	5.5% 1/6 of 3σ region GFs disagree
δ/π	1.15	23%

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Significant qualitative changes within uncertainties

Clarity on the flavor puzzle will rely on predictions of remaining parameters in the neutrino sector

Long History of Predictions

An incomplete discussion:

- Texture zeros:

$$\theta_C = \sqrt{\frac{m_d}{m_s}}$$

H. Fritzsch PLB (1977)

- SU(5) motivated, above 10^{15} GeV:

$$m_b = m_\tau \qquad m_s = \frac{1}{3} m_\mu \qquad m_d = 3 m_e \qquad \tan^4 \theta_C = \frac{m_d}{m_s}$$

H. Georgi, C. Jarlskog PLB (1979)
"The factors of 3 appearing in eq. (5)
are suggestive because 3 is an important number
in these theories - it is the number of colors."

  Starts with: $M_d = M_e^T$, uses a 45_H rep to get 1, $\frac{1}{3}$, 3

- $U(1)$ with a vev of the flavon $\Rightarrow \lambda = \langle \Phi \rangle / \Lambda$
  Couplings are then λ^n

C. Froggatt, H. Nielsen NPB (1979)

Long History of Predictions: Post 1998

- Tribimaximal

$$s_{12}^2 = \frac{1}{3} \qquad s_{23}^2 = 0.5 \qquad s_{13}^2 = 0$$

P. Harrison, D. Perkins, W. Scott hep-ph/0202074

- TBM derived from spontaneous breaking of non-Abelian discrete symmetries

$$A_4,\ S_4,\ A_5$$

E.g. G. Altarelli, F. Feruglio hep-ph/0504165

Long History of Predictions: Post 2012

- Anarchy: uniformly sampled across Harr measure:

$$dU = ds_{12}^2 \wedge dc_{13}^4 \wedge ds_{23}^2 \wedge d\delta$$

L. Hall, H. Murayama, N. Weiner hep-ph/9911341
A. de Gouvea, H. Murayama 1204.1249

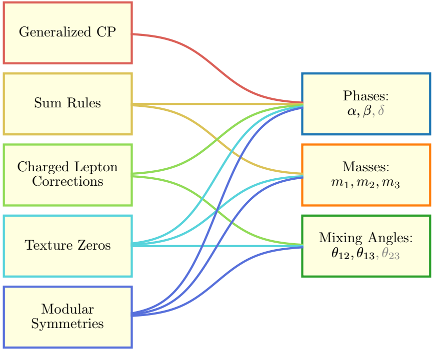
Modifications for Dirac, Majorana, Seesaw, etc.

- Modular symmetries, generalization of discrete symmetries
  vev of complex modulus τ leads to flavor symmetry breaking
  Based on $SL(2,\mathbb{Z})$, symmetry of a torus
  Fewer parameters than FN
  Can connect to mass sum rule formalism

F. Feruglio 1706.08749
J. Gehrlein, M. Spinrath 2012.04131

Flavor Model Prediction Classes

- Generalized CP
- Mass sum rules
- Charged lepton corrections
- Texture zeros
- Modular symmetries
- Constrained sequential dominance



Generalized CP

If CP is preserved in the lepton sector, Majorana phases:

$$\alpha, \beta \in \{0, \pi\}$$

L. Wolfenstein PLB (1981)
B. Kayers PRD (1984)

Others predict $\alpha, \beta \in \{\pi/2, 3\pi/2\}$ from $\Delta(48)$, $\Delta(6n^2)$ family symmetry

G. Ding, Y. Zhou 1312.5222
S. King, T. Neder 1403.1758

We generically consider Majorana phases $\alpha, \beta \in \{0, \pi/2, \pi, 3\pi/2\}$

J. Penedo, S. Petcov 1806.03203

Mass Sum Rules

- Generic structure of the form:

$$C_1 e^{i\chi_1}\bar{m}_1^d + C_2 e^{i\chi_2}\bar{m}_2^d + m_3^d = 0$$

$\bar{m}_1 = m_1 e^{i\alpha}$, $\bar{m}_2 = m_2 e^{i\beta}$

- Provide effective prediction from many underlying models

J. Barry, W. Rodejohann 1007.5217

- Given Δm^2's, for a given m_{lightest}, two constraints ($\Re$, $\Im$) and two unknowns (α, β)

- Also shuffle all parameters together

A collection of models mapped on to sum rules yields:

c_1	c_2	d	χ_1	χ_2
1	1	1	π	π
1	2	1	π	π
1	2	1	π	0
1/2	1/2	1	π	π
$\frac{2}{\sqrt{3}+1}$	$\frac{\sqrt{3}-1}{\sqrt{3}+1}$	1	0	π
1	1	-1	π	π
1	2	-1	π	0
1	2	-1	0	π
1	2	-1	π	$\pi/2, 3\pi/2$
1	2	1/2	$\pi, 0, \pi/2$	$0, \pi, \pi/2$
1/3	1	1/2	π	0
1/2	1/2	$-1/2$	π	π

J. Gehrlein, M. Spinrath 1704.02371

Charged Lepton Corrections

- What is measured, U, is actually $U=U_e^\dagger U_\nu$
- Perhaps $U_\nu\sim$TBM,... and $U_e\sim U_{\rm CKM}$
- U_e could come from $SU(5)$, $SO(10)$, ...

H. Georgi, S. Glashow PRL (1974)
H. Georgi AIP (1975)
H. Fritzsch, P. Minkowski AP (1975)

- Structure of U_ν typically has $\theta_{23}^\nu=45^\circ$ and

$$\sin^2\theta_{12}^\nu\in\{1/3,1/2,1/(2+\phi_g),(3-\phi_g)/4,1/4\}$$

$\phi_g=(1+\sqrt{5})/2$

Also consider:

$$\theta_{13}^\nu\{\pi/10,\pi/20,\arcsin(1/3)\}$$

Free complex phases
Finally, we consider 1 or 2 CL rotations, but no more than 4 rotations total
Never over-cover the space

Texture Zeros

- Assume some elements in symmetric mass matrix are zero
- Connections to extended scalar sector with Abelian symmetries

  W. Grimus, et al. hep-ph/0405016
- All three texture zeros ruled out early

  P. Frampton, S. Glashow, D. Marfatia hep-ph/0201008
- Note that if $M_{ee}=0$ then $\Gamma_{0\nu\beta\beta}=0$

Modular Symmetries

- Symmetry breaking leads to mass sum rule constraints
- "Constants" now depend on other mixing parameters
- Often two free parameters predict 9 parameters (3 masses, 3 mixing angles, 3 complex phases)
- In practice, one is usually constrained by measurement of θ_{13}

Modular Symmetries

One example based on A_4:

$$\sin^2\theta_{12}(\theta) = \frac{1}{3-2\sin^2\theta} \quad , \qquad \sin^2\theta_{13}(\theta) = \frac{2}{3}\sin^2\theta$$

$$\sin^2\theta_{23}(\theta,\phi) = \frac{1}{2} + \frac{\sin\theta_{13}(\theta)}{2}\frac{\sqrt{2-3\sin^2\theta_{13}(\theta)}}{1-\sin^2\theta_{13}(\theta)}\cos\phi$$

$$\delta(\theta,\phi) = \arcsin\left(-\frac{\sin\phi}{\sin 2\theta_{23}(\theta,\phi)}\right)$$

Mass sum rule takes form:

$$C_1 e^{i\chi_1} = -\mathrm{e}^{-2i\phi}\frac{\sqrt{3}\sin(2\theta)-\cos\phi\cos(2\theta)-i\sin\phi}{\sqrt{3}\sin(2\theta)-\cos\phi\cos(2\theta)+i\sin\phi}$$

$$C_2 e^{i\chi_2} = -\mathrm{e}^{-i\phi}\frac{2}{\sqrt{3}\sin(2\theta)-\cos\phi\cos(2\theta)+i\sin\phi}$$

$$d = 1$$

θ, ϕ are free parameters

P. Novichkov, S. Petcov, M. Tanimoto 1812.11289

Constrained Sequential Dominance

Two RH neutrinos in a seesaw

AKA Littlest See-Saw

Defined as $CSD(n)$ where first two columns of mass matrix are:

$$M_\nu = a \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & -1 & 1 \end{pmatrix} + be^{i\eta} \begin{pmatrix} 1 & n & 2-n \\ n & n^2 & n(2-n) \\ 2-n & n(2-n) & (2-n)^2 \end{pmatrix}$$

Given n, parameters are a/b and η

η is sometimes fixed by $\mathbb{Z}_3$; we take it as free

S. King hep-ph/9904210

Based on literature, we consider:

$n=3$ S. King 1304.6264, P. Ballet, et al. 1612.01999
$n=4$ S. King 1305.4846; $n=-1/2$ P. Chen, et al. 1906.11414
$n=1\pm\sqrt{6}$ I. Varzielas, S. King, M. Levy 2211.00654

Neutrinoless Double Beta Decay

If neutrinos are Majorana, they may exhibit $(A,Z) \to (A,Z+2) + e^- + e^-$

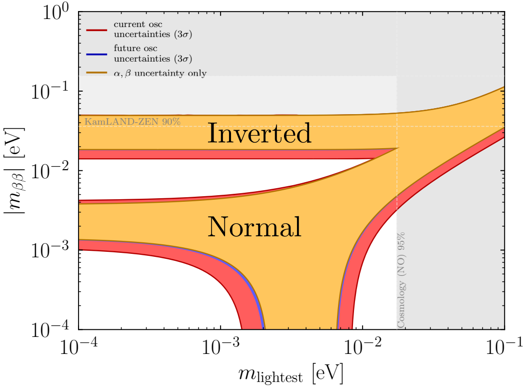
$$\Gamma_{0\nu\beta\beta} = G^{0\nu}|\mathcal{M}^{0\nu}|^2|m_{\beta\beta}|^2$$

$$|m_{\beta\beta}| = \sum_i m_i U_{ei}^2$$

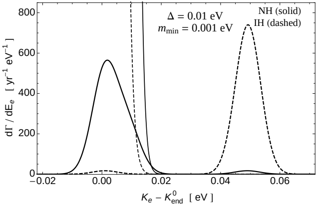
Depends on Majorana phases α, β

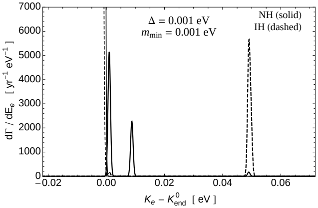
Mass Ordering: Broad Implications

- Affects cosmology
- Affects $0\nu\beta\beta$
- Affects end point measurements
- Affects CνB



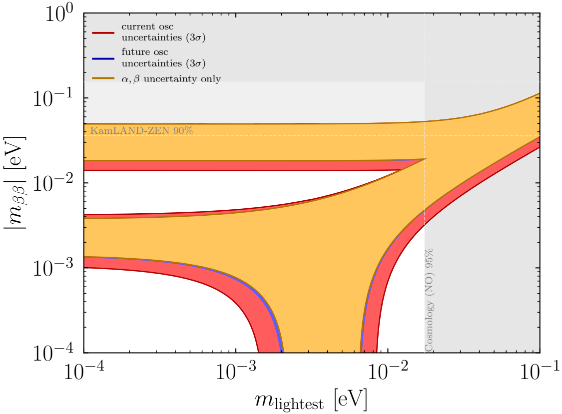
PBD, J. Gehrlein 2308.09737





A. Long, C. Lunardini, E. Sabancilar 1405.7654

Neutrinoless Double Beta Decay Parameter Space



Funnel below $|m_{\beta\beta}| = 10^{-3}$ eV is essentially impossible to probe

Neutrinoless Double Beta Decay Analysis Strategy

1. Compute significances correctly: $\Delta\chi^2 = 11.83$ (3σ for 2 dof)

   Most studies vary each parameter
   up to $\Delta\chi^2 = 9$ (3σ for 1 dof)
   which can both over and under cover

2. If experiments reach the funnel, should we anticipate that that is a predicted area of parameter space?
3. First determine if model predictions are viable
4. Determine if model reaches into funnel at all
5. Determine what fraction of the model predictions are in the funnel

Numerical Approach

- Each class depends on underlying parameters in fundamentally different ways
- Define the funnel fraction as:

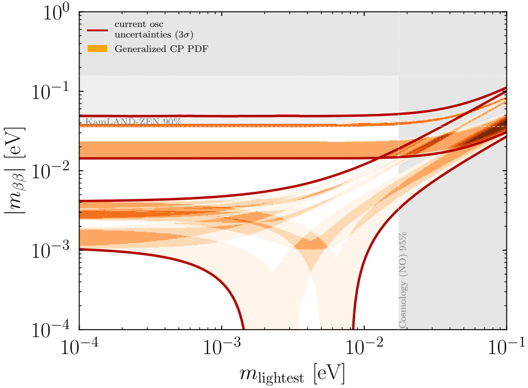
$$f = \frac{\int_{\text{funnel}} d\log m_{\text{lightest}} d\log m_{\beta\beta}}{\int d\log m_{\text{lightest}} d\log m_{\beta\beta}}$$

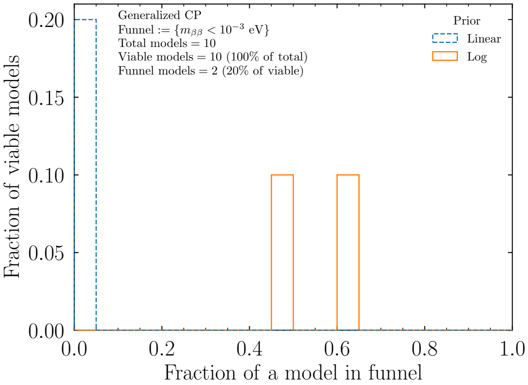
  Integrate only over NO

  MO will be well known by then

- Consider both log space for masses and linear space

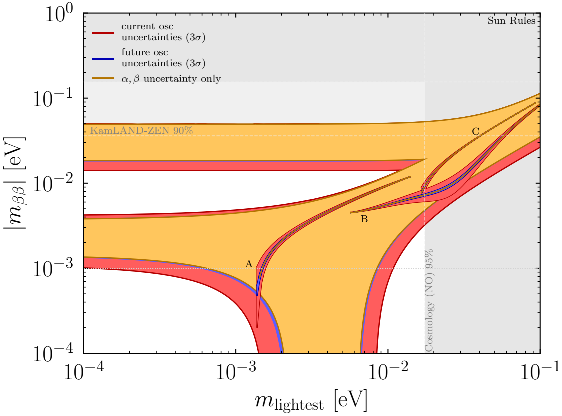
Funnel: Generalized CP





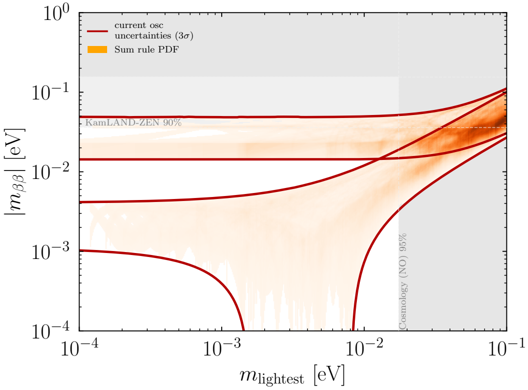
$$(\alpha, \beta) = (0, \pi) \text{ or } = (\pi, 0) \Rightarrow \text{funnel}$$

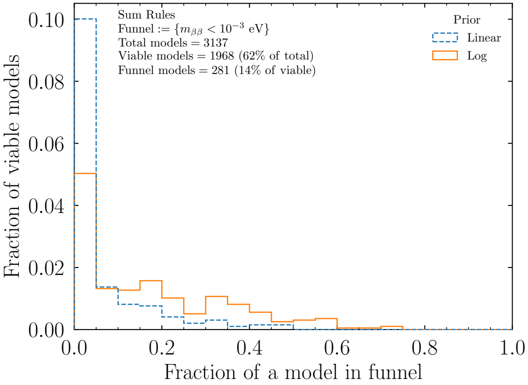
Funnel: Mass Sum Rules: Example



	c_1	c_2	d	χ_1	χ_2
A	1	2	$\frac{1}{2}$	π	$\frac{\pi}{2}$
B	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	π	π
C	1	2	1	π	0

Funnel: Mass Sum Rules

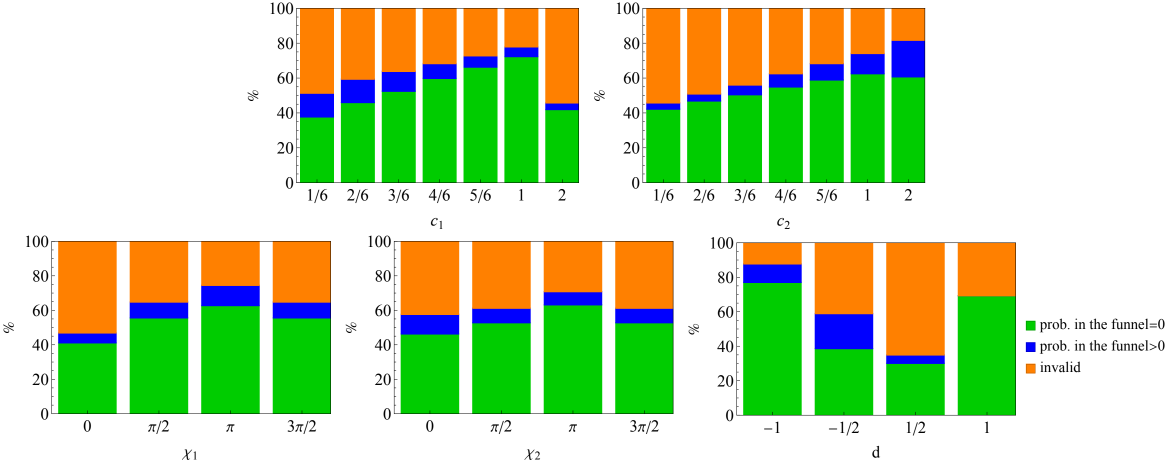




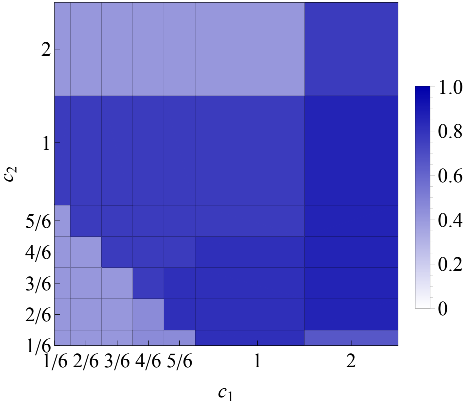
Funnel: Mass Sum Rules

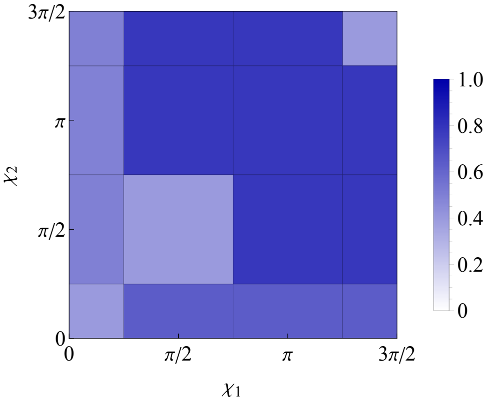
c_1	c_2	d	χ_1	χ_2	Fraction in funnel
1	2	$-1/2$	$\pi/2$	0	0.74
1	2	$-1/2$	$3\pi/2$	0	0.74
4/6	1	$-1/2$	$3\pi/2$	0	0.67
4/6	1	$-1/2$	$\pi/2$	0	0.62
5/6	1	$-1/2$	$3\pi/2$	0	0.59
5/6	1	$-1/2$	$\pi/2$	0	0.58
5/6	2	$-1/2$	$\pi/2$	0	0.58
5/6	2	$-1/2$	$3\pi/2$	0	0.58
1	2	$-1/2$	0	$\pi/2$	0.58
1	2	$-1/2$	0	$3\pi/2$	0.58
1/3	1	$-1/2$	0	$\pi/2$	0.56
4/6	5/6	$-1/2$	$\pi/2$	0	0.54
4/6	5/6	$-1/2$	$3\pi/2$	0	0.54
1/6	1/6	$-1/2$	π	π	0.54
1/3	1	$-1/2$	0	$3\pi/2$	0.54
1/6	1/2	-1	0	0	0.51
1/6	4/6	-1	0	0	0.51

Funnel: Mass Sum Rules Parameters



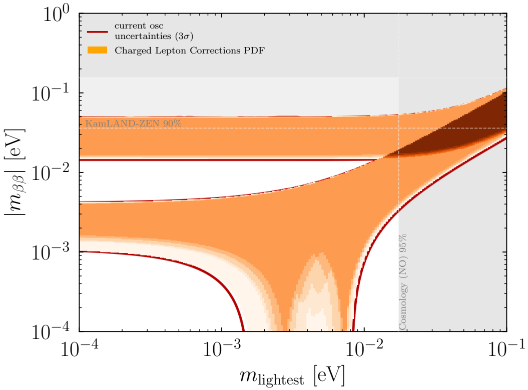
Funnel: Mass Sum Rules Parameters and Correlations

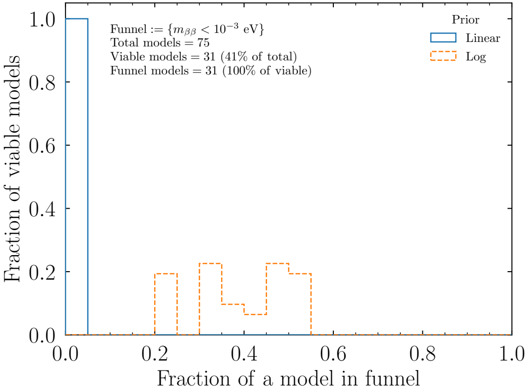




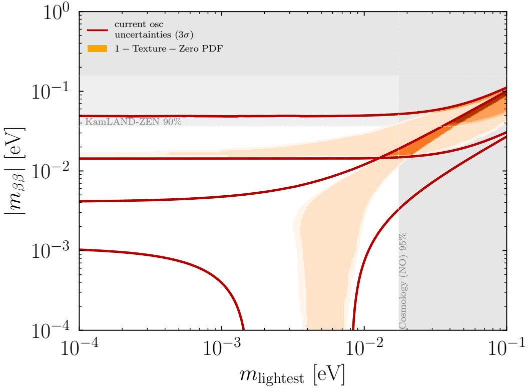
Validity fraction

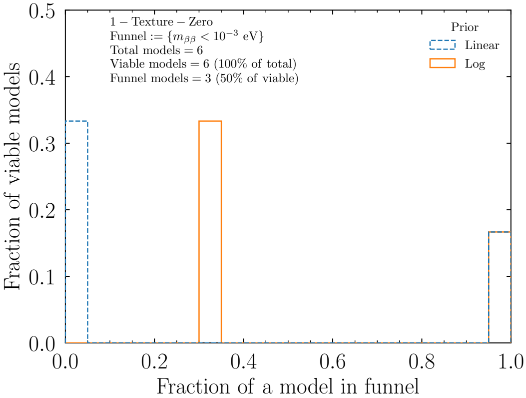
Funnel: Charged Lepton Corrections





Funnel: One Texture Zeros



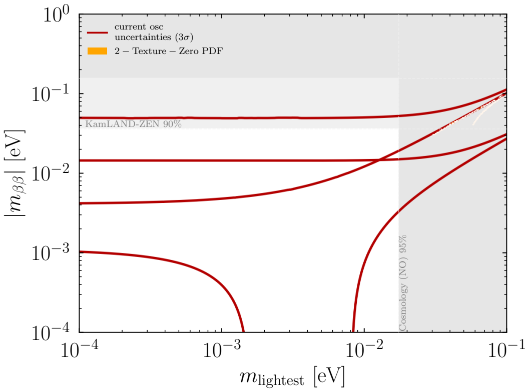


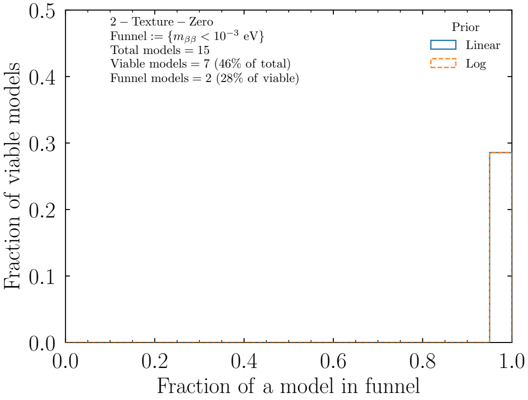
Funnel: One Texture Zeros

	Fraction in funnel
M_{ee}	1
$M_{e\mu}$	0.31
$M_{e\tau}$	0.30
$M_{\mu\mu}$	0
$M_{\mu\tau}$	0
$M_{\tau\tau}$	0

- All one texture zeros are valid, but with MO dependence
- $M_{ee} = 0 \Rightarrow$ no $0\nu\beta\beta$
- $M_{e\mu} = 0$ and $M_{e\tau} = 0$ have significant parameter space in funnel
- $M_{e\mu} = 0$ and $M_{e\tau} = 0$ have lower bound on $m_{\rm lightest} \sim 4 \times 10^{-3}$ eV in NO

Funnel: Two Texture Zeros





Funnel: Two Texture Zeros

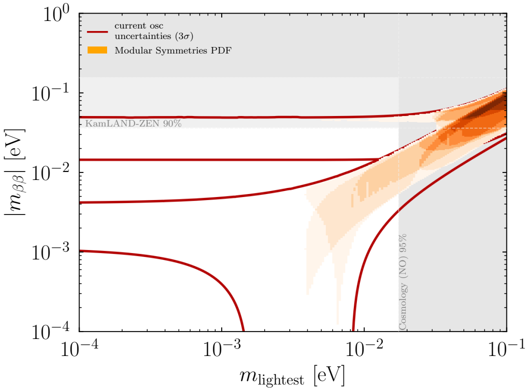
	$M_{e\mu}$	$M_{e\tau}$	$M_{\mu\mu}$	$M_{\mu\tau}$	$M_{\tau\tau}$
M_{ee}	1	1	X	X	X
$M_{e\mu}$		X	0	X	0
$M_{e\tau}$			0	X	0
$M_{\mu\mu}$				X	0
$M_{\mu\tau}$					X

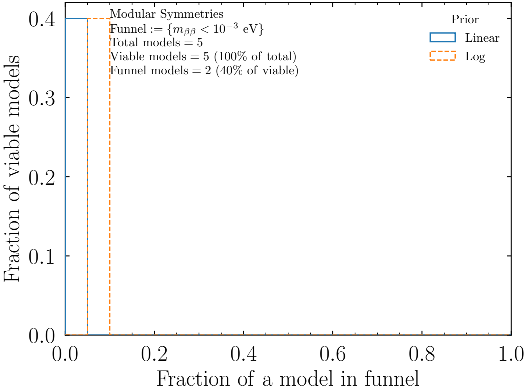
- 7/15 two texture zeros valid with oscillation data
- Showed that, due to improvements in cosmology and precision in oscillations, only two remain valid:

$$M_{ee} = M_{e\mu} = 0 \quad , \quad M_{ee} = M_{e\tau} = 0$$

Both predict no $0\nu\beta\beta$

Funnel: Modular Symmetries





Funnel: Summary

Model	Total	Viable	Fraction of viable models in funnel
Generalized CP	10	10	0.20
Mass sum rules	3137	1968	0.14
Charged lepton corrections	75	31	1.00
1-Texture zeros	6	6	0.50
2-Texture zeros	15	7	0.28
Modular symmetries	5	5	0.40

- Carefully calculated validity of many model predictions
- Most valid models in the NO predict large $|m_{\beta\beta}|$
- Significant fraction have some parameter space with $|m_{\beta\beta}|$ very small
- Some model predictions are dominantly in the funnel

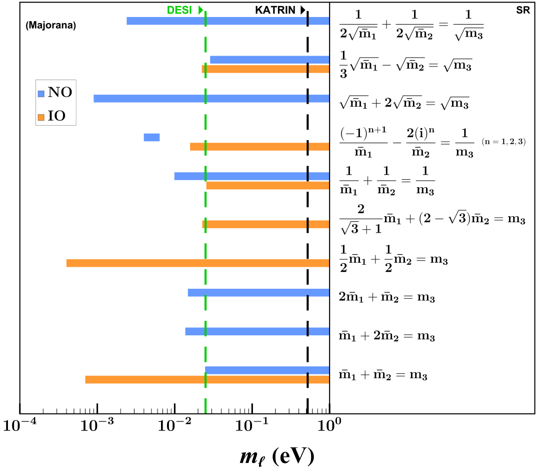
Role of Future Neutrino Measurements

In the coming years we will learn:

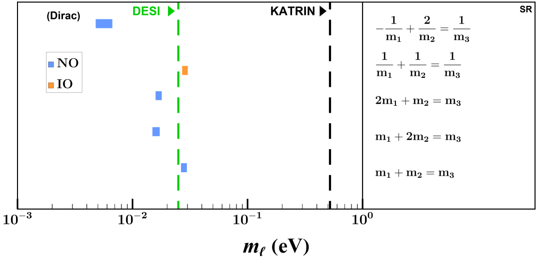
1. The atmospheric mass ordering
2. Improved precision on θ_{12}, Δm^2_{21}, and $|\Delta m^2_{31}|$
3. The octant of θ_{23}
4. The complex CP violating phase δ
5. The absolute mass scale

How will this inform our ability to identify flavor model predictions?

Future: Mass Sum Rules



Majorana



Dirac

Future: Comparing Model Predictions

Define an overlap function:

$$R(A||B) = \int \frac{\mathcal{L}(D|\theta_i, B)}{\mathcal{L}(D|\theta_i, A)} P(\theta_i|D, A) d\theta_i$$

- A is true model, B is test model, D is data, θ_i are the neutrino parameters
- $R \to 0 \Rightarrow$ models are completely disjoint; if A is true, then B can be disfavored
- $R \to 1 \Rightarrow$ models completely overlap; every scenario where A is true, B is also true
- $R(A||B) \neq R(B||A)$
- Use 6 years of JUNO for solar parameters, 10 years of DUNE/HK for θ_{23}, δ

Future: Mass Sum Rules: Differentiation

Majorana: NO

TRUE / TEST	$\{1,1,1\}$	$\{1,2,1\}$	$\{2,1,1\}$	$\{1,1,-1\}$	$\{1,2,-1\}$	$\{1,2,\frac{1}{2}\}$	$\{\frac{1}{3},1,\frac{1}{2}\}$	$\{\frac{1}{2},\frac{1}{2},-\frac{1}{2}\}$
$\{1,1,1\}$	—	**0.80**	**0.81**	**0.70**	**0**	**0.76**	**0.97**	**0.58**
$\{1,2,1\}$	**1**	—	**1**	**0.88**	**0**	**0.96**	**1**	**0.72**
$\{2,1,1\}$	**1**	**0.98**	—	**0.86**	**0**	**0.94**	**1**	**0.71**
$\{1,1,-1\}$	**1**	**1**	**1**	—	**0**	**1**	**1**	**0.82**
$\{1,2,-1\}$	**0**	**0**	**0**	**0**	—	**0**	**0**	**0.02**
$\{1,2,\frac{1}{2}\}$	**1**	**1**	**1**	**0.92**	**0**	—	**1**	**0.76**
$\{\frac{1}{3},1,\frac{1}{2}\}$	**0.98**	**0.78**	**0.80**	**0.70**	**0**	**0.74**	—	**0.55**
$\{\frac{1}{2},\frac{1}{2},-\frac{1}{2}\}$	**1**	**1**	**1**	**1**	**0.12**	**1**	**1**	—

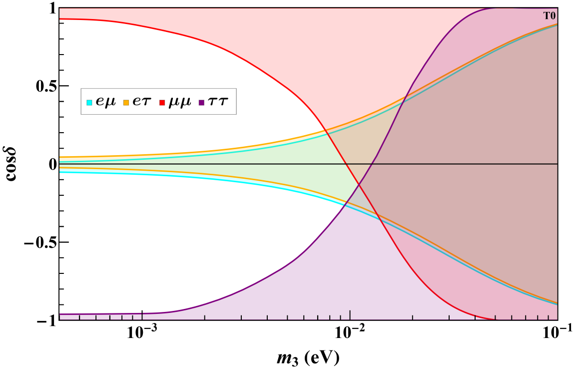
Future: Mass Sum Rules: Differentiation

Majorana: IO

TEST \ TRUE	$\{1,1,1\}$	$\{\frac{1}{2},\frac{1}{2},1\}$	$\{\frac{2}{\sqrt{3}+1},2-\sqrt{3},1\}$	$\{1,1,-1\}$	$\{1,2,-1\}$	$\{\frac{1}{3},1,\frac{1}{2}\}$
$\{1,1,1\}$	—	**0.90**	**1**	**1**	**1**	**1**
$\{\frac{1}{2},\frac{1}{2},1\}$	**1**	—	**1**	**1**	**1**	**1**
$\{\frac{2}{\sqrt{3}+1},2-\sqrt{3},1\}$	**0.39**	**0.35**	—	**1**	**0.86**	**1**
$\{1,1,-1\}$	**0.37**	**0.33**	**0.95**	—	**0.82**	**0.95**
$\{1,2,-1\}$	**0.45**	**0.40**	**1**	**1**	—	**1**
$\{\frac{1}{3},1,\frac{1}{2}\}$	**0.39**	**0.34**	**1**	**1**	**0.85**	—

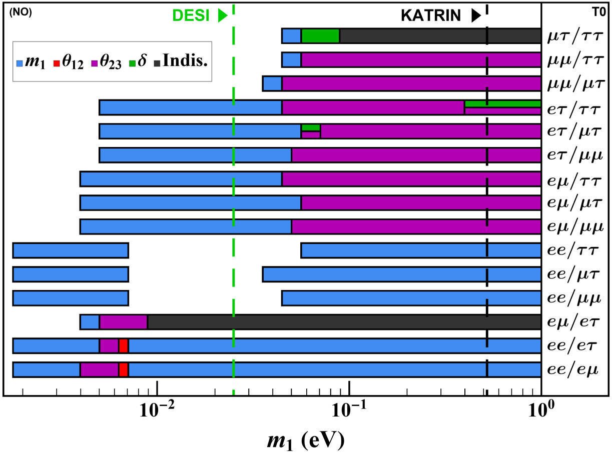
Future: One Texture Zeros

IO:



If mass scale is low, measuring $\cos\delta$ is key

Future: One Texture Zeros



Future: One Texture Zeros: Differentiation

NO

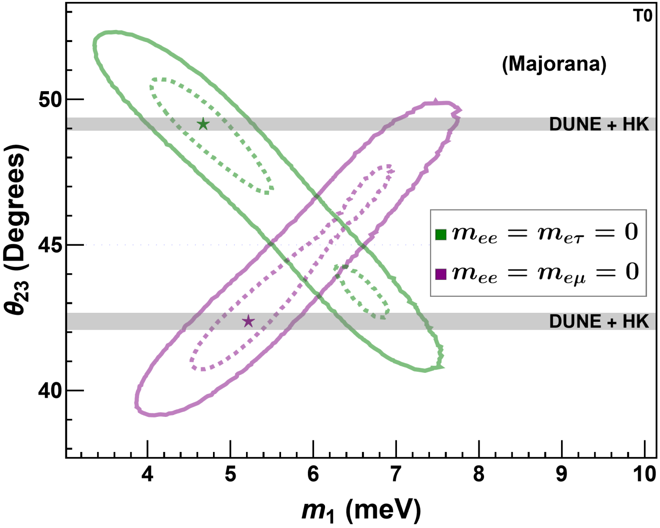
TRUE / TEST	m_{ee}	$m_{e\mu}$	$m_{e\tau}$	$m_{\mu\mu}$	$m_{\mu\tau}$	$m_{\tau\tau}$
m_{ee}	—	**0.08**	**0.24**	**0**	**0**	**0**
$m_{e\mu}$	**0.03**	—	**0.97**	**0.08**	**0.17**	**0.23**
$m_{e\tau}$	**0.09**	**0.89**	—	**0.07**	**0.02**	**0.21**
$m_{\mu\mu}$	**0**	**1**	**1**	—	$< \mathbf{0.01}$	$< \mathbf{0.01}$
$m_{\mu\tau}$	**0**	**1**	**1**	$< \mathbf{0.01}$	—	**1**
$m_{\tau\tau}$	**0**	**1**	**1**	$< \mathbf{0.01}$	**0.70**	—

Future: One Texture Zeros: Differentiation

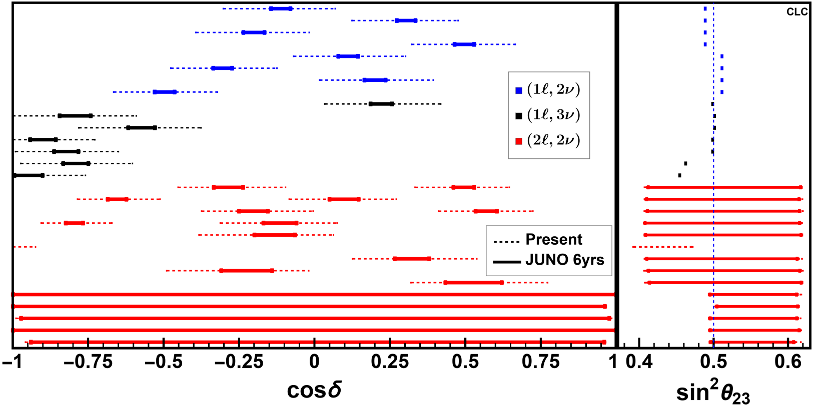
IO

TEST \ TRUE	$m_{e\mu}$	$m_{e\tau}$	$m_{\mu\mu}$	$m_{\mu\tau}$	$m_{\tau\tau}$
$m_{e\mu}$	—	**0.97**	**0.38**	**0.77**	**0.71**
$m_{e\tau}$	**0.97**	—	**0.40**	**0.79**	**0.72**
$m_{\mu\mu}$	**0.31**	**0.31**	—	**0.61**	**0.31**
$m_{\mu\tau}$	**0.56**	**0.57**	**0.51**	—	**0.64**
$m_{\tau\tau}$	**0.58**	**0.58**	**0.32**	**0.73**	—

Future: Two Texture Zeros



Future: Charged Lepton Corrections



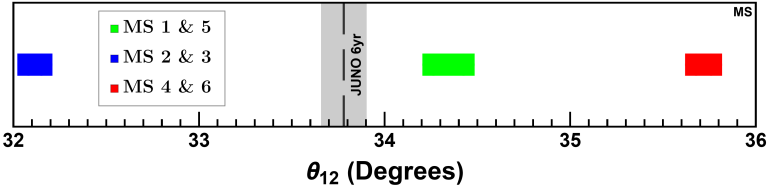
1. Some ruled out by recent JUNO data: $(U^e_{12}, U^e_{23}, \theta^{\nu,\mathrm{BM}}_{12})$, $(U^e_{12}, \pi/20, \theta^{\nu,\mathrm{GRA}}_{12})$
2. Measuring $\cos\delta$, and $\mathrm{sgn}(\cos\delta)$ is important PBD 2309.03262
3. Precise measurements of δ and θ_{23}, coming from full DUNE and HK, essential

Future: Modular Symmetries

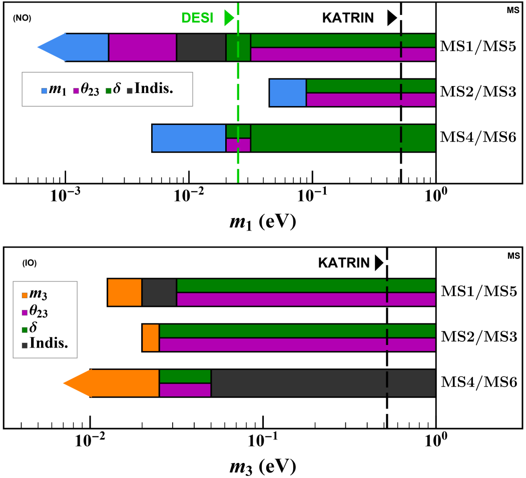
Simple predictions relate θ_{12} and θ_{13}:

Model	Group	Prediction	Current Status
MS 1 & 5	$\mathcal{A}_4$	$s_{12}^2 = \frac{1}{3c_{13}^2}$	$0.309 \pm 0.009 = 0.341 \pm 0.0002$
MS 2 & 3	$\mathcal{S}_4$	$t_{12}^2 = \frac{1-3s_{13}^2}{2}$	$0.448 \pm 0.018 = 0.467 \pm 0.0009$
MS 4 & 6	$\mathcal{A}_5$	$t_{12}^2 = \frac{1/x}{x/5-s_{13}^2}$	$0.448 \pm 0.018 = 0.394 \pm 0.0003$

$x \equiv \sqrt{5}\phi_g$



Future: Modular Symmetries: Differentiation

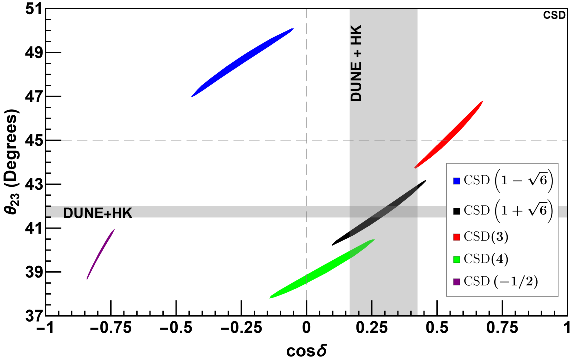


Good measurements of θ_{23} and δ, along with a good measurement of θ_{12}, will likely allow for complete discrimination

Future: Constrained Sequential Dominance

Same $\theta_{13} \to \theta_{12}$ relation as the $\mathcal{S}_4$ modular symmetries:

$$t_{12}^2 = \frac{1 - 3s_{13}^2}{2}$$



Future: Summary

$\boldsymbol{m_{\text{lightest}}}$

- DESI+Planck+Lyα should achieve 0.02 eV precision — DESI 1611.00036
- To disfavor 1 meV requires $\sum_i m_i > 93$ meV, near current limits

MO

- Will be well measured and will have dramatic impact on flavor models

$\boldsymbol{\theta_{12}}$

- JUNO data is already putting tension on models — Many recent papers discussing this
- More data will significantly help with two-texture zeros and modular symmetries

$\vdots$

Future: Summary

$\boldsymbol{\theta_{23}}$

- Octant determination significantly cuts down space everywhere except mass sum rules
- Charged lepton corrections often predict θ_{23} within 0.5° of 45° making ν_μ disappearance precision essential to (dis)favor maximal

$\boldsymbol{\cos\delta}$

- Key discriminator in many cases except mass sum rules
- Breaking the sign degeneracy is essential

$\boldsymbol{|\Delta m^2_{ij}|}$, $\boldsymbol{\theta_{13}}$

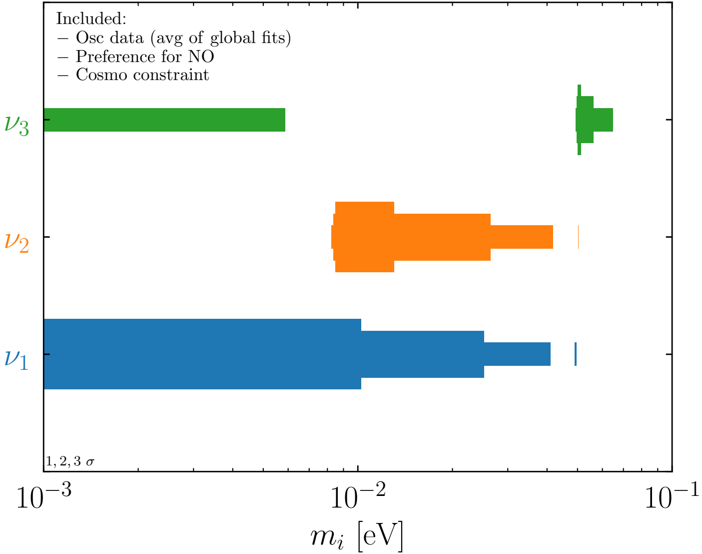
- Further precision has modest to little impact across the board

Summary

- Flavor model predictions provide an approach to understanding majority free parameters in SM
- Evolution of model building informed by new measurements:
  - Old approaches ruled out
  - New ideas generated
- Some mass sum rules, texture zeros and others anticipate a vanishing $0\nu\beta\beta$ rate
- Mapped out which future measurements will most impact ability to (dis)favor and differentiate models

Backups

Absolute Masses



Charged Lepton Corrections

- two rotations in the neutrino sector, one charged lepton rotation (15 cases)

$$U_{\rm PMNS}=(U^e_{ij})^\dagger\Psi U^\nu_{23}(\pi/4)U^\nu_{12}(\theta^{\nu,k}_{12})Q$$
$$\text{where } (ij)\in\{12,\ 13,\ 23\}$$

- two rotations in the neutrino sector, two charged lepton rotations (15 cases)

$$U_{\rm PMNS}=(U^e_{ij})^\dagger(U^e_{lm})^\dagger\Psi U^\nu_{23}(\pi/4)U^\nu_{12}(\theta^{\nu,k}_{12})Q$$
$$\text{where } (ij)\in\{12,\ 13\},\ (lm)\in\{13,\ 23\},\ (ij)\neq(lm)$$

- three rotations in the neutrino sector, one charged lepton rotation (45 cases)

$$U_{\rm PMNS}=(U^e_{ij})^\dagger\Psi U^\nu_{23}(\pi/4)U^\nu_{13}(\theta^{\nu,p}_{13})U^\nu_{12}(\theta^{\nu,k}_{12})Q$$
$$\text{where } (ij)\in\{12,\ 13\ ,\ 23\}$$
$$\text{and } p\in\{\text{T13-1, T13-2, T13-3}\}$$

$$k\in\{\text{TBM, BM, GRA, GRB, HG}\}$$

Future: Summary

Observable		SR	T0	CLC	MS	CSD
m_ℓ	Measuring $\lesssim 10$ meV will exclude many model predictions; a precise measurement will differentiate many.	$\bigstar$	✓	×	✓	×
MO	Resolving the ordering will instantly rule out a significant proportion of all model predictions.	✓	✓	×	✓	$\bigstar$
θ_{12}	Increased precision alone will rule out some model predictions and will tighten parameter space for others.	×	✓	✓	$\bigstar$	$\bigstar$
θ_{23}	Resolving octant degeneracy alone will have a large impact on most model predictions.	×	✓	✓	✓	✓
$\cos\delta$	Impact will depend on the true value, but can be leveraged with the other mixing parameters.	×	✓	✓	✓	✓

×: Increased precision will not have significant impact.
✓: Increased precision and synergistic measurements will have notable impact.
$\bigstar$: Increased precision alone will have significant impact.