Charm-quark production in deep-inelastic scattering: preparing for the EIC

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Plan

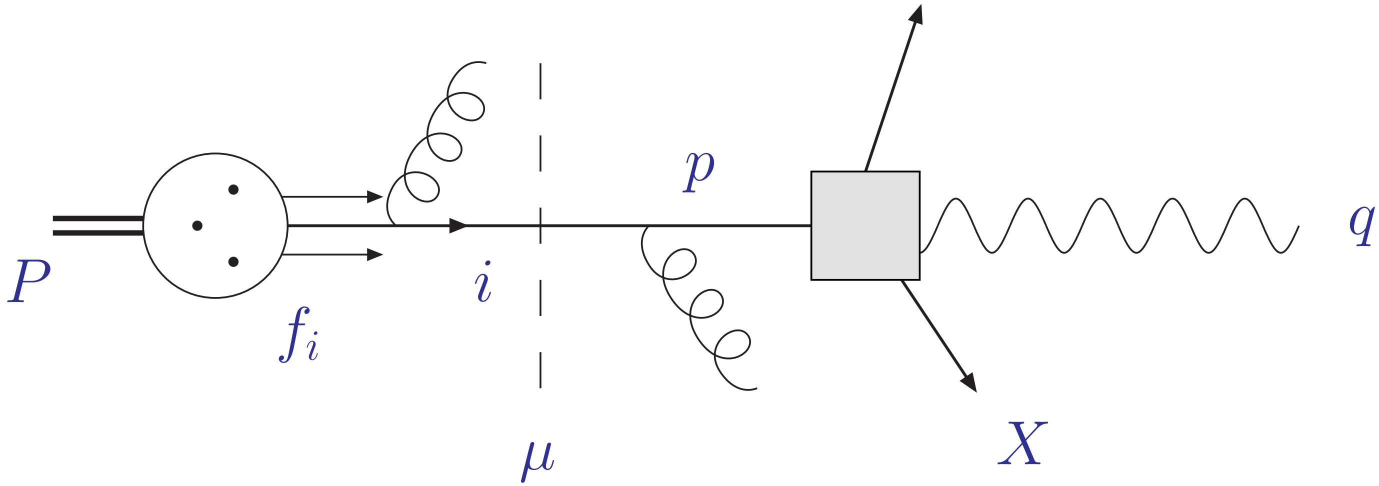
Work on heavy-quark DIS

- *Heavy-quark production in deep-inelastic scattering Mellin moments of structure functions*
  M. Klann, S. M., and K. Schönwald arXiv:2602.04455
- *Heavy-quark pair-production in DIS at NLO QCD matched to a parton shower*
  S. Castro, C. Del Pio, A. Kardos S. M., and A. Spourdalakis
  arXiv:2606.21510

Once upon a time ...

- Deep-inelastic electron-proton collisions with HERA collider
  - deep structure of proton at highest Q^2 and smallest x

QCD factorization

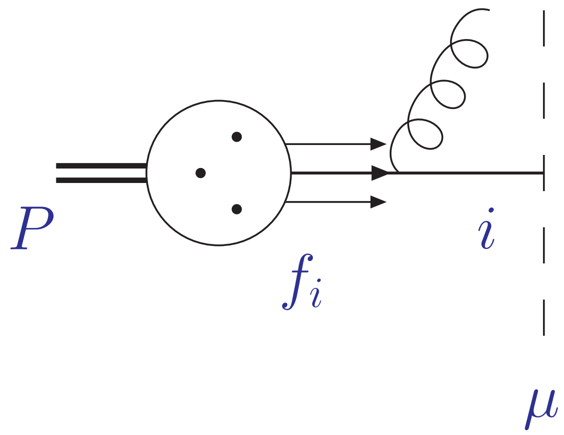


$$\sigma_{\gamma p \to X} = \sum_i f_{i/p}(\mu^2) \otimes \hat{\sigma}_{\gamma i \to X}\left(\alpha_s(\mu^2), Q^2, \mu^2\right)$$

- Factorization at scale μ
  - separation of sensitivity to dynamics from long and short distances
- Hard parton cross section $\hat{\sigma}_{\gamma i \to X}$ calculable in perturbation theory
  - cross section $\hat{\sigma}_{\gamma j \to X}$ for parton types i and hadronic final state X
- Non-perturbative parameters: parton distribution functions f_i, strong coupling α_s, particle masses m_X
  - known from global fits to exp. data, lattice computations, . . .

Parton luminosity

- Long distance dynamics due to proton structure



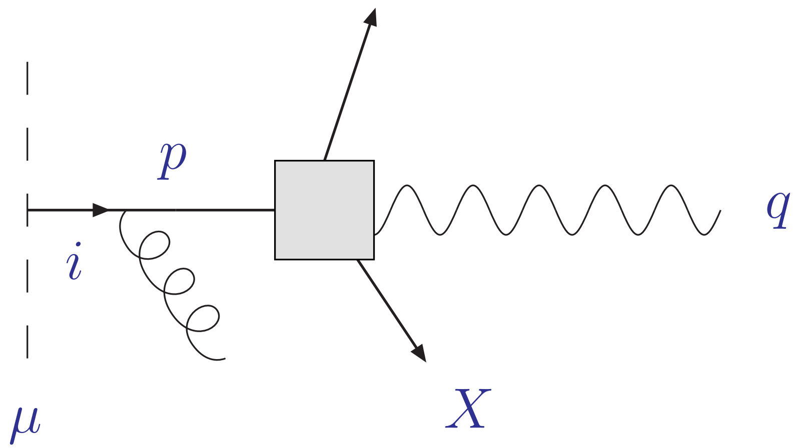
- Cross section depends on parton distributions f_i

$$\sigma_{ep\to X} = \sum_i f_i(\mu^2) \otimes \Big[\dots \Big]$$

- Parton distributions known from global fits to exp. data
  - available fits accurate to NNLO (N^3LO) ABMP; CT; MSHT; NNPDF; . . .
  - information on proton structure depends on kinematic coverage

Hard scattering cross section

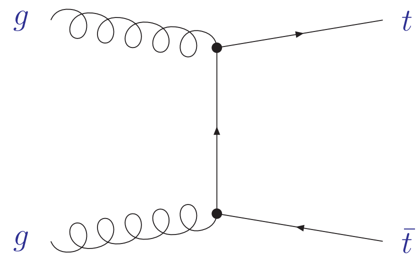
- Parton cross section $\hat{\sigma}_{\gamma i \to X}$ calculable pertubatively in powers of α_s
  - known to NLO, NNLO, . . . ($\mathcal{O}(\text{few}\%)$ theory uncertainty)



- Accuracy of perturbative predictions
  - LO (leading order) ($\mathcal{O}(50 - 100\%)$ unc.)
  - NLO (next-to-leading order) ($\mathcal{O}(10 - 30\%)$ unc.)
  - NNLO (next-to-next-to-leading order) ($\lesssim$ $\mathcal{O}(10\%)$ unc.)
  - N^3LO (next-to-next-to-next-to-leading order)
  - N^4LO (next-to-next-to-next-to-next-to-leading order)
  - . . .

Perturbative QCD corrections

State of the art



- Top-quark hadro-production
  - amplitude with scales s, t, u and quark mass m_t
- NNLO QCD differential predictions
  Czakon, Heymes, Mitov ‘15; Catani, Devoto, Grazzini, Kallweit, Mazzitelli ‘19
- NNLO event generation for top-quark pair production
  Mazzitelli, Monni, Nason, Re, Wiesemann and Zanderighi ‘20, ‘21

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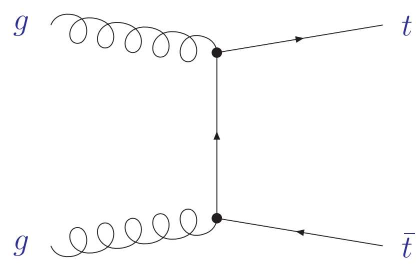

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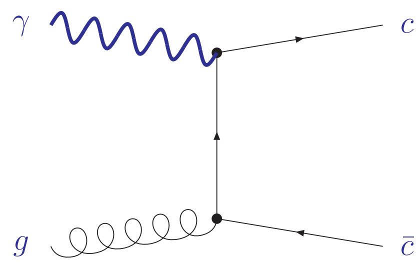


- Heavy quark DIS
  - amplitude with scales s, t, u, quark mass m_c
    and $\boxed{\textit{photon virtuality } Q^2}$
- NLO QCD differential predictions
  Riemersma, Smith, van Neerven '93; Harris, Smith '97
- limit $Q^2 \gg m_c^2$ at NNLO
  Ablinger, Behring, Blümlein, De Freitas, von Manteuffel, Schneider, Schönwald '26
- NNLO structure function approximations
  Lo Presti, Kawamura, S.M., Vogt '12; Barontini, Bonvini, Laurenti '26

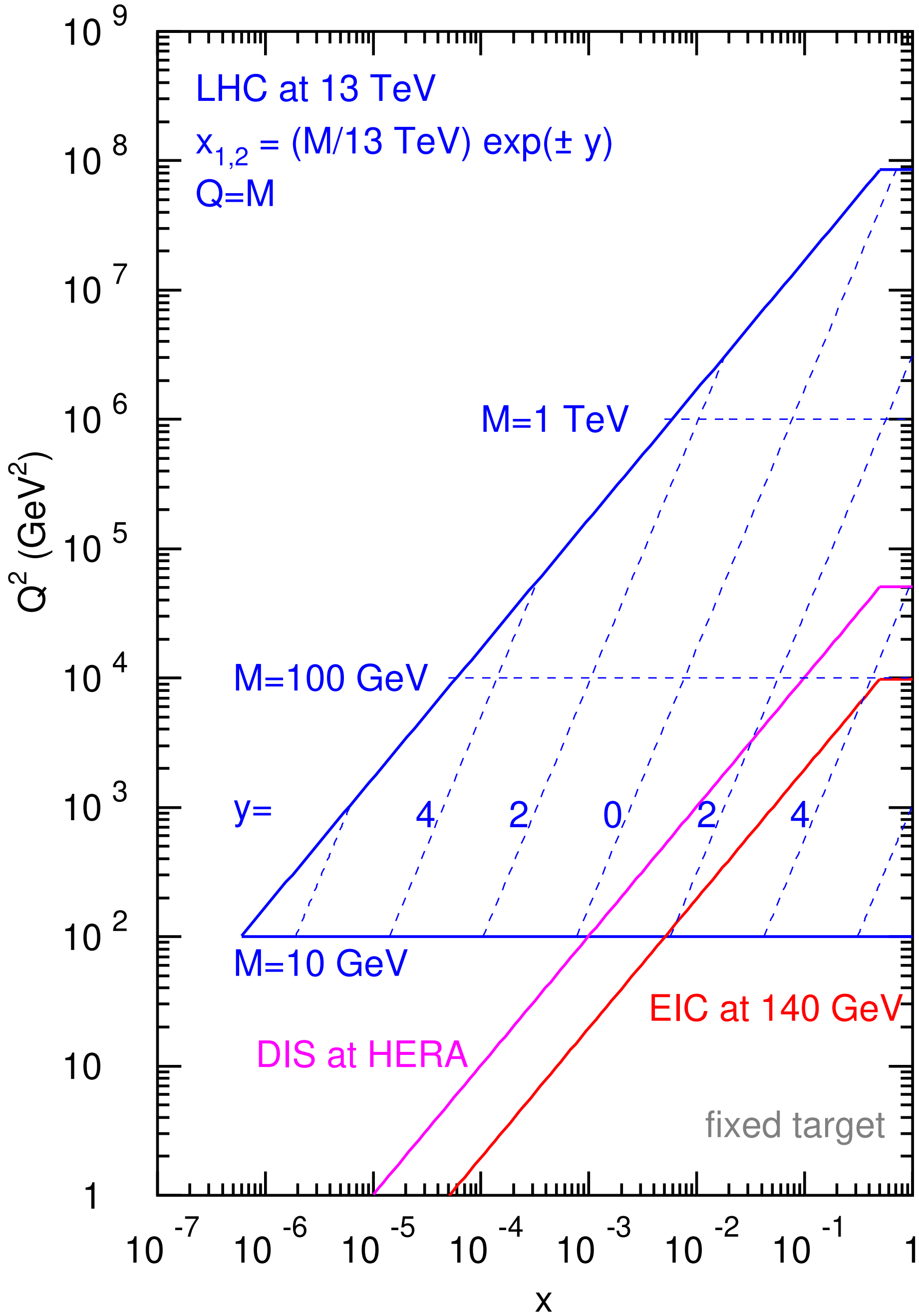
Uses of heavy quark DIS at EIC

- Direct probe of gluon PDF
  - charm and bottom production in DIS are driven mainly by gluons
  - strong constraints on the gluon PDF and heavy-flavor PDFs at medium/small x
- Precision QCD
  - heavy-quark masses provide hard scale for perturbative QCD tests
  - tests mass effects, threshold behavior, and higher-order corrections
- Unique EIC capability
  - high luminosity
  - excellent vertexing enable precise charm/bottom tagging

Upshot

- Heavy-quark DIS at EIC offers precision access to the gluon PDF and powerful test of QCD at medium/small-x

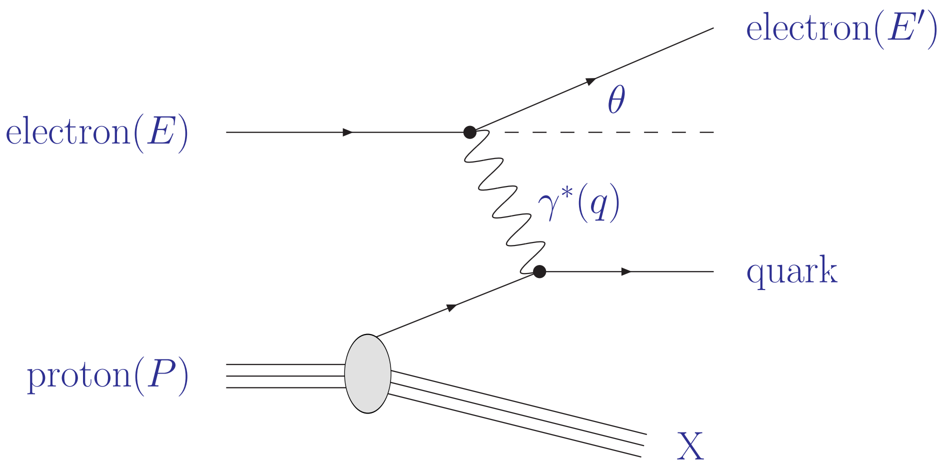
Parton kinematics: present and future



- LHC run at $\sqrt{s}$ = 13 TeV
  - parton kinematics well covered by HERA and fixed target experiments
  - momentum fractions $x_{1,2} = M/\sqrt{S}e^{\pm y}$
  - forward rapidities sensitive to small-x
  - high scales $Q \simeq 1$ TeV probe large $x \simeq 0.1 \ldots 0.8$
- EIC run at $\sqrt{s}$ = 140 GeV
  - ep-collisions at EIC cover large part of phase space relevant for LHC
  - overlap with HERA and fixed target experiments

Structure functions

Deep-inelastic scattering



Kinematic variables

- momentum transfer $Q^2 = -q^2$
- Bjorken variable $x = Q^2/(2p \cdot q)$

- $\boxed{\text{Massless}}$ DIS structure function F_2^p (up to order $\mathcal{O}(1/Q^2)$)

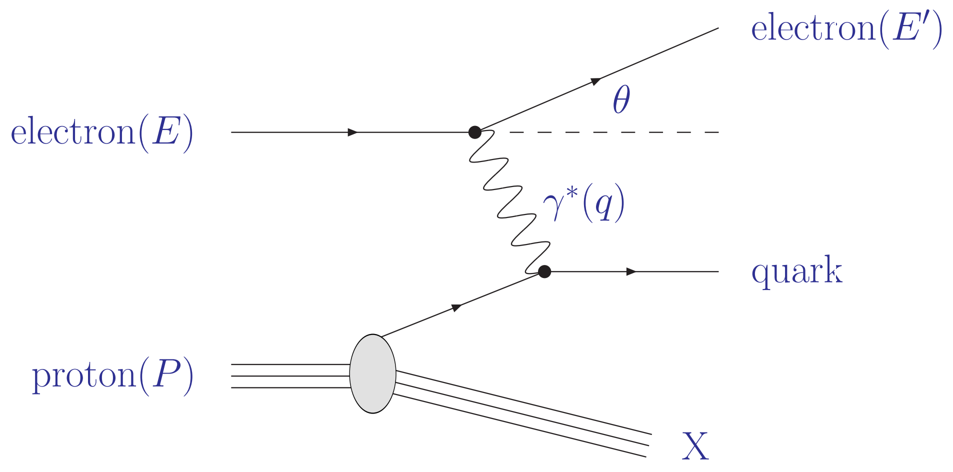
$$x^{-1} F_2^p(x, Q^2) \; = \; \sum_i \int_x^1 \frac{d\xi}{\xi} \, C_{2,i}\left(\frac{x}{\xi}, \alpha_s(\mu^2), \frac{\mu^2}{Q^2}\right) \, f_i^p(\xi, \mu^2)$$

- Massless coefficient functions

$$C_{a,i} = \alpha_s^n \left(c_{a,i}^{(0)} + \alpha_s c_{a,i}^{(1)} + \alpha_s^2 c_{a,i}^{(2)} + \alpha_s^3 c_{a,i}^{(3)} + \alpha_s^4 c_{a,i}^{(4)} + \ldots \right)$$

  - $c_{a,i}^{(3)}$ at N^{3}LO known — Vermaseren, S.M., Vogt ‘05
  - work in progress at N^{4}LO — S.M., Ruijl, Ueda, Vermaseren, Vogt ‘22

Deep-inelastic scattering



Kinematic variables

- momentum transfer $Q^2 = -q^2$
- Bjorken variable $x = Q^2/(2p \cdot q)$

- $\boxed{\text{Massive charged current}}$ DIS structure function F_2^p (up to order $\mathcal{O}(1/Q^2)$)

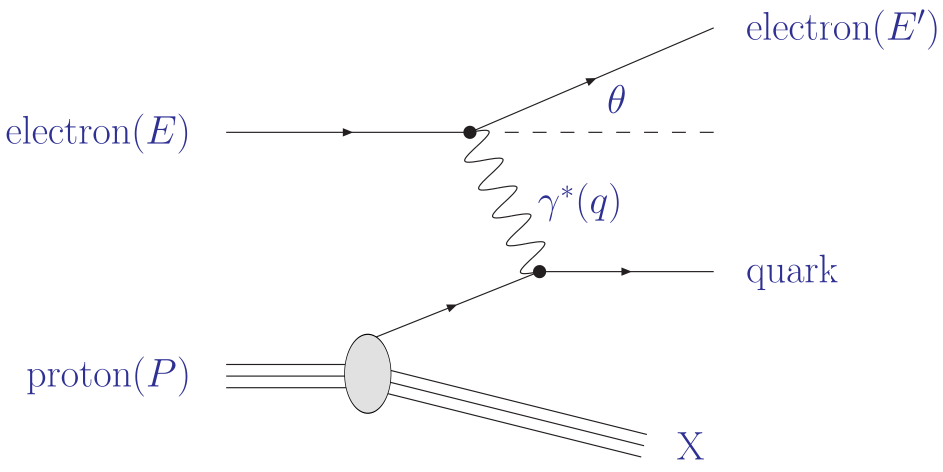
$$x^{-1} F_2^p(x, Q^2, m^2) = \sum_i \int_{ax}^1 \frac{d\xi}{\xi} C_{2,i}\left(\frac{x}{\xi}, \alpha_s(\mu^2), \frac{\mu^2}{Q^2}, \frac{m^2}{Q^2}\right) f_i^p(\xi, \mu^2)$$

- Massive coefficient functions

$$C_{a,i} = \alpha_s^n \left(c_{a,i}^{(0)} + \alpha_s c_{a,i}^{(1)} + \alpha_s^2 c_{a,i}^{(2)} + \dots\right)$$

  - $c_{a,i}^{(1)}$ at NLO known since long — Gottschalk '81; Glück, Kretzer, Reya '96
  - NNLO recently accomplished — Caola, Gambuti, Link '26; Klann, S.M., Mukherjee, Rana to appear

Deep-inelastic scattering



Kinematic variables

- momentum transfer $Q^2 = -q^2$
- Bjorken variable $x = Q^2/(2p \cdot q)$

- $\boxed{\text{Massive neutral current}}$ DIS structure function F_2^p (up to order $\mathcal{O}(1/Q^2)$)

$$x^{-1} F_2^p(x, Q^2, m^2) \;=\; \sum_i \int_{ax}^1 \frac{d\xi}{\xi}\, C_{2,i}\left(\frac{x}{\xi}, \alpha_s(\mu^2), \frac{\mu^2}{Q^2}, \frac{m^2}{Q^2}\right) f_i^p(\xi, \mu^2)$$

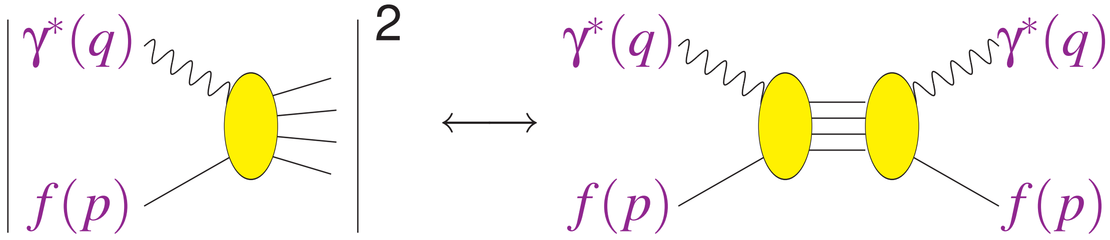
- Massive coefficient functions
  $C_{a,i} = \alpha_s^n \left(c_{a,i}^{(0)} + \alpha_s c_{a,i}^{(1)} + \alpha_s^2 c_{a,i}^{(2)} + \dots \right)$
  - $c_{a,i}^{(1)}$ at NLO known since long — Laenen, Riemersma, Smith, van Neerven ‘93
  - NNLO current frontier of research

Operator product expansion (I)

- Direct computation of physical observable
  - structures functions in deep-inelastic scattering (DIS)

Optical theorem

- Total cross section related to imaginary part of Compton amplitude
  - Bjorken variable $x = Q^2/(2p \cdot q)$ and momentum transfer $Q^2 = -q^2$



- Optical theorem relates hadronic tensor $W_{\mu\nu}$ to imaginary part of Compton amplitude $T_{\mu\nu} = \mathrm{i} \int d^4z\, \mathrm{e}^{\mathrm{i}q\cdot z} \langle \mathrm{P}|\, T\left(j_\mu^\dagger(z) j_\nu(0)\right) |\mathrm{P}\rangle$

$$W_{\mu\nu} = e_{\mu\nu} \frac{1}{2x} F_L(x, Q^2) + d_{\mu\nu} \frac{1}{2x} F_2(x, Q^2) + \mathrm{i}\epsilon_{\mu\nu\alpha\beta} \frac{p^\alpha q^\beta}{p \cdot q} F_3(x, Q^2)$$

- OPE of $T_{\mu\nu}$ for short distances $z^2 \simeq 0$ in Bjorken limit $Q^2 \to \infty$, x fixed

  Wilson '72; Christ, Hasslacher, Mueller '72

Operator product expansion (II)

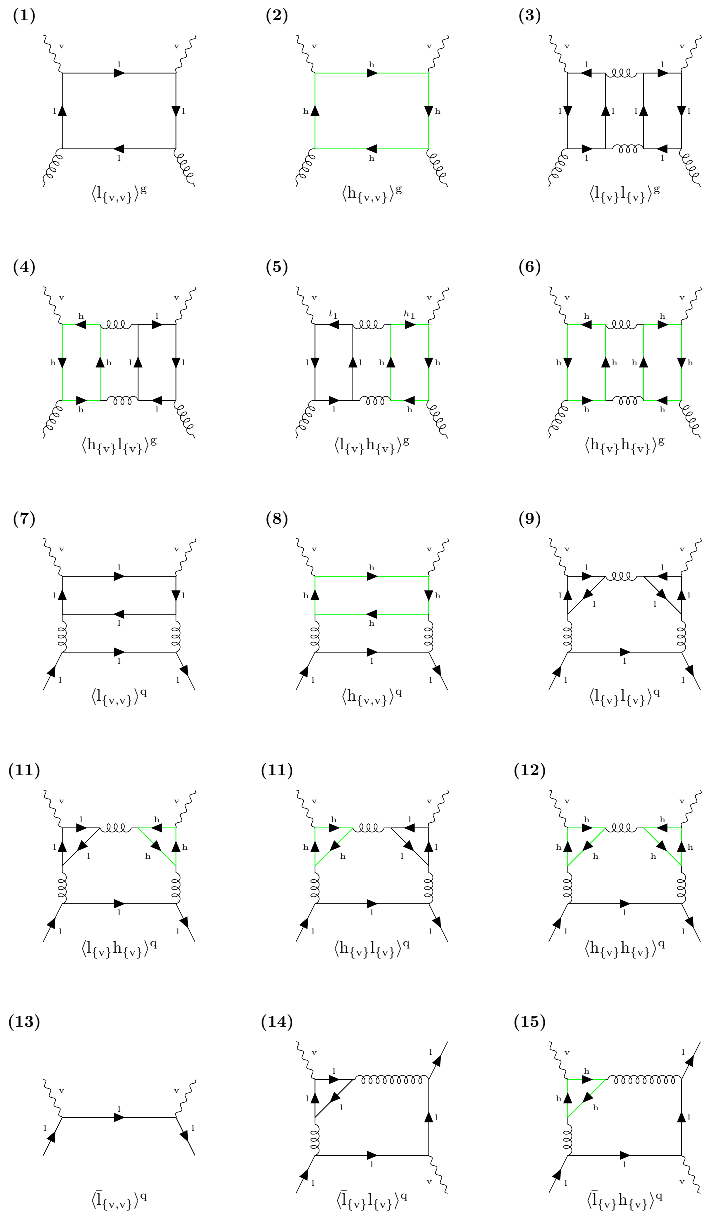
- OPE for parton states gives coefficient functions in Mellin space $C^N_{a,i}$

$$T_{\mu\nu,\mathrm{k}} \;=\; \sum_{N,\mathrm{j}} \left(\frac{1}{2x}\right)^N \Bigg[e_{\mu\nu}\, C^N_{L,\mathrm{j}} \left(\frac{Q^2}{\mu^2}, \alpha_s\right) + d_{\mu\nu}\, C^N_{2,\mathrm{j}} \left(\frac{Q^2}{\mu^2}, \alpha_s\right) + \mathrm{i}\epsilon_{\mu\nu\alpha\beta} \frac{p^\alpha q^\beta}{p\cdot q} C^N_{3,\mathrm{j}} \left(\frac{Q^2}{\mu^2}, \alpha_s\right) \Bigg] A^N_{\mathrm{jk}} \left(\mu^2\right) \;+\; \text{higher twists}$$

- Operator matrix elements $A^N_{\mathrm{ij}} = \langle j | O^N_{\mathrm{i}} | j \rangle$ in parton state
- Anomalous dimensions $\gamma_{\mathrm{ij}}(N)$ from collinear singularities of Compton amplitude $T_{\mu\nu}$ after mass factorization
  - established computational approach through four loops
    one loop Buras '80; two loops Kazakov, Kotikov '90; S.M., Vermaseren '99; three loops S.M., Vermaseren, Vogt '04; four loops Davies, Vogt, Ruijl, Ueda, Vermaseren '17; S.M., Ruijl, Ueda, Vermaseren, Vogt to appear
- Versatile calculation method
  - photon-DIS $\longrightarrow \gamma_{\mathrm{qq}}\,, \gamma_{\mathrm{qg}}$
  - Higgs (scalar)-DIS $\longrightarrow \gamma_{\mathrm{gq}}\,, \gamma_{\mathrm{gg}}$
  - graviton-DIS $\longrightarrow \Delta\gamma_{\mathrm{ij}}$ (polarized quantities) S.M., Vermaseren, Vogt '14

Operator product expansion (III)

Forward Compton amplitude with massive quarks



- flavor classes for DIS at three loops
  Klann, S.M. Schönwald '26
- forward Compton amplitude for heavy quark DIS up to three loops
- massless partons (black) massive quarks (green)

NLO coefficient functions (I)

- Mellin moment $N = 2$ for $c_{2,g}$ to NLO

  Klann, S.M. Schönwald '26
  - massless DIS (blue)
  - heavy quark DIS with complete mass dependence (black)

$$
\begin{aligned}
C_{2,\mathrm{g}}^{2} = \quad & a_{s,\mathrm{ren}}\Bigg[T_F\langle \mathrm{h}_{\{\mathrm{v,v}\}}\rangle^{\mathrm{g}} n_{\mathrm{h}}\left(\frac{4\mathrm{H}_0\mathrm{P}_{\mathrm{g},2,02}}{3(\lambda-1)(\lambda+1)^3}+\frac{\mathrm{P}_{\mathrm{g},2,01}}{(\lambda+1)^2}\right)-T_F\langle \mathrm{l}_{\{\mathrm{v,v}\}}\rangle^{\mathrm{g}} n_{\mathrm{l}}\Bigg] \\
& + a_{s,\mathrm{ren}}^2\Bigg[C_A T_F\langle \mathrm{h}_{\{\mathrm{v,v}\}}\rangle^{\mathrm{g}} n_{\mathrm{h}}\Bigg(\frac{4\mathrm{H}_{-1,0}\mathrm{P}_{\mathrm{g},2,13}}{45(\lambda-1)\lambda^3(\lambda+1)^3}-\frac{2\mathrm{H}_{1,0}\mathrm{P}_{\mathrm{g},2,14}}{45(\lambda-1)\lambda^3(\lambda+1)^3} \\
& +\frac{\mathrm{H}_0\mathrm{P}_{\mathrm{g},2,11}}{135(\lambda-1)\lambda^2(\lambda+1)^3}-\frac{88\mathrm{H}_{0,1}\mathrm{P}_{\mathrm{g},2,02}}{9(\lambda-1)(\lambda+1)^3}+\frac{22\mathrm{H}_1\mathrm{P}_{\mathrm{g},2,08}}{3(\lambda+1)^2}+\frac{16\mathrm{H}_{-1,0,0}\mathrm{P}_{\mathrm{g},2,09}}{15(\lambda-1)^2} \\
& -\frac{16\mathrm{H}_{0,-1,0}\mathrm{P}_{\mathrm{g},2,09}}{15(\lambda-1)^2}+\frac{8\mathrm{H}_{0,1,0}\mathrm{P}_{\mathrm{g},2,09}}{15(\lambda-1)^2}-\frac{8\mathrm{H}_{1,0,0}\mathrm{P}_{\mathrm{g},2,09}}{15(\lambda-1)^2}-\frac{2\mathrm{H}_{0,0}\mathrm{P}_{\mathrm{g},2,12}}{45(\lambda-1)^2(\lambda+1)^4} \\
& +\frac{\mathrm{P}_{\mathrm{g},2,10}}{90\lambda^2(\lambda+1)^2}-\frac{4\zeta_3\mathrm{P}_{\mathrm{g},2,09}}{5(\lambda-1)^2}\Bigg)+C_F T_F\langle \mathrm{h}_{\{\mathrm{v,v}\}}\rangle^{\mathrm{g}} n_{\mathrm{h}}\Bigg(\frac{4\mathrm{H}_{-1,0}\mathrm{P}_{\mathrm{g},2,06}}{45(\lambda-1)\lambda^3(\lambda+1)} \\
& -\frac{2\mathrm{H}_{1,0}\mathrm{P}_{\mathrm{g},2,06}}{45(\lambda-1)\lambda^3(\lambda+1)}+\frac{\mathrm{H}_0\mathrm{P}_{\mathrm{g},2,05}}{45(\lambda-1)\lambda^2(\lambda+1)^3}-\frac{64\mathrm{H}_{-1,0,0}\mathrm{P}_{\mathrm{g},2,03}}{15(\lambda-1)^2}+\frac{64\mathrm{H}_{0,-1,0}\mathrm{P}_{\mathrm{g},2,03}}{15(\lambda-1)^2} \\
& -\frac{32\mathrm{H}_{0,1,0}\mathrm{P}_{\mathrm{g},2,03}}{15(\lambda-1)^2}+\frac{32\mathrm{H}_{1,0,0}\mathrm{P}_{\mathrm{g},2,03}}{15(\lambda-1)^2}-\frac{2\mathrm{H}_{0,0}\mathrm{P}_{\mathrm{g},2,07}}{45(\lambda-1)^2(\lambda+1)^4}+\frac{\mathrm{P}_{\mathrm{g},2,04}}{90\lambda^2(\lambda+1)^2} \\
& +\frac{16\zeta_3\mathrm{P}_{\mathrm{g},2,03}}{5(\lambda-1)^2}\Bigg)+\left(\frac{115}{162}-4\zeta_3\right)C_A T_F\langle \mathrm{l}_{\{\mathrm{v,v}\}}\rangle^{\mathrm{g}} n_{\mathrm{l}}+\left(\frac{32\zeta_3}{5}-\frac{4799}{405}\right)C_F T_F\langle \mathrm{l}_{\{\mathrm{v,v}\}}\rangle^{\mathrm{g}} n_{\mathrm{l}}\Bigg]
\end{aligned}
$$

NLO coefficient functions (II)

- Numerical comparisions of Mellin moments for $c_{2,g}$ at NLO
  - exact results KMS — Klann, S.M. Schönwald '26
  - numerical grid RSvN — Riemersma, Smith, van Neerven '93
  - numerical grid LeProHQ — Hekhorn '19
- Numerical grids with precision of few per mill

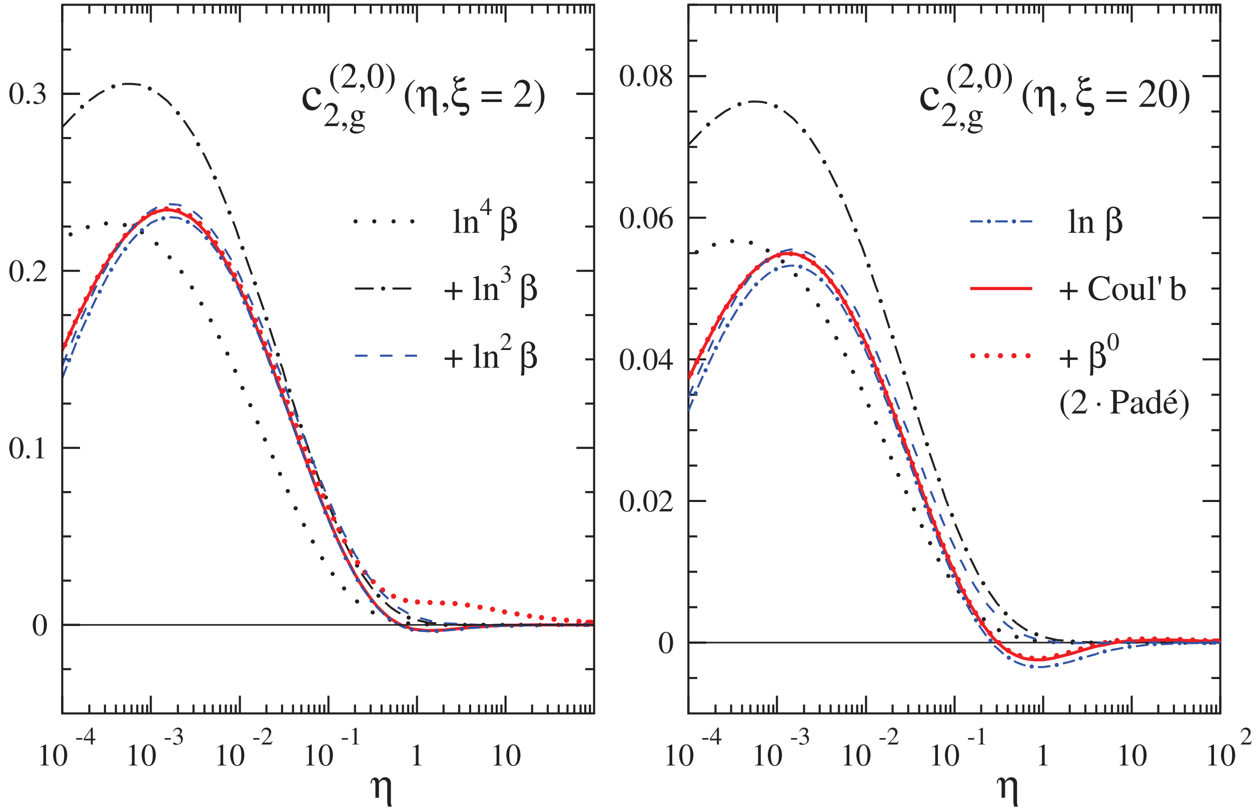
N	KMS	RSvN	LeProHQ	KMS/RSvN	KMS/LeProHQ
2	$1.606527957e+01$	$1.6121e+01$	$1.6047e+01$	0.9966	1.0011
4	$3.680483878e+00$	$3.6872e+00$	$3.6743e+00$	0.9982	1.0017
6	$9.457069517e-01$	$9.4677e-01$	$9.4356e-01$	0.9989	1.0023
8	$2.623839998e-01$	$2.6259e-01$	$2.6164e-01$	0.9992	1.0029
10	$7.663126763e-02$	$7.6681e-02$	$7.6370e-02$	0.9993	1.0034
12	$2.319720217e-02$	$2.3211e-02$	$2.3106e-02$	0.9994	1.0039
14	$7.208613114e-03$	$7.2126e-03$	$7.1769e-03$	0.9994	1.0044
16	$2.285220881e-03$	$2.2865e-03$	$2.2742e-03$	0.9995	1.0049
18	$7.358664768e-04$	$7.3627e-04$	$7.3200e-04$	0.9995	1.0053
20	$2.399582635e-04$	$2.4009e-04$	$2.3860e-04$	0.9995	1.0057
22	$7.906076590e-05$	$7.9104e-05$	$7.8587e-05$	0.9994	1.0060

Strategies beyond NLO

Approximate NNLO coefficient functions

- Relevant kinematics:
  - high-energy limit $s \gg m^2$ with BFKL logarithms $\ln(s/m^2)$
    Catani, Ciafaloni, Hautmann '91
  - partonic threshold $s \simeq 4m^2$ with Sudakov logarithms $\ln \beta$
    (velocity of heavy quark $\beta = \sqrt{1 - 4m^2/s}$)
    Laenen, S.M. '98; Alekhin, S.M. '08; Lo Presti, Kawamura, S.M., Vogt '10
  - large $Q^2 \gg m^2$ Bierenbaum, Blümlein, Klein '09;
    Ablinger, Behring, Blümlein, De Freitas, von Manteuffel, Schneider, Schönwald '26
- Approximate expressions at NNLO are constructed by merging all known limits
  Lo Presti, Kawamura, S.M., Vogt '12
  Barontini, Bonvini, Laurenti '26
  Bruschke, Klann, S.M. Schönwald to appear

Approximate coefficient functions at NNLO



- Distance from threshold $\eta = s/(4m^2) - 1$
  - Sudakov logarithms dominant for $\eta \ll 1$ known to NNLL

Lo Presti, Kawamura, S.M., Vogt '10

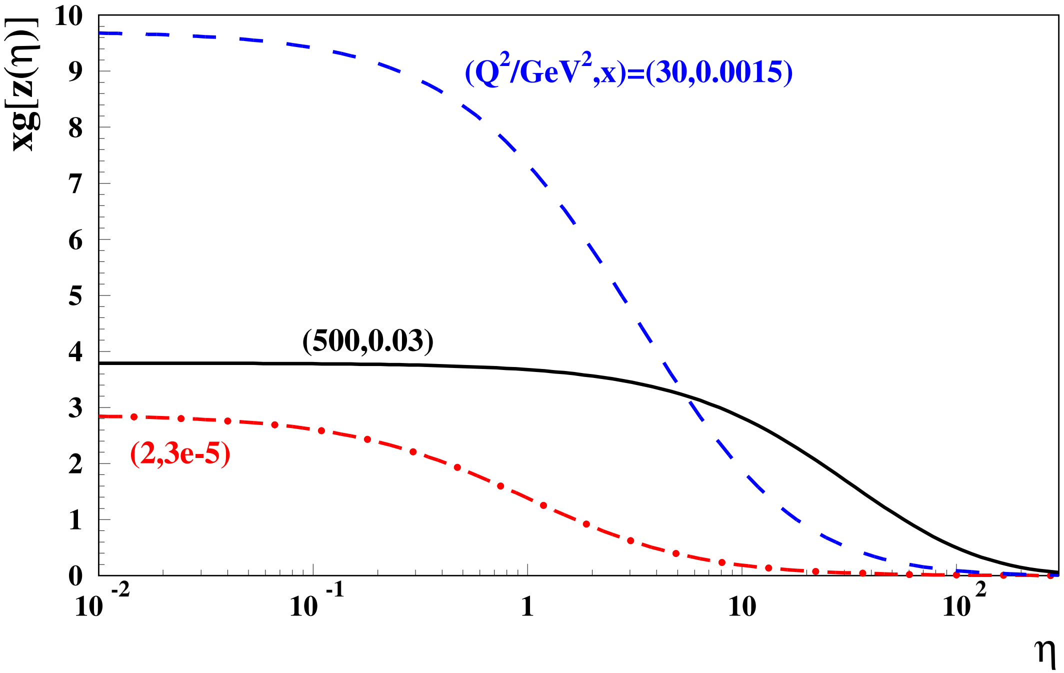
- Small impact of subleading terms $\mathcal{O}(\beta^0)$ (Padè estimate)

Convolution with gluon PDF

- Recall QCD factorization

$$F_2^c(x, Q^2, m^2) \propto \sum_{i=q,\bar{q},g} \int_0^{\eta_{max}} d\eta \; f_i\left(z(\eta), \mu^2\right) \; c_{i,k}\left(\eta, \xi, \mu^2\right)$$

  - gluon PDF gives large weight to parton dynamics near threshold for $Q^2 \lesssim 10 - 30$ GeV^2



Asymptotics beyond NLO

- Exact factorization at large $Q^2 \gg m^2$ ($\xi \to \infty$)

  Buza, Matiounine, Smith, Migneron, van Neerven '95

  - decompose heavy-quark $c_{k,i}$ into light-quark $C_{k,j}^{\mathrm{light}}$ and operator matrix elements (OMEs) A_{ji}

$$c_{k,i}(\eta, \xi, \mu^2) \to \left[A_{ji}\left(\frac{m^2}{\mu^2}\right) \otimes C_{k,j}^{\mathrm{light}}\left(\frac{Q^2}{\mu^2}\right) \right](x)$$

- NNLO corrections require three-loop results
  - massless coefficient functions Vermaseren, S.M., Vogt '05
  - heavy-quark OMEs; currently fixed Mellin moments available and complete asymptotics for pure-singlet part Bierenbaum, Blümlein, Klein '09; Ablinger, Behring, Blümlein, De Freitas, von Manteuffel, Schneider '14; Ablinger, Behring, Blümlein, De Freitas, von Manteuffel, Schneider, Schönwald '26

High-energy limit beyond NLO

- Well-known k_t-factorization at large η (small x)

  Catani, Ciafaloni, Hautmann '91

  - decompose heavy-quark structure function $F_2^c(x, Q^2, m^2)$ into gluon PDF un-integrated in k_t and heavy-quark impact factor
- Schematic expansion up to NNLO

$$\begin{aligned} c_{k,i}^{(1)}(\eta, \xi, \mu^2) &\approx c_{\mathrm{LL}_x}^{(1)} \mathrm{const}_\eta \\ c_{k,i}^{(2)}(\eta, \xi, \mu^2) &\approx c_{\mathrm{LL}_x}^{(2)} \ln \eta + c_{\mathrm{NLL}_x}^{(2)} \mathrm{const}_\eta \end{aligned}$$

- Small-x dependence governed by BFKL anomalous dimension
  - prediction of $c_{\mathrm{LL}_x}^{(l)}$
- Result valid for all ξ;
  - for $c_{\mathrm{LL}_x}^{(2)}$ known exactly — Catani, Ciafaloni, Hautmann '91
  - for $c_{\mathrm{NLL}_x}^{(2)}$ at large ξ from asymptotic factorization

High-energy limit

- NNLO coefficients $c^{(2)}_{\mathrm{NLL}_x}\mathrm{const}_\eta$ of large η (small x) expansion at large ξ from OMEs A_{ji}

$$a_{Qg}^{(3)}|_{\mathrm{NLL}_x} \longrightarrow \lim_{\xi\to\infty}\, c_{2,g}^{(2)}|_{\mathrm{NLL}_x}\mathrm{const}_\eta$$

$$a_{Qq}^{(3)\,\mathrm{ps}}|_{\mathrm{NLL}_x} \longrightarrow \lim_{\xi\to\infty}\, c_{2,q}^{(2)\,\mathrm{ps}}|_{\mathrm{NLL}_x}\mathrm{const}_\eta$$

High-energy limit

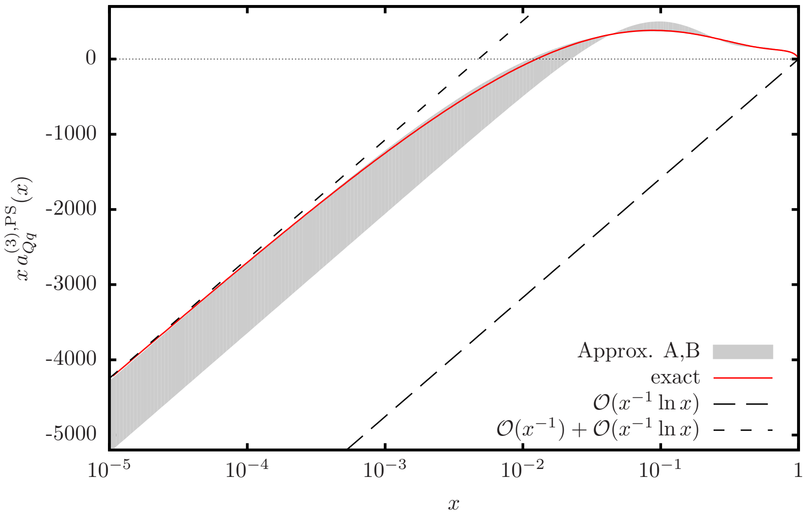
- NNLO coefficients $c^{(2)}_{\mathrm{NLL}_x}\,\mathrm{const}_\eta$ of large η (small x) expansion at large ξ from OMEs A_{ji}

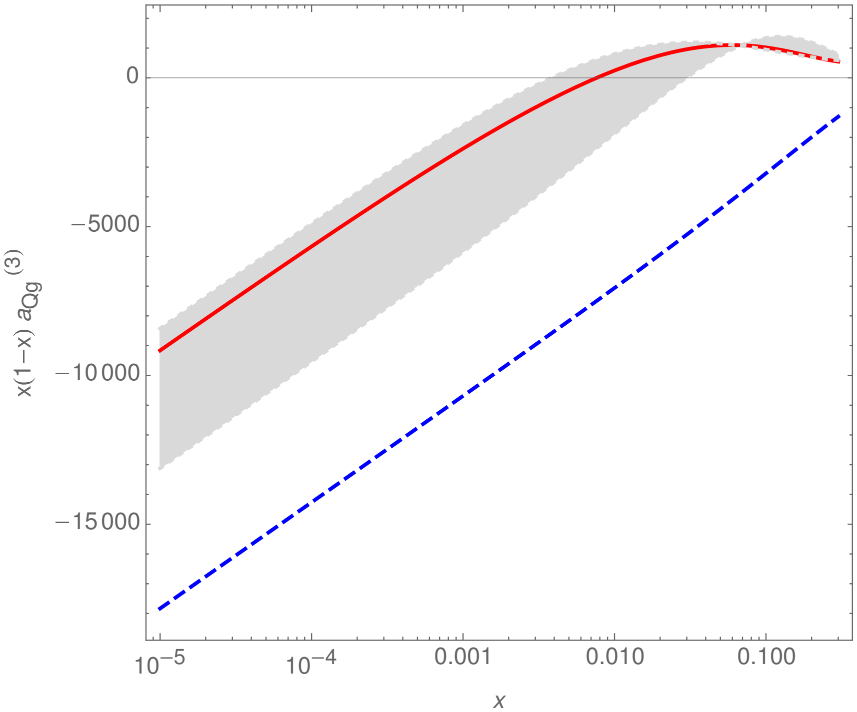
$$a^{(3)}_{Qg}|_{\mathrm{NLL}_x} \longrightarrow \lim_{\xi\to\infty} c^{(2)}_{2,g}|_{\mathrm{NLL}_x}\mathrm{const}_\eta$$

$$a^{(3)\,\mathrm{ps}}_{Qq}|_{\mathrm{NLL}_x} \longrightarrow \lim_{\xi\to\infty} c^{(2)\,\mathrm{ps}}_{2,q}|_{\mathrm{NLL}_x}\mathrm{const}_\eta$$

- Exact result for $a^{(3)}_{Qg}|_{\mathrm{NLL}_x}$ and $a^{(3)\,\mathrm{ps}}_{Qq}|_{\mathrm{NLL}_x}$ known and comaptible with approximation from fixed Mellin moments

Ablinger, Behring, Blümlein, De Freitas, von Manteuffel, Schneider '14;
Ablinger, Behring, Blümlein, De Freitas, von Manteuffel, Schneider, Schönwald '26



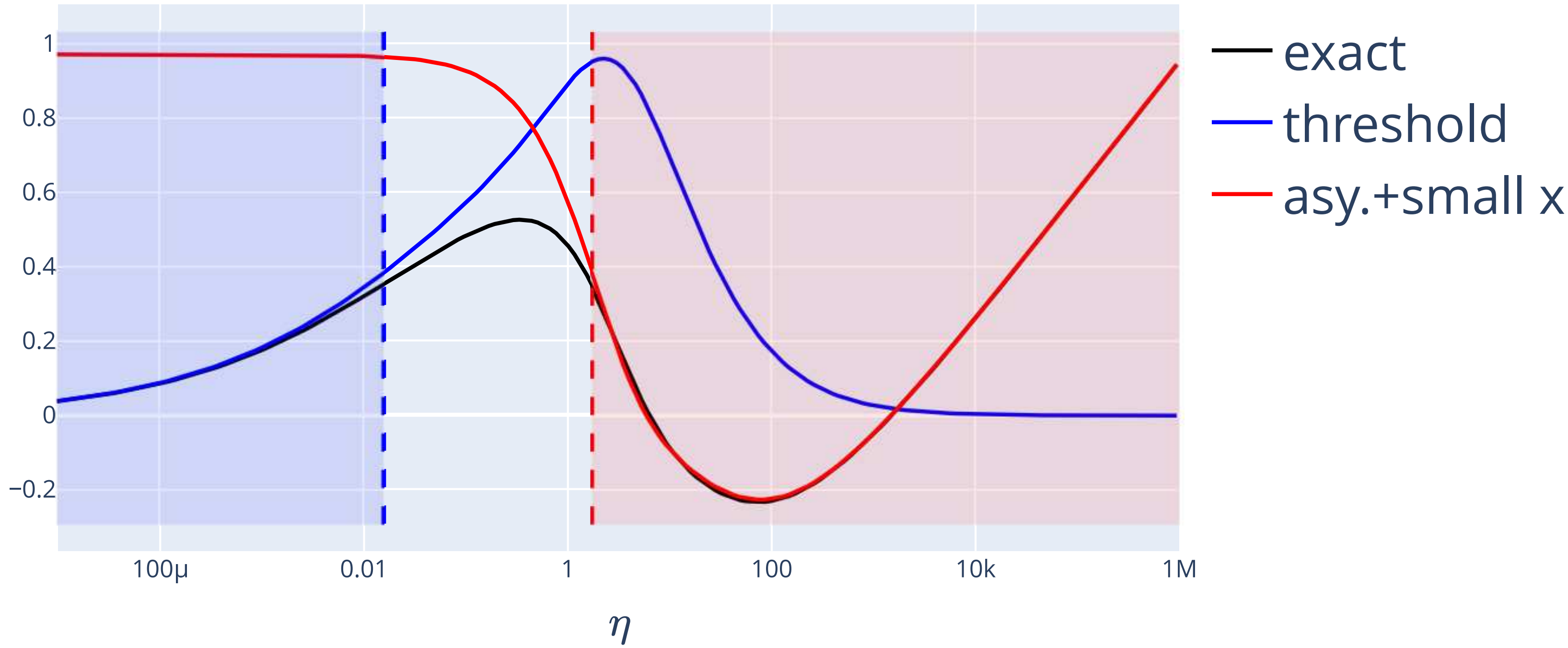


Interpolation

Bruschke '26

$$c_{2,g} \simeq \begin{cases} c_{2,g}^{\mathrm{thr}} \pm \text{uncertainty} & \text{if } \eta \leqslant \eta_1 \\ \text{spline+Gaussian process regression} & \text{if } \eta_1 < \eta < \eta_2 \\ c_{2,g}^{\mathrm{asy+small}\,x} \pm \text{uncertainty} & \text{if } \eta \geqslant \eta_2 \end{cases}$$

$c_{2,g}^{(2,2)}$, $\xi = 20$

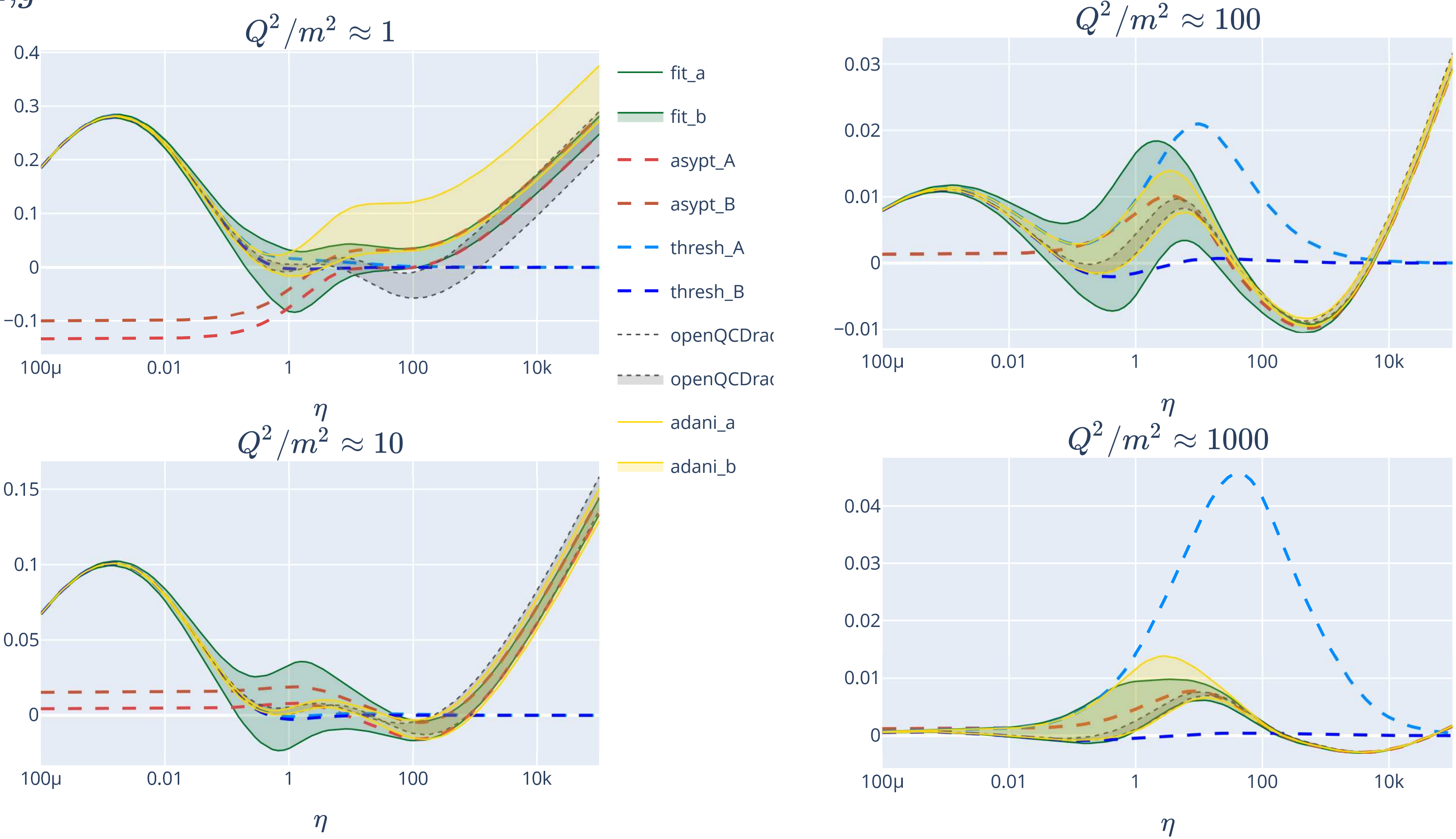


NNLO coefficient functions

Barontini, Bonvini, Laurenti '26

Bruschke, Klann, S.M. Schönwald to appear

$c_{2,g}^{c\bar{c},(2,0)}$



Outlook

Mellin moments at three loops

- Mellin moment $N = 2$ for $c_{2,g}$ to NNLO   Klann, S.M. Schönwald to appear
- Feynman diagram computation
  - master integrals: 2216
  - full Mellin moment: 142 iterated integrals
  - large cancellations are typical in amplitude calculations
- Elliptic integrals
  - despite elliptic master integrals, final Mellin moment is much simpler
  - many elliptic structures cancel in full physical combination
- Numerical evaluation of moment is difficult

Computer algebra

- Higher moments still too expensive at this order
- Heavy use of computer algebra for integral reductions   Kira (v3); FORM (v5)

Going differential

Parton shower for deep inelastic scattering (I)

NLO QCD matched to parton showers

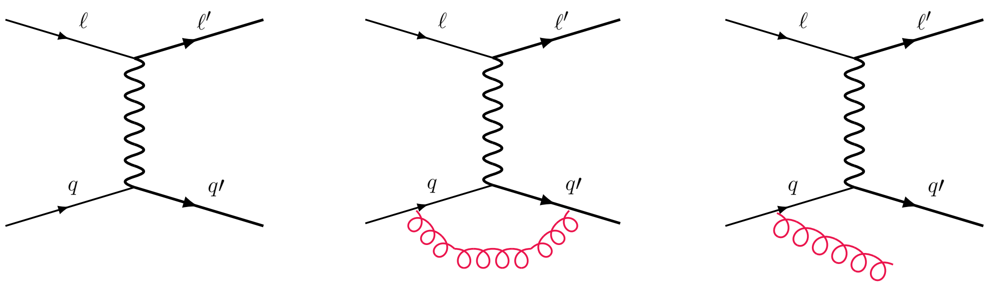
- `POWHEG` event generator for DIS
  - originally developed for hadron-hadron collisions
  - required substantial extensions for lepton-hadron kinematics
- Main technical innovation: new momentum mappings for DIS kinematics
  - FKS subtraction in `POWHEG-BOX` Frixione, Kunszt, Signer '95
- Massless DIS for both neutral-current and charged-current processes

  Banfi, Ferrario Ravasio, Jäger, Karlberg, Reichenbach, Zanderighi '23
- Heavy-quark charged-current DIS

  Buonocore, Limatola, Nason, Tramontano '24

Limitation

- Born level $2 \to 1$ kinemtics for all QCD processes



Parton shower for deep inelastic scattering (II)

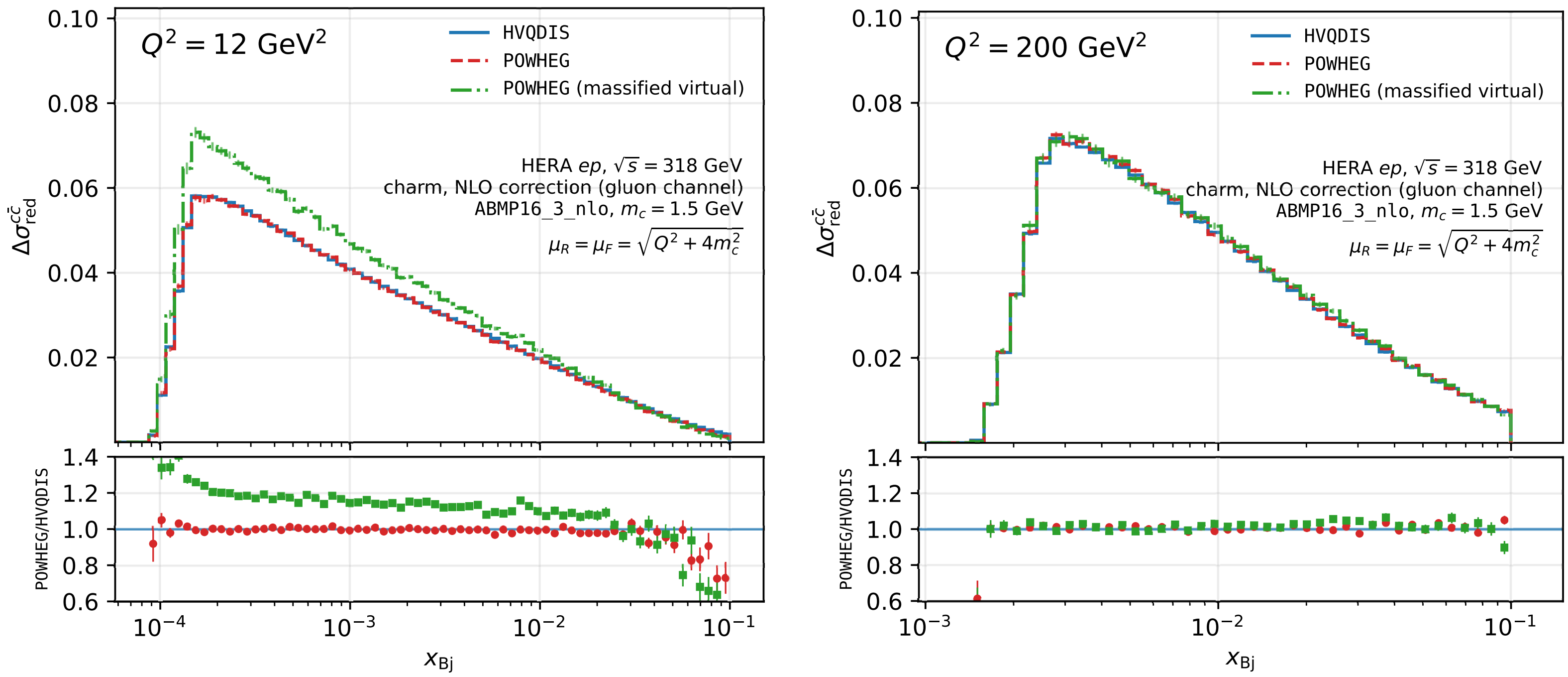
- POWHEG implementation for heavy-quark neutral-current DIS
  Castro, Del Pio, Kardos, S.M., Spourdalakis '26

Main technical challenges

- QCD process with $2 \to 2$ kinematics at Born level
- Reviving legacy code from HERA for validation
  - HVQDIS Harris, Smith '97 for fixed order fully differential NLO QCD corrections
- Checks of NLO virtual amplitudes
  - azimuthal average of massive amplitudes with GoSam
    Cullen et al. '14
  - start from massless amplitudes and use massification procedure for mapping collinear poles $1/\epsilon$ to $\ln m_h$ Mitov, S.M. '06

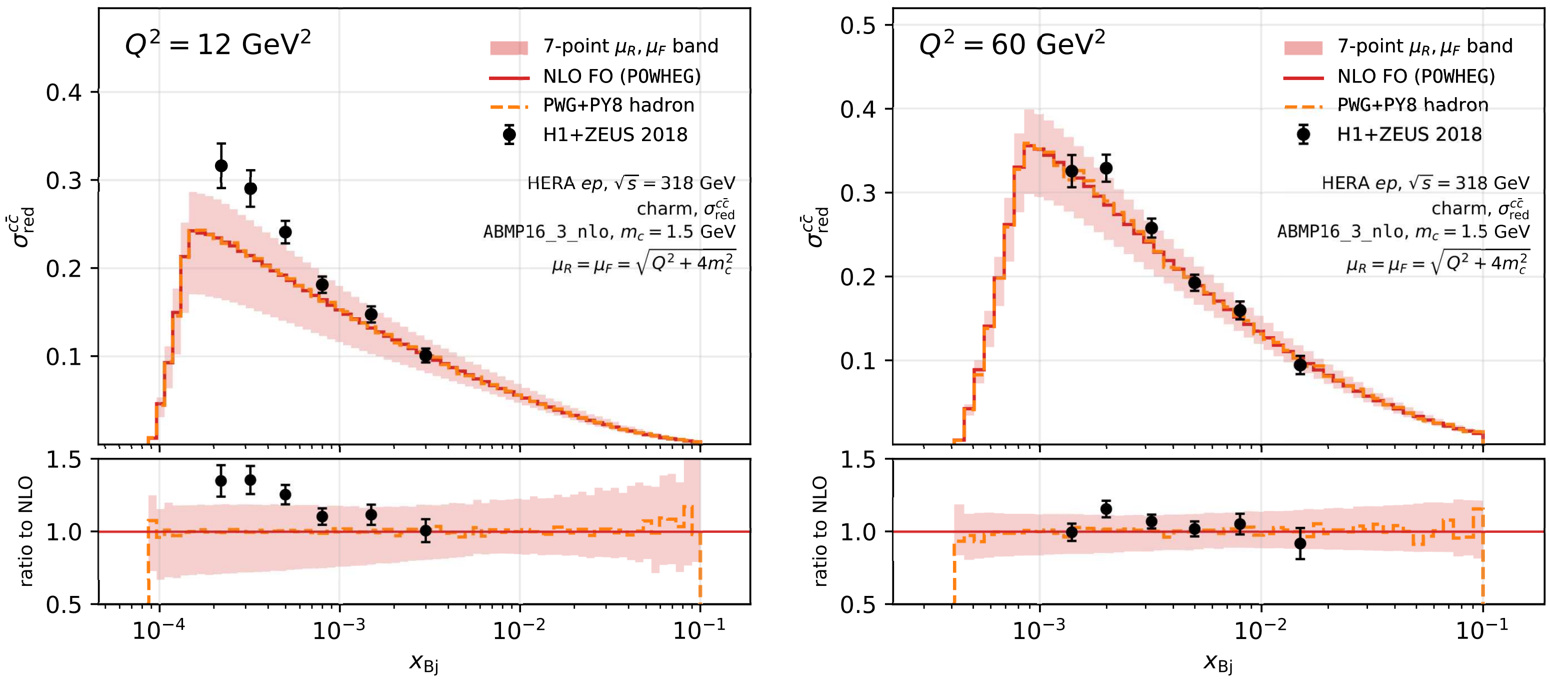
$$\left|\mathcal{M}_m^{(1)}\right\rangle = \left|\mathcal{M}_0^{(1)}\right\rangle + \boldsymbol{I}_m \left|\mathcal{M}_m^{(0)}\right\rangle - \boldsymbol{I}_0 \left|\mathcal{M}_0^{(0)}\right\rangle$$

DIS $c\bar{c}$ production and parton shower (I)



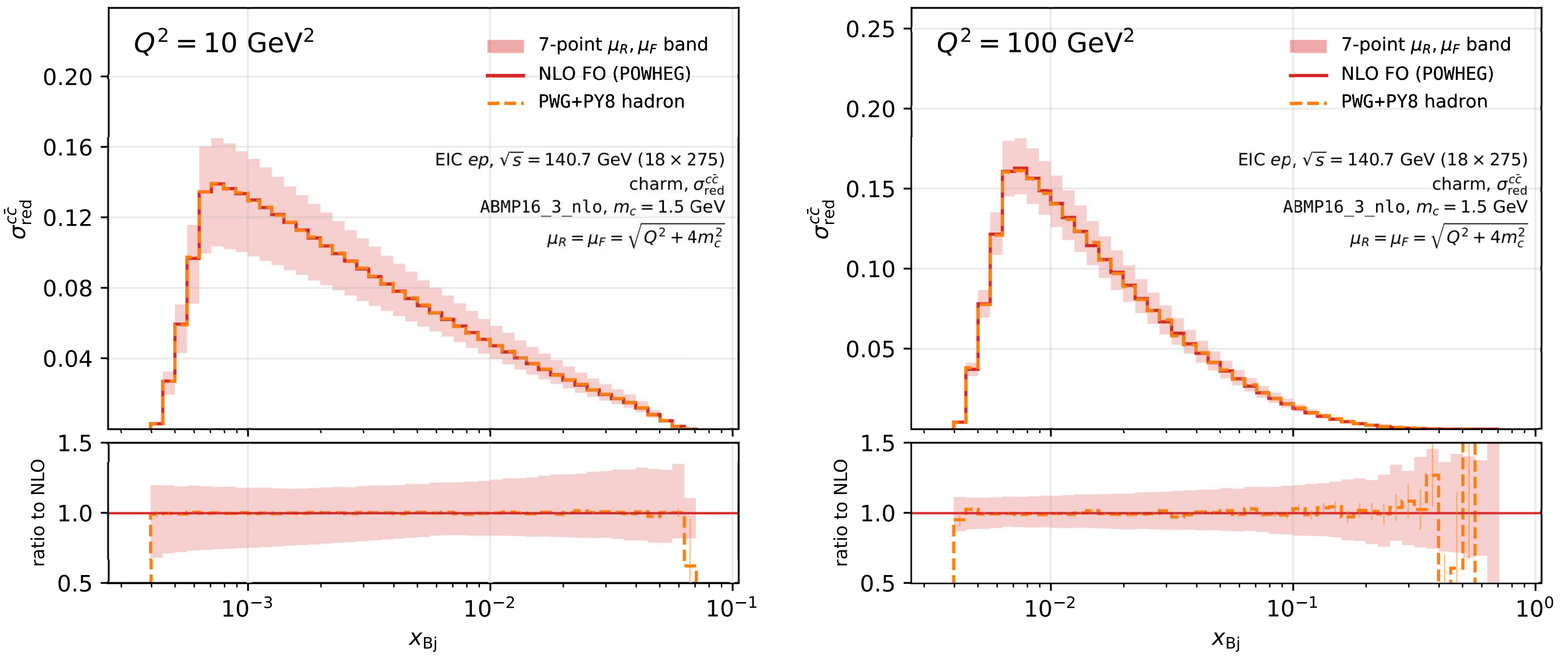
- NLO correction to reduced charm cross section $\Delta\sigma_{\rm red}^{c\bar{c}}$
  - `HVQDIS` (solid) — Harris, Smith '97
  - `POWHEG` (dashed) — Castro, Del Pio, Kardos, S.M., Spourdalakis '26
  - `POWHEG` with massified virtual amplitude (dash-dotted)

DIS $c\bar{c}$ production and parton shower (II)



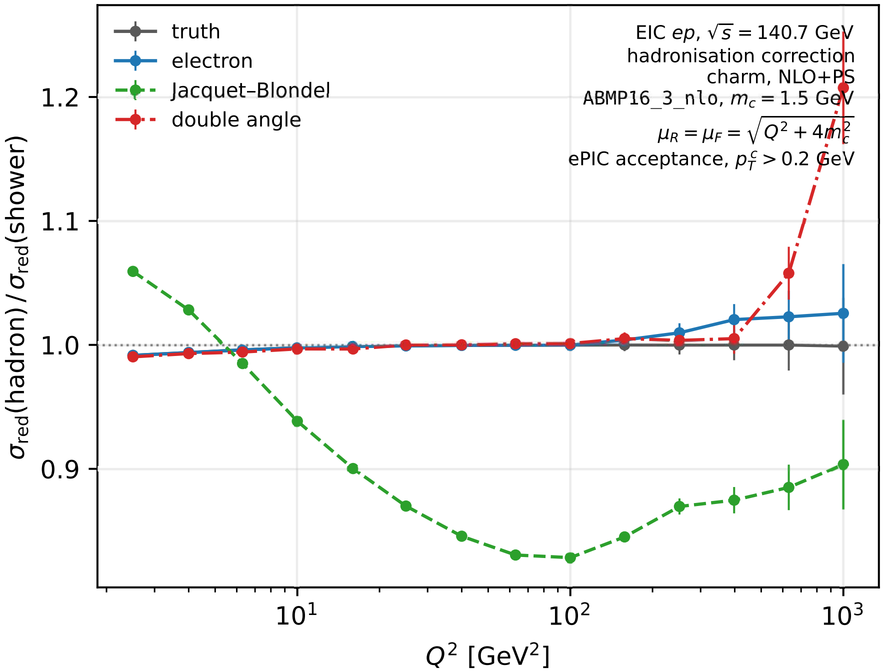
- Reduced charm cross section $\sigma_{\rm red}^{c\bar{c}}$ with 7-point (μ_R, μ_F) scale-variation band compared to HERA data (filled circles) H1+ZEUS combination '18
  - POWHEG at fixed order NLO (solid)
  - POWHEG+PYTHIA8 at particle-level (dashed)

DIS $c\bar{c}$ production and parton shower (III)



- Reduced charm cross section $\sigma_{\rm red}^{c\bar{c}}$ with 7-point (μ_R, μ_F) scale-variation band for EIC kinematics at $\sqrt{s} = 140.7\,\text{GeV}$
  - `POWHEG` at fixed order NLO (solid)
  - `POWHEG+PYTHIA8` at particle-level (dashed)

Outlook



- Hadronization corrections for reduced charm cross section $\sigma_{\rm red}^{c\bar{c}}$ for EIC kinematics at $\sqrt{s} = 140.7$ GeV — Castro, Del Pio, Kardos, S.M. to appear
- Reconstruction of DIS kinematics x, Q^2, y, W using different methods
  - Jacquet-Blondel method — Jaquet, Blondel '79
  - double angle — Bentvelsen, Engelen, Kooijman '92

Summary

Heavy-quark DIS structure functions

- Coefficient functions are now pushed to NNLO frontier
- NNLO approximations built by combining threshold limit, small-x / high-energy limit and large-Q^2asymptotics
- New three-loop Mellin moments provide additional constraints

Parton shower with POWHEG

- Dedicated POWHEG implementation for heavy-quark neutral-current DIS
- Matching NLO QCD predictions to parton shower for realistic event simulation
- Showered predictions for HERA and EIC phenomenology

Outlook

- More three-loop Mellin moments for F_2^c
- Acceptance studies for heavy-quark DIS at EIC and ePIC detector
- ...
- Complete fully differentail NNLO QCD predictions for heavy-quark DIS