Theoretical Constraints on New Interactions

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Dark Interactions

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Outline

- What Particles?
- What Interactions?
- What Else?

1. What Particles?

Starting Assumption:
Poincaré Invariance

Poincaré symmetry is the full symmetry of **Special Relativity**

$$ds^2 = -dt^2 + dx^2 + dy^2 + dz^2$$

Poincaré transformations leave invariant the interval between two events

Starting Assumption: Poincaré Invariance

$$ds^2 = -dt^2 + dx^2 + dy^2 + dz^2$$

10 Such Transformations: $P_\mu, \; J_{\mu\nu}$

- 4 Translations P_μ
- 3 Rotations $\vec{\mathbf{J}} = \{J^{23}, J^{31}, J^{12}\}$
- 3 Boosts $\vec{\mathbf{K}} = \{J^{01}, J^{02}, J^{03}\}$

Starting Assumption: Poincaré Invariance

Classify one-particle states according to their transformations under the Poincaré Group

$$C_1 = -P_\mu P^\mu$$
$$C_2 = W_\mu W^\mu$$

Pauli-Lubanski $W^\mu = \frac{1}{2}\epsilon^{\mu\nu\rho\sigma} J_{\nu\rho} P_\sigma$

massive

$$C_1 = m^2$$
$$C_2 = m^2 s(s+1)$$

spin $s = 0, \frac{1}{2}, 1, \ldots$

massless

$$C_1 = 0$$
$$J_3 = \sigma$$

helicity $\sigma = 0, \pm\frac{1}{2}, \pm 1, \ldots$

Starting Assumption: Poincaré Invariance

QM + Poincaré invariance + Cluster Decomposition

➞ ***fields***

spin-0 $\mathcal{L} = -\frac{1}{2}\partial_\mu\phi\,\partial^\mu\phi - \frac{1}{2}m^2\phi^2$

spin-1 $\mathcal{L} = -\frac{1}{4}(\partial_\mu A_\nu - \partial_\nu A_\mu)(\partial^\mu A^\nu - \partial^\nu A^\mu) - \frac{1}{2}m^2 A_\mu A^\mu$

spin-2 $\mathcal{L} = -\frac{1}{2}\partial_\lambda h_{\mu\nu}\partial^\lambda h^{\mu\nu} + \partial_\mu h_{\nu\lambda}\partial^\nu h^{\mu\lambda} - \partial_\mu h^{\mu\nu}\partial_\nu h + \frac{1}{2}\partial_\lambda h\partial^\lambda h - \frac{1}{2}m^2(h_{\mu\nu}h^{\mu\nu} - h^2)$

spin≥3 $\mathcal{L} = \ldots$

2. What Interactions?

Massless Particles

Interactions of *massless* particles are remarkably constrained

spin-0 $\mathcal{L} = -\frac{1}{2}\partial_\mu\phi\,\partial^\mu\phi + ???$

spin-1 $\mathcal{L} = -\frac{1}{4}(\partial_\mu A_\nu - \partial_\nu A_\mu)(\partial^\mu A^\nu - \partial^\nu A^\mu) + ???$

spin-2 $\mathcal{L} = -\frac{1}{2}\partial_\lambda h_{\mu\nu}\partial^\lambda h^{\mu\nu} + \partial_\mu h_{\nu\lambda}\partial^\nu h^{\mu\lambda} - \partial_\mu h^{\mu\nu}\partial_\nu h + \frac{1}{2}\partial_\lambda h\partial^\lambda h + ???$

spin≥3 $\mathcal{L} = \ldots + ???$

Massless Particles

Poincaré invariance + masslessness
$\Rightarrow$ *gauge invariance**

*except for spin-0 $\mathcal{L} = -\frac{1}{2}\partial_\mu\phi\,\partial^\mu\phi$

$\delta\phi = c \;\rightarrow\; \delta\mathcal{L} = 0$

spin-1 $\mathcal{L} = -\frac{1}{4}(\partial_\mu A_\nu - \partial_\nu A_\mu)(\partial^\mu A^\nu - \partial^\nu A^\mu)$

$\delta A_\mu = \partial_\mu\Lambda \;\rightarrow\; \delta\mathcal{L} = 0$

spin-2 $\mathcal{L} = -\frac{1}{2}\partial_\lambda h_{\mu\nu}\partial^\lambda h^{\mu\nu} + \partial_\mu h_{\nu\lambda}\partial^\nu h^{\mu\lambda} - \partial_\mu h^{\mu\nu}\partial_\nu h + \frac{1}{2}\partial_\lambda h\partial^\lambda h$

$\delta h_{\mu\nu} = \partial_\mu\xi_\nu + \partial_\nu\xi_\mu \;\rightarrow\; \delta\mathcal{L} = 0$

spin≥3 $\mathcal{L} = \ldots$

$\ldots$

Massless Particles

Interacting theories must *preserve* or *extend* the gauge symmetries of the free theories

spin-0 $\mathcal{L} = -\frac{1}{2}\partial_\mu\phi\,\partial^\mu\phi\ +???$

$\delta\phi = c\ +???$

spin-1 $\mathcal{L} = -\frac{1}{4}(\partial_\mu A_\nu - \partial_\nu A_\mu)(\partial^\mu A^\nu - \partial^\nu A^\mu)\ +???$

$\delta A_\mu = \partial_\mu \Lambda +???$

spin-2 $\mathcal{L} = -\frac{1}{2}\partial_\lambda h_{\mu\nu}\partial^\lambda h^{\mu\nu} + \partial_\mu h_{\nu\lambda}\partial^\nu h^{\mu\lambda} - \partial_\mu h^{\mu\nu}\partial_\nu h + \frac{1}{2}\partial_\lambda h \partial^\lambda h +???$

$\delta h_{\mu\nu} = \partial_\mu \xi_\nu + \partial_\nu \xi_\mu +???$

spin≥3 $\mathcal{L} = \ldots\ +???$

$\ldots\ +???$

Massless Particles

Can easily write theories which preserve the linear symmetry

spin-0 $\mathcal{L} = -\frac{1}{2}\partial_\mu\phi\,\partial^\mu\phi + \sum_n c_n f(X^n)$

$\delta\phi = c \qquad X \equiv \partial_\mu\phi$

spin-1 $\mathcal{L} = -\frac{1}{4}(\partial_\mu A_\nu - \partial_\nu A_\mu)(\partial^\mu A^\nu - \partial^\nu A^\mu) + \sum_n c_n f(F^n)$

$\delta A_\mu = \partial_\mu\Lambda \qquad F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu$

spin-2 $\mathcal{L} = -\frac{1}{2}\partial_\lambda h_{\mu\nu}\partial^\lambda h^{\mu\nu} + \partial_\mu h_{\nu\lambda}\partial^\nu h^{\mu\lambda} - \partial_\mu h^{\mu\nu}\partial_\nu h + \frac{1}{2}\partial_\lambda h\partial^\lambda h + \sum_n c_n f(R^n)$

$\delta h_{\mu\nu} = \partial_\mu\xi_\nu + \partial_\nu\xi_\mu \qquad R_{\mu\nu\rho\sigma} \equiv \partial_\sigma\partial_{[\mu}h_{\nu]\rho} - \partial_\rho\partial_{[\mu}h_{\nu]\sigma}$

spin≥3 $\mathcal{L} = \ldots$

$\ldots$

Massless Particles

"*Interesting*" theories are the ones where the gauge symmetry is extended by the interactions

$$\delta\phi = c + ??? \qquad \delta A_\mu = \partial_\mu \Lambda + ??? \qquad \delta h_{\mu\nu} = \partial_\mu \xi_\nu + \partial_\nu \xi_\mu + ???$$

CONSTRAINT:

the product of two gauge transformations must itself be a gauge transformation

simple analogue: translations

$$T_a f(x) = f(x+a) \quad \rightarrow \quad T_a T_b f(x) = T_{a+b} f(x)$$

infinitesimal transformations of fields:

$$[\mathcal{O}_a, \mathcal{O}_b]\phi_{\mu_1,\ldots,\mu_n} = \mathcal{O}_{c(a,b)}\phi_{\mu_1,\ldots,\mu_n}$$

Massless Particles

"*Interesting*" theories are the ones where the gauge symmetry is extended by the interactions

spin-1: $\delta A^a_\mu = \partial_\mu \Lambda^a + g f^{abc} A^b_\mu \Lambda^c$ **Yang-Mills!**

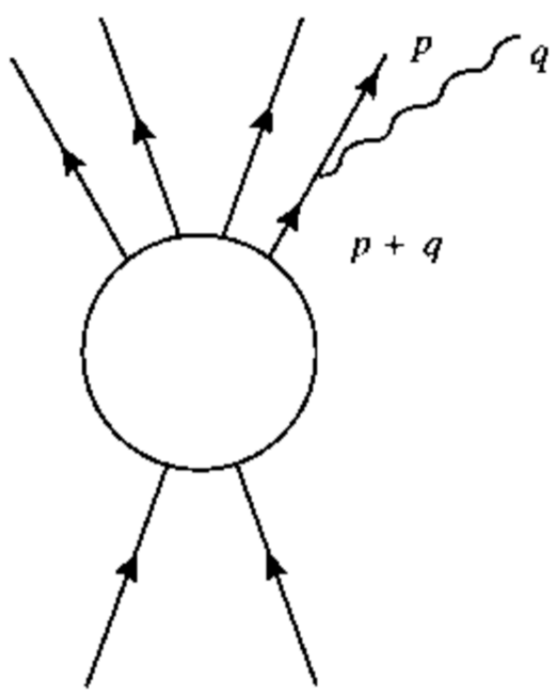
spin-2: $\delta g_{\mu\nu} = \nabla_\mu \xi_\nu + \nabla_\nu \xi_\mu$ **General Relativity!**

spin≥3: $none$

(Wald 1986)

Weinberg (1964)

soft theorems for massless particles



$q \to 0$

- spin-1: electric charge is conserved $$\sum_n e_n = 0$$
- spin-2: coupling is the same for all forms of energy and momentum $$\sum_n g_n \, p_n^\mu = 0$$
- spin $\geq$ 3: couplings don't survive at low energies $$\sum_n g_n \, p_n^\mu \, p_n^\nu \ldots = 0$$

Summary: Massless Particles

only assumed Poincaré invariance

- **spin-0: no problem**
- **spin-1: only have E&M, Yang-Mills; electric charge is conserved**
- **spin-2: only have GR; can derive equivalence principle**
- **spin $\geq$ 3: no non-trivial theories at low energies**

consistent with Standard Model + gravity

What about massive particles?

Anything goes?

Massive Particles

spin-0: $\mathcal{L} = -\frac{1}{2}\partial_\mu\phi\,\partial^\mu\phi - \frac{1}{2}m^2\phi^2 + \frac{1}{4}\lambda\phi^4 + \ldots$

Exists! E.g., Higgs boson…

Massive Particles

spin-0: $\mathcal{L} = -\frac{1}{2}\partial_\mu\phi\,\partial^\mu\phi - \frac{1}{2}m^2\phi^2 + \frac{1}{4}\lambda\phi^4 + \ldots$

Exists! E.g., Higgs boson…

spin-1: $\mathcal{L} = -\frac{1}{4}(\partial_\mu A_\nu - \partial_\nu A_\mu)(\partial^\mu A^\nu - \partial^\nu A^\mu) - \frac{1}{2}m^2 A_\mu A^\mu + e^2\phi^2 A_\mu A^\mu + \ldots$

Exists! E.g., W & Z bosons…

Massive Particles

spin-0: $\mathcal{L} = -\frac{1}{2}\partial_\mu\phi\,\partial^\mu\phi - \frac{1}{2}m^2\phi^2 + \frac{1}{4}\lambda\phi^4 + \ldots$

Exists! E.g., Higgs boson…

spin-1: $\mathcal{L} = -\frac{1}{4}(\partial_\mu A_\nu - \partial_\nu A_\mu)(\partial^\mu A^\nu - \partial^\nu A^\mu) - \frac{1}{2}m^2 A_\mu A^\mu + e^2\phi^2 A_\mu A^\mu + \ldots$

Exists! E.g., W & Z bosons…

spin-2:

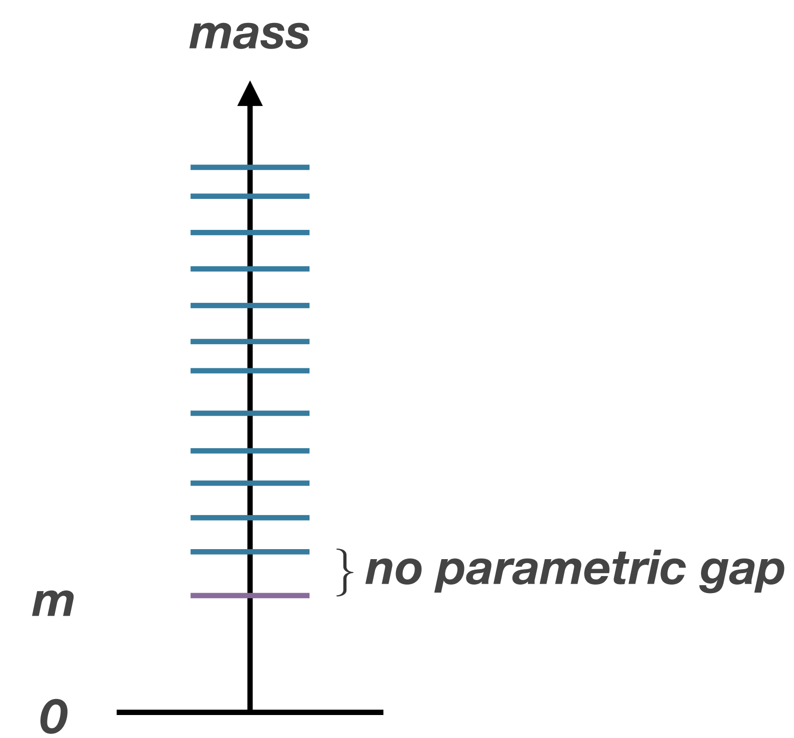
$$\mathcal{L} = -\frac{1}{2}\partial_\lambda h_{\mu\nu}\partial^\lambda h^{\mu\nu} + \partial_\mu h_{\nu\lambda}\partial^\nu h^{\mu\lambda} - \partial_\mu h^{\mu\nu}\partial_\nu h + \frac{1}{2}\partial_\lambda h\partial^\lambda h - \frac{1}{2}m^2(h_{\mu\nu}h^{\mu\nu} - h^2) + ???$$

Exists???

Massive Particles

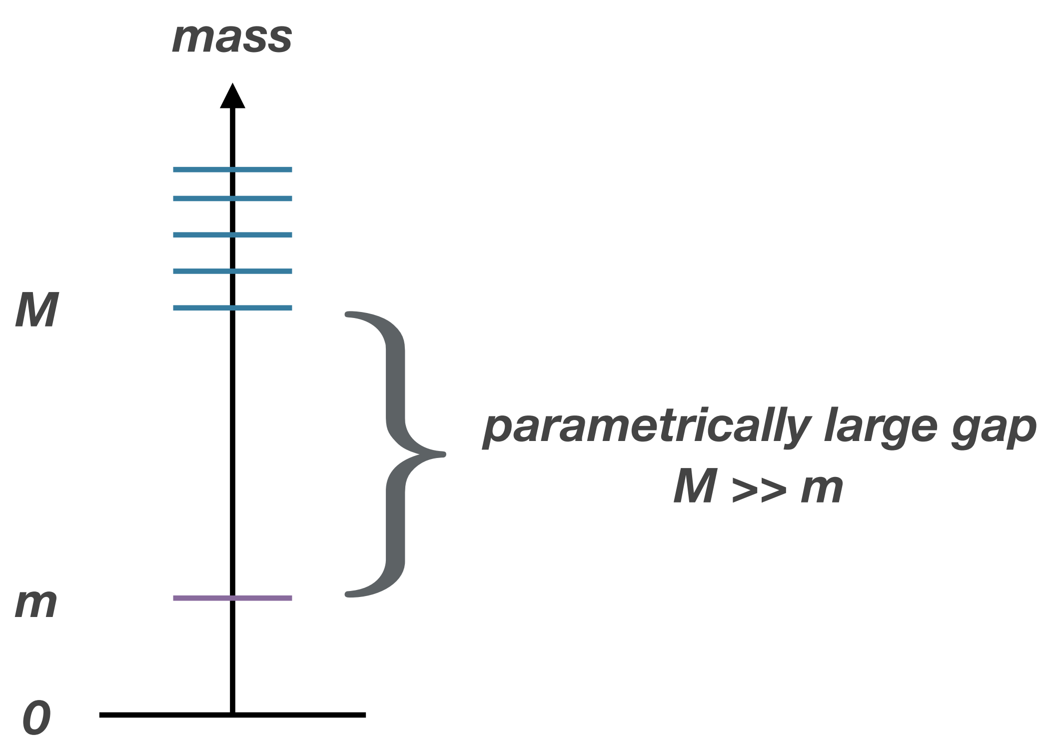
Interacting, massive high-spin particles *do* exist

- **QCD:** $m^2 \sim \Lambda_{QCD}^2$
- **Kaluza-Klein Theory:** $m^2 \sim \dfrac{1}{R^2}$
- **String Theory:** $m^2 \sim \dfrac{1}{\alpha'}$



Massive Particles

Can the spectrum look like this?



spin-0: yes!

spin-1: yes!

spin≥2: ???

Need to construct a low energy EFT with cut-off >> mass

Massive Particles

Massive Spin-2: the Free Theory

$$\mathcal{S}_2 = \int d^4x \, \left(-\tfrac{1}{2}\partial_\lambda h_{\mu\nu}\partial^\lambda h^{\mu\nu} + \partial_\mu h_{\nu\lambda}\partial^\nu h^{\mu\lambda} - \partial_\mu h^{\mu\nu}\partial_\nu h + \tfrac{1}{2}\partial_\lambda h \partial^\lambda h \right.$$
$$\left. + m_1^2 \, h_{\mu\nu}h^{\mu\nu} + m_2^2 h^2\right)$$

add a mass!

Massive Particles

Massive Spin-2: the Free Theory

$$\mathcal{S}_2 = \int d^4x \, \left(-\frac{1}{2}\partial_\lambda h_{\mu\nu}\partial^\lambda h^{\mu\nu} + \partial_\mu h_{\nu\lambda}\partial^\nu h^{\mu\lambda} - \partial_\mu h^{\mu\nu}\partial_\nu h + \frac{1}{2}\partial_\lambda h \partial^\lambda h \right.$$
$$\left. + m_1^2 \, h_{\mu\nu}h^{\mu\nu} + m_2^2 h^2\right)$$

add a mass!

$h_{\mu\nu}$ has **10** independent components

for the *massless* spin-2, gauge invariance

$$\delta h_{\mu\nu} = \partial_\mu \xi_\nu + \partial_\nu \xi_\mu$$

leaves **2** physical degrees of freedom: 

BREAK DIFFEOMORPHISM INVARIANCE → extra DOF!

Massive Particles

Massive Spin-2: the Free Theory

$$\mathcal{S}_2 = \int d^4x \, \left(-\tfrac{1}{2}\partial_\lambda h_{\mu\nu}\partial^\lambda h^{\mu\nu} + \partial_\mu h_{\nu\lambda}\partial^\nu h^{\mu\lambda} - \partial_\mu h^{\mu\nu}\partial_\nu h + \tfrac{1}{2}\partial_\lambda h \partial^\lambda h \right.$$
$$\left. + m_1^2 \, h_{\mu\nu}h^{\mu\nu} + m_2^2 h^2\right)$$

- 4 Bianchi constraints: $m_1^2\partial^\mu h_{\mu\nu} + m_2^2 \partial_\nu h = 0$

$10 - 4 = 6$ DOF

massive graviton should have $2s + 1 = 5$ DOF

Massive Particles

Massive Spin-2: the Free Theory

$$\mathcal{S}_2 = \int d^4x \, \left(-\tfrac{1}{2}\partial_\lambda h_{\mu\nu}\partial^\lambda h^{\mu\nu} + \partial_\mu h_{\nu\lambda}\partial^\nu h^{\mu\lambda} - \partial_\mu h^{\mu\nu}\partial_\nu h + \tfrac{1}{2}\partial_\lambda h \partial^\lambda h \right.$$
$$\left. -\tfrac{1}{2}m^2(h_{\mu\nu}h^{\mu\nu} - h^2)\right)$$

← FIERZ-PAULI MASS TERM

- 4 Bianchi constraints: $m_1^2\partial^\mu h_{\mu\nu} + m_2^2\partial_\nu h = 0$
- additional constraint: $m^2 h = 0$

$10 - 4 - 1 = 5$ DOF ✓

Massive Particles

Massive Spin-2: the Interacting Theory

$$\mathcal{S}_2 = \int d^4x \, \big(-\tfrac{1}{2}\partial_\lambda h_{\mu\nu}\partial^\lambda h^{\mu\nu} + \partial_\mu h_{\nu\lambda}\partial^\nu h^{\mu\lambda} - \partial_\mu h^{\mu\nu}\partial_\nu h + \tfrac{1}{2}\partial_\lambda h \partial^\lambda h$$
$$- \tfrac{1}{2}m^2(h_{\mu\nu}h^{\mu\nu} - h^2) + \mathcal{O}(h^3) + \mathcal{O}(h^4) + \dots \big)$$

add interaction terms

- Is there a *non*-linear theory of a massive spin-2 particle that maintains a constraint at the fully non-linear level and thus avoids an extra, pathological DOF?

Boulware, Deser (1972)

YES!

Massive Particles

Massive Spin-2: the Interacting Theory

$$\mathcal{S} = \frac{M_{Pl}^2}{2}\left\{\int d^4x \, \det e \, R[e] - m^2 \int \sum_{n=0}^{4} \beta_n S_n[e]\right\}$$

$$g_{\mu\nu} = e_\mu{}^a e_\nu{}^b \eta_{ab}$$

$$e^a \equiv e_\mu{}^a dx^\mu$$

$$\eta_{\mu\nu} = \delta_\mu{}^a \delta_\nu{}^b \eta_{ab}$$

$$\mathbf{1}^a \equiv \delta_\mu{}^a dx^\mu$$

$$S_0[e] = \epsilon_{abcd}\, e^a \wedge e^b \wedge e^c \wedge e^d$$

$$S_1[e] = \epsilon_{abcd}\, e^a \wedge e^b \wedge e^c \wedge \mathbf{1}^d$$

$$S_2[e] = \epsilon_{abcd}\, e^a \wedge e^b \wedge \mathbf{1}^c \wedge \mathbf{1}^d$$

$$S_3[e] = \epsilon_{abcd}\, e^a \wedge \mathbf{1}^b \wedge \mathbf{1}^c \wedge \mathbf{1}^d$$

$$S_4[e] = \epsilon_{abcd}\, \mathbf{1}^a \wedge \mathbf{1}^b \wedge \mathbf{1}^c \wedge \mathbf{1}^d$$

C. de Rham, G. Gabadadze, A. J. Tolley (2010) S. F. Hassan, RAR (2011) K. Hinterbichler, RAR (2012)

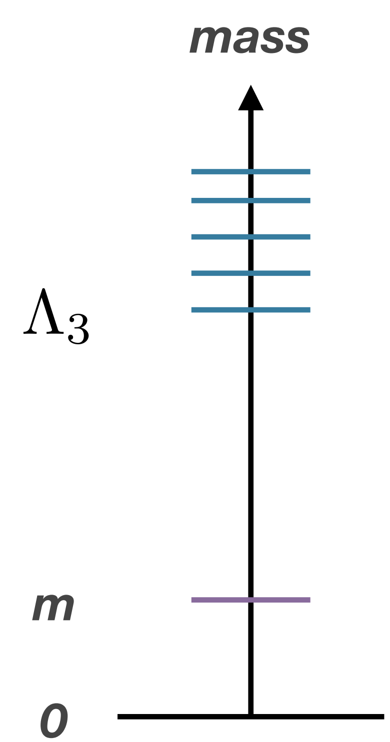
What about the cut-off?

$$\mathcal{S} = \frac{M_{Pl}^2}{2}\left\{\int d^4x \, \det e \, R[e] - m^2 \int \sum_{n=0}^{4} \beta_n S_n[e]\right\}$$

EFT cut-off:

$$\Lambda_3 = (M_{Pl} m^2)^{1/3}$$

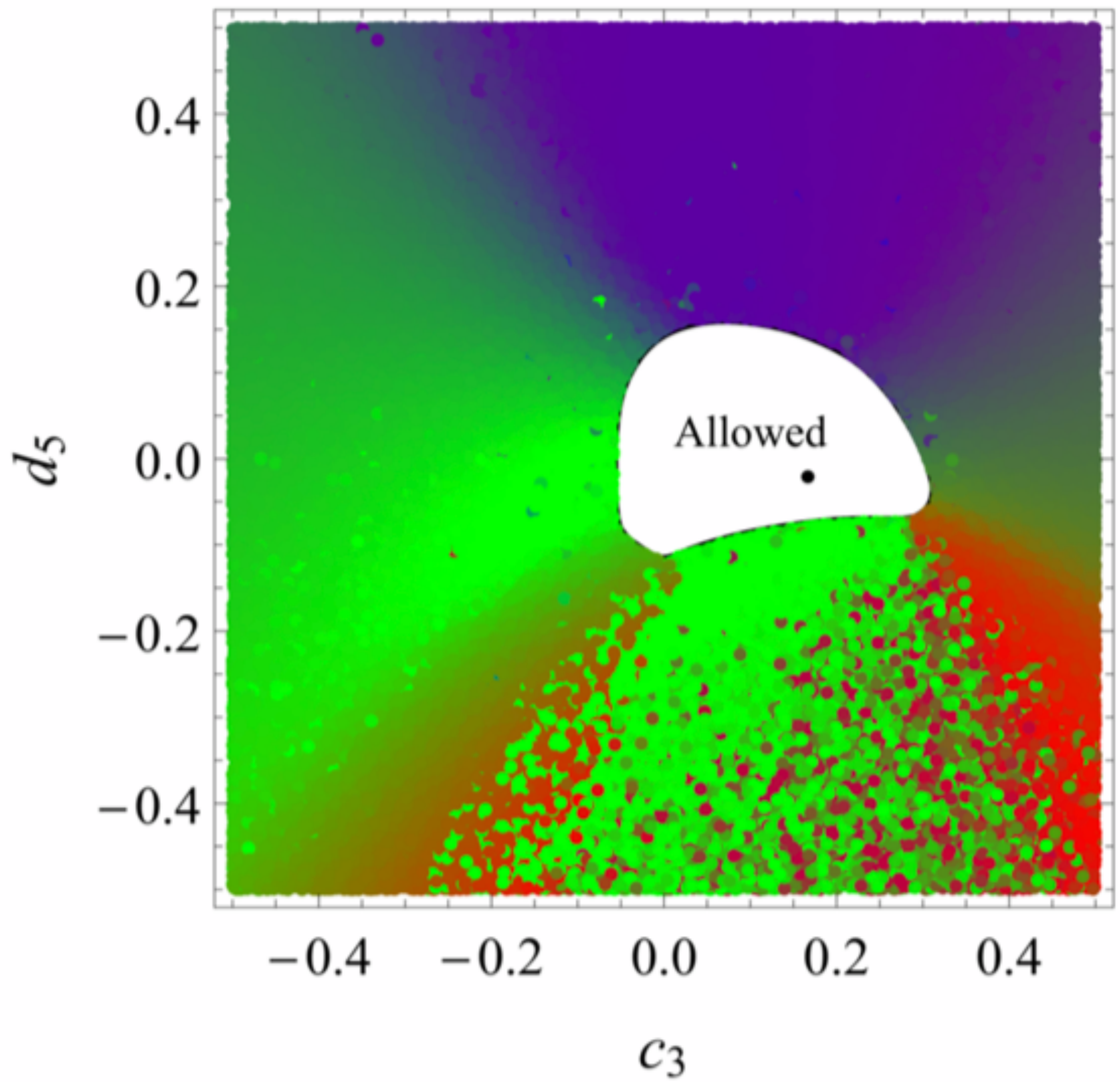
$$\Lambda_3 \gg m$$



hierarchy is stable under quantum corrections

Beyond Effective Field Theory

Analyticity of the S-Matrix

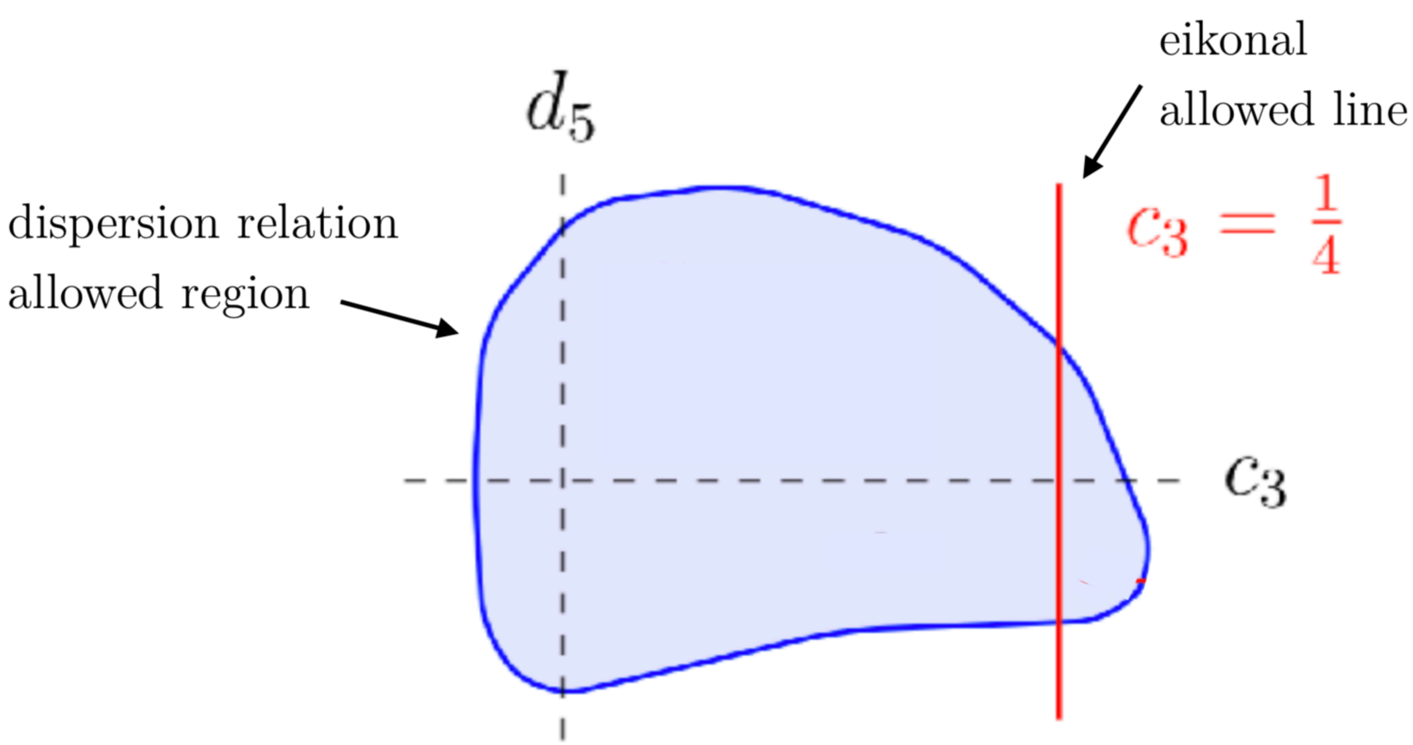


Consistency of scattering amplitudes in the forward limit constrains the free parameters of a massive spin-2 particle

(Cheung, Remmen 2016)

Beyond Effective Field Theory

Absence of superluminality in the eikonal limit



Consider scattering at high energy, large impact parameter

(Joyce, Hinterbichler, RAR 2017)

Can this be our graviton?

Observational Bounds:

- Yukawa suppression in gravitationally bound clusters:

$m < 10^{-29} eV$ Goldhaber and Nieto (1974)

- Lunar Laser Ranging:

$m < 10^{-32} eV$ Dvali, Gruzinov and Zaldarriaga (2003)

- LIGO: GW150914

$m < 10^{-22} eV$ Abbott *et al* (2016)

C. de Rham, J. T. Deskins, A. J. Tolley, S-Y Zhou (2016)

Outlook for Massive Spin-2

- Black hole solutions are still poorly understood
- Viable cosmologies seem to require strong coupling, very large scale inhomogeneities/ anisotropies
- UV completion is outstanding issue

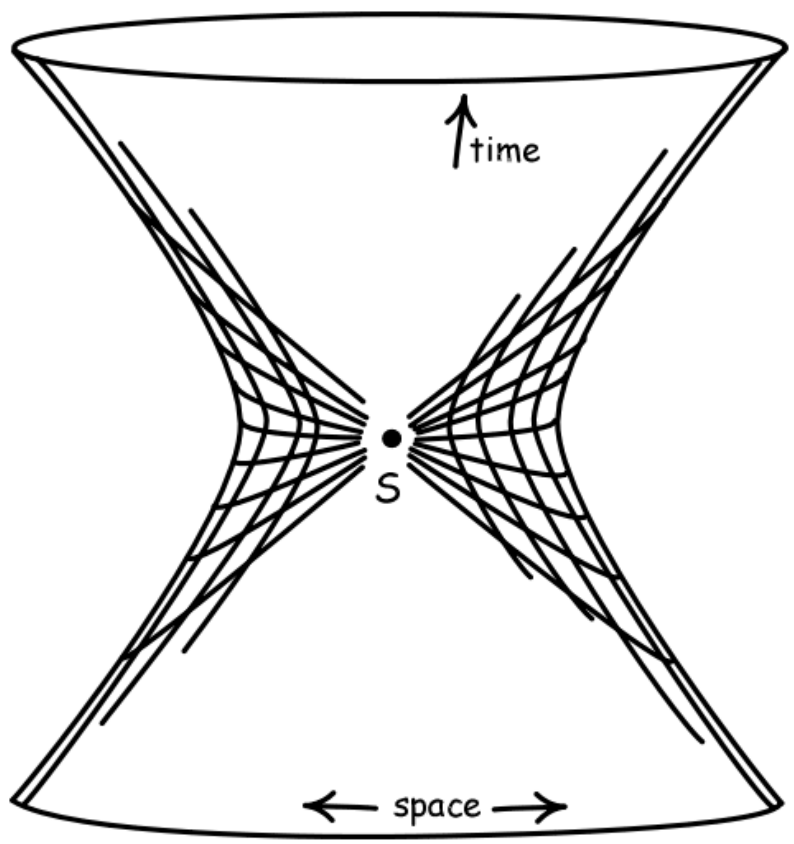
Summary: Massive Particles

- spin-0: no problem
- spin-1: need a Higgs mechanism
- spin-2: non-trivial, UV completion?
- spin $\geq$ 3: well, there's no no-go…

consistent with Standard Model + gravity

3. What Else?

Representations of the de Sitter Group



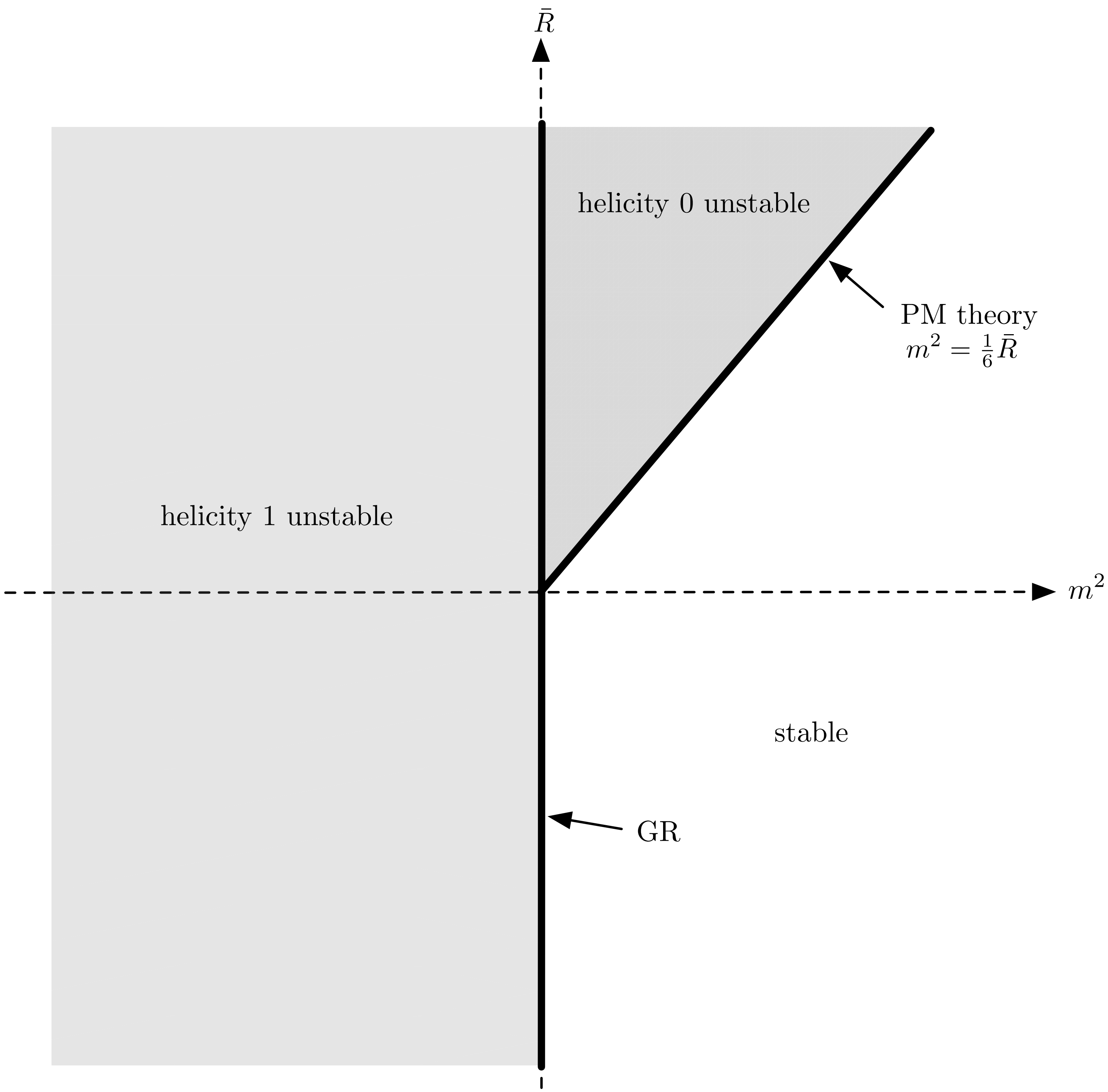
$$R_{\mu\nu} - \frac{1}{2} R\, g_{\mu\nu} + \Lambda\, g_{\mu\nu} = 0$$

Minkowski: Poincare
de Sitter: SO(1,4)

} Maximally symmetric spaces
10 isometries

Representations of the de Sitter Group

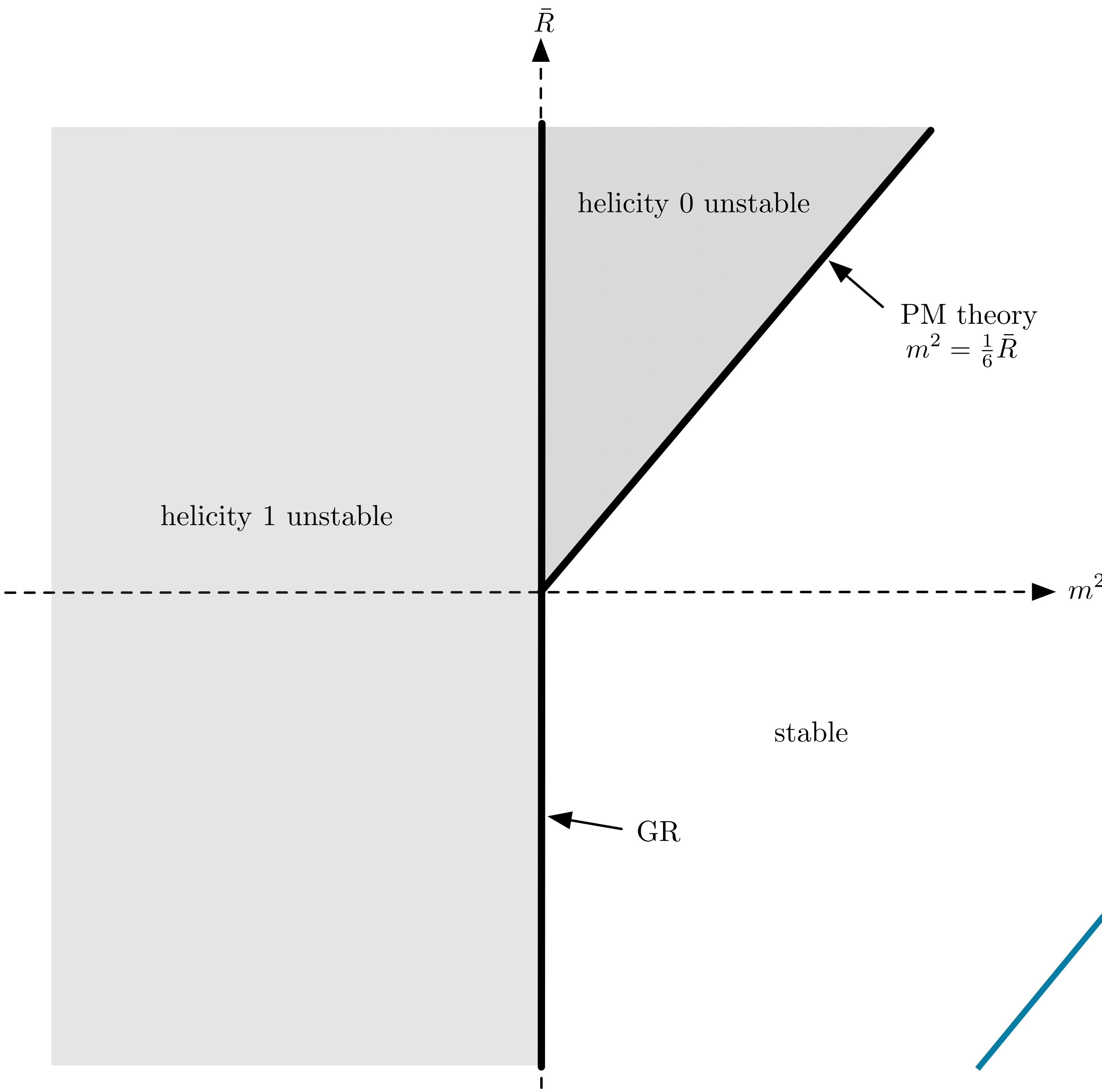
Spin-2 Particles



- $m^2 = 0$ 2 dof
- $m^2 < \frac{1}{6}\bar{R}$ unstable
- $m^2 = \frac{1}{6}\bar{R}$ 4 dof
- $m^2 > \frac{1}{6}\bar{R}$ 5 dof

Representations of the de Sitter Group

Spin-2 Particles



- $m^2 = 0$ — 2 dof
- $m^2 < \frac{1}{6}\bar{R}$ — unstable
- $m^2 = \frac{1}{6}\bar{R}$ — 4 dof
- $m^2 > \frac{1}{6}\bar{R}$ — 5 dof

Deser, Higuchi, Nepomechie, Waldron

"partially massless"

Representations of the de Sitter Group

Partially Massless Spin-2 Particles

$$\mathcal{L}_2 = \sqrt{-\bar{g}}\left[-\tfrac{1}{2}\bar{\nabla}_\alpha h_{\mu\nu}\,\bar{\nabla}^\alpha h^{\mu\nu} + \bar{\nabla}_\alpha h_{\mu\nu}\,\bar{\nabla}^\nu h^{\mu\alpha} - \bar{\nabla}_\mu h\,\bar{\nabla}_\nu h^{\mu\nu} + \tfrac{1}{2}\bar{\nabla}_\mu h\bar{\nabla}^\mu h \right.$$

$$\left. + \tfrac{\bar{R}}{4}\left(h^{\mu\nu}h_{\mu\nu} - \tfrac{1}{2}h^2\right) - \tfrac{1}{2}m^2(h_{\mu\nu}h^{\mu\nu} - h^2)\right]$$

gauge symmetry: $\delta h_{\mu\nu} = \left(\bar{\nabla}_\mu\bar{\nabla}_\nu + \tfrac{1}{12}\bar{R}\,\bar{g}_{\mu\nu}\right)\phi$

Representations of the de Sitter Group

Partially Massless Spin-2 Particles

$$\mathcal{L}_2 = \sqrt{-\bar{g}}\,\Big[-\tfrac{1}{2}\bar{\nabla}_\alpha h_{\mu\nu}\,\bar{\nabla}^\alpha h^{\mu\nu} + \bar{\nabla}_\alpha h_{\mu\nu}\,\bar{\nabla}^\nu h^{\mu\alpha} - \bar{\nabla}_\mu h\,\bar{\nabla}_\nu h^{\mu\nu} + \tfrac{1}{2}\bar{\nabla}_\mu h \bar{\nabla}^\mu h$$

$$+\frac{\bar{R}}{4}\left(h^{\mu\nu}h_{\mu\nu} - \tfrac{1}{2}h^2\right) - \tfrac{1}{2}m^2(h_{\mu\nu}h^{\mu\nu} - h^2)\Big]$$

gauge symmetry: $\delta h_{\mu\nu} = \left(\bar{\nabla}_\mu \bar{\nabla}_\nu + \frac{1}{12}\bar{R}\,\bar{g}_{\mu\nu}\right)\phi$

- *no non-trivial extensions of the gauge symmetry*

  $[\delta_\phi, \delta_\psi]h_{\mu\nu} = \delta_{\chi(\phi,\psi)}h_{\mu\nu}$ (Garcia-Saenz, RAR 2014)

- *consistency conditions???*

Summary

- Various theoretical consistency conditions constrain the possible building blocks of our universe and their interactions
- Most powerful when they can help us constrain physics we can’t constrain observationally
- Possible applications: constraining an overabundance of low energy EFTs in the dark sector; finding consistent UV completions

Thank you,

Rachel A Rosen
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