Brookhaven National Laboratory

SMEFT dipoles from low and high energy constraints

--- A story of light quarks

Zhite Yu (BNL, High Energy Physics)

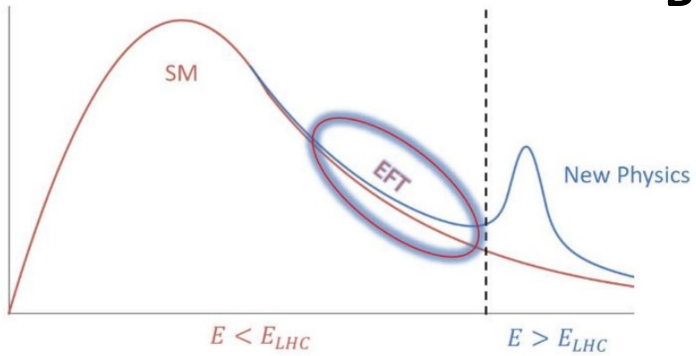
Work with Sally Dawson, in progress. All results preliminary!

BNL HET Lunch Seminar

BNL, Sep 4, 2026

SMEFT and dipole operators

$$\mathcal{L} = \mathcal{L}_{\text{SM}} + \sum_i \left(\frac{C_i}{\Lambda^2} O_i + \text{h.c.} \right)$$



Bottom-up approach

$$\Lambda = \mathcal{O}\,(\text{TeV})$$

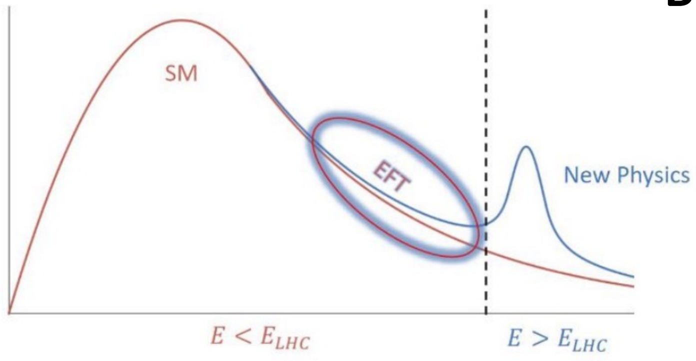
SMEFT

B. Grzadkowski et al, JHEP 10 (2010) 085

$X^2\varphi^2$		$\psi^2 X\varphi$	
$Q_{\varphi G}$	$\varphi^\dagger\varphi\, G^A_{\mu\nu}G^{A\mu\nu}$	Q_{eW}	$(\bar{l}_p\sigma^{\mu\nu}e_r)\tau^I\varphi W^I_{\mu\nu}$
$Q_{\varphi\widetilde{G}}$	$\varphi^\dagger\varphi\, \widetilde{G}^A_{\mu\nu}G^{A\mu\nu}$	Q_{eB}	$(\bar{l}_p\sigma^{\mu\nu}e_r)\varphi B_{\mu\nu}$
$Q_{\varphi W}$	$\varphi^\dagger\varphi\, W^I_{\mu\nu}W^{I\mu\nu}$	Q_{uG}	$(\bar{q}_p\sigma^{\mu\nu}T^A u_r)\widetilde{\varphi}\, G^A_{\mu\nu}$
$Q_{\varphi\widetilde{W}}$	$\varphi^\dagger\varphi\, \widetilde{W}^I_{\mu\nu}W^{I\mu\nu}$	Q_{uW}	$(\bar{q}_p\sigma^{\mu\nu}u_r)\tau^I\widetilde{\varphi}\, W^I_{\mu\nu}$
$Q_{\varphi B}$	$\varphi^\dagger\varphi\, B_{\mu\nu}B^{\mu\nu}$	Q_{uB}	$(\bar{q}_p\sigma^{\mu\nu}u_r)\widetilde{\varphi}\, B_{\mu\nu}$
$Q_{\varphi\widetilde{B}}$	$\varphi^\dagger\varphi\, \widetilde{B}_{\mu\nu}B^{\mu\nu}$	Q_{dG}	$(\bar{q}_p\sigma^{\mu\nu}T^A d_r)\varphi\, G^A_{\mu\nu}$
$Q_{\varphi WB}$	$\varphi^\dagger\tau^I\varphi\, W^I_{\mu\nu}B^{\mu\nu}$	Q_{dW}	$(\bar{q}_p\sigma^{\mu\nu}d_r)\tau^I\varphi\, W^I_{\mu\nu}$
$Q_{\varphi\widetilde{W}B}$	$\varphi^\dagger\tau^I\varphi\, \widetilde{W}^I_{\mu\nu}B^{\mu\nu}$	Q_{dB}	$(\bar{q}_p\sigma^{\mu\nu}d_r)\varphi\, B_{\mu\nu}$

SMEFT and dipole operators

$$\mathcal{L} = \mathcal{L}_{\rm SM} + \sum_i \left(\frac{C_i}{\Lambda^2} O_i + \text{h.c.} \right)$$

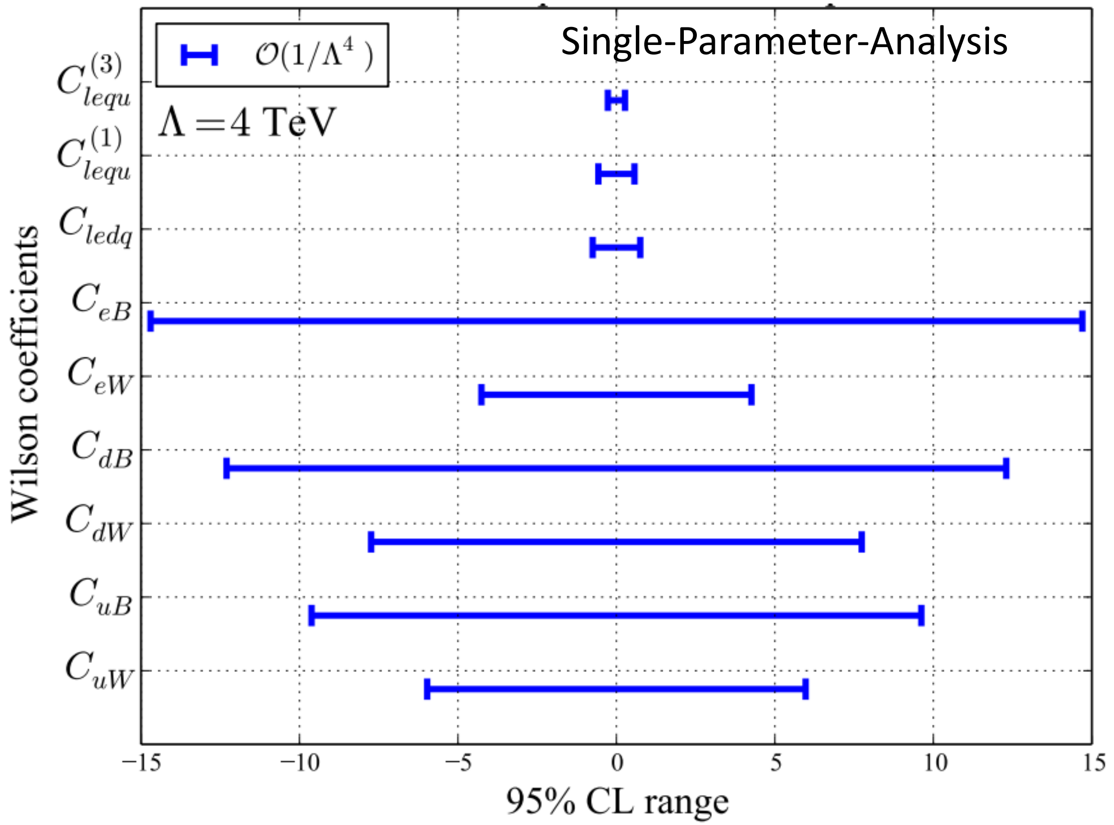


Bottom-up approach

$$\Lambda = \mathcal{O}\,(\text{TeV})$$

SMEFT

Boughezal et al, `21 PRD 104, 095022



Dipole operators and dipole moments: lepton case

❑ Example: photon dipole operator for electron

$$\mathcal{L} = \mathcal{L}_{\text{QED}} - \frac{e}{2\Lambda}\,\bar{\psi}\,\sigma^{\mu\nu}\big(\Gamma_R + i\,\Gamma_I\,\gamma_5\big)\psi\,F_{\mu\nu}$$

❑ Matched to low-energy non-relativistic case (at tree level)

Example: photon dipole Hamiltonian $\Delta\mathcal{H}_D = \frac{e}{2\Lambda}\bar{\psi}\,\sigma^{\mu\nu}(\Gamma_R + i\Gamma_I\gamma_5)\psi\,F_{\mu\nu}$

Applied to a particle state at constant EM field:

$$\Delta E_D = \lim_{p'\to p}\frac{\langle p', s|\int d^3\vec{x}\,\Delta\mathcal{H}_D(x)|p, s\rangle}{\langle p', s|p, s\rangle} = -\frac{2e}{\Lambda}\big(\Gamma_R\,\vec{B}\cdot\vec{S} + \Gamma_I\,\vec{E}\cdot\vec{S}\big) + \mathcal{O}(v)$$

⇒ Magnetic and electric **dipole moments**:

$$\Delta\vec{\mu} = \frac{2e}{\Lambda}\Gamma_R\vec{S}, \qquad \Delta\vec{d} = \frac{2e}{\Lambda}\Gamma_I\vec{S}$$

Dipole operators and dipole moments: quark case

❑ Example: photon dipole operator for quarks

$$\mathcal{L} = \mathcal{L}_{\text{QED}} - \frac{1}{4}\sum_q \bar{\psi}_q\,\sigma^{\mu\nu}\big(\mu_q + i\,d_q\,\gamma_5\big)\psi_q\,F_{\mu\nu}$$

❑ Does NOT directly apply to quarks due to confinement!

Instead, they match to **nucleon** dipoles $\Delta\mathcal{H}_D = \frac{1}{4}\sum_q \bar{\psi}_q\,\sigma^{\mu\nu}(\mu_q + i d_q\gamma_5)\psi_q\,F_{\mu\nu}$

Applied to a **proton** state at constant EM field: $\langle p, s'|\,\bar{\psi}_q\,\sigma^{\mu\nu}\,\psi_q\,|p, s\rangle = g_T^q\,\bar{u}(p, s')\sigma^{\mu\nu}u(p, s)$

$$\Delta E_D = \lim_{p'\to p}\frac{\langle p', s|\int d^3\vec{x}\,\Delta\mathcal{H}_D(x)|p, s\rangle}{\langle p', s|p, s\rangle} = \sum_q g_T^q\big(\mu_q\,\vec{B}\cdot\vec{S} + d_q\,\vec{E}\cdot\vec{S}\big)$$

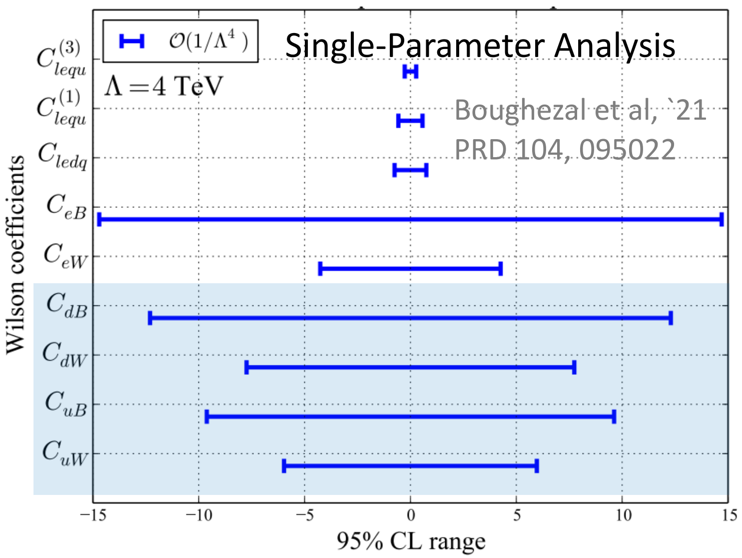
➡ Magnetic and electric **dipole moments**:

$$\Delta\mu_p = \sum_q g_T^q\,\mu_q, \qquad \Delta d_p = \sum_q g_T^q\,d_q$$

- LHS = low-energy measurements
- RHS: g_T^q from LQCD/transversity fits

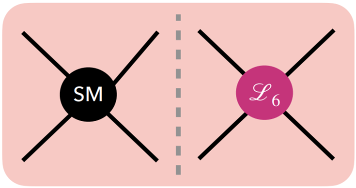
Special at high energy

❑ Suppressed interference with SM



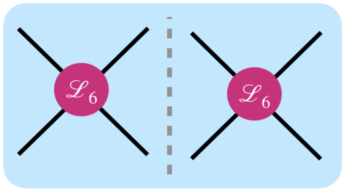
$$\sigma \propto |\mathcal{M}_{\rm SM} + \Lambda^{-2}\mathcal{M}_{\rm NP}|^2$$

Interference effects



$$\mathcal{O}\left(\Lambda^{-2}\right)$$

But, dipole operators flip chirality



$$\mathcal{O}\left(\Lambda^{-4}\right)$$

→ *Weak* constraints!

❑ Chiral-odd property

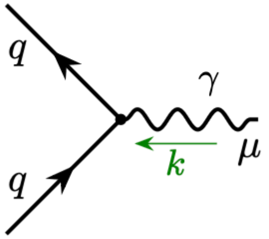
$$\begin{aligned} 16\pi^2 \frac{dC_{uW}}{d\ln\mu} &= \left[2C_{F,3}g_3^2 + \mathcal{O}(g_2^2, g_1^2)\right] C_{uW} + \mathcal{O}(g_1 g_2) C_{uB} + 2C_{F,3} g_2 g_3 C_{uG} \\ &\quad - Y_u^\dagger \left[\mathcal{O}(g_2)\left(C_{HW} + i\, C_{H\widetilde{W}}\right) + \mathcal{O}(g_1)\left(C_{HWB} + i\, C_{H\widetilde{W}B}\right)\right] \\ &\simeq 2C_{F,3} g_3^2 C_{uW} \end{aligned}$$

Mix with ***dipole*** SMEFTs, or with others via ***Yukawa*** couplings. → Self-contained subset for light fermions.

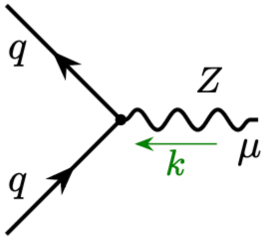
Light quark dipole operators

- **Dipoles in (γ, Z) basis: 4 dipole couplings = 8 real d.o.f.**

$$\mathcal{L} \supset \mathcal{L}_{\mathrm{SM}} - \frac{v+h}{2v^2}\Big[eA_{\mu\nu}\bar{u}\,\sigma^{\mu\nu}\big(\mathrm{Re}\,\Gamma^u_\gamma + i\,\mathrm{Im}\,\Gamma^u_\gamma\,\gamma_5\big)u + g_Z Z_{\mu\nu}\bar{u}\,\sigma^{\mu\nu}\big(\mathrm{Re}\,\Gamma^u_Z + i\,\mathrm{Im}\,\Gamma^u_Z\,\gamma_5\big)u + (u \to d)\Big]$$



$$= -ie\left\{e_q\gamma^\mu + \frac{i\sigma^{\mu\nu}k_\nu}{v}\left[\mathrm{Re}(\Gamma^q_\gamma) + i\,\mathrm{Im}(\Gamma^q_\gamma)\gamma_5\right]\right\},$$



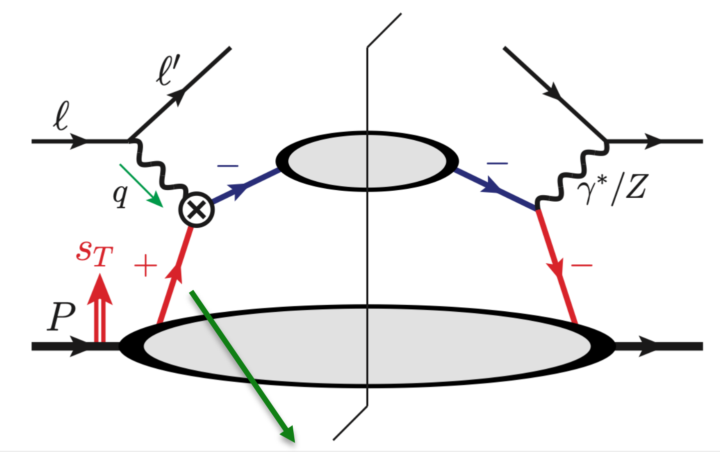
$$= -ig_Z\left\{\gamma^\mu(g^q_V - g^q_A\gamma_5) + \frac{i\sigma^{\mu\nu}k_\nu}{v}\left[\mathrm{Re}(\Gamma^q_Z) + i\,\mathrm{Im}(\Gamma^q_Z)\gamma_5\right]\right\},$$

$$g_Z = \frac{g_2}{2\cos\theta_W}$$

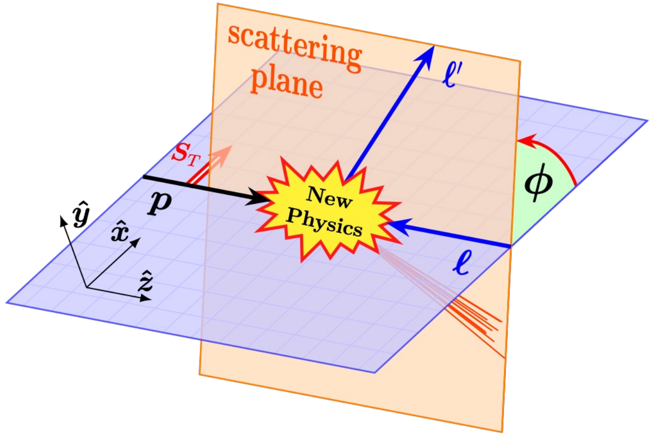
Chiral-odd probe yields linear sensitivity

❑ Transverse spin asymmetry

R. Boughezal, et al, PRD 107 (2023) 075028



$$\frac{\not k}{2} \to \frac{\not k}{2}(1+\gamma_5 \not s_T)$$



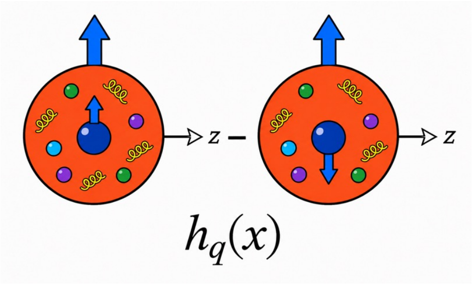
Observable:

lepton's **azimuthal** asymmetry

in the Lab frame

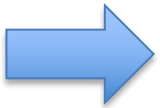
s_T introduces another γ-matrix → **Chiral-odd!**

❑ Quark transverse spin comes from proton via transversity PDF



$$h_q(x)\boldsymbol{S}_T = \frac{dN_q^\uparrow}{dx} - \frac{dN_q^\downarrow}{dx}$$

$$f_q(x) = \frac{dN_q^\uparrow}{dx} + \frac{dN_q^\downarrow}{dx}$$



Quark spin

$$\boldsymbol{s}_{T,q}(x) = \frac{h_q(x)}{f_q(x)}\boldsymbol{S}_T$$

Proton spin

Observable forms

❑ Transverse spin asymmetry with azimuthal modulation

- Spin $\boldsymbol{S_T}$ enters along with dipole couplings Γ_V^q
- Constrained by rotation and parity: $\boldsymbol{S}_T \to \boldsymbol{S}_T, \text{ but } (\boldsymbol{p}, \boldsymbol{\ell}'_T) \to (-\boldsymbol{p}, -\boldsymbol{\ell}'_T)$

Only two forms: $\underbrace{\boldsymbol{S}_T \cdot \boldsymbol{\ell}'_T \propto \cos(\phi - \phi_S)}_{\text{P-odd}}, \quad \underbrace{\boldsymbol{p} \cdot (\boldsymbol{S}_T \times \boldsymbol{\ell}'_T) \propto \sin(\phi - \phi_S)}_{\text{P-even}}$

❑ Constraint from chiral symmetry

$$W_{\alpha\alpha'}(\phi) = \mathcal{M}_\alpha(\phi)\mathcal{M}^*_{\alpha'}(\phi) = e^{i(\alpha-\alpha')\phi} W_{\alpha\alpha'}(0)$$

$W^q_{+-}(\phi) = e^{i\phi} W^q_{+-}(0) \propto \Gamma^q_V$ flips quark helicity from R to L in the amplitude. $\quad \Delta\phi \equiv \phi - \phi_S$

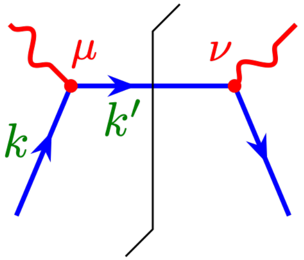
$$\frac{d\sigma}{dx\, dQ^2\, d\phi} \propto \mathrm{Re}\left[\rho_{+-}(\boldsymbol{s}_T)\, W^q_{+-}(\phi)\right] = s_T\, \mathrm{Re}\left[\Gamma^q_V e^{i(\phi-\phi_S)}\right] = s_T \Big[\underbrace{\mathrm{Re}(\Gamma^q_V)\cos(\Delta\phi) - \mathrm{Im}(\Gamma^q_V)\sin(\Delta\phi)}_{\text{P-odd}} \Big]$$

The coefficient function must be *parity-odd.*

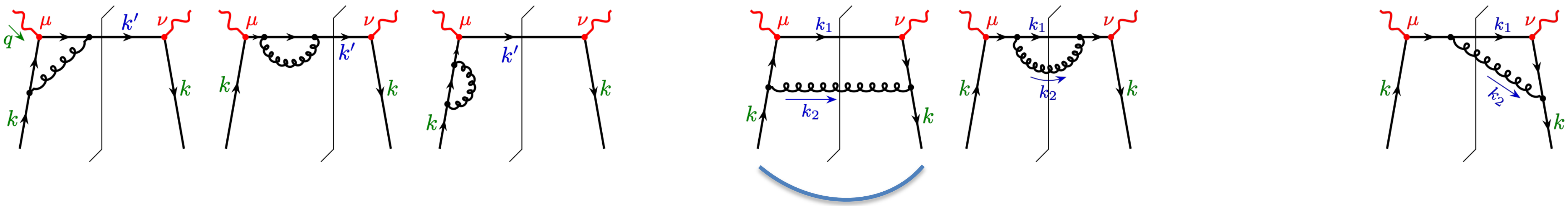
Results for $\gamma\gamma$ channel

❑ Leading order

$$\frac{d\sigma_\gamma^{(0)}}{dx_B\,dQ^2\,d\phi} = \frac{\alpha_e^2}{Q^4}\Bigg\{(1+(1-y)^2)\cdot\sum_q e_q^2\,[f_q(x_B,\mu)+f_{\bar q}(x_B,\mu)]$$
$$+\underbrace{\lambda_e S_T}_{\text{Double spin asymmetry: (LT)}}\cdot\frac{2Q}{v}\cdot y\sqrt{1-y}\sum_{q=u,d} e_q\,\mathrm{Re}\Big[\Gamma_\gamma^q\,e^{i(\phi-\phi_S)}\Big]\,[h_q(x_B,\mu)+h_{\bar q}(x_B,\mu)]\Bigg\}$$



❑ Next-to-leading order of QCD

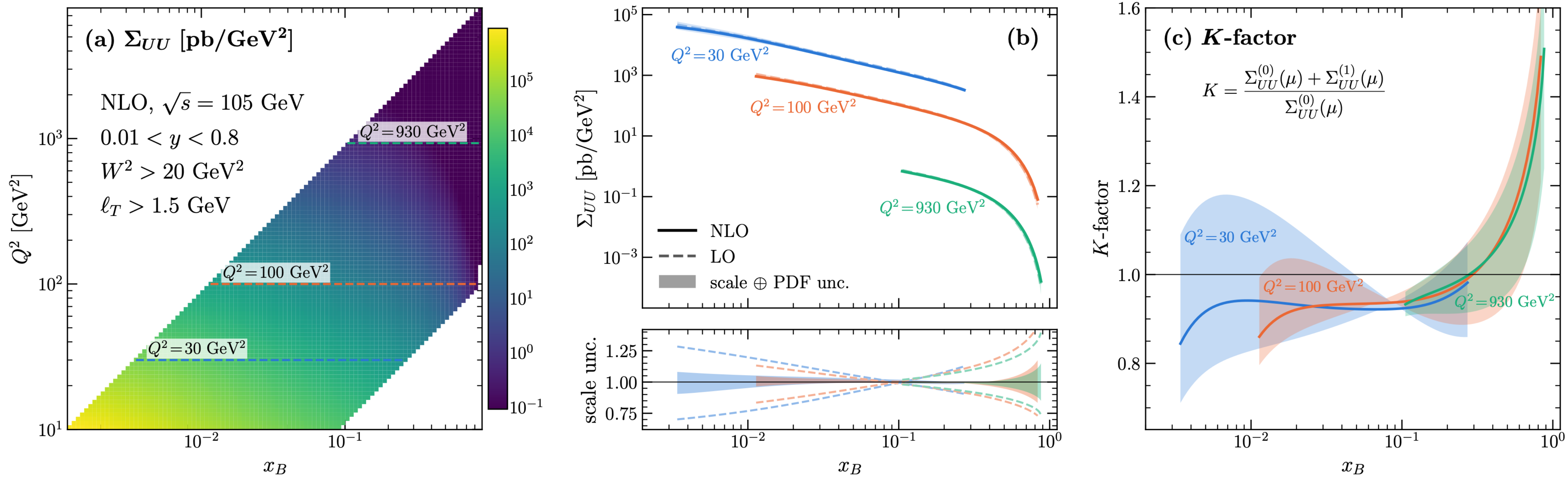


Kreimer's scheme: reading point → $\frac{\not k}{2}(1+\gamma_5 \not s_T)$

Preserve chiral symmetry: $\mathrm{Re}\Big[\Gamma_\gamma^q\,e^{i(\phi-\phi_S)}\Big]$

Cross section: unpolarized part

$$\frac{d\sigma}{dx_B\, dQ^2\, d\Delta\phi} = \frac{1}{2\pi}\left[\Sigma_{UU}(x_B, Q^2) + \lambda_e\, \Sigma_{LU}(x_B, Q^2) + S_T\, \Sigma_{UT}(x_B, Q^2, \Delta\phi) + \lambda_e S_T\, \Sigma_{LT}(x_B, Q^2, \Delta\phi)\right]$$



Unpolarized cross section $= f_{i/p} \otimes$ unpolarized hard coefficients of (AA, AZ, ZZ)

PDF uncertainty $\lesssim 1\%$

JAM22-PDF NLO, 96 replica PDFs

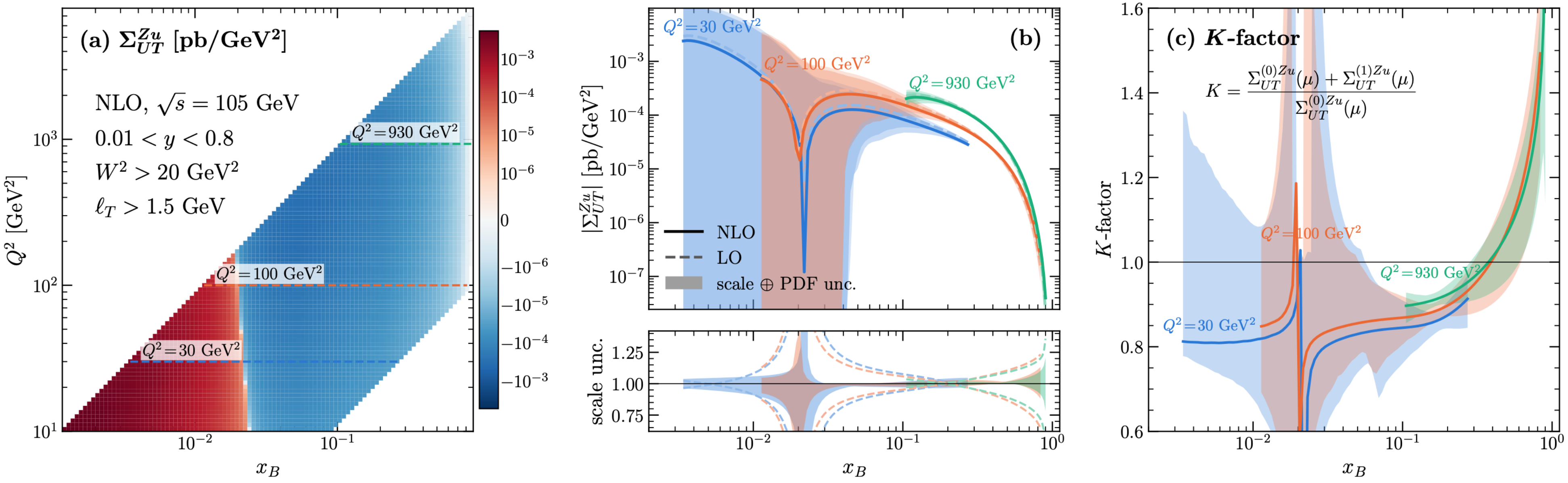
Cross section: single transverse spin

$$\frac{d\sigma}{dx_B\, dQ^2\, d\Delta\phi} = \frac{1}{2\pi}\left[\Sigma_{UU}(x_B,Q^2) + \lambda_e\,\Sigma_{LU}(x_B,Q^2) + S_T\,\Sigma_{UT}(x_B,Q^2,\Delta\phi) + \lambda_e S_T\,\Sigma_{LT}(x_B,Q^2,\Delta\phi)\right]$$

$$\Sigma_{UT}(x_B,Q^2,\Delta\phi) = \sum_{q=u,d}\left[\mathrm{Re}\left(\Gamma_Z^q e^{i\Delta\phi}\right)\,\Sigma_{UT}^{Zq}(x_B,Q^2) + \mathrm{Re}\left(\Gamma_\gamma^q e^{i\Delta\phi}\right)\,\Sigma_{UT}^{\gamma q}(x_B,Q^2)\right]$$

$(\gamma Z, ZZ)$ channels

γZ channel



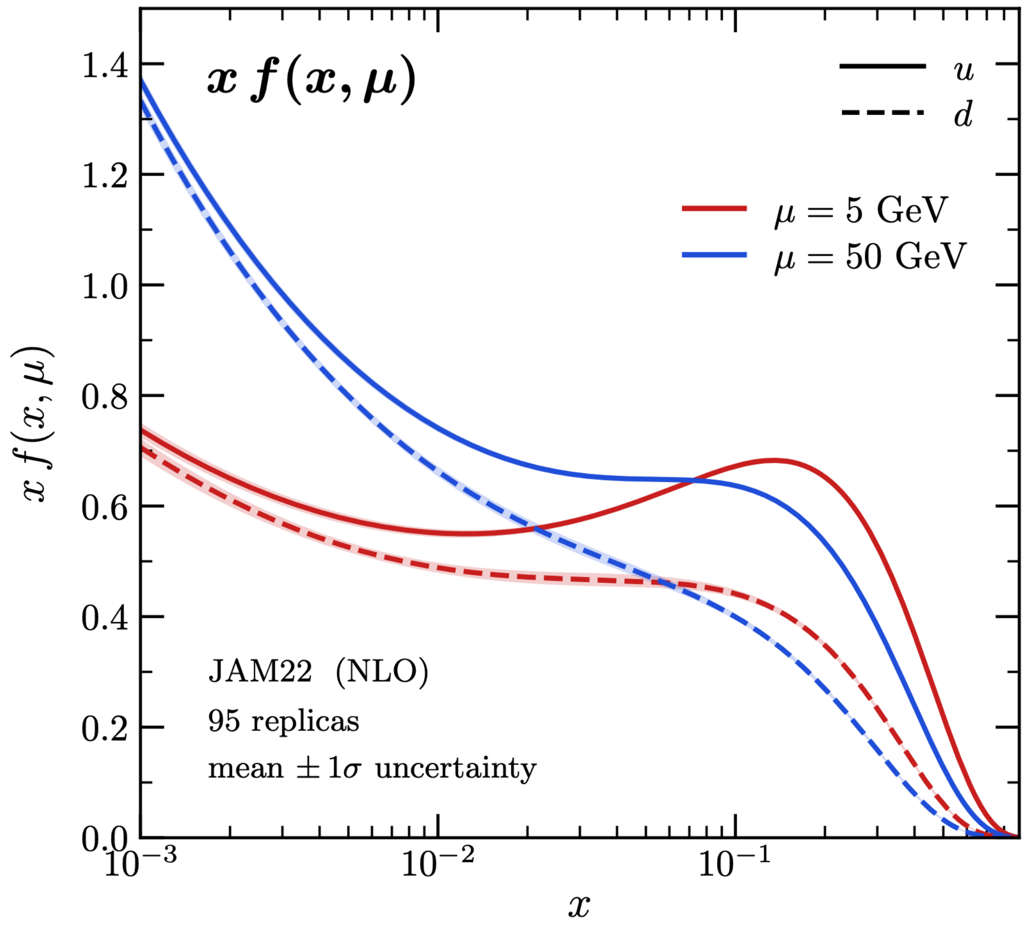
Σ_{UT}^{Zu} dominates

Single polarized cross section = $h_{q/p} \otimes$ polarized hard coefficients of $(\gamma Z, ZZ)$

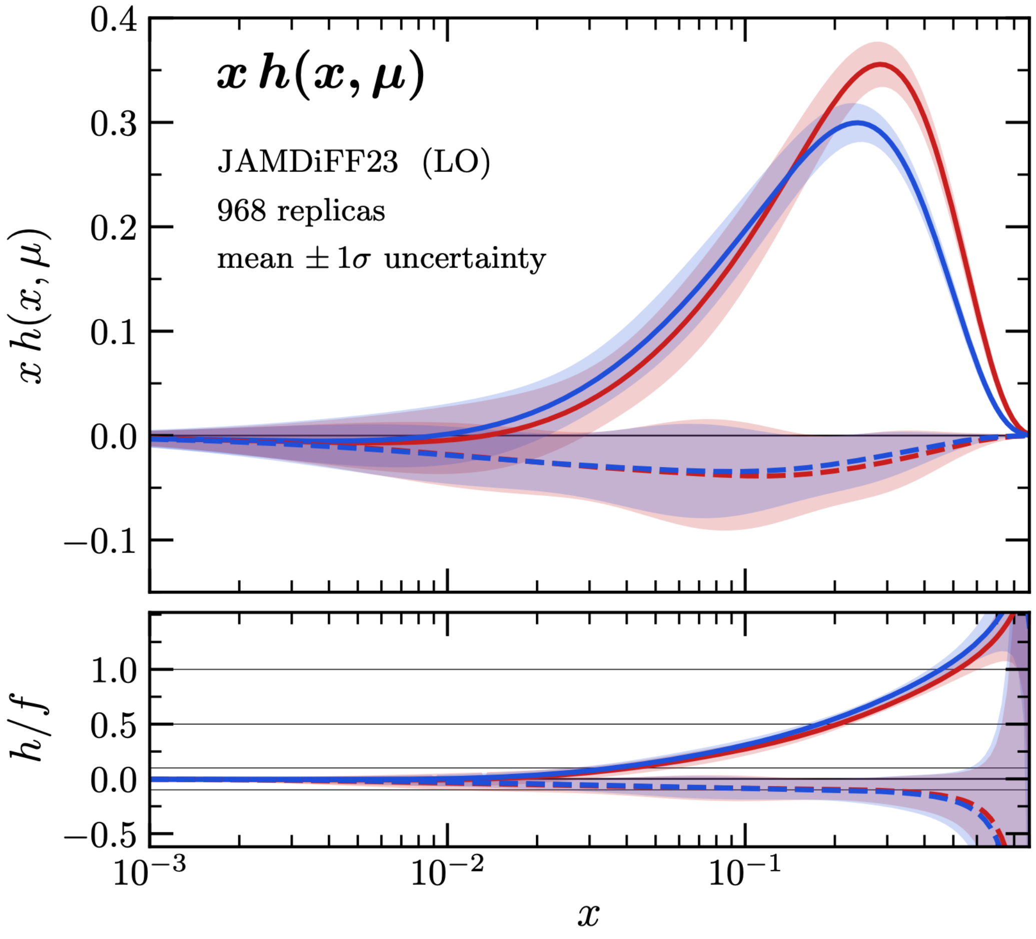
PDF uncertainty dominates!

JAMDiff23-transversity LO, 969 replica PDFs

JAM PDF



$x\,f(x,\mu)$
u
d
$\mu = 5$ GeV
$\mu = 50$ GeV
JAM22 (NLO)
95 replicas
mean $\pm\,1\sigma$ uncertainty
$x\,f(x,\mu)$
x



$x\,h(x,\mu)$
JAMDiFF23 (LO)
968 replicas
mean $\pm\,1\sigma$ uncertainty
$x\,h(x,\mu)$
h/f
x

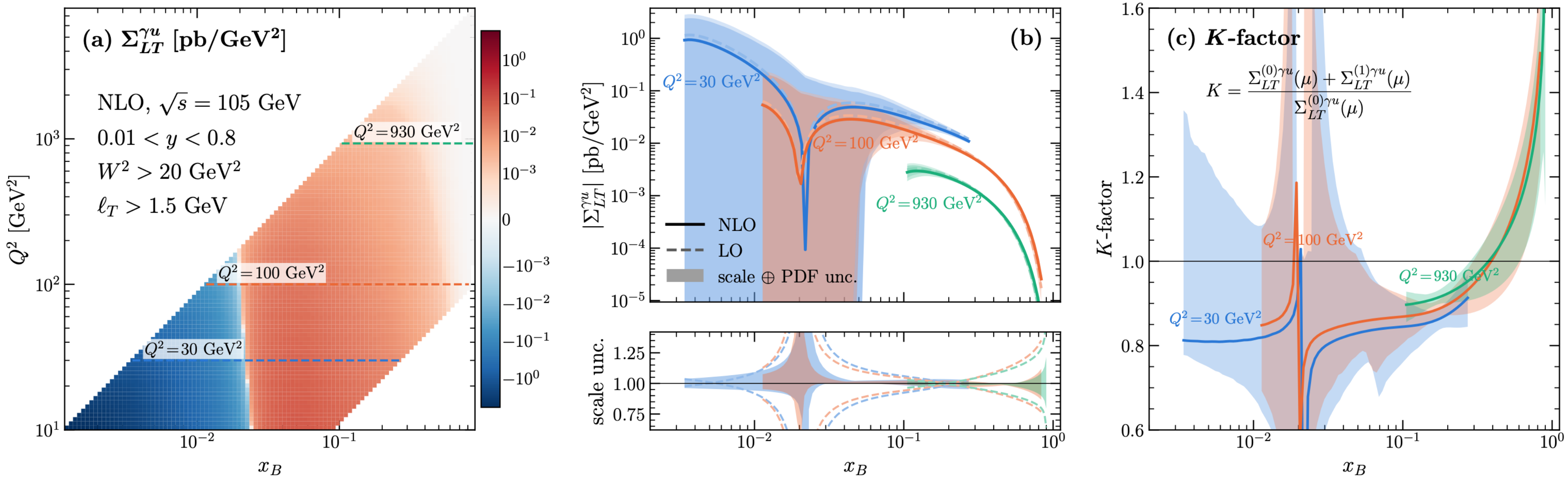
Cross section: longitudinal-transverse spin asymmetry

$$\frac{d\sigma}{dx_B\, dQ^2\, d\Delta\phi} = \frac{1}{2\pi}\left[\Sigma_{UU}(x_B, Q^2) + \lambda_e\, \Sigma_{LU}(x_B, Q^2) + S_T\, \Sigma_{UT}(x_B, Q^2, \Delta\phi) + \lambda_e S_T\, \Sigma_{LT}(x_B, Q^2, \Delta\phi)\right]$$

$$\Sigma_{LT}(x_B, Q^2, \Delta\phi) = \sum_{q=u,d}\left[\mathrm{Re}\left(\Gamma_Z^q e^{i\Delta\phi}\right)\, \Sigma_{LT}^{Zq}(x_B, Q^2) + \mathrm{Re}\left(\Gamma_\gamma^q e^{i\Delta\phi}\right)\, \Sigma_{LT}^{\gamma q}(x_B, Q^2)\right]$$

$(\gamma Z, ZZ)$ channels

$(\gamma\gamma, \gamma Z)$ channels



$\Sigma_{LT}^{\gamma u}$ dominates

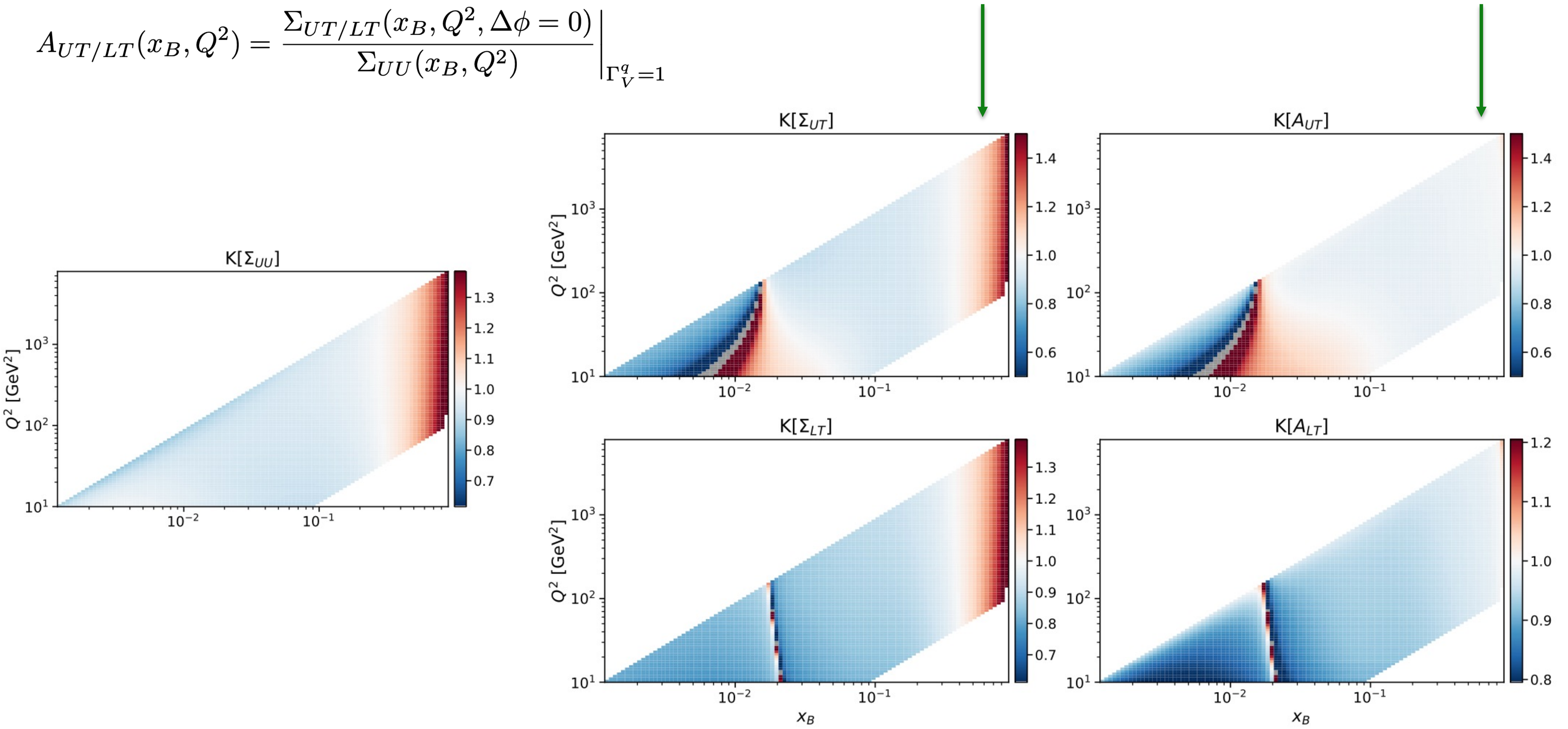
Double polarized cross section = $h_{q/p} \otimes$ polarized hard coefficients of $(\gamma\gamma, \gamma Z, ZZ)$

PDF uncertainty dominates!

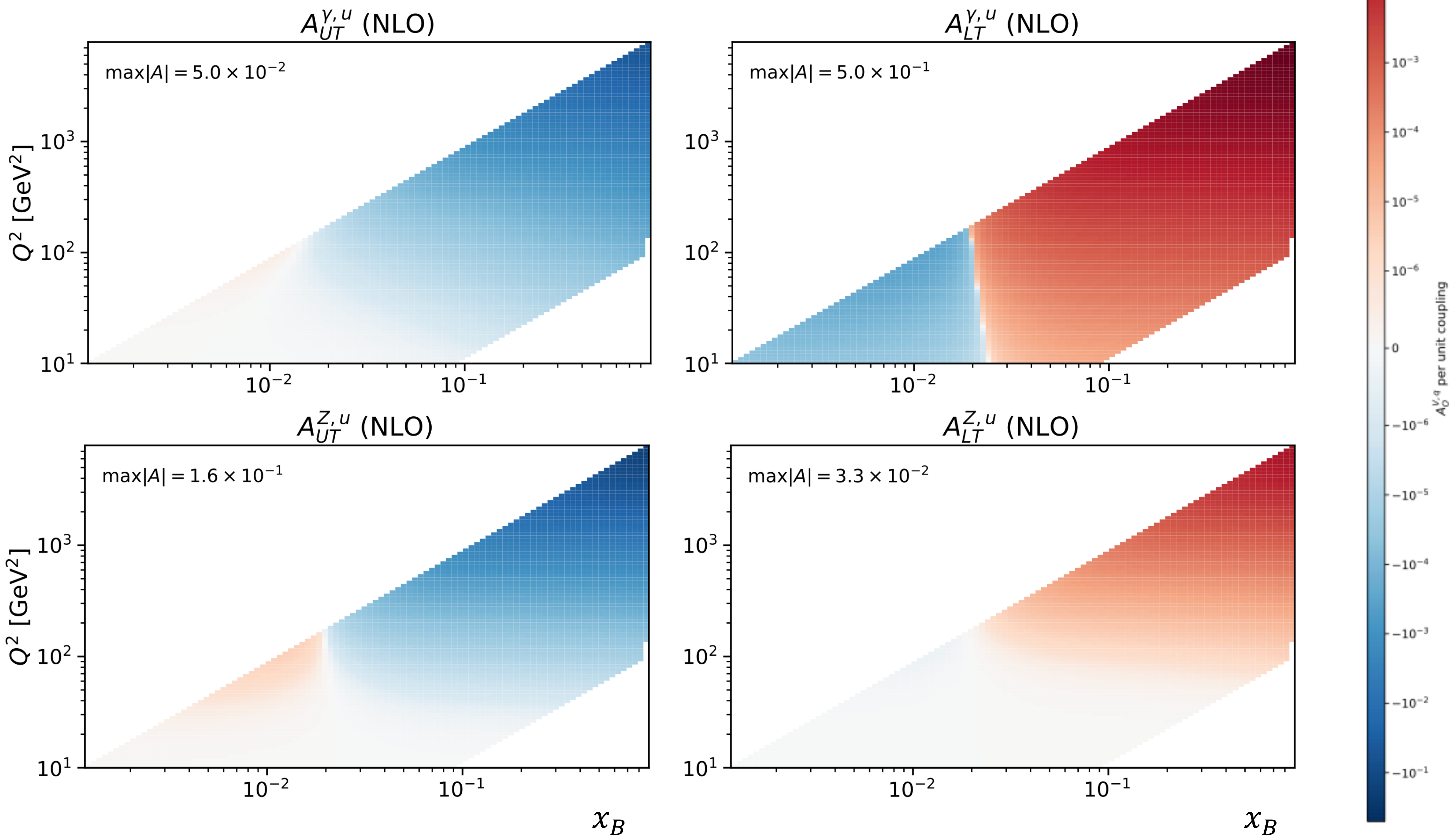
JAMDiff23-transversity LO, 969 replica PDFs

K-factors cancel in asymmetries

$$A_{UT/LT}(x_B, Q^2) = \frac{\Sigma_{UT/LT}(x_B, Q^2, \Delta\phi = 0)}{\Sigma_{UU}(x_B, Q^2)}\Bigg|_{\Gamma_V^q = 1}$$



Asymmetry distribution



Statistical methods for bounding dipole couplings

❑ For a fixed PDF replica

Take to be real

Either polarized cross section takes the form $$\frac{d\sigma}{dx_B\, dQ^2\, d\phi} = U(x_B, Q^2) + P\,\Gamma_a\, V^a(x_B, Q^2)\cos\phi$$

Polarization degree

- Divide the measured region of (x_B, Q^2, ϕ) into many small bins, i
- Expected event yield $\boldsymbol{\mu_i}$ in each bin: $\mu_i = \mu_i(\Gamma_a) = \mathcal{L}\,\Delta x_B\,\Delta Q^2\,\Delta\phi \cdot \left[U_i + P\,\Gamma_a\,V_i^a \cos\phi_i\right]$
- Actual event number $\boldsymbol{n_i}$ in bin i follows a Poisson distribution: $p\left(n_i|\mu_i\right) = e^{-\mu_i}\frac{\mu_i^{n_i}}{n_i!}$

  For theory estimates, take $n_i = \mu_i(\Gamma_a = 0) = \mathcal{L}\,\Delta x_B\,\Delta Q^2\,\Delta\phi \cdot U_i$

- **Likelihood** $L = L(\Gamma_a) = \prod_i p\left(n_i|\mu_i(\Gamma_a)\right)$
- **Log-likelihood ratio** $q(\Gamma_a) = -2\ln\left[\frac{L(\Gamma_a)}{L(0)}\right]$

Statistical methods for bounding dipole couplings

❑ To include scale uncertainty

$$q(\Gamma_a) \;\rightarrow\; \min_{\mu \in \{Q, 2Q, Q/2\}} q(\Gamma_a, \mu)$$

❑ One-parameter bounds from LLR

Keep only one parameter Γ_a, setting all others to 0. $|q(\Gamma_a)| \leq 1$ 68% CL. bound

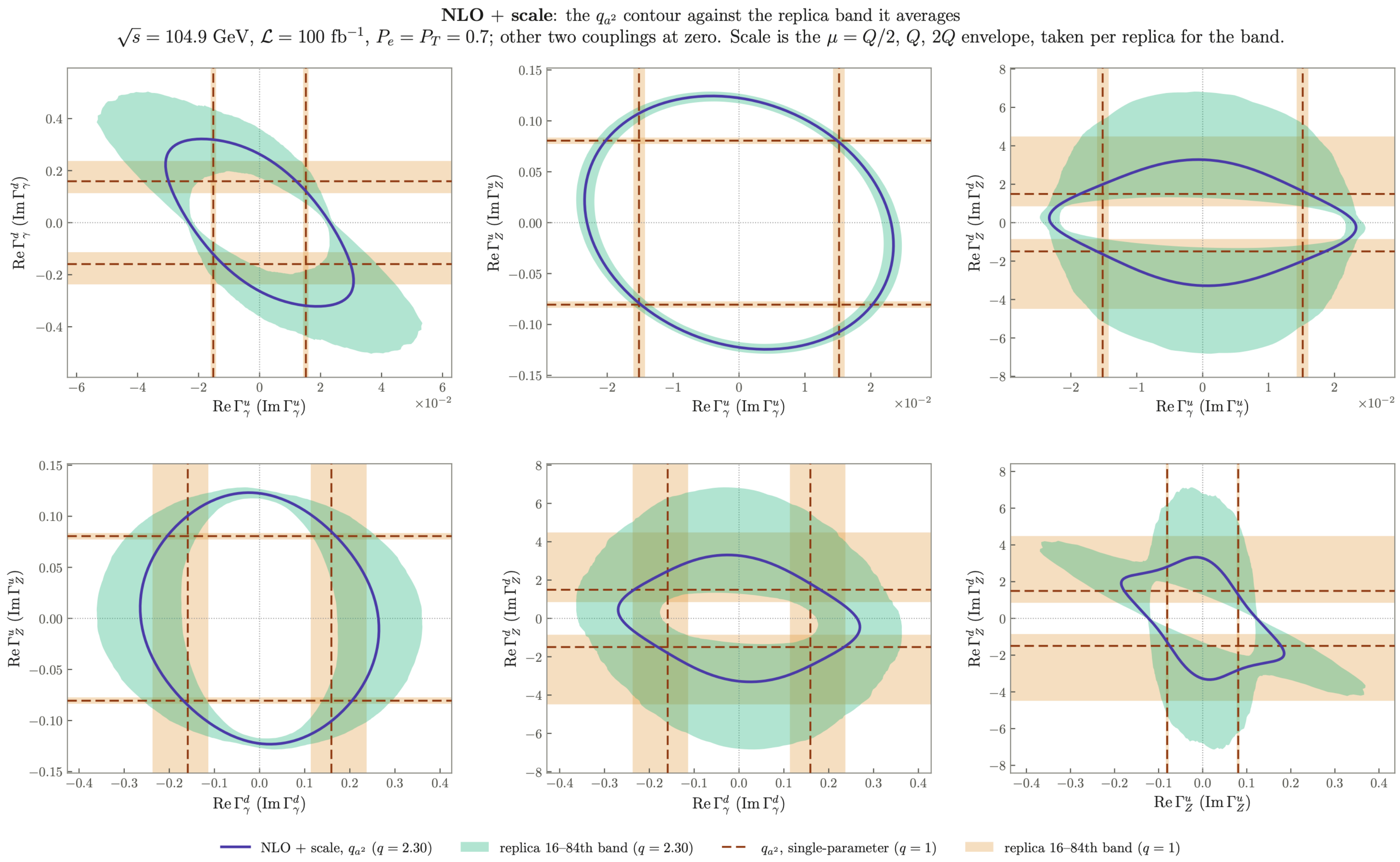
❑ Two-parameter bounds from LLR

Keep only two parameters (Γ_a, Γ_b), setting all others to 0. $|q(\Gamma_a, \Gamma_b)| \leq 2.3$ 68% CL. bound

❑ To include PDF uncertainty…

Loop through all replicas, and find [16%, 84%] quantiles as the “68% C.L. of the bounds”

Bounds on dipole couplings at NLO

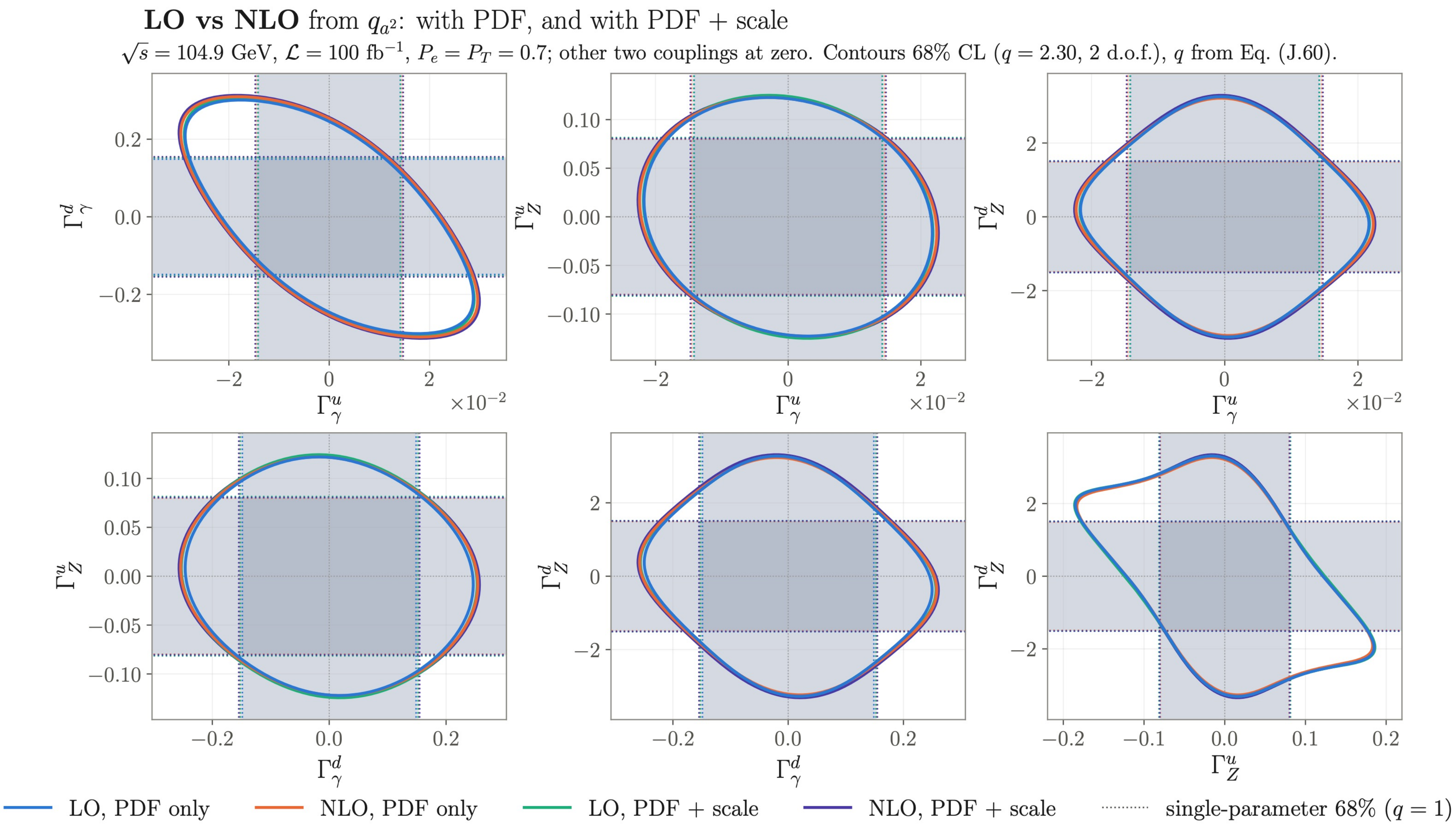


- ✓ Scale uncertainty negligible
- ✓ Photon and Z dipoles almost not correlated
- ✓ u and d dipoles highly correlated
- ✓ d dipoles are very affected by PDF uncertainty
- ✓ Constraints:

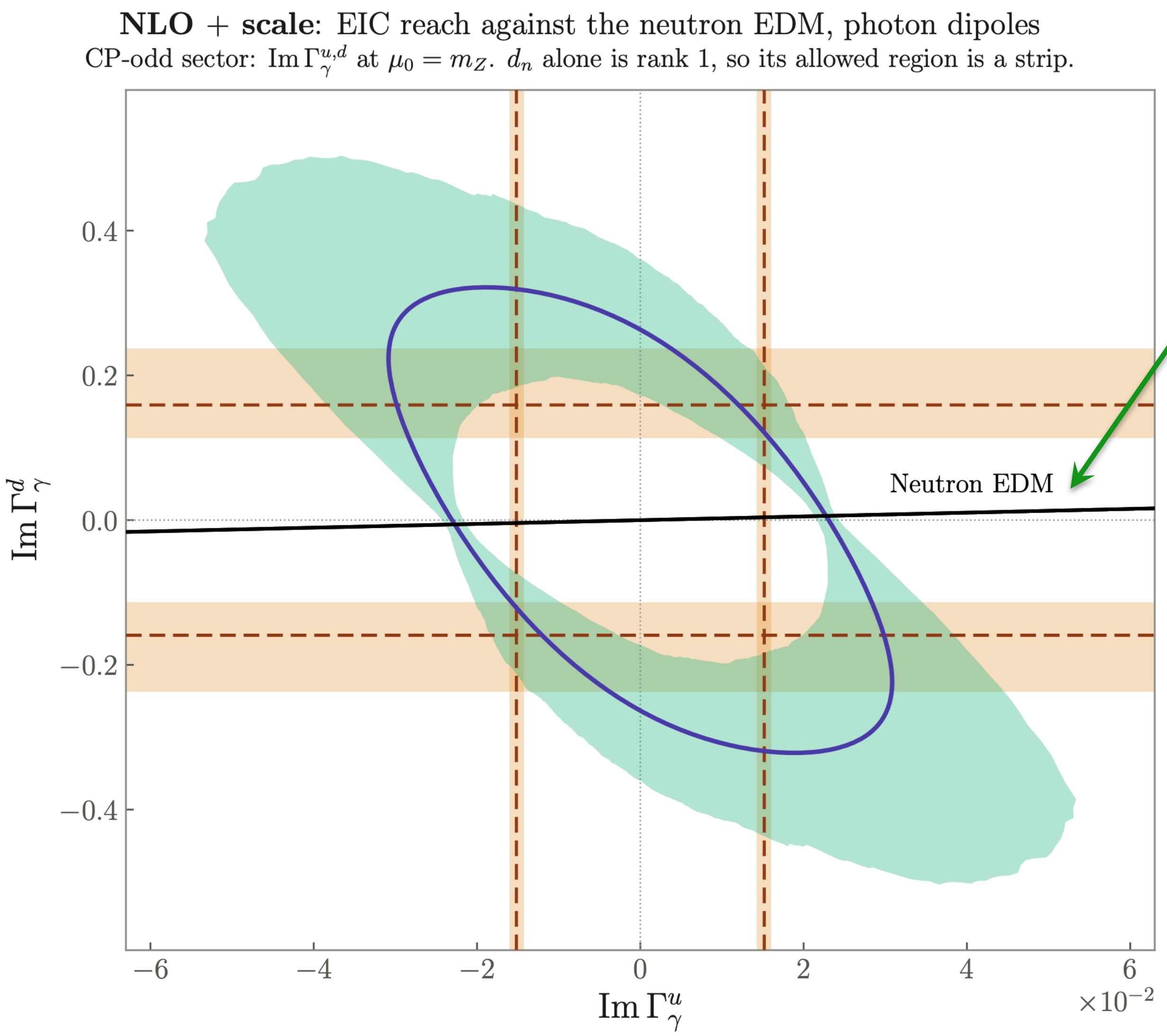
$$|\Gamma^u_\gamma| < |\Gamma^u_Z| < |\Gamma^d_\gamma| < |\Gamma^d_Z|$$

- ✓ Γ^d_Z almost unconstrained.

Bounds on dipole couplings: NLO vs. LO



Comparison with low-energy neutron EDM constraint



Neutron EDM

$$d_n = g_T^d\, d_u + g_T^u\, d_d$$

$$\overset{e\cdot\mathrm{cm}}{=} \frac{\hbar c}{v}\left(g_T^d\ \mathrm{Im}\,\Gamma_\gamma^u + g_T^u\ \mathrm{Im}\,\Gamma_\gamma^d\right)$$

At $\mu = 2\mathrm{GeV}$, tensor charges from Lattice

$$g_T^u = +0.784 \pm 0.030$$

$$g_T^d = -0.204 \pm 0.015$$

At EIC, TSA contains extra QED charge

$$A \sim \underbrace{e_u\, h_u}_{+}\ \mathrm{Re}\big[\Gamma_\gamma^u\, e^{i\Delta\phi}\big] + \underbrace{e_d\, h_d}_{+}\ \mathrm{Re}\big[\Gamma_\gamma^d\, e^{i\Delta\phi}\big]$$

Complementary, and also to real part

Comparison with Drell-Yan measurement at LHC