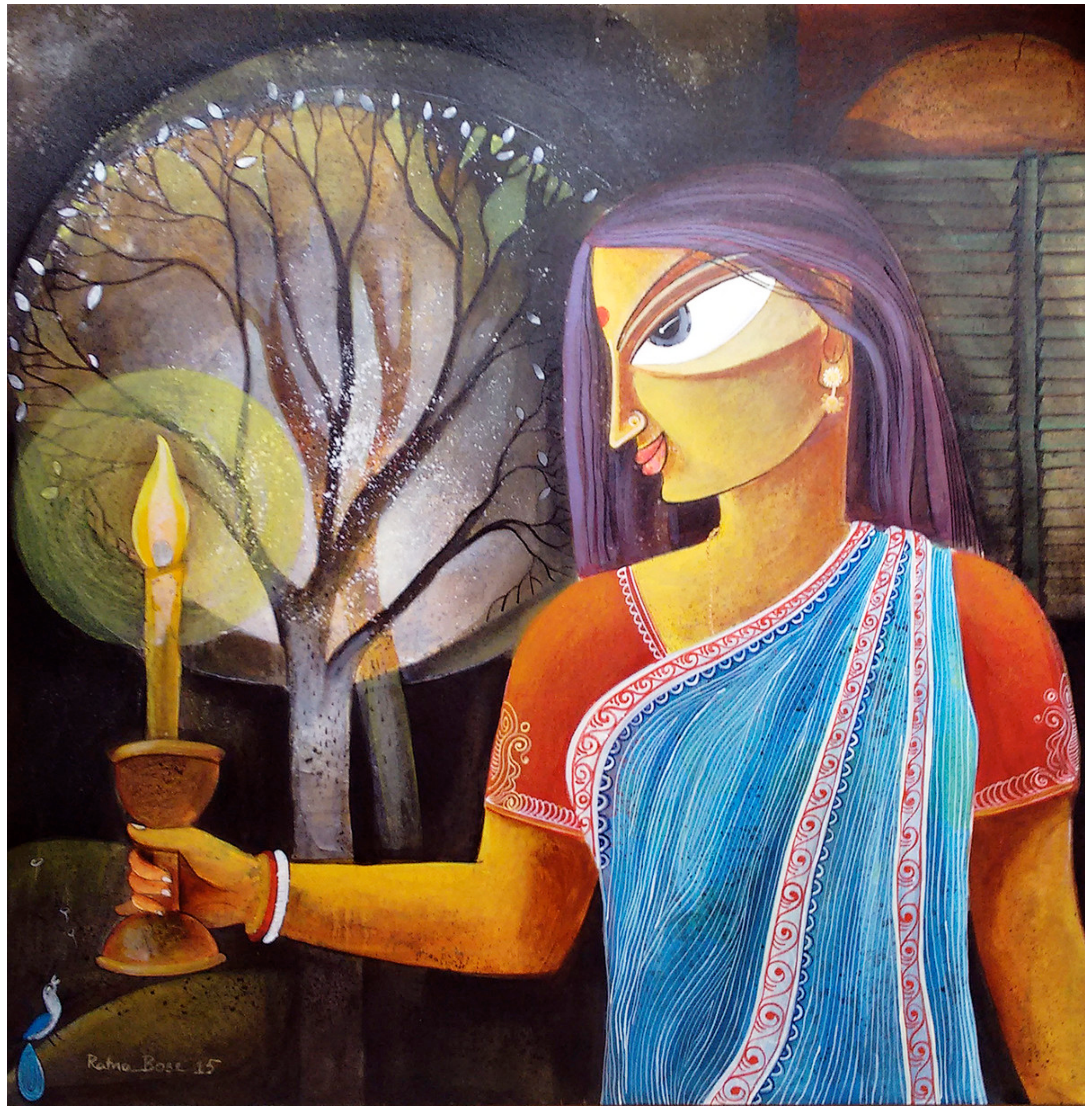

ratna bose

Photomultiplier tube, cable + electronics, charge and time measurement basics

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7/15/2026

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Library of materials from this and other lectures
https://www.phy.bnl.gov/~diwan/#photo

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Outline

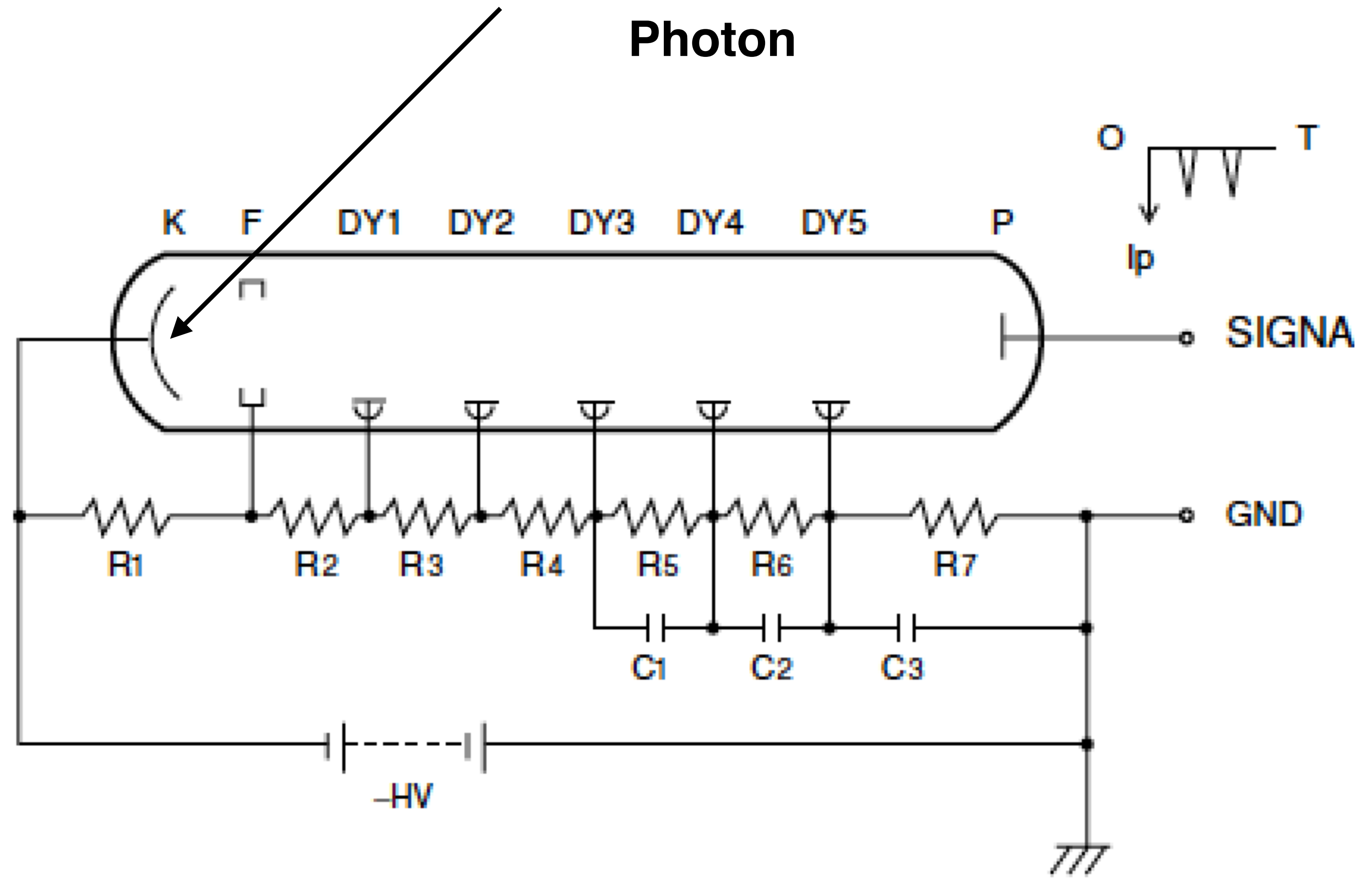
- ***In this lecture we are going to learn the basics of how to analyze detector functions.***

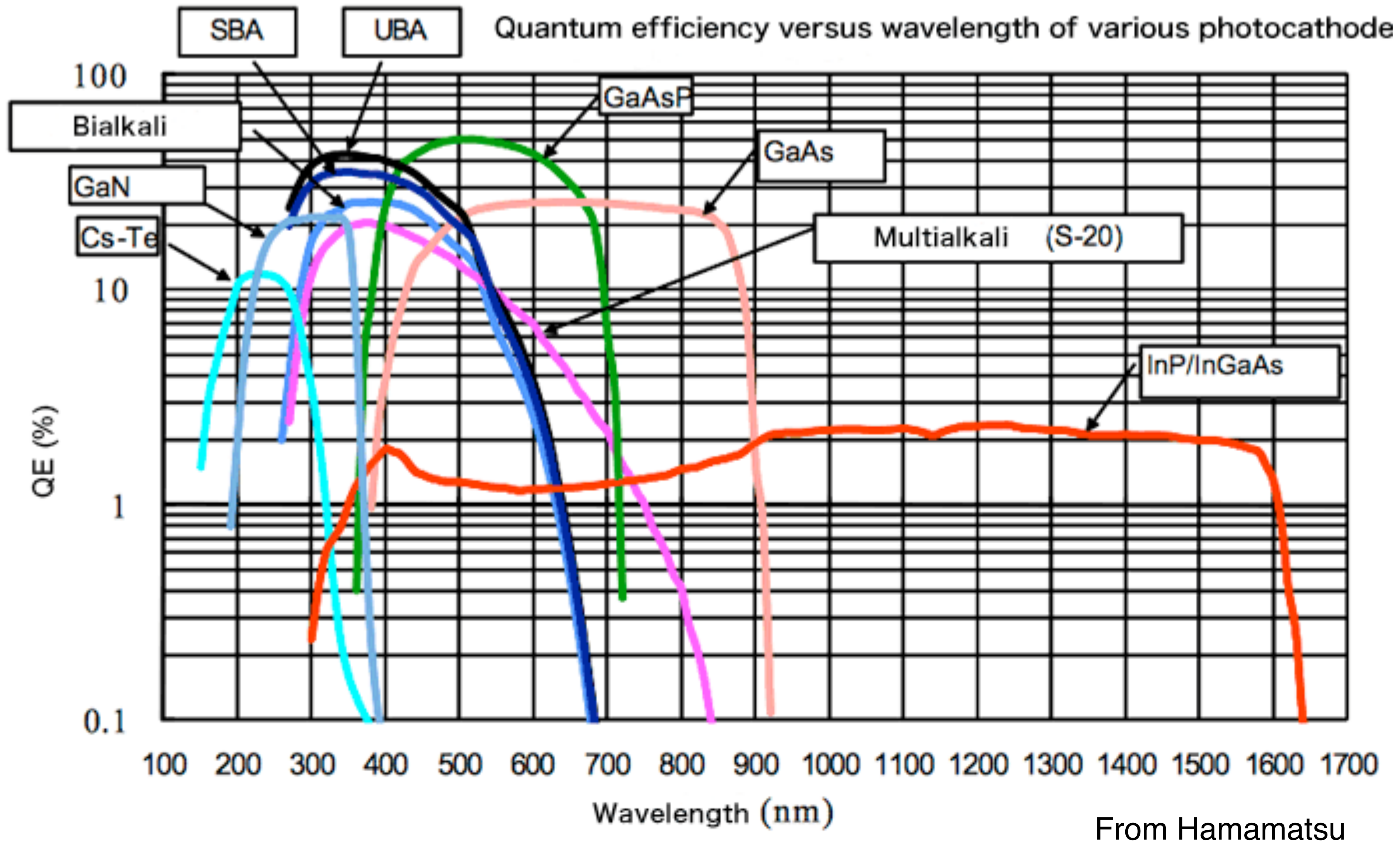
  1. ***We will learn the basics of how a photomultiplier tube works. This is a very important sensor.***
  2. ***We will learn the chain of signal generation and detection.***
  3. ***We will learn about the statistics of gain through multiplication.***
  4. ***We will also learn about time measurement and its resolution from such devices.***
  5. ***And we will learn about statistics of a counting process.***
  6. ***To derive these we will need some powerful mathematical tools. We will get an introduction to these tools.***

- ***Reference: See the guide from Hamamatsu, Photomultiplier tubes: Basics and Applications (2007) 3rd edition. There are many papers in journals such as Nuclear Instruments and Methods.***

Most of this can be found in textbooks.

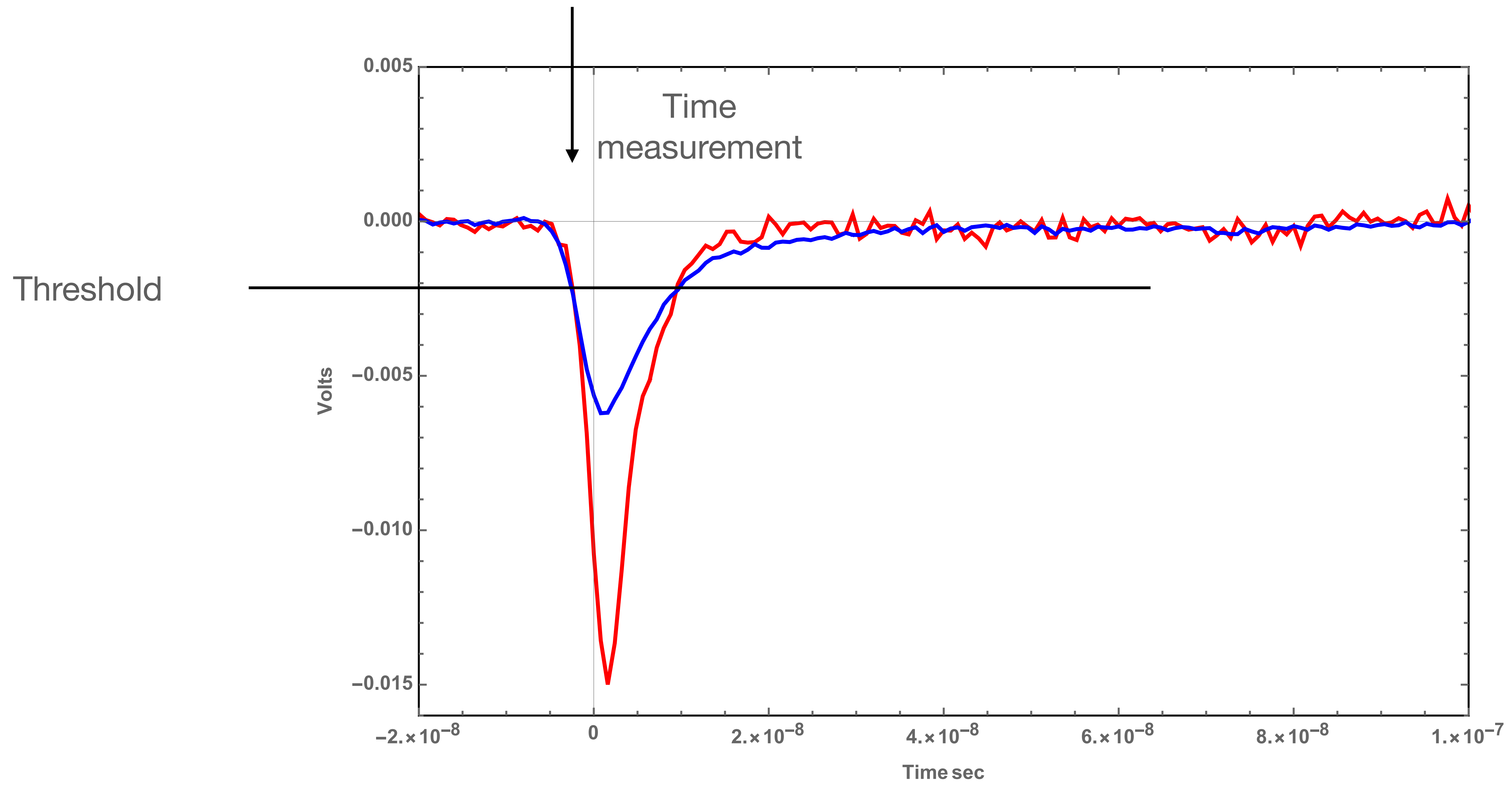
Photo-Multiplier Tube





From Hamamatsu

- The photoelectric effect(Hertz 1887!) causes metals to eject electrons in response to light.
- Einstein's Nobel prize was for explaining the photoelectric effect with quantum mechanics !
- Electric fields accelerate and multiply the primary electron in several stages. Each stage has multiplication of ~4-5.
- Typical Gain = $A^* V^n \sim 10^6$- 10^7 where V is the typical voltage ~ few 1000 V.
- Time resolution < 10 ns. From varying transit time.
- Transit time can be <1 microsec
- PMT first stage is sensitive to small magnetic fields.
- Many clever geometries.



$$Q \approx \frac{-0.007\ Volt \times 5 \times 10^{-9}\ sec}{50\ Ohm} = 0.7\ pC$$

4 million electrons

These are a couple of typical single pe pulses. Red is with a short cable, and Blue is with a 40 meter RG316 cable. A cable will smooth out a pulse and reduce its amplitude. What happens to the charge ?

We are going to integrate pulses like these to get the total charge.

The time is obtained by looking for the pulse to go above threshold. Typically this is set to see a single photo-electron. We will deal with time resolution and counting later in this lecture.

R has two grandparents. Each is going to give R some money for her birthday. \$10, \$20, \$30, or \$40 with probability of 0.1, 0.2, 0.3, 0.4. What is the probability that R gets a total of \$50 ?

How do we make a method that provides an answer quickly ? It is called a characteristic function. It keeps track of numbers. Gauss described this as a clothes-line to find the number that you need.

$$\phi(s)_R = 0.1e^{10s} + 0.2e^{20s} + 0.3e^{30s} + 0.4e^{40s}$$

To get the total you multiply the char. functions.

$$\phi(s)_{R1+R2} = (0.1e^{10s} + 0.2e^{20s} + 0.3e^{30s} + 0.4e^{40s})^2$$

$$\phi(s)_{R1+R2} = 0.01e^{20s} + 0.04e^{30s} + 0.1e^{40s} + 0.2e^{50s} + 0.25e^{60s} + 0.24e^{70s} + 0.16e^{80s}$$

Notice that you can just read off the probability that R gets \$50

The characteristic function always exists and encodes all information about a probability density.

Some definitions (much of this is the same concept for Laplace and Fourier transforms also known as integral transforms)

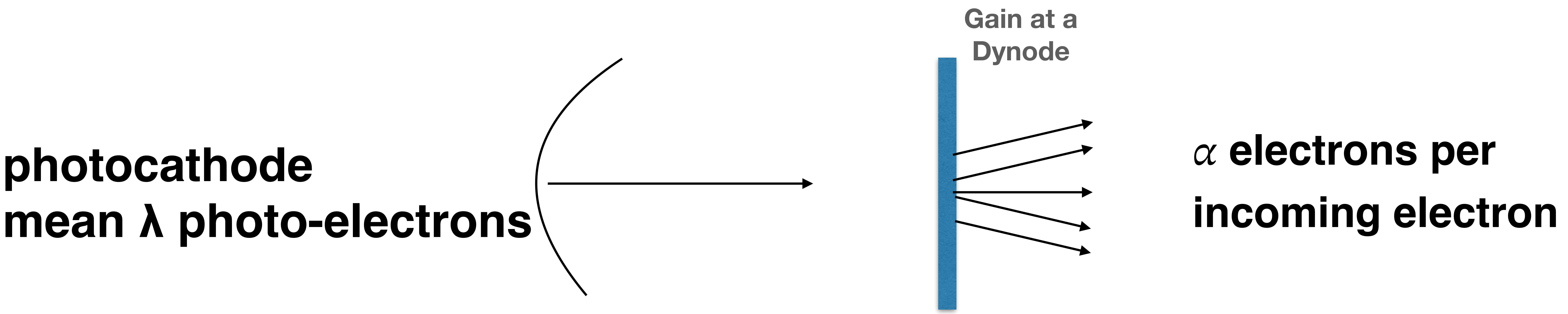
- X — **Continuous random variable with probability density** $P_X(x)$
- $\varphi_X(k) \equiv E[e^{ikx}] = \int_{-\infty}^{+\infty} P_X(x) e^{ikx} dx$ **is the characteristic function.**
- x **is a realization of** X **over the its domain.** $E[]$ **is the expectation value of its argument.**
- **Some special points**
  - **The characteristic function tags the probability at** x **with a unique identifier** e^{ikx}
  - **If all the probability is only at some value** $x = 5$**, for example then** $\varphi_X(k) = e^{ik5}$
  - $\varphi_X(k=0) = 1$ **since it is simply the total probability**
  - $\langle x \rangle = -i \dfrac{\partial \varphi}{\partial k}(k=0)$; **higher derivatives lead to high moments.**
- **Imagine** X, Y **are two random variables and** $z = f(x, y)$ **then the characteristics function for Z is**
- $\varphi_Z(k) = \int\int e^{ikf(xy)} P_X(x) Q_Y(y) dx dy$ **=> for** $z = x + y$ **then** $\varphi_Z(k) = \varphi_X(k)\varphi_Y(k)$

Basics of photomultiplier charge

Mean of λ photo electrons come from the photo-cathode to the first dynode.

Each electron generates α electrons at the first dynode.

Each subsequent stage produces gain of few electrons per incoming electrons leading to gains of ~$10^{6\text{-}7}$ after ~10 stages



We are going to assume that the stages after the first stage do not contribute to the variance of the gain. This is a good assumption as long as the gain at the first stage is reasonable > 3. We will deal with this later.

If mean of λ photons convert in a photo-sensor with a mean gain of α electrons per photon what is the distribution of the output number of electrons ?

Basically, an average of λ packets arrive each with an average of α items in each packet. What is the mean and variance of the total number of items ? How do we calculate this…

Simple calculation of mean and variance

- λ and α are the Poisson parameters for the incident photoelectrons and the first stage gain. Total charge random variable will be called Q
- Obviously, $\langle Q \rangle = \lambda \times \alpha$

$Q = \sum_{i=1}^{K} L_i$ where K is the random number of incident photoelectrons and L_i is the random number of secondaries for each photoelectrons.

$$\frac{Var[Q]}{\langle Q \rangle^2} = \frac{Var[K]}{\langle K \rangle^2} + \frac{1}{K}\frac{Var[L]}{\langle L \rangle^2} = \frac{1}{\lambda} + \frac{1}{\lambda \times \alpha}$$

$$Var[Q] = \lambda\alpha(\alpha + 1)$$

suppose L is actually Normally (Gaussian) distributed with parameters α (mean) and σ (standard deviation)

$$\frac{Var[Q]}{\langle Q \rangle^2} = \frac{1}{\lambda} + \frac{1}{\lambda}\frac{\sigma^2}{\alpha^2}$$

$$Var[Q] = \lambda(\sigma^2 + \alpha^2)$$

The full calculation of the PDF for the charge is at https://www.phy.bnl.gov/~diwan/#photo. There is also a video lecture.

Couple of definitions

Let's define some PDFs to get a compact expression

The Normal PDF

$$N(x:\mu,\sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}}\ \mathrm{e}^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

The Exponential modified Normal PDF

$$E_N(x:\mu,\sigma^2,\lambda) = \frac{\lambda}{2} e^{\frac{\lambda^2\sigma^2}{2}} e^{-\lambda(x-\mu)} Erfc\left[\frac{1}{\sqrt{2}}\left(\lambda\sigma - \frac{x-\mu}{\sigma}\right)\right]$$

μ : mean of the Gaussian

σ^2: variance of the Gaussian

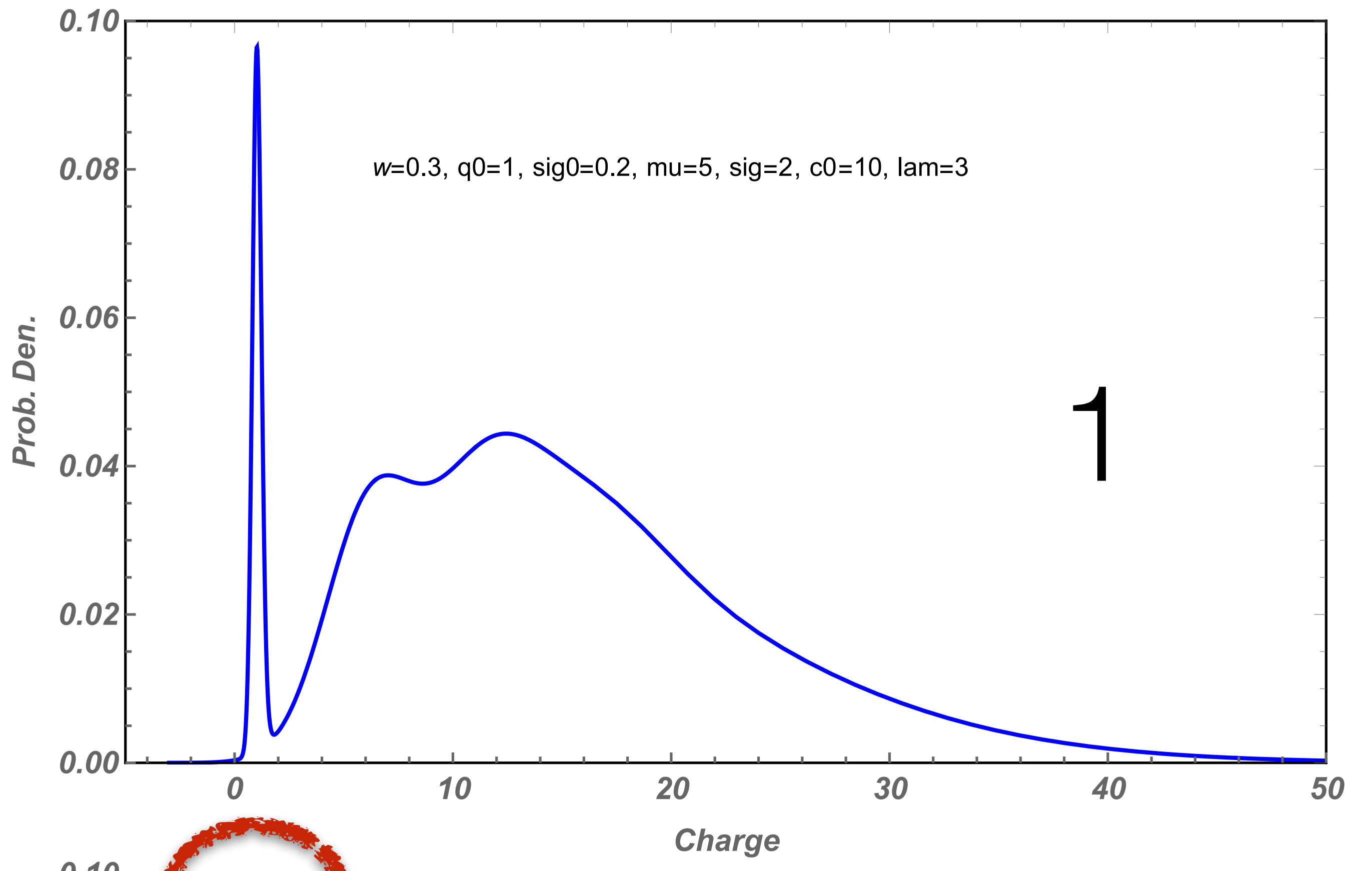
λ: exponential decay parameter

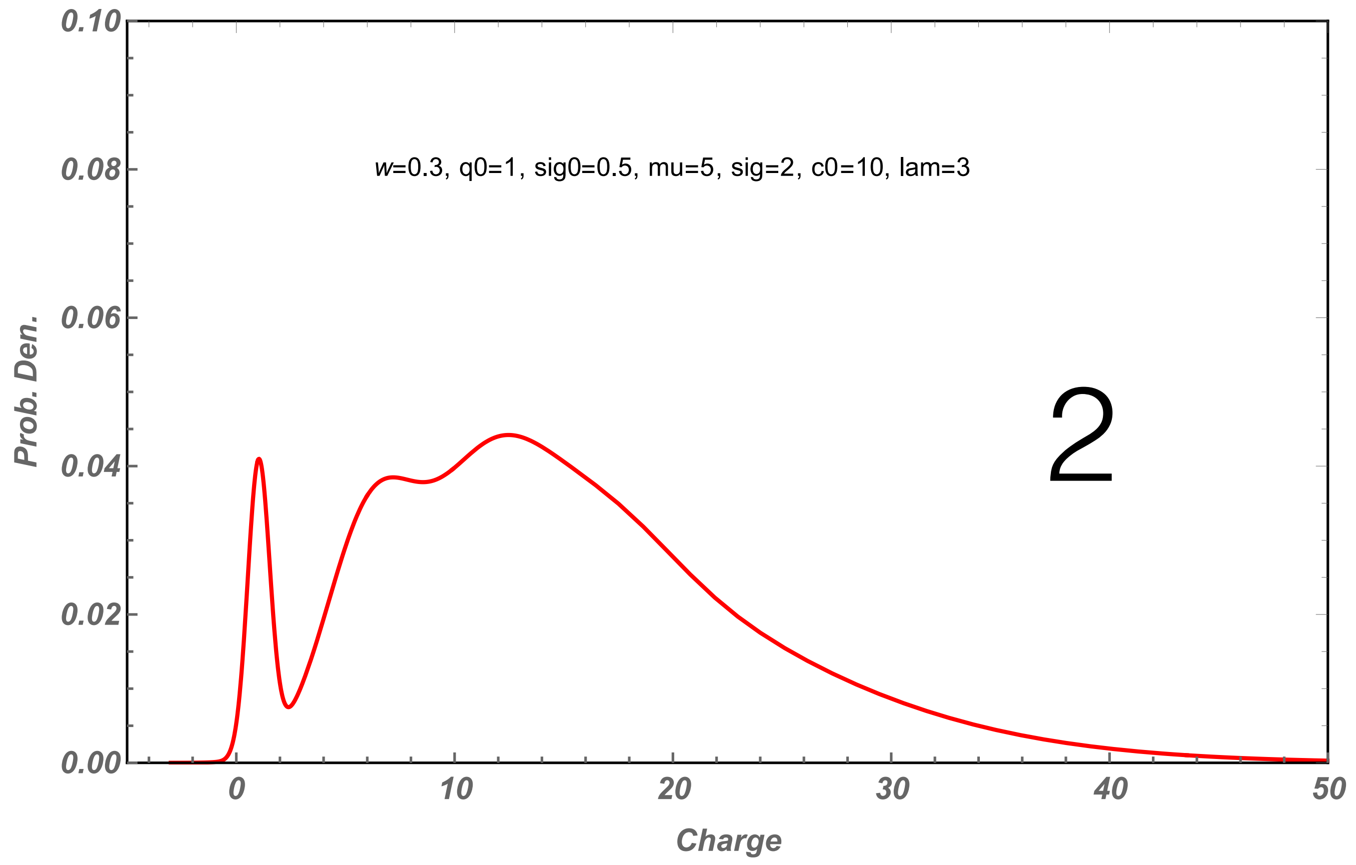
All terms broken out for the PDF in compact notation

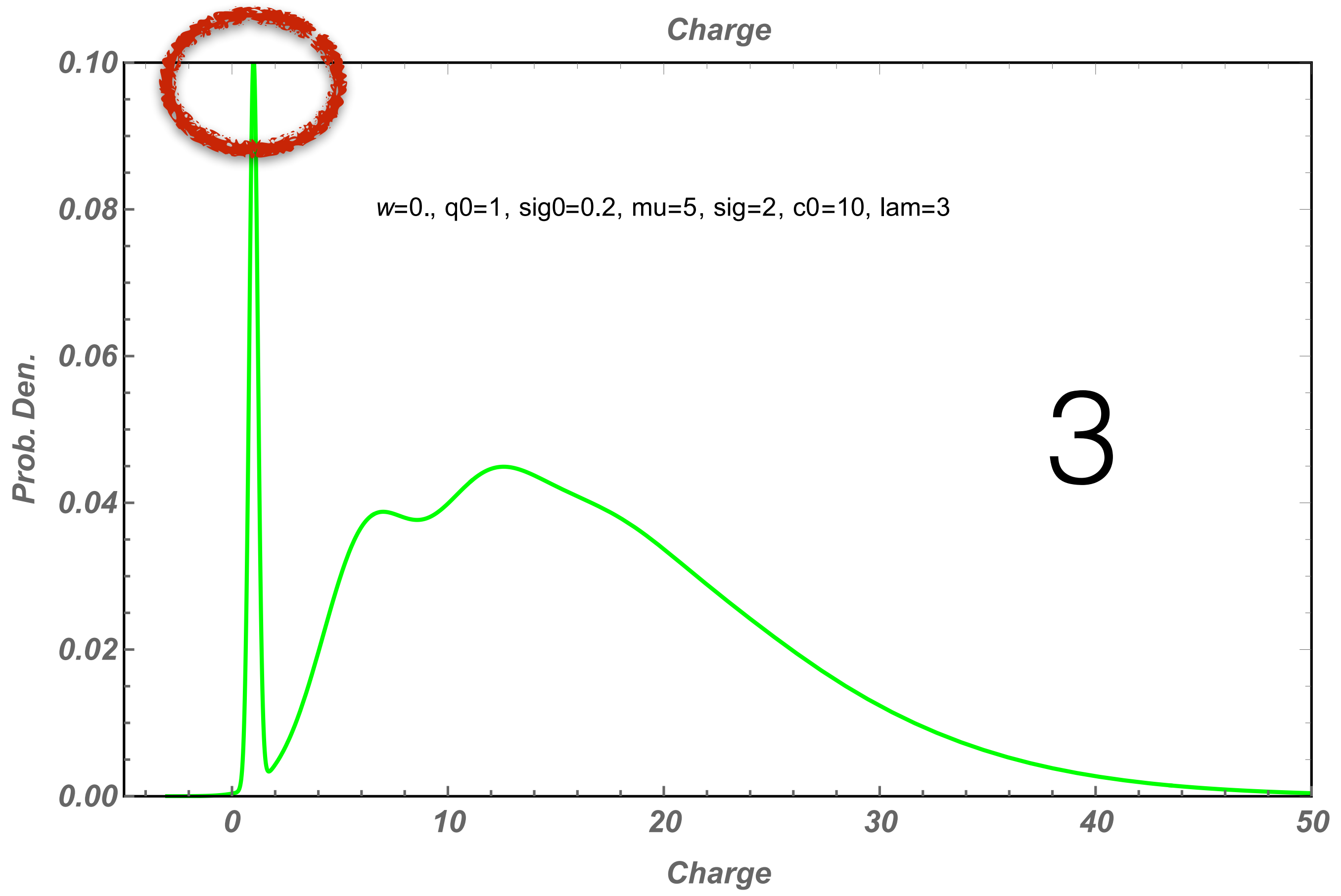
	Terms	$(1-w)\times$	$w\times$
no signal	$e^{-\lambda}\times$	$N(x:q_0,\sigma_0^2)$	$E_N(x:q_0,\sigma_0^2,c_0)$
single pe	$\lambda e^{-\lambda}\times$	$N(x:\mu+q_0,\sigma^2+\sigma_0^2)$	$E_N(x:\mu+q_0,\sigma^2+\sigma_0^2,c_0)$
many pe	$\sum_{k=2}^{\infty}\frac{e^{-\lambda}\lambda^k}{k!}\times$	$N(x:\mu k+q_0,\sigma^2 k+\sigma_0^2)$	$E_N(x:\mu k+q_0,\sigma^2 k+\sigma_0^2,c_0)$
		no dark current	with dark current

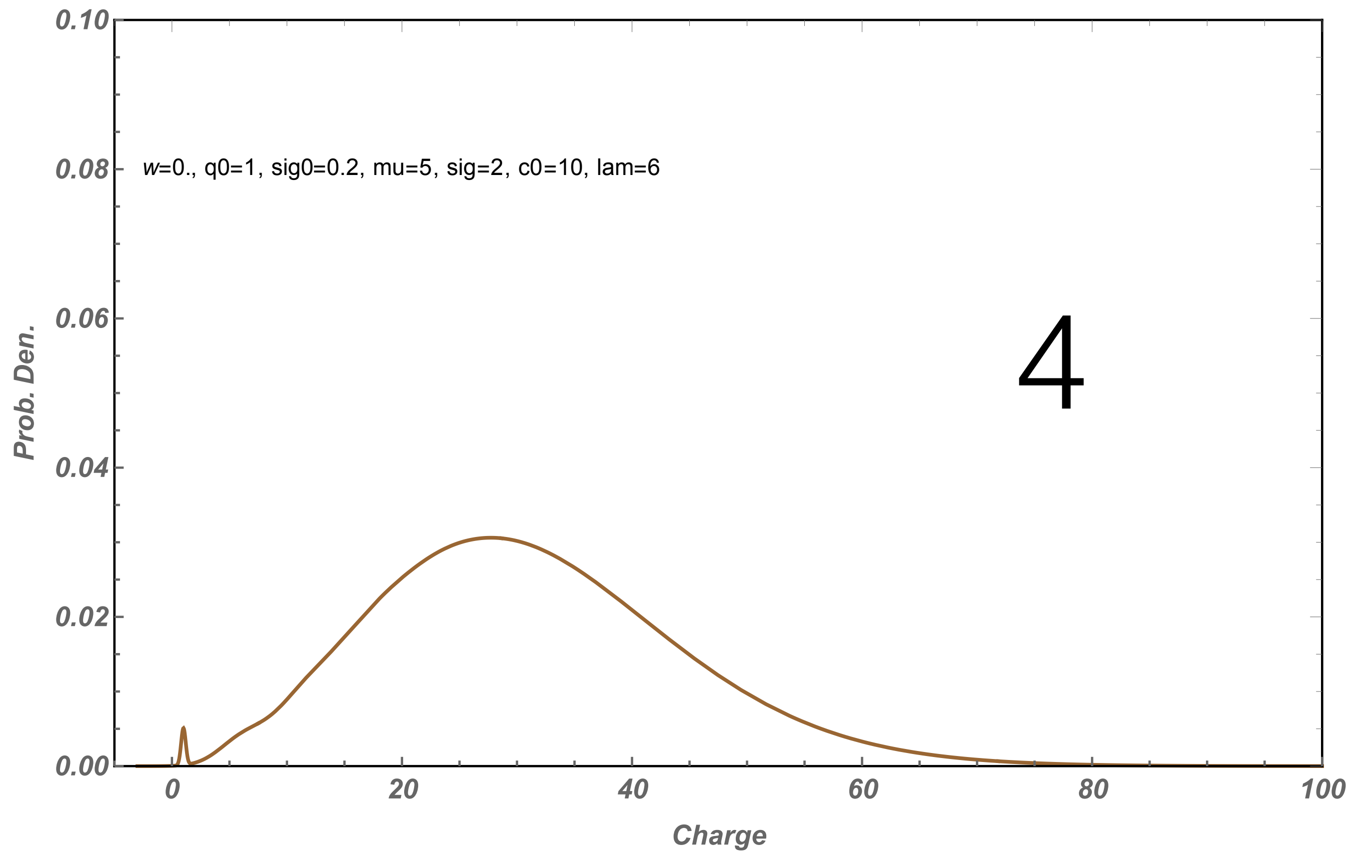
all these are to be added together to get the full PDF.
The sum is applied across the row.

Plot some examples







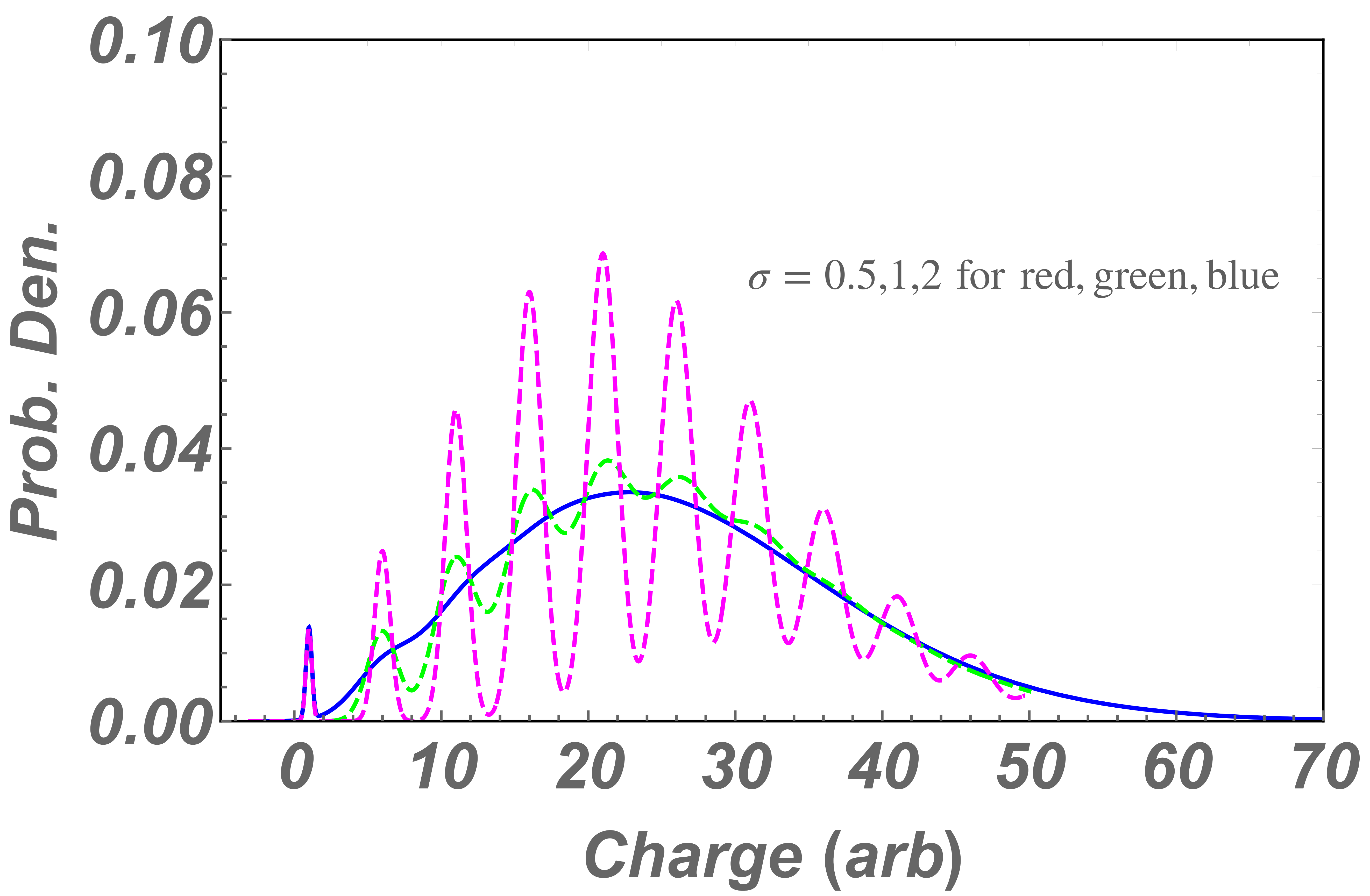


plot	λ	w	q_0	σ_0	μ	σ	c_0
1	3	0.3	1	0.2	5	2	10
2	3	0.3	1	0.5	5	2	10
3	3	0	1	0.2	5	2	10
4	6	0	1	0.2	5	2	10

with dark current, narrow/wide pedestal

no dark current, less/more p.e.

Example where the gain has a narrow variance; and individual photoelectrons can be separated and counted.



The probability of dark pulses is set to zero,w=0. The values of other parameters are set to beλ=5(mean photo-electrons),q0=1(baseline shift),σ0=0.2(baseline fluctuation),μ=5(mean gain),σ=0.5, 1, 2(gain fluctuation) for the magenta-dashed, green-dashed, and blue-solid curves, respectively

A little bit about electronics

Photomultiplier (PMT) basics

Electrot rode	Nominal Ratio	Resistor	Voltage	dampng or load resistor	stabiliza tion capacito	Voltage drop	Estimated Gain	Power Watt	capacitor charge C
K			-2000						
	16.8	1650000				619.136960	7	-0.23232156	
Dy1			-1380.863039						
	4	540000				202.626641	5.73323236	-0.07603251	
Dy2			-1178.236398						
	5	780000				292.682926	6.35809185	-0.10982473	
DY3			-885.5534709						
	3.33	440000				165.103189	5.12356714	-0.06195241	
DY4			-720.4502814						
	1.67	270000				101.313320	3.61557169	-0.03801625	
dy5			-619.1369606						
	1	150000				56.2851782	2.19357822	-0.02112014	
dy6			-562.8517824						
	1.2	180000				67.5422138	2.57681568	-0.02534417	
dy7			-495.3095685	0					
	1.5	220000				82.5515947	3.05903249	-0.03097620	
dy8			-412.7579737	100					
	2.2	330000			0	123.827392	4.21561205	-0.04646431	0
dy9			-288.9305816	100					
	3	440000			0.000000	165.103189	5.12356714	-0.06195241	7.26454E-06
dy10			-123.8273921	100					
	3.4	330000			0.000000	123.827392	4.21561205	-0.04646431	5.44841E-06
P			0	open					
SUM		5330000					7441972.67	0.75 W	
Current		-0.000375235	amp						

Example of a base for Hamamatsu R5912 10 stage tube.

Considerations

Max voltage.

Max voltage drop from K to the first dynode determines the first stage gain.

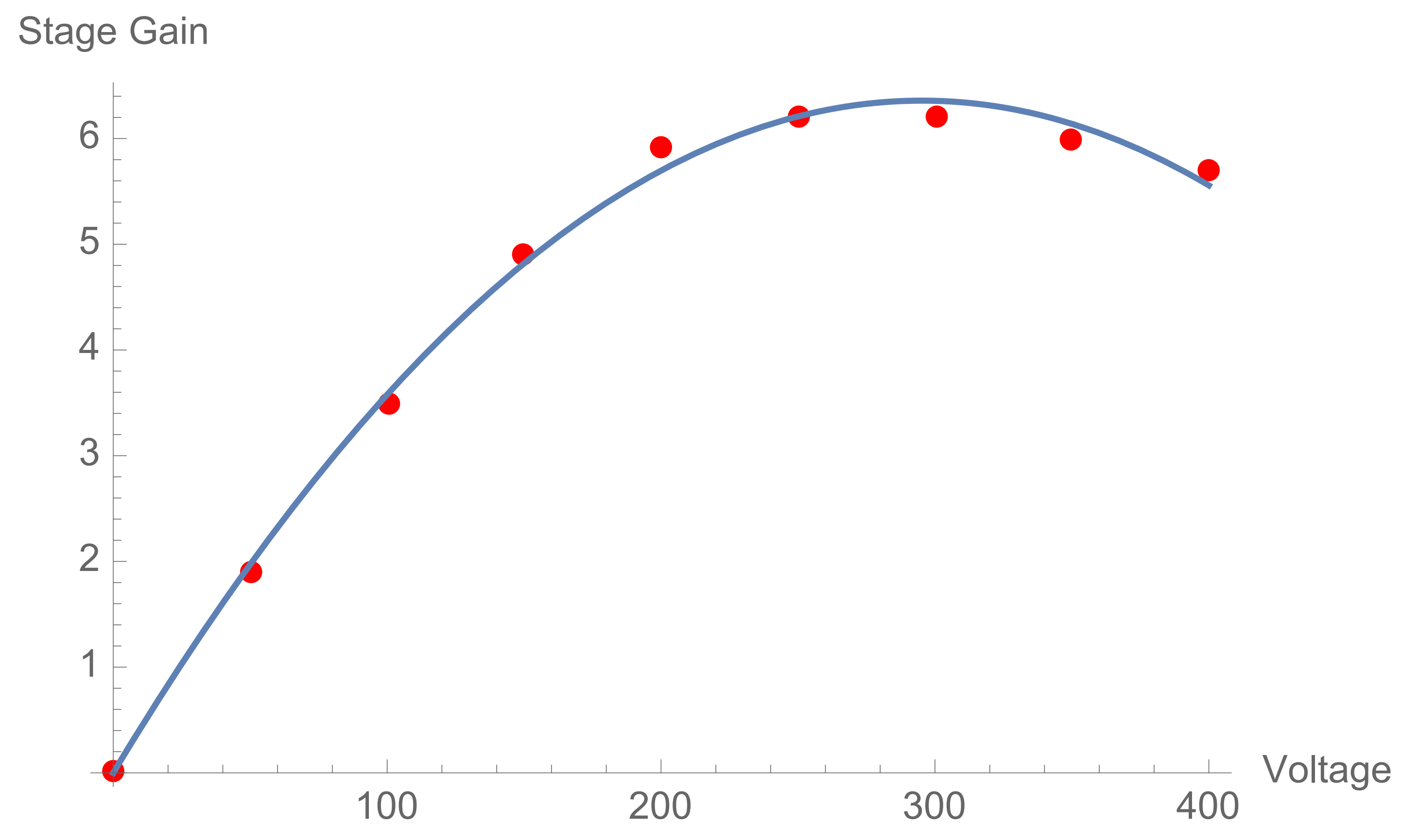
Total current should be high enough compared to expected current through tube.

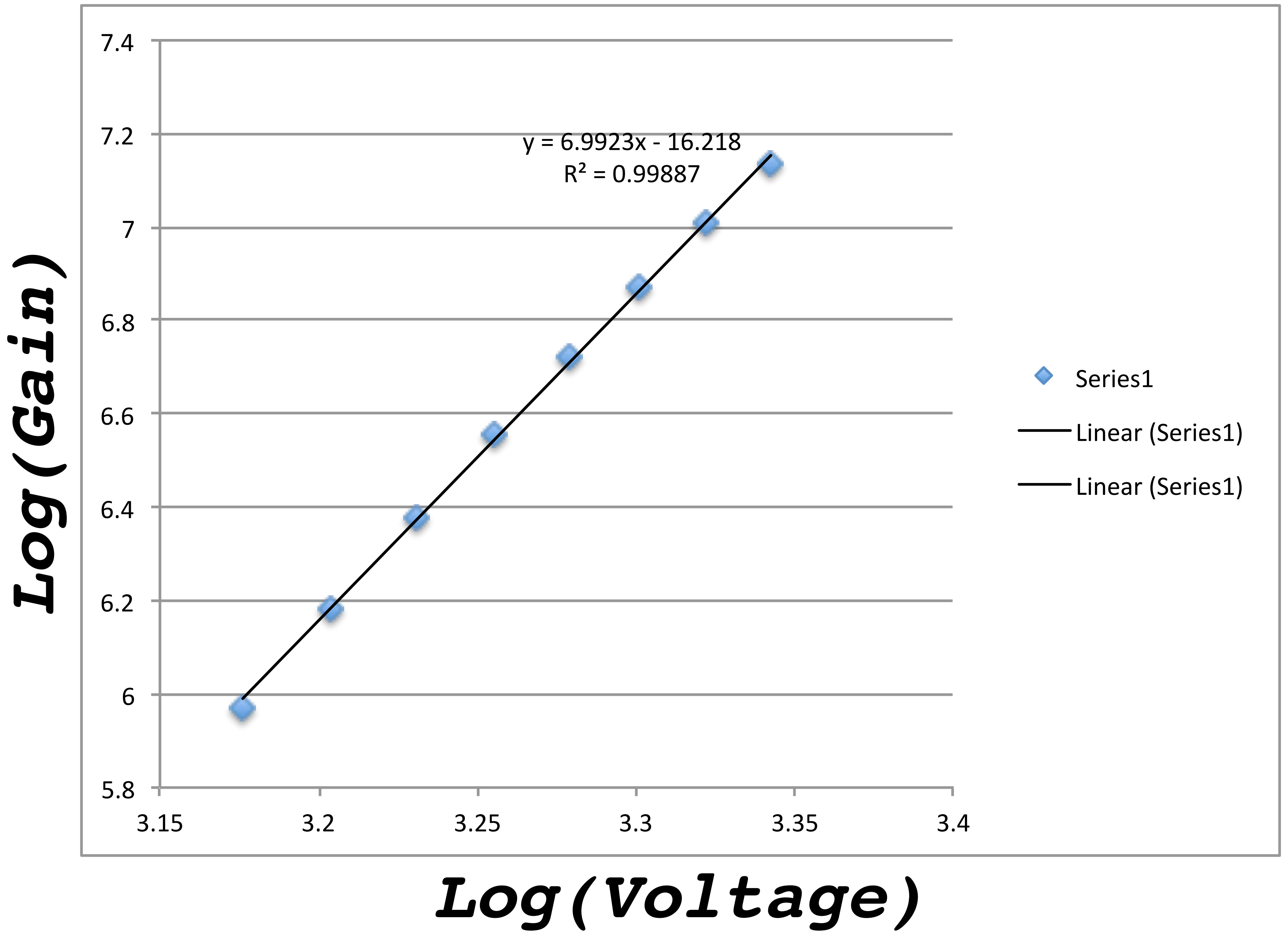
The charge in the stabilization capacitors should be larger than expected charge from a signal pulse.

If power is too high it can heat up the base.

Gain at each stage is calculated using $g(V) = 0.04*V-0.0007*V^2$

Basic calculation of gain



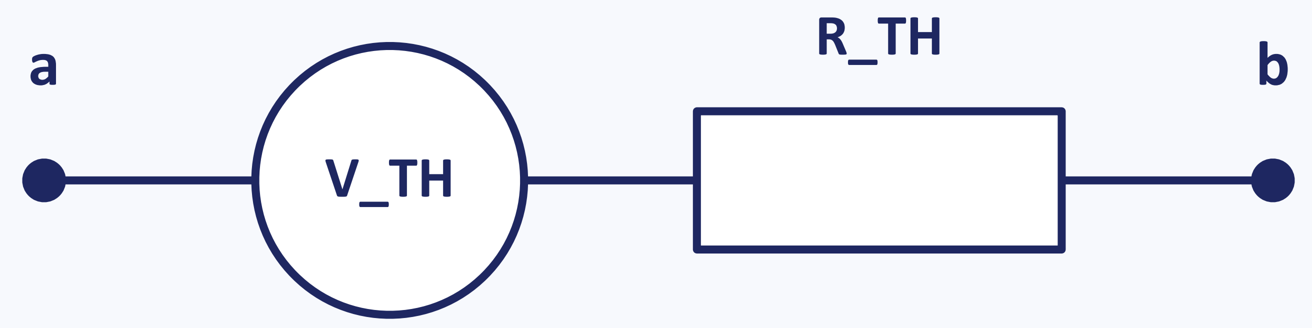


- *For the previous calculation a model for gain was used from a handbook. The model is different for each type of photo-cathode material. This one is for bialkali.*
- *This resulted in a voltage versus gain curve on the right. The slope of the log/log curve is the exponent of the voltage n= ~7.0*
- *This also means that if voltage changes by 1% => gain changes by 7 %*

Thévenin & Norton Equivalent Circuits

Any linear two-terminal network reduces to a single source and a single resistor

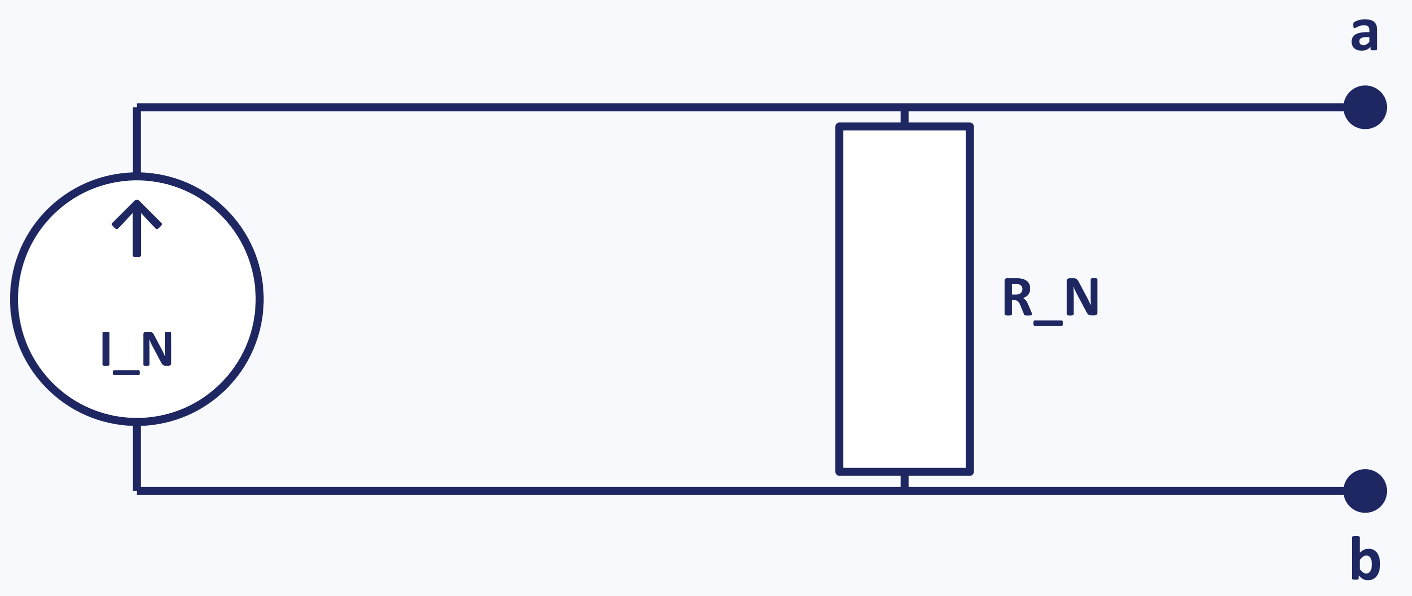
THÉVENIN EQUIVALENT



$V_{TH} = V_{oc}$ $R_{TH} = V_{oc} / I_{sc}$

- Series voltage source V_{TH} with resistance R_{TH}
- Best for finding the voltage across a variable load
- OC: open circuit voltage SC: Short circuit current.

NORTON EQUIVALENT



$I_N = I_{sc}$ $R_N = V_{oc} / I_{sc}$

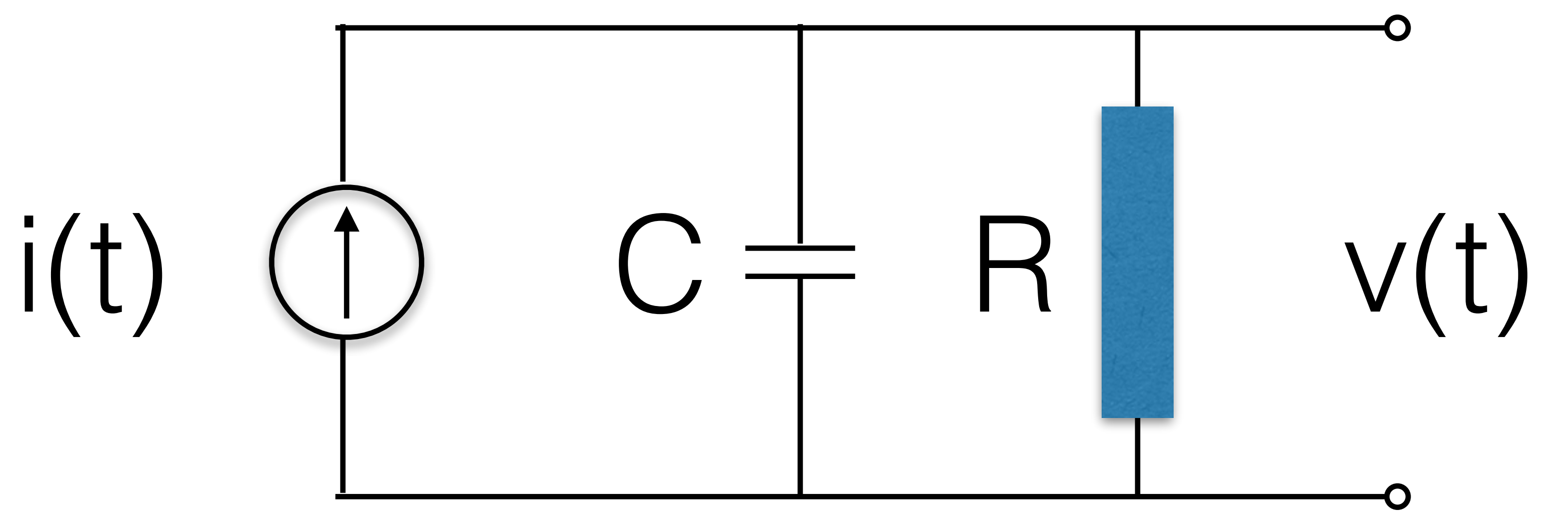
- Parallel current source I_N with resistance R_N
- Best for finding the current into a variable load

Source Transformation: $R_{TH} = R_N$ $V_{TH} = I_N \times R_N$ $I_N = V_{TH} / R_{TH}$

R_{TH} (= R_N) is found by zeroing all independent sources and computing the resistance seen at terminals a–b — or via $R_{TH} = V_{oc} / I_{sc}$.

PMT equivalent circuit

- *The circuit on slide 3 for the PMT base looks complicated. How is it to be modeled ? Surely the shape of the pulse depends on all those resisters and capacitors ?*
- *We use Norton's theorem for passive circuits: it states that no matter how complicated a network the equivalent circuit is a current source in parallel with a resistor (for DC circuits) or impedance for AC circuits.*
- *In the case of a PMT it would be a resistor and a capacitor. The values of these can be calculated from the network, but we don't have to. We can just measure them.*
- *For the current source we can assume it is a delta function for a single electron pulse; but it is better to assume that the current also has an exponential shape to account for scintillation lifetime.*



PMT equivalent

Assumptions

- **No inductances**
- **C is a capacitance that corresponds to all the capacitances in the base.**
- **R is generally some resistance that terminates the cable. Could be 50 ohm.**
- $\tau = RC$ **becomes the typical time constant for the pulse.**
- **i(t) also has some time constant. Assume it to be exponential as well.**

PMT signal solution

$i(t) = \dfrac{q_0}{\tau_s} e^{-t/\tau_s}$ for $t > 0$, and 0 for $t < 0$

Set $\tau = RC$

There are two ways to solve this. First we do the differential equation.

$$i(t) = C\frac{dv(t)}{dt} + \frac{v(t)}{R}$$

$v(t) = -\dfrac{q_0 R}{(\tau - \tau_s)} \times \left(e^{-t/\tau_s} - e^{-t/\tau}\right)$ for $\tau \neq \tau$ and $t > 0$

$v(t) = \dfrac{q_0 R}{\tau_s^2} t \times e^{-t/\tau_s}$ for $\tau = \tau_s$

Second way is by Fourier transform. This will allow much more flexibility in the long run. We replace the differential equation by its Fourier transform.

$$i(t) \Rightarrow I(\omega) = \frac{q_0}{\tau_s}\frac{\tau_s}{(1 + i\omega\tau_s)}$$

Recall that the impedance of C and R in parallel is $\dfrac{R}{(1 + i\omega RC)}$

$$I(\omega) = V(\omega) \times \frac{(i\omega\tau + 1)}{R} \Rightarrow V(\omega) = q_0 R \times \frac{1}{(1 + i\omega\tau_s)(1 + i\omega\tau)}$$

PMT signal solution by Fourier transform

We need to take inverse Fourier transform of

$$V(\omega) = \frac{q_0 R}{(1+i\omega\tau_s)(1+i\omega\tau)}$$

use partial fraction separation first.

$$V(\omega) = -\frac{q_0 R}{(\tau - \tau_s)}\left[\frac{\tau_s}{(1+i\omega\tau_s)} - \frac{\tau}{(1+i\omega\tau)}\right]$$

by inspection we get

$$v(t) \; = -\frac{q_0 R}{(\tau - \tau_s)}\left(e^{-t/\tau_s} - e^{-t/\tau}\right)u(t) \text{ where } u(t) \text{ is the unit step function.}$$

if $\tau = \tau_s$ then

$$t_{max} = \frac{Log(\tau/\tau_s)(\tau\tau_s)}{(\tau - \tau_s)}$$

$$V(\omega) = \frac{q_0 R}{(1+i\omega\tau)^2} = \frac{q_0 R}{\tau^2}\frac{1}{(1/\tau + i\omega)^2}$$

by inspection (use standard tables of Fourier transforms)

$$v(t) = -\frac{q_0 R}{\tau^2} t \times e^{-t/\tau}$$

example PMT signal plots

Here we will plot what the PMT pulse looks like. First some numbers: Recall that q_0 is the charge at the anode and so it includes the gain from the PMT.

$q_0 = GNe$ where G is the gain and Ne is the charge at the cathode.

Assume $G = 10^7$ and $N = 1 \Rightarrow q_0 = 1.6\times10^{-12}C = 1.6pC$

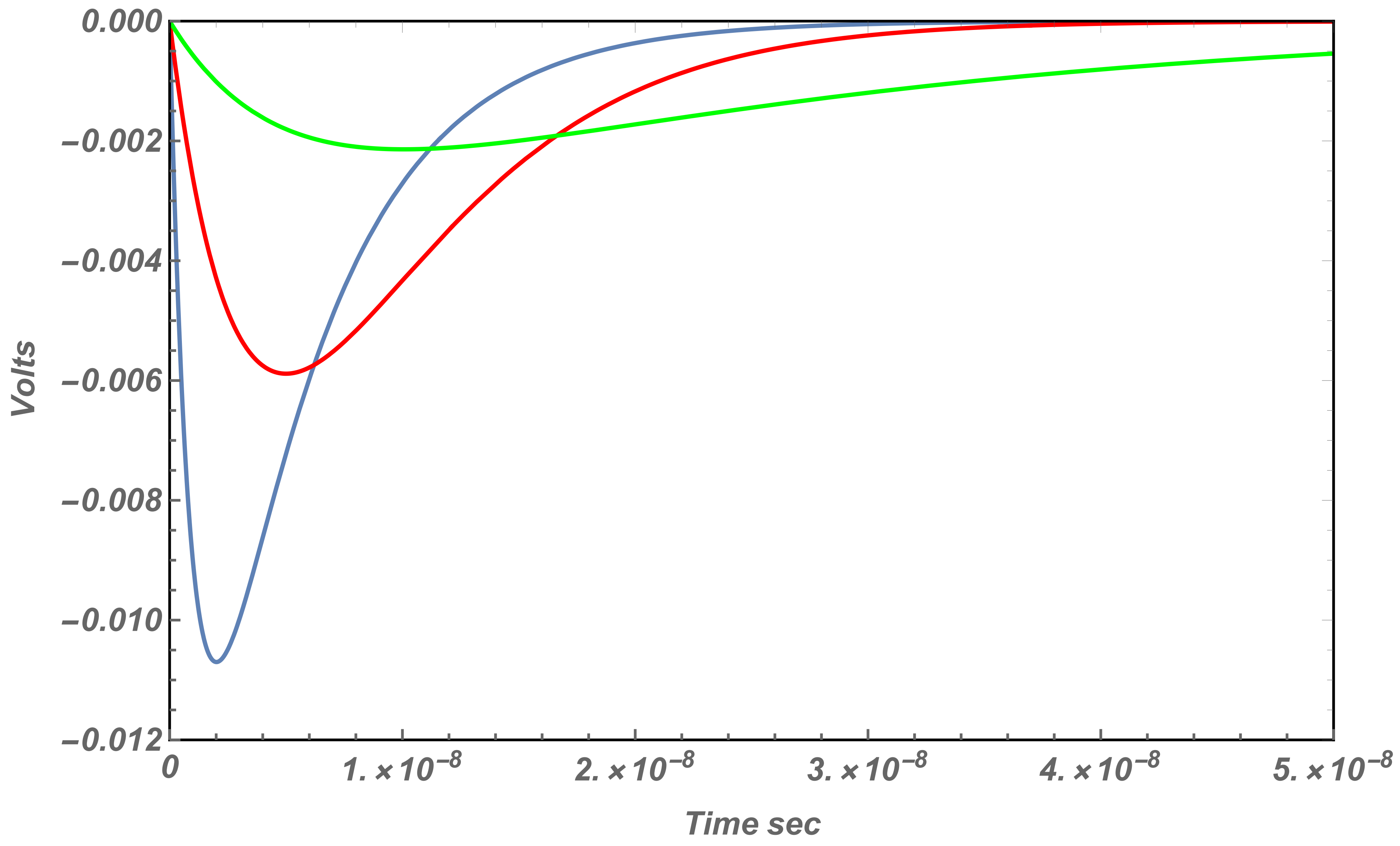
We now plot 3 cases

$\tau_s < \tau$: set $\tau_s = 1ns$ and τ=5 ns

$RC = 5ns \Rightarrow C = 0.1nF$ for R = 50 Ω

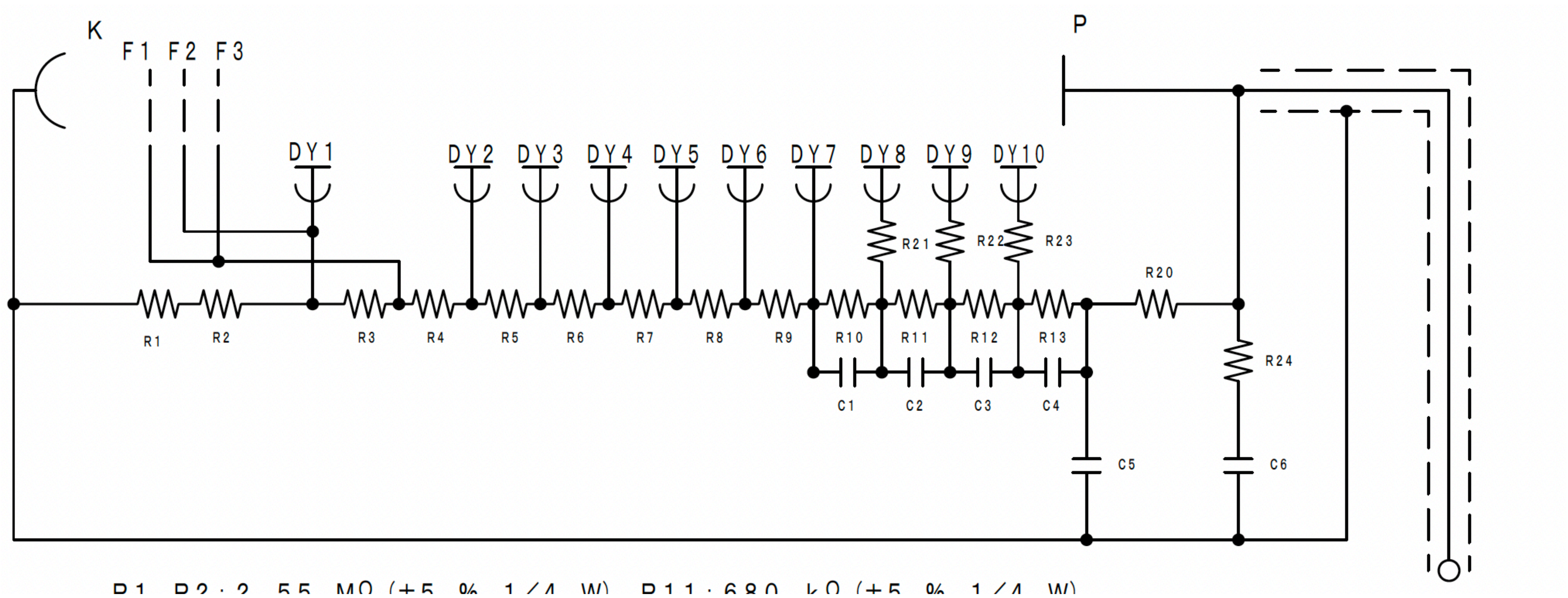
$\tau_s = \tau = 5ns$

$\tau_s > \tau$: set $\tau_s = 25ns$ and τ=5 ns



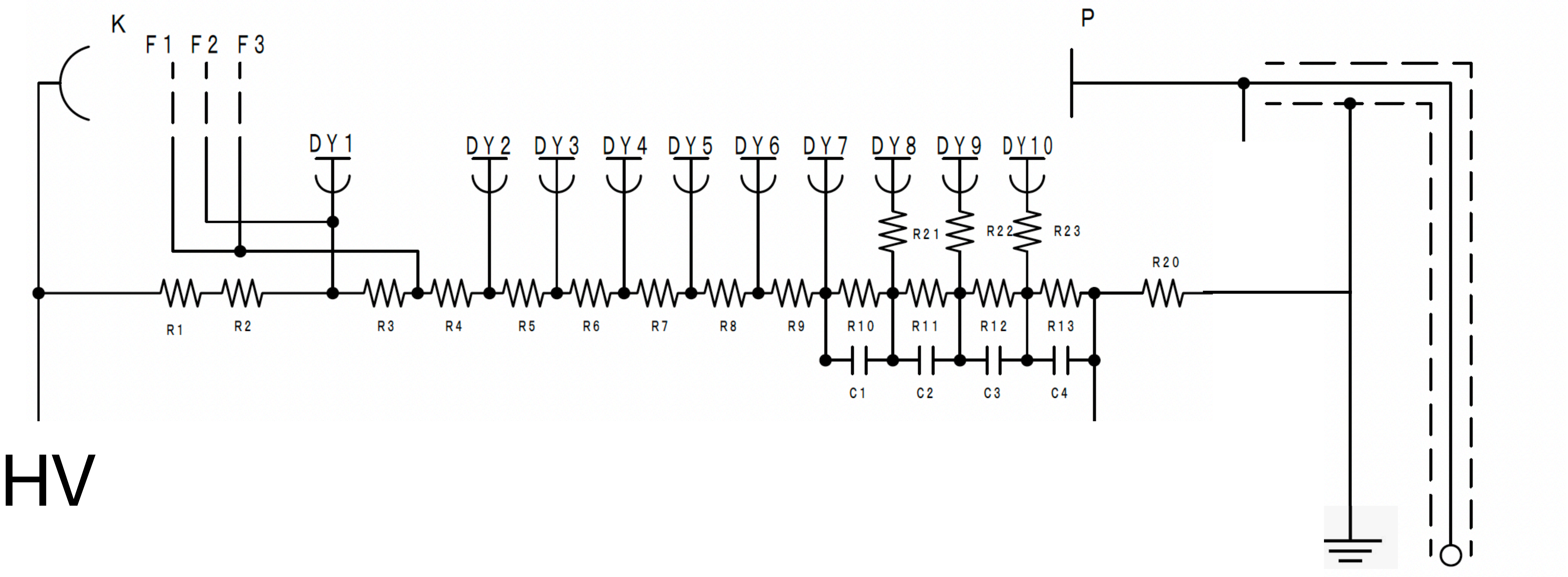
Rise time is the shorter time and fall time is the longer time

AC coupled PMT



This is a PMT base circuit in which a positive HV is applied at the anode. Signals have to separated externally by using a capacitor splitter.

A termination R24=50 ohm is included.



-HV

This is a PMT base circuit in which a negative HV is applied at the cathode. Signal is at anode and notice that it directly goes into a cable that is 50 ohm. This requires no splitter and all DC current from the anode is measured.

Notice that this scheme requires 2 cables. And the cathode will be at HV compared to its surroundings.

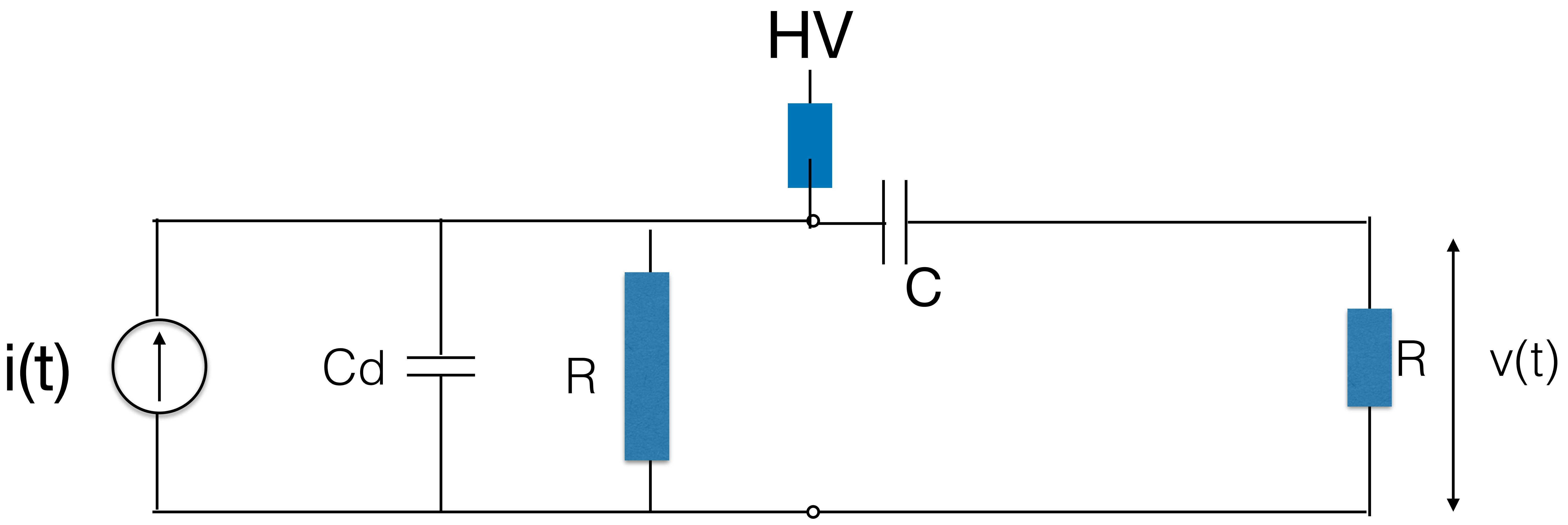
Detectors and readout

A detector generates charge; when charge moves it creates an electrical current that can be measured externally.

For many applications with PMTs we use negative HV to move the electrons, but AC coupling is much more common for many detectors.

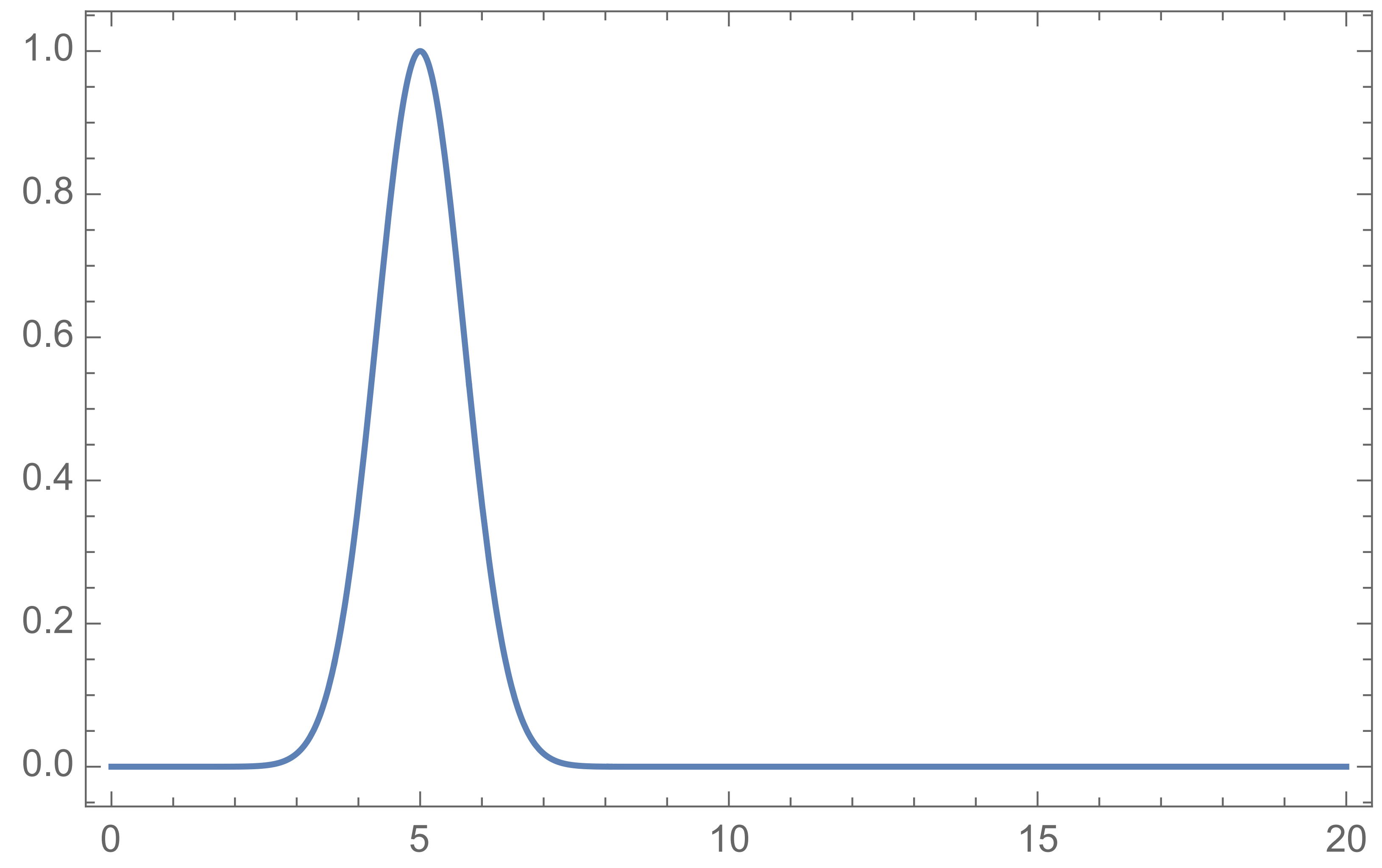
But to move the charge, we need to apply some bias voltage. This bias voltage is the source of energy for the detector to generate the signals that we want.

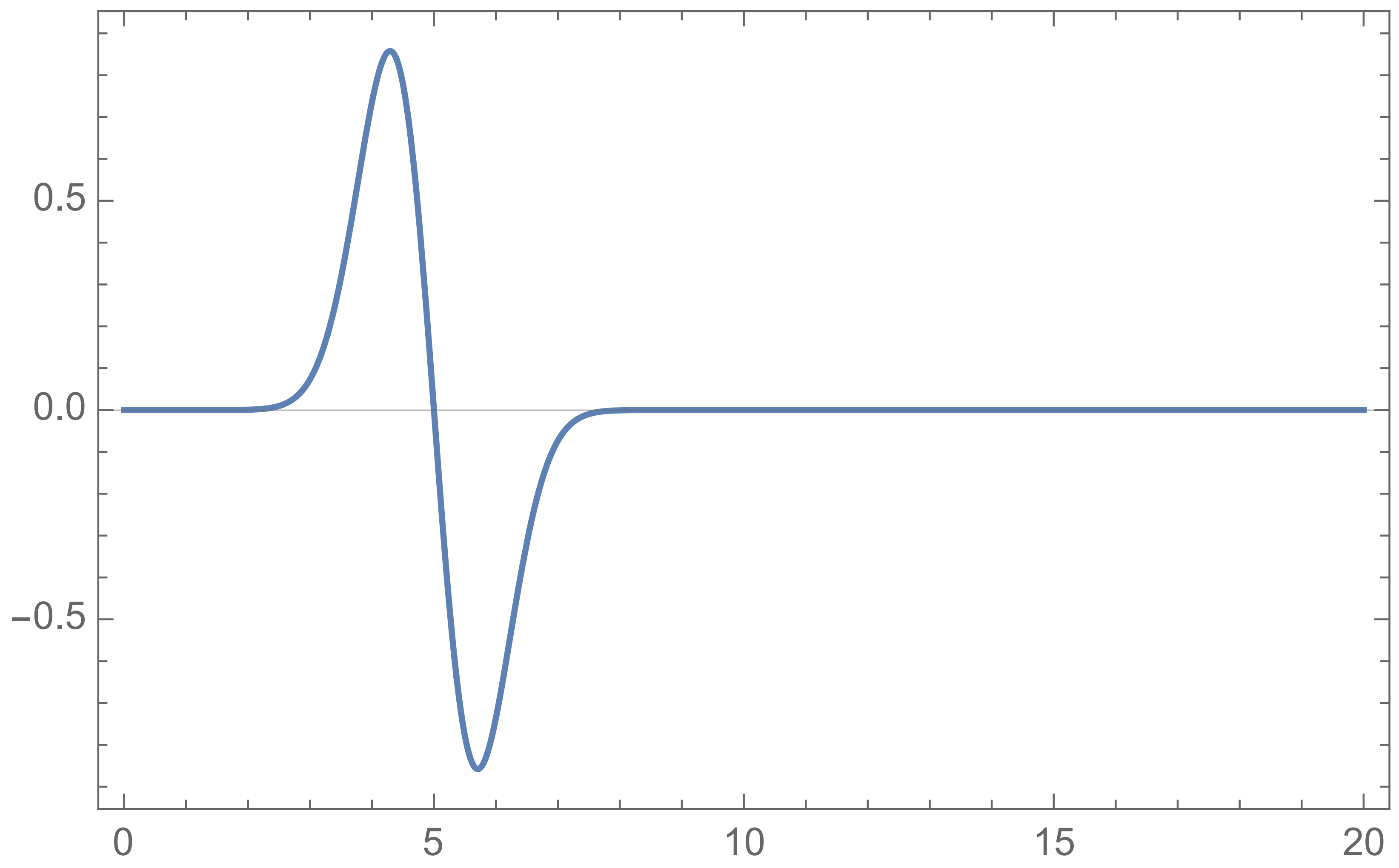
A common problem is how to separate the bias voltage from the signal.
This is usually done with a capacitor that can hold the HV.   A resistor limits the current from the HV.



There are many schemes for doing this and it can define the shape of the signal at output

Different types of pulses





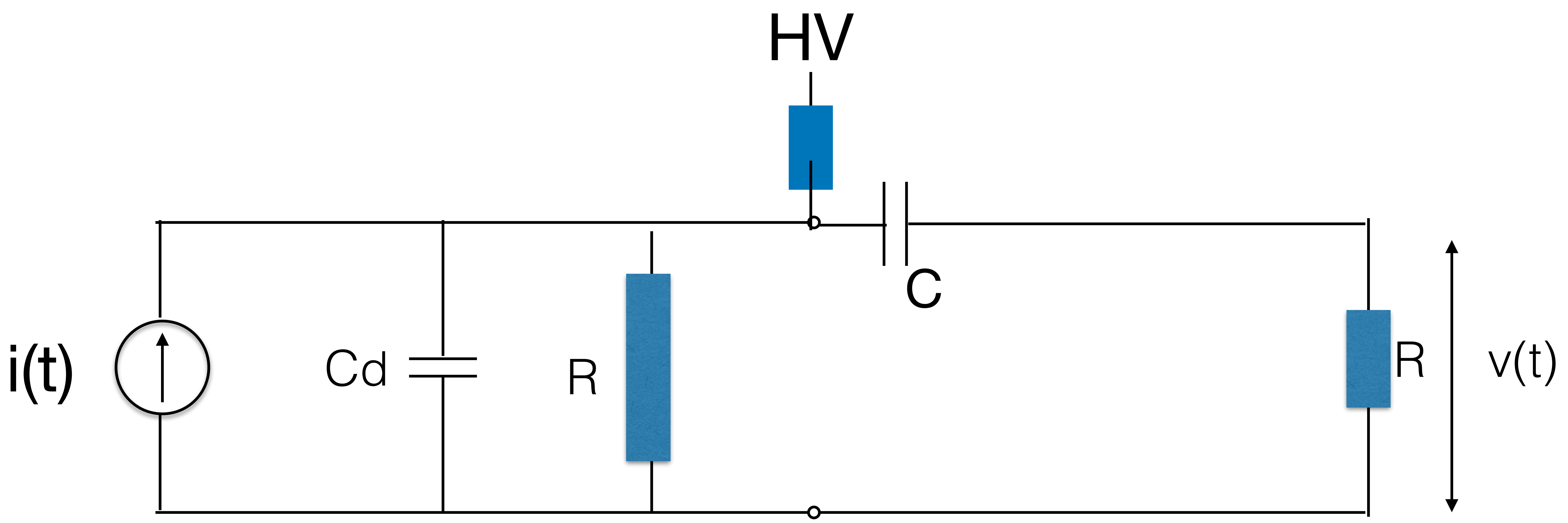
- A Gaussian unipolar pulse is preferred because it is less subject to noise.
- Also it can be integrated to provide the charge from the detector.
- But if the pulse is differentiated then it is much more subject to noise, and cannot be integrated to provide the charge.
- Detector electronics is arranged to make the pulse shape to allow both time and charge measurements.
- It is possible to choose the coupling capacitor so that the second negative pulse is extended to very long times and it becomes effectively a single pulse.
- This is called Pole_zero cancellation.

Safety issue

Notice that C is charged up to the high voltage and so has a charge on it of Q = C x HV.
The energy in the charge is E = (1/2) C x HV^2
You will get a shock and cause a spark if this exceeds ~ 0.25 J (safety standard)
For HV = 1000 V and C = 200 nF E = 0.1 J. This will certainly cause a shock.

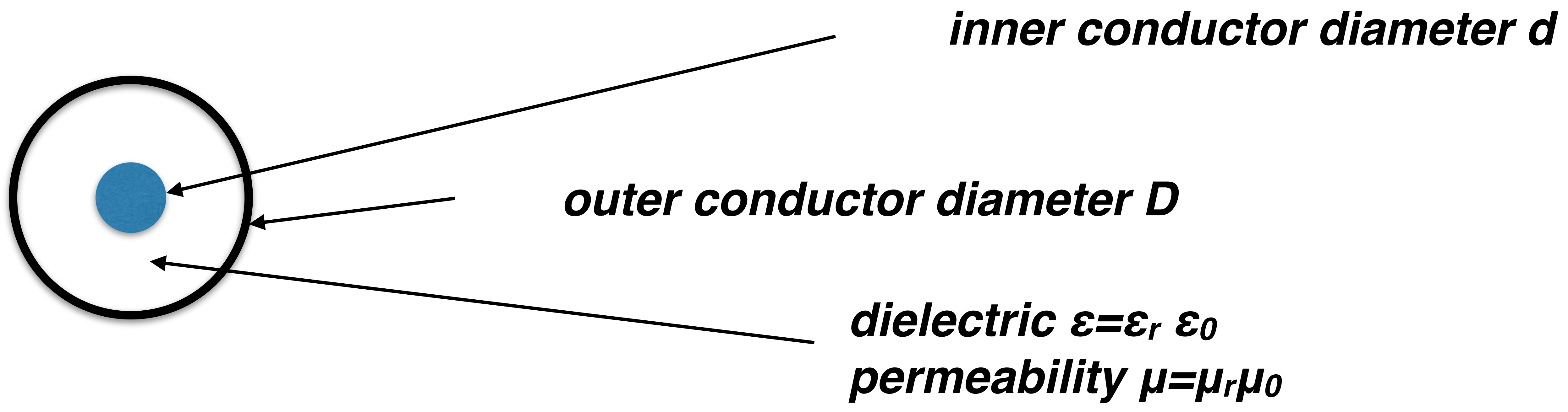
Rules: the circuit must be enclosed and no touching or removing the cable while it is hot.

Turn off the HV and wait for the capacitor to drain for a few minutes.



Issue of the cable

Coaxial Cable



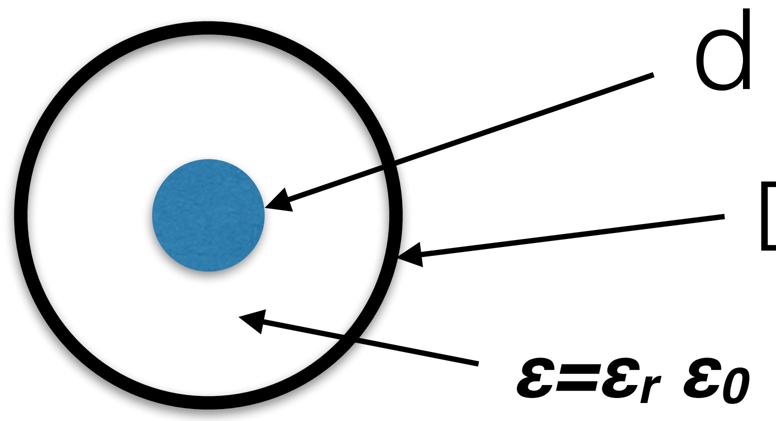
We will do some elementary calculations first, and then perform a more detailed analysis.

The idea is to use these notes as reference when needed.

Recall that the same treatment can be used for any pair of conductors that are parallel to each other and are used to transmit an electrical signal.

Signal gets transmitted by alternating electric and magnetic fields between the two conductors.

Capacitance of coaxial cable



Assume a charge of λ per unit length on the inner conductor. Then for length dl the total charge is $q = \lambda dl$

The electrical field at radius $d < r < D$ is radial and given by

$$2\pi r dl E_r \varepsilon_0 \varepsilon_r = q = \lambda dl$$

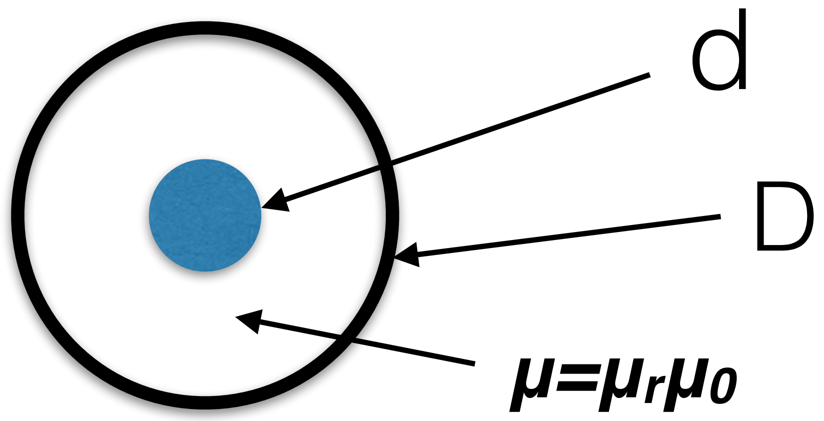
$$E_r = \frac{\lambda}{2\pi r \varepsilon_0 \varepsilon_r}$$

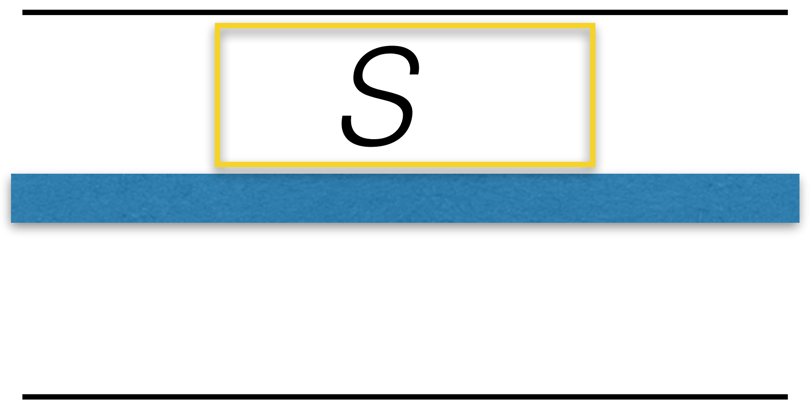
Voltage difference

$$\Delta V = \frac{\lambda}{2\pi\varepsilon_0\varepsilon_r}\int_d^D \frac{1}{r}dr = \frac{\lambda}{2\pi\varepsilon_0\varepsilon_r}\log(D/d)$$

Capacitance per unit length $c = \dfrac{\lambda}{\Delta V} = \dfrac{2\pi\varepsilon_0\varepsilon_r}{\log(D/d)}$

Inductance of coaxial cable





Assume there is current I in the inner conductor and $-I$ in outer.

The magnetic field in the cable for $d < r < D$ is given by.

$$2\pi r B = \mu_0 \mu_r I$$

$$B = \frac{\mu_0 \mu_r I}{2\pi r}$$

we take a rectangular surface area S (length ι) normal to the B field which is circular around the central conductor.

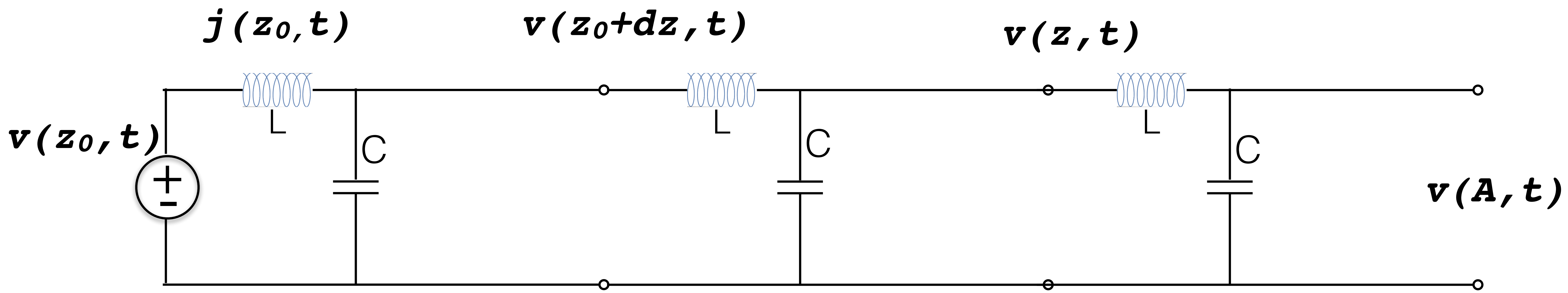
$$\phi = \int_S B\, ds = l \int_d^D \frac{\mu_0 \mu_r I}{2\pi r} dr$$

$$\text{Inductance per unit length} = \frac{\phi}{\iota * I} = \frac{\mu_0 \mu_r}{2\pi} \log(D/d)$$

Recall some numbers

$$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$$

$\mu_0 = 4\pi\ 10^{-7} H/m$ and so $\varepsilon_0 = 1/(\mu_0 c^2) \approx 8.85\ 10^{-12} F/m$



Cable is of length A, and each section has inductance $L = \ell dz$
and capacitance $C = cdz$. Now we get two coupled equations. $j(z,t)$ is the current at z, and t.

$$v(z+dz,t) - v(z,t) = -\ell dz \frac{dj(z,t)}{dt}$$ This has to do with the voltage drop across the inductor.

$$j(z+dz,t) - j(z,t) = -cdz \frac{dv(z,t)}{dt}$$ This is the current going to ground through the capacitor

$$\frac{dv(z,t)}{dz} = -\ell \frac{dj(z,t)}{dt}$$

$$\frac{dj(z,t)}{dz} = -c \frac{dv(z,t)}{dt} \qquad \Rightarrow \qquad \frac{d^2v(z,t)}{dz^2} = \ell c \frac{d^2v(z,t)}{dt^2}$$

We are going to use a double Fourier transform for the variables z,t. The conjugate variable for length is the wave number, k and for time it is frequency ω.

$$v(z,t) = \int V(k,\omega) e^{+i\omega t} e^{-ikz} dk d\omega; \quad j(z,t) = \int J(k,\omega) e^{+i\omega t} e^{-ikz} dk d\omega$$

Cable transmission (lossless) example

Plugging into the wave equation (this is true only when ω and k are not 0)

$$-k^2 V(k,\omega) = -\ell c \omega^2 V(k,\omega)$$

The condition of the wave equation is that

$$k^2 = \ell c \omega^2 \quad \text{or} \quad \beta = \omega / k = 1/\sqrt{\ell c}$$

The velocity of the wave through the cable is given by $1/\sqrt{\ell c}$. Going back to the coupled equations.

$$ikV(k,\omega) = i\omega \ell J(k,\omega)$$

This gives us the cable impedance which is real.

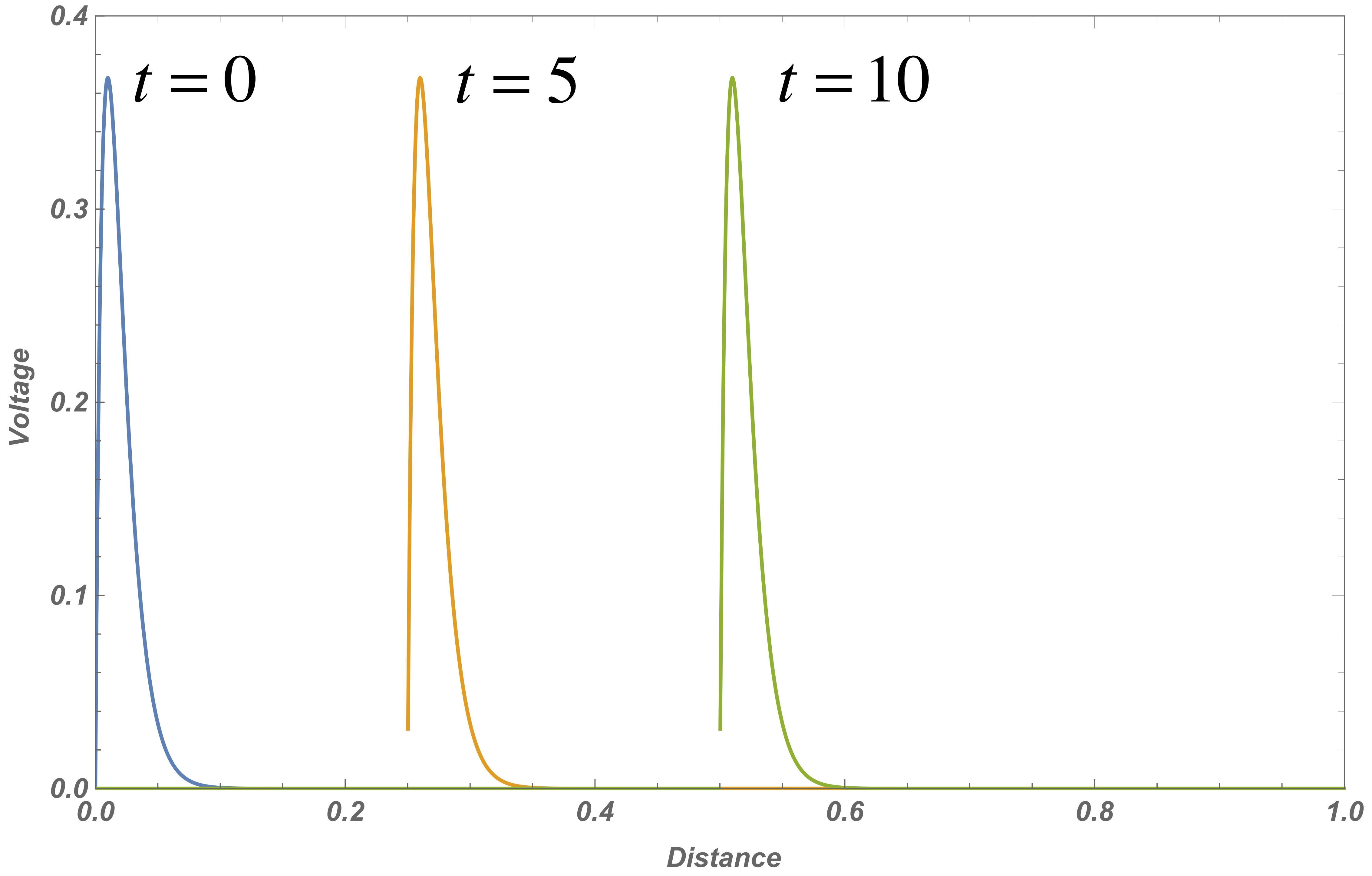
$$Z_0 = \frac{V(k,\omega)}{J(k,\omega)} = \ell \frac{\omega}{k} = \sqrt{\frac{\ell}{c}} \quad \text{.....} \quad \text{We now use the formulas for } \ell \text{ and c.}$$

$$\beta = 1/\sqrt{\ell c} = 1/\sqrt{\frac{\mu_0 \mu_r}{2\pi} \log(D/d) \frac{2\pi \varepsilon_0 \varepsilon_r}{\log(D/d)}} = \frac{c_{light}}{\sqrt{\mu_r \varepsilon_r}} \quad \text{... only depends on dielectric.}$$

$$Z_0 = \sqrt{\frac{\frac{\mu_0 \mu_r}{2\pi} \log(D/d)}{\frac{2\pi \varepsilon_0 \varepsilon_r}{\log(D/d)}}} = \frac{\log(D/d)}{2\pi} \sqrt{\frac{\mu_0 \mu_r}{\varepsilon_0 \varepsilon_r}} = \frac{\log(D/d)}{2\pi} \mu_0 c_{light} \sqrt{\frac{\mu_r}{\varepsilon_r}}$$

$$Z_0 = 60\Omega \times \sqrt{\frac{\mu_r}{\varepsilon_r}} \log(\frac{D}{d})$$

Graphical Solution



$k = 100$

$\omega = 5$

$v = \omega / k = 0.05$

$CableLeng = 1.0$

The solution to the wave equation is a continuous function with the following form

$v(z,t) = f(kz \pm \omega t)$

The sign of k determines the direction of the pulse movement.

Solution

We can also think about the solution in Laplace or Fourier space

(also known as Phaser space)

$v(z,t) = f(kz \pm \omega t)$

Notice that this could be considered a function with a time shift $t \rightarrow t \pm z / \upsilon$

The time shift is z dependent. i.e. given a location along the cable, the

signal is time shifted by $\pm z / \upsilon$. In Fourier space this is just a phase shift.

On page 8 we obtained the solution to a PMT pulse. In case of a cable

attached to the PMT with a velocity constant υ, the solution at z is

$$V(\omega) \rightarrow V(\omega) \times e^{-i\omega(z/\upsilon)} = \frac{q_0 R}{(1+i\omega\tau_s)(1+i\omega\tau)} e^{-i\omega(z/\upsilon)}$$

This is going to be very useful. We can call the phase factor the propagator along the

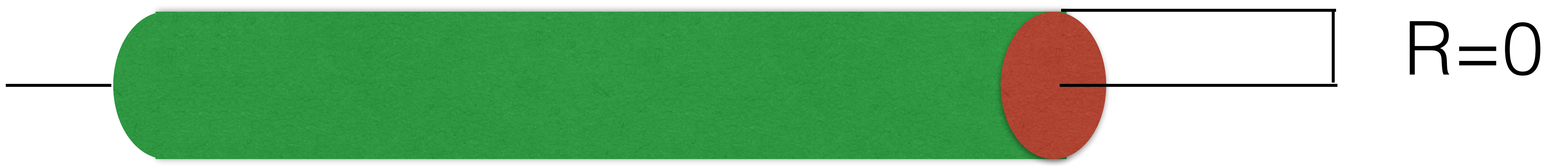
cable. If a cable is of length l, then every time the signal goes from one end to the

other it gets a phase factor $\mathrm{e}^{-i\omega(l/\upsilon)}$

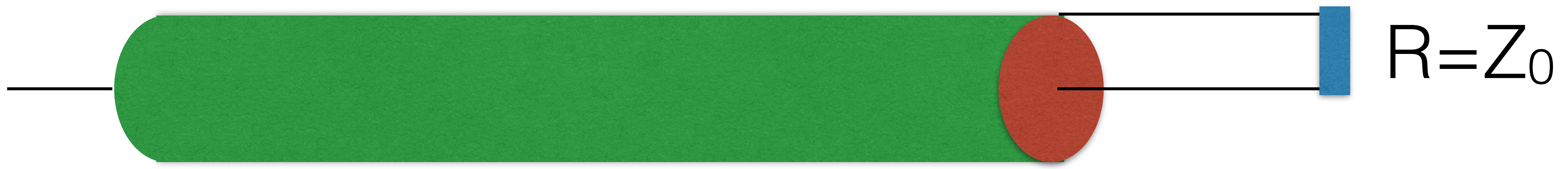
Some notes

- *It is useful to understand the cable in terms of some analogies.*
- *The inductance represents the tendency of the cable to oppose a change in current. It slows down the movement of charge down a cable.*
- *The capacitance can be thought of as the tendency of the cable to oppose a change in voltage. The characteristic impedance is the balance of the L and C.*
- *Any shunt resistances (through the dielectric) along the cable act as dissipators of the charge, somewhat like leaks in a pipe.*
- *As the charge flows through the inductor (if the rate is increased too fast it is slowed), it fills the capacitor until the voltage increases so that the charge can flow forward again.*
- *For an infinite cable, if the voltage is turned on suddenly at one end, the current flows continuously filling up the capacitors. Therefore an infinite cable is the same as a resistor.*
- *As the current reaches the end of the cable, if it sees a resistor of the same value as the cable impedance then it is as if the cable is infinite. This is called proper cable termination.*
- *The impedance of the cable is finite only for high frequencies or small wavelength (compared to its length).*

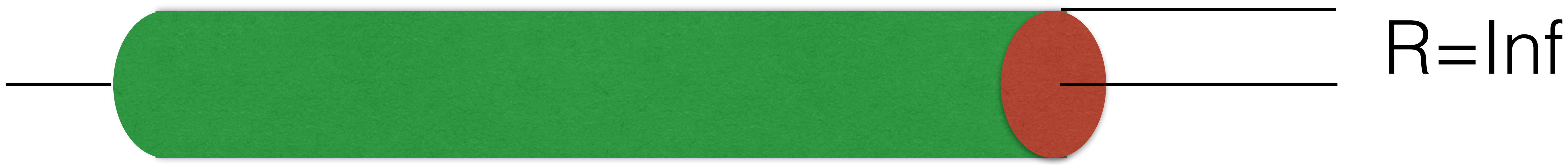
Imperfect termination



If the cable is shorted, charge traveling on the center will return on the shield. A voltage pulse will return with reversed polarity.



If the cable is terminated correctly charge traveling on the center will dissipate in the load. A voltage pulse will not return.



If the cable is open, charge traveling on the center will return on the center. A voltage pulse will return with same polarity.

Time measurement

Time probability distribution and resolution in scintillation counters.

- *Scintillation counters often are coupled to photo-multipliers.*
- *Scintillation photons have an exponential probability density distribution in time.*
- *A collection of n scintillation photons, each independent, will form a time series. Such a series will form a signal from a photo-multiplier tube.*
- *For our purposes here, we will assume that the detector has fast time response, and low noise. When we measure the time of the pulse, it is essentially the time of the first photo-electron in the time series of n photo-electrons*
- *Time resolution from such a device will have 3 components: 1) fluctuation in the time of the photon, 2) fluctuation from transit time spread, 3) fluctuation from the gain process in the PMT.*
- *For the time PDF, We will combine (2) and (3) into a single Normal distribution.*

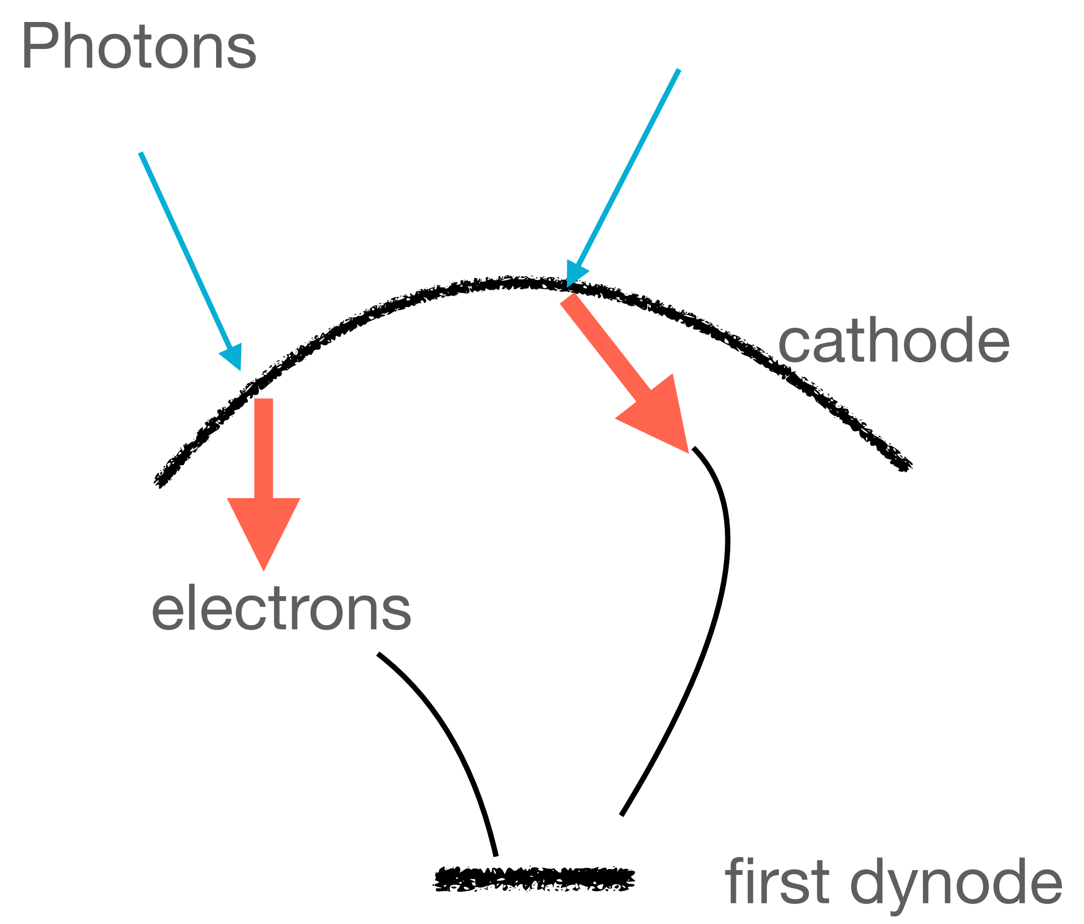


Photo-electrons from the cathode will arrive at different times at the first dynode where they are multiplied

Time resolution formula.

Proof can be found in my detailed notes.

- If $P_X(t)$ is the PDF of events in time, then $F_X(t) = \int_0^t P_X(x)dx$ is the cumulative probability that $(X \leq t)$
- If there are n events then the PDF for the minimum is given by

  $$P_T(t) = nP_X(t)[1 - F_X(t)]^{n-1}$$

  for an exponential PDF for X, $P_T = \frac{n}{\tau}e^{-nt/\tau}$

  The time resolution improves as $\sigma = \tau/n$

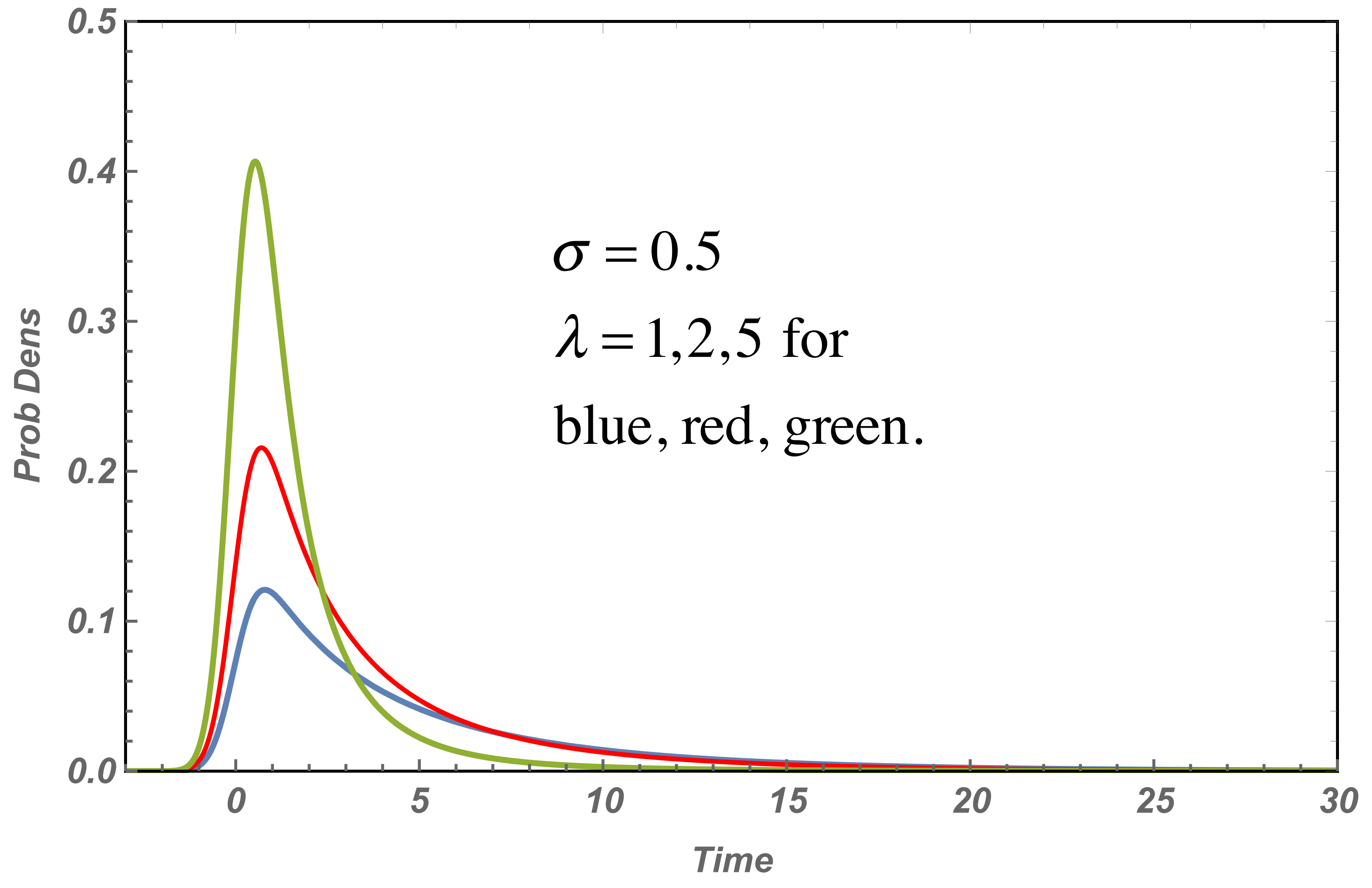
  We have to convolute this with Gaussian resolution due to noise and transit time spread.

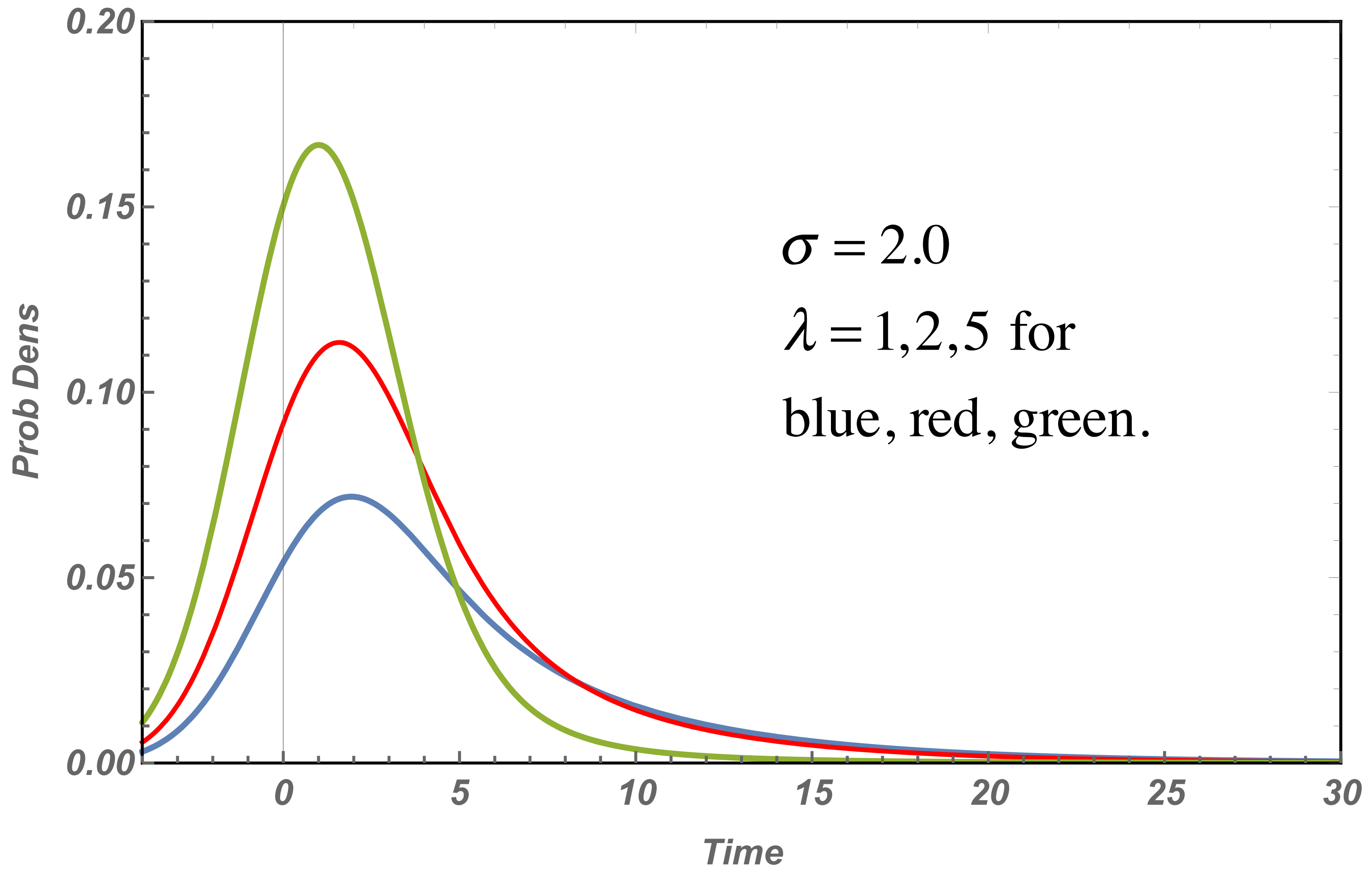
  We also have to include the Poisson probability for n events.

The answer for the distribution of first photon time including the effects of noise is now given by

$$\text{Probability(t or no-t)} = \left[\begin{array}{ll} e^{-\lambda} & \text{for no definite event time.} \\ \sum_{n=1}^{\infty} e^{-\lambda} \frac{\lambda^n}{n!} \times E_N(t:0,\sigma^2,n/\tau) & \text{when there is a time} \end{array} \right.$$

We plot this for some examples. We set τ=6; σ=0.5, 2 and λ=1, 2, 5



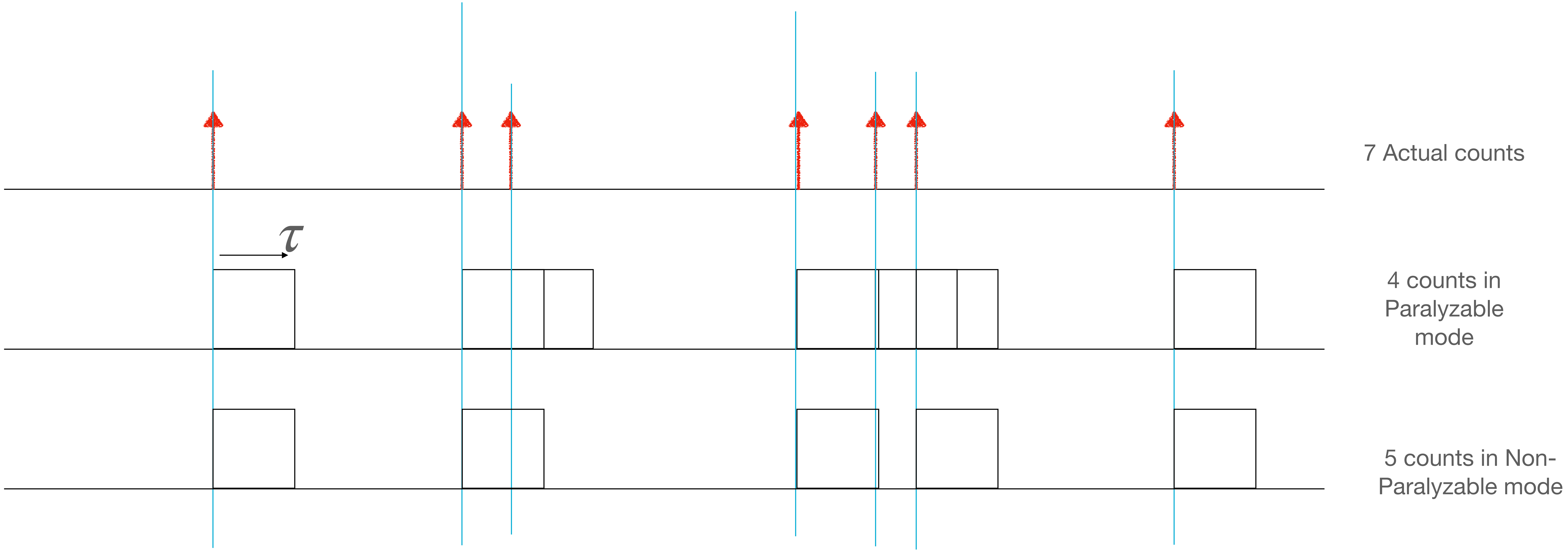


Notice how there is a tail for low number of photons. We leave it to the reader to calculate the mean and variance of these.

Random processes and Pulse Counting in typical experiments

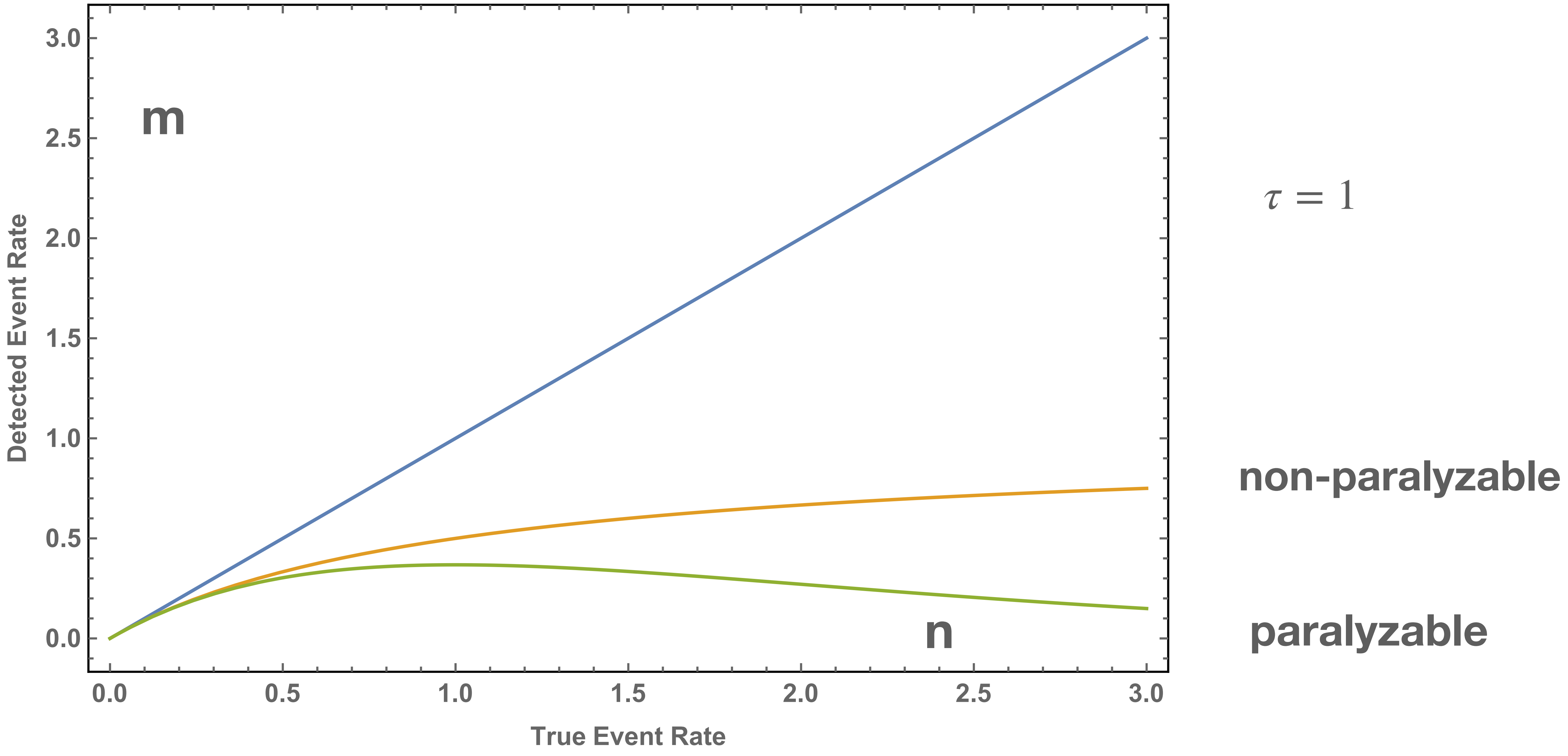
Using a thresholding algorithm

- **In typical experiments, such as accelerator or underground radiation counters, we count events after some processing.**
- **A continuous measurement in time may contain a signal as pulses, and some random noise. A simple pulse finding method will count a signal pulse when the measurement makes a transition from low to high across a given threshold.**
- **The statistics of such a thresholding filter are non-trivial, but calculable under some assumptions.**
- **We will do a few calculations for reference. This is actually a subject of interest in both the statistics and electrical engineering communities.**



- ***First we will do some simple stuff covered in textbooks (see Glen F. Knoll, for example). Feller (1948) Courant Lectures has a complete description. Feller originally identified these are Type 1, and Type 2 processes.***
- ***Let's imagine that there are n real events in a unit of time.***
- ***The pulse that is produced and used for counting has a holding time (width) of τ .... And m pulses are actually counted.***
- ***The relation between $m < n$ depends on the nature of the pulses: if the filter produces pulses of arbitrary length (thus not counting events when the events are closer than the resolving time) then it is called paralyzable. If the filter produced pulses of fixed length no matter what then it is called Non-paralyzable.***
- ***The real situation can be more complex and could be based on the shape of the event pulse. But these are useful extremes to analyze.***

Non-paralyzable (Type 1) and Paralyzable (type 2) dead time loss



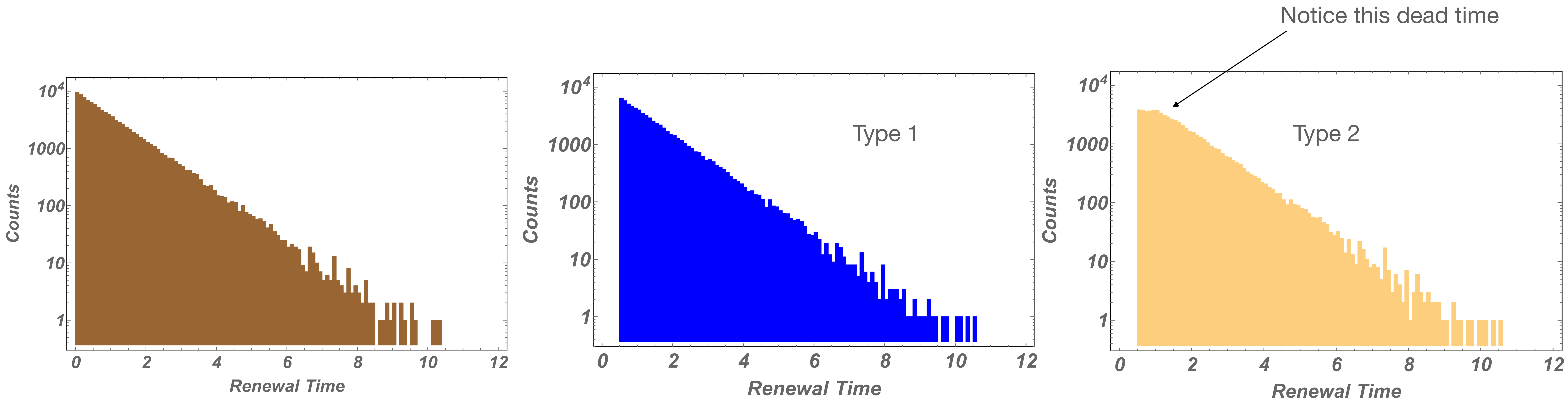
- **For a randomly distributed event rate of r what is the probability density function for the time intervals between events ?** $P_{gap}(t) = re^{-rt}$
- **Non-paralyzable dead time formula:** $m = \dfrac{n}{(1 + n\tau)}$
- **Paralyzable dead time formula:** $m = ne^{-nt}$

More generally: what can we monitor to make sure the detector and pulse counting system are working as intended ?

- Clearly we cannot monitor the ratio m/n since we do not know n
- We could make a plot of the rate m and make sure it stays constant if that is what we expect in a data taking run.
- We could plot the width of the pulses, the minimum will be τ and then it will drop with some PDF. This PDF is known, what is it?
- We could plot the time between the edges of the pulses ? This would be called the renewal time distribution. This is actually the best monitor for each channel and also for the trigger. It should be exponential except for small times where it will deviate because of dead time.
- What if the rate n is not random in the sense of a Poisson process ? What if some of the rate is correlated. e.g, the pulses are more probable if they are close to each other.

Simulation of renewal times and pulse widths

1 Hz,   holding time = 0.5 sec



- You can find the proof of how to analytically calculate the renewal time PDF from Feller.
- You can also take a lecture on *Renewal theory* from MIT opencourses.
- Type 2 renewal times get pushed out longer because of the multiple overlapping pulses.
- Plot on right: The minimum pulse width is 0.5, When there is another pulse within 0.5 sec, this extends to 1 sec, a third pulse has to be present within the second 0.5 sec internal. Therefore there will be lower probability >1 sec.

Conclusion.

- ***We talked about the charge spectrum from a typical photo-multiplier.***
- ***We reviewed the basic mechanism for signal generation and transmission from a tube (could be any detector)***
- ***We examined time resolution as well as the statistics of counting from detectors.***
- ***Also look at https://www.phy.bnl.gov/~diwan/#photo for much more details.***