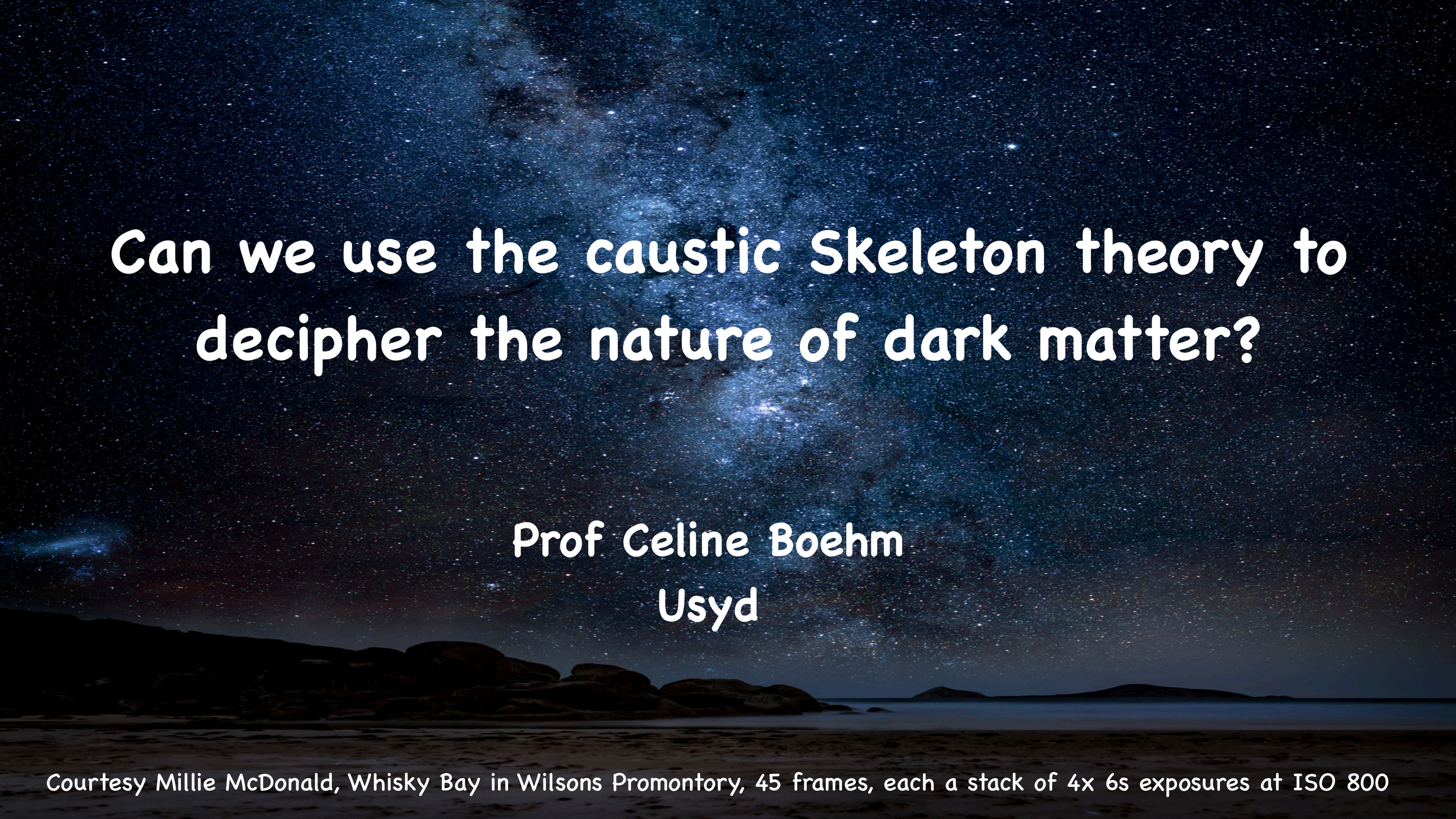
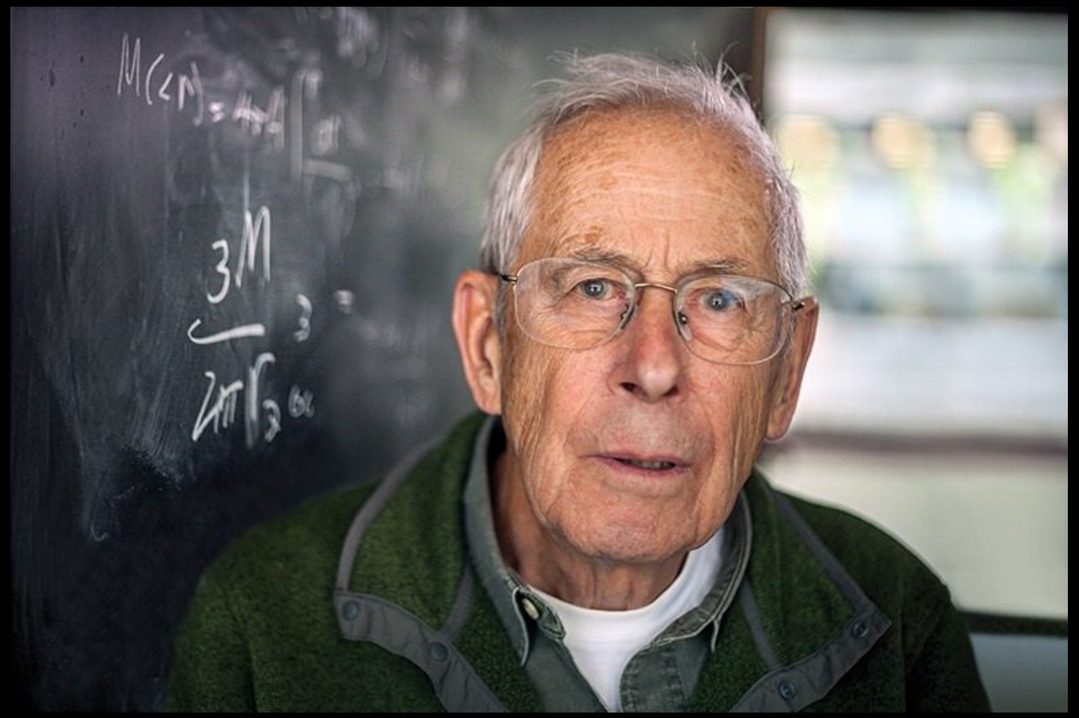


Can we use the caustic Skeleton theory to decipher the nature of dark matter?

Prof Celine Boehm
Usyd

Courtesy Millie McDonald, Whisky Bay in Wilsons Promontory, 45 frames, each a stack of 4x 6s exposures at ISO 800

What is the origin of structure in the Universe?



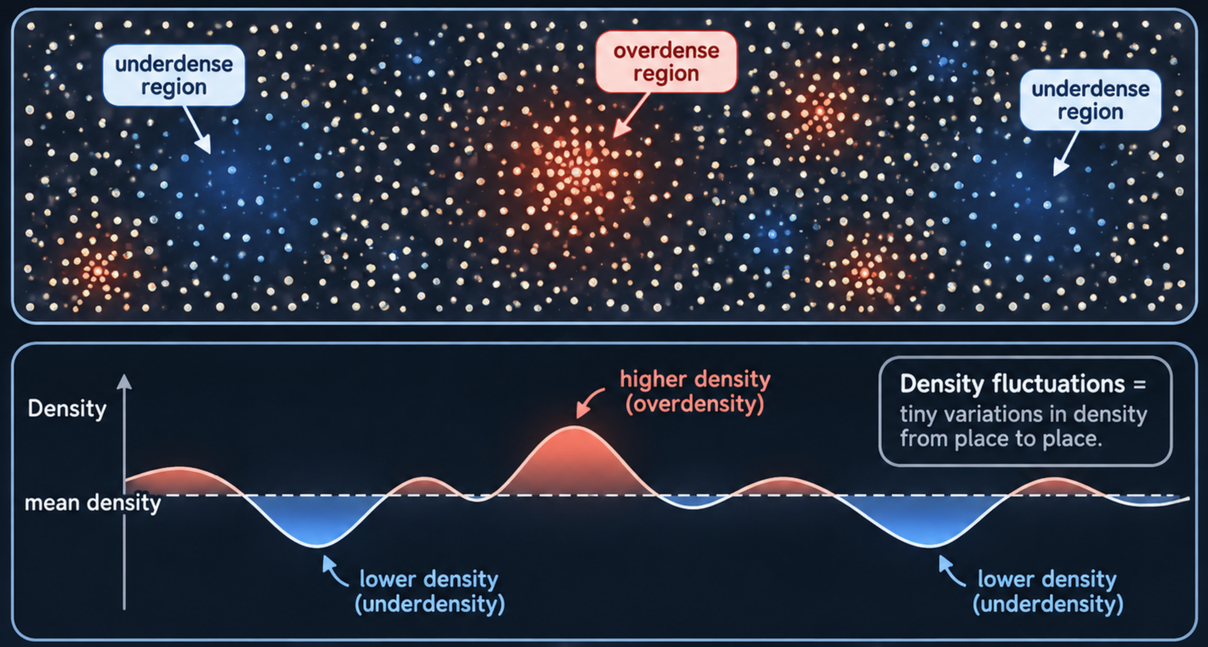
Jim Peebles

There must have existed primordial fluctuations of matter density in the early Universe that grew with gravity to form galaxies eventually.

What is the origin of structure in the Universe?

Density fluctuations

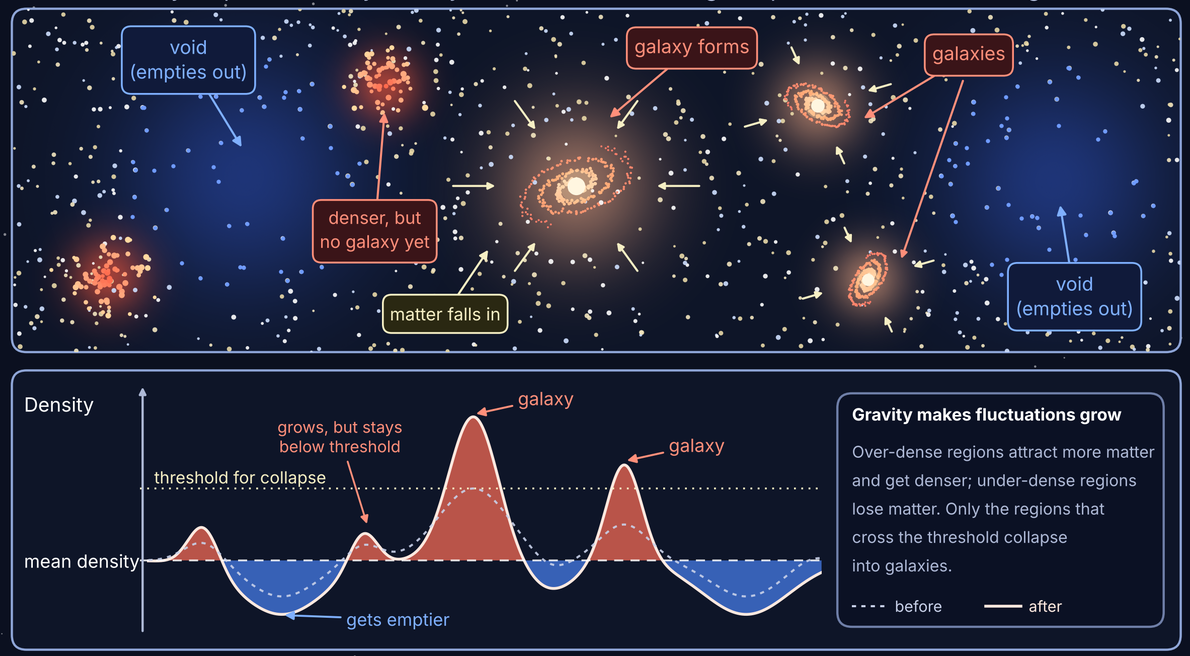
The Universe is not perfectly uniform. It has tiny variations in density from place to place.



Some regions contain slightly more matter than average, while others contain slightly less.

Adding gravity

Gravity amplifies the tiny density fluctuations: dense regions pull in more matter and grow.



Gravity pulls matter into the denser regions: the densest ones collapse into galaxies, while underdense regions become voids.

What is the origin of structure in the Universe?

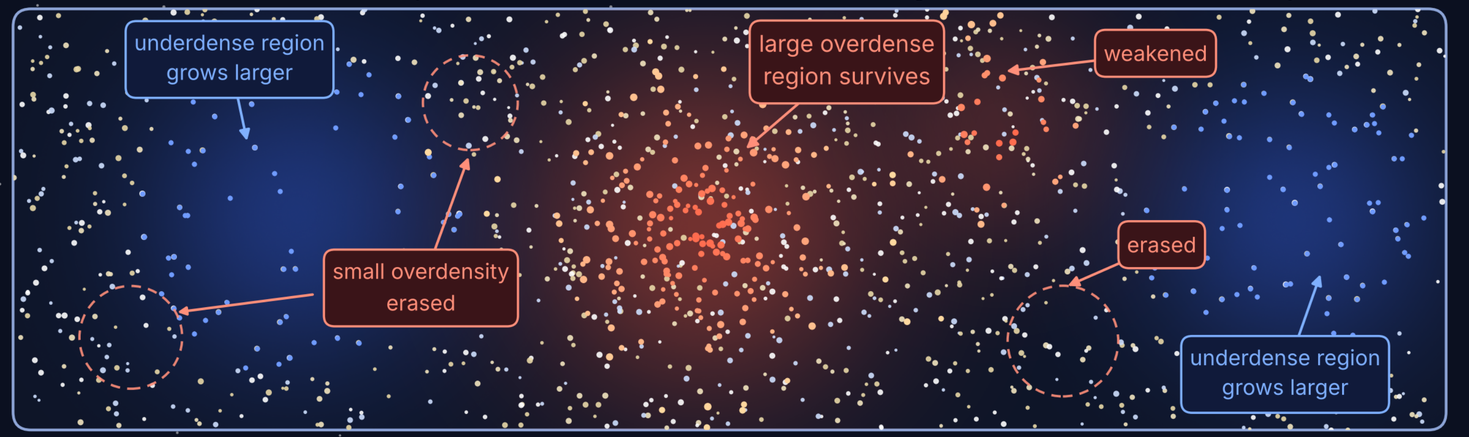


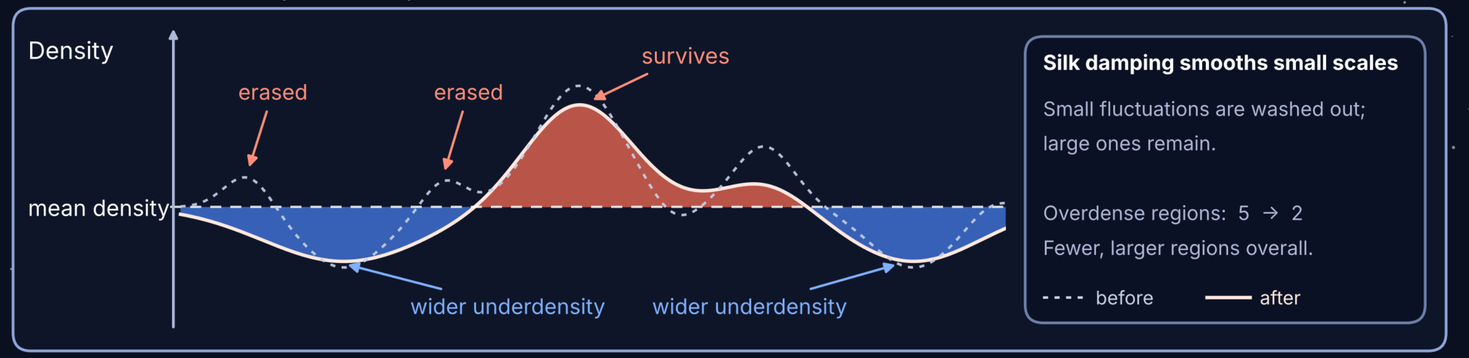
Joe Silk

Perhaps but there is a serious limitation: the Thomson cross section!

Silk damping

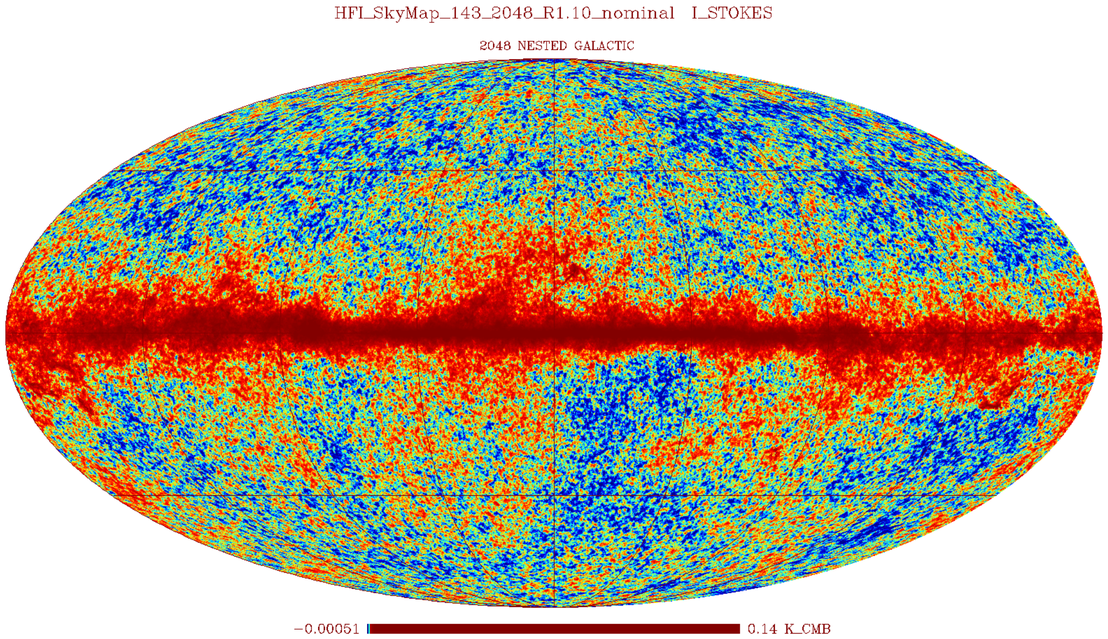
Before atoms form, photons diffuse out of small dense regions and smooth them away.

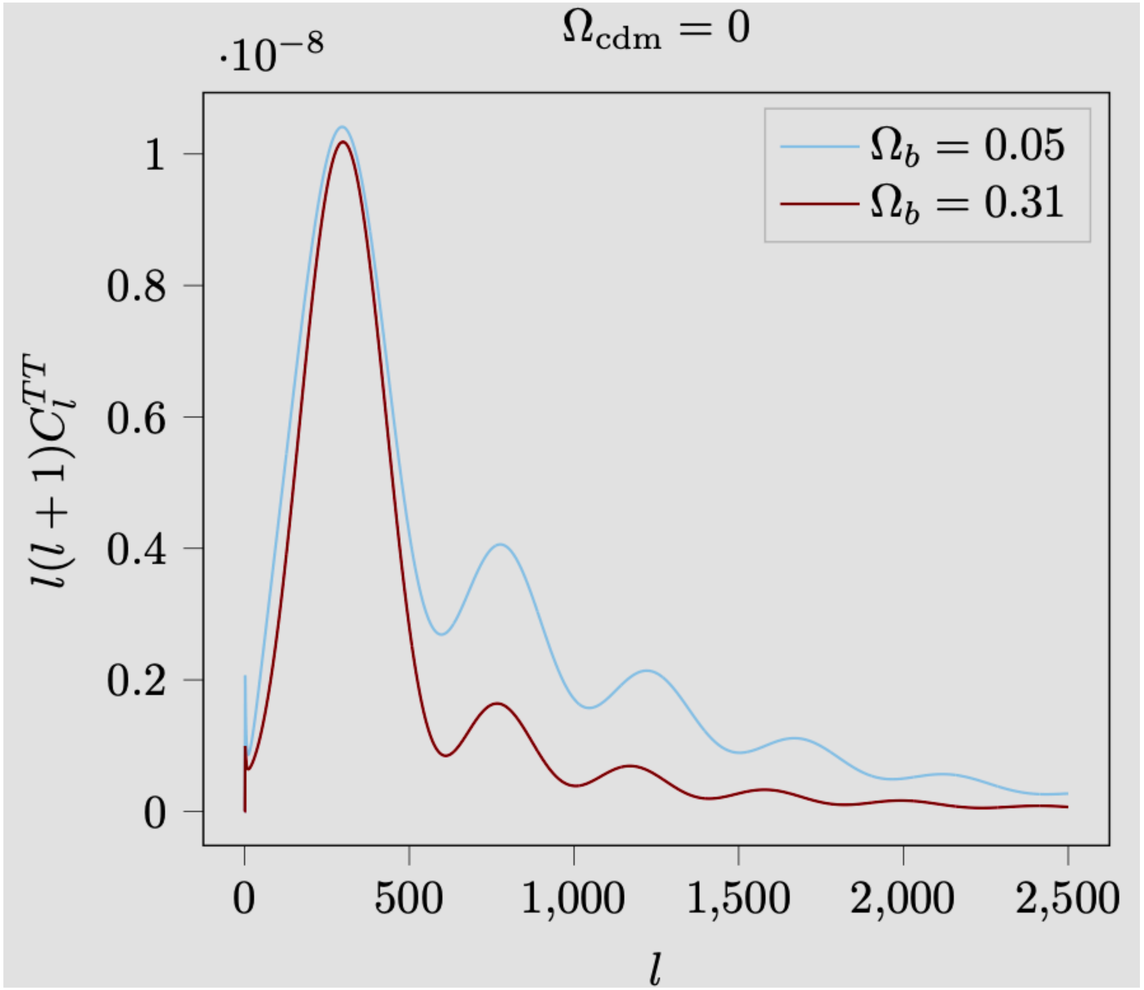




Small overdense regions are erased and underdense regions widen: fewer, larger regions remain after Silk damping.

What is the origin of structure in the Universe?





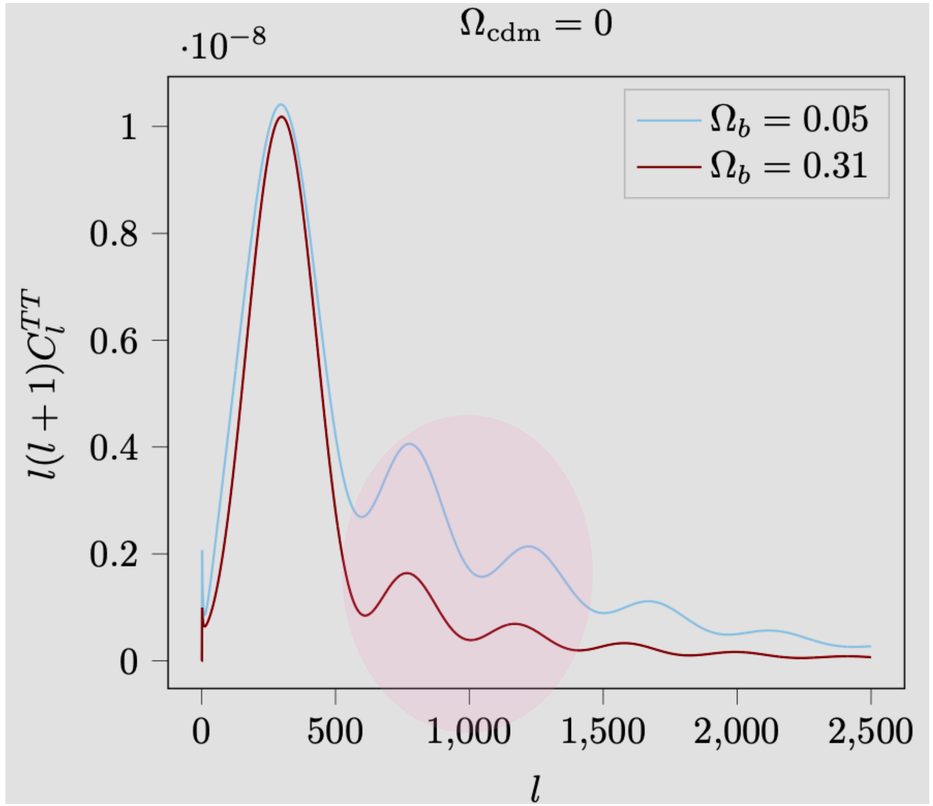
CB & Schaeffer 2000, 2004 using Weinberg 1971 & Chapman, Cowling

$$\dot{\theta}_{\rm b} = k^2\psi - \mathcal{H}\theta_{\rm b} + c_s^2 k^2 \delta_{\rm b} - R^{-1}\dot{\kappa}(\theta_{\rm b} - \theta_\gamma) \qquad \text{Ordinary matter}$$

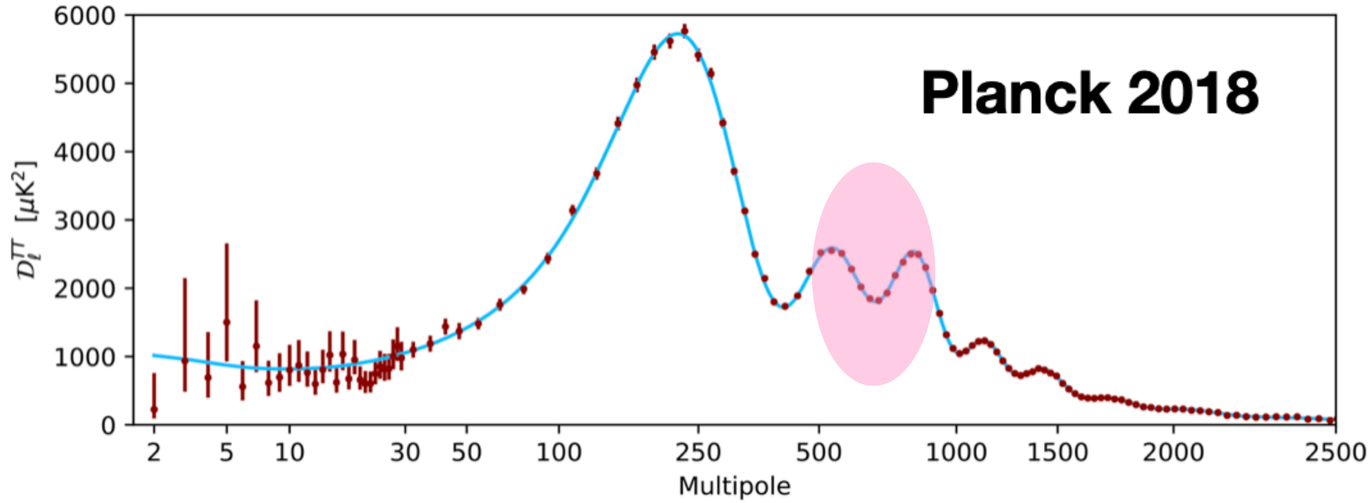
$$\dot{\theta}_\gamma = k^2\psi + k^2\left(\frac{1}{4}\delta_\gamma - \sigma_\gamma\right) - \dot{\kappa}(\theta_\gamma - \theta_{\rm b})\,, \qquad \text{Photons}$$

$$l_{Silk}^2 \simeq \frac{2\pi^2}{3}\int^{t_{dec(b-\gamma)}} \frac{c^2\rho_\gamma}{\rho_{tot}\, a^2\, \Gamma_\gamma}\left(1+\Theta_\gamma\right) dt$$

What is the origin of structure in the Universe?



Expectation



Observation

Ordinary matter can't evade Thomson scattering...

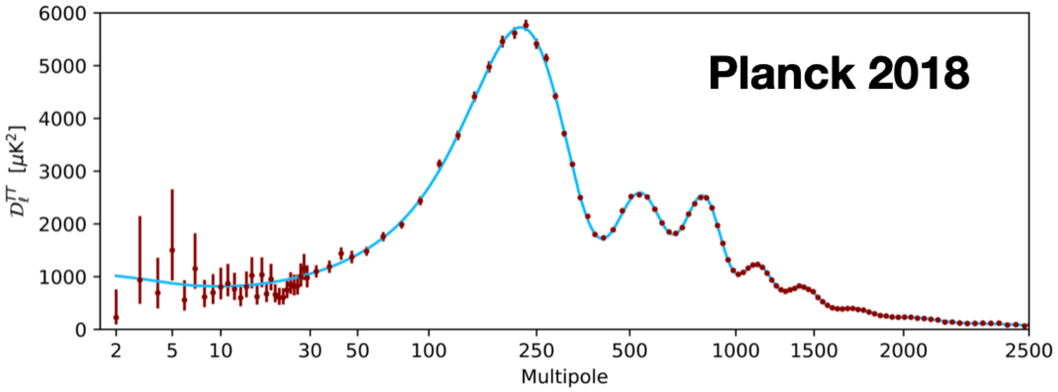
And yet there is a form of matter in the Universe that does!

Looking for something that does not couple to photons...



Chatgpt

CMB & Structure formation



$$\dot{\theta}_b = k^2\psi - \mathcal{H}\theta_b + c_s^2 k^2 \delta_b - R^{-1}\dot{\kappa}(\theta_b - \theta_\gamma)$$
$$\dot{\theta}_\gamma = k^2\psi + k^2\left(\frac{1}{4}\delta_\gamma - \sigma_\gamma\right) - \dot{\kappa}(\theta_\gamma - \theta_b)\ ,$$
$$\dot{\theta}_{DM} = k^2\psi - \mathcal{H}\theta_{DM}\ ,$$

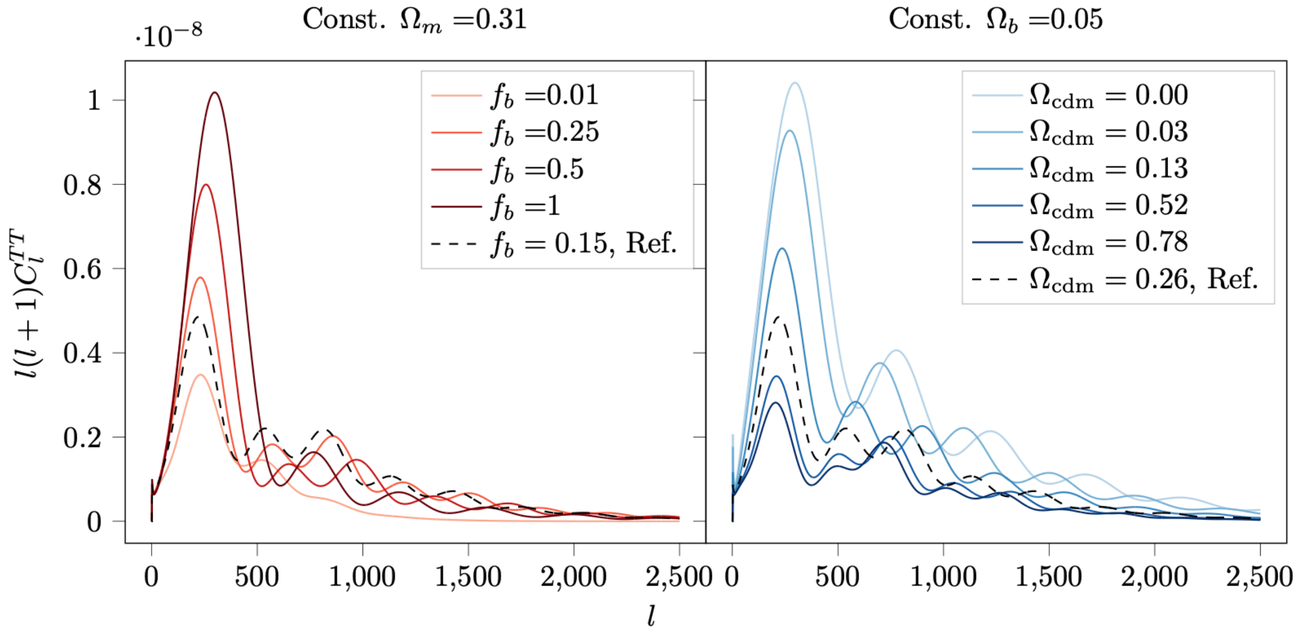


Figure 1.6: *The impact on the CMB TT spectrum of changing the baryon fraction while keeping the total matter density constant (left) as well as changing the dark matter density while keeping the baryon density constant (right). In the latter, the Hubble rate today is kept constant, while Ω_Λ varies to maintain flatness. Ref. notes the values inferred from observations [5].*

Adding non interacting particles explains the 3rd peak of the CMB

This is the so-called Cold Dark Matter scenario

Is dark matter really collisionless?

What cosmology assumes vs. what particle physics tells us

Traditional CDM Cosmology

(CMB / LSS)

$$\sigma = 0$$

dark matter feels gravity only

?

consistent?

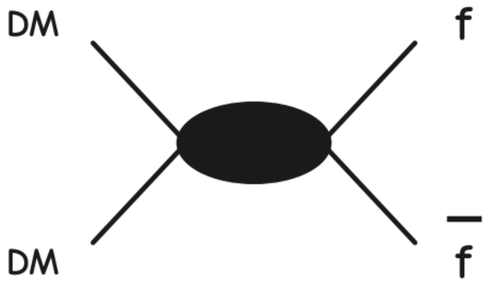
Particle Physics

$$\sigma_{ann} \sim \sigma_{weak}$$

needed to get the observed abundance

Same interaction, seen from three sides:

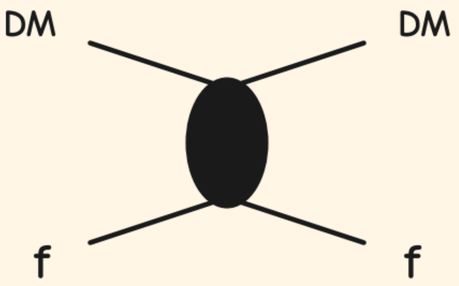
Annihilation



sets the relic abundance

rotate

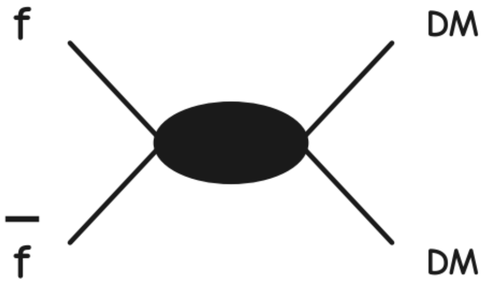
Scattering



$\sigma_{el} \neq 0$: not collisionless!

rotate

Production



searched for at colliders

If dark matter annihilates into ordinary particles, it must also scatter off them:
$\sigma = 0$ is an assumption to be tested, not a given.

Which interactions then?

DM-photon

DM-neutrino

DM-baryon

DM-DM
DM-dark sector

$$
\begin{aligned}
\dot{\theta}_b &= k^2\psi - \mathcal{H}\theta_b + c_s^2 k^2 \delta_b - R^{-1}\dot{\kappa}(\theta_b - \theta_\gamma) \\
\dot{\theta}_\gamma &= k^2\psi + k^2\left(\frac{1}{4}\delta_\gamma - \sigma_\gamma\right) \\
&\quad -\dot{\kappa}(\theta_\gamma - \theta_b) - \dot{\mu}(\theta_\gamma - \theta_{DM}) \,, \\
\dot{\theta}_{DM} &= k^2\psi - \mathcal{H}\theta_{DM} - S^{-1}\dot{\mu}(\theta_{DM} - \theta_\gamma) \,.
\end{aligned}
$$

Which interactions then?

DM-photon | **DM-neutrino** | **DM-baryon** | **DM-DM**
DM-dark sector

$$
\begin{aligned}
\dot{\theta}_{\rm b} &= k^2\psi - \mathcal{H}\theta_{\rm b} + c_s^2 k^2 \delta_{\rm b} - R^{-1}\dot{\kappa}(\theta_{\rm b} - \theta_\gamma) \\
\dot{\theta}_\gamma &= k^2\psi + k^2\left(\frac{1}{4}\delta_\gamma - \sigma_\gamma\right) \\
&\quad -\dot{\kappa}(\theta_\gamma - \theta_{\rm b}) - \dot{\mu}(\theta_\gamma - \theta_{\rm DM}) \,, \\
\dot{\theta}_{\rm DM} &= k^2\psi - \mathcal{H}\theta_{\rm DM} - S^{-1}\dot{\mu}(\theta_{\rm DM} - \theta_\gamma) \,.
\end{aligned}
$$

$$
\begin{aligned}
\dot{\theta}_{\rm b} &= k^2\psi - \mathscr{H}\theta_{\rm b} + c_s^2 k^2 \delta_{\rm b} - R^{-1}\dot{\kappa}(\theta_{\rm b} - \theta_\gamma) \\
\dot{\theta}_\gamma &= k^2\psi + k^2\left(\frac{1}{4}\delta_\gamma - \sigma_\gamma\right) - \dot{\kappa}(\theta_\gamma - \theta_{\rm b}) \,, \\
\dot{\theta}_\nu &= k^2\psi + k^2\left(\frac{1}{4}\delta_\nu - \sigma_\nu\right) - \dot{\kappa}_{\nu{\rm DM}}(\theta_\nu - \theta_{\rm DM}) \,, \\
\dot{\theta}_{\rm DM} &= k^2\psi - \mathscr{H}\theta_{\rm DM} - R\,\dot{\kappa}_{\nu{\rm DM}}(\theta_{\rm DM} - \theta_\nu) \,.
\end{aligned}
$$

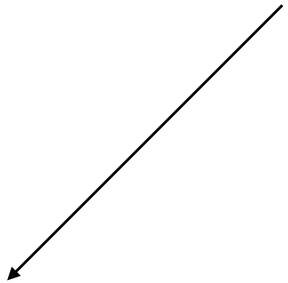
Which interactions then?

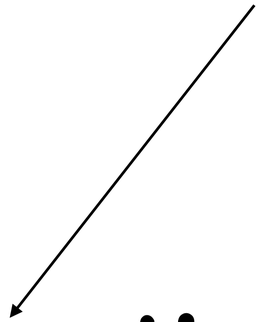
DM-photon

DM-neutrino

DM-baryon

DM-DM
DM-dark sector





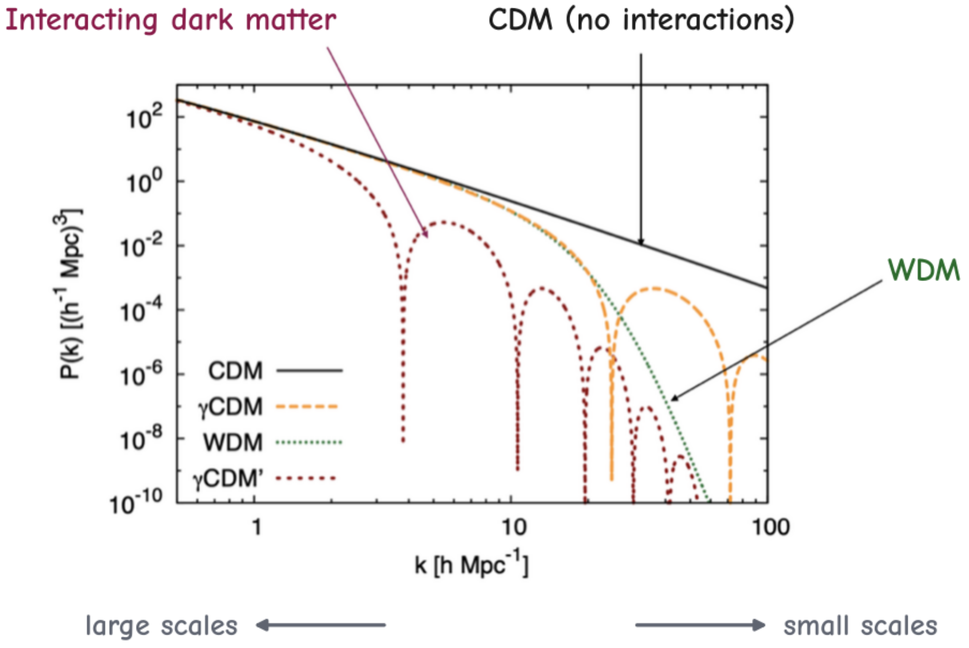
Edges

Self-Interacting DM

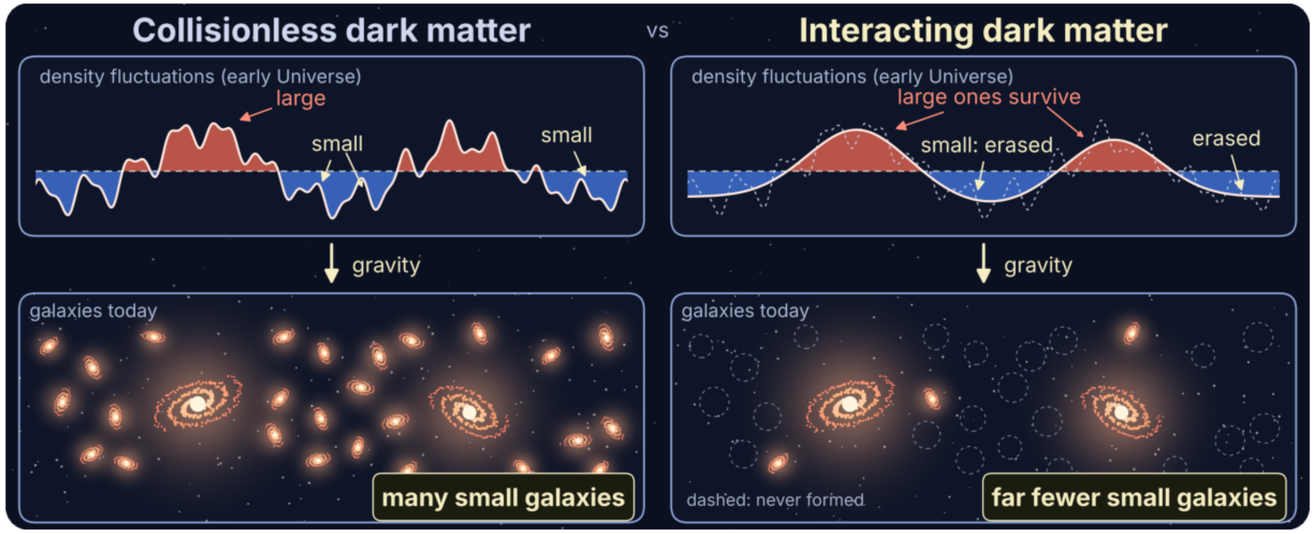
Primordial effect of dark matter interactions

Like Silk damping, interactions erase the small-scale fluctuations

1. Less power on small scales



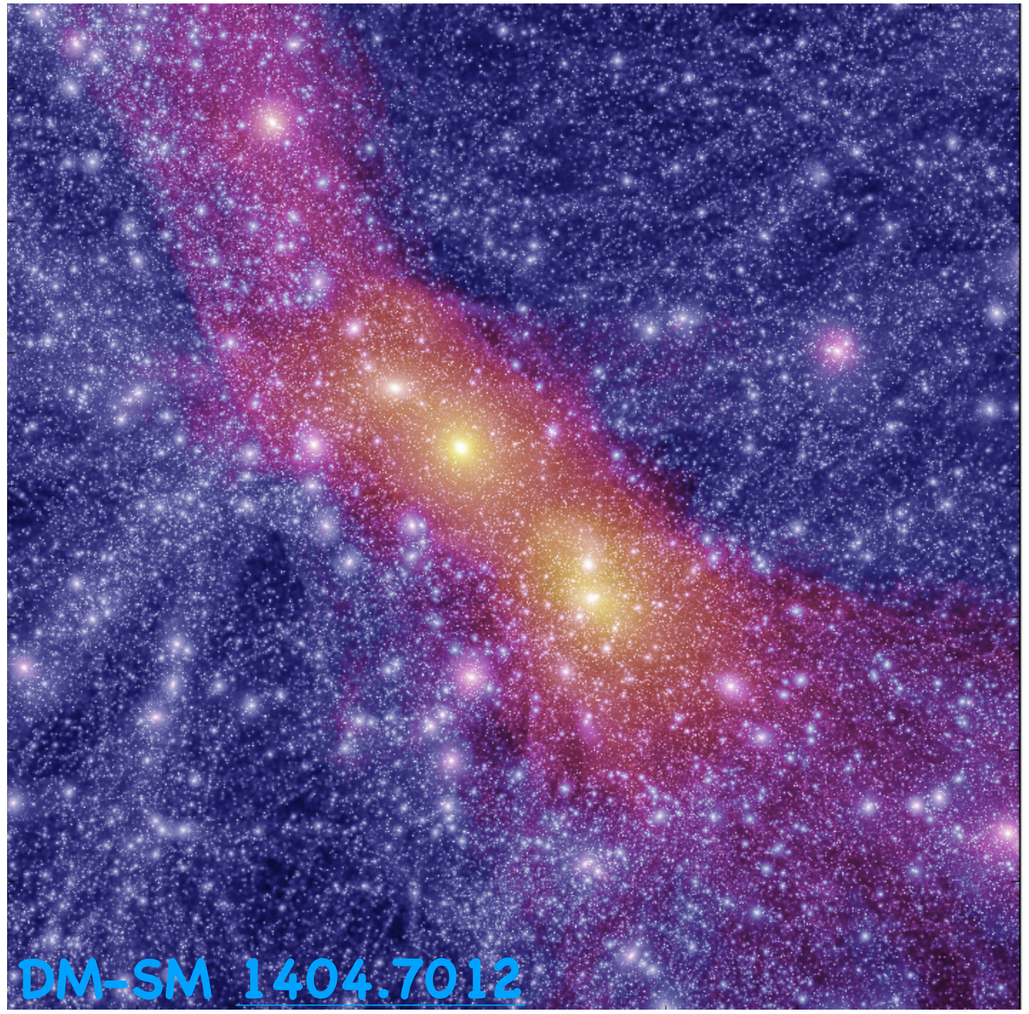
2. Fewer small structures



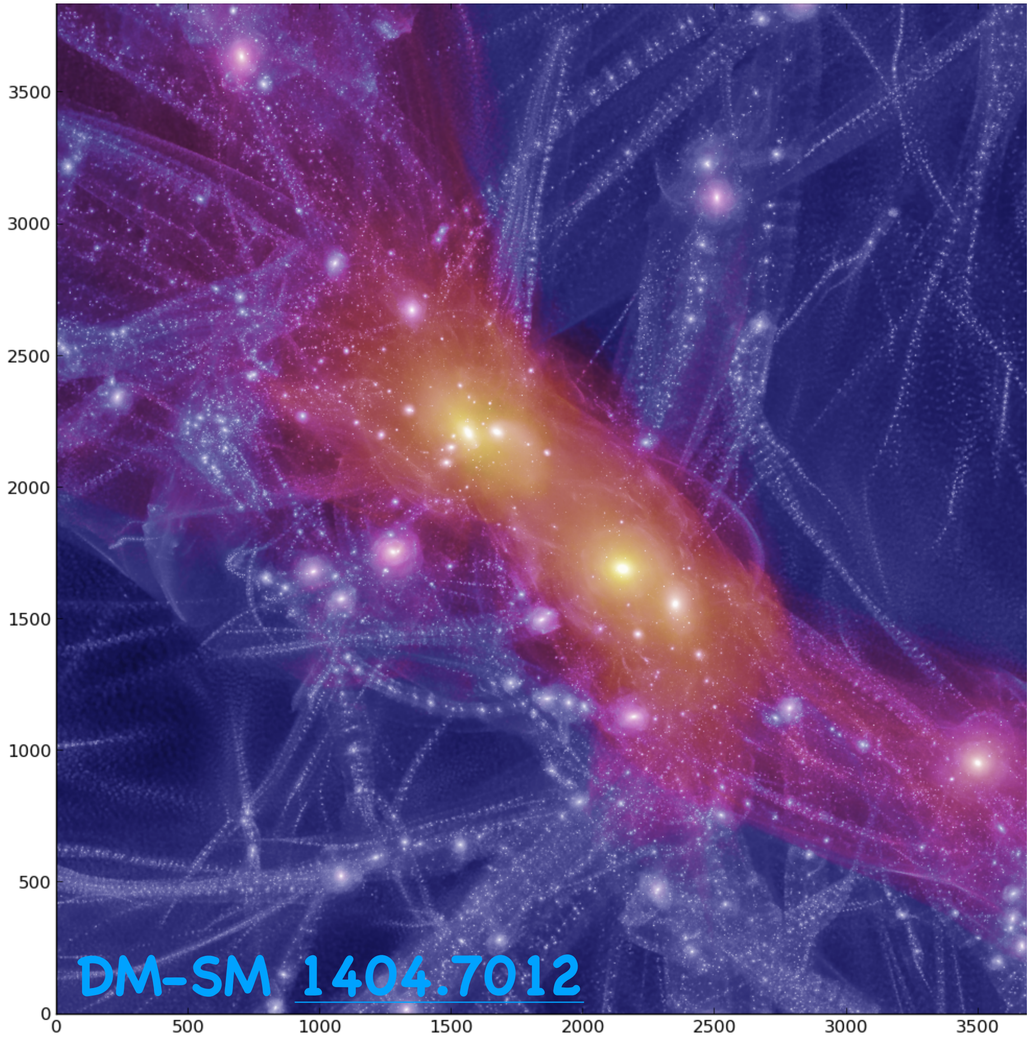
Interactions damp the fluctuations on small scales (large k), as Silk damping does:
the large structures are unchanged, but in principle far fewer small galaxies form.

Dark Matter interactions & large scales

DM with no interactions



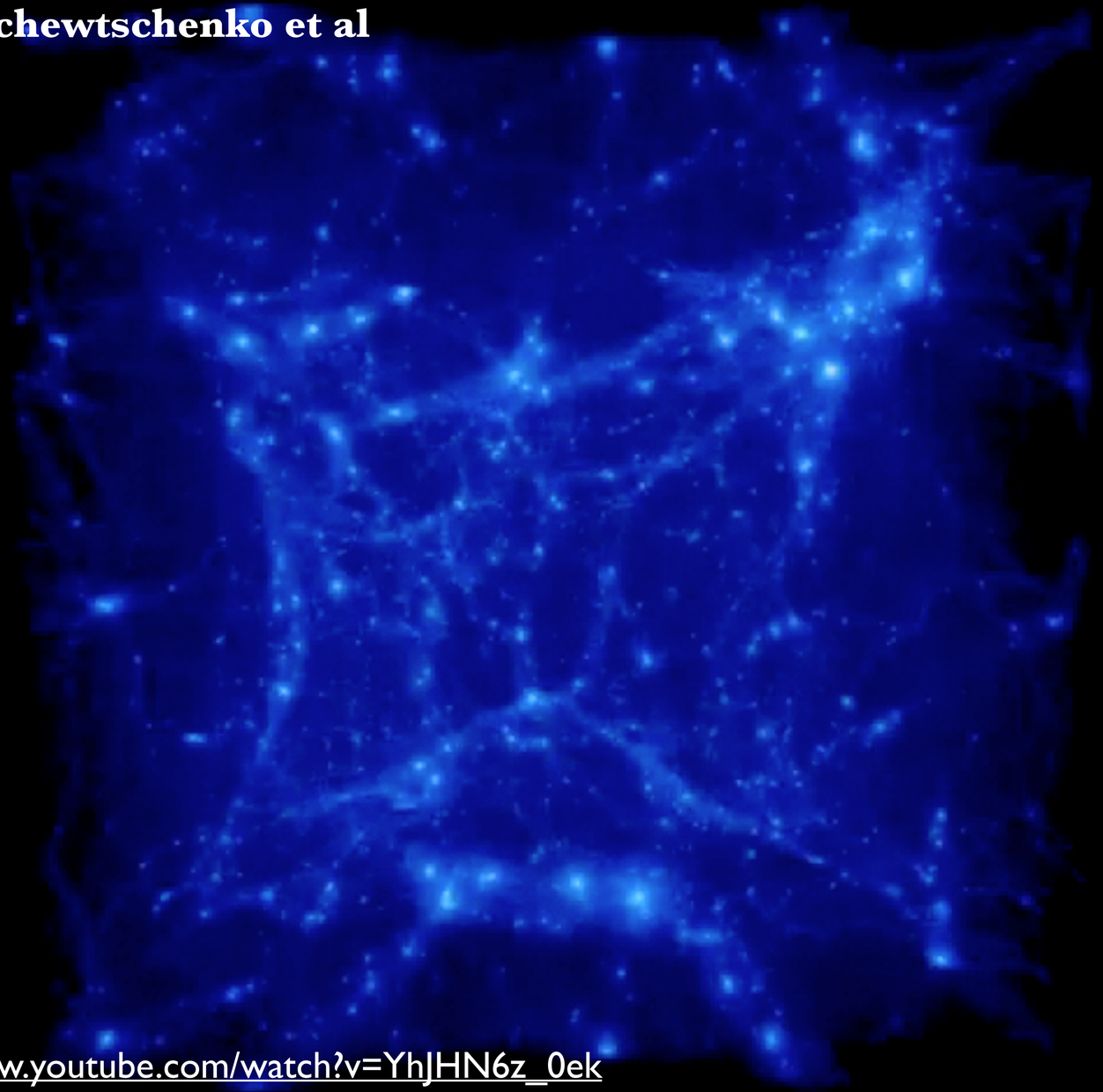
Very strongly Interacting DM



We observe a Universe that is kind of closer to the one on the left

The Milky Way for Interacting DM

C.B., J. Schewtschenko et al



http://www.youtube.com/watch?v=YhJHN6z_0ek

Self-interactions vs other interactions

Early Universe

Late Universe

Time evolution

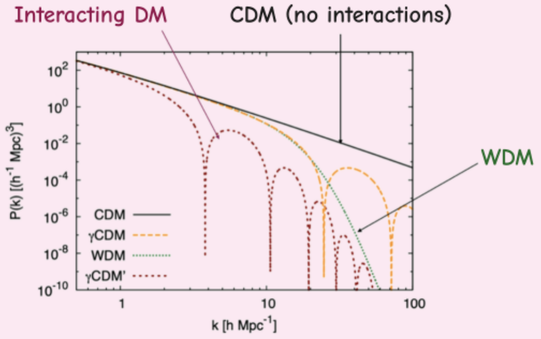
Interactions with ordinary particles



f = photons, neutrinos, ...

main effect

Damps the small-scale fluctuations



No other effects

(except perhaps for the coupling with neutrinos)

Self-interactions



dark matter with itself

Early-time effects

often neglected

main effect

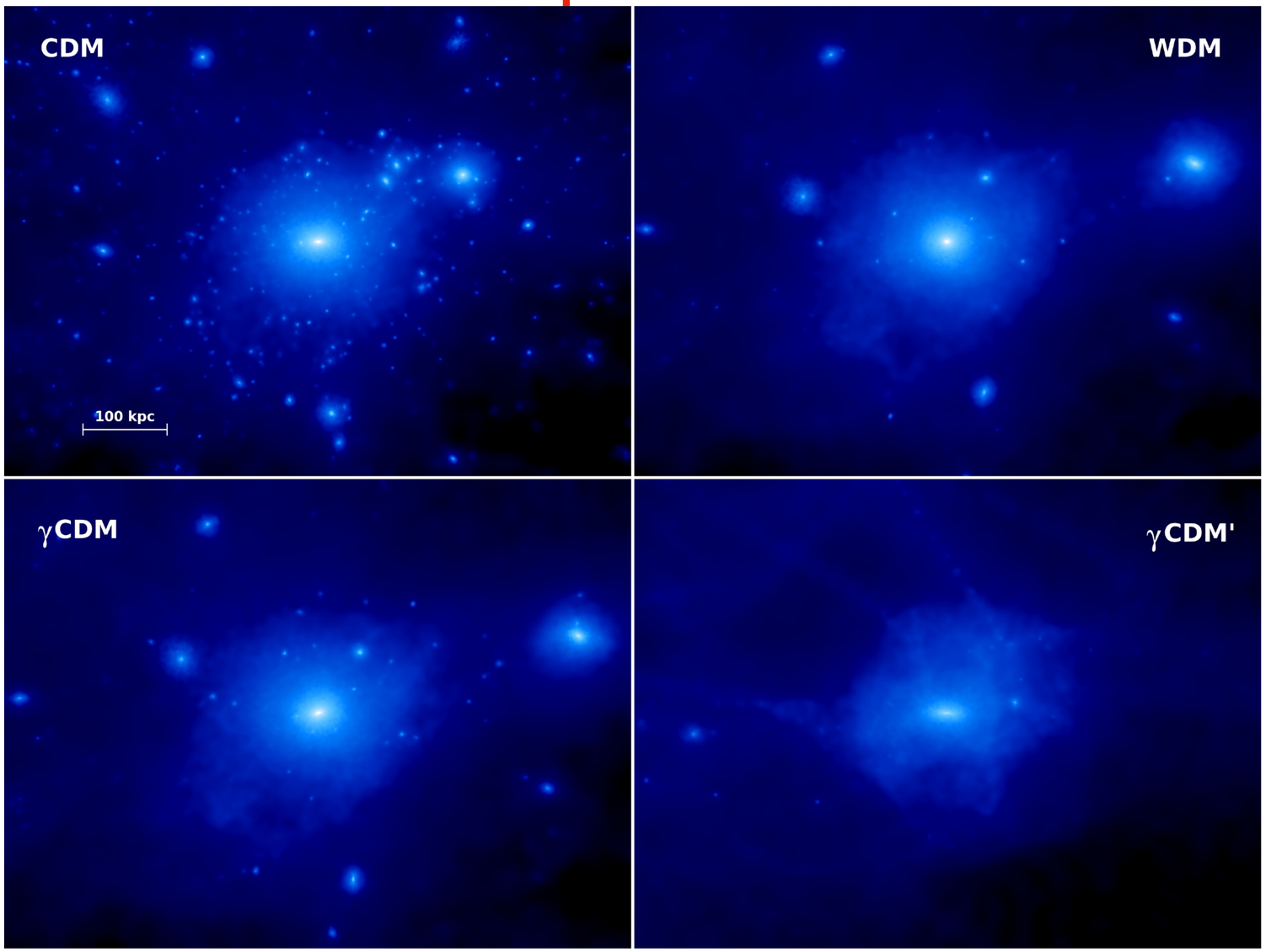
Late-time effects that change the dark matter density inside objects

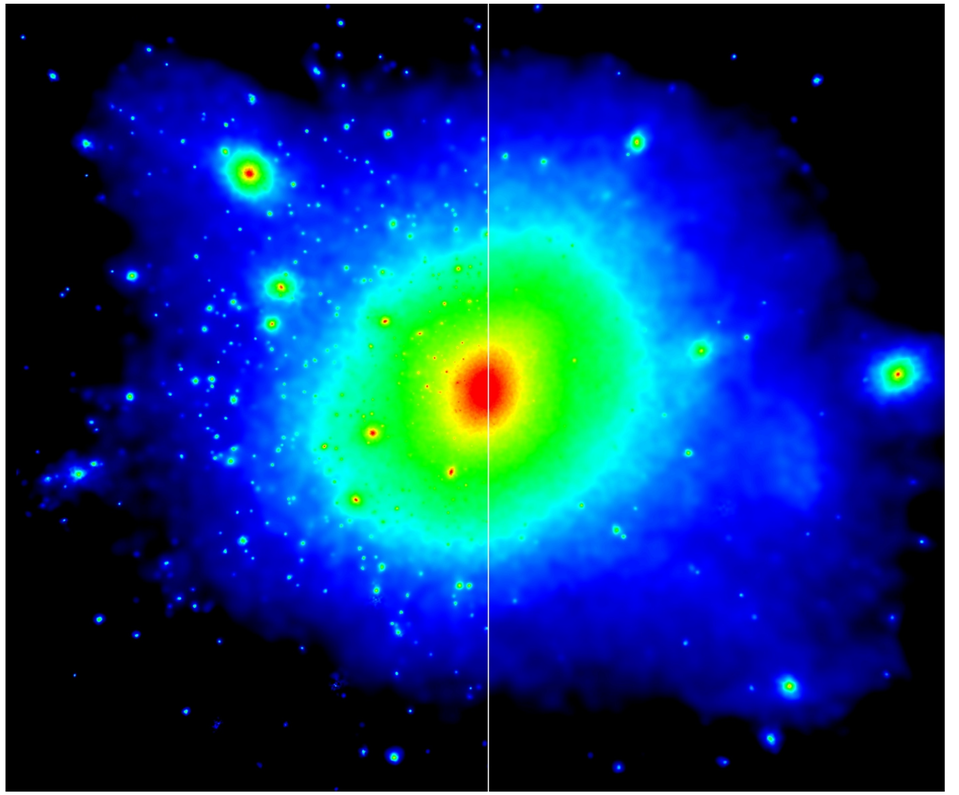


Can we distinguish between models?

The most promising route so far:

Less satellite companions

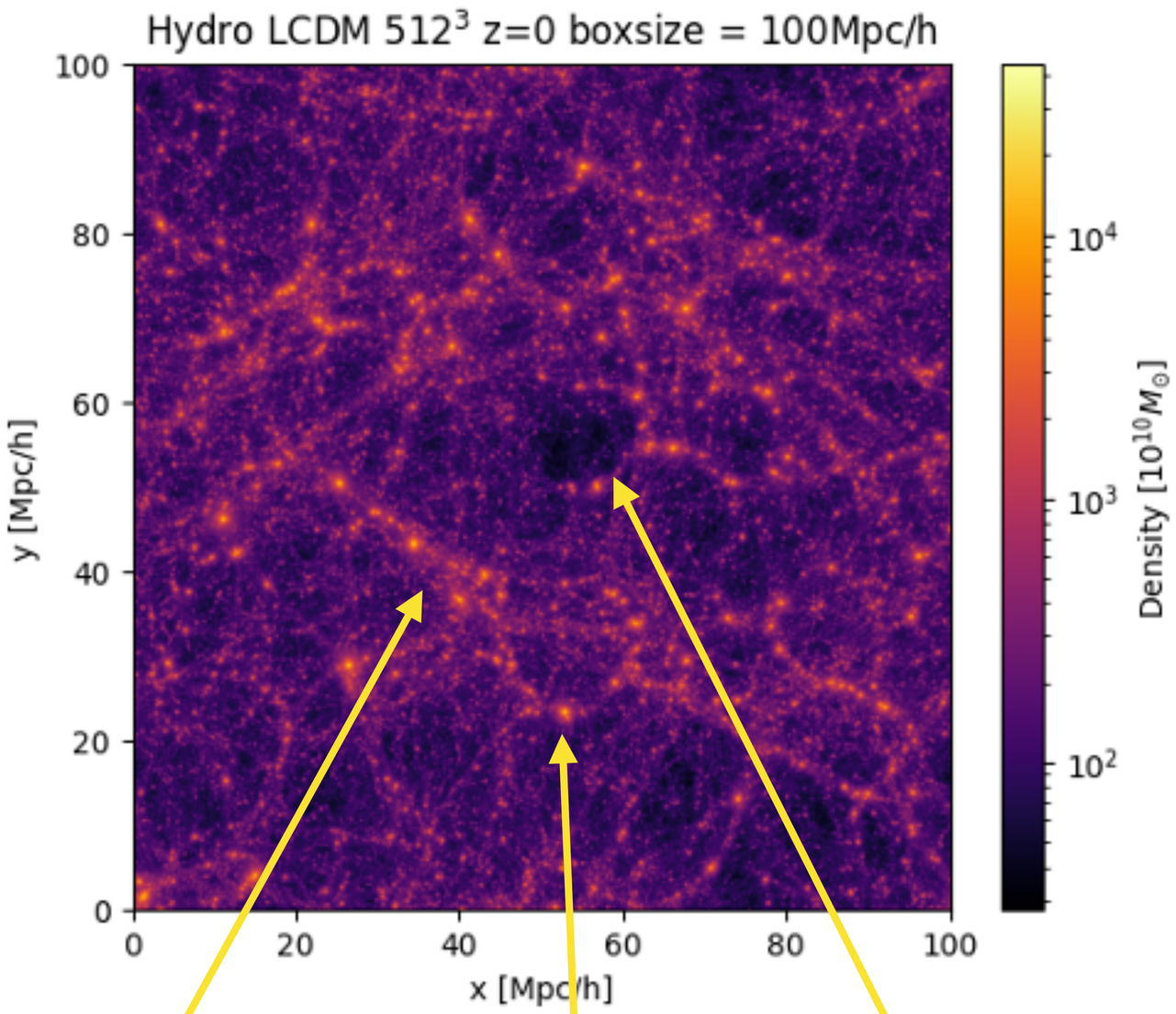


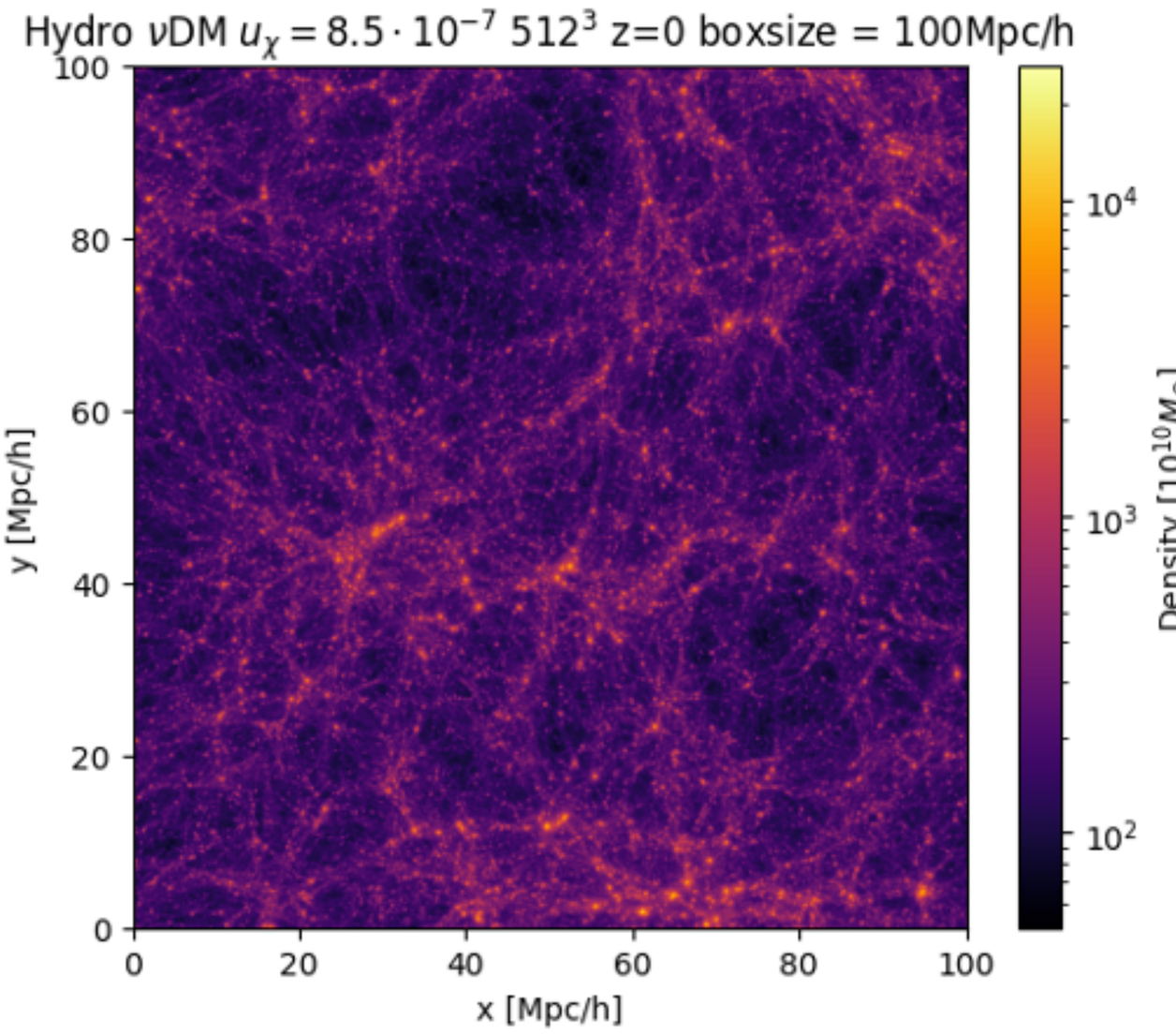


$$\sigma v \lesssim 10^{-36}\ \mathrm{cm}^2 \left(\frac{\mathrm{m_{DM}}}{\mathrm{MeV}}\right)$$

Are there subtle differences at large-scales?

Differences between models



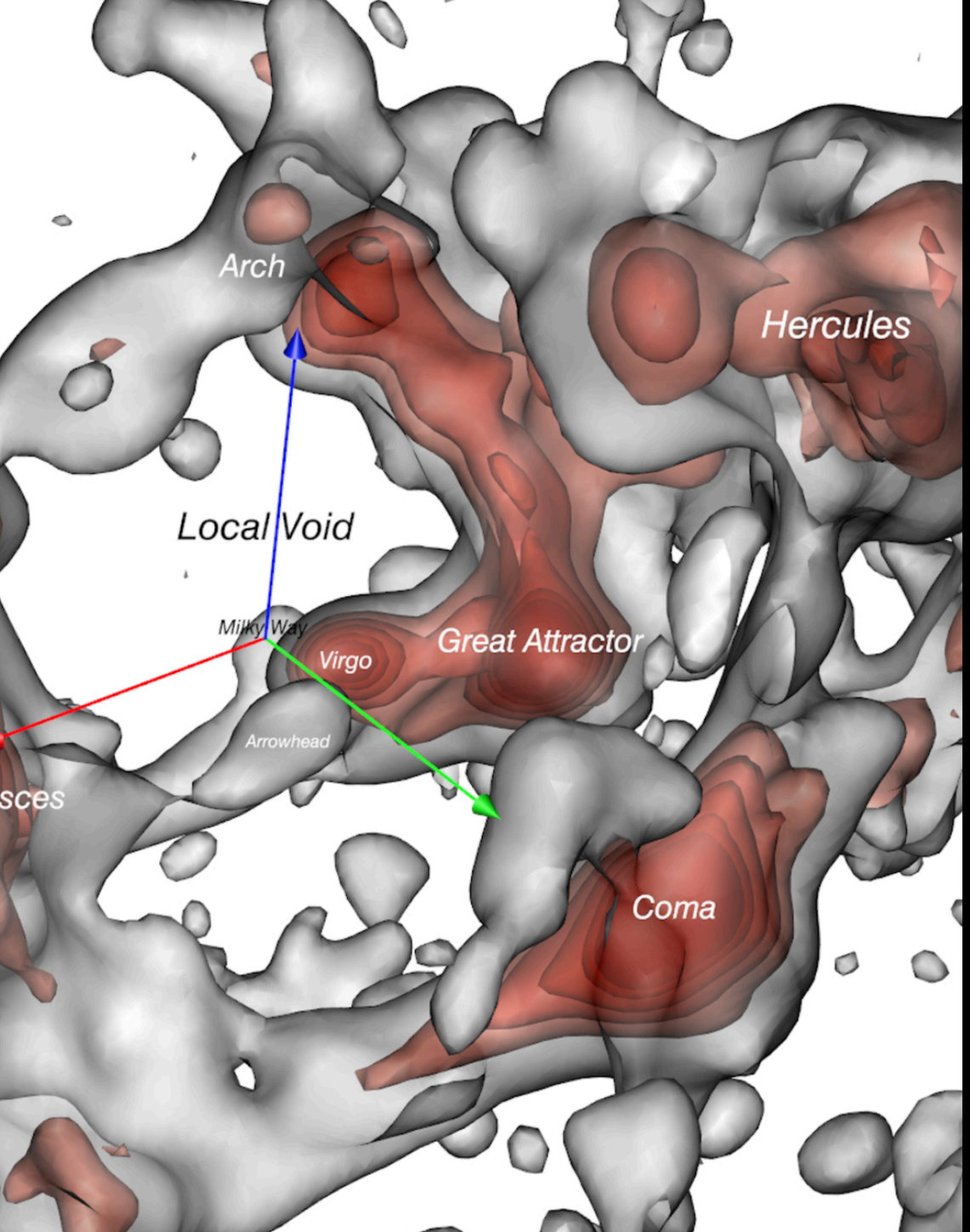


Filament

Cluster

Void

We need a very precise and predicting tool!



Locally,
we should see the following:
1. A wall separating us
from a void

2. Some filaments

Could we predict these from initial conditions?

Zel’dovich pancakes: the first analytic cosmic web

1970 · Zel’dovich

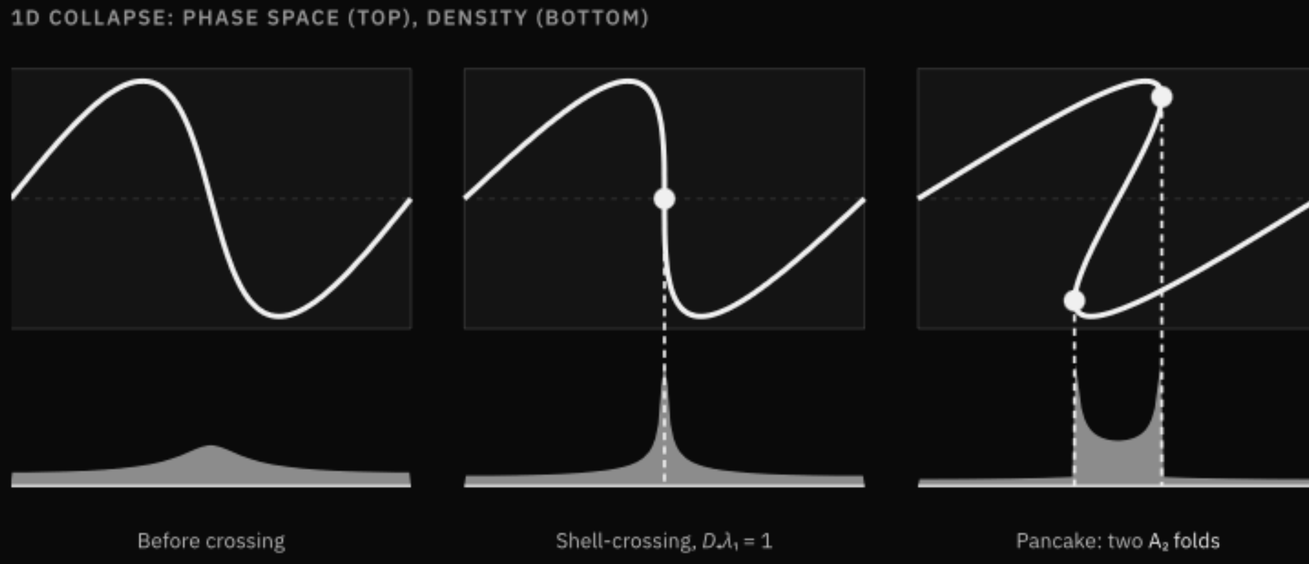
Collapse first along the largest λ_1 makes sheets. With small scales erased by **Silk damping**, vast pancakes form first, then fragment into galaxies: top-down

1982 · Arnol’d, Shandarin & Zel’dovich

Pancakes are caustics: catastrophe theory classifies them, but only in 1D and 2D

1980s · Cold dark matter

No Silk damping, so growth is bottom-up, but the folds survive: smoothing Ψ at σ plays the role of the damping cut-off



1982 · GEOPHYS. ASTROPHYS. FLUID DYN. 20, 111

The Large Scale Structure of the Universe I. General Properties. One- and Two-Dimensional Models

V. I. Arnol’d · S. F. Shandarin · Ya. B. Zel’dovich



Based on catastrophe theory by V. Arnold

Caustics formation

Example of caustics



Folding of dark matter sheet

Slide credit Amelie Read

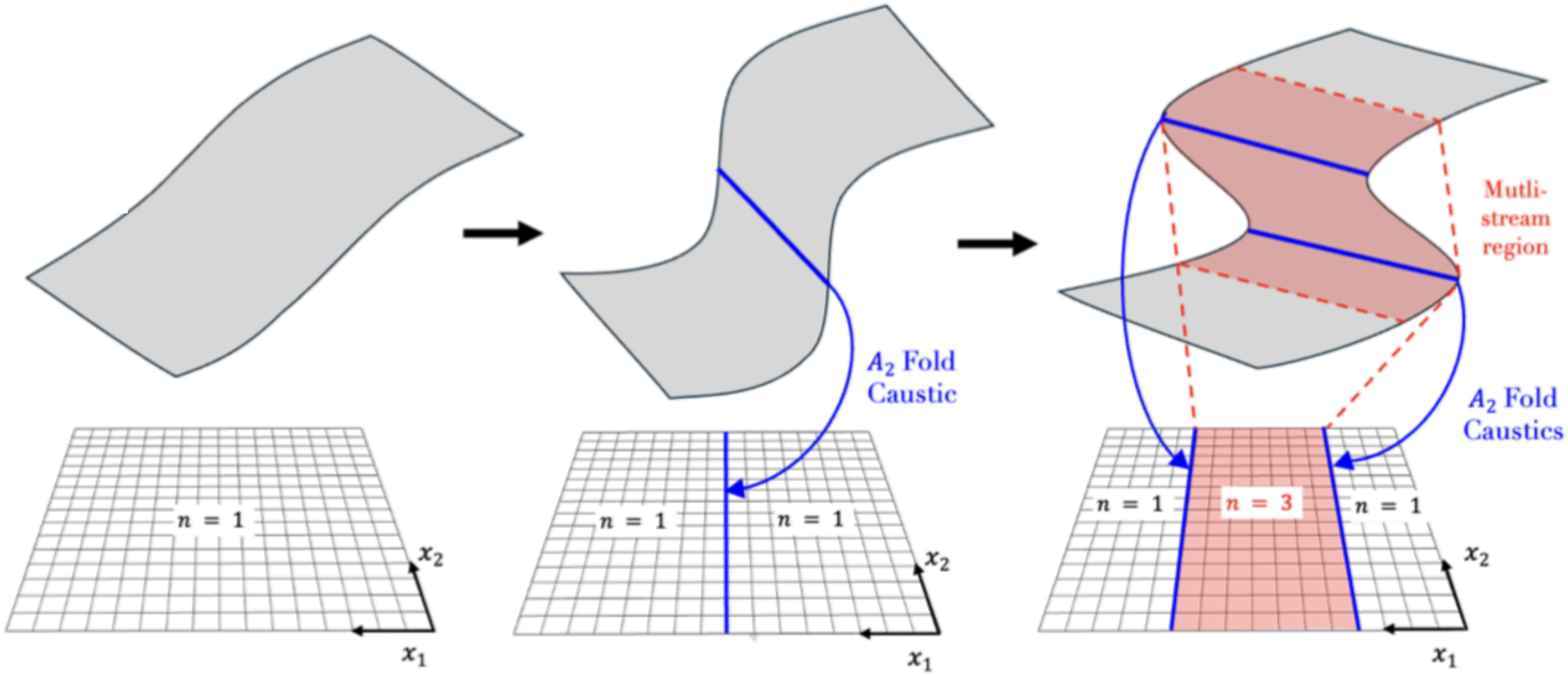


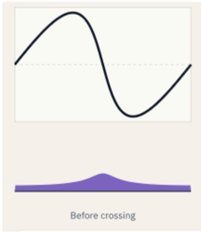


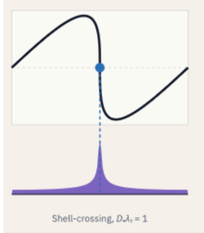


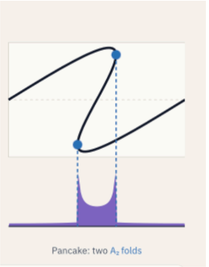


Folding of dark matter sheet







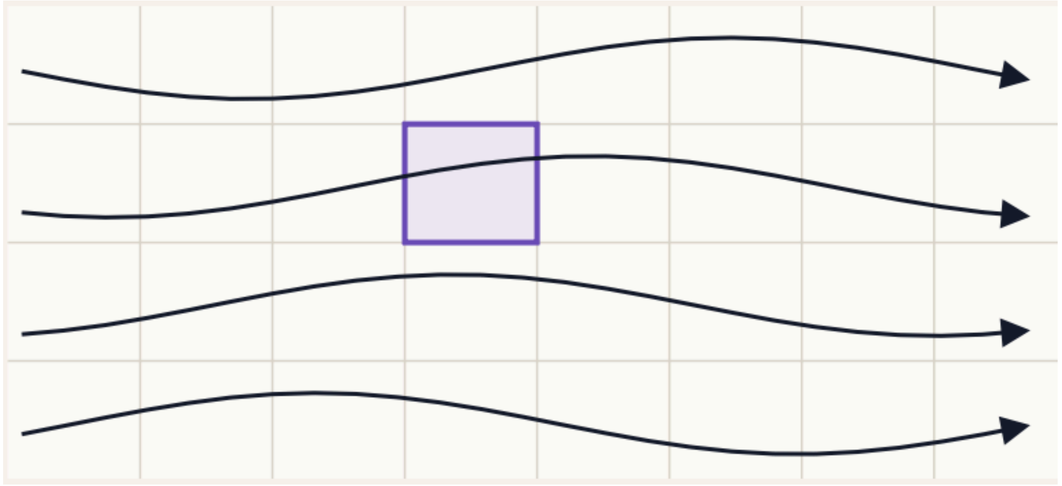


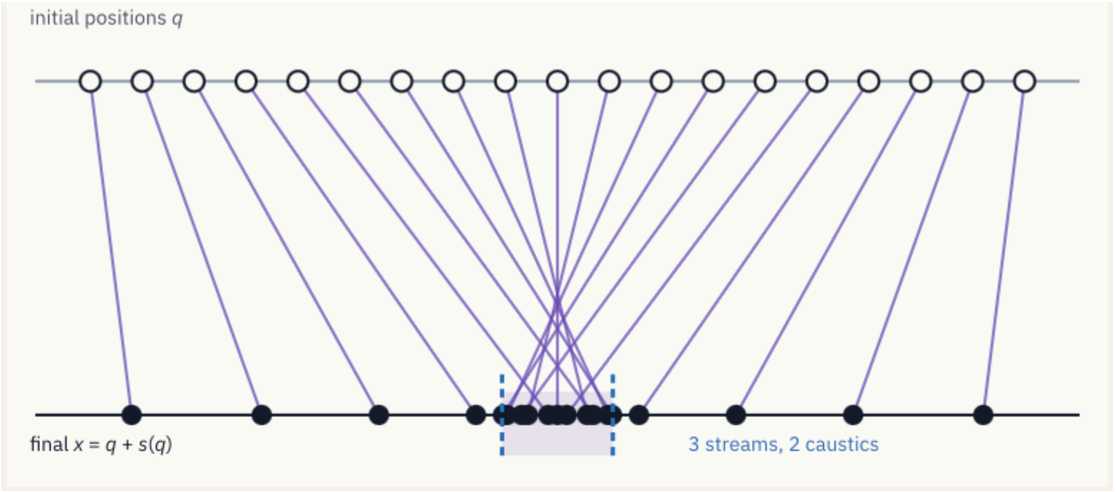
Caustics Skeleton Theory

Starting point

Euler: sit at a fixed point x

Lagrange: follows particles from q





Caustics live where the Lagrangian map folds

Lagrangian fluid dynamics

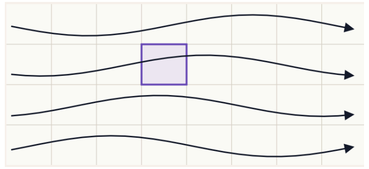
Eulerian (final) coordinates

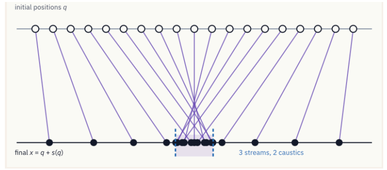
Lagrangian (initial) coordinates

Displacement field

Flowing with the dark matter fluid

$$\mathbf{x}_t(\mathbf{q}) = \mathbf{q} + \mathbf{s}_t(\mathbf{q})$$





Evolution of primordial fluctuations

Zel'dovich approximation:

$$\mathbf{s}_t(\mathbf{q}) = -\, b_+(t)\, \nabla \Psi(\mathbf{q}) \quad \text{with} \quad \frac{\mathrm{d}^2 b_+(t)}{\mathrm{d}t^2} + 2\frac{\dot{a}}{a}\frac{\mathrm{d}b_+(t)}{\mathrm{d}t} = 4\pi G \rho_u b_+(t)$$

$$\nabla \mathbf{s}_t(\mathbf{q}) \mathbf{v}_i(\mathbf{q}) = \mu_i(\mathbf{q}) \mathbf{v}_i(\mathbf{q})$$

Arnol'd, Shandarin, Zel'dovich (1982)

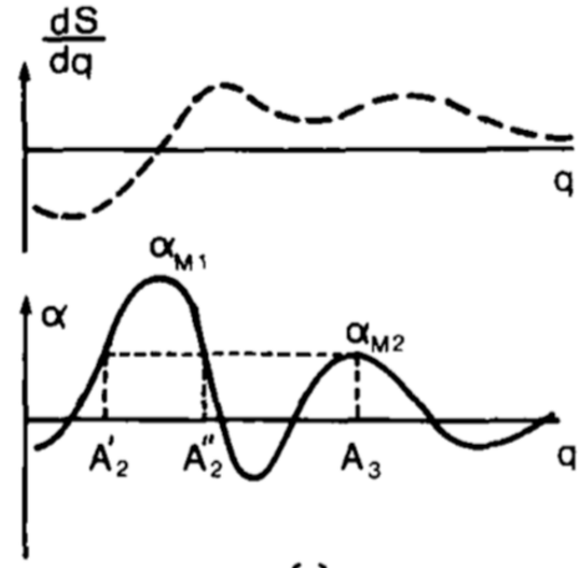
How do caustics arise in the density field?

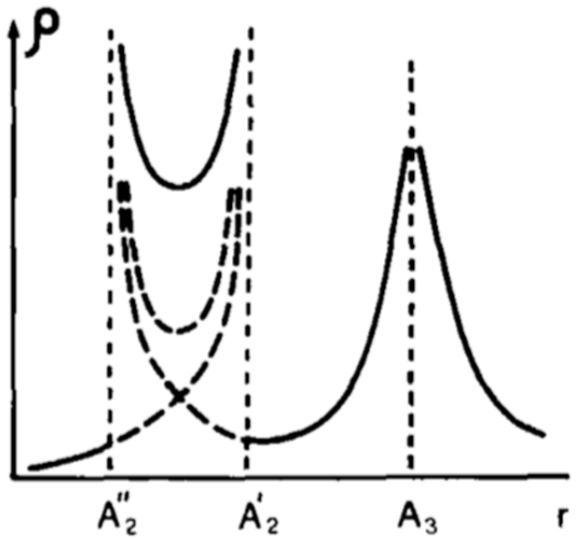
$$\nabla \mathbf{s}_t(\mathbf{q})\mathbf{v}_i(\mathbf{q}) = \mu_i(\mathbf{q})\mathbf{v}_i(\mathbf{q})$$

$$\mathbf{s}_t(\mathbf{q}) = -\, b_+(t)\,\nabla\Psi(\mathbf{q})$$

$$\frac{\mathrm{d}^2 b_+(t)}{\mathrm{d}t^2} + 2\frac{\dot{a}}{a}\frac{\mathrm{d}b_+(t)}{\mathrm{d}t} = 4\pi G\rho_u b_+(t)$$

$$\rho_t(\mathbf{x}') = \sum_{\mathbf{q}\in\mathbf{x}_t^{-1}(\mathbf{x}')} \frac{\bar{\rho}}{|\det \nabla\mathbf{x}_t(\mathbf{q})|} = \sum_{\mathbf{q}\in\mathbf{x}_t^{-1}(\mathbf{x}')} \frac{\bar{\rho}}{|1+\mu_1(\mathbf{q})||1+\mu_2(\mathbf{q})||1+\mu_3(\mathbf{q})|}$$





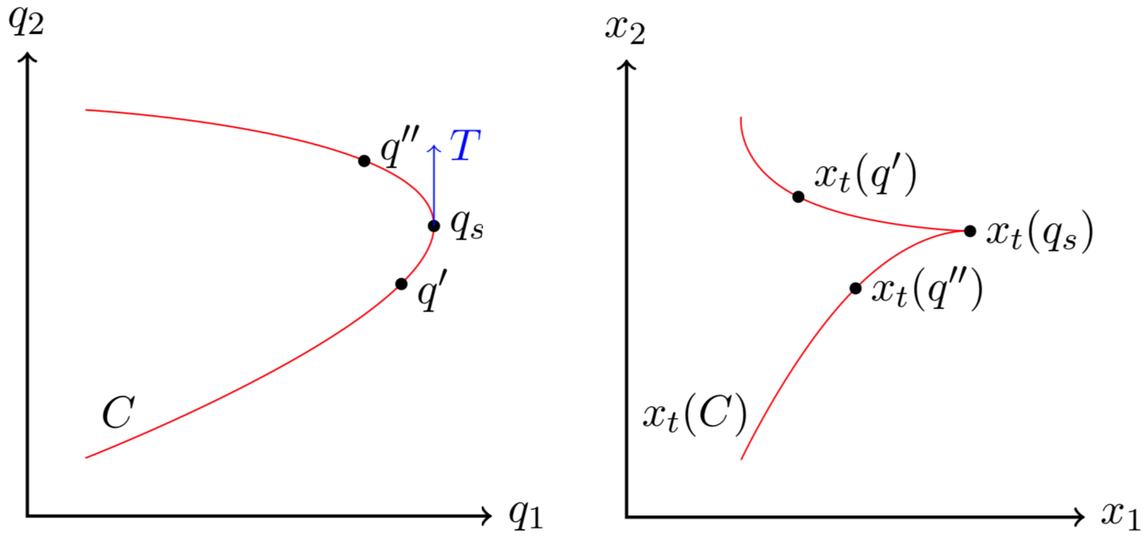
Arnol'd, Shandarin, Zel'dovich (1982)

Generalisation to 3D?

$$\rho_t(\mathbf{x}') = \sum_{\mathbf{q}\in\mathbf{x}_t^{-1}(\mathbf{x}')} \frac{\bar{\rho}}{|\det \nabla \mathbf{x}_t(\mathbf{q})|} = \sum_{\mathbf{q}\in\mathbf{x}_t^{-1}(\mathbf{x}')} \frac{\bar{\rho}}{|1+\mu_1(\mathbf{q})||1+\mu_2(\mathbf{q})||1+\mu_3(\mathbf{q})|}$$

The breakthrough by Job Feldbrugge was to consider a manifold





(a) Lagrangian space L

(b) Eulerian space E

$$(1+\mu_{it}(q_s))v_{it}^*(q_s)\cdot T = 0$$

(

Slide credit Job Feldbrugge

Caustic conditions

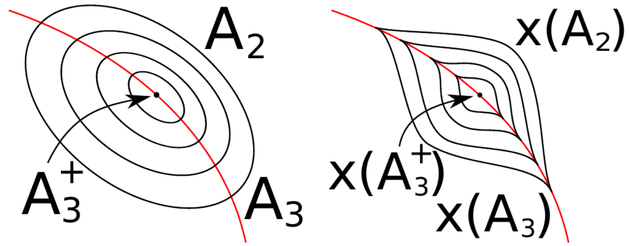
When applying the caustic condition to a Lagrangian space, we obtain the fold condition

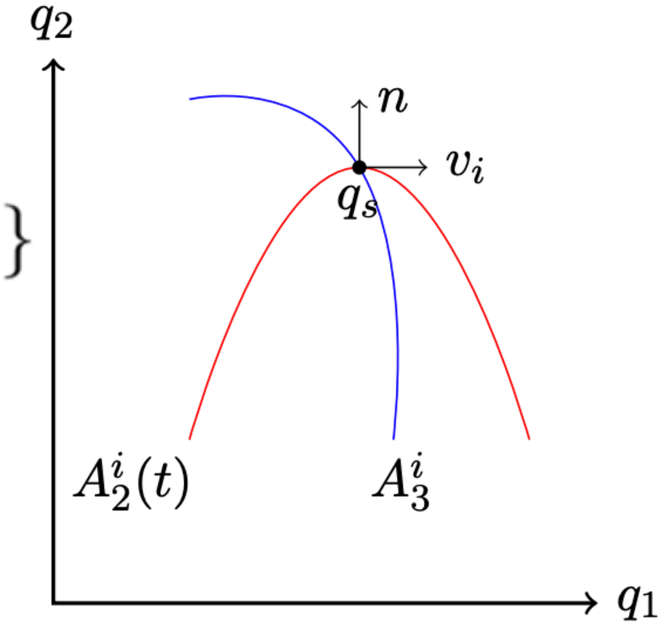
$$A_2^i(t) = \{\mathbf{q} \in L \mid 1 + \mu_{it}(\mathbf{q}) = 0\}$$

Applying the caustic condition a second time, we obtain the cusp caustic in terms of both the eigenvalue and eigenvector fields

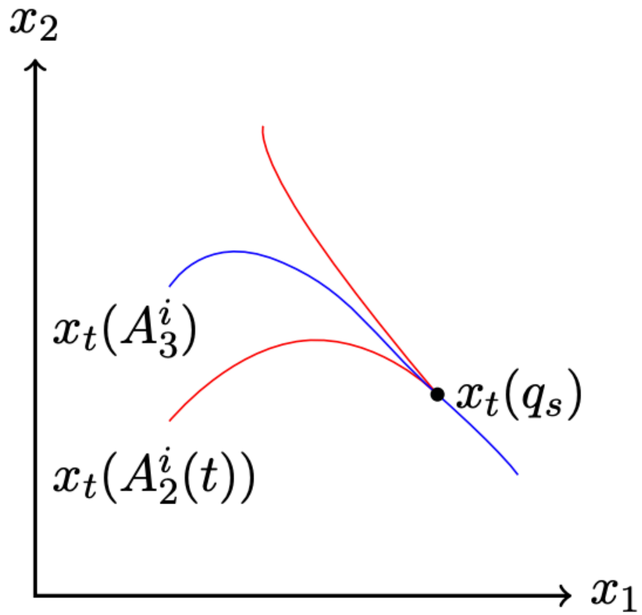
$$A_3^i(t) = \{\mathbf{q} \in L \mid \mathbf{q} \in A_2^i(t), \mathbf{v}_i \cdot \nabla \mu_{it} = 0\}$$

$$(1 + \mu_{it}(q_s))v_{it}^*(q_s) \cdot T = 0$$





(a) Lagrangian space



(b) Eulerian space

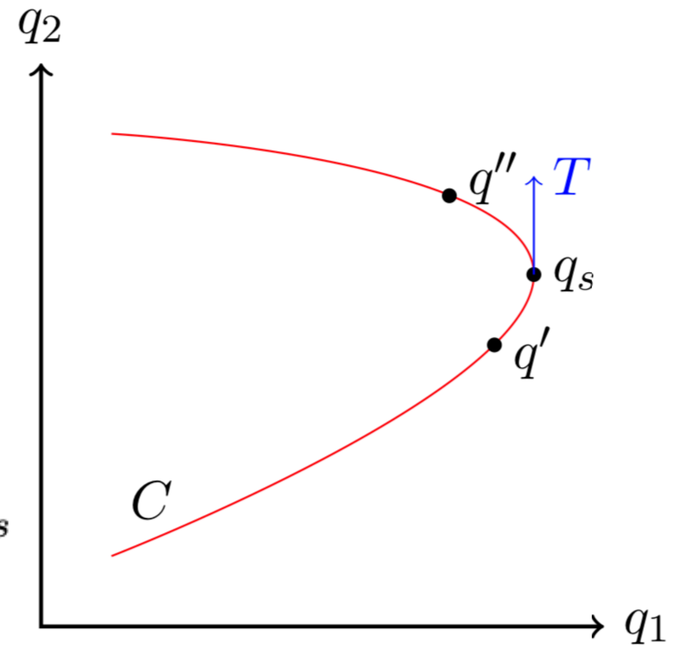
Feldbrugge et al (2019)

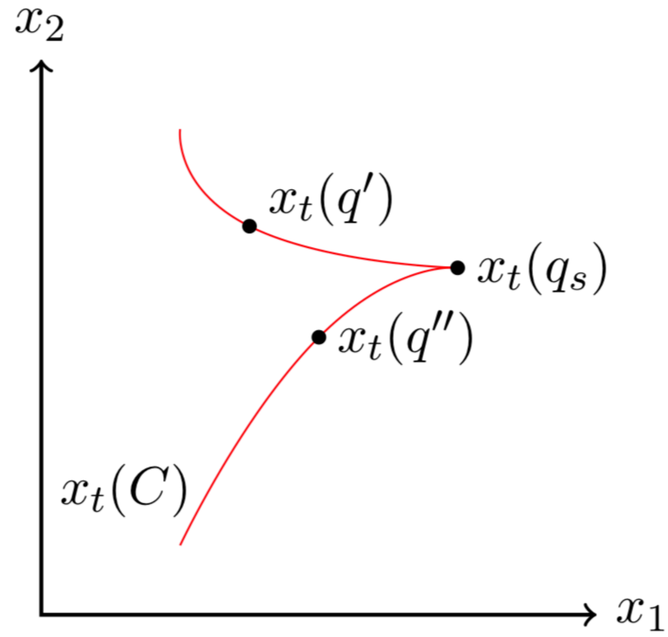
Caustic conditions

Slide credit Job Feldbrugge

Job Feldbrugge managed to generalise the evaluation of any caustic in any dimension

$$\frac{\Delta x}{|\Delta q|} = \frac{\|x_t(q') - x_t(q'')\|}{\|q' - q''\|} \to 0 \qquad q', q'' \to q_s$$





Theorem: *A manifold $M \subset L$ forms a singularity under the mapping x_t in the point $x_t(q_s) \in x_t(M) \subset E$ at time t, meaning that $x_t(M)$ is not smooth in $x_t(q_s)$, if and only if there exists at least one nonzero tangent vector $T \in T_{q_s}M$ satisfying*

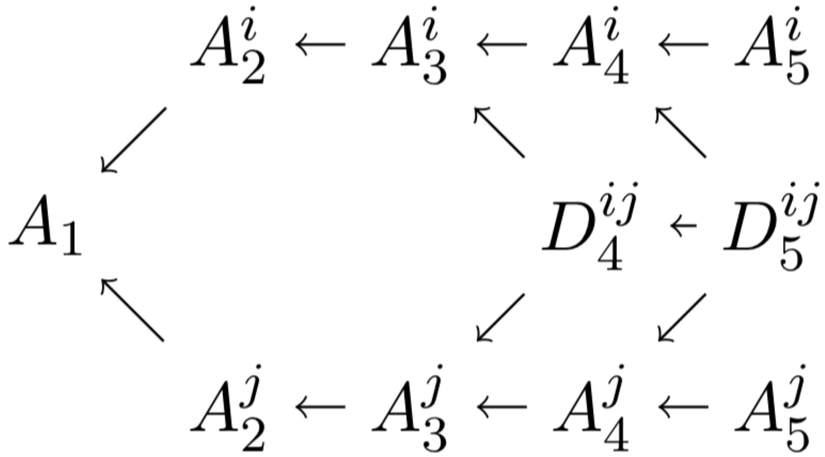
$$(1 + \mu_{it}(q_s))v_{it}^*(q_s) \cdot T = 0$$

for all $i = 1, 2, \ldots, dim(L)$.

Feldbrugge et al (2019)

)

Caustic conditions



Iterative application of the shell-crossing condition

$$(1 + \mu_{it}(q_s))v_{it}^*(q_s) \cdot T = 0$$

leads to the caustic conditions on both the eigenvalue and eigenvector fields:

Fold:	$A_2^i(t) = \{\mathbf{q} \in L \mid 1 + \mu_{it}(\mathbf{q}) = 0\}$
Cusp:	$A_3^i(t) = \{\mathbf{q} \in L \mid \mathbf{q} \in A_2^i(t), \mathbf{v}_i \cdot \nabla \mu_{it} = 0\}$
Swallowtail:	$A_4^i(t) = \{\mathbf{q} \in L \mid \mathbf{q} \in A_3^i(t), \mathbf{v}_i \cdot \nabla(\mathbf{v}_i \cdot \nabla \mu_{it}) = 0\}$
Butterfly:	$A_5^i(t) = \{\mathbf{q} \in L \mid \mathbf{q} \in A_4^i(t), \mathbf{v}_i \cdot \nabla(\mathbf{v}_i \cdot \nabla(\mathbf{v}_i \cdot \nabla \mu_{it})) = 0\}$
Umbilic:	$D_4^{ij}(t) = \{\mathbf{q} \in L \mid 1 + \mu_{it}(\mathbf{q}) = 1 + \mu_{jt}(\mathbf{q}) = 0\}$
Parabolic:	$D_5^{ij}(t) = \{\mathbf{q} \in L \mid q \in D_4^{ij}(t), \mathbf{v}_i \cdot \nabla \mu_i = \mathbf{v}_j \cdot \nabla \mu_j = 0\}$

Slide credit Job Feldbrugge

Feldbrugge et al (2019)

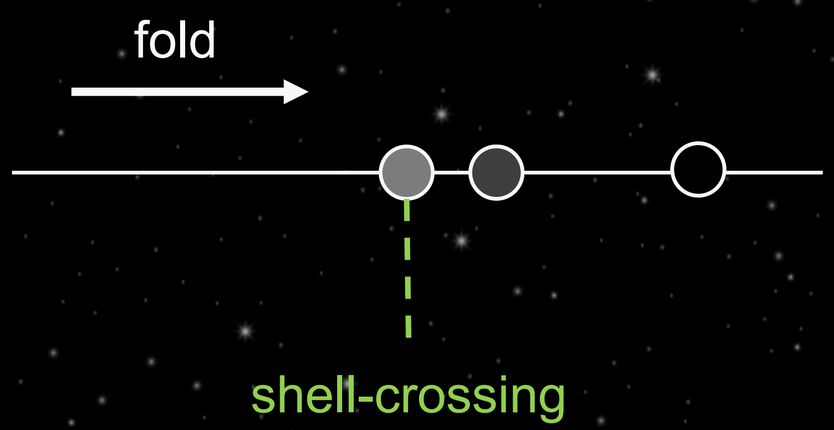
Caustics in 1D

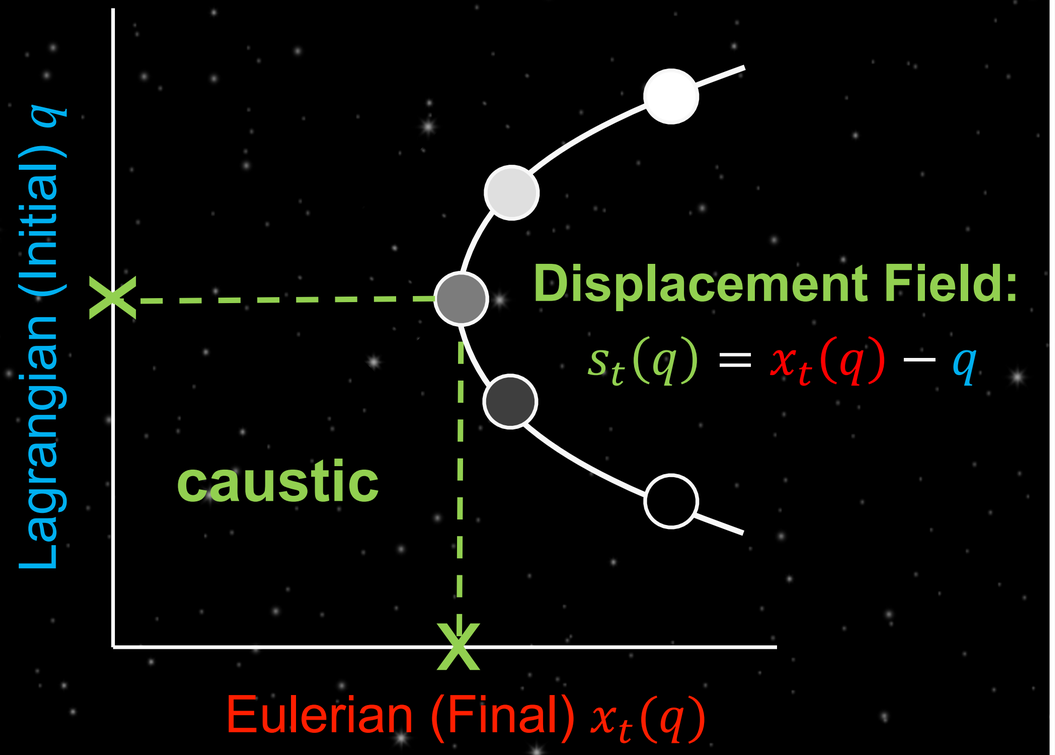
Slide credit Amelie Read

Caustics

Critical Points in $x_t(q)$

In 1D:





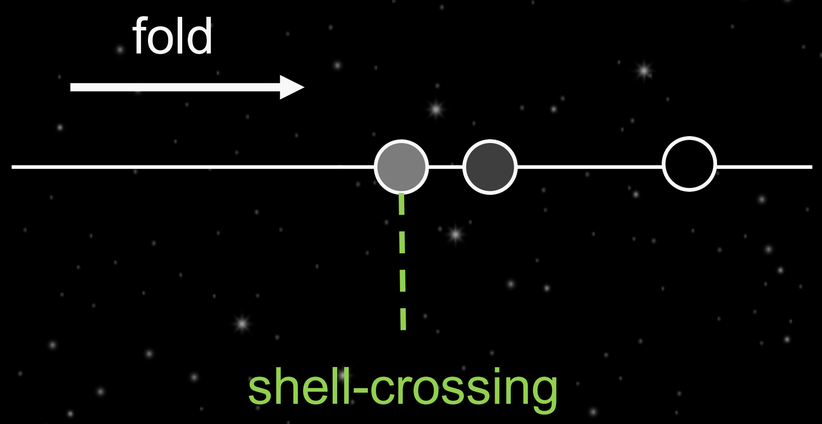
During shell-crossing, the density field spikes to infinity => caustics

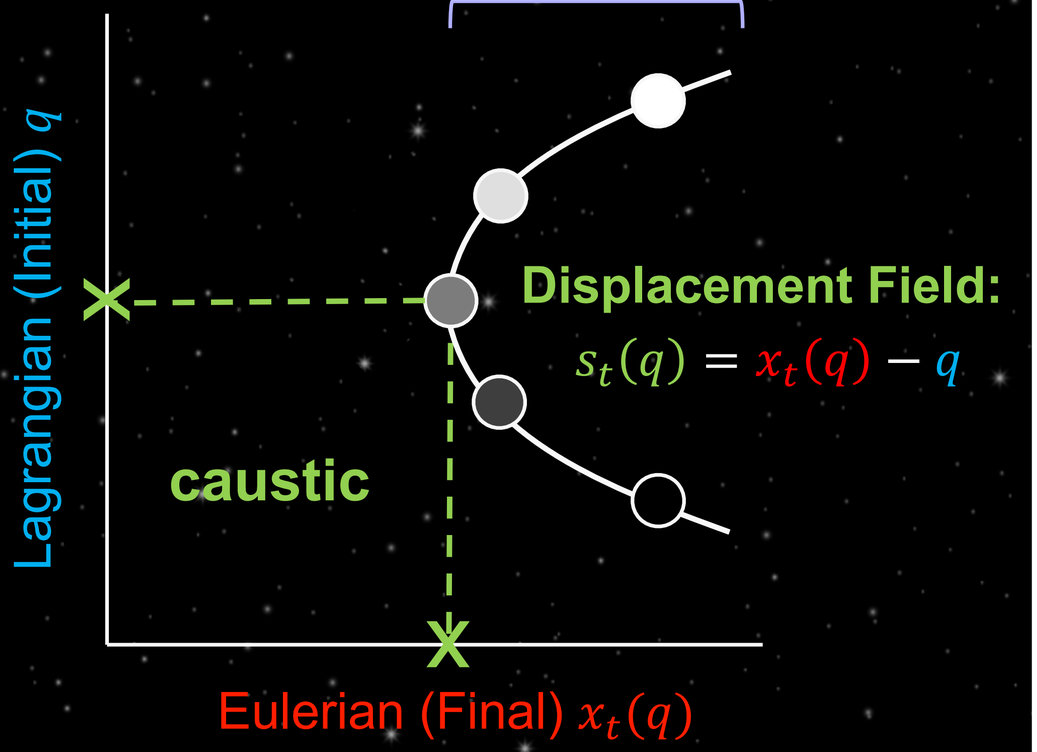
Slide credit Amelie Read

Caustics

Critical Points in $x_t(q)$

In 1D:





Multi-stream regions mark beginning of gravitational collapse

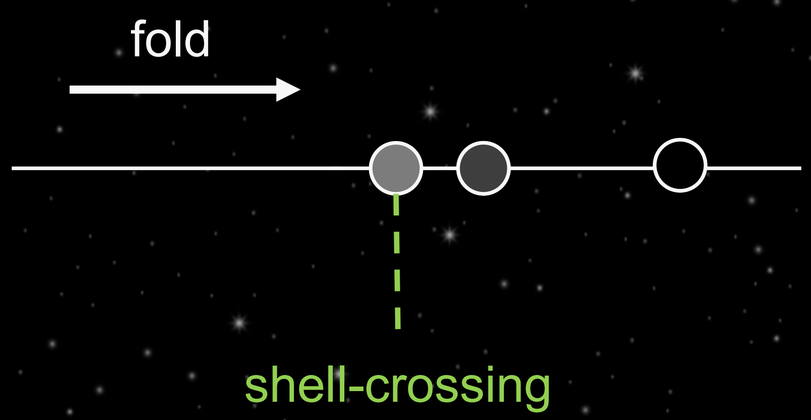
~ caustics should be good tracers of structure ~

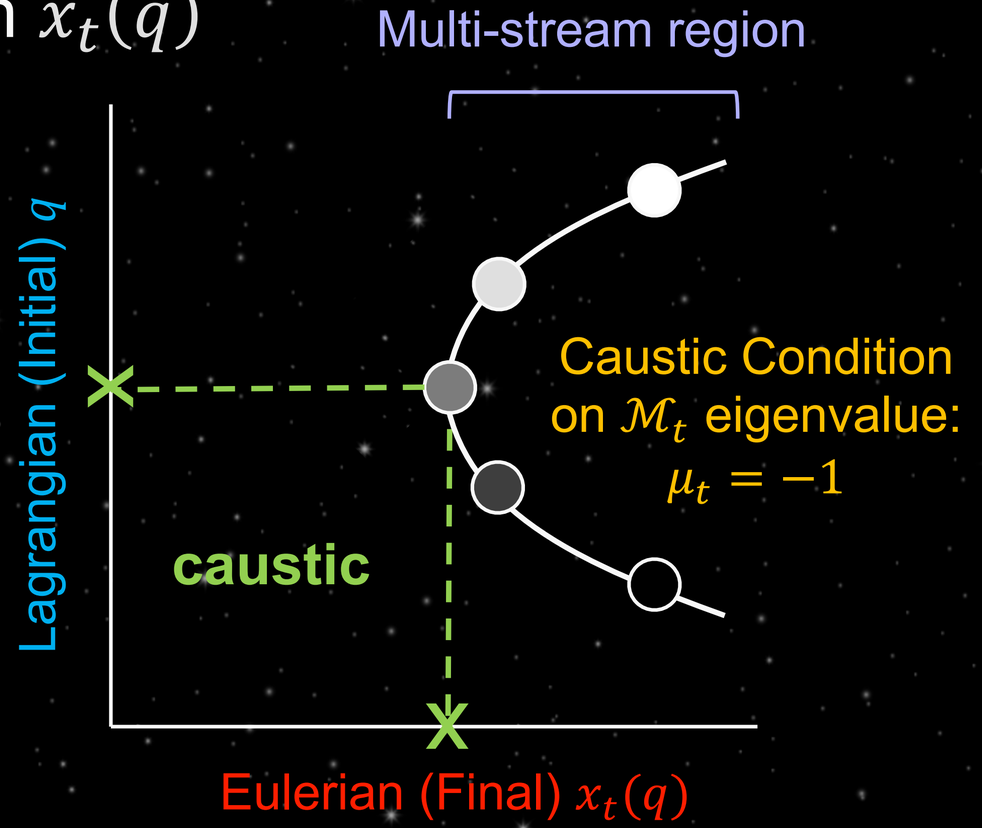
Slide credit Amelie Read

Caustics

Critical Points in $x_t(q)$

In 1D:





Multi-stream regions mark beginning of gravitational collapse

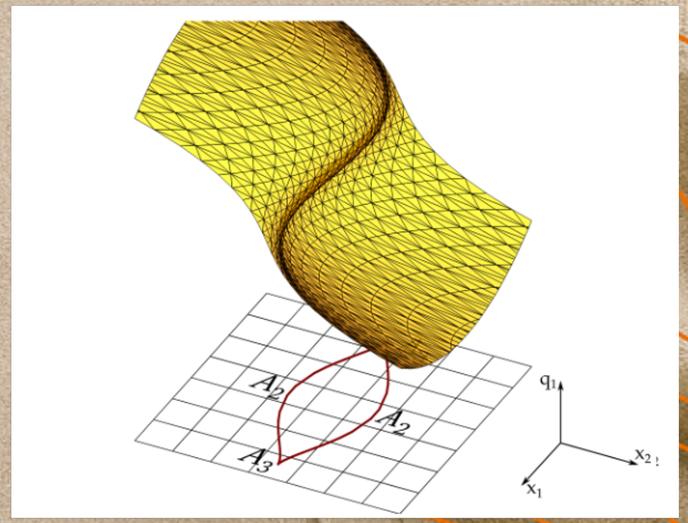
~ caustics should be good tracers of structure ~

Caustics in 2D

Slide credit Amelie Read

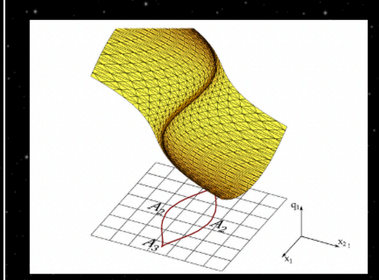
Caustics

Name	Symbol	2D Cosmic Web	Caustic Conditions on $\mathcal{M}_t$
Fold	A_2	Shell-Crossing	$\mu_{ti} = -1$
Cusp	A_3	Filament	$q \in A_2^i$ and $\boldsymbol{v}_{ti} \cdot \nabla \mu_{ti} = 0$
Swallowtail	A_4	Cluster	$q \in A_3^i$ and $\boldsymbol{v}_{ti} \cdot \nabla(\boldsymbol{v}_i \cdot \nabla \mu_{ti}) = 0$
Elliptic/Hyperbolic Umbilic	$D_4^{\pm}$	Cluster	$\mu_{ti} = \mu_{tj} = -1$

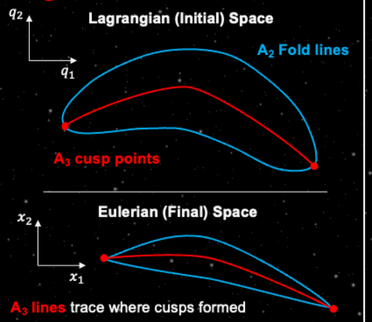


Hidding et al. (2013)

2D Filaments: A_3 cusp lines



Hidding et al. (2013)



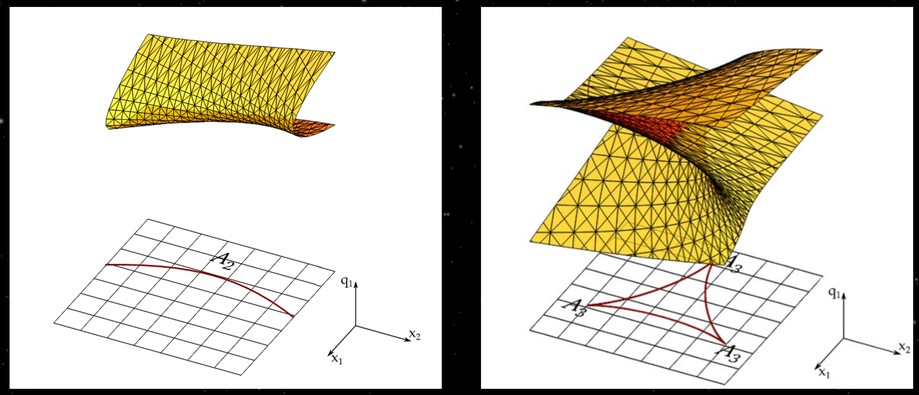
A_3 lines trace where cusps formed

19

2D Clusters:

A_4 Swallowtail Points

D_4 Umblilic Points

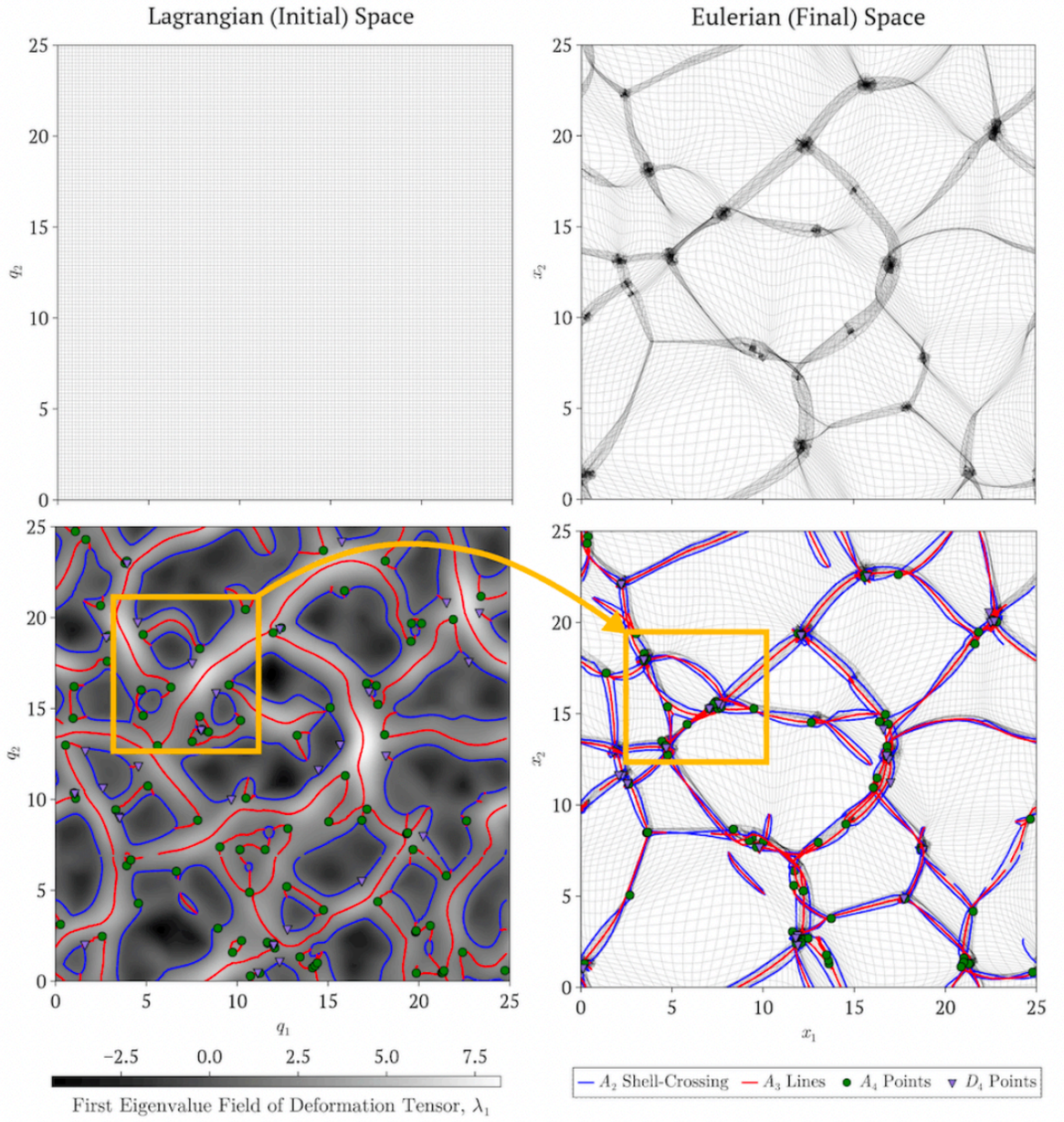


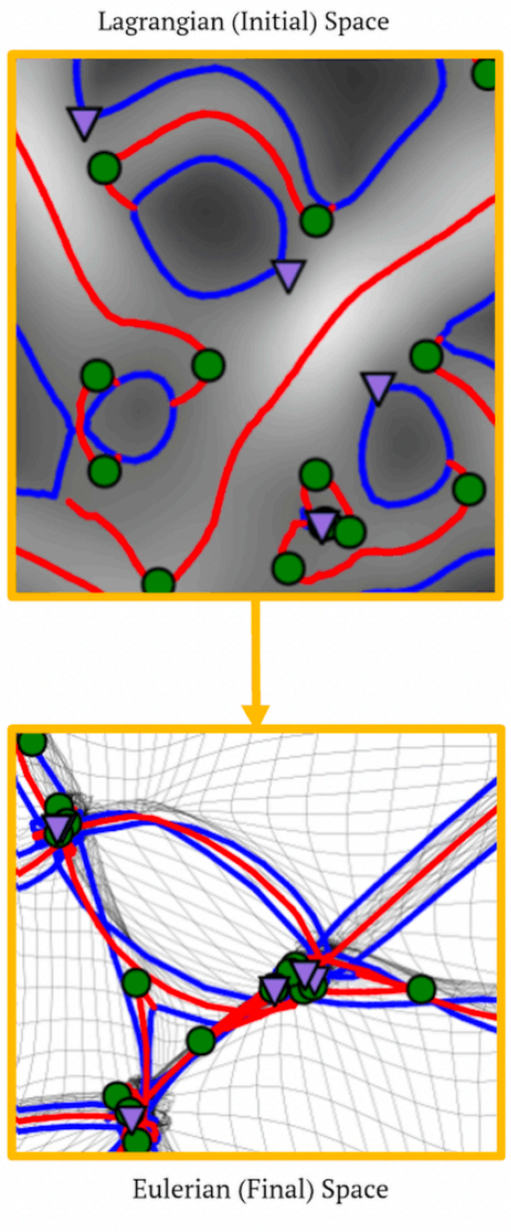
Hidding et al. (2013)

Name	Symbol	2D Cosmic Web	Caustic Conditions on $\mathcal{M}_t$
Fold	A_2	Shell-Crossing	$\mu_{ti} = -1$
Cusp	A_3	Filament	$q \in A_2^i$ and $\boldsymbol{v}_{ti} \cdot \nabla \mu_{ti} = 0$
Swallowtail	A_4	Cluster	$q \in A_3^i$ and $\boldsymbol{v}_{ti} \cdot \nabla(\boldsymbol{v}_i \cdot \nabla \mu_{ti}) = 0$
Elliptic/Hyperbolic Umbilic	$D_4^{\pm}$	Cluster	$\mu_{ti} = \mu_{tj} = -1$

Slide credit Amelie Read

Caustics in 2D simulations





Caustics in 3D

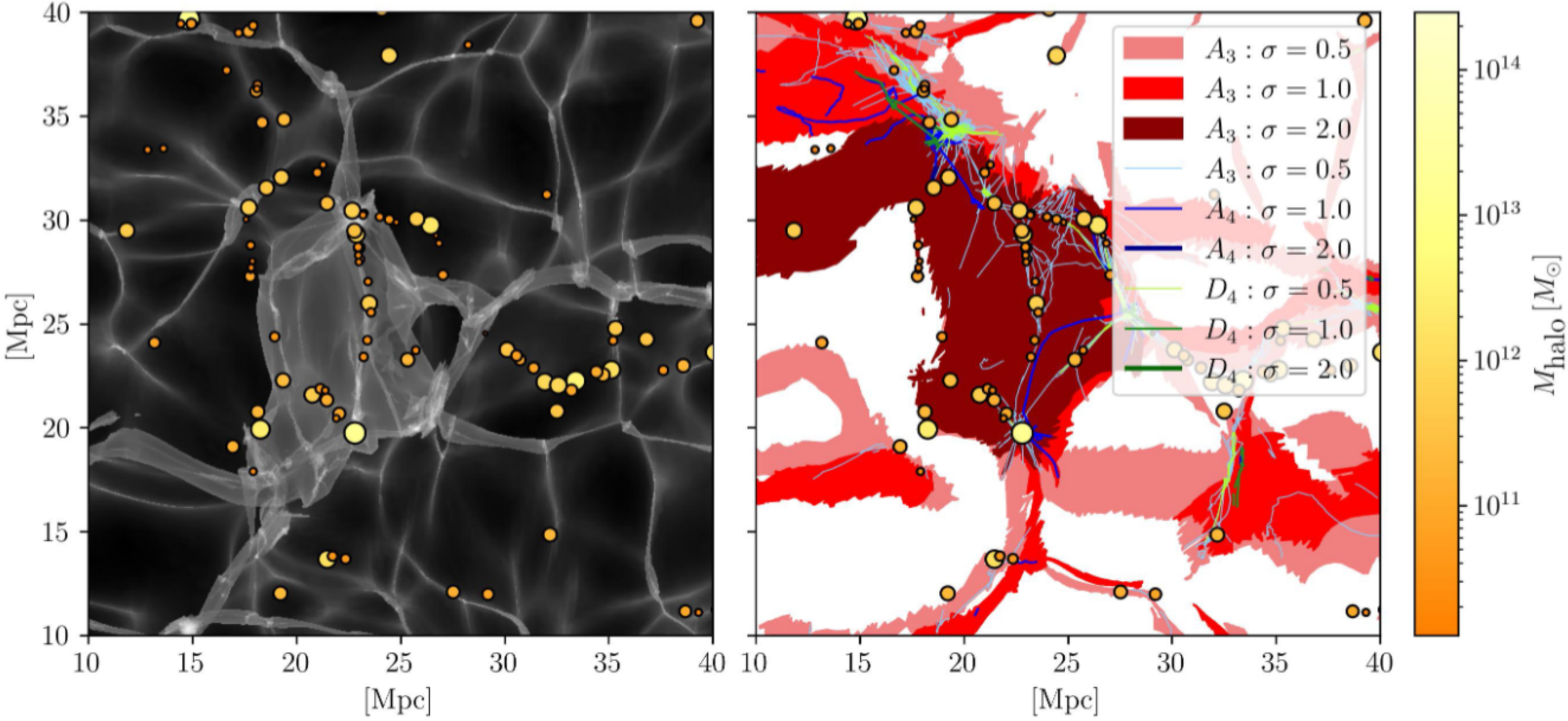
Slide credit Amelie Read

Caustics in the Cosmic Web

Name	Symbol	3D Cosmic Web	Caustic Conditions on $\mathcal{M}_t$
Fold	A_2	Shell-Crossing	$\mu_{ti} = -1$
Cusp	A_3	Wall	$q \in A_2^i$ and $v_{ti} \cdot \nabla \mu_{ti} = 0$
Swallowtail	A_4	Filament	$q \in A_3^i$ and $v_{ti} \cdot \nabla (v_{ti} \cdot \nabla \mu_{ti}) = 0$
Butterfly	A_5	Cluster	$q \in A_4^i$ and $v_{ti} \cdot \nabla (v_{ti} \cdot \nabla (v_{ti} \cdot \nabla \mu_{ti})) = 0$
Elliptic/Hyperbolic Umbilic	$D_4^{\pm}$	Filament	$\mu_{ti} = \mu_{tj} = -1$
Parabolic Umbilic	D_5	Cluster	$q \in D_4^{ij}$ and $v_{ti} \cdot \nabla(\mu_{ti} - \mu_{tj}) = v_{tj} \cdot \nabla(\mu_{ti} - \mu_{tj}) = 0$

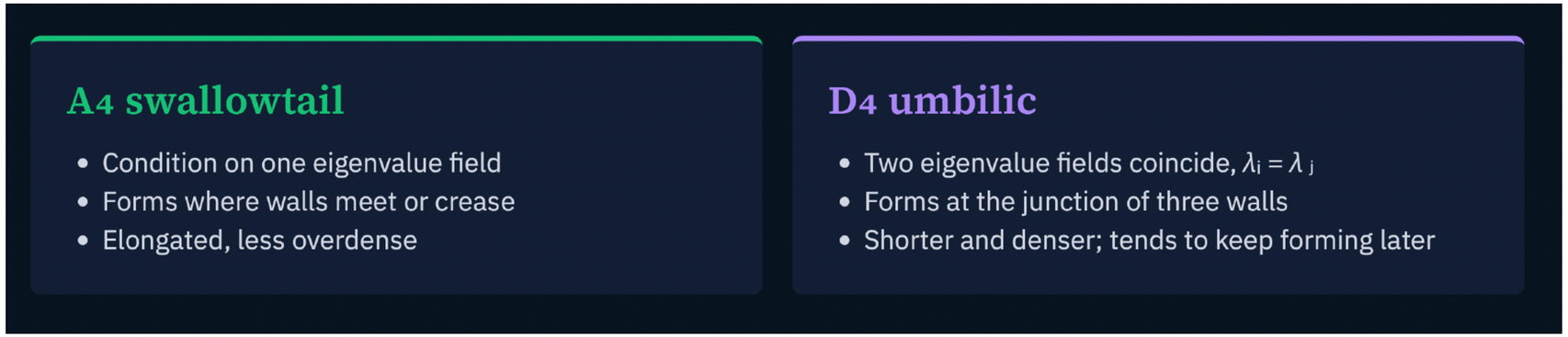
Slide credit Job Feldbrugge

Caustics in 3D



Caustics in 3D

2 types of filaments



2 types of clusters

The caustic skeleton of the local web

Applying to Observations

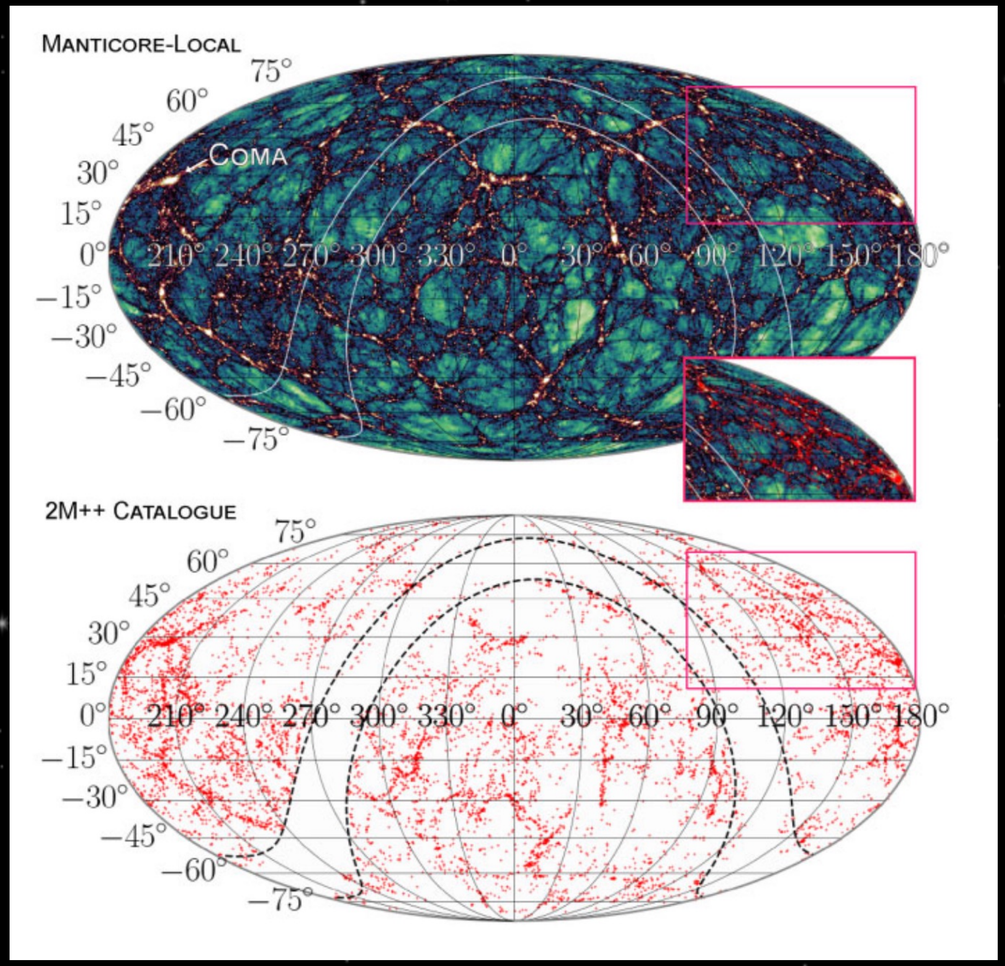
And the local Universe

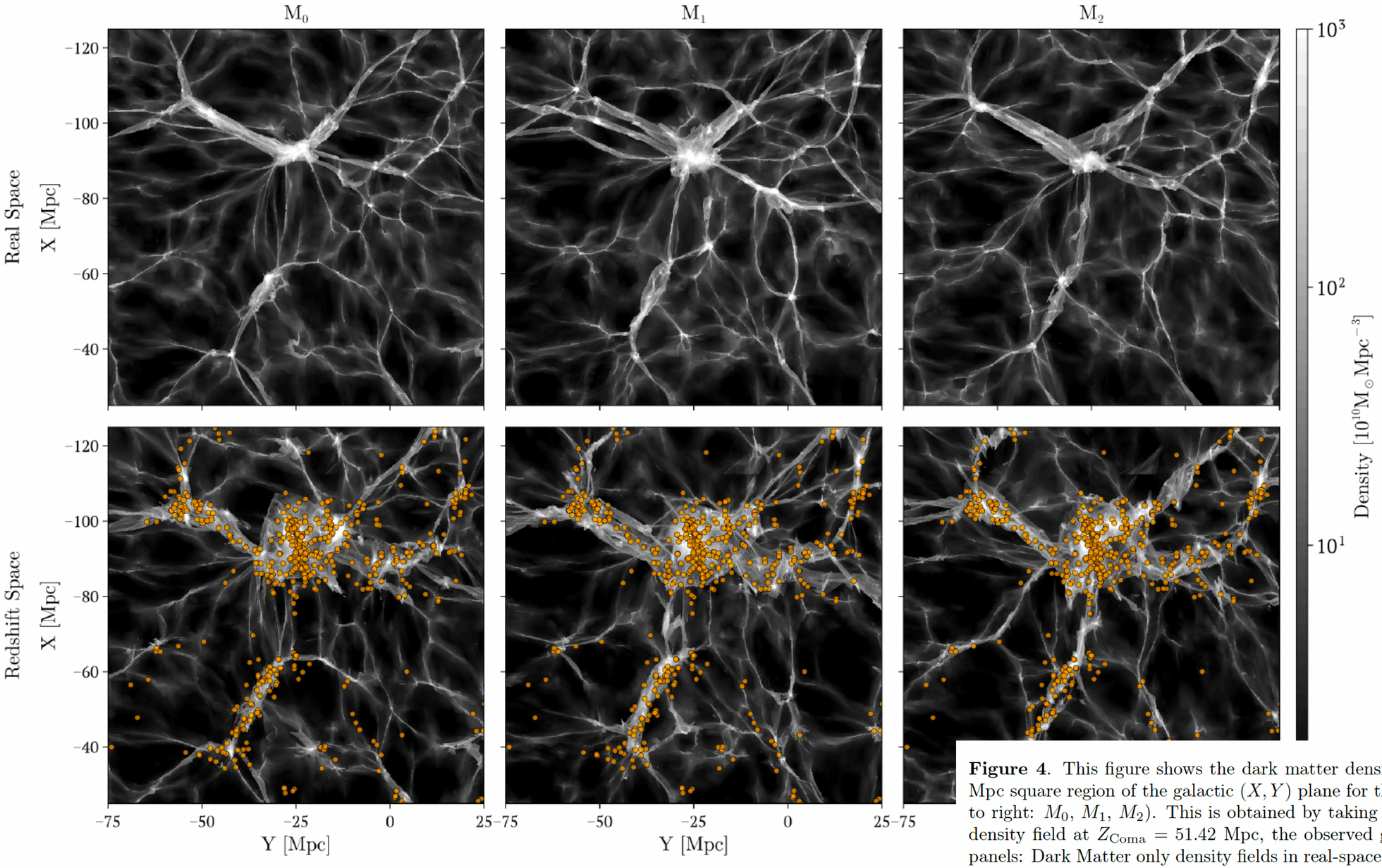
Manticore Project:
McAlpine et al. (2025)

1. Bayesian Constrained Reconstruction of local universe initial conditions from the 2M++ galaxy catalogue

2. Dark-Matter only N-body resimulations of local universe

Assuming ΛCDM cosmology
H_0 = 68.1 km/s/Mpc (DES Yr 3)







Amelie Read

Figure 4. This figure shows the dark matter density field around the Coma Supercluster in a 100 Mpc square region of the galactic (X, Y) plane for three different `Manticore-Local` simulations (left to right: M_0, M_1, M_2). This is obtained by taking a 2D slice through the calculated 3D simulation density field at $Z_{\rm Coma} = 51.42$ Mpc, the observed galactic Z-coordinate of the Coma Cluster. Top panels: Dark Matter only density fields in real-space coordinates given by `SWIFT` final $z = 0$ snapshot. Bottom panels: Dark Matter only density fields projected into redshift space with simulation peculiar velocity fields. Redshift space slices are taken at same Z-coordinate. Orange points show 2M++ galaxies - which the `Manticore-Local` density fields were reconstructed from - within a 10 Mpc thick slice around $Z_{\rm Coma}$. The density is estimated with the Phase-Space Delaunay Tessellation Field Estimator [118, 119].

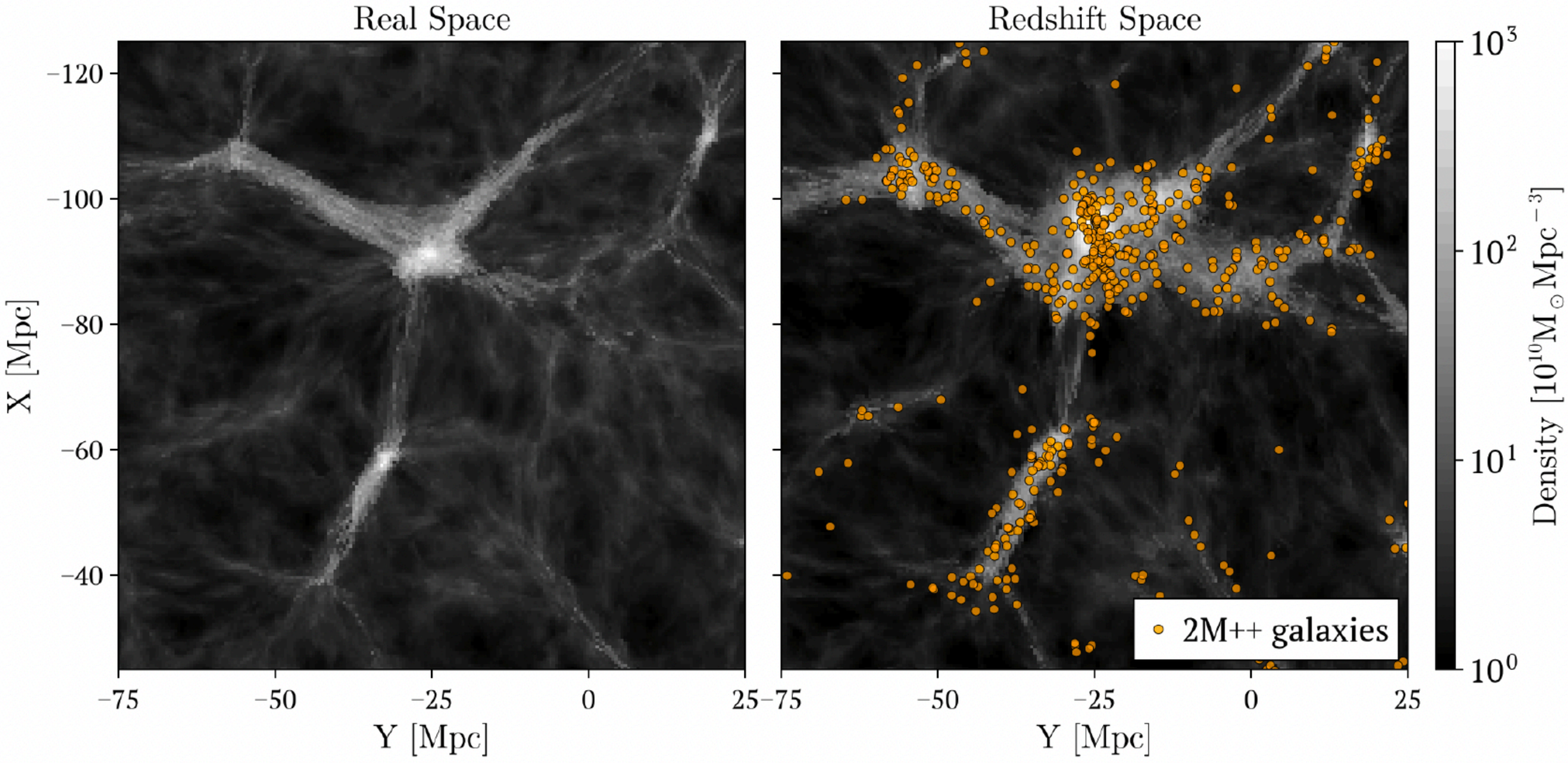
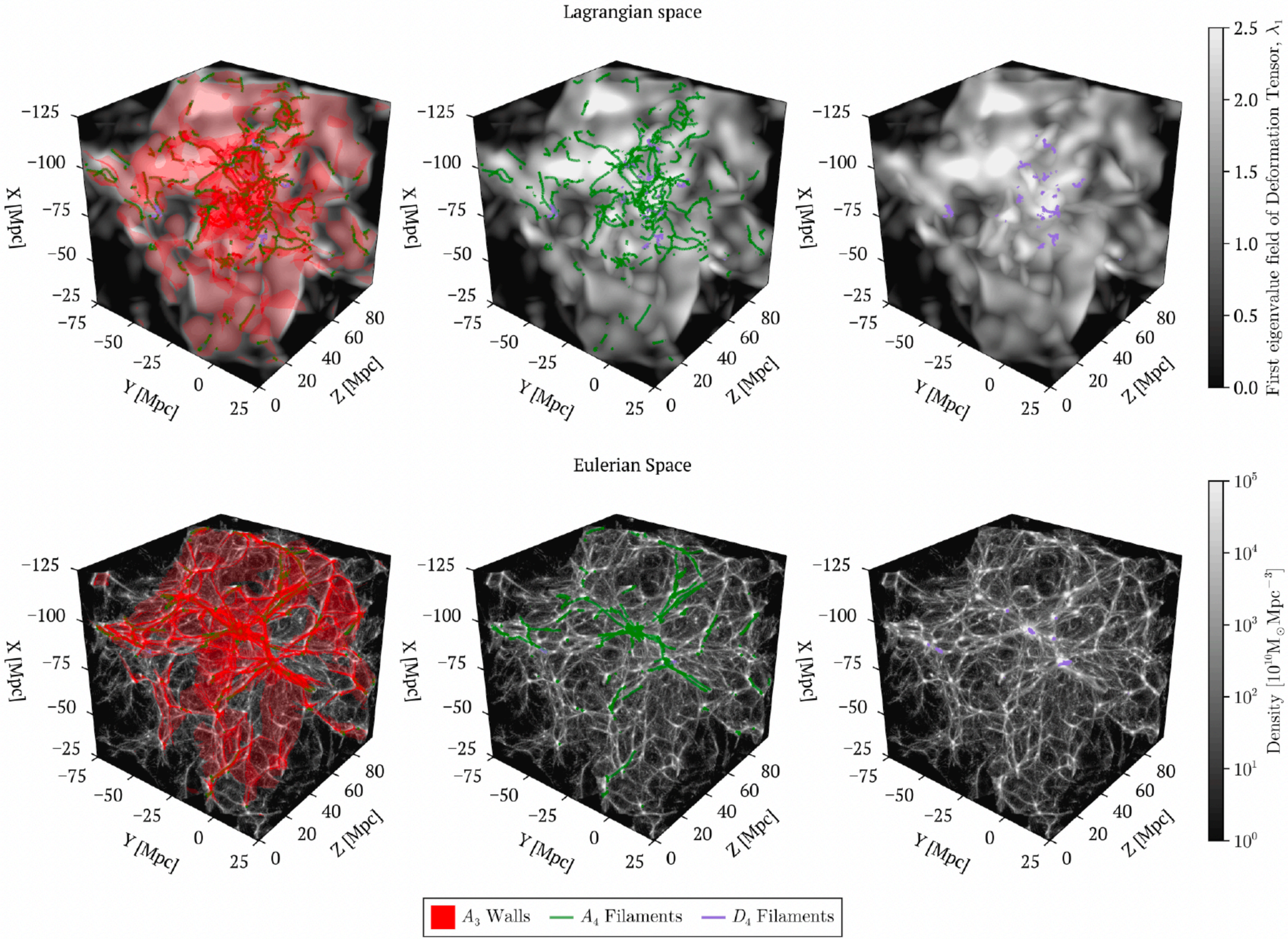
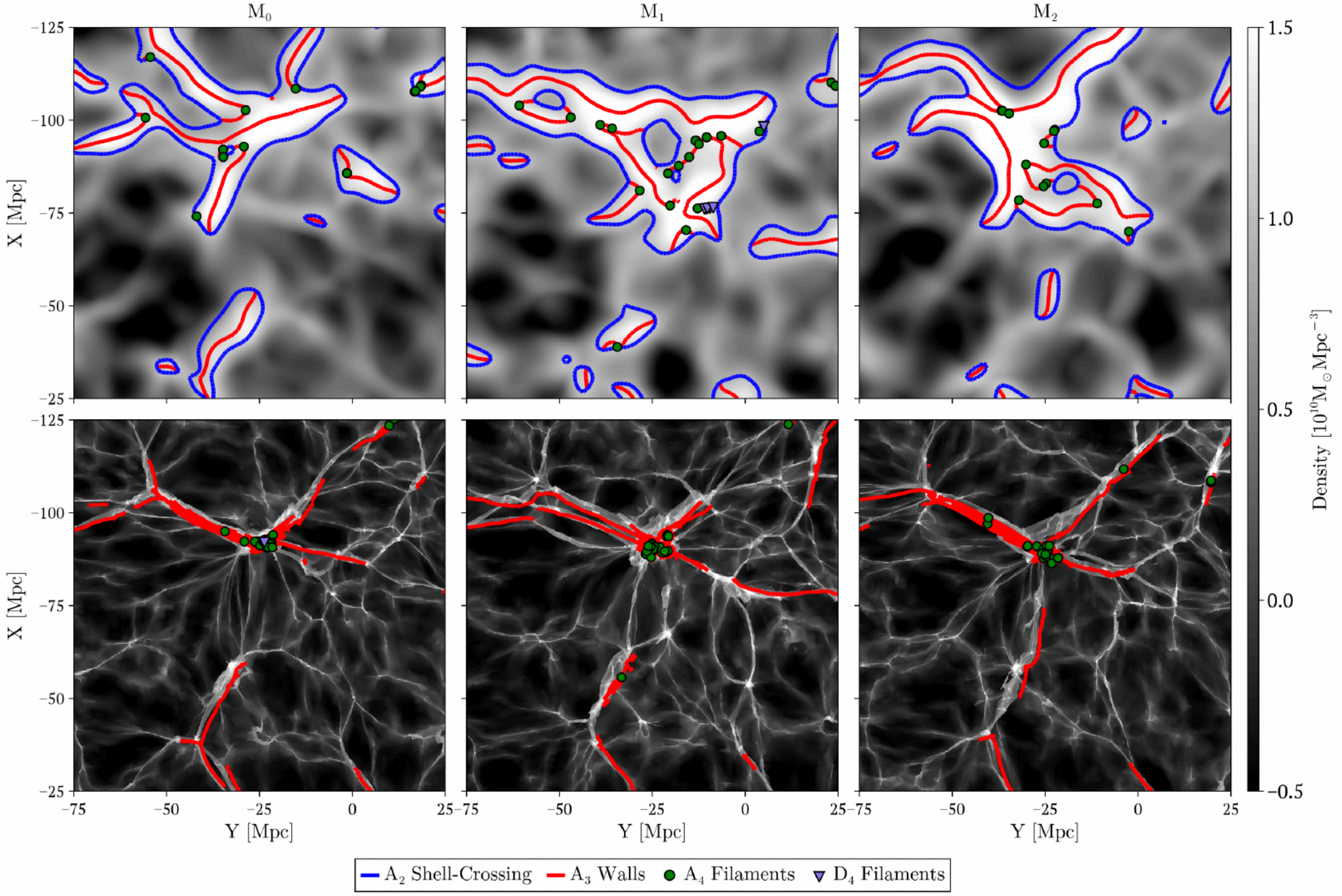


Figure 5. This figure shows the median density field around the Coma Cluster of 10 different `Manticore-Local` realisations in real space (left) and redshift space (right). The density in a 2d slice at Z_{Coma} is estimated for each realisation with the Phase-Space Delaunay Tessellation Field Estimator [118, 119] as shown in fig. 4, and the median value of the density in each pixel is shown here. Orange points show 2M++ galaxies within a 10 Mpc thick slice around Z_{Coma}.


Lagrangian space
Eulerian Space
X [Mpc]
Y [Mpc]
Z [Mpc]
−125
−100
−75
−50
−25
0
25
20
40
60
80
First eigenvalue field of Deformation Tensor, λ_1
2.5
2.0
1.5
1.0
0.5
0.0
Density $[10^{10}\mathrm{M}_\odot\mathrm{Mpc}^{-3}]$
10^5
10^4
10^3
10^2
10^1
10^0
A_3 Walls
A_4 Filaments
D_4 Filaments

Lagrange

Euler



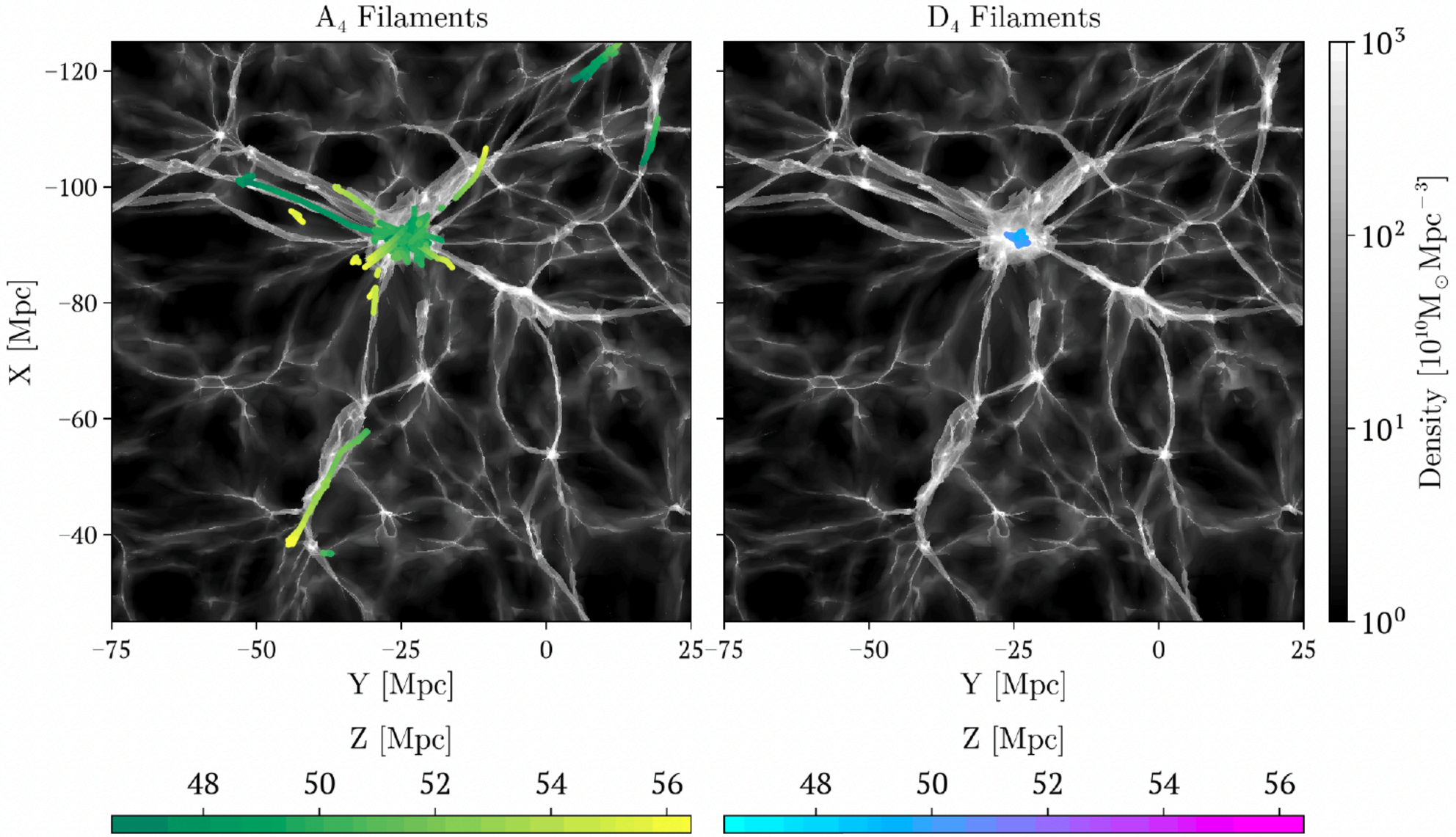
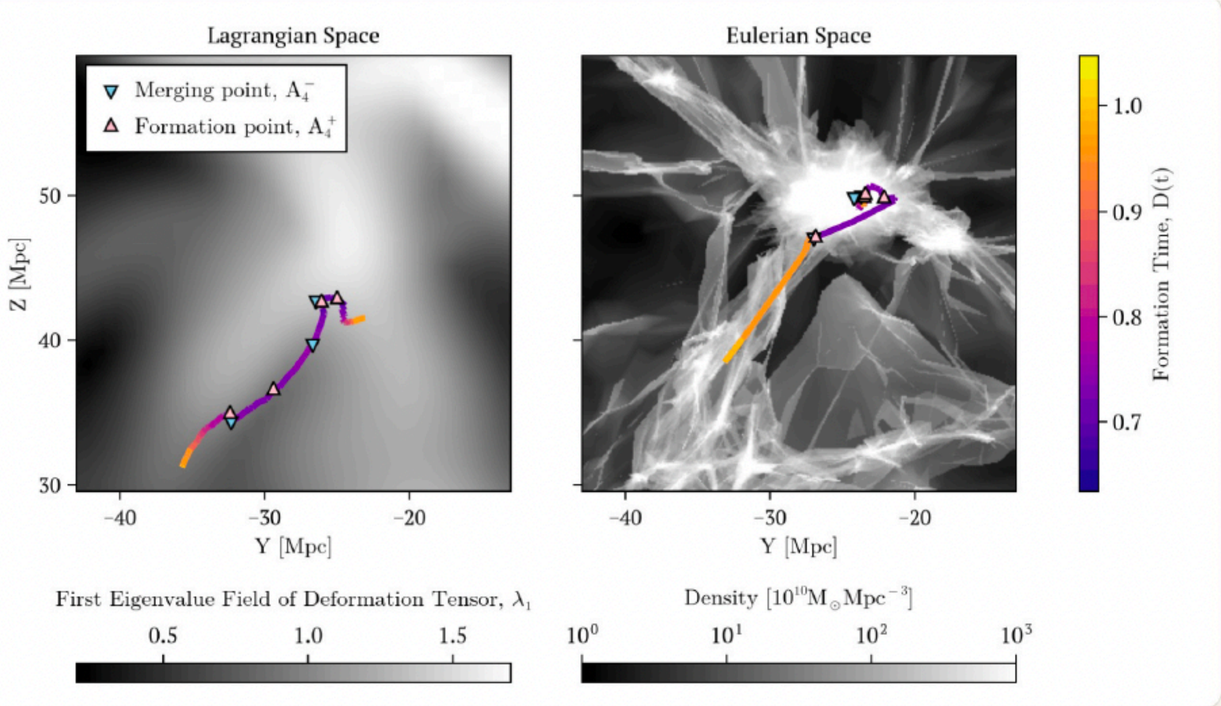
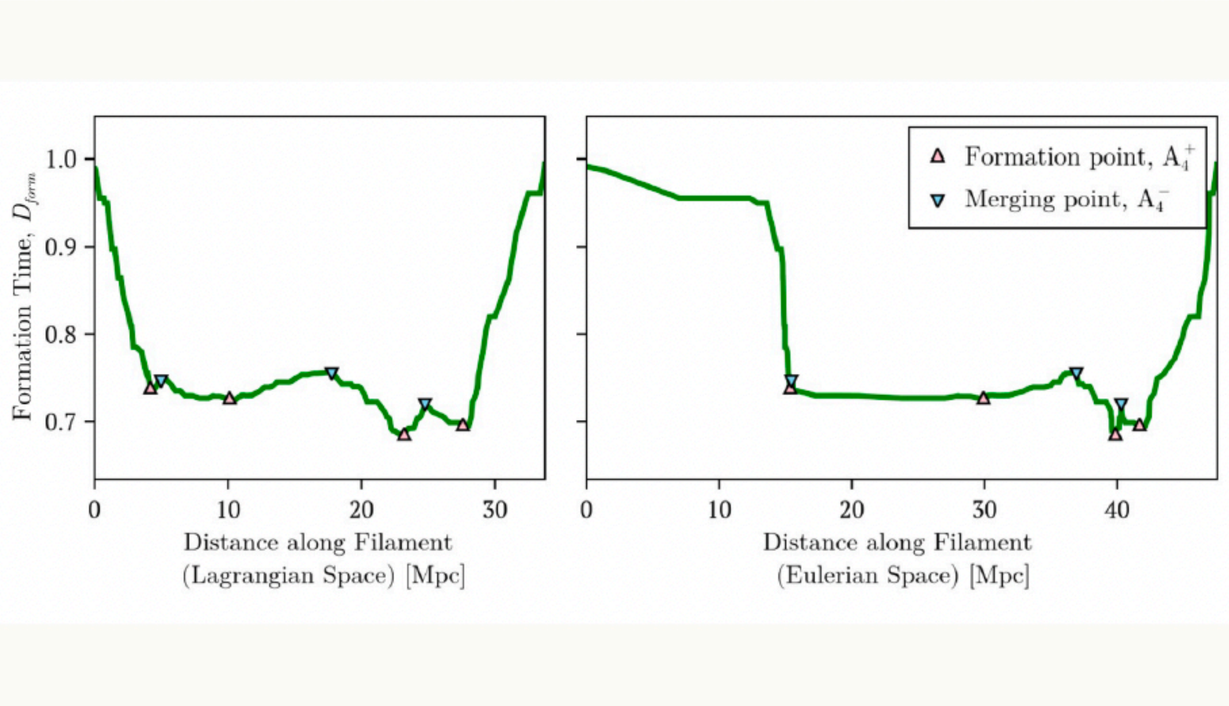


Figure 7. Filamentary ($\sigma = 3$ Mpc) caustics of the Coma Supercluster for M_1 simulation within 10 Mpc of $Z_{\rm Coma} = 51.42$ Mpc projected onto X-Y galactic plane in 100 Mpc square region. Z-coordinates of filaments are shown for A_4 filaments and D_4 filaments on the bottom left and right, respectively. 2D real-space density field slice of M_1 simulations underlaid, same as upper middle panel of fig. 4. The density is estimated with the Phase-Space Delaunay Tessellation Field Estimator [118, 119].

Formation History

A filament is born in pieces that later merge





Can we discriminate among
Dark Matter models?

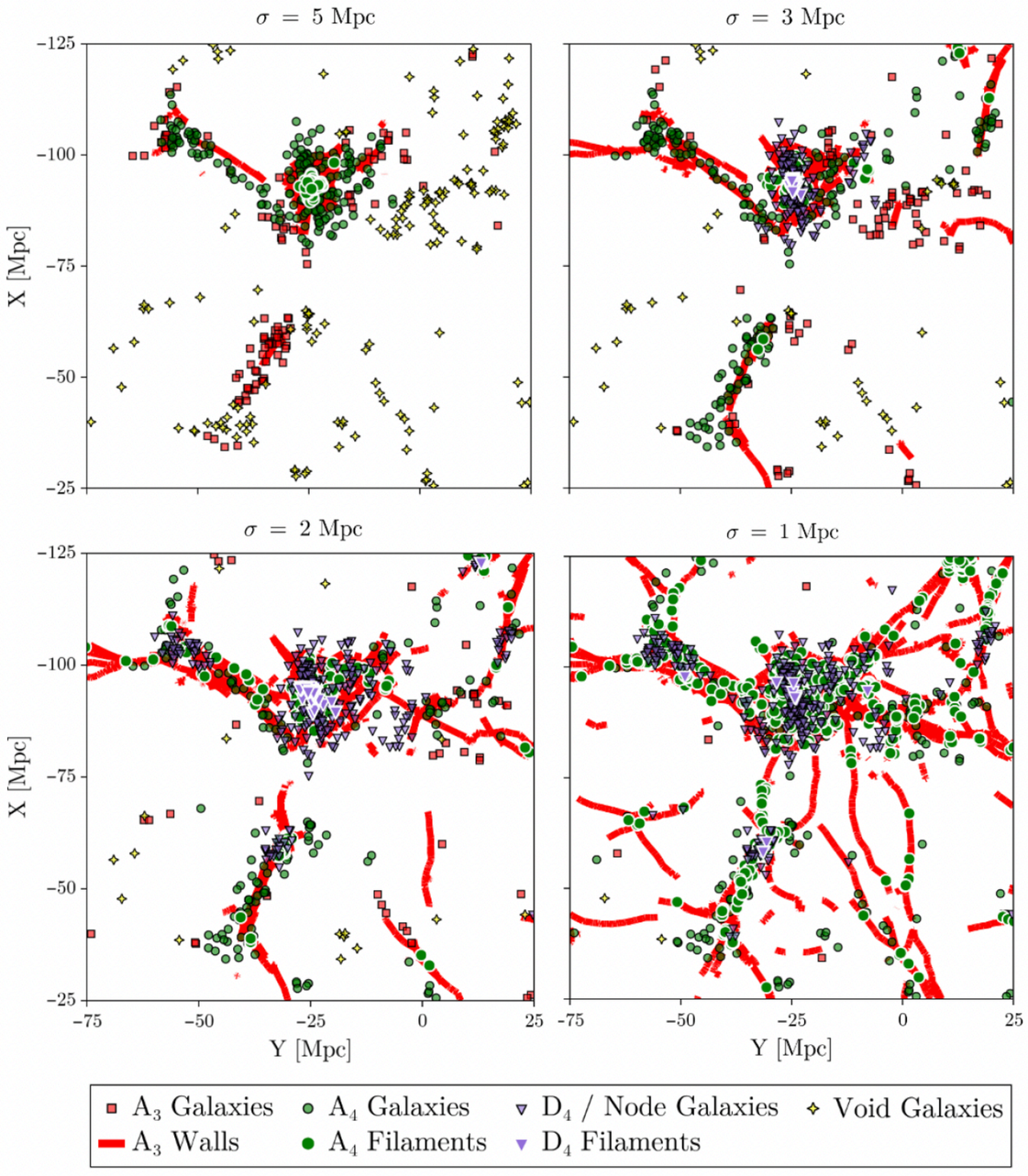
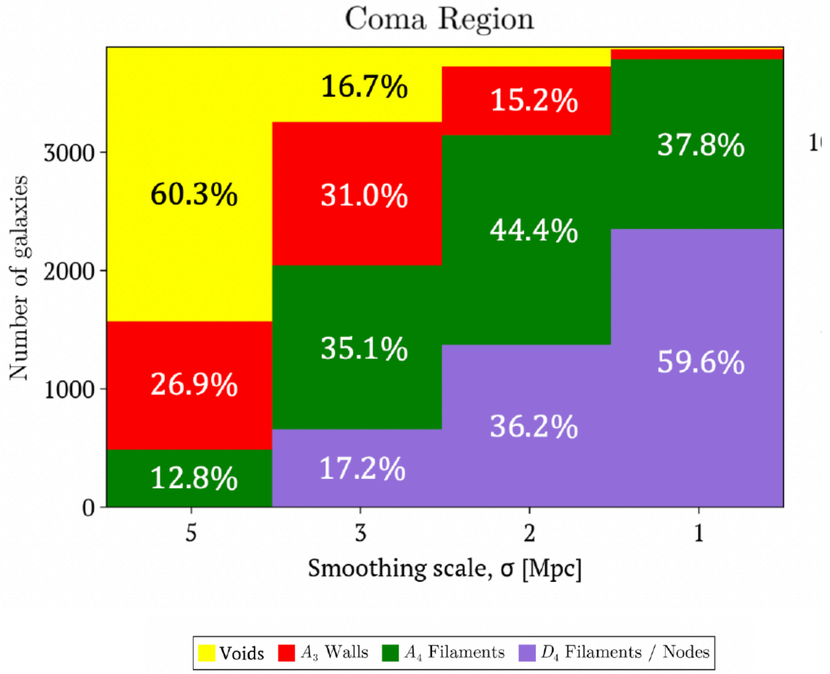
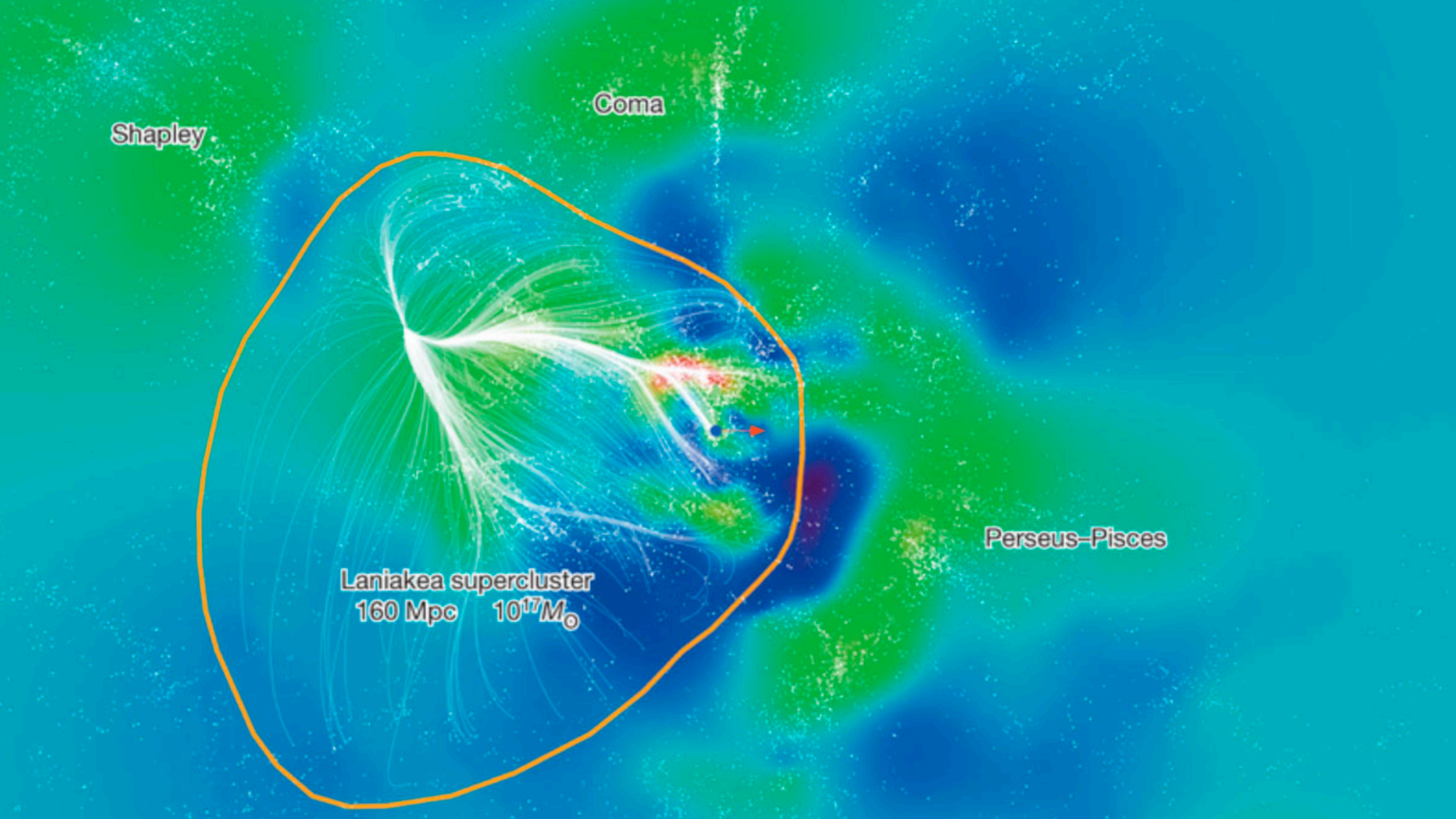
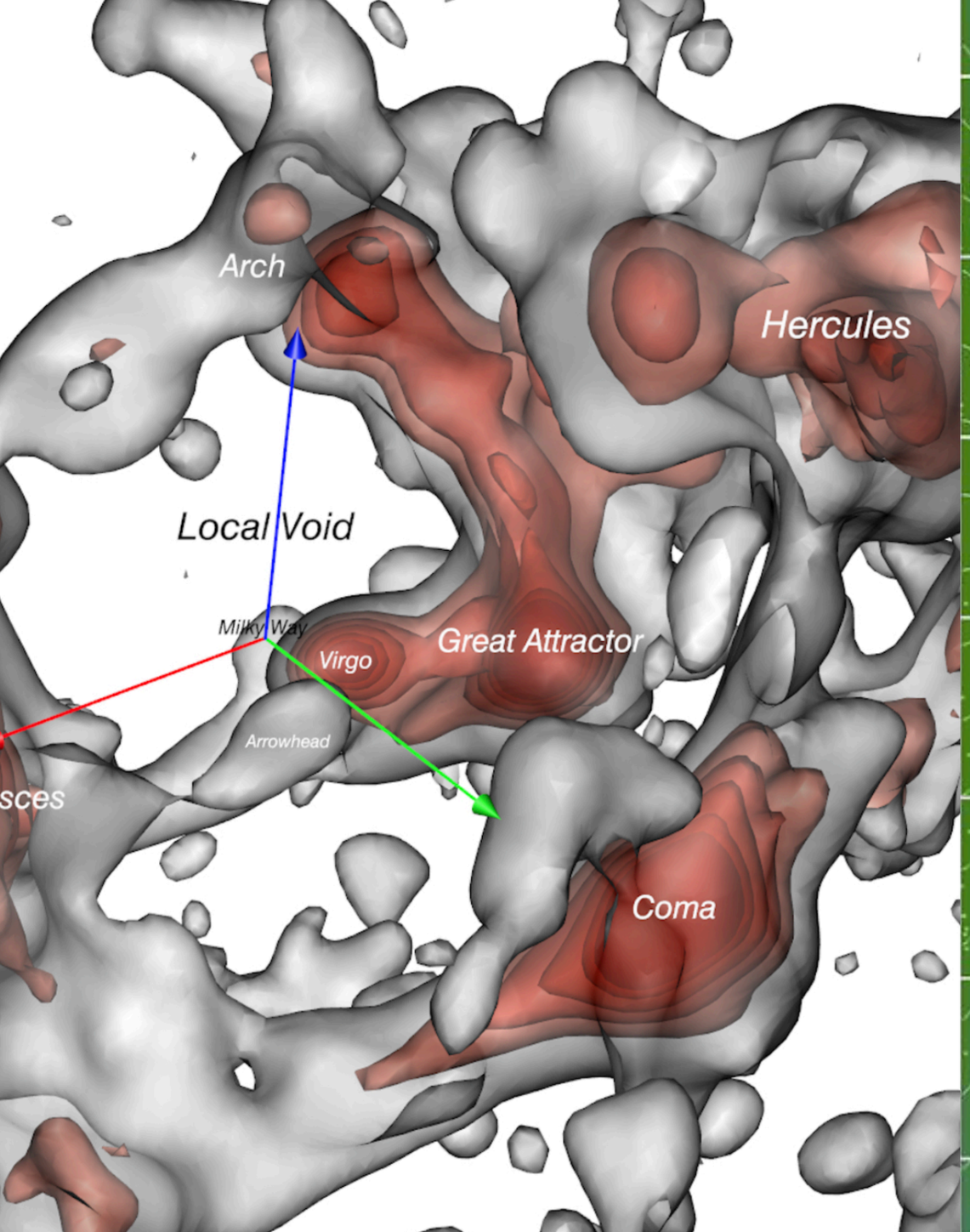


Figure 14. Classification scheme outlined in section 5 applied to 2M++ galaxies in 10 Mpc thick slice around $Z_{\rm Coma}$ (this is the same subset of galaxies shown in fig. 9) using caustic skeleton extracted from the M_1 realisation. Since the large-scale environment of a galaxy is scale dependent, the galaxy classification scheme is applied at smoothing scales $\sigma = 5, 3, 2, 1$, shown in the top left, top right, bottom left and bottom right, respectively. The large-scale environment of a galaxy at a given smoothing scale is classified as either dominated by A_3 wall caustics (red squares), A_4 filaments (green points) or D_4 filaments/Nodes (purple down triangles). In the case that no caustic structure is within 5 Mpc of a galaxy, it is classified as a void galaxy (yellow 4-pointed stars) A slice of the corresponding redshift-space caustic skeleton from simulation M_1 for each smoothing scale is also included: A_3 walls appear as red lines, and A_4 and D_4 filaments appear as green points and purple down triangles, both with a white outline. The order of layering of elements in this figure reflects the hierarchy of the classification scheme - i.e. the D_4 filaments are likely associated with cluster nodes and thereby have the densest and have the strongest sphere of influence
, followed by A_4 filaments, then A_3 walls.

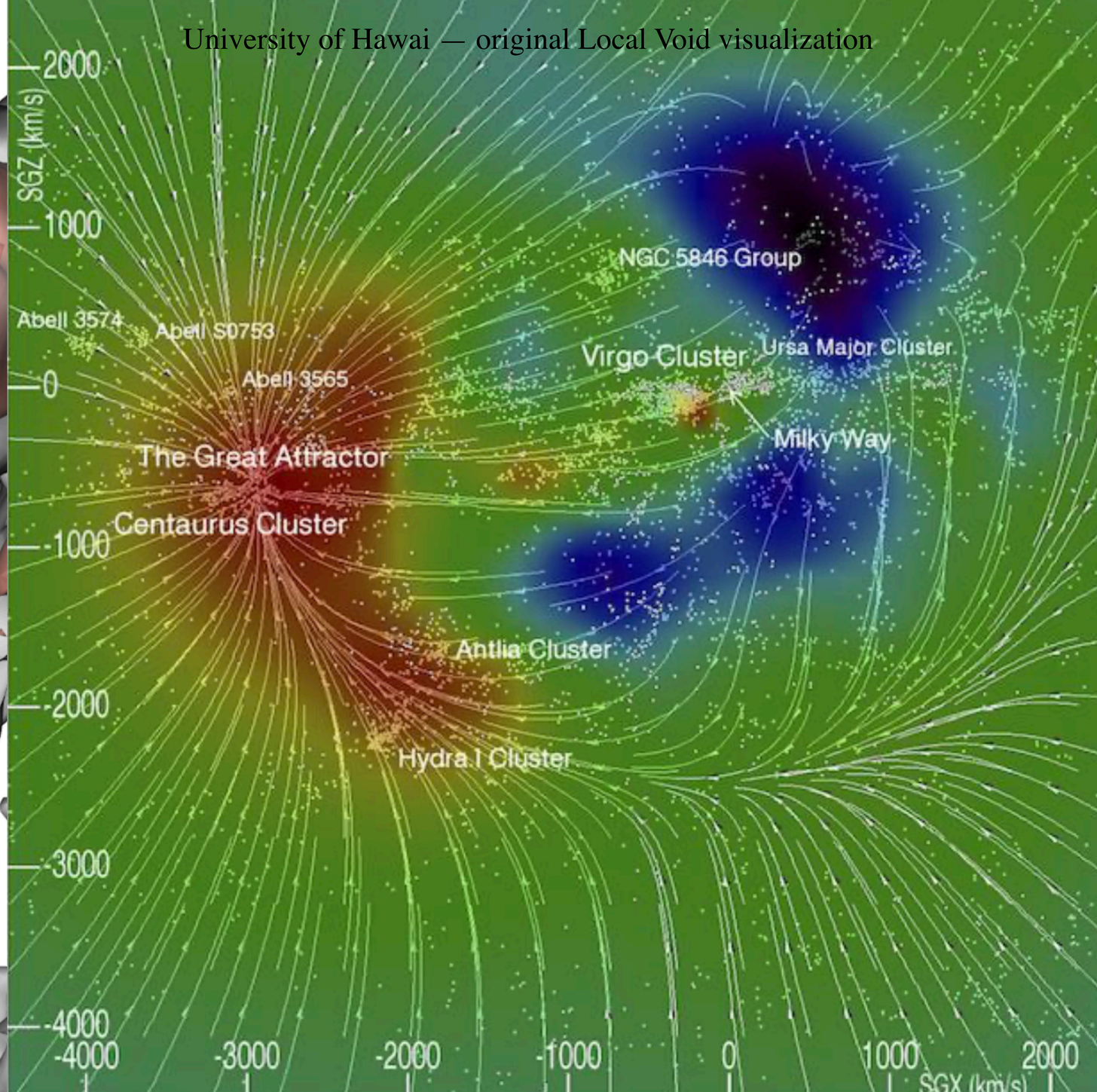


Implications for future work


Coma
Shapley
Laniakea supercluster
160 Mpc $10^{17}M_{\odot}$
Perseus–Pisces

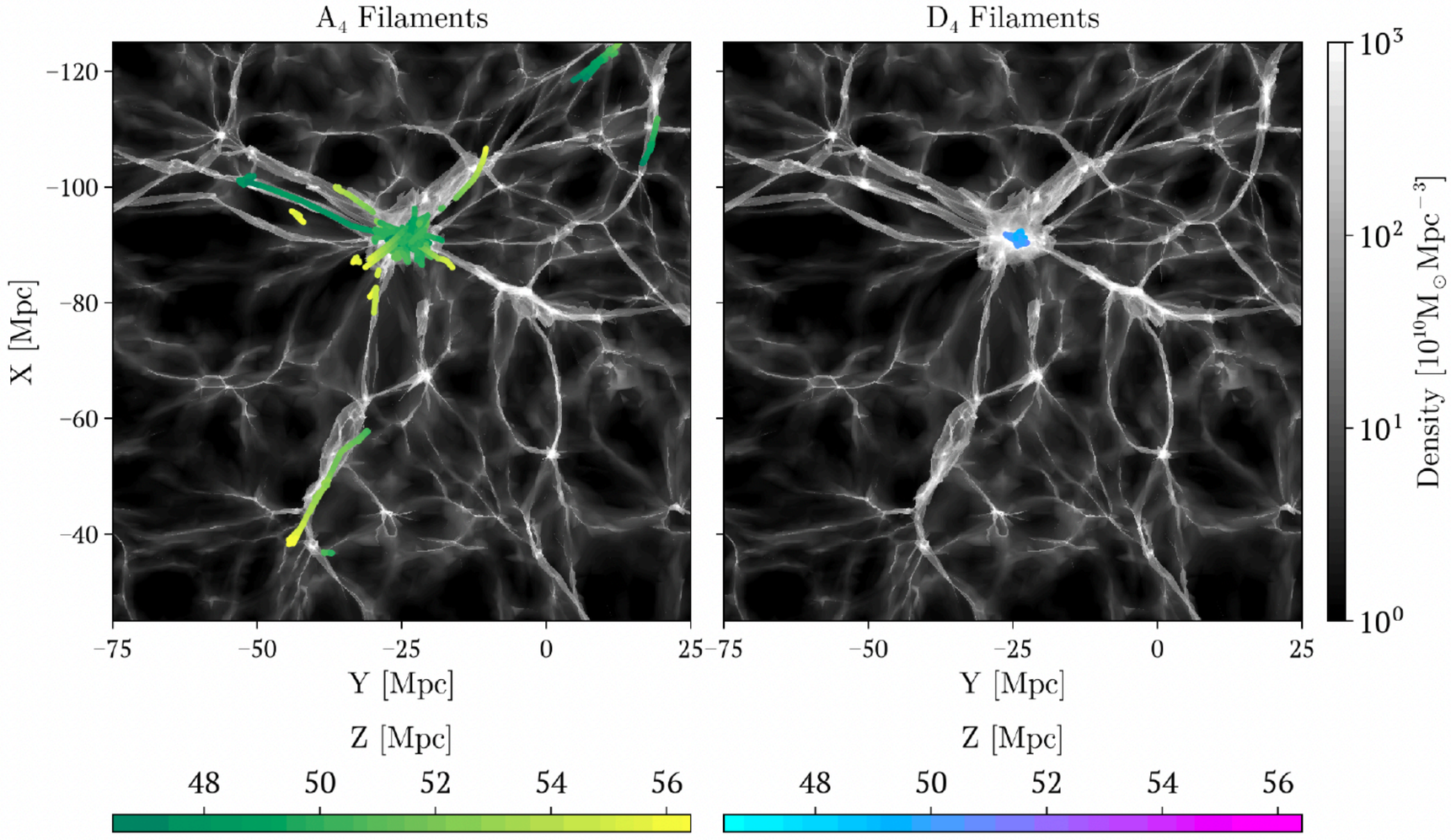


University of Hawai — original Local Void visualization



Conclusion

We got an incredible tool



Just at the beginning… stay tuned!