Constraints on Exotic Spin-Dependent Long-Range Interactions from Spin-Independent Experiments

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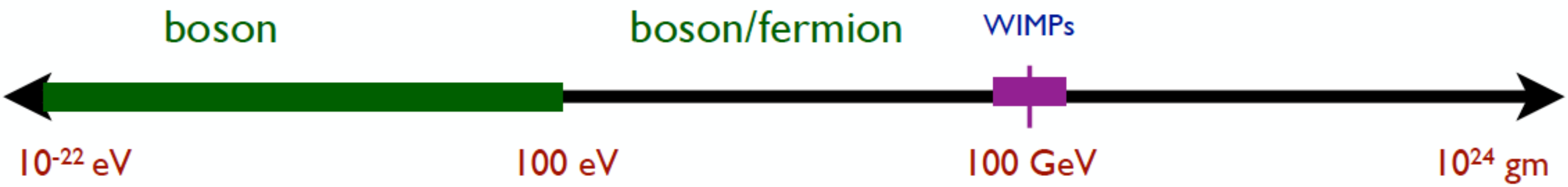
Indiana University

Talk @ 3rd Biennial Workshop on Dark Interactions, BNL
October 02-05, 2018

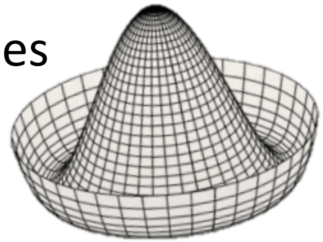
With William Michael Snow, Dennis E. Krause, Joshua C. Long
Phys. Rev. D 95, 096005 (2017) [arXiv:1611.01580] & work in progress

New Ultralight Bosons



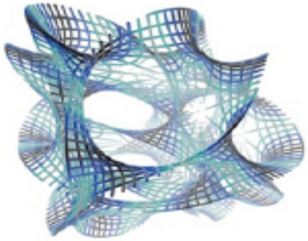
Weakly-interacting bosons are consistent with dark matter abundance in the keV - 10^{-22} eV mass range

Pseudo Nambu-Goldstone bosons (psGbs) of broken symmetries



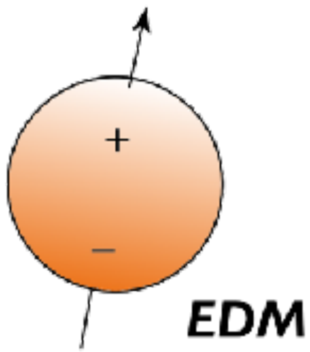
Stueckelberg Mechanism: gauge symmetry broken at high scale, but with weak coupling.

String theory or extra dimensions naturally give rise to psGbs from non-trivial topology.



The QCD axion and the strong CP problem

$$L \supset \frac{g_s^2}{32\pi^2}\bar{\theta}\; G^a_{\mu\nu}\tilde{G}^{\mu\nu}_a$$

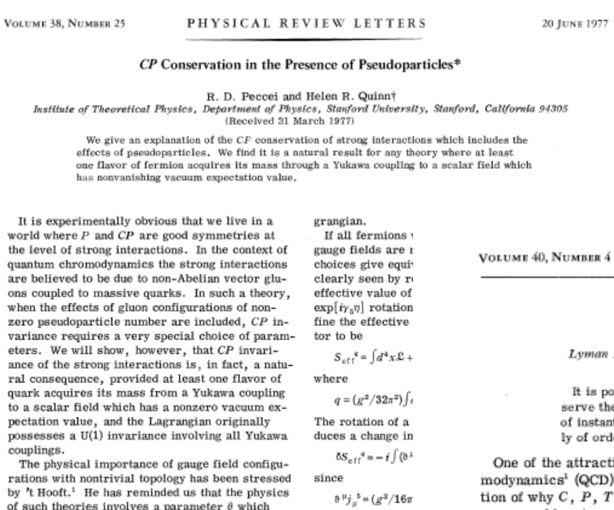


The neutron electric dipole moment

$$d_N \sim \frac{e}{m_N}\left(\frac{m_q}{m_N}\right)\left|\bar{\theta}\right| \sim 10^{-16}\left|\bar{\theta}\right| \text{ e-cm}$$

Current bound on nEDM: $\bar{\theta} \leq 10^{-10}$

Solution:
Turn $\bar{\theta}$ into a dynamical field
An axion



VOLUME 38, NUMBER 25 PHYSICAL REVIEW LETTERS 20 JUNE 1977

CP Conservation in the Presence of Pseudoparticles*

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(Received 31 March 1977)

We give an explanation of the *CP* conservation of strong interactions which includes the effects of pseudoparticles. We find it is a natural result for any theory where at least one flavor of fermion acquires its mass through a Yukawa coupling to a scalar field which has nonvanishing vacuum expectation value.

It is experimentally obvious that we live in a world where *P* and *CP* are good symmetries at the level of strong interactions. In the context of quantum chromodynamics the strong interactions are believed to be due to non-Abelian vector gluons coupled to massive quarks. In such a theory, when the effects of gluon configurations of nonzero pseudoparticle number are included, *CP* invariance requires a very special choice of parameters. We will show, however, that *CP* invariance of the strong interactions is, in fact, a natural consequence, provided at least one flavor of quark acquires its mass from a Yukawa coupling to a scalar field which has a nonzero vacuum expectation value, and the Lagrangian originally possesses a U(1) invariance involving all Yukawa couplings.

The physical importance of gauge field configurations with nontrivial topology has been stressed by 't Hooft.[1] He has reminded us that the physics of such theories involves a parameter θ which

grangian.
If all fermions
gauge fields are
choices give equi
clearly seen by r
effective value of
$\exp[i\gamma_5\eta]$ rotation
fine the effective
tor to be

$S_{eff}^{\theta} = \int d^4x \mathcal{L} +$

where

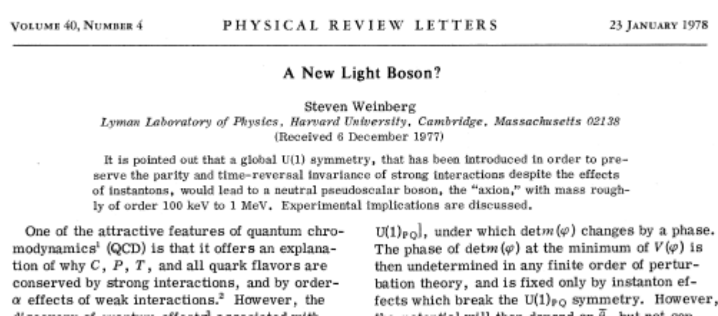
$q = (g^2/32\pi^2)\int$

The rotation of a
duces a change in

$\delta S_{eff}^{\theta} = -i\int(\partial$

since

$\partial^\mu j_\mu^5 = (g^2/16\pi$



VOLUME 40, NUMBER 4 PHYSICAL REVIEW LETTERS 23 JANUARY 1978

A New Light Boson?

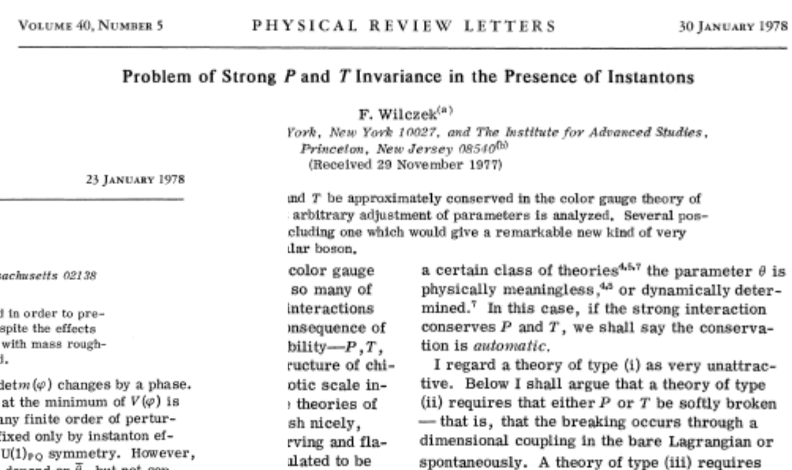
Steven Weinberg
Lyman Laboratory of Physics, Harvard University, Cambridge, Massachusetts 02138

(Received 6 December 1977)

It is pointed out that a global U(1) symmetry, that has been introduced in order to preserve the parity and time-reversal invariance of strong interactions despite the effects of instantons, would lead to a neutral pseudoscalar boson, the "axion," with mass roughly of order 100 keV to 1 MeV. Experimental implications are discussed.

One of the attractive features of quantum chromodynamics[1] (QCD) is that it offers an explanation of why *C*, *P*, *T*, and all quark flavors are conserved by strong interactions, and by order-α effects of weak interactions.[2] However, the

$U(1)_{PQ}$], under which $\det m(\varphi)$ changes by a phase. The phase of $\det m(\varphi)$ at the minimum of $V(\varphi)$ is then undetermined in any finite order of perturbation theory, and is fixed only by instanton effects which break the $U(1)_{PQ}$ symmetry. However,



VOLUME 40, NUMBER 5 PHYSICAL REVIEW LETTERS 30 JANUARY 1978

Problem of Strong *P* and *T* Invariance in the Presence of Instantons

F. Wilczek[a]
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(Received 29 November 1977)

nd *T* be approximately conserved in the color gauge theory of
arbitrary adjustment of parameters is analyzed. Several pos-
cluding one which would give a remarkable new kind of very
lar boson.

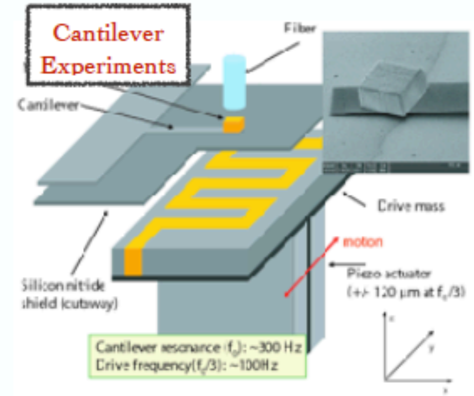
color gauge
so many of
interactions
nsequence of
bility—*P*, *T*,
ructure of chi-
otic scale in-
theories of
sh nicely,
rving and fla-
lated to be

a certain class of theories[4,5,7] the parameter θ is physically meaningless,[4,5] or dynamically determined.[7] In this case, if the strong interaction conserves *P* and *T*, we shall say the conservation is *automatic*.

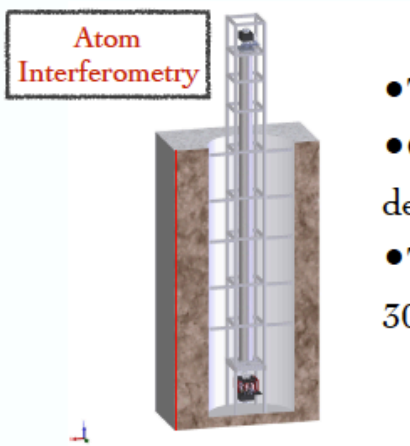
I regard a theory of type (i) as very unattractive. Below I shall argue that a theory of type (ii) requires that either *P* or *T* be softly broken — that is, that the breaking occurs through a dimensional coupling in the bare Lagrangian or spontaneously. A theory of type (iii) requires

Opportunities to probe the low energy frontier

- Short Distance Tests of Gravity
- Extra Dimensions



Dimopoulos, Kapitulnik (1997)

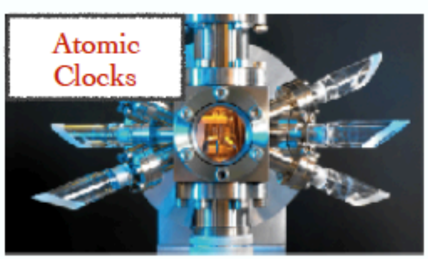


- Tests of Gravity
- Gravitational Wave detection at low frequencies
- Tests of Atom Neutrality at 30 decimals

Dimopoulos, Geraci (2003)
Dimopoulos, Kasevich et. al.(2006-2008)

- Axion Dark Matter Detection
- Axion Force Detection

Graham et. al. (2012)
AA, Geraci (2014)

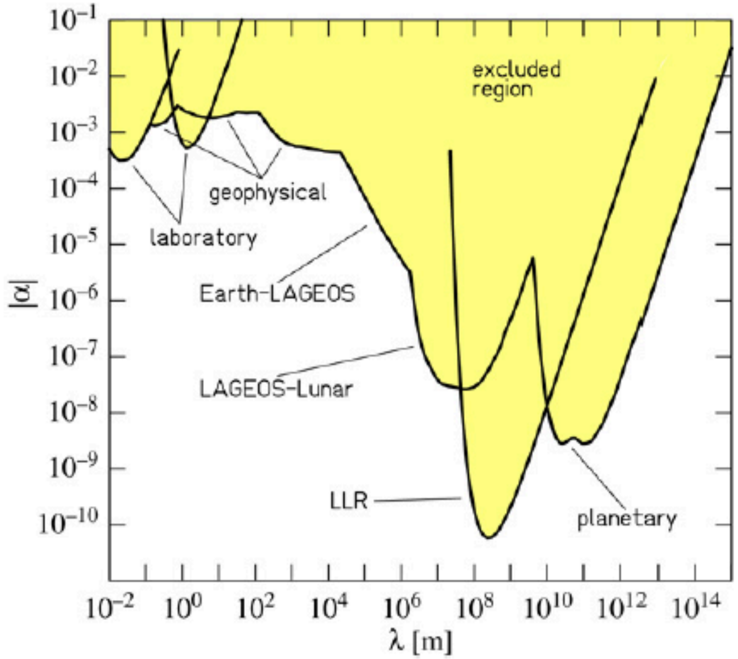


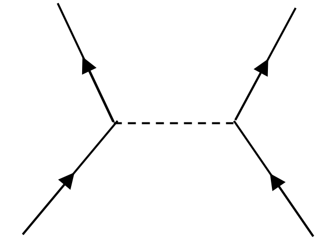
- Setting the Time Standard
- Dilaton Dark Matter Detection

AA, Huang, Van Tilburg (2014)

A. Arvanitaki

Limits on ultra-light bosons from laboratory experiments

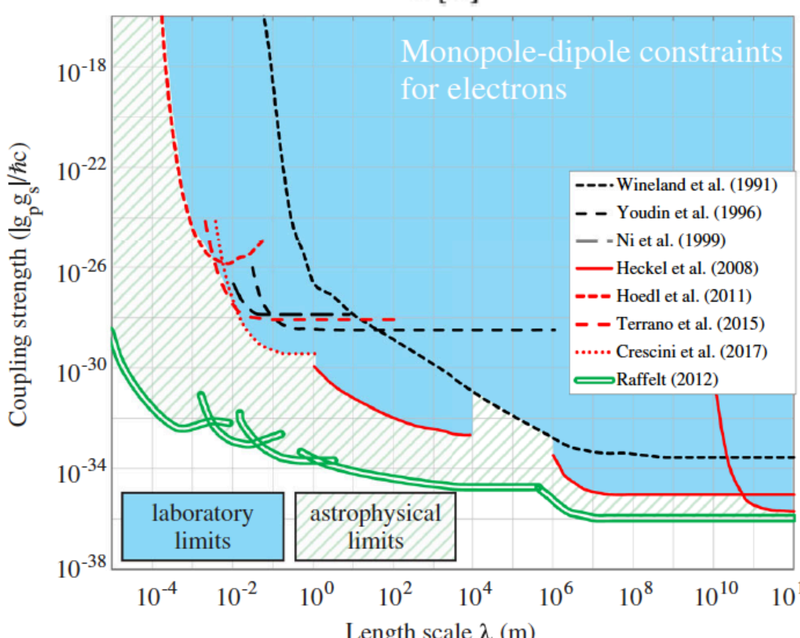


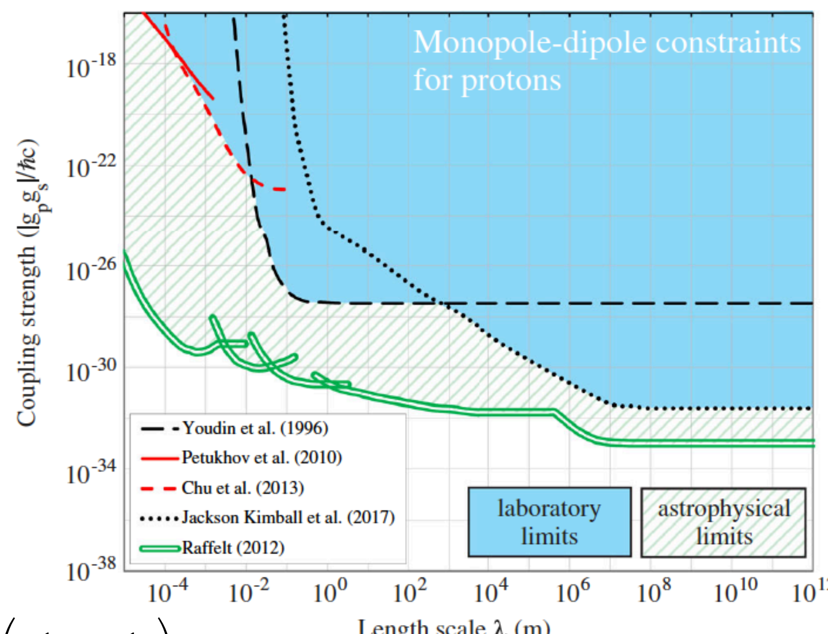


$$V(r) = -\frac{Gm_1 m_2}{r}\left(1 + \alpha e^{-r/\lambda}\right)$$

$$\alpha = \frac{\hbar c}{4\pi G m_1 m_2}\left(g_S^X g_S^Y - g_V^X g_V^Y\right)$$

$$\lambda = \frac{1}{\mu}$$





$$V_{S-P} = \frac{g_S g_P}{8\pi m}(\vec{\sigma}\cdot\hat{r})\left(\frac{1}{\lambda r} + \frac{1}{r^2}\right)e^{-r/\lambda}$$

Reviews of Modern Physics, 90, 025008 (2018).

Search for new physics with atoms and molecules

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and the University of Maryland, College Park, Maryland 20742, USA

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University of California, Berkeley, California 94720, USA,
and Nuclear Science Division, Lawrence Berkeley National Laboratory,
Berkeley, California 94720, USA

D. DeMille

Yale University, New Haven, Connecticut 06520, USA

Derek F. Jackson Kimball

California State University, East Bay, Hayward, California 94542, USA

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Joint Quantum Institute, National Institute of Standards and Technology
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(published 29 June 2018)

This article reviews recent developments in tests of fundamental physics using atoms and molecules, including the subjects of parity violation, searches for permanent electric dipole moments, tests of the *CPT* theorem and Lorentz symmetry, searches for spatiotemporal variation of fundamental constants, tests of quantum electrodynamics, tests of general relativity and the equivalence principle, searches for dark matter, dark energy, and extra forces, and tests of the spin-statistics theorem. Key results are presented in the context of potential new physics and in the broader context of similar investigations in other fields. Ongoing and future experiments of the next decade are discussed.

DOI: 10.1103/RevModPhys.90.025008

Reviews of Modern Physics, 90, 025008 (2018).

Search for new physics with atoms

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DOI: 10.1103/RevModPhys.90.025008

VII. Review of Laboratory Searches for Exotic Spin-dependent Interactions

Long-Range Interactions from Sub-GeV Dark Matter

S. Fichet, PRL, 120, 131801 (2018)

$$\mathcal{O}_a^0 = \frac{1}{\Lambda}\bar{N}N|\phi|^2, \qquad \mathcal{O}_a^{1/2} = \frac{1}{\Lambda^2}\bar{N}N\bar{\chi}\chi,$$

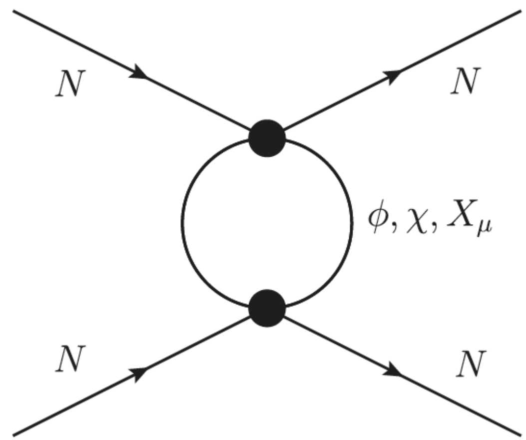
$$\mathcal{O}_b^0 = \frac{1}{\Lambda^2}\bar{N}\gamma^\mu N\phi^* i\overleftrightarrow{\partial}_\mu\phi, \qquad \mathcal{O}_b^{1/2} = \frac{1}{\Lambda^2}\bar{N}\gamma^\mu N\bar{\chi}\gamma^\mu\chi,$$

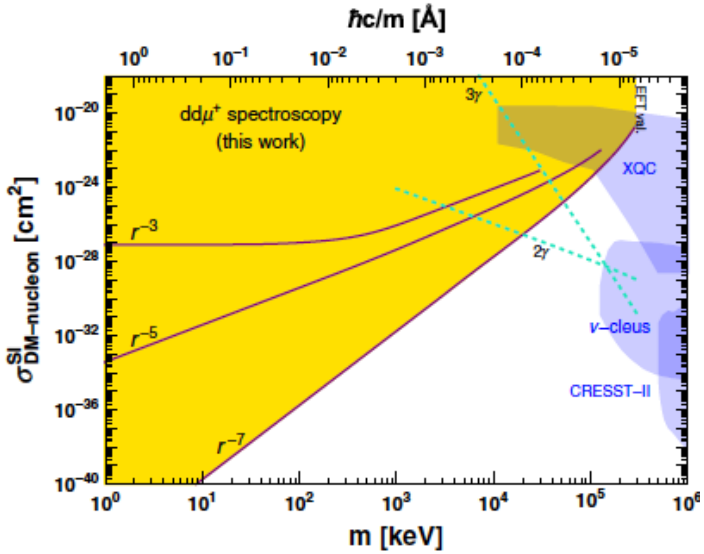
$$\mathcal{O}_c^0 = \frac{1}{\Lambda^3}\bar{N}N\partial^\mu\phi^*\partial_\mu\phi, \qquad \mathcal{O}_c^{1/2} = \frac{1}{\Lambda^2}\bar{N}\gamma^\mu N\bar{\chi}\gamma^\mu\gamma^5\chi,$$

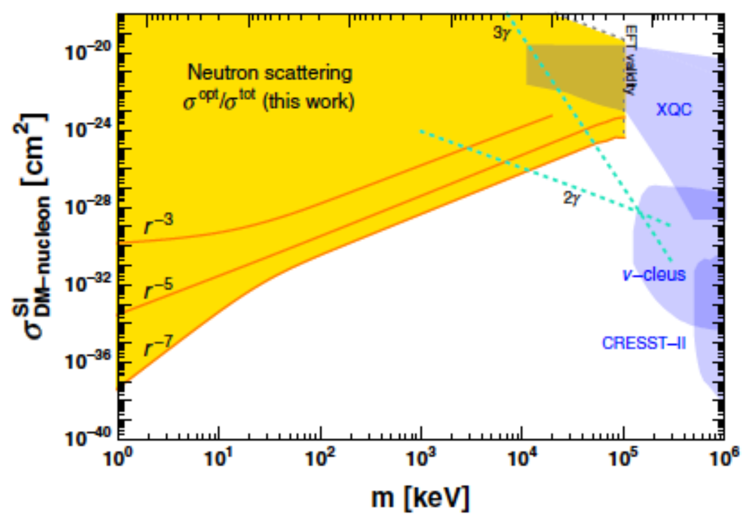
$$\mathcal{O}_a^1 = \frac{m^2}{\Lambda^3}\bar{N}N|X^\mu + \partial^\mu\pi|^2,$$

$$\mathcal{O}_b^1 = \frac{1}{\Lambda^2}2\bar{N}\gamma^\mu N\mathrm{Im}[X^*_{\mu\nu}X^\nu + \partial^\nu(X_\nu X^*_\mu) + \partial^\mu\bar{c}c^*],$$

$$\mathcal{O}_c^1 = \frac{1}{\Lambda^3}\bar{N}N|X^{\mu\nu}|^2, \qquad \mathcal{O}_d^1 = \frac{1}{\Lambda^3}\bar{N}NX^{\mu\nu}\tilde{X}^{\mu\nu},$$







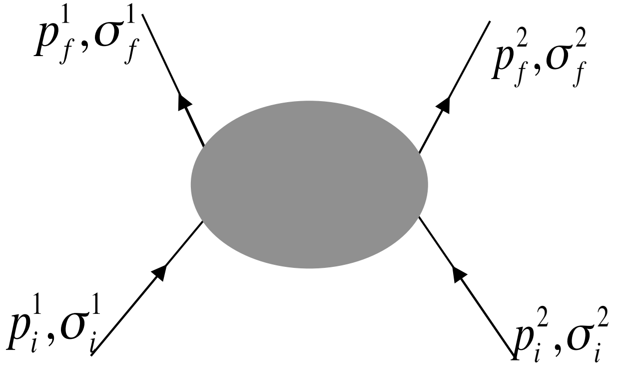
Outline:

- Current limits on exotic spin-dependent long-range interactions.

- Calculations of spin-independent interactions from double-boson exchange processes with spin-dependent couplings to fermions.

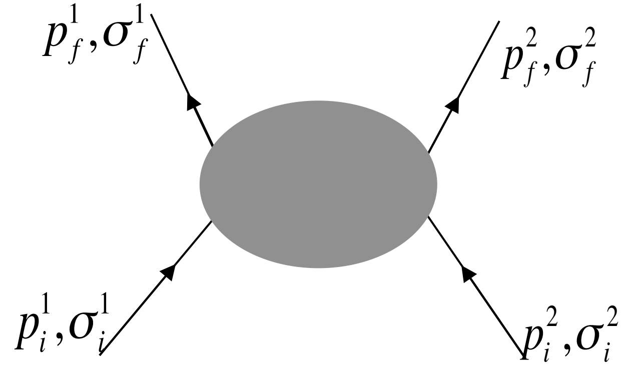
- Improving the constraints using spin-independent experiments

Model-independent forms of long-range interactions between a pair of non-relativistic fermions



- Interaction is rotationally invariant
- Only depend on $\mathbf{r}^{12}$, $\mathbf{p}^{(i)}$, and $\boldsymbol{\sigma}^{(i)}$, with i=1,2
- Local
- Satisfy energy-momentum conservation

Model-independent forms of long-range interactions between a pair of non-relativistic fermions



- Interaction is rotationally invariant
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- Local
- Satisfy energy-momentum conservation

$$\mathcal{O}_1 = 1$$

$$\mathcal{O}_2 = \vec{\sigma} \cdot \vec{\sigma}'$$

$$\mathcal{O}_3 = \frac{1}{m^2} (\vec{\sigma} \cdot \vec{q}) \, (\vec{\sigma}' \cdot \vec{q})$$

$$\mathcal{O}_{4,5} = \frac{i}{2m^2} (\vec{\sigma} \pm \vec{\sigma}') \cdot \left(\vec{P} \times \vec{q}\right)$$

$$\mathcal{O}_{6,7} = \frac{i}{2m^2} \left[\left(\vec{\sigma} \cdot \vec{P}\right) (\vec{\sigma}' \cdot \vec{q}) \pm (\vec{\sigma} \cdot \vec{q}) \left(\vec{\sigma}' \cdot \vec{P}\right)\right]$$

$$\mathcal{O}_8 = \frac{1}{m^2} \left(\vec{\sigma} \cdot \vec{P}\right) \left(\vec{\sigma}' \cdot \vec{P}\right)$$

$$\mathcal{O}_{9,10} = \frac{i}{2m} (\vec{\sigma} \pm \vec{\sigma}') \cdot \vec{q}$$

$$\mathcal{O}_{11} = \frac{i}{m} (\vec{\sigma} \times \vec{\sigma}') \cdot \vec{q}$$

$$\mathcal{O}_{12,13} = \frac{1}{2m} (\vec{\sigma} \pm \vec{\sigma}') \cdot \vec{P}$$

$$\mathcal{O}_{14} = \frac{1}{m} (\vec{\sigma} \times \vec{\sigma}') \cdot \vec{P}$$

$$\mathcal{O}_{15} = \frac{1}{2m^3} \left\{ \left[\vec{\sigma} \cdot \left(\vec{P} \times \vec{q}\right)\right] (\vec{\sigma}' \cdot \vec{q}) + (\vec{\sigma} \cdot \vec{q}) \left[\vec{\sigma}' \cdot \left(\vec{P} \times \vec{q}\right)\right] \right\}$$

$$\mathcal{O}_{16} = \frac{i}{2m^3} \left\{ \left[\vec{\sigma} \cdot \left(\vec{P} \times \vec{q}\right)\right] \left(\vec{\sigma}' \cdot \vec{P}\right) + \left(\vec{\sigma} \cdot \vec{P}\right) \left[\vec{\sigma}' \cdot \left(\vec{P} \times \vec{q}\right)\right] \right\}$$

$$V(\vec{r},\vec{p}) = \sum_i f_i^{X,Y} \, \mathrm{O}_i(\vec{q},\vec{p}) \, \mathrm{y(r)}$$

XY= ee, ep, en, pp, nn, np.

M: mass the fermion, μ: boson mass

r: separation between the fermions

B. Dobrescu and I. Mocioiu J. High Energy Phys. 11 (2006) 005.

J. E. Moody and F. Wilczek, Phys. Rev. D 30, 130 (1984).

Model-independent forms of long-range interactions between a pair of non-relativistic fermions

- Interaction is rotationally invariant
- Only depend on $\mathbf{r}^{12}$, $\mathbf{p}^{(i)}$, and $\boldsymbol{\sigma}^{(i)}$, with i=1,2
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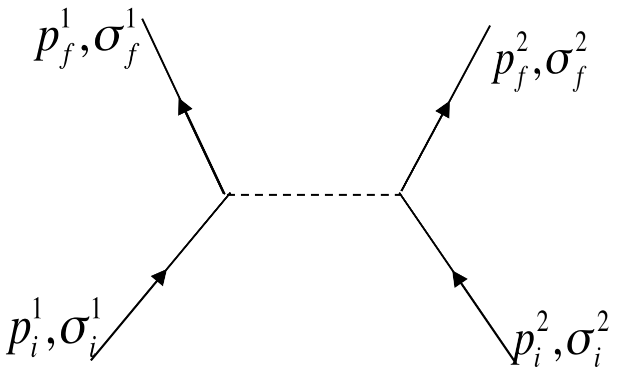

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$$\mathcal{O}_{11} = \frac{i}{m} (\vec{\sigma} \times \vec{\sigma}') \cdot \vec{q}$$

$$\mathcal{O}_{12,13} = \frac{1}{2m} (\vec{\sigma} \pm \vec{\sigma}') \cdot \vec{P}$$

$$\mathcal{O}_{14} = \frac{1}{m} (\vec{\sigma} \times \vec{\sigma}') \cdot \vec{P}$$

$$\mathcal{O}_{15} = \frac{1}{2m^3} \left\{\left[\vec{\sigma} \cdot \left(\vec{P} \times \vec{q}\right)\right] (\vec{\sigma}' \cdot \vec{q}) + (\vec{\sigma} \cdot \vec{q}) \left[\vec{\sigma}' \cdot \left(\vec{P} \times \vec{q}\right)\right]\right\}$$

$$\mathcal{O}_{16} = \frac{i}{2m^3} \left\{\left[\vec{\sigma} \cdot \left(\vec{P} \times \vec{q}\right)\right] \left(\vec{\sigma}' \cdot \vec{P}\right) + \left(\vec{\sigma} \cdot \vec{P}\right) \left[\vec{\sigma}' \cdot \left(\vec{P} \times \vec{q}\right)\right]\right\}$$

spin-0 and spin-1
single boson exchange

$$y(r) = \frac{e^{-\mu r}}{4\pi r}$$

$$V(\vec{r},\vec{p}) = \sum_i f_i^{X,Y} O_i(\vec{q},\vec{p})\, y(r)$$

XY= ee, ep, en, pp, nn, np.
M: mass the fermion, μ: boson mass
r: separation between the fermions

B. Dobrescu and I. Mocioiu J. High Energy Phys. 11 (2006) 005.
J. E. Moody and F. Wilczek, Phys. Rev. D 30, 130 (1984).

Coefficients f_i in terms of physical couplings g

Parameter	$s=0$	$s=1$
f_2^{ee}	0	$(g_A^e)^2$
f_3^{ee}	$-\frac{1}{4}(g_P^e)^2$	$\frac{1}{4}[(g_V^e)^2+(g_A^e)^2]$
f_{11}^{ee}	0	$g_A^e g_V^e$
f_{6+7}^{ee}	0	$g_A^e g_V^e$[a]
f_8^{ee}	0	$-\frac{5}{4}(g_A^e)^2$
f_{14}^{ee}	0	$(g_A^e)^2$[a]
f_{15}^{ee}	0	$(g_V^e)^2$[a]
f_{16}^{ee}	0	$g_A^e g_V^e$[a]
$f_\perp^{ee}+f_\perp^{ep}+f_\perp^{en}$	$\frac{1}{2}g_S^e[g_S^e+g_S^p+g_S^n]$	$\frac{1}{2}[3(g_V^e)^2+(g_A^e)^2+g_V^e g_V^p+g_V^e g_V^n]$
$f_r^{ee}+f_r^{ep}+f_r^{en}$	$g_P^e[g_S^e+g_S^p+g_S^n]$	$g_A^e[g_V^e+g_V^p+g_V^n]$[a]
$f_v^{ee}+f_v^{ep}+f_v^{en}$	0	$2g_A^e[2g_V^e+g_V^p+g_V^n]$

T. M. Leslie, E. Weisman, R. Khatiwada, and J. C. Long, Phys. Rev. D 89, 114022 (2014).

Current Limits from Laboratory Experiments

- Very stringent limits on possible interactions arising from scalar and vector couplings from spin-independent experiments. In contrast, limits on pseudoscalar and axial couplings are many orders of magnitude weaker

- Limits on pseudoscalar couplings to nucleons are more suppressed than their electron analogs.

Type of coupling	μ≈[100 μeV, 10 meV]	
$g_S^2,\ g_V^2$	$10^{-40}-10^{-35}$	1
$\left(g_P^e\right)^2$	$10^{-16}-10^{-8}$	2
$\left(g_P^N\right)^2$	$10^{-4}-10^{-6}$	3
$\left(g_A^N\right)^2$	$10^{-15}-10^{-12}$	4
$\left(g_A^e\right)^2$	$10^{-32}-10^{-29}$	5
$g_S g_P^e$	$10^{-28}-10^{-21}$	6
$g_S g_P^N$	$10^{-25}-10^{-17}$	7
$g_V g_A^e$	$10^{-26}-10^{-23}$	8
$g_V g_A^N$	$10^{-29}-10^{-25}$	9

[1] R. S. Decca, et. al, PRL 116 (2016) 221102.
[2] W. A. Terrano, et. al, PRL 115 (2015) 20801.
[3] M. P. Ledbetter et. al, PRL 110, 040402 (2013), N. F. Ramsey, Physica A (Amsterdam) 96, 285 (1979).
[4] C. Haddock, et. al, PLB 783, (2018) 227.
[5,8] T. M. Leslie, et. al, Phys. Rev. D 89 (2014) 114022.
[6] B. R. Heckel, et. al, PRL 111 (2013) 15802.
[7] K. Tullney, et. al, PRL 111 (2013)100801, M. Bulatowicz, et. al, PRL 111(2013)102001(IU), A. K. Petukov, et. al, PRL 105 (2010) 170401.
[9] H. Yan, et. al, PRL 110 (2013) 082003.

Why are Limits on Pseudoscalar and Axial Couplings Poor?

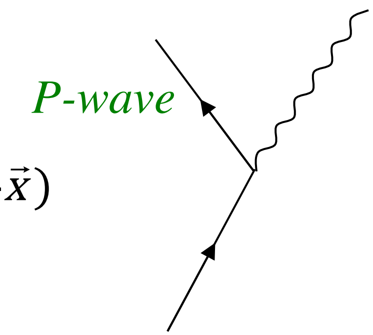
1. Pseudoscalar and axial interactions are suppressed by factors of μ / m per vertex relative to scalar and vector interactions for nonrelativistic motion

$$g_S \bar{\psi}\psi\varphi(t,\vec{x}) \xrightarrow{\mathbf{P}} g_S \bar{\psi}\psi\varphi(t,-\vec{x})$$

$$g_V \bar{\psi}\gamma^\mu\psi A_\mu(t,\vec{x}) \xrightarrow{\mathbf{P}} g_V \bar{\psi}\gamma^\mu\psi A_\mu(t,-\vec{x})$$

$$g_P i\bar{\psi}\gamma_5\psi\varphi(t,\vec{x}) \xrightarrow{\mathbf{P}} -g_P i\bar{\psi}\gamma_5\psi\varphi(t,-\vec{x})$$

$$g_A \bar{\psi}i\gamma^\mu\gamma_5\psi A_\mu(t,\vec{x}) \xrightarrow{\mathbf{P}} -g_A \bar{\psi}i\gamma_5\gamma^\mu\psi A_\mu(t,-\vec{x})$$



Why are Limits on Pseudoscalar and Axial Couplings Poor?

1. Pseudoscalar and axial interactions are suppressed by factors of μ / m per vertex relative to scalar and vector interactions for nonrelativistic motion

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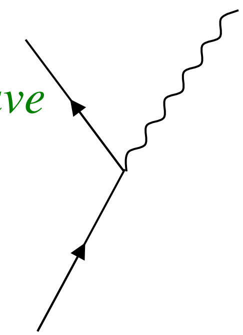
P-wave



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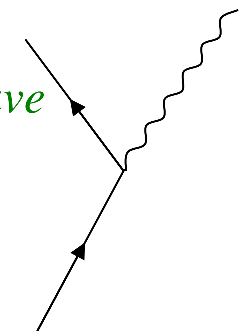


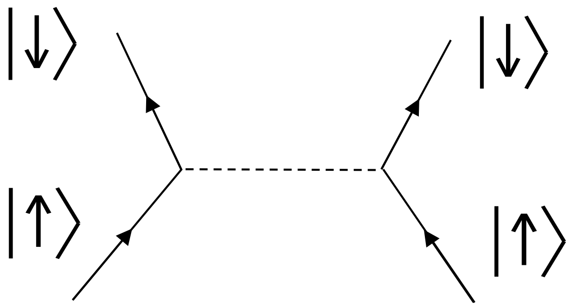
ex: for a nucleon and a 1 meV boson $\frac{\mu}{m} \approx 10^{-15}$

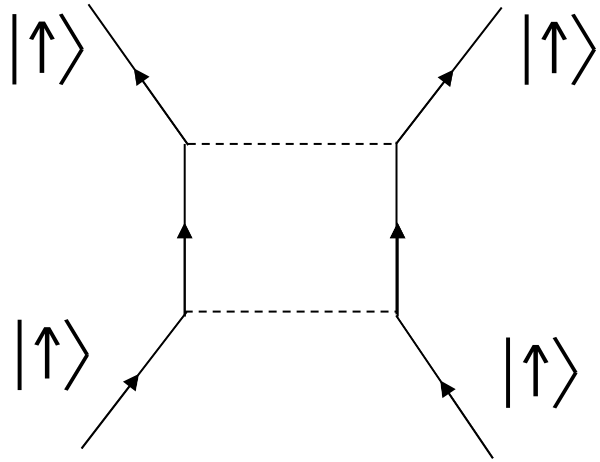
2. Necessarily spin-dependent at lowest order

Experimental difficulties associated with polarization:

- Only the valence fermions are accessible
- Large external magnetic fields.
- Polarization techniques vary widely in efficiency.
- Difficult to maintain polarization of members of the ensemble.
- Both internal and external magnetic fields produce large systematic effects in delicate experiments.
- ….

Two-Boson Exchange





Aim of this work:

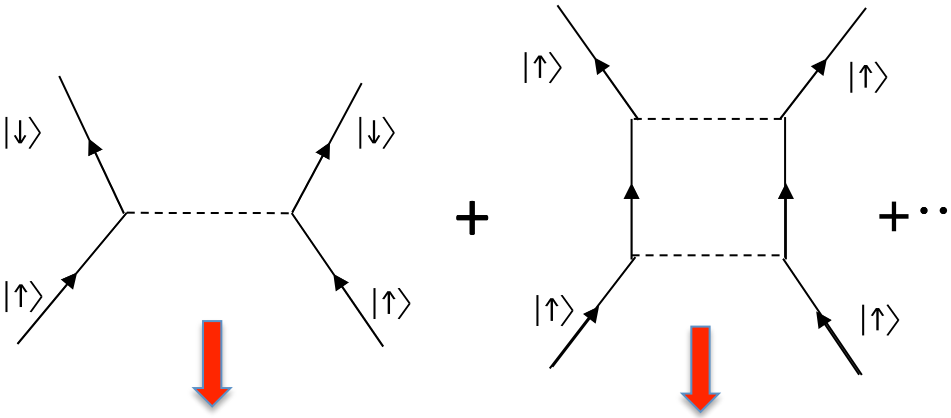
- derive all long-range *spin-independent* interactions from double boson exchange from spin-0 and spin-1 with *spin-dependent* couplings between a pair of NR Dirac fermions

- Use the derived expressions with the strong spin-independent limits to help improve constraints on interactions involving pseudoscalar and axial couplings.

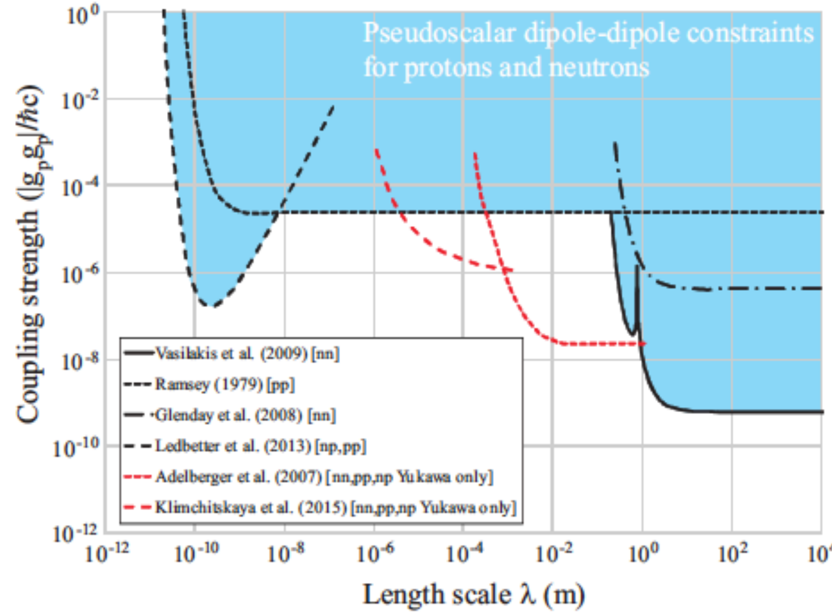
$$L_P = -g_P \; \bar{\psi} i \gamma_5 \psi \varphi$$

Previous Work

$K_1(x)$: modified Bessel function of the second kind



$$V_P = \left[\frac{g_P^2}{16\pi m^2} \vec{\sigma}_a \cdot \nabla \vec{\sigma}_b \cdot \nabla \frac{e^{-\mu r}}{r} - \frac{g_P^4}{m^2} \mu \frac{K_1(2\mu r)}{4\pi^2 r^2} + \cdots \right.$$



Blue → spin-dependent

Red → spin-independent

[E. Fischbach and D. E. Krause, (1999)]

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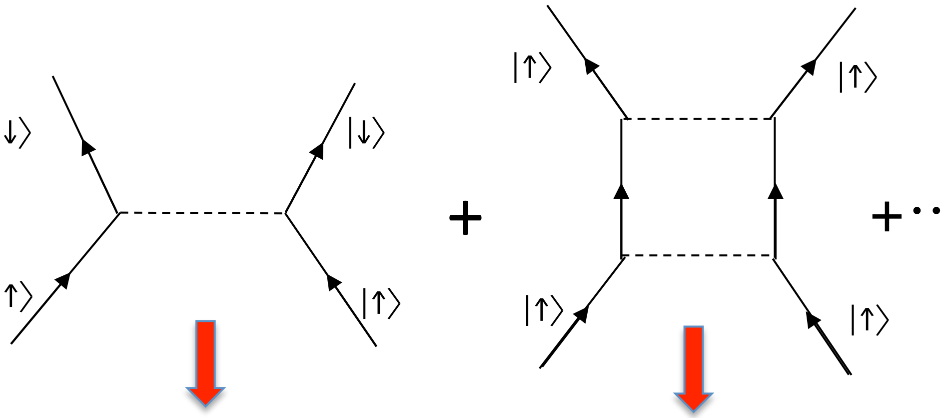

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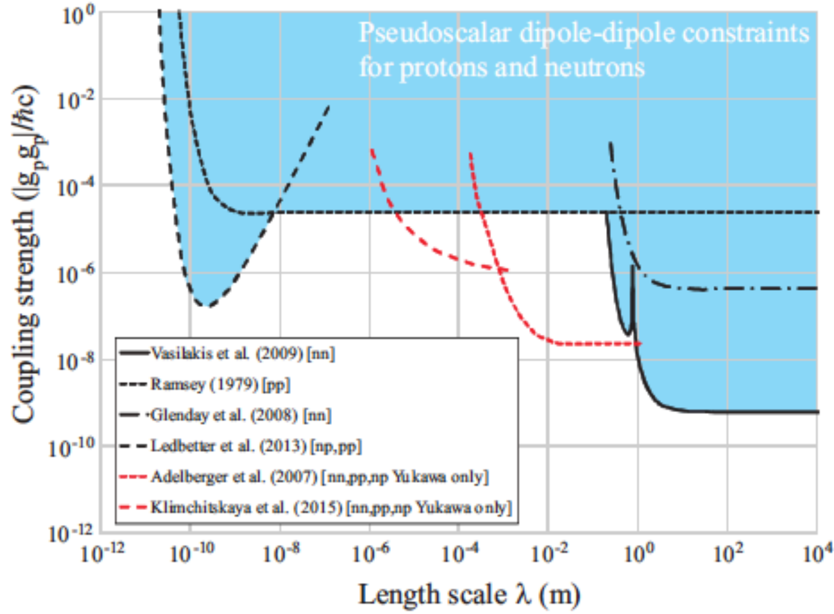

- No similar analysis done and no functional forms exist for interactions $\propto g_S^2 g_P^2, g_V^2 g_A^2,\ g_{PD}^4,\ g_S^2 g_{PD}^2,$ and g_A^4

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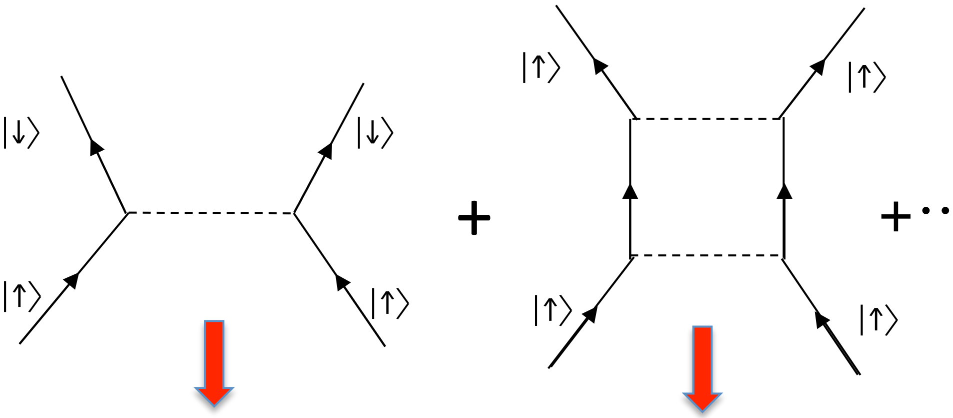

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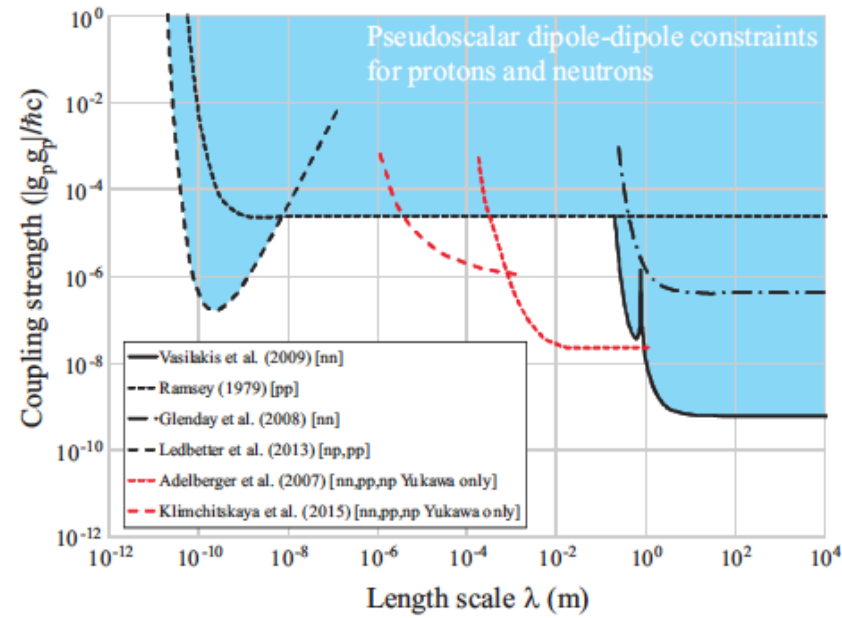

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- Limits do not necessarily apply to the pseudoscalar derivative coupling

KSVZ axion

$$m\,\bar{\psi} e^{i\gamma_5 \varphi / f_a} \psi \approx -i \frac{m}{f_a} \bar{\psi}\gamma_5 \psi \varphi + \frac{m}{2f_a^2} \bar{\psi}\psi\varphi^2 + \cdots$$

field redefinitions →

$$\frac{1}{2f_a} \bar{\psi}\gamma^\mu \gamma_5 \psi \partial_\mu \varphi$$

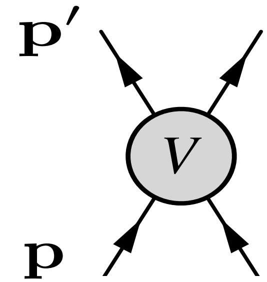
$$g_{PD} = \frac{m}{f_a}$$

Calculating an Interaction Energy

- Use non-relativistic Old-Fashioned Perturbation Theory (OFPT) to calculate a transition amplitude and relate to a potential via the Lippmann-Schwinger (LS) equation

$$\langle f|\,\mathcal{T}\,|i\rangle = \langle f|\,V\,|i\rangle + \sum_n \frac{\langle f|\,V\,|n\rangle\,\langle n|\,\mathcal{T}\,|i\rangle}{E_i - E_n + i\epsilon}.$$

The Green function of two fermions G_o



- No self-energy terms are considered since masses and couplings are taken from experiments.

- Methods based on UT: Okubo (Prog. Theor. Phys. **12**, 603 1954), Epelbaum (Nucl. Phys. A **637**, 107 1998)

- Methods based on covariant PT: Dispersion Method (Feinberg and Sucher), EFT(Holstein), Chiral EFT (Kaiser, Machledit, etc.).

Effective Hamiltonian

$$H_{int} = \int d^3\vec{x}\,\overline{\psi}(\vec{x},t)[(g_S + ig_P\gamma_5)\phi(\vec{x},t) + (g_V\gamma^\mu + g_A\gamma^\mu\gamma_5)A_\mu(\vec{x},t)]\psi(\vec{x},t)$$

$$H_{PD} = \int d^3\vec{x}\left[\frac{g_{PD}}{2m}\overline{\psi}(\vec{x},t)\gamma_5\gamma_\mu\partial^\mu\phi(\vec{x},t)\psi(\vec{x},t) + \frac{1}{2}\left(\frac{g_{\mathrm{PD}}}{2m}\overline{\psi}(\vec{x},t)\gamma^0\gamma_5\psi(\vec{x},t)\right)^2\right]$$

Up to *O(1/m)* and *O(1/m²)* in the expansion :

$$H_S^{eff} = g_S\phi + \frac{1}{8m^2}\vec{p}\cdot\vec{\nabla}\phi + \frac{1}{8m^2}\,\vec{\sigma}\cdot(\vec{\nabla}\times\vec{p})\phi - \frac{g_S}{4m^2}\{p^2,\phi\},$$

$$H_P^{eff} = -\frac{g_P}{2m}\vec{\sigma}\cdot\vec{\nabla}\phi + \frac{g_P^2}{2m}\phi^2,$$

$$H_V^{eff} = g_V A_0 - \frac{g_V}{2m}\{\vec{p},\cdot\vec{A}\} - \frac{g_V}{2m}\vec{\sigma}\cdot\vec{\nabla}\times\vec{A} + \frac{g_V^2}{2m}\vec{A}^2,$$

$$H_A^{eff} = -g_A\vec{\sigma}\cdot\vec{A} + \frac{g_A}{2m}\{\vec{\sigma}\cdot\vec{p},A_0\} + \frac{g_A^2}{2m}A_0^2,$$

$$H_{PD}^{eff} = -\frac{g_{\mathrm{PD}}}{2m}\,\vec{\sigma}\cdot\vec{\nabla}\,\phi - \frac{g_{\mathrm{PD}}}{4m^2}\,\{\vec{\sigma}\cdot\vec{p},\,\partial_0\phi\} + \frac{g_{\mathrm{PD}}^2}{8m^3}\,(\partial_0\phi)^2 + \frac{g_{\mathrm{PD}}}{8m^3}\,\{p^2,\,\vec{\sigma}\cdot\vec{\nabla}\phi\}$$

$$-i\frac{g_{\mathrm{PD}}}{16m^3}\,\{\vec{\sigma}\cdot\vec{p},\,\{\vec{p}\cdot,\,\vec{\nabla}\phi\}\} + \frac{g_{\mathrm{PD}}}{16m^3}\,\vec{\sigma}\cdot\vec{\nabla}\,\partial_0^2\phi.$$

Power counting

- An order by order expansion in 1/m:

$$T = T^{(-1)} + T^{(0)} + T^{(1)} + T^{(2)} + ...,$$

$$V = V^{(-1)} + V^{(0)} + V^{(1)} + V^{(2)} + ...,$$

- Iterations of the LS eq

$$V + VG_oV + VG_oVG_oV + ...$$

Type of interaction	number of contributing TOD diagrams
g_P^4	2
$g_S^2 g_P^2$	6
$g_S^2 g_{PD}^2$	≥ 24
$g_V^2 g_A^2$	≥ 60
g_{PD}^4	≥ 158
g_A^4	≥ 78

- To order g^4 :

$$V^{(-1)} = T^{(-1)} - [V^{(0)}G_oV^{(0)}]$$

$$V^{(0)} = T^{(0)} - [V^{(0)}G_oV^{(1)} + V^{(1)}G_oV^{(0)}]$$

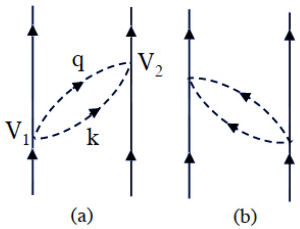
$$V^{(1)} = T^{(1)} - [V^{(1)}G_oV^{(1)} + V^{(2)}G_oV^{(0)} + V^{(0)}G_oV^{(2)}]$$

$$V^{(2)} = T^{(2)} - [V^{(2)}G_oV^{(1)} + V^{(1)}G_oV^{(2)} + V^{(3)}G_oV^{(0)} + V^{(0)}G_oV^{(3)}]$$

Results for the spin-independent long-range interaction from two boson exchange

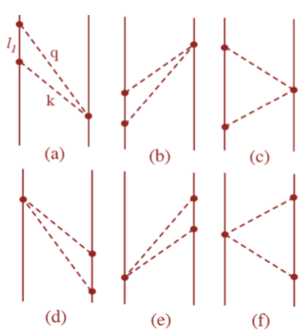
Range of applicability:
$r \geq \frac{1}{\mu} >> \frac{1}{m}$
with finite boson mass

$$V^{(0)} = T^{(0)}$$



$$V_{P-P} = -\frac{g_{P,1}^2 g_{P,2}^2}{4m_1 m_2}\frac{\mu K_1(2\mu r)}{8\pi^3 r^2}$$

Agrees with
S. D. Drell and K. Huang, Phys. Rev. 91, 1527 (1953).
F. Ferrer and J. A. Grifols, Phys. Rev. D 58, 096006 (1998).



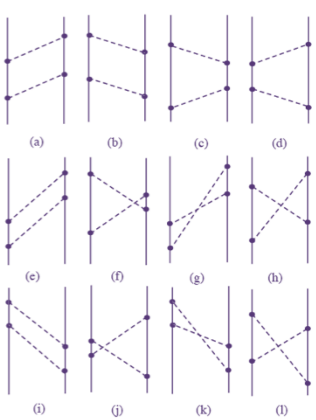
$$V_{S-P} = \left(\frac{g_{S,1}^2 g_{P,2}^2}{2m_2} + \frac{g_{S,2}^2 g_{P,1}^2}{2m_1}\right)\frac{e^{-2\mu r}}{16\pi^2 r^2}$$

$$V_{V-A}(r) \simeq \left[\frac{g_{V,1}^2 g_{A,2}^2}{2m_1} + \frac{g_{V,2}^2 g_{A,1}^2}{2m_2} + \left(\frac{g_{V,2}^2 g_{A,1}^2}{2m_1} + \frac{g_{V,1}^2 g_{A,2}^2}{2m_2}\right)\right]\frac{e^{-2\mu r}}{16\pi^2 r^2} + \ldots\ldots$$

S. Aldaihan, D. E. Krause, J. C. Long, and W. M. Snow, Phys. Rev. D 95, 096005 (2017).

$$V^{(1)} = T^{(1)} - [V^{(1)}G_o V^{(1)} + V^{(2)}G_o V^{(0)} + V^{(0)}G_o V^{(2)}]$$

$$x = \mu r$$



$$\overline{V}_{PD-PD} = -\frac{g_{PD}^4}{512 m^5}\frac{e^{-2x}}{2r^6}\left[5\left(6 + 12x + 10x^2 + 4x^3 + x^4\right) + 5x^2(x+1)^2 + 2x^5\right] + \ldots\ldots$$

$$\overline{V}_{S-PD} = \frac{g_S^2\, g_{PD}^2}{128\pi^2 m^3}\frac{e^{-2x}}{r^4}(x+1)[2x^2 - 3x - 3] + \ldots\ldots$$

$$\overline{V}_{A-A} = \frac{3g_A^4}{64\pi^2 m}\frac{e^{-2x}}{r^2}(5 - 2x) + \ldots\ldots$$

Agrees (up to a minor discrepancy) with
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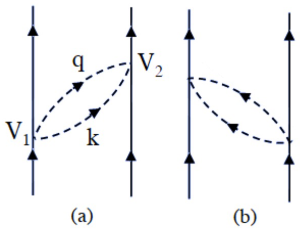

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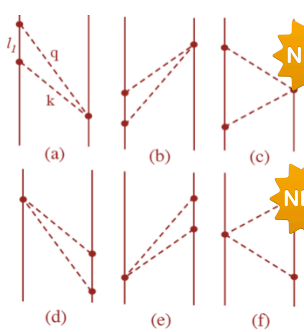

NEW $$V_{S-P} = \left(\frac{g_{S,1}^2 g_{P,2}^2}{2m_2} + \frac{g_{S,2}^2 g_{P,1}^2}{2m_1}\right)\frac{e^{-2\mu r}}{16\pi^2 r^2}$$

S. Aldaihan, D. E. Krause, J. C. Long, and W. M. Snow, Phys. Rev. D 95, 096005 (2017).

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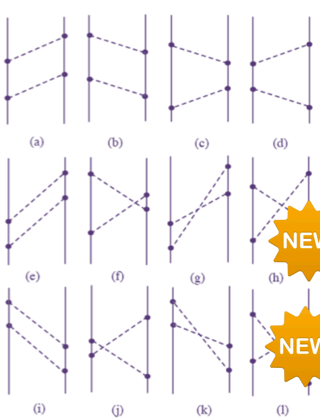

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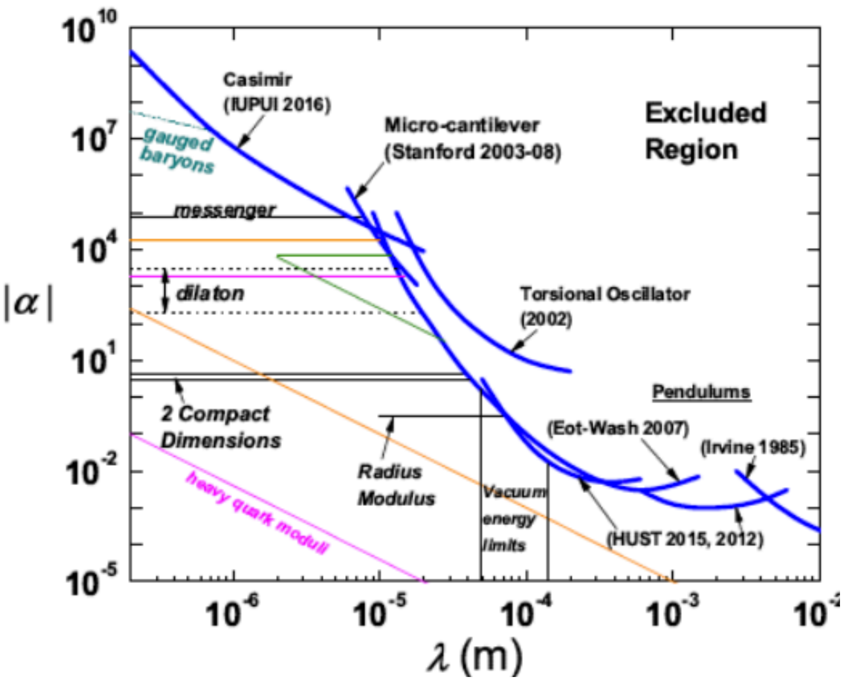
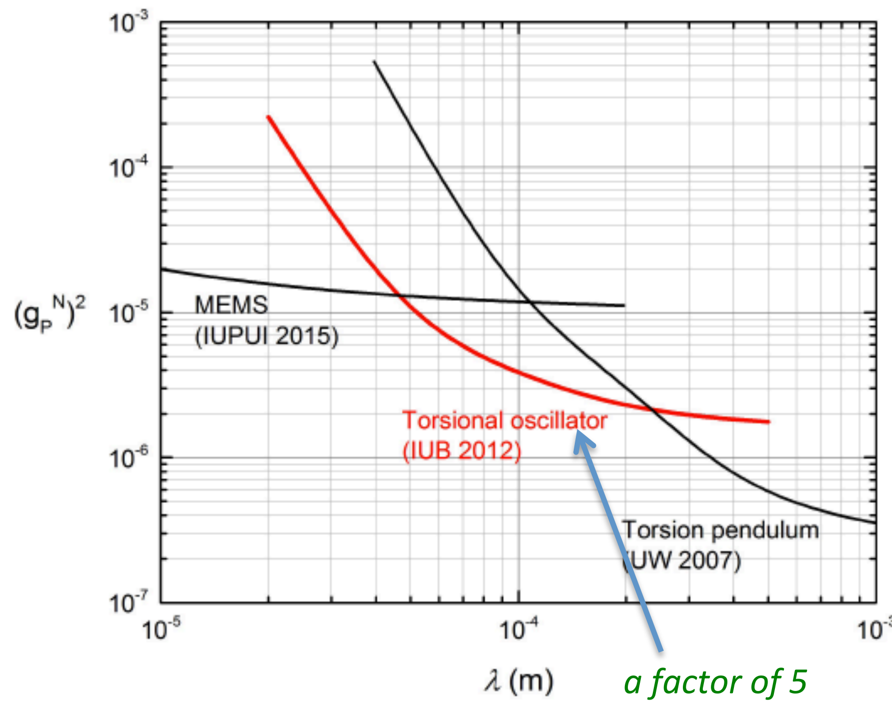
Constraints from Spin-Independent Experiments





a factor of 5 improvement

$$V(r) = -\frac{G m_1 m_2}{r}\left(1 + \alpha e^{-r/\lambda}\right)$$

$$\alpha = \frac{\hbar c}{4\pi G m_1 m_2}\left(g_S^X g_S^Y - g_V^X g_V^Y\right)$$

$$\approx 10^{-38}\left(g_S^X g_S^Y - g_V^X g_V^Y\right)$$

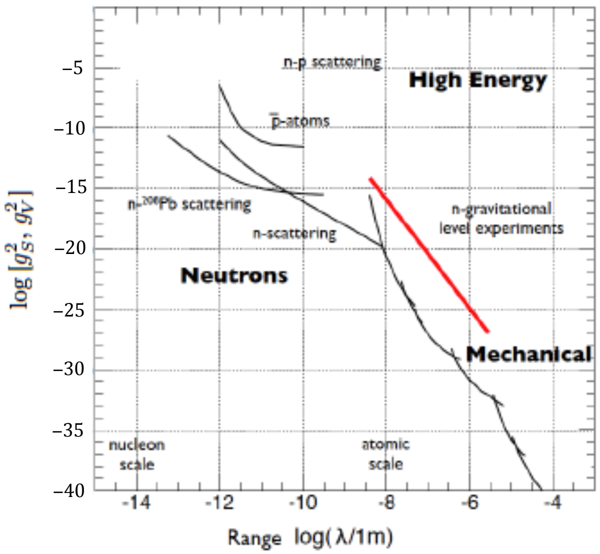
$$\lambda = 1/\mu$$

$$V_{P-P}(r) = -\frac{g_P^4}{4m^2 r^2}\frac{K_1(2r/\lambda)}{8\pi^2\lambda}$$

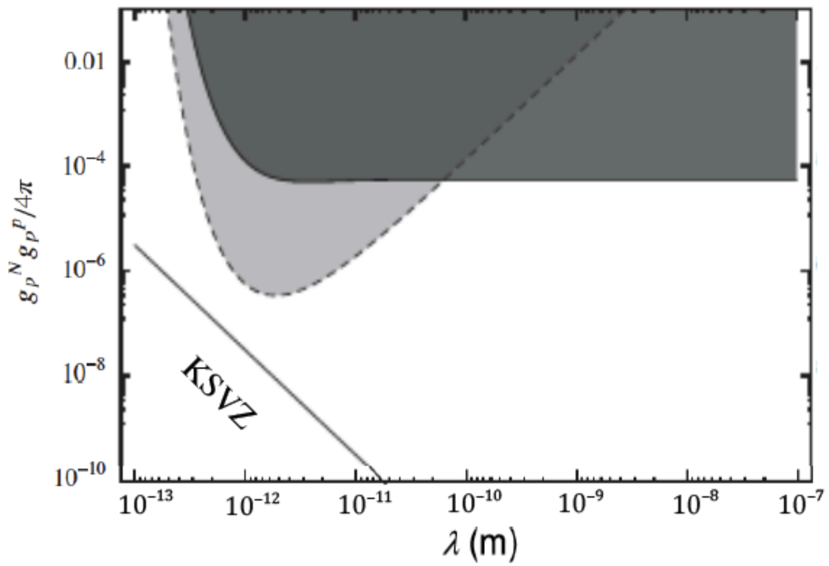
an existence proof that sensitive experimental searches for spin-independent interactions can also yield interesting limits on spin-dependent interactions at certain distance scales.

S. Aldaihan, D. E. Krause, J. C. Long, and W. M. Snow, Phys. Rev. D 95, 096005 (2017).

Further Opportunities



Constraints on the scalar coupling from spin-independent experiments testing interactions between normal matter across a wide length scale



Constraints from spin-dependent experiments between protons and neutrons

- $V_{PD\text{-}PD} \propto 1/r^6$ ⇨ sensitive to short-length scales.
- No constrains on $(g^N_{PD})^2$ below 10^{-12} m, a theoretically interesting regime.

$$V_P = \frac{g_P^2}{4\pi}\left(\frac{\mu}{2m}\right)^2\left[(\vec{\sigma}_1\cdot\vec{\sigma}_2)+S_{12}\left(\frac{1}{3}+\frac{1}{x}+\frac{1}{x^2}\right)\right]\frac{e^{-\mu r}}{r}$$

$$S_{12}=3(\vec{\sigma}_1\cdot\hat{r})(\vec{\sigma}_2\cdot\hat{r})-\vec{\sigma}_1\cdot\vec{\sigma}_2$$

M. P. Ledbetter, M.V. Romalis. PRL 110, 040402 (2013).

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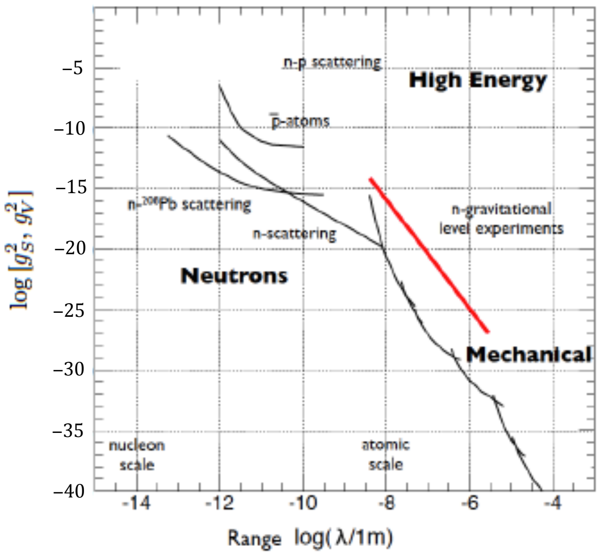

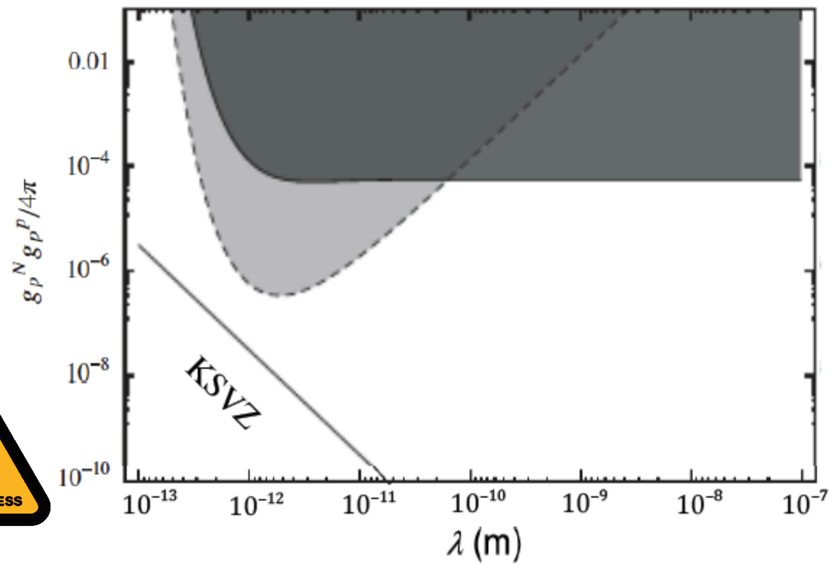

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Conclusion:

- Ultralight bosons, including axions, generate long-range interactions of various types that can be probed with precision laboratory experiments.

- Experiments testing spin-dependent interactions experience additional challenges that do not exist in spin-independent experiments.

- The functional forms derived from 2-boson exchange processes open up an opportunity to constrain, using spin-independent data, spin-dependent couplings over new length scales that are outside the sensitivity of current spin-dependent experiments.

Conclusions:

- ❖ We derived the leading spin-independent long-range contribution to the position-dependent part of the potential arising from the exchange of two light bosons involving pseudoscalar and axial Yukawa couplings between two massive Dirac fermions.

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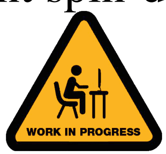

- ❖ The calculation of non-static terms is more involved and evaluating all contributions for all possible combinations is still a work in progress.

- ❖ It is desirable to avoid ambiguities with the potential by performing a calculation of the interaction energy instead.

- ❖ Analysis of the derived functional forms with data from spin-independent experiments over a broad length scale and across various species of fermions.

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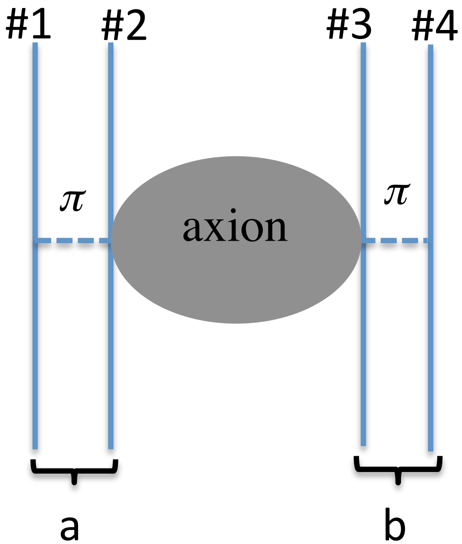
Two Bound state Systems?

Advantage over 2-free body interaction:

- A large part of the interaction is absorbed into the mean field of the nucleus since

$$\left(\frac{1}{m_a r_a}\right)^2 \approx 1$$

- Of order $g_{.P}^2 g_{strong}^4$



$$V_{ab}(r_{ab}) = g_P^2 \langle Q_a \rangle \langle Q_b \rangle \frac{e^{-\mu r}}{4\pi r}$$

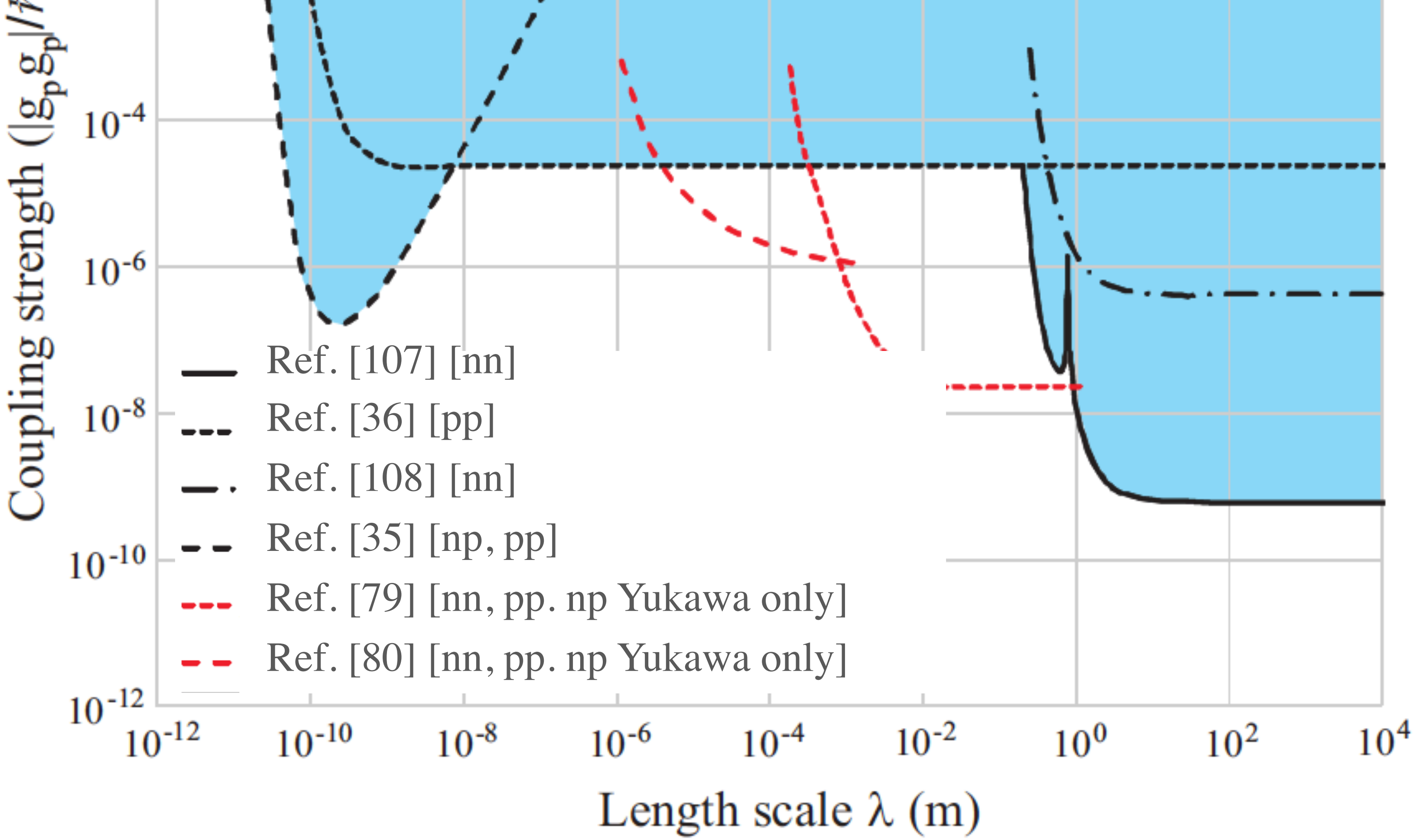
while form of V_{ab} is simple, calculation of $\langle Q_a \rangle$ isn`t, however a general shell model calculation may give a reasonable estimation.

D. Krause, et. al, Proc. XXVIIIth Recontre De Moriond , 1993, p.455.

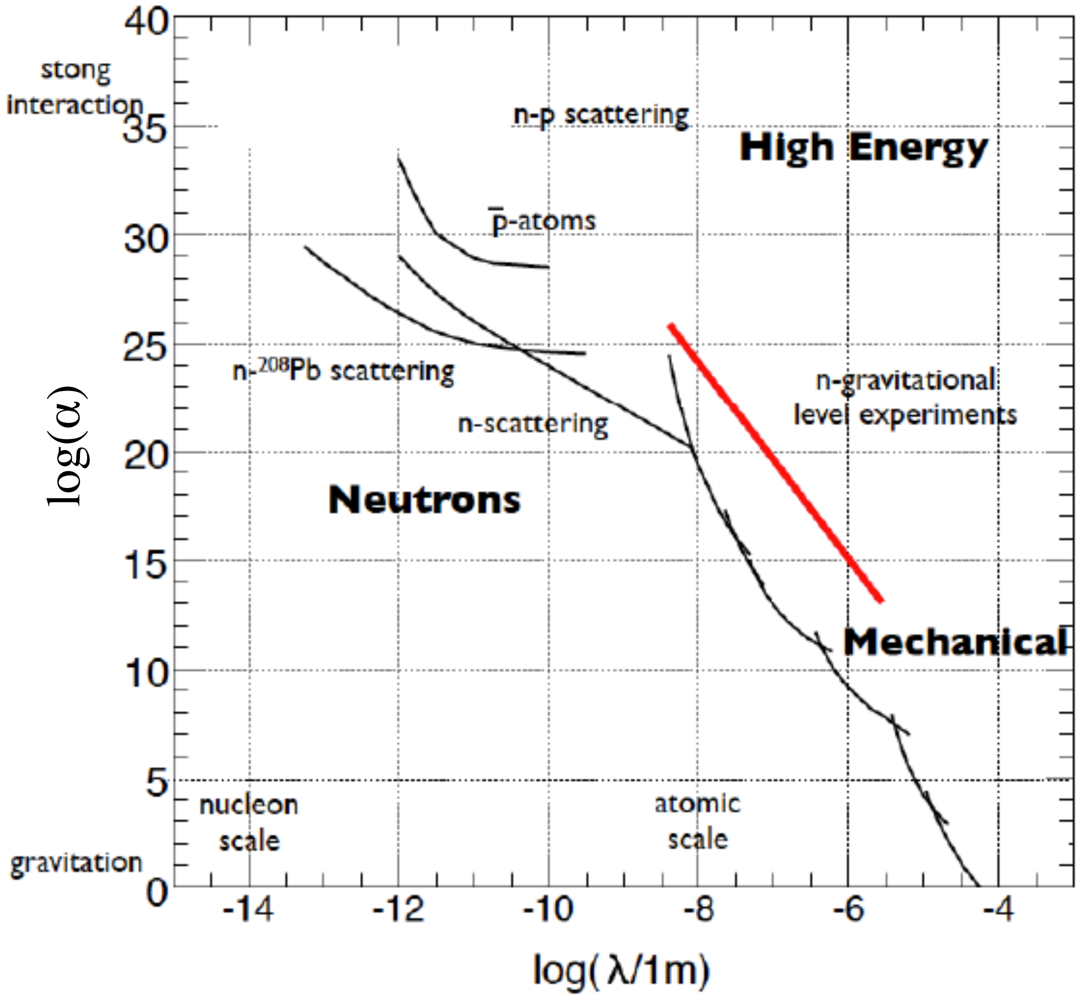
Useful resources

Sabine Hossenfelder on lost in math and SM

https://wicn.org/podcasts/audio/sabine-hossenfelder-lost-math

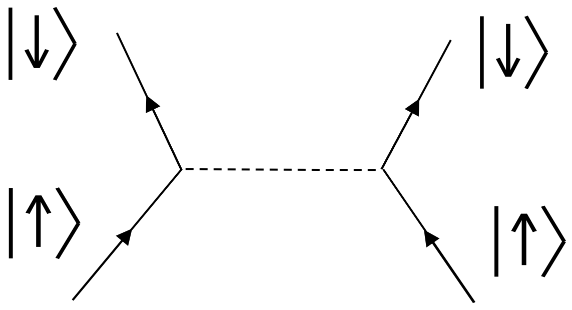


Coupling strength (|g_s g_p g_p|/ħ
10^-4
10^-6
10^-8
10^-10
10^-12
10^-12
10^-10
10^-8
10^-6
10^-4
10^-2
10^0
10^2
10^4
Length scale λ (m)
Ref. [107] [nn]
Ref. [36] [pp]
Ref. [108] [nn]
Ref. [35] [np, pp]
Ref. [79] [nn, pp. np Yukawa only]
Ref. [80] [nn, pp. np Yukawa only]

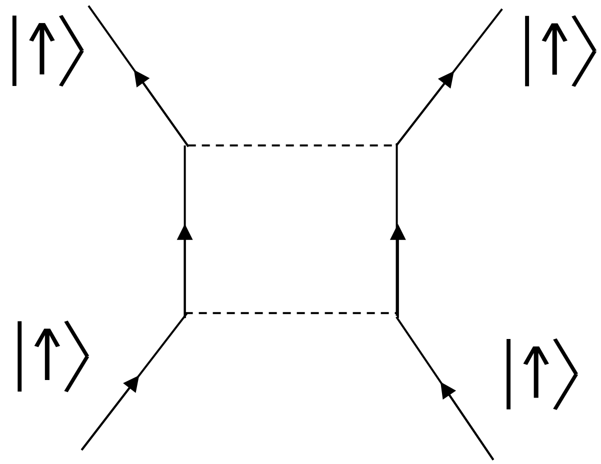


40
35
30
25
20
15
10
5
0
stong
interaction
gravitation
log(α)
n-p scattering
High Energy
p̄-atoms
n-208Pb scattering
n-scattering
n-gravitational
level experiments
Neutrons
Mechanical
nucleon
scale
atomic
scale
-14
-12
-10
-8
-6
-4
log(λ/1m)

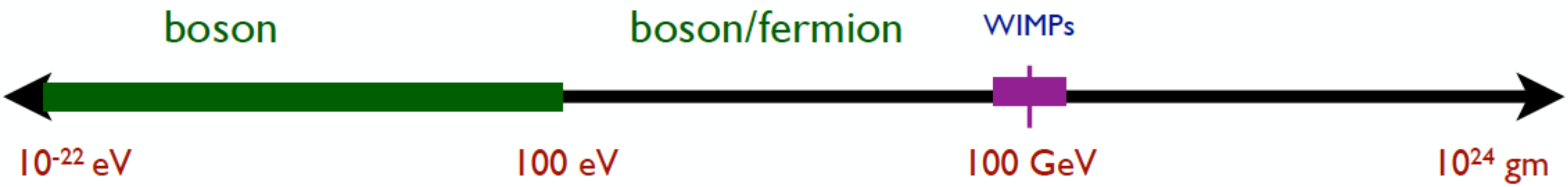
Two-Boson Exchange



(a)



(b)



Weakly-interacting bosons are consistent with dark matter abundance in the keV - 10^{-22} eV mass range

Give rise to many phenomena on the low energy scale offering feasible experimental probes!

US Cosmic Vision: New Ideas in Dark Matter. arXiv:1707.04591 [hep-ph].

Particle Physics Motivation: Ultralight Spin-1 Particles

- Many theories beyond the SM give rise to interactions with vector as well as axial couplings

$$L = \bar{\psi}\left(g_V \gamma^\mu + g_A i \gamma^\mu \gamma_5\right)\psi A_\mu$$

- supersymmetric and grand-unifed theories require two or more Higgs doublet which possess an extra U(1) symmetry generator of the form:

$$F = a\,B + bL + cY + dF_{ax}$$

Baryon and lepton numbers *Hypercharge* *Axial U(1) generator*

a, b, c, and d depend on the details of the theory

P. Fayet, Nucl. Phys. B 347, 743 (1990).

Possible couplings

Axions (and ALPs) effects

Fermions

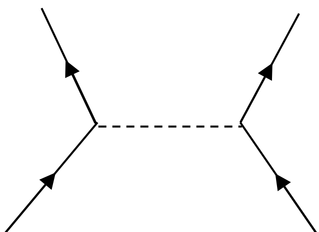
$\bar{\psi}\gamma^{\mu}\gamma_5\psi\partial_{\mu}\varphi$ — dipole

$\bar{\psi}\psi\varphi$ — monopole

Gauge Fields

$F_{\mu\nu}F^{\mu\nu}\varphi$ — Current

$G_{\mu\nu}G^{\mu\nu}\varphi$ — QCD axion



Moody and Wilczek (1987)
B. Dobrescu and I. Mocioiu (2006)

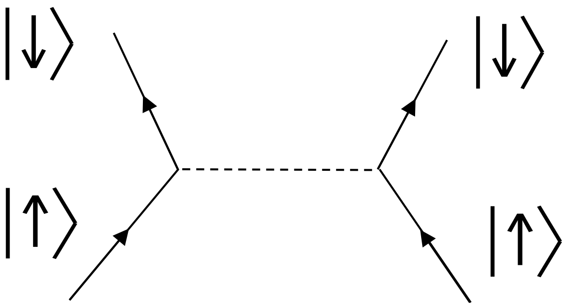
$$V_{S-S} = -\frac{g_S^2}{4\pi r}e^{-\mu r}$$

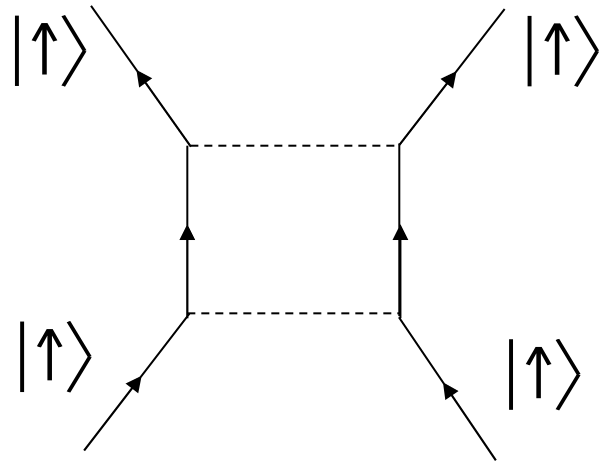
$$V_{S-P} = \frac{g_S g_P}{8\pi m}\vec{\sigma}\cdot\nabla\frac{e^{-\mu r}}{r}$$

$$V_{P-P} = \frac{g_P^2}{16\pi m^2}\vec{\sigma}_a\cdot\nabla\vec{\sigma}_b\cdot\nabla\frac{e^{-\mu r}}{r}$$

Torsion balance, casimir experiments
molecular spectroscopy, NMR, microwave cavity….etc

Two-Boson Exchange





Aim of this work:

- derive all long-range *spin-independent* interaction energies from double boson exchange from spin-0 and spin-1 with *spin-dependent* couplings between a pair of NR Dirac fermions

- Use the derived expressions with the strong spin-independent limits to help improve constraints on interactions involving pseudoscalar and axial couplings.

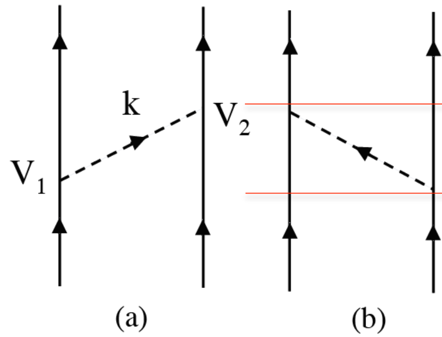
A Single-Boson Exchange Potential in TOPT

$$\omega_k = \sqrt{k^2 + \mu^2}$$

$$\Delta E_i = E - E_I = \frac{p'^2_i - p_i^2}{2m_i}$$

$$H^{eff}_{PD} = -\frac{g_{PD}}{2m}\, \vec{\sigma}\cdot\vec{\nabla}\,\phi$$

$$f(r) = e^{-\mu r}/4\pi r$$



$$|f\rangle = |\vec{p}_1';\vec{p}_2'\rangle$$

$$\frac{1}{-\omega_k + E_i - E_I} = \frac{-1}{\omega_k}\left[1 + \frac{E_i - E_I}{\omega_k} + \frac{(E_i - E_I)^2}{\omega_k^2} + \dots\right]$$

$$|i\rangle = |\vec{p}_1, \vec{p}_2\rangle$$

$$T_{fi} = \sum_\alpha \langle f|H_{PD}|\alpha\rangle \frac{1}{E_i - E_\alpha + i\epsilon}\langle\alpha|H_{PD}|i\rangle$$

$$\xrightarrow[\text{static}]{} \sum_{\vec{k}} \{V_2 \frac{-1}{\omega_k} V_1 + V_1 \frac{-1}{\omega_k} V_2\}$$

$$= \frac{g_1 g_2}{4m_1 m_2} \frac{\vec{\sigma}_1\cdot\vec{\nabla}\,\vec{\sigma}_2\cdot\vec{\nabla}}{\omega_k^2}\,\delta(\vec{p'}_2 + \vec{p}_l' - \vec{p}_1 - \vec{p}_2)$$

- velocity in most experimental searches is small
- it is desirable to obtain functional forms with operators that are either purely static or purely velocity-dependent

- Vertex corrections: $-i\frac{g_{PD}}{16m^3}\{\vec{\sigma}\cdot\vec{p},\ \{\vec{p}\cdot,\ \vec{\nabla}\phi\}\} + \frac{g_{PD}}{8m^3}\{p^2,\ \vec{\sigma}\cdot\vec{\nabla}\phi\}$
- Propagator corrections: $\frac{(E_i - E_I)^2}{\omega_k^2}$

$$V_{OBE}(r) = \frac{g^2}{4m^2}\Big[\Big(1 - \frac{p'^2 + p^2}{2m^2}\Big)\,\vec{\sigma}_1\cdot\vec{\nabla}\vec{\sigma}_2\cdot\vec{\nabla}f(r)$$
$$+ i\Big[\frac{p^2}{8m^2}, \Big\{\vec{\sigma}_1\cdot\vec{p}, \vec{\sigma}_2\cdot\vec{\nabla}f(r)\Big\} + \Big\{\vec{\sigma}_2\cdot\vec{p}, \vec{\sigma}_1\cdot\vec{\nabla}f(r)\Big\}\Big] + i\vec{\sigma}_2\cdot\vec{\nabla}\vec{\sigma}_1\cdot\vec{\nabla}\Big[\frac{p^2}{4m^2}, \Big\{\vec{p}\cdot, \vec{r}f(r)\Big\}\Big]\Big]$$

Agrees with J.L Friar, Ann. Phys. (N.Y) 104, 380 (1977)

- Shortcomings: Non-local, non-hermitian, a function of r, p^2, r. p

Unitary Transformation

$$H = T + V_{OBE}$$
$$H_o = T + V^o_{OBE}$$
$$T = \sum_i \sqrt{m_i^2 + \vec{p}_i^{\,2}}$$

$$H = e^{-iU} H_o e^{iU} = H_o + i[H_o, U] + \ldots$$

$$V_{OBE} = V^o_{OBE} + i[T, U] + i[V^o_{OBE}, U] + \ldots.$$

$O(g^4)$

where

$$iU = iU_1 + iU_2 = i\frac{g^2}{32m^3}\Big[\Big(\{\vec{\sigma}_1 \cdot \vec{p}, \vec{\sigma}_2 \cdot \vec{\nabla} f(r)\} + 1 \leftrightarrow 2\Big) + \vec{\sigma}_2 \cdot \vec{\nabla} \vec{\sigma}_1 \cdot \vec{\nabla}\{\vec{p}, \cdot \vec{r} f(r)\}\Big].$$

$$V^o_{OBE} = \frac{g^2}{4m^2} \vec{\sigma}_2 \cdot \vec{\nabla} \vec{\sigma}_1 \cdot \vec{\nabla}\Big[1 - \Big(\frac{p'^2 + p^2}{2m^2}\Big)\Big]\frac{e^{-\mu r}}{4\pi r}$$

Local and a function of r and p^2 only!

The range of applicability

A few issues in the massless limit (not related to our work):

- $V_{PD\text{-}PD}$, $V_{2\pi} \longrightarrow 1/r^6$ as $\mu \to 0$ whereas the result calculated by Ferrer and Grifols in PRD 58, 096006 (1998) shows a $1/r^5$ dependence. A discontinuity in the massless limit could be due to the lack of conserved charges associated with a spin-0 field.

- The results derived with a massive spin-1 exchange diverge as $\mu \to 0$ due to the existence of non-conserved currents associated with the longitudinal polarization component. Such limit is not realized in our case.