Constraints on Exotic Spin-Dependent Long-Range Interactions from Spin-Independent Experiments

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Talk @ 3rd Biennial Workshop on Dark Interactions, BNL
October 02-05, 2018

With William Michael Snow, Dennis E. Krause, Joshua C. Long
Weakly-interacting bosons are consistent with dark matter abundance in the keV - $10^{-22}$ eV mass range.

Pseudo Nambu-Goldstone bosons (psGbs) of broken symmetries

Stueckelberg Mechanism: gauge symmetry broken at high scale, but with weak coupling.

String theory or extra dimensions naturally give rise to psGbs from non-trivial topology.
The QCD axion and the strong CP problem

\[ L \supset \frac{g_s}{32\pi^2} \partial \theta \, G^a \tilde{G}^{\mu\nu} \]

The neutron electric dipole moment

\[ d_N \sim \frac{\alpha}{m_N^2} \frac{m_q}{m_N} |\bar{\theta}| \sim 10^{-16} |\bar{\theta}| \text{ e-}{\text{cm}} \]

Current bound on nEDM: \( \bar{\theta} \leq 10^{-10} \)

Solution:

Turn \( \bar{\theta} \) into a dynamical field

An axion

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The physical importance of gauge field configurations with non-trivial topology has been stressed by 't Hooft. 1 It has been suggested that the physics of such theories involves a parameter \( \bar{\theta} \) which

gauges.

If all fermionic

gauge fields are

choices give equi-

arity seen by a

ubiquitous value

of \( \bar{\theta} \), it

suffices to

\[ \phi(x) = \bar{\theta} \phi(x) \]

where

\[ \phi(x) = \bar{\theta} \phi(x) \]

The rotation of a

suffices to

\[ \phi(x) = \bar{\theta} \phi(x) \]
Opportunities to probe the low energy frontier

- Short Distance Tests of Gravity
- Extra Dimensions

- Tests of Gravity
- Gravitational Wave detection at low frequencies
- Tests of Atom Neutrality at 50 decimals

Dimopoulos, Kasevich et. al.(2006-2008)

- Axion Dark Matter Detection
- Axion Force Detection

Graham et. al. (2012)
AA, Geraci (2014)

- Setting the Time Standard
- Dilaton Dark Matter Detection

AA, Huang, Van Tilburg (2014)
Limits on ultra-light bosons from laboratory experiments

\[ V(r) = -\frac{Gm_1 m_2}{r} \left( 1 + \alpha e^{-r/\lambda} \right) \]

\[ \lambda = \frac{1}{\mu} \]

Monopole-dipole constraints for electrons

\[ V_{S-P} = \frac{g_S g_P}{8\pi m} \left( \vec{\sigma} \cdot \hat{r} \right) \left( \frac{1}{\lambda r} + \frac{1}{r^2} \right) e^{-r/\lambda} \]
Search for new physics with atoms and molecules

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(published 29 June 2018)

This article reviews recent developments in tests of fundamental physics using atoms and molecules,
including the subjects of purity violation, searches for permanent electric dipole moments, tests of the
CPT theorem and Lorentz symmetry, searches for spatiotemporal variation of fundamental constants,
tests of quantum electrodynamics, tests of general relativity and the equivalence principle, searches
for dark matter, dark energy, and extra forces, and tests of the spin-statistics theorem. Key results are
presented in the context of potential new physics and in the broader context of similar investigations
in other fields. Ongoing and future experiments of the next decade are discussed.

DOI: 10.1103/RevModPhys.90.025008
VII. Review of Laboratory Searches for Exotic Spin-dependent Interactions

A. Early work
1. Torsion in gravity
2. Electric dipole moments
3. Axions and axionlike particles (ALPs)
4. Early experiments

B. Theoretical motivation
1. Axionlike particles in string theory
2. The hierarchy problem
3. Dark energy
4. Unparticles
5. Paraphotons, dark photons, hidden photons, and new $Z'$ bosons
6. Conclusions

C. Parametrization
1. Introduction
2. Moody-Wilczek-Dobrescu-Mocioiu formalism
3. MWDM formalism for Lorentz-invariant, single-boson exchange
4. Contact interactions
5. Position representation and permutation symmetry
Long-Range Interactions from Sub-GeV Dark Matter

\[ \mathcal{O}_a^0 = \frac{1}{\Lambda^2} \bar{N}N|\phi|^2, \quad \mathcal{O}_a^{1/2} = \frac{1}{\Lambda^2} \bar{N}N\bar{\chi}\chi, \]

\[ \mathcal{O}_b^0 = \frac{1}{\Lambda^2} \bar{N} \gamma^\mu N\phi^* \gamma^\mu \phi, \quad \mathcal{O}_b^{1/2} = \frac{1}{\Lambda^2} \bar{N} \gamma^\mu N\bar{\chi}\gamma^\mu \chi, \]

\[ \mathcal{O}_c^0 = \frac{1}{\Lambda^3} \bar{N}N\partial^\mu \phi^* \partial_\mu \phi, \quad \mathcal{O}_c^{1/2} = \frac{1}{\Lambda^2} \bar{N} \gamma^\mu N\bar{\chi}\gamma^\mu \gamma^5 \chi, \]

\[ \mathcal{O}_d^1 = \frac{m^2}{\Lambda^3} \bar{N}N|X^\mu + \partial^\mu \pi|^2, \]

\[ \mathcal{O}_b^1 = \frac{1}{\Lambda^2} 2\bar{N} \gamma^\nu \gamma^\mu N \text{Im}[X_{\mu \nu} X_{\nu}^* + \partial^\nu (X_\nu X_{\mu}^*) + \partial^\nu \bar{\pi} c^*], \]

\[ \mathcal{O}_c^1 = \frac{1}{\Lambda^3} \bar{N}N|X_{\mu \nu}|^2, \quad \mathcal{O}_d^1 = \frac{1}{\Lambda^3} \bar{N}N X_{\mu \nu} \bar{X}^{\mu \nu}, \]
Outline:

- Current limits on exotic spin-dependent long-range interactions.
- Calculations of spin-independent interactions from double-boson exchange processes with spin-dependent couplings to fermions.
- Improving the constraints using spin-independent experiments
Model-independent forms of long-range interactions between a pair of non-relativistic fermions

- Interaction is rotationally invariant
- Only depend on $r^{12}$, $p^{(i)}$, and $\sigma^{(i)}$, with $i=1,2$
- Local
- Satisfy energy-momentum conservation
Model-independent forms of long-range interactions between a pair of non-relativistic fermions

- Interaction is rotationally invariant
- Only depend on $r^{12}$, $p^{(i)}$, and $\sigma^{(i)}$, with $i=1,2$
- Local
- Satisfy energy-momentum conservation

$$O_1 = 1$$
$$O_2 = \sigma \cdot \sigma'$$
$$O_3 = \frac{1}{m^2} (\sigma \cdot \vec{q}) (\sigma' \cdot \vec{q})$$
$$O_{4,5} = \frac{i}{2m^2} (\sigma \pm \sigma') \cdot (\vec{P} \times \vec{q})$$
$$O_{6,7} = \frac{i}{2m^2} [ (\sigma \cdot \vec{P}) (\sigma' \cdot \vec{q}) \pm (\sigma \cdot \vec{q}) (\sigma' \cdot \vec{P}) ]$$
$$O_8 = \frac{1}{m^2} (\sigma \cdot \vec{P}) (\sigma' \cdot \vec{P})$$
$$O_{9,10} = \frac{i}{2m} (\sigma \pm \sigma') \cdot \vec{q}$$
$$O_{11} = \frac{i}{m} (\sigma \times \sigma') \cdot \vec{q}$$
$$O_{12,13} = \frac{1}{2m} (\sigma \pm \sigma') \cdot \vec{P}$$
$$O_{14} = \frac{1}{m} (\sigma \times \sigma') \cdot \vec{P}$$
$$O_{15} = \frac{1}{2m^3} \left\{ [\sigma \cdot (\vec{P} \times \vec{q})] (\sigma' \cdot \vec{q}) + (\sigma \cdot \vec{q}) [\sigma' \cdot (\vec{P} \times \vec{q})] \right\}$$
$$O_{16} = \frac{i}{2m^3} \left\{ [\sigma \cdot (\vec{P} \times \vec{q})] (\sigma' \cdot \vec{P}) + (\sigma \cdot \vec{P}) [\sigma' \cdot (\vec{P} \times \vec{q})] \right\}$$

$$V(\vec{r}, \vec{p}) = \sum_i f_i^{XY} O_i(\vec{q}, \vec{p}) y(r)$$

$XY= ee, ep, en, pp, nn, np.$
$M$: mass the fermion, $\mu$: boson mass
$r$: separation between the fermions

Interaction is rotationally invariant
Only depend on $r^{12}$, $p^{(i)}$, and $\sigma^{(i)}$, with i=1,2
Local
Satisfy energy-momentum conservation

$O_1 = 1$

$O_2 = \bar{\sigma} \cdot \bar{\sigma}'$

$O_3 = \frac{1}{m^2} (\bar{\sigma} \cdot \bar{q}) (\bar{\sigma}' \cdot \bar{q})$

$O_{4,5} = \frac{i}{2m^2} (\bar{\sigma} \pm \bar{\sigma}') \cdot (\bar{P} \times \bar{q})$

$O_{6,7} = \frac{i}{2m^2} \left[ (\bar{\sigma} \cdot \bar{P}) (\bar{\sigma}' \cdot \bar{q}) \pm (\bar{\sigma} \cdot \bar{q}) (\bar{\sigma}' \cdot \bar{P}) \right]$

$O_8 = \frac{1}{m^2} (\bar{\sigma} \cdot \bar{P}) (\bar{\sigma}' \cdot \bar{P})$

$O_{9,10} = \frac{i}{2m} (\bar{\sigma} \pm \bar{\sigma}') \cdot \bar{q}$

$O_{11} = \frac{i}{m} (\bar{\sigma} \times \bar{\sigma}') \cdot \bar{P}$

$O_{12,13} = \frac{1}{2m} (\bar{\sigma} \pm \bar{\sigma}') \cdot \bar{P}$

$O_{14} = \frac{1}{m} (\bar{\sigma} \times \bar{\sigma}') \cdot \bar{P}$

$O_{15} = \frac{1}{2m^3} \left\{ [\bar{\sigma} \cdot (\bar{P} \times \bar{q})] (\bar{\sigma}' \cdot \bar{q}) + (\bar{\sigma} \cdot \bar{q}) [\bar{\sigma}' \cdot (\bar{P} \times \bar{q})] \right\}$

$O_{16} = \frac{i}{2m^3} \left\{ [\bar{\sigma} \cdot (\bar{P} \times \bar{q})] (\bar{\sigma}' \cdot \bar{P}) + (\bar{\sigma} \cdot \bar{P}) [\bar{\sigma}' \cdot (\bar{P} \times \bar{q})] \right\}$

$V(\vec{r}, \vec{p}) = \sum_i f^{XY}_i (\vec{q}, \vec{p}) O_i(\vec{q}, \vec{p}) y(r)$

$X= ee, ep, en, pp, nn, np.$

$Y$: mass the fermion, $\mu$: boson mass

$r$: separation between the fermions

Coefficients $f_i$ in terms of physical couplings $g$

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$s = 0$</th>
<th>$s = 1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f^{ee}_{2}$</td>
<td>0</td>
<td>$(g^e_A)^2$</td>
</tr>
<tr>
<td>$f^{ee}_{3}$</td>
<td>$-\frac{1}{4}(g^e_p)^2$</td>
<td>$\frac{1}{4}[(g^e_V)^2 + (g^e_A)^2]$</td>
</tr>
<tr>
<td>$f^{ee}_{11}$</td>
<td>0</td>
<td>$g^e_A g^e_V$</td>
</tr>
<tr>
<td>$f^{ee}_{6+7}$</td>
<td>0</td>
<td>$g^e_A g^e_V^a$</td>
</tr>
<tr>
<td>$f^{ee}_{8}$</td>
<td>0</td>
<td>$-\frac{5}{4}(g^e_A)^2$</td>
</tr>
<tr>
<td>$f^{ee}_{14}$</td>
<td>0</td>
<td>$(g^e_A)^2a$</td>
</tr>
<tr>
<td>$f^{ee}_{15}$</td>
<td>0</td>
<td>$(g^e_V)^2a$</td>
</tr>
<tr>
<td>$f^{ee}_{16}$</td>
<td>0</td>
<td>$g^e_A g^e_V^a$</td>
</tr>
<tr>
<td>$f^{ee}_1 + f^{ep}_1 + f^{en}_1$</td>
<td>$\frac{1}{2}g^e_S[g^e_S + g^p_S + g^n_S]$</td>
<td>$\frac{1}{2}[3(g^e_V)^2 + (g^e_A)^2 + g^p_V g^p_V + g^n_V g^n_V]$</td>
</tr>
<tr>
<td>$f^{ee}_r + f^{ep}_r + f^{en}_r$</td>
<td>$g^e_p[g^e_S + g^p_S + g^n_S]$</td>
<td>$g^e_A[g^e_V + g^p_V + g^n_V]^a$</td>
</tr>
<tr>
<td>$f^{ee}_v + f^{ep}_v + f^{en}_v$</td>
<td>0</td>
<td>$2g^e_A[2g^e_V + g^p_V + g^n_V]$</td>
</tr>
</tbody>
</table>

Current Limits from Laboratory Experiments

- Very stringent limits on possible interactions arising from scalar and vector couplings from spin-independent experiments. In contrast, limits on pseudoscalar and axial couplings are many orders of magnitude weaker.

- Limits on pseudoscalar couplings to nucleons are more suppressed than their electron analogs.

<table>
<thead>
<tr>
<th>Type of coupling</th>
<th>$\mu \approx [100 , \mu \text{eV}, 10 , \text{meV}]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$g_S^2, g_V^2$</td>
<td>$10^{-40} - 10^{-35}$</td>
</tr>
<tr>
<td>$(g_P^e)^2$</td>
<td>$10^{-16} - 10^{-8}$</td>
</tr>
<tr>
<td>$(g_P^N)^2$</td>
<td>$10^{-4} - 10^{-6}$</td>
</tr>
<tr>
<td>$(g_A^e)^2$</td>
<td>$10^{-15} - 10^{-12}$</td>
</tr>
<tr>
<td>$g_S g_P^e$</td>
<td>$10^{-32} - 10^{-29}$</td>
</tr>
<tr>
<td>$g_S g_P^N$</td>
<td>$10^{-28} - 10^{-21}$</td>
</tr>
<tr>
<td>$g_V g_A^e$</td>
<td>$10^{-25} - 10^{-17}$</td>
</tr>
<tr>
<td>$g_V g_A^N$</td>
<td>$10^{-26} - 10^{-23}$</td>
</tr>
</tbody>
</table>
Why are Limits on Pseudoscalar and Axial Couplings Poor?

1. Pseudoscalar and axial interactions are suppressed by factors of $\mu/m$ per vertex relative to scalar and vector interactions for nonrelativistic motion.

\[
\begin{align*}
&g_s \bar{\psi} \psi \phi(t, \bar{x}) \quad \text{P} \quad g_s \bar{\psi} \psi \phi(t, -\bar{x}) \\
&g_v \bar{\psi} \gamma^\mu \psi A_\mu(t, \bar{x}) \quad \text{P} \quad g_v \bar{\psi} \gamma^\mu \psi A_\mu(t, -\bar{x}) \\
&g_p i \bar{\psi} \gamma_5 \psi \phi(t, \bar{x}) \quad \text{P} \quad -g_p i \bar{\psi} \gamma_5 \psi \phi(t, -\bar{x}) \quad \text{P-wave} \\
&g_A \bar{\psi} i \gamma^\mu \gamma_5 \psi A_\mu(t, \bar{x}) \quad \text{P} \quad -g_A \bar{\psi} i \gamma^\mu \gamma_5 \psi A_\mu(t, -\bar{x})
\end{align*}
\]
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&g_p i \bar{\psi} \gamma_5 \psi \varphi(t, \bar{x}) \quad \overset{P}{=} \quad -g_p i \bar{\psi} \gamma_5 \psi \varphi(t, -\bar{x}) \\
&g_A i \bar{\psi} i \gamma^\mu \gamma_5 \psi A_\mu(t, \bar{x}) \quad \overset{P}{=} \quad -g_A i \bar{\psi} i \gamma^\mu \gamma_5 \psi A_\mu(t, -\bar{x})
\end{align*}
\]

ex: for a nucleon and a 1 meV boson $\frac{\mu}{m} \approx 10^{-15}$
Why are Limits on Pseudoscalar and Axial Couplings Poor?

1. Pseudoscalar and axial interactions are suppressed by factors of $\mu/m$ per vertex relative to scalar and vector interactions for nonrelativistic motion:

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    g_S \bar{\psi}\gamma^\mu\phi(t,\bar{x}) & \quad \mathbf{P} \quad g_S \bar{\psi}\gamma^\mu\phi(t,-\bar{x}) \\
    g_V \bar{\psi}\gamma^5\gamma^\mu\psi A_\mu(t,\bar{x}) & \quad \mathbf{P} \quad g_V \bar{\psi}\gamma^5\gamma^\mu\psi A_\mu(t,-\bar{x}) \\
    g_P i\bar{\psi}\gamma_5\psi \phi(t,\bar{x}) & \quad \mathbf{P} \quad -g_P i\bar{\psi}\gamma_5\psi \phi(t,-\bar{x}) \\
    g_A \bar{\psi}i\gamma^\mu\gamma_5\gamma^\mu\psi A_\mu(t,\bar{x}) & \quad \mathbf{P} \quad -g_A \bar{\psi}i\gamma_5\gamma^\mu\gamma^\mu\psi A_\mu(t,-\bar{x})
\end{align*}
\]

ex: for a nucleon and a 1 meV boson $\frac{\mu}{m} \approx 10^{-15}$

2. Necessarily spin-dependent at lowest order

---

**Experimental difficulties associated with polarization:**
- Only the valence fermions are accessible.
- Large external magnetic fields.
- Polarization techniques vary widely in efficiency.
- Difficult to maintain polarization of members of the ensemble.
- Both internal and external magnetic fields produce large systematic effects in delicate experiments.
- ….
Aim of this work:

- derive all long-range *spin-independent* interactions from double boson exchange from spin-0 and spin-1 with *spin-dependent* couplings between a pair of NR Dirac fermions

- Use the derived expressions with the strong spin-independent limits to help improve constraints on interactions involving pseudoscalar and axial couplings.
\[ L_P = -g_P \bar{\psi} i \gamma_5 \psi \varphi \]

\[ V_P = \left[ \frac{g_P^2}{16\pi m^2} \vec{\sigma}_a \cdot \nabla \vec{\sigma}_b \cdot \nabla e^{-\mu r} \right] \frac{g_P^4}{m^2} \frac{K_1(2\mu r)}{4\pi^2 r^2} + \cdots \]

Previous Work

\[ K_1(x): \text{modified Bessel function of the second kind} \]

Blue ➔ spin-dependent
Red ➔ spin-independent

[E. Fischbach and D. E. Krause, (1999)]
\[ L_P = -g_P \bar{\psi} i\gamma_5 \psi \phi \]

\[ V_P = \frac{g_P^2}{16\pi m^2} \tilde{\sigma}_a \cdot \nabla \tilde{\sigma}_b \cdot \nabla \frac{e^{-\mu r}}{r} - \frac{g_P^4}{m^2} \mu \frac{K_1(2\mu r)}{4\pi^2 r^2} + \cdots \]

- No similar analysis done and no functional forms exist for interactions
  \[ \propto g_S^2 g_P^2, g_V^2, g_A^4, g_{PD}^2, g_S^2 g_{PD}^2, \text{ and } g_A^4 \]

Previous Work

\[ K_1(x): \text{modified Bessel function of the second kind} \]
\[ L_p = -g_p \bar{\psi} i\gamma_5 \psi \phi \]

\[ V_p = \left[ \frac{g_p^2}{16\pi m^2} \vec{\sigma}_a \cdot \nabla \vec{\sigma}_b \cdot \nabla e^{-\mu r} \right] - \frac{g_p^4}{m^2} \mu \frac{K_1(2\mu r)}{4\pi^2 r^2} + \cdots \]

- No similar analysis done and no functional forms exist for interactions \[ \propto g_S g_P, g_V g_A, g_P^2, g_P^2, \text{ and } g_A^4 \]

- Limits do not necessarily apply to the pseudoscalar derivative coupling

\[ m \bar{\psi} e^{i\gamma_5 \varphi / f_a} \psi \approx -\frac{i m}{f_a} \bar{\psi} \gamma_5 \psi \varphi + \frac{m}{2f_a^2} \bar{\psi} \psi \varphi^2 + \cdots \]

\[ g_{PD} = \frac{m}{f_a} \]

Previous Work

\[ K_1(x): \text{ modified Bessel function of the second kind} \]
Calculating an Interaction Energy

- Use non-relativistic Old-Fashioned Perturbation Theory (OFPT) to calculate a transition amplitude and relate to a potential via the Lippmann-Schwinger (LS) equation

\[
\langle f \mid T \mid i \rangle = \langle f \mid V \mid i \rangle + \sum_n \frac{\langle f \mid V \mid n \rangle \langle n \mid T \mid i \rangle}{E_i - E_n + i\epsilon}.
\]

- No self-energy terms are considered since masses and couplings are taken from experiments.


- Methods based on covariant PT: Dispersion Method (Feinberg and Sucher), EFT (Holstein), Chiral EFT (Kaiser, Machledit, etc.).
Effective Hamiltonian

\[ H_{\text{int}} = \int d^3 \vec{x} \overline{\psi}(\vec{x}, t)[(g_S + ig_P \gamma_5)\phi(\vec{x}, t) + (g_V \gamma^\mu + g_A \gamma^\mu \gamma_5)A_\mu(\vec{x}, t)]\psi(\vec{x}, t) \]

\[ H_{PD} = \int d^3 \vec{x} \left[ \frac{g_{PD}}{2m} \overline{\psi}(\vec{x}, t)\gamma_5 \gamma_\mu \partial^\mu \phi(\vec{x}, t)\psi(\vec{x}, t) + \frac{1}{2} \left( \frac{g_{PD}}{2m} \overline{\psi}(\vec{x}, t)\gamma^0 \gamma_5 \psi(\vec{x}, t) \right)^2 \right] \]

Up to \(O(1/m)\) and \(O(1/m^2)\) in the expansion:

\[ H_{S}^{\text{eff}} = g_S \phi + \frac{1}{8m^2} \vec{p} \cdot \vec{\nabla} \phi + \frac{1}{8m^2} \vec{\sigma} \cdot (\vec{\nabla} \times \vec{p}) \phi - \frac{g_S}{4m^2} \{p^2, \phi\} \]

\[ H_{P}^{\text{eff}} = -\frac{g_P}{2m} \vec{\sigma} \cdot \vec{\nabla} \phi + \frac{g_P^2}{2m} \phi^2 \]

\[ H_{V}^{\text{eff}} = g_V A_0 - \frac{g_V}{2m} \{\vec{p}, \vec{A}\} - \frac{g_V}{2m} \vec{\sigma} \cdot \vec{\nabla} \times \vec{A} + \frac{g_V^2}{2m} \vec{A}^2 \]

\[ H_{A}^{\text{eff}} = -g_A \vec{\sigma} \cdot \vec{A} + \frac{g_A}{2m} \{\vec{\sigma} \cdot \vec{p}, A_0\} + \frac{g_A^2}{2m} A_0^2 \]

\[ H_{PD}^{\text{eff}} = -\frac{g_{PD}}{2m} \vec{\sigma} \cdot \vec{\nabla} \phi - \frac{g_{PD}^2}{4m^2} \{\vec{\sigma} \cdot \vec{p}, \partial_0 \phi\} + \frac{g_{PD}^2}{8m^3} (\partial_0 \phi)^2 + \frac{g_{PD}}{8m^3} \{p^2, \vec{\sigma} \cdot \vec{\nabla} \phi\} \]

\[-i \frac{g_{PD}}{16m^3} \{\vec{\sigma} \cdot \vec{p}, \{\vec{p}, \vec{\nabla} \phi\}\} + \frac{g_{PD}}{16m^3} \vec{\sigma} \cdot \vec{\nabla} \partial_0^2 \phi. \]
Power counting

- An order by order expansion in $1/m$:

$$ T = T^{(-1)} + T^{(0)} + T^{(1)} + T^{(2)} + ... $$

$$ V = V^{(-1)} + V^{(0)} + V^{(1)} + V^{(2)} + ... $$

- Iterations of the LS eq

$$ V + V G_o V + V G_o V G_o V + .... $$

- To order $g^4$:

$$ V^{(-1)} = T^{(-1)} - [V^{(0)} G_o V^{(0)}] $$

$$ V^{(0)} = T^{(0)} - [V^{(0)} G_o V^{(1)} + V^{(1)} G_o V^{(0)}] $$

$$ V^{(1)} = T^{(1)} - [V^{(1)} G_o V^{(1)} + V^{(2)} G_o V^{(0)} + V^{(0)} G_o V^{(2)}] $$

$$ V^{(2)} = T^{(2)} - [V^{(2)} G_o V^{(1)} + V^{(1)} G_o V^{(2)} + V^{(3)} G_o V^{(0)} + V^{(0)} G_o V^{(3)}] $$

<table>
<thead>
<tr>
<th>Type of interaction</th>
<th>number of contributing TOD diagrams</th>
</tr>
</thead>
<tbody>
<tr>
<td>$g_P^4$</td>
<td>2</td>
</tr>
<tr>
<td>$g_S^2 g_P^2$</td>
<td>6</td>
</tr>
<tr>
<td>$g_S^2 g_{PD}^2$</td>
<td>$\geq 24$</td>
</tr>
<tr>
<td>$g_V^2 g_A^2$</td>
<td>$\geq 60$</td>
</tr>
<tr>
<td>$g_{PD}^4$</td>
<td>$\geq 158$</td>
</tr>
<tr>
<td>$g_A^4$</td>
<td>$\geq 78$</td>
</tr>
</tbody>
</table>
Results for the spin-independent long-range interaction from two boson exchange

\[ V^{(0)} = T^{(0)} \]

Agrees with

\[ V_{P-P} = -\frac{g_{P,1}^2 g_{P,2}^2 \mu K_1(2\mu r)}{4m_1 m_2} \frac{1}{8\pi^3 r^2} \]

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\[ V_{S-P} = \left( \frac{g_{S,1}^2 g_{P,2}^2}{2m_2} + \frac{g_{S,2}^2 g_{P,1}^2}{2m_1} \right) \frac{e^{-2\mu r}}{16\pi^2 r^2} \]

Range of applicability:
\[ r \geq \frac{1}{\mu m} \]
with finite boson mass

\[ V_{V-A}(r) \simeq \left[ \frac{g_{V,1}^2 g_{A,2}^2}{2m_1} + \frac{g_{V,2}^2 g_{A,1}^2}{2m_2} + \left( \frac{g_{V,2}^2 g_{A,1}^2}{2m_1} + \frac{g_{V,1}^2 g_{A,2}^2}{2m_2} \right) \right] \frac{e^{-2\mu r}}{16\pi^2 r^2} + \ldots \]

\[ V^{(1)} = T^{(1)} - [V^{(1)}G_0V^{(1)} + V^{(2)}G_0V^{(0)} + V^{(0)}G_0V^{(2)}] \]

\[ \overline{V}_{PD-PD} = -\frac{g_{PD}^4}{512m^5} \frac{e^{-2x}}{2r^6} [5(6 + 12x + 10x^2 + 4x^3 + x^4) + 5x^2(x+1)^2 + 2x^5] + \ldots \]

Agrees (up to a minor discrepancy) with
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\[ V^{(1)} = T^{(1)} - [V^{(1)} G_o V^{(1)} + V^{(2)} G_o V^{(0)} + V^{(0)} G_o V^{(2)}] \]

Range of applicability:
\[ r \geq \frac{1}{\mu} \gg \frac{1}{m} \]
with finite boson mass

NEW \[ \overline{V}_{P-D-PD} = -\frac{g_{P-D}^4}{512m^5} \frac{e^{-2x}}{2r^6} [5(6 + 12x + 10x^2 + 4x^3 + x^4) + 5x^2(x + 1)^2 + 2x^5] \]

Agrees (up to a minor discrepancy) with

NEW \[ \overline{V}_{S-PD} = \frac{g_{S}^2 g_{P-D}^2}{128\pi^2 m^3} \frac{e^{-2x}}{r^4} (x + 1)[2x^2 - 3x - 3] + ... \]

NEW \[ \overline{V}_{A-A} = \frac{3g_A^4}{64\pi^2 m} \frac{e^{-2x}}{r^2} (5 - 2x) + ... \]

Agrees with
Constraints from Spin-Independent Experiments

\[ \lambda = 1 / \mu \]

\[ V_{P-P}(r) = -\frac{g_P^4}{4m^2r^2} \frac{K_1(2r / \lambda)}{8\pi^2 \lambda} \]

an existence proof that sensitive experimental searches for spin-independent interactions can also yield interesting limits on spin-dependent interactions at certain distance scales.

Further Opportunities

Constraints on the scalar coupling from spin-independent experiments testing interactions between normal matter across a wide length scale

- $V_{\text{PD-PD}} \propto 1/r^6$ sensitive to short-length scales.
- No constrains on $(g_{\text{PD}}^N)^2$ below $10^{-12}$ m, a theoretically interesting regime.

$$V_p = \frac{g_p^2}{4\pi} \left( \frac{\mu}{2m} \right)^2 \left[ (\sigma_1 \cdot \sigma_2) + S_{12} \left( \frac{1}{3} + \frac{1}{x} + \frac{1}{x^2} \right) \right] e^{-ur}$$

$$S_{12} = 3(\sigma_1 \cdot \hat{r})(\sigma_2 \cdot \hat{r}) - \sigma_1 \cdot \sigma_2$$

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\[ S_{12} = 3(\vec{\sigma}_1 \cdot \hat{r})(\vec{\sigma}_2 \cdot \hat{r}) - \vec{\sigma}_1 \cdot \vec{\sigma}_2 \]

Conclusion:

- Ultralight bosons, including axions, generate long-range interactions of various types that can be probed with precision laboratory experiments.

- Experiments testing spin-dependent interactions experience additional challenges that do not exist in spin-independent experiments.

- The functional forms derived from 2-boson exchange processes open up an opportunity to constrain, using spin-independent data, spin-dependent couplings over new length scales that are outside the sensitivity of current spin-dependent experiments.
Conclusions:

- We derived the leading spin-independent long-range contribution to the position-dependent part of the potential arising from the exchange of two light bosons involving pseudoscalar and axial Yukawa couplings between two massive Dirac fermions.

- The result we derived for the interaction with pseudoscalar Yukawa couplings is in agreement with the limiting case found in the literature. The partial result we obtained for the interaction with pseudoscalar derivative coupling agrees with the overall shape of the potential due to pion-nucleon couplings up to a minor discrepancy in the numerical factors.

- The functional forms we derived open up an opportunity to constrain, using existing spin-independent data, spin-dependent couplings over new length scales that are outside the sensitivity of current spin-dependent experiments.

Remaining Work:

- The calculation of non-static terms is more involved and evaluating all contributions for all possible combinations is still a work in progress.

- It is desirable to avoid ambiguities with the potential by performing a calculation of the interaction energy instead.

- Analysis of the derived functional forms with data from spin-independent experiments over a broad length scale and across various species of fermions.
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Two Bound state Systems?

Advantage over 2-free body interaction:

- A large part of the interaction is absorbed into the mean field of the nucleus since

\[
\left( \frac{1}{m_a r_a} \right)^2 \approx 1
\]

- Of order \( g_p^2 g_{\text{strong}}^4 \)

while form of \( V_{ab} \) is simple, calculation of \( \langle Q_a \rangle \) isn`t, however a general shell model calculation may give a reasonable estimation.

\[
V_{ab}(r_{ab}) = g_p^2 \langle Q_a \rangle \langle Q_b \rangle \frac{e^{-\mu r}}{4\pi r}
\]

Useful resources

Sabine Hossenfelder on lost in math and SM
https://wicn.org/podcasts/audio/sabine-hossenfelder-lost-math
log(\(\alpha\))

log(\(\lambda/1m\))

- Strong interaction
- n-p scattering
- \(\bar{p}\)-atoms
- n-\(^{208}\)Pb scattering
- n-scattering
- n-gravitational level experiments

**Neutrons**

**High Energy**

**Mechanical**

- Nucleon scale
- Atomic scale
- Gravitation scale
Two-Boson Exchange

(a)

(b)
Weakly-interacting bosons are consistent with dark matter abundance in the keV - $10^{-22}$ eV mass range

Give rise to many phenomena on the low energy scale offering feasible experimental probes!

Particle Physics Motivation: Ultralight Spin-1 Particles

- Many theories beyond the SM give rise to interactions with vector as well as axial couplings

\[ L = \bar{\psi} \left( g_V \gamma^\mu + g_A i \gamma^\mu \gamma_5 \right) \psi A_\mu \]

- Supersymmetric and grand-unified theories require two or more Higgs doublet which possess an extra U(1) symmetry generator of the form:

\[ F = a \, B + bL + cY + dF^a_x \]

- \( a, b, c, \) and \( d \) depend on the details of the theory

Possible couplings
Axions (and ALPs) effects

Fermions

\[ \bar{\psi} \gamma^\mu \gamma_5 \psi \partial_\mu \varphi \]
dipole

\[ \bar{\psi} \psi \varphi \]
monopole

Gauge Fields

\[ F_{\mu \nu} F^{\mu \nu} \varphi \]
Current

\[ G_{\mu \nu} G^{\mu \nu} \varphi \]
QCD axion

Moody and Wilczek (1987)
B. Dobrescu and I. Mocioiu (2006)

\[ V_{s-s} = -\frac{g_s^2}{4\pi r} e^{-\mu r} \]

\[ V_{s-p} = \frac{g_s g_p}{8\pi m} \vec{\sigma} \cdot \nabla \frac{e^{-\mu r}}{r} \]

\[ V_{p-p} = \frac{g_p^2}{16\pi m^2} \vec{\sigma}_a \cdot \nabla \vec{\sigma}_b \cdot \nabla \frac{e^{-\mu r}}{r} \]

Torsion balance, casimir experiments
molecular spectroscopy, NMR, microwave cavity…etc
Two-Boson Exchange

Aim of this work:

- derive all long-range *spin-independent* interaction energies from double boson exchange from spin-0 and spin-1 with *spin-dependent* couplings between a pair of NR Dirac fermions
- Use the derived expressions with the strong spin-independent limits to help improve constraints on interactions involving pseudoscalar and axial couplings.
A Single-Boson Exchange Potential in TOPT

\[ \omega_k = \sqrt{k^2 + \mu^2} \]

\[ \Delta E_i = E - E_I = \frac{p_i^2 - p_i'^2}{2m_i} \]

\[ H_{PD}^{eff} = -\frac{g^{PD}}{2m} \hat{\sigma} \cdot \vec{\nabla} \phi \]

\[ f(r) = e^{-\mu r} / 4\pi r \]

\[ T_{fi} = \sum_{\alpha} \langle f | H_{PD} | \alpha \rangle \frac{1}{E_i - E_{\alpha} + i\epsilon} \langle \alpha | H_{PD} | i \rangle \]

\[ \longrightarrow \ \text{static} \sum_k \left\{ V_2 \frac{1}{\omega_k} V_1 + V_1 \frac{1}{\omega_k} V_2 \right\} \]

\[ = \frac{g_1 g_2}{4m_1 m_2} \frac{\hat{\sigma}_1 \cdot \vec{\nabla} \hat{\sigma}_2 \cdot \vec{\nabla}}{\omega_k^2} \delta(p_2' + p_1' - p_1 - p_2) \]

- Vertex corrections: \(-i\frac{g^{PD}}{16m^3} \{ \hat{\sigma} \cdot \vec{p}, \{ \vec{p}', \vec{\nabla} \phi \} \} + \frac{g^{PD}}{8m^3} \{ p^2, \hat{\sigma} \cdot \vec{\nabla} \phi \} \)

- Propagator corrections: \(e^{\frac{(E_i - E_I)^2}{\omega_k^2}}\)

\[ V_{OBE}(r) = \frac{g^2}{4m^2} \left[ \left( 1 - \frac{p'^2 + p^2}{2m^2} \right) \hat{\sigma}_1 \cdot \vec{\nabla} \hat{\sigma}_2 \cdot \vec{\nabla} f(r) \right. \]

\[ \left. + i \left[ \frac{p^2}{8m^2} \left\{ \hat{\sigma}_1 \cdot \vec{p}, \hat{\sigma}_2 \cdot \vec{\nabla} f(r) \right\} + \left\{ \hat{\sigma}_2 \cdot \vec{p}, \hat{\sigma}_1 \cdot \vec{\nabla} f(r) \right\} \right] + i \hat{\sigma}_2 \cdot \vec{\nabla} \hat{\sigma}_1 \cdot \vec{\nabla} \left[ \frac{p^2}{4m^2}, \{ \vec{p}', \vec{r} f(r) \} \right] \right] \]

- Shortcomings: Non-local, non-hermitian, a function of \( r, p^2, r, p \)

- velocity in most experimental searches is small

- it is desirable to obtain functional forms with operators that are either purely static or purely velocity-dependent

Unitary Transformation

\[ H = e^{-iU} H_o e^{iU} = H_o + i[H_o, U] + \ldots \]

\[ V_{OBE} = V_{OBE}^o + i[T, U] + i[V_{OBE}^o, U] + \ldots \]

where

\[ iU = iU_1 + iU_2 = \frac{g^2}{32m^3} \left[ \left\{ \mbox{\ss}_1 \cdot \vec{p}, \mbox{\ss}_2 \cdot \vec{\nabla} f(r) \right\} + 1 \leftrightarrow 2 \right] + \mbox{\ss}_2 \cdot \vec{\nabla} \mbox{\ss}_1 \cdot \vec{\nabla} \{\vec{p}, \vec{r} f(r)\} \] .

\[ V_{OBE}^o = \frac{g^2}{4m^2} \mbox{\ss}_2 \cdot \vec{\nabla} \mbox{\ss}_1 \cdot \vec{\nabla} \left[ 1 - \left( \frac{p'^2 + p^2}{2m^2} \right) \right] e^{-\mu r} \]

Local and a function of \( r \) and \( p^2 \) only!
The range of applicability

A few issues in the massless limit (not related to our work):

- $V_{PD-PD}, V_{2\pi} \to 1/r^6$ as $\mu \to 0$ whereas the result calculated by Ferrer and Grifols in PRD 58, 096006 (1998) shows a $1/r^5$ dependence. A discontinuity in the massless limit could be due to the lack of conserved charges associated with a spin-0 field.

- The results derived with a massive spin-1 exchange diverge as $\mu \to 0$ due to the existence of non-conserved currents associated with the longitudinal polarization component. Such limit is not realized in our case.