Momentum-space structure of hadrons and nuclei at high energy

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BNL

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Thanks to the Leona Woods Lecture Committee!

Thanks to BNL and the Nuclear Theory Group for hosting me these three weeks!

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A CENTURY OF SERVICE

Transverse-momentum-dependent (TMD) factorization

Color Glass Condensate (CGC)

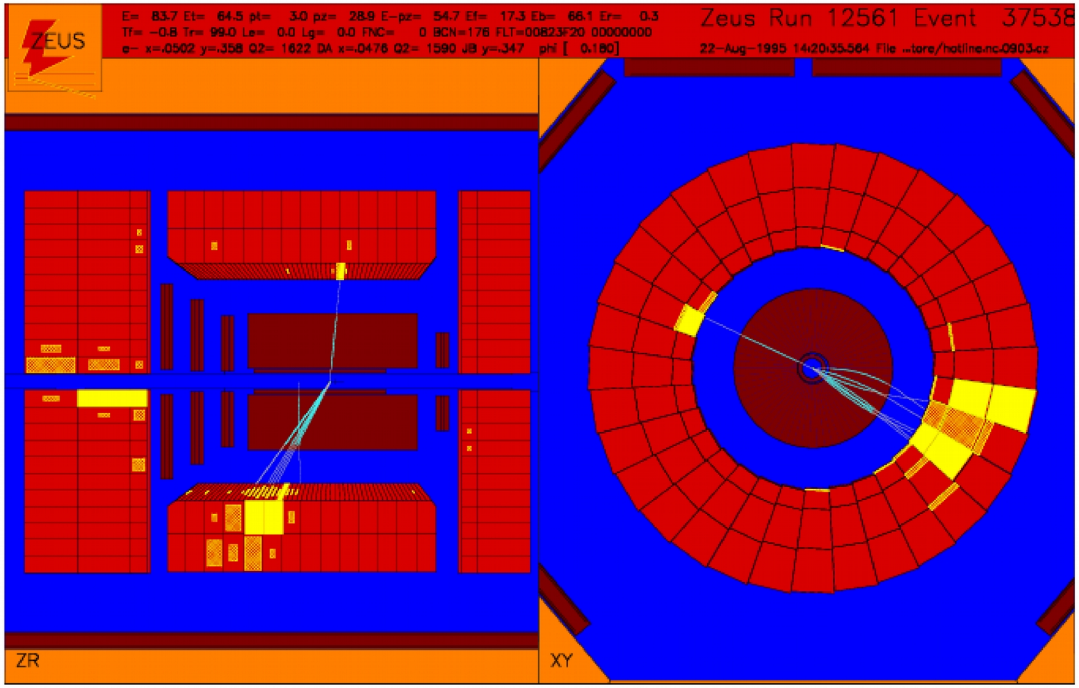
Factorization:

Non-perturbative parton densities

X

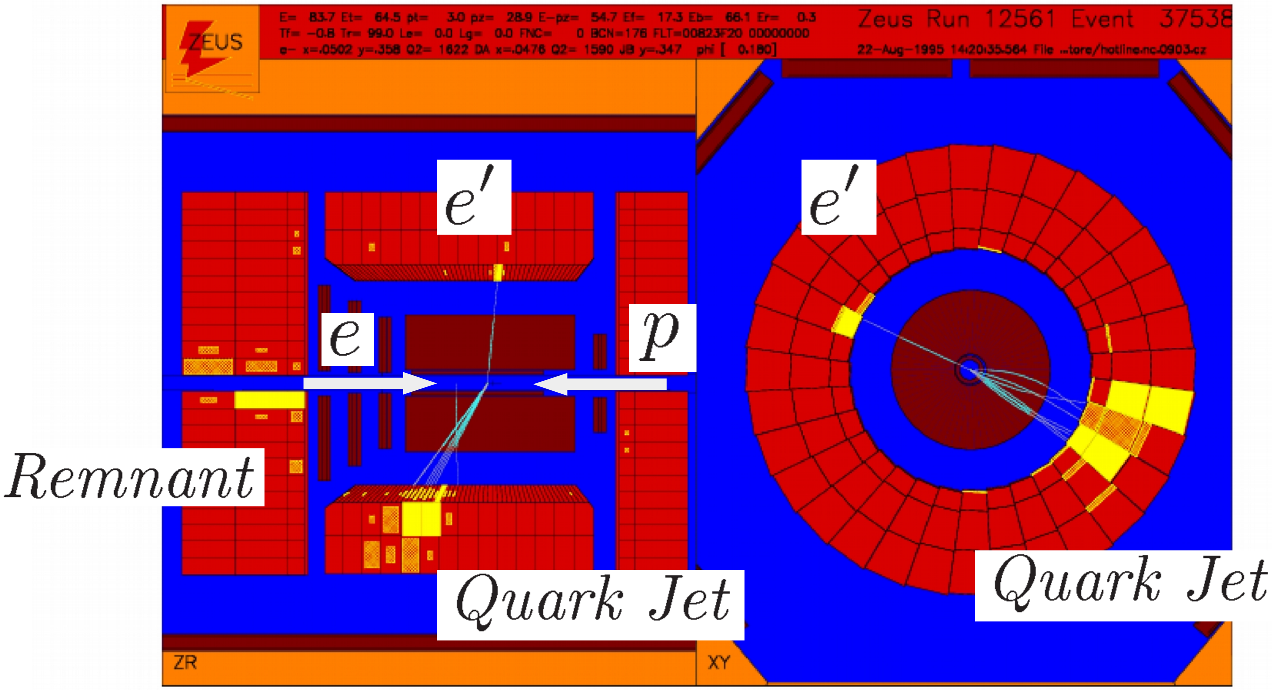
Perturbative partonic cross section

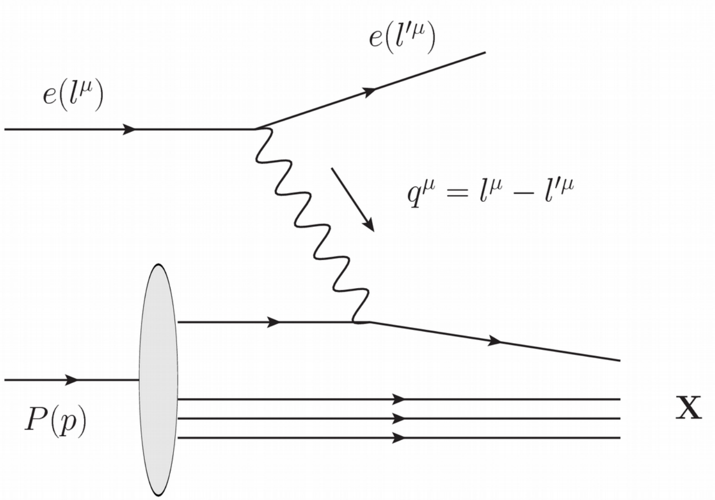
Deep Inelastic Scattering (DIS) $e(l) + p(P) \rightarrow e(l') + X$



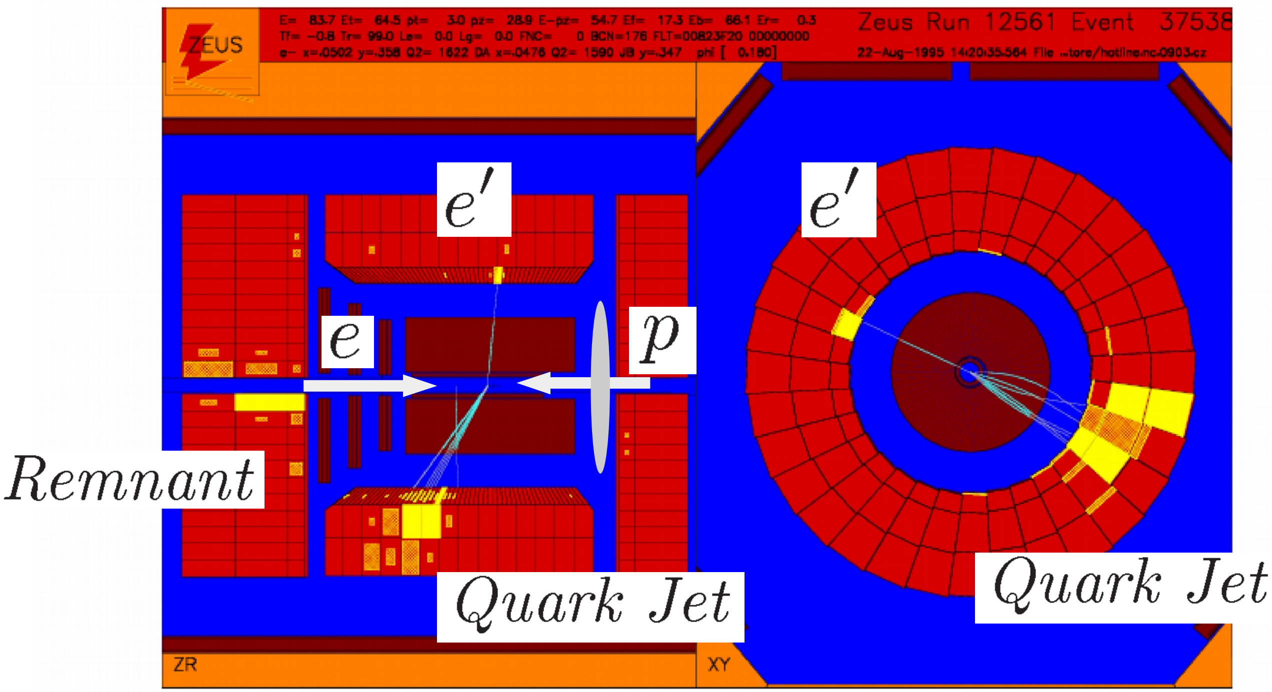
HERA event; arXiv:1301.7572

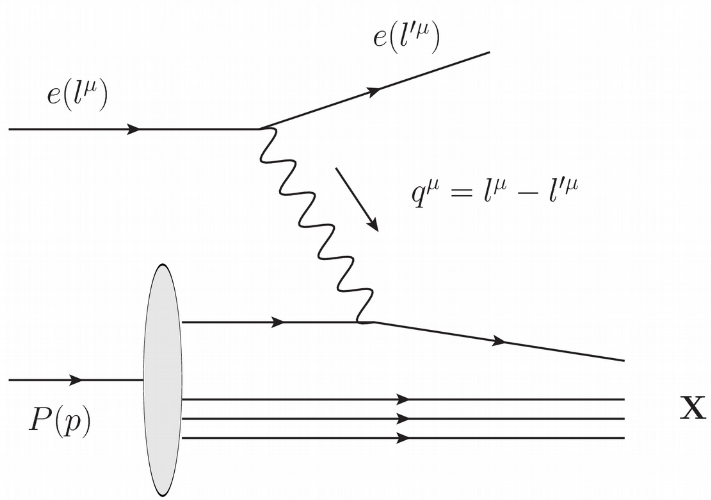
Deep Inelastic Scattering (DIS) $e(l) + H(P) \rightarrow e(l') + X$





Deep Inelastic Scattering (DIS) $e(l) + p(P) \to e(l') + X$





Momentum transferred:

$Q \equiv \sqrt{-q^2}$

$Q = 100 GeV$

Distance scale probed:

$\sim 1/Q$

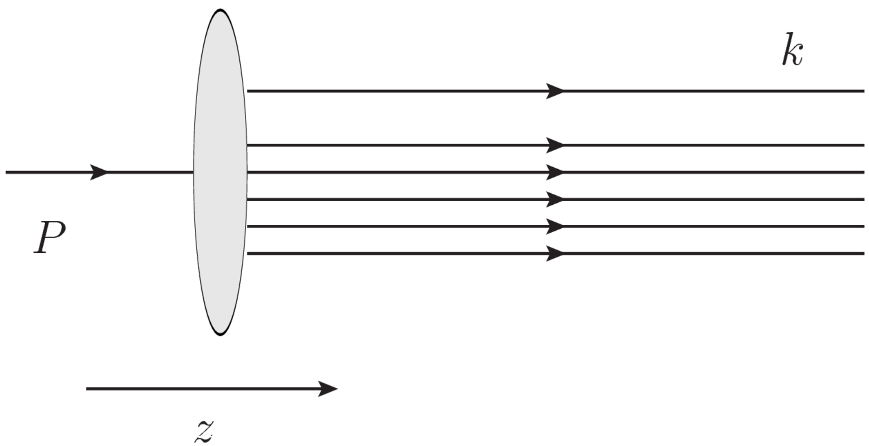
$\sim 0.01\ fm$

Proton's rest frame: Time of parton interaction $\sim 1 fm/c$.

Boosted frame: Time dilation factor ~ 150;

Last parton interaction happened $\sim$100 fm before eq int.

Infinite Momentum Frame

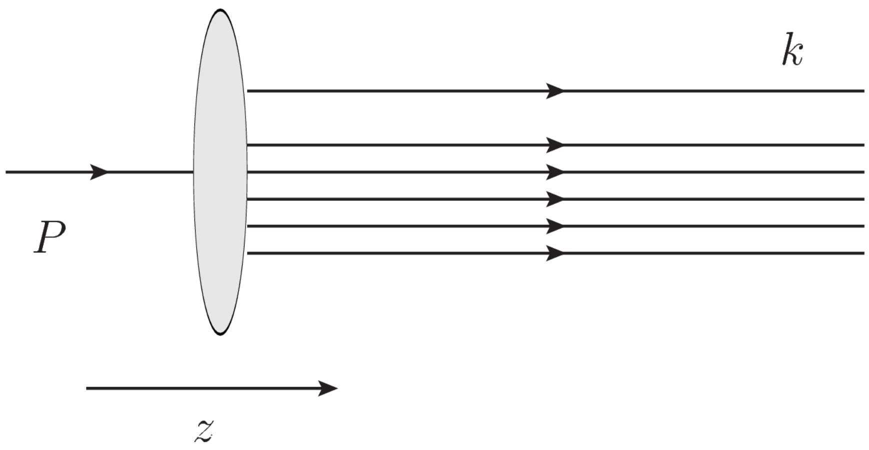


$$P^\mu = \big(P^+, M_p^2/(2P^+), 0_T\big)$$

$$k^\mu = \big(k^+, k^-, k_T\big)$$

$$P^\pm = \frac{1}{\sqrt{2}}\left(P^0 \pm P^z\right)$$

Infinite Momentum Frame



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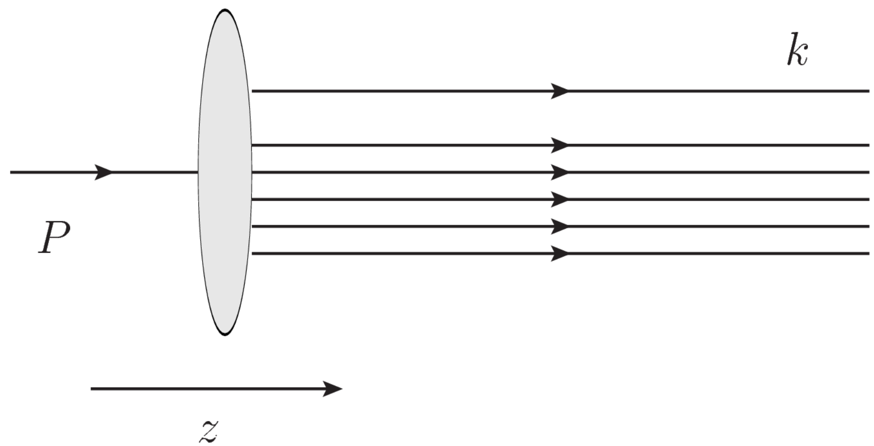
Infinite momentum frame: $k^\mu \sim (Q, 0, 0_T)$

Infinite Momentum Frame



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Infinite momentum frame: $k^\mu \sim (Q, 0, 0_T)$

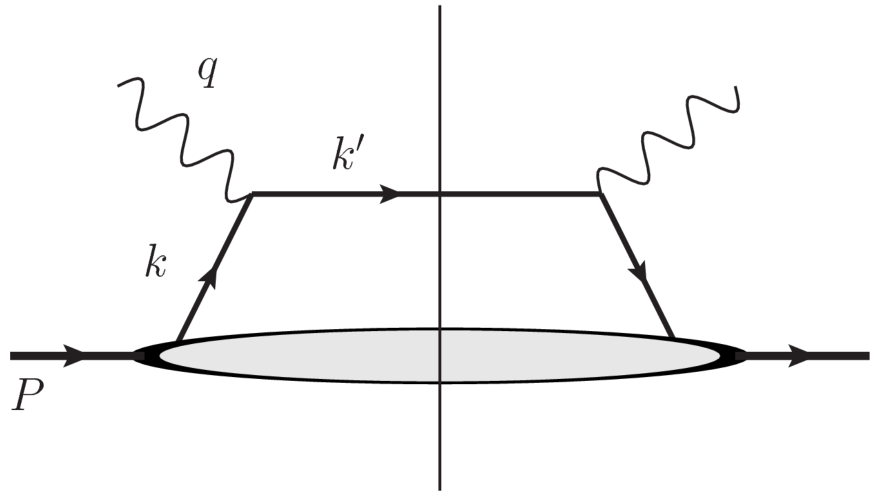
Define:

Fractional momentum of the parton: $\xi = k^+/P^+$

Parton distribution function (PDF)

Number distribution of partons: $f_i(\xi)$

Momentum distribution of partons: $\xi f_i(\xi)$

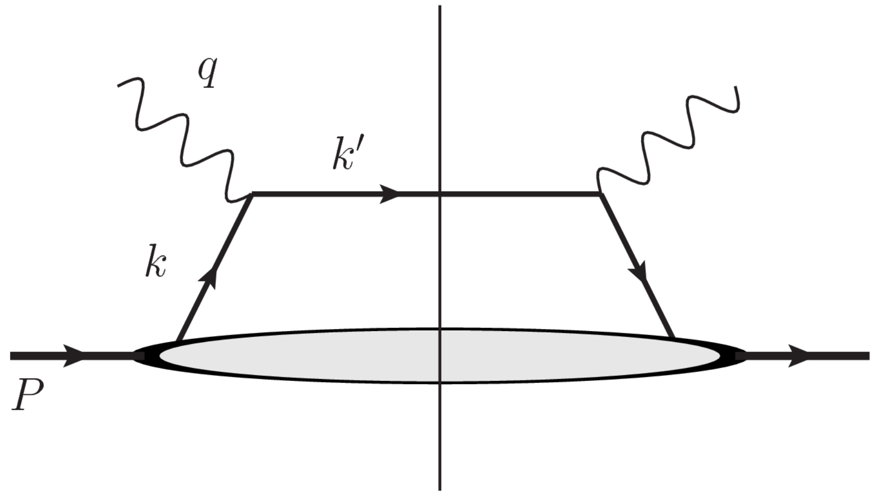


$$\frac{d^2\sigma}{dx dQ^2} = \sum_i \int_0^1 d\xi\, f_i(\xi) \frac{d^2\hat{\sigma}_i}{dx dQ^2}$$

$$Q \equiv \sqrt{-q^2}$$

$$x \equiv \frac{Q^2}{2P \cdot q}$$

$Q^2 \to \infty$, $\quad x$ is fixed.



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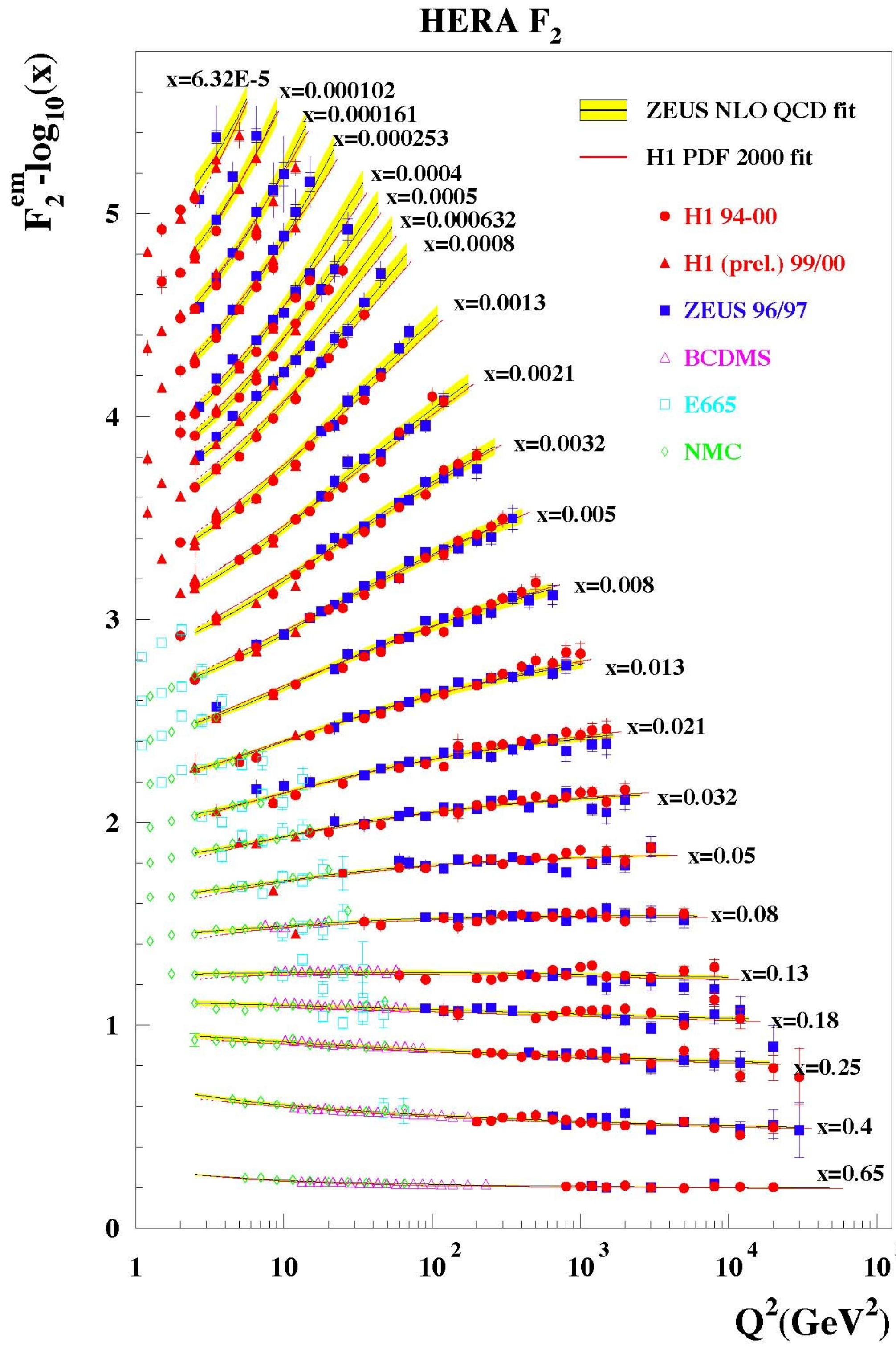
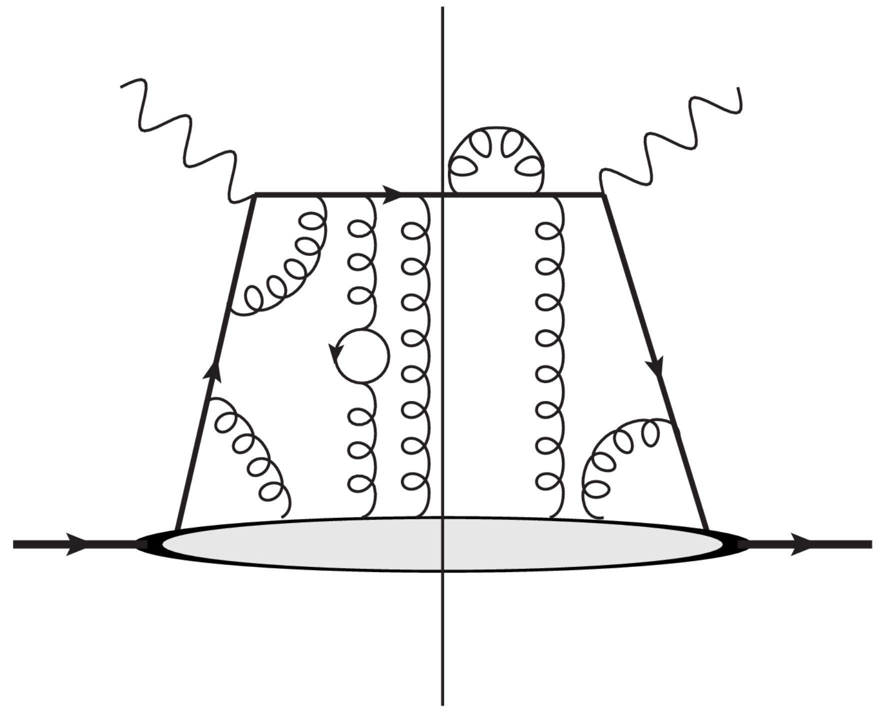
$$\xi = x$$

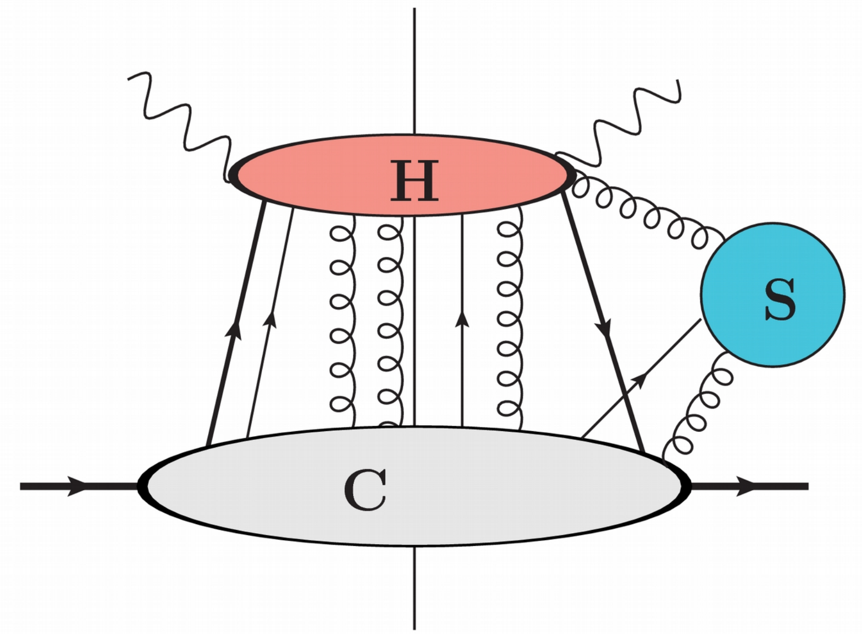
Bjorken scaling



QCD corrections to the parton model



Leading regions of momenta



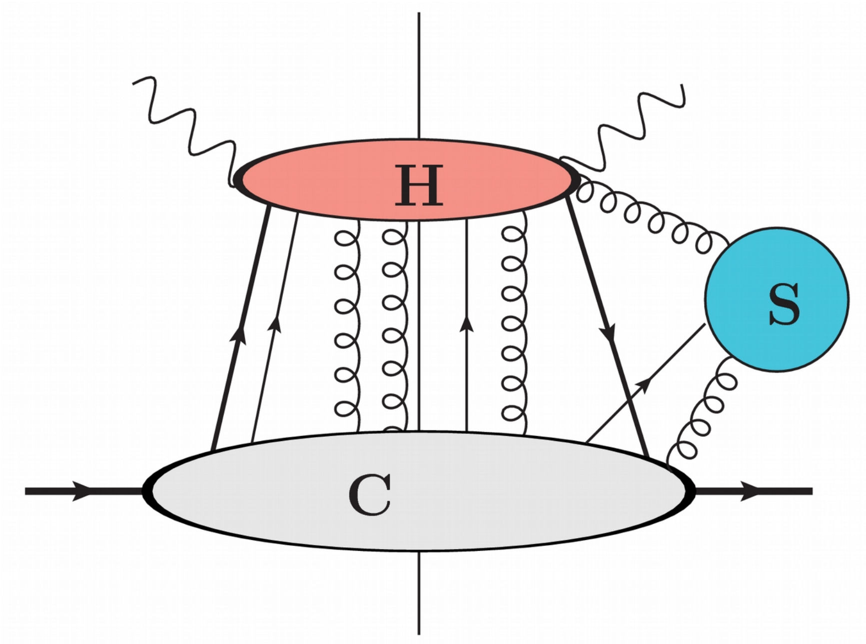
1)Hard;

2)Collinear;

3)Soft.

S. B. Libby and G. F. Sterman, 1978

Leading regions of momenta



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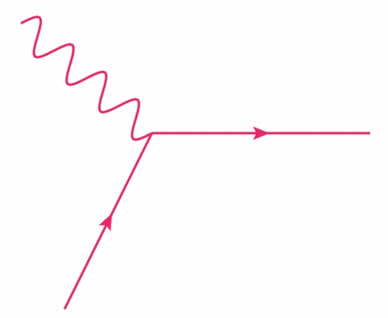
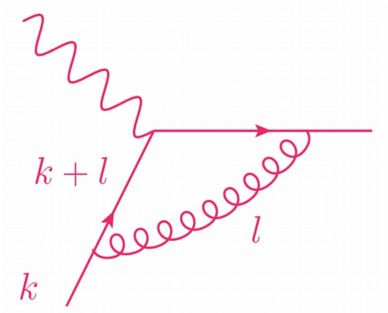
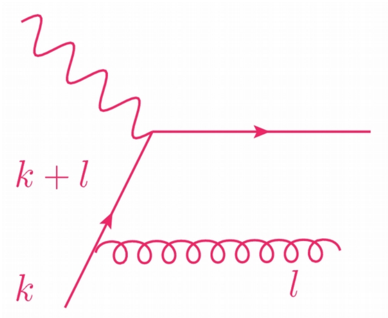
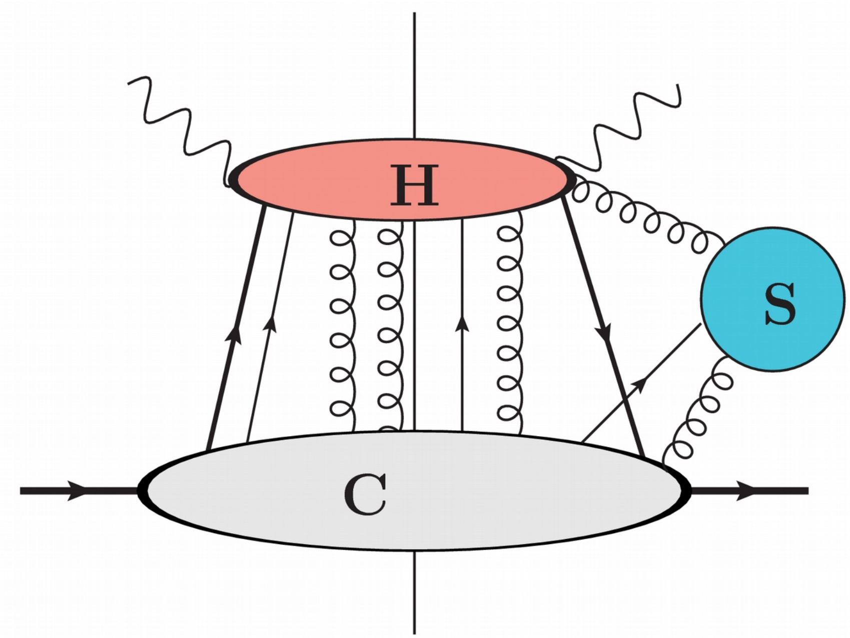






Leading regions of momenta



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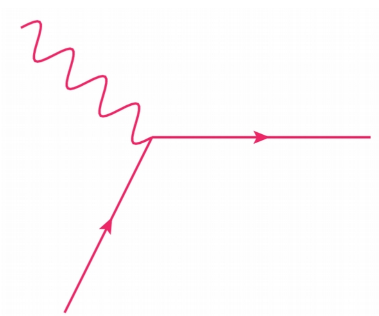
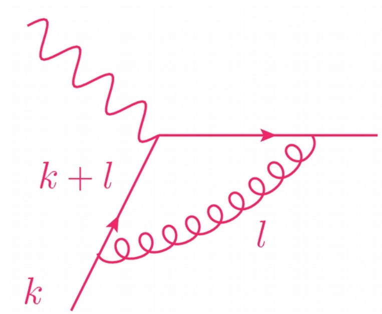
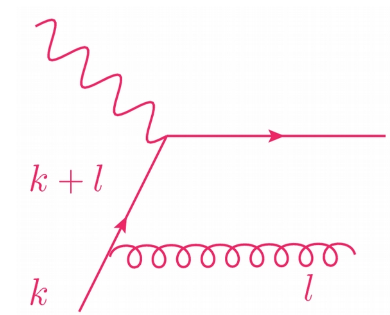
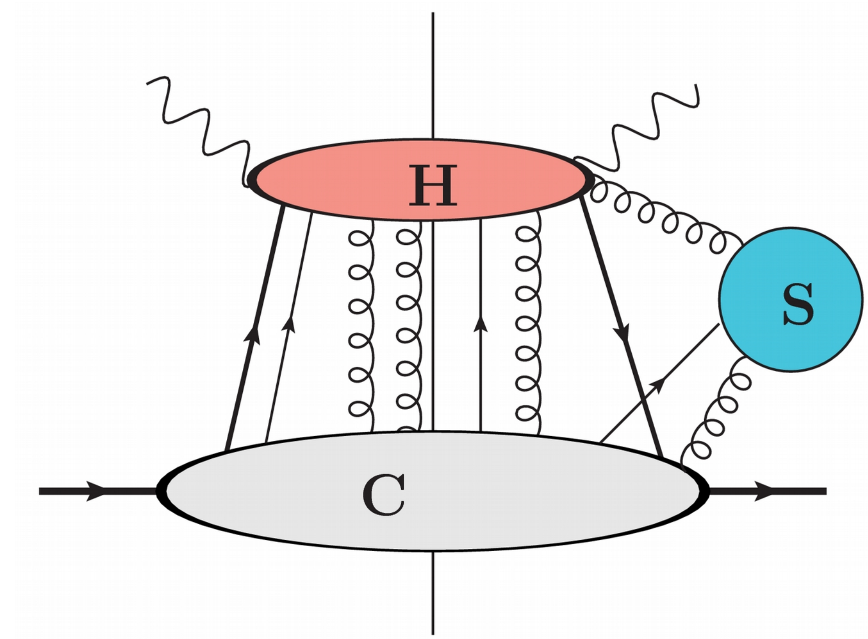






$$\text{Hard region} \sim w^{(0)} + \alpha_s(Q)w^{(1)} + \alpha_s^2(Q)w^{(2)} + \ldots$$

Leading regions of momenta



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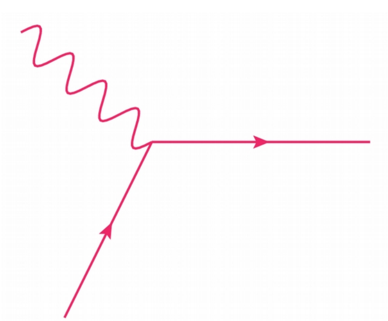
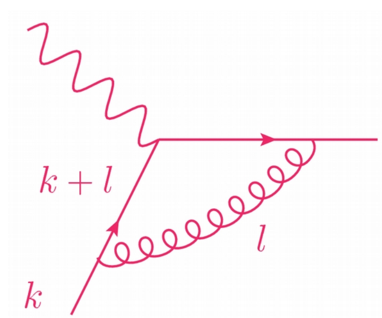
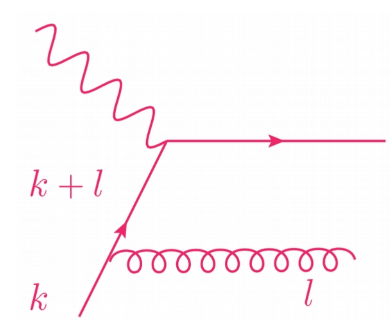






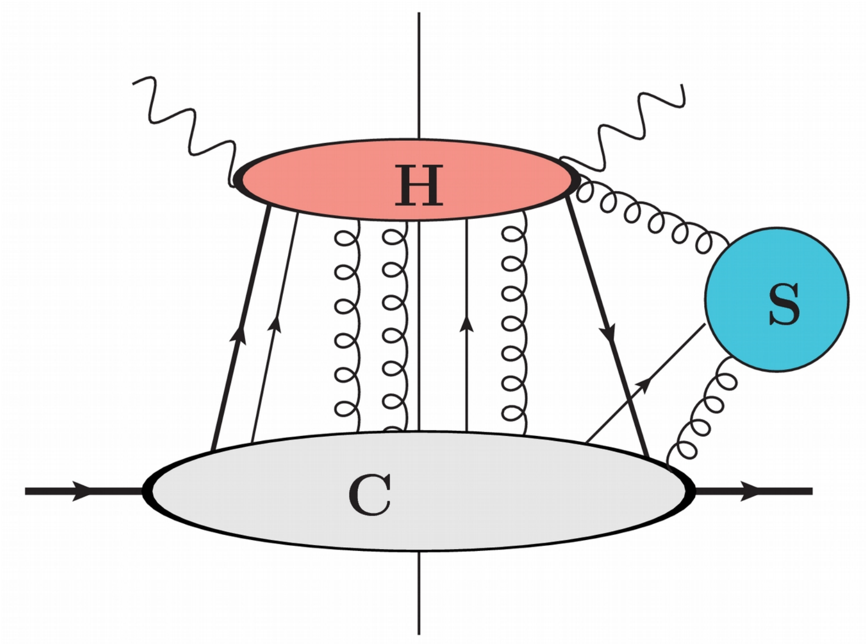
Singularities in the massless limit $m \to 0$:

- Soft infrared divergence: $E_k \to 0,\ E_l \to 0;$
- Collinear infrared divergence: $\theta_{kl} \to 0.$

$$\frac{1}{(k+l)^2 - m^2} = \frac{1}{2E_k E_l (1 - \beta \cos\theta_{kl})}$$

$$\beta = \sqrt{1 + \frac{m^2}{E_k^2}}$$

Leading regions of momenta



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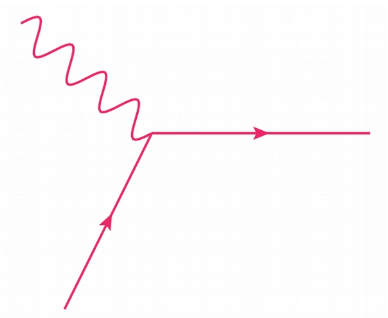

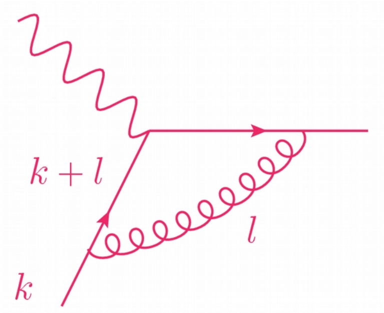

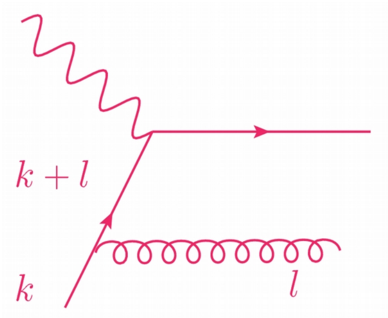

Factorization scale μ

Leading regions of momenta

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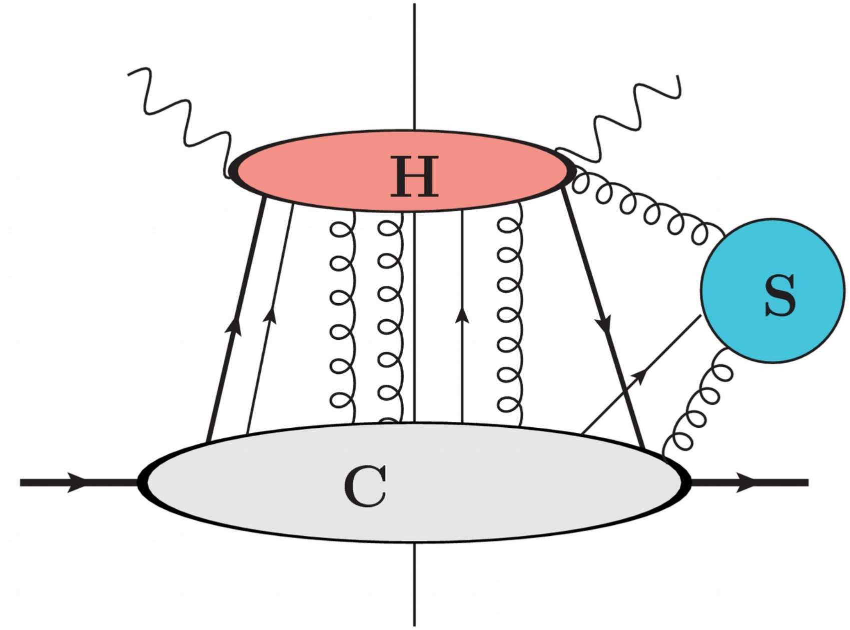

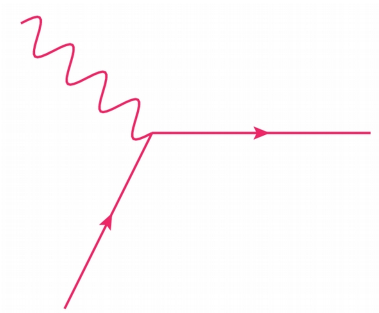

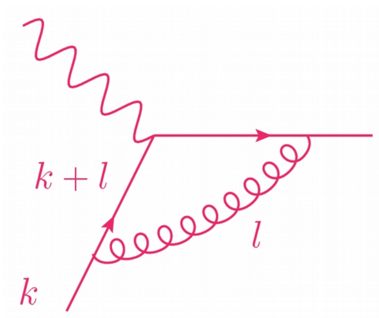

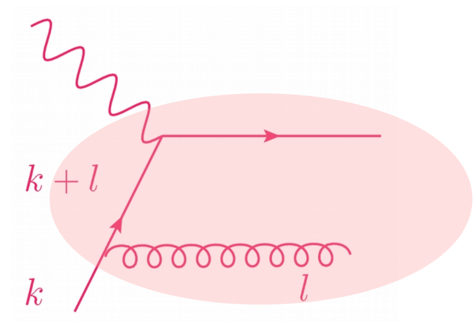

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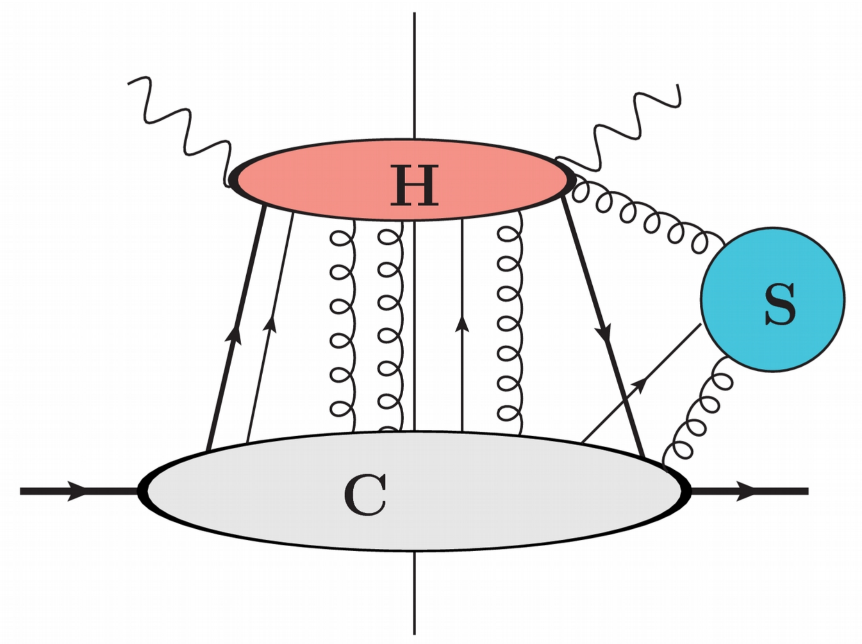

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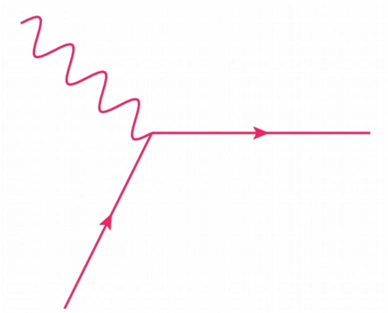
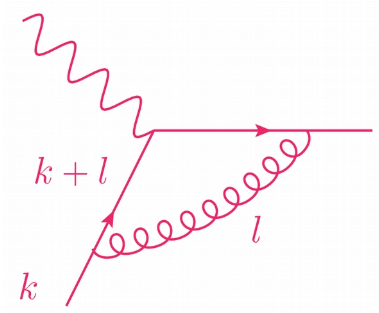
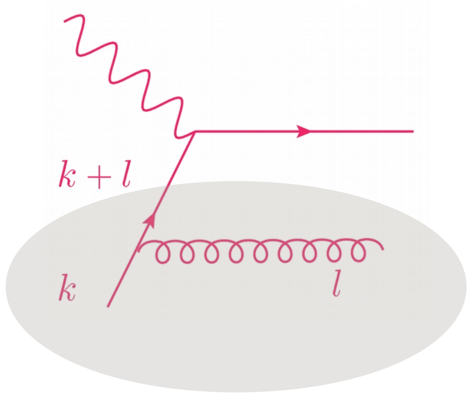
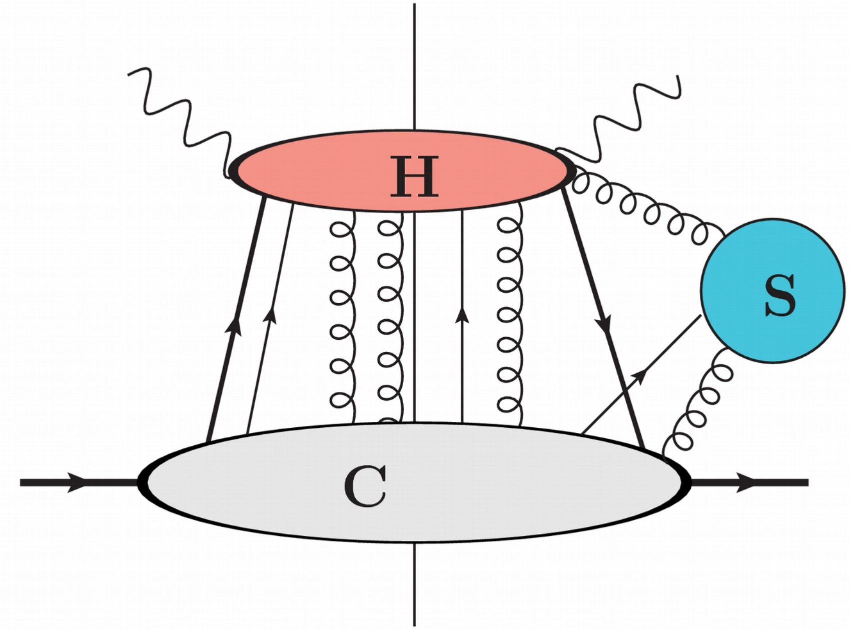






Factorization scale μ

Leading regions of momenta



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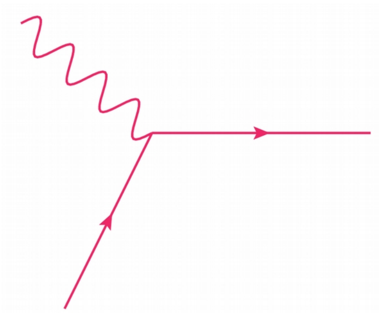
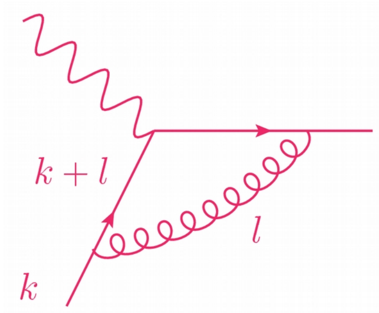
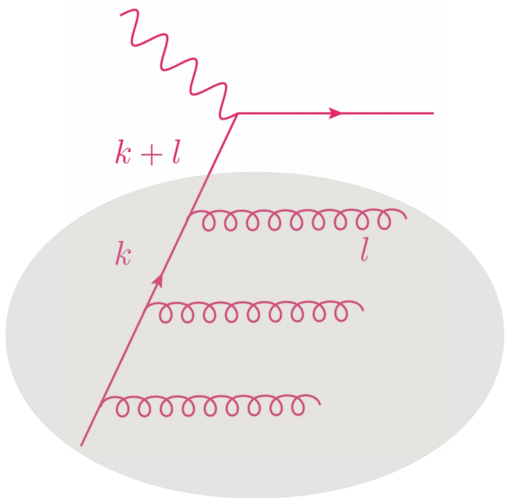
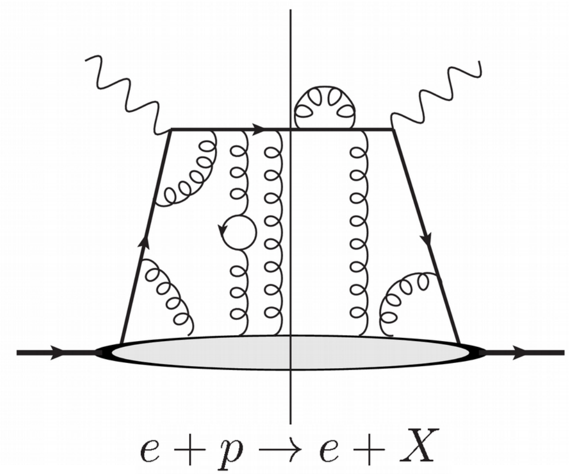




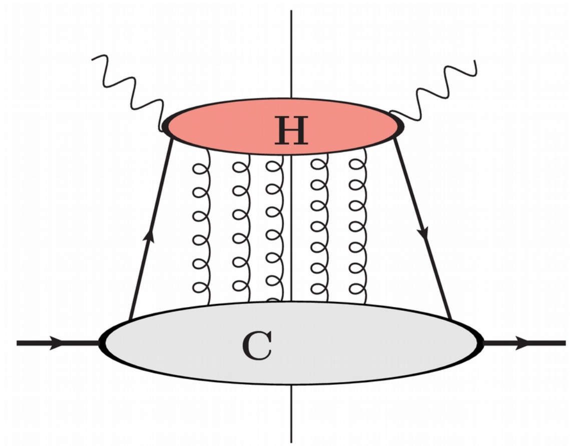


Factorization scale $\mu \sim Q$

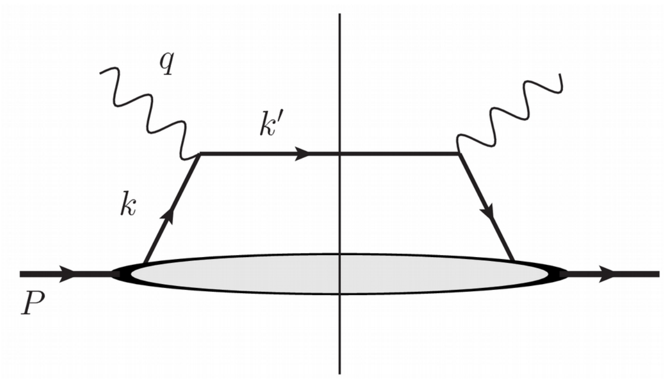
DGLAP evolution equations, $Q^2 \to \infty$, x is fixed. Dokshitzer–Gribov–Lipatov–Altarelli–Parisi

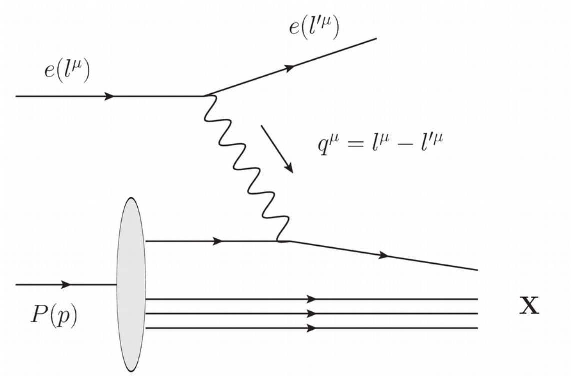


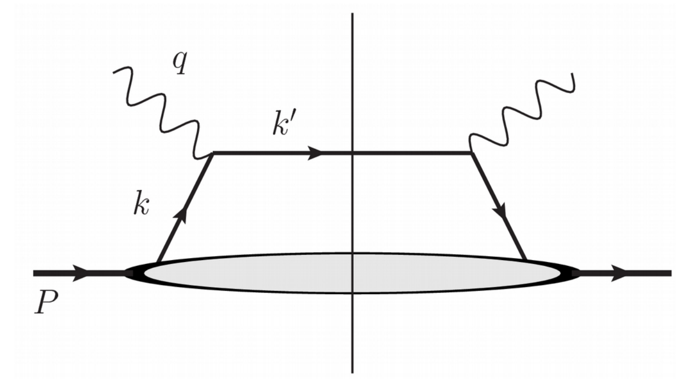
Leading contributions



Parton model







Electromagnetic interaction Lagrangian for the quark fields: $\mathcal{L}_{\text{em-q}} = -eA^{\mu} J_{\mu}(x)$

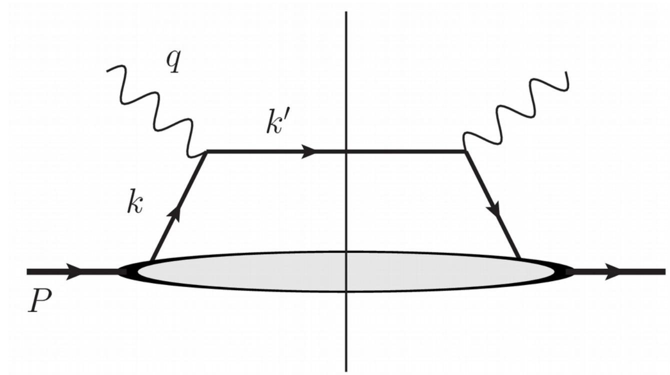
Quark-electromagnetic current:

$$J_{\mu}(x) = \sum_{f} e_f \bar{\psi}_f \gamma^{\mu} \psi_f$$

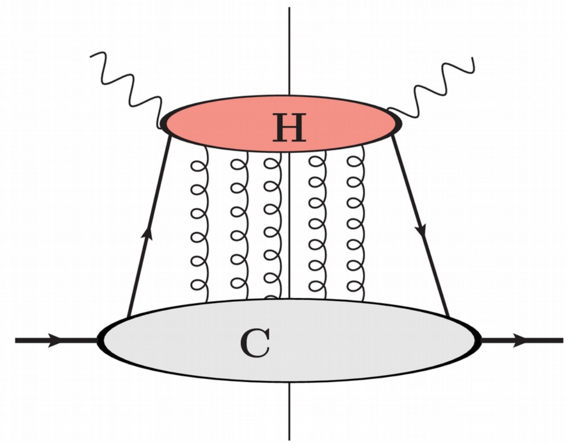
Hadronic part:

$$W^{\mu\nu} \sim \sum_{X} \langle P|J^{\mu}(0)|X\rangle \langle X|J^{\mu}(0)|P\rangle \, \delta^{(4)} \left(P_X - P - q\right)$$

$$\sim \int d^4x \, e^{iq\cdot x} \langle P|J^{\mu}(x)J^{\nu}(0)|P\rangle$$

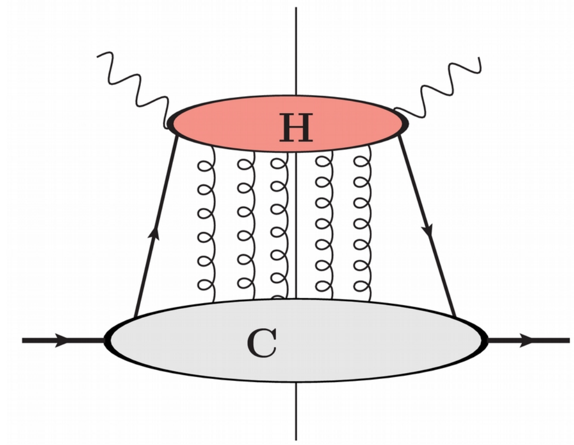


$$f(\xi) = \int \frac{dx^-}{2\pi} e^{-i\xi P^+ x^-} \left\langle P|\bar{\psi}(x^-)\frac{\gamma^+}{2}\psi(0)|P\right\rangle$$



$$f(\xi) = \int \frac{dx^-}{2\pi} e^{-i\xi P^+ x^-} \left\langle P|\bar{\psi}(x^-)W(x^-,0)\frac{\gamma^+}{2}\psi(0)|P\right\rangle$$

$$W(a,b) = \mathcal{P}\exp\left[ig\int_a^b dz^- A_a^+(z^-)t^a\right]$$



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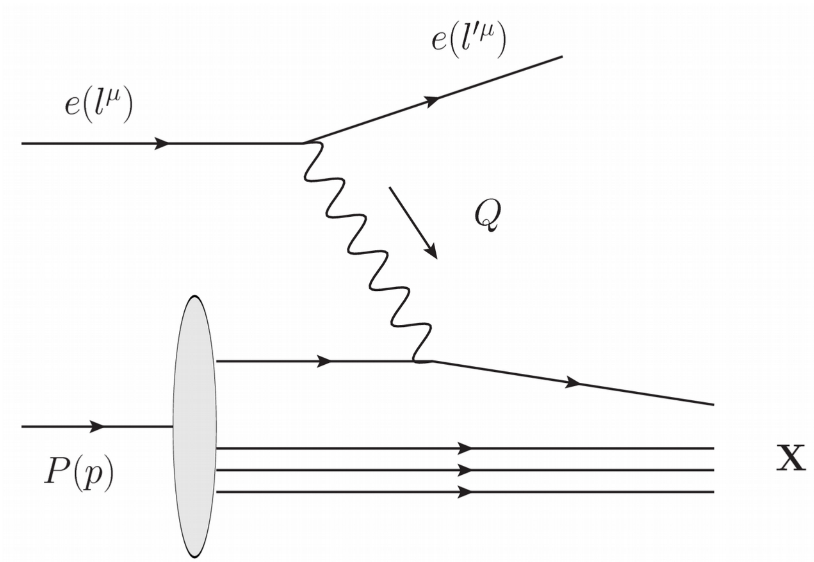
Number density interpretation:

$$f(\xi) = \frac{1}{\xi(2\pi)^3} \sum_\alpha \int d^2k_T \frac{\left\langle P|b^\dagger_{k,\alpha} b_{k,\alpha}|P\right\rangle}{\langle P|P\rangle} \sim \langle N\rangle$$

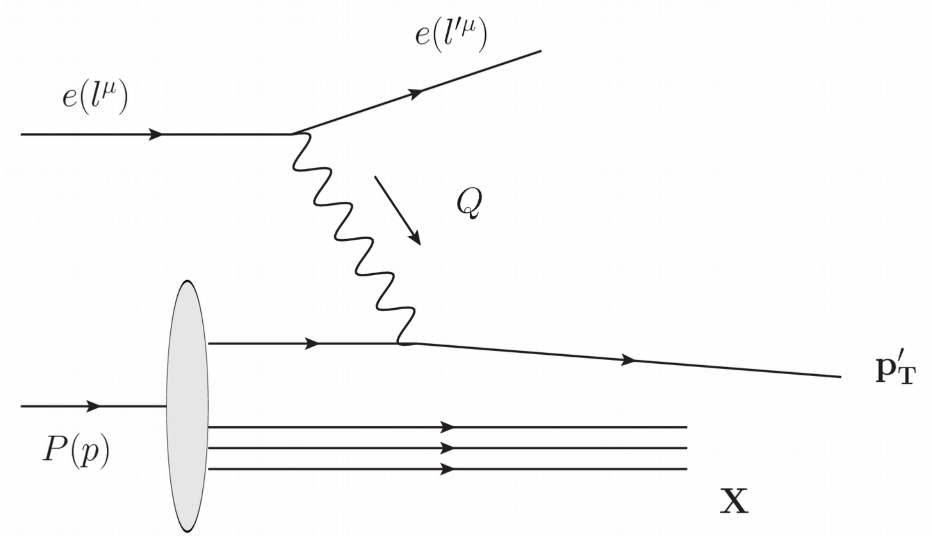
Transverse-momentum-dependent (TMD) distributions

Transverse-momentum-dependent (TMD) distributions



DIS

$$e + p \to e + X$$

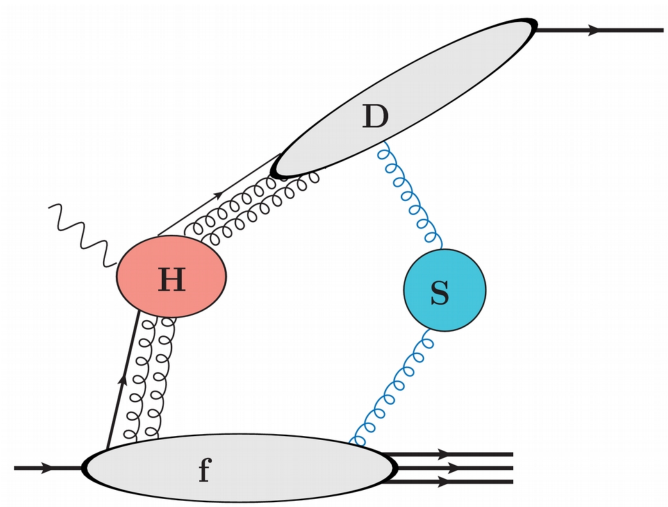


SIDIS

$$e + p \to e + q + X$$

Transverse-momentum-dependent (TMD) factorization

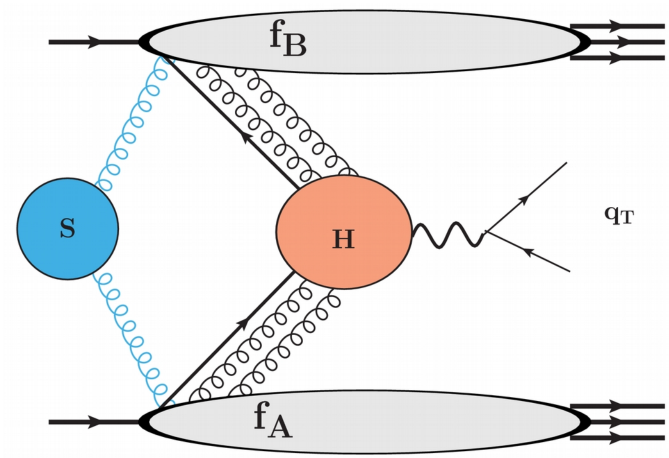
$e + p \to e + h + X$ SIDIS



$$d\sigma \sim |H|^2 \otimes f_{SIDIS}(\xi, k_T) \otimes D(z, p_T) \otimes S(l_T)$$

$$\times \delta^{(2)}(P'_{hT} - zk_T - p_T - l_T)$$

$p + p \to \mu^+ + \mu^- + X$ Drell-Yan



$$d\sigma \sim |H|^2 \otimes f_{A_{DY}}(\xi_a, k_{Ta}) \otimes f_{B_{DY}}(\xi_b, k_{Tb}) \otimes S(l_T)$$

$$\times \delta^{(2)}(q_T - k_{Ta} - k_{Tb} - l_T)$$

J. C. Collins and A. Metz, 2004
X. D. Ji, J. P. Ma and F. Yuan, 2005

J.C. Collins, D.E. Soper and G. Sterman, 1985
X. D. Ji, J. P. Ma and F. Yuan, 2004

Definition of quark and gluon TMDs

$$\mathcal{F}_q(x,|k_t|) \sim \int dz^+ d^2\boldsymbol{z} e^{ixP^- z^+ - ik_t\cdot \boldsymbol{z}} \langle P| \left[\bar{\psi}\left(z^+,\boldsymbol{z}_t\right) W_{process}(z,0)\psi\left(0^+,\boldsymbol{0}_t\right)\right] |P\rangle$$

$$\mathcal{F}_g(x,|k_t|) \sim \int dz^+ d^2\boldsymbol{z} e^{ixP^- z^+ - ik_t\cdot \boldsymbol{z}} \langle P|\mathrm{Tr}\left[F^{i-}\left(z^+,\boldsymbol{z}_t\right) W_{process}(z,0)F^{i-}\left(0^+,\boldsymbol{0}_t\right)\right] |P\rangle$$

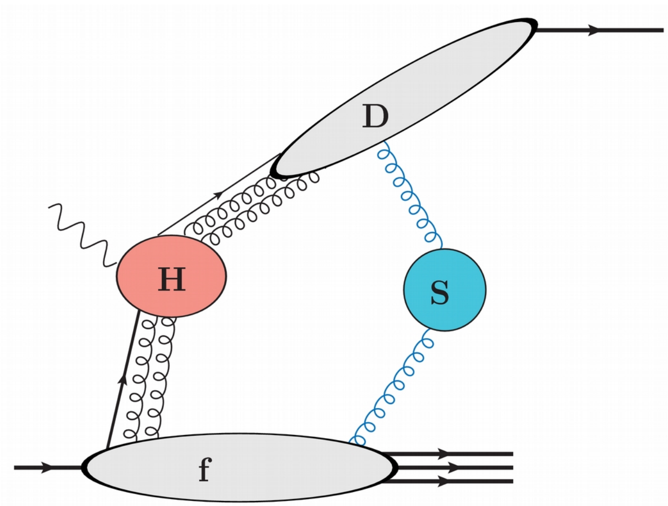
$$F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu - ig\left[A^\mu, A^\nu\right]$$

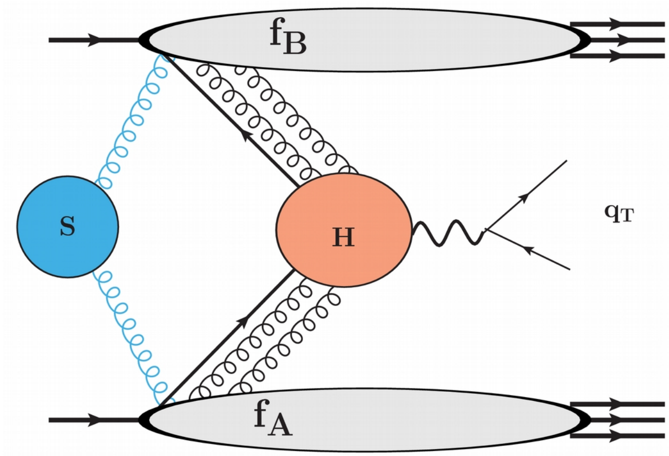
$$W_{process} = \mathcal{P}\exp\left[ig\int_a^b dx_\mu A_a^\mu\right]$$

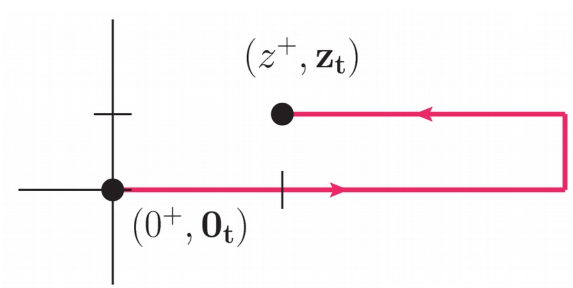
Transverse-momentum-dependent (TMD) factorization

$e + p \to e + h + X$ SIDIS

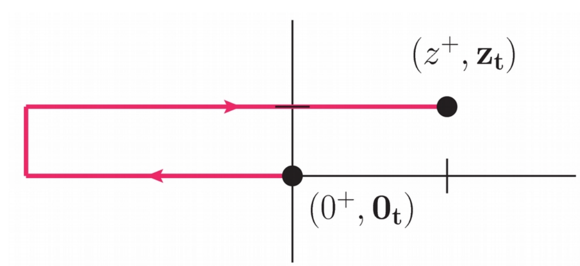
$p + p \to \mu^+ + \mu^- + X$ Drell-Yan







$U^{[+]}$



$U^{[-]}$

S. J. Brodsky, D. S. Hwang, and I. Schmidt, 2002; J.C. Collins, 2002

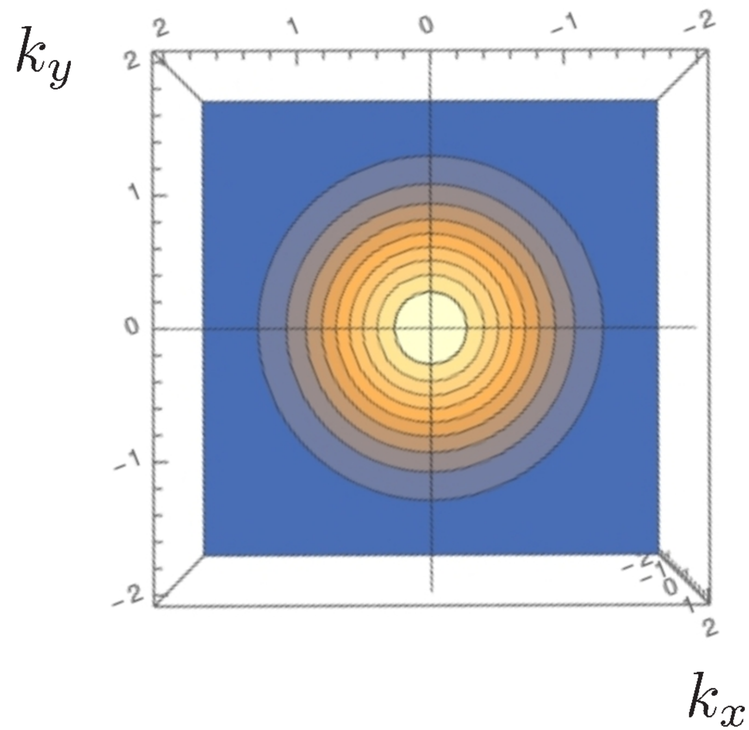
A. V. Belitsky, X. Ji, and F. Yuan, 2003; D. Boer, P. J. Mulders, and F. Pijlman, 2003

Transverse-momentum-dependent (TMD) distributions

- Distribution of partons in three-dimensional momentum space

Transverse-momentum-dependent (TMD) distributions

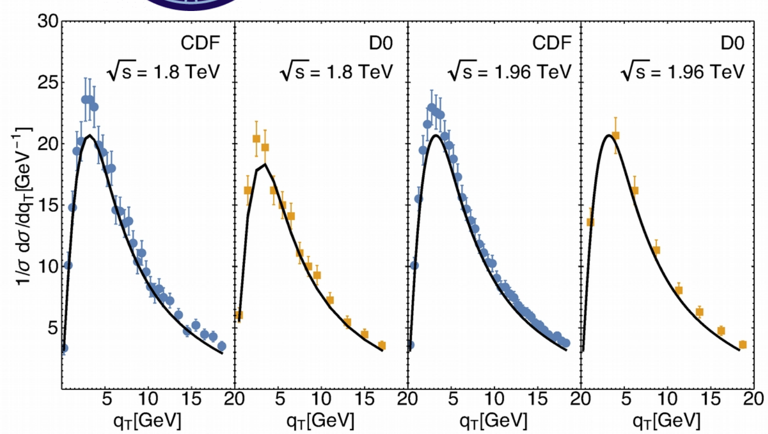
- Models;
- Global fits;
- Lattice.



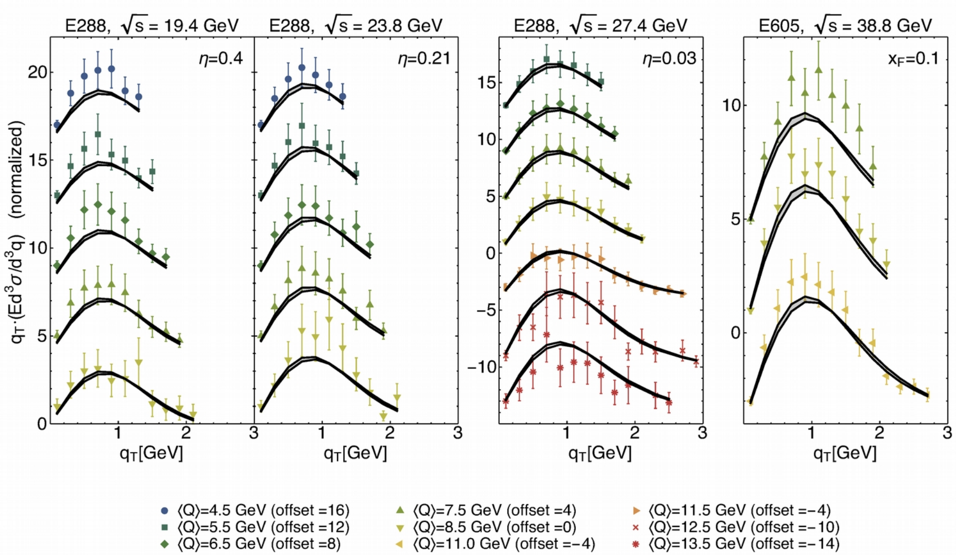
Transverse-momentum-dependent (TMD) distributions

- Models;
- Global fits;
- Lattice.

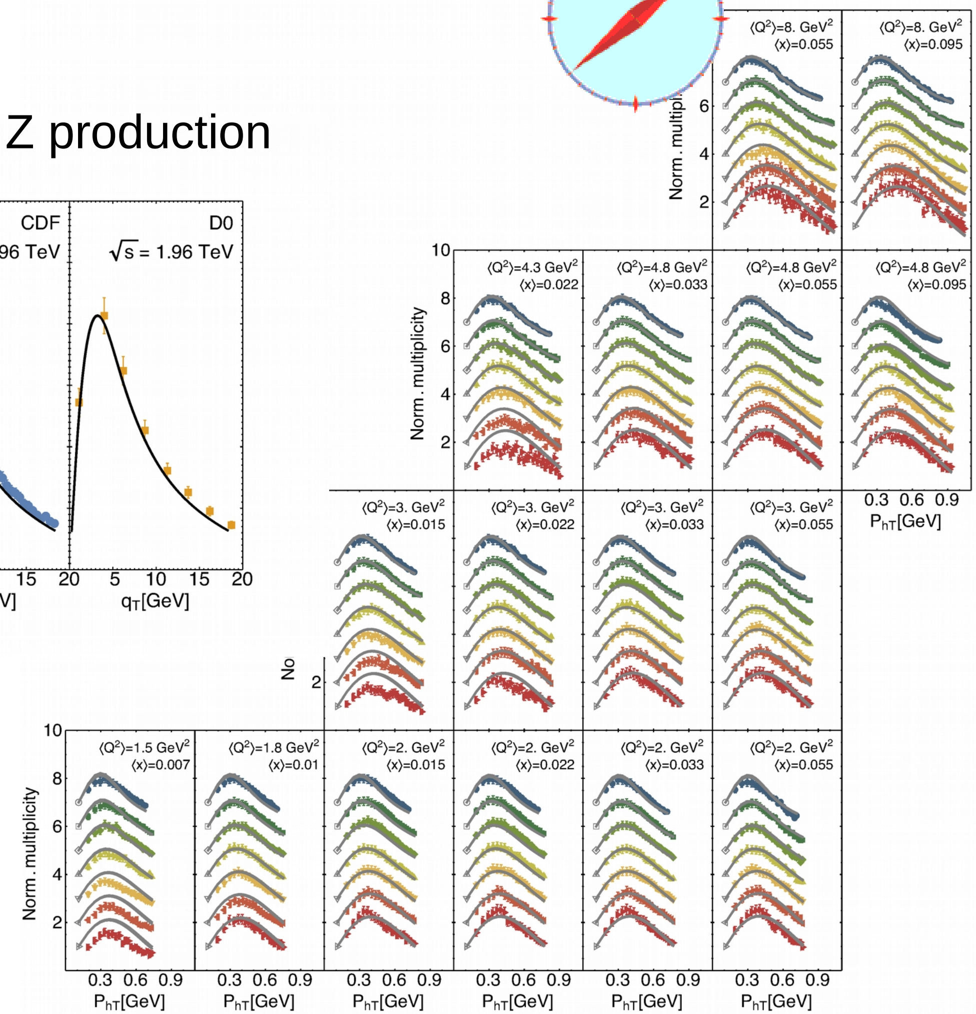
Z production



DY



SIDIS



A. Bacchetta, F. Delcarro, C.Pisano, M. Radici, A. Signori, 2017

Transverse-momentum-dependent (TMD) distributions

- Distribution of partons in three-dimensional momentum space

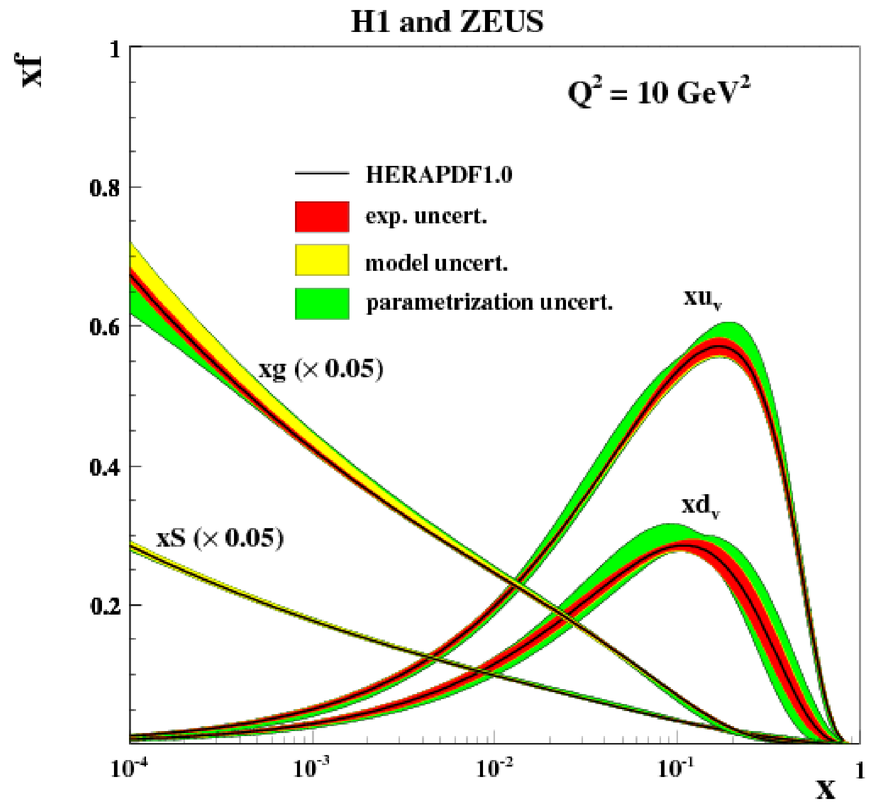
- TMD factorization and non-universality

    - Semi-inclusive DIS;
    - Drell-Yan;
    - Jets or hadrons produced back-to-back in DIS;
    - Jets or hadrons produced back-to-back in e^+e^- annihilation;

    ? Jets or hadrons produced back-to-back in pp and pA collisions.

Color Glass Condensate for gluon distributions at small x

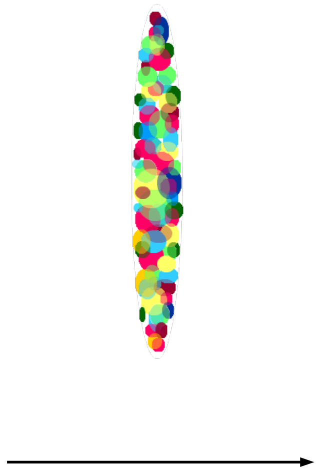
Color Glass Condensate for gluon distributions at small x

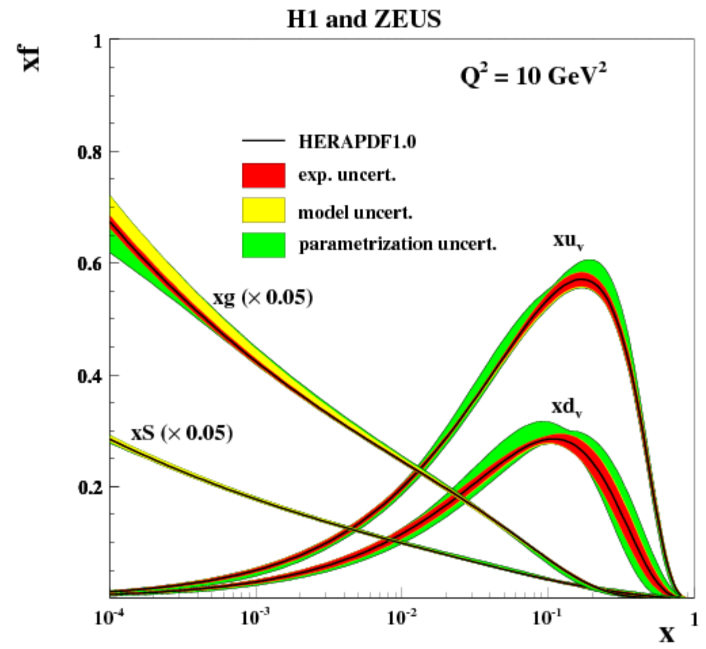


H1 and ZEUS Collaborations (2010)

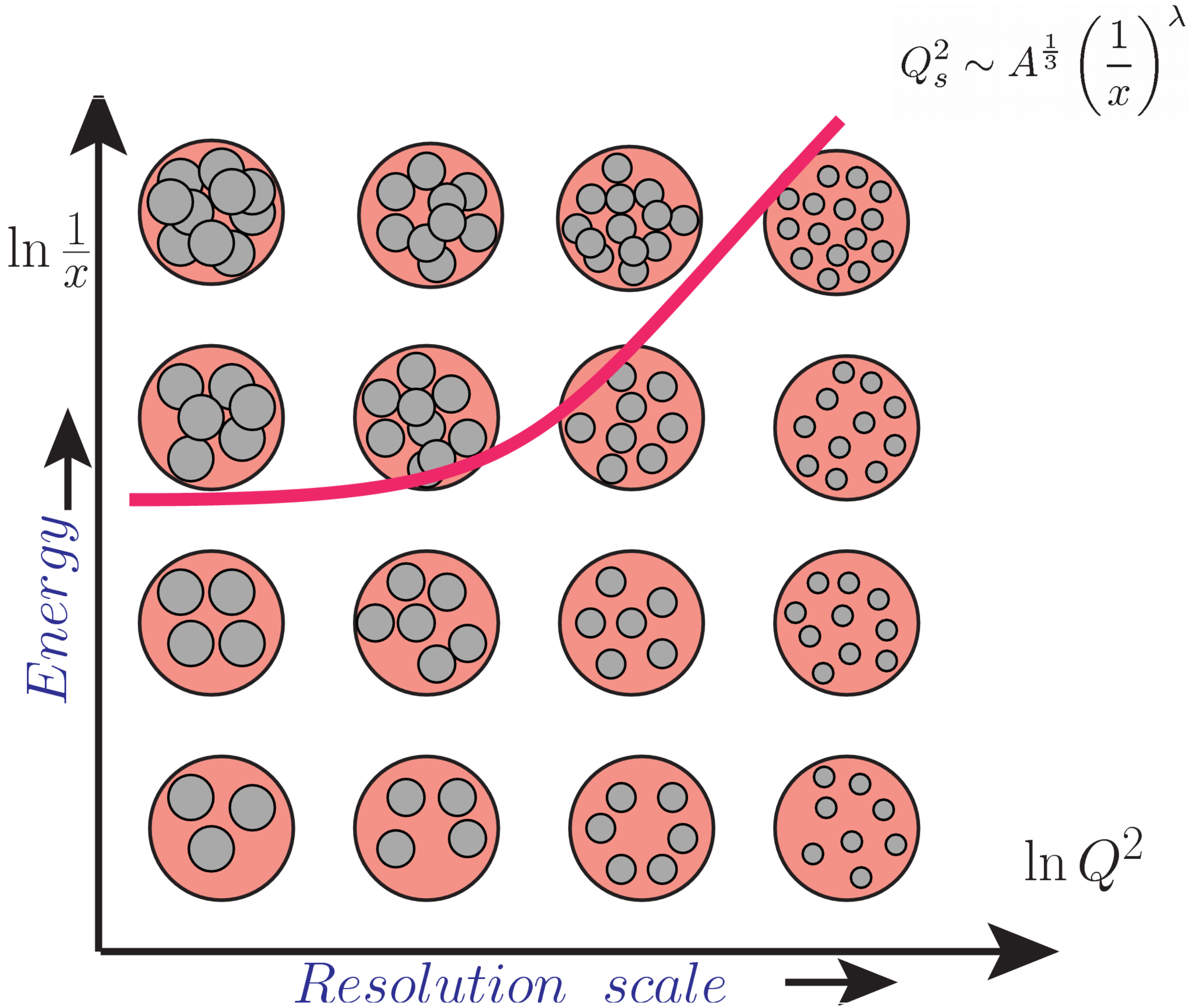
High-energy limit = Small-x limit

$$Q^2 \text{ is fixed}\ , \quad s \to \infty\ , \quad x \to 0$$





H1 and ZEUS Collaborations (2010)



$Q_s^2 \sim A^{\frac{1}{3}} \left(\frac{1}{x}\right)^{\lambda}$
$\ln \frac{1}{x}$
$Energy$
$\ln Q^2$
$Resolution\ \ scale$

Yang-Mills Lagrangian:

$$\mathcal{L}_{\mathrm{YM}} = -\frac{1}{2}\partial^{\mu} A^{a,\nu}\partial_{\mu} A_{a,\nu} + \frac{1}{2}\partial^{\mu} A^{a,\nu}\partial_{\nu} A_{a,\mu}$$

$$-g f_{abc}\left(\partial_{\mu} A^{a}_{\nu}\right) A^{b,\mu} A^{c,\nu} - \frac{g^2}{4} f_{eab} f_{ecd} A^{a}_{\mu} A^{b}_{\nu} A^{c,\mu} A^{d,\nu}$$

Saturation for a strong gluon field: $A_{\mu} \sim \frac{1}{g}$

High occupation number of gluons => Classical gluon field at small x

Saturation scale $Q_s \gg \Lambda_{QCD}$ => $\alpha_s(Q_s) \ll 1$

L. D. McLerran and R. Venugopalan, 1994

Separation of collective degrees of freedom:

Partons with large *x* act as sources for the gluons with small *x*

Solve classical Yang-Mills equations for the gluon field with current generated by the sources ρ at large x:

$$[D_\mu, F^{\mu\nu}] = J^\nu \qquad\qquad J_a^\mu = \delta^{\mu +}\delta(x^-)\rho_a(x_t)$$

L. D. McLerran and R. Venugopalan, 1994

The color sources are random, described by a distribution $W[\rho]$.

Observables:

$$\langle \mathcal{O} \rangle \equiv \int [\mathcal{D}\rho]\, W[\rho] \mathcal{O}[\rho]$$

- A model, Gaussian distribution:

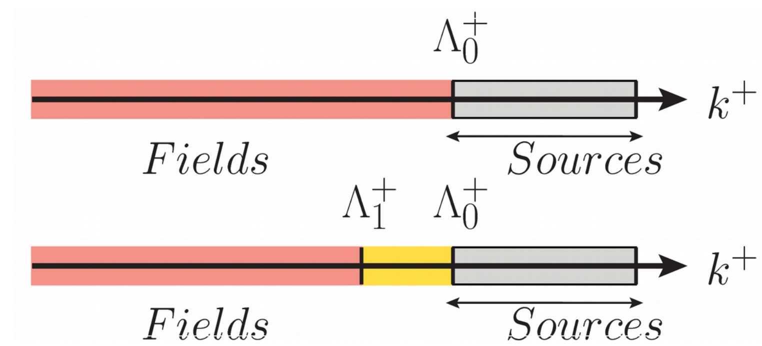
$$W[\rho] = \exp\left\{ -\int d^3x \frac{\rho_a \rho_a}{2\mu^2} \right\}$$

L. D. McLerran and R. Venugopalan, 1994

Quantum corrections enter through renormalization group equation.

Separation scale Λ^+

$$\langle \mathcal{O} \rangle = \int [\mathcal{D}\rho]\, W_{\Lambda^+}[\rho] \left\{ \frac{\int^{\Lambda^+} [\mathcal{D}A]\, \mathcal{O}[\rho] e^{iS[A,\rho]}}{\int^{\Lambda^+} [\mathcal{D}A]\, e^{iS[A,\rho]}} \right\}$$



Small-x evolution equation:

$$\frac{\partial W_{\Lambda^+}[\rho]}{\partial \ln (\Lambda^+)} = -\mathcal{H}\left[\rho, \frac{\delta}{\delta\rho}\right] W_{\Lambda^+}[\rho]$$

Resummation of high-energy logarithms:

$$\alpha_s \ln \frac{1}{x}$$

Jalilian-Marian–Iancu–McLerran–Weigert–Leonidov–Kovner (JIMWLK) $\rightarrow$ Large N_c Balitsky-Kovchegov (BK)

- A Gaussian (MV) model for the initial condition:

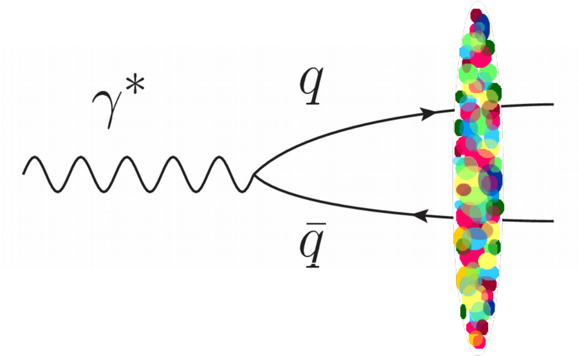
$$W[\rho] = \exp\left\{ - \int d^3x \frac{\rho_a \rho_a}{2\mu^2} \right\}$$

CGC can be applied in:

- Electron – Proton
- Proton – Proton
- Proton – Nucleus
- Nucleus – Nucleus

Electron – Proton

$$\sigma \sim |\psi_{\gamma\to q\bar{q}}|^2 \otimes \sigma_{\text{dipole}}$$



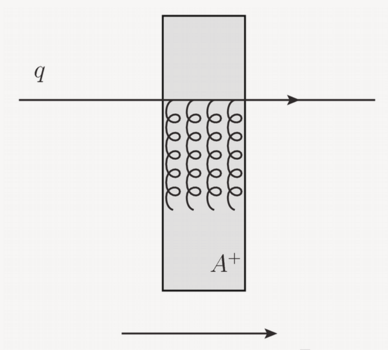
- Classical MV resums $\alpha_s^2 A^{1/3}$:

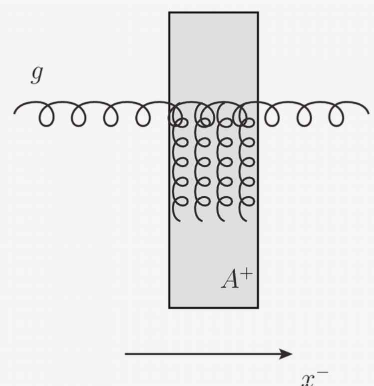
$$\sigma_{\text{dipole}}(x, r_t) \sim \int [D\rho]\, W_{\Lambda^+}[\rho] \left(1 - \frac{1}{N_c} \text{tr}\left[U(x_t)U^\dagger(y_t)\right]\right)$$

- Quantum evolution resums $\alpha_s \ln \frac{1}{x}$:

$$\frac{\partial W_{\Lambda^+}[\rho]}{\partial \ln(\Lambda^+)} = -\mathcal{H}\left[\rho, \frac{\delta}{\delta\rho}\right] W_{\Lambda^+}[\rho]$$

- Eikonal approximation:

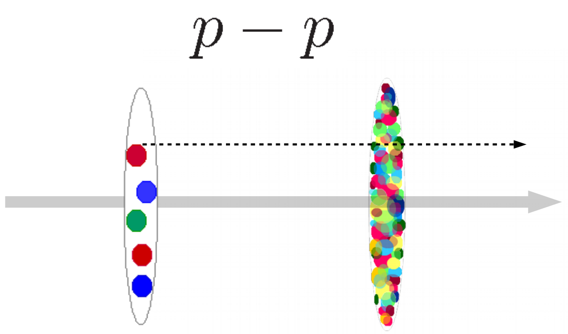


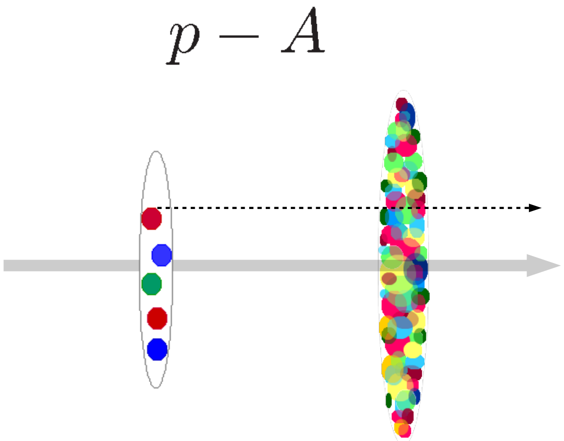


$$U(\mathbf{x}) = \mathcal{P}\exp\left[ig\int dx^- A_a^+(x^-, \mathbf{x})t^a\right] \qquad V(\mathbf{x}) = \mathcal{P}\exp\left[ig\int dx^- A_a^+(x^-, \mathbf{x})T^a\right]$$

Fundamental/Adjoint eikonal Wilson line

Proton – Proton
Proton – Nucleus
Nucleus – Nucleus





$$U(\mathbf{x}) = \mathcal{P}\exp\left[ig\int dx^- A_a^+(x^-,\mathbf{x})t^a\right]$$

$$V(\mathbf{x}) = \mathcal{P}\exp\left[ig\int dx^- A_a^+(x^-,\mathbf{x})T^a\right]$$

Proton – Proton
Proton – Nucleus
Nucleus – Nucleus

Classical Yang-Mills EOM:

$$[D_\mu, F^{\mu\nu}] = J^\nu$$

$$J^\mu = \delta^{\mu +}\rho_1 + \delta^{\mu +}\rho_2$$

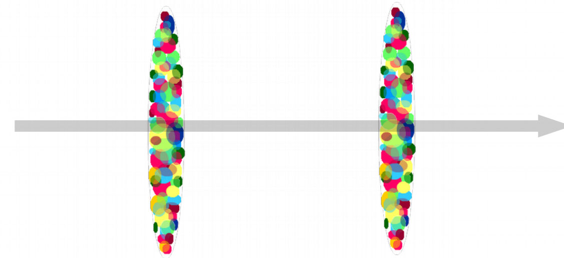
Observables:

$$\langle \mathcal{O} \rangle \equiv \int [\mathcal{D}\rho_1 \mathcal{D}\rho_2]\, W_1[\rho_1] W_2[\rho_2] \mathcal{O}[\rho]$$

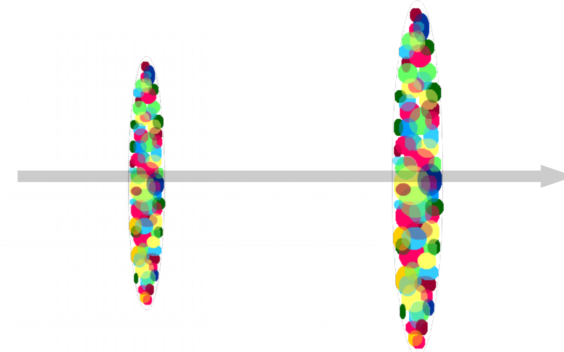
Quantum corrections through small-*x* evolution:

$$\frac{\partial W_{\Lambda^\pm}[\rho]}{\partial \ln(\Lambda^\pm)} = -\mathcal{H}_{1,2}\, W_{\Lambda^\pm}[\rho]$$

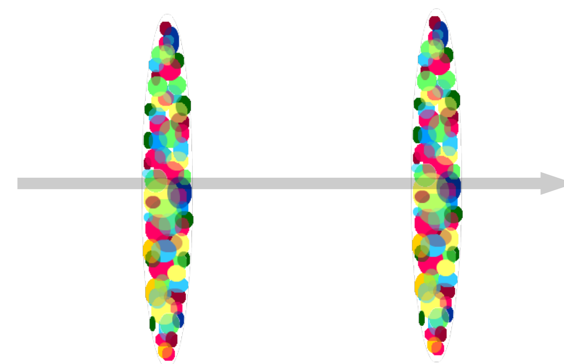
$p - p$



$p - A$

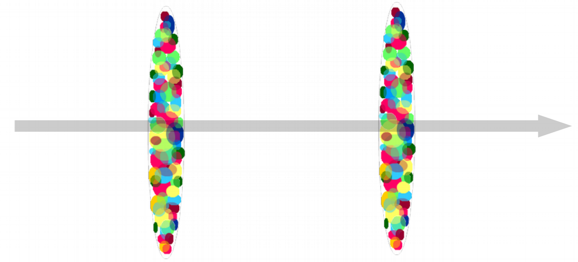


$A - A$

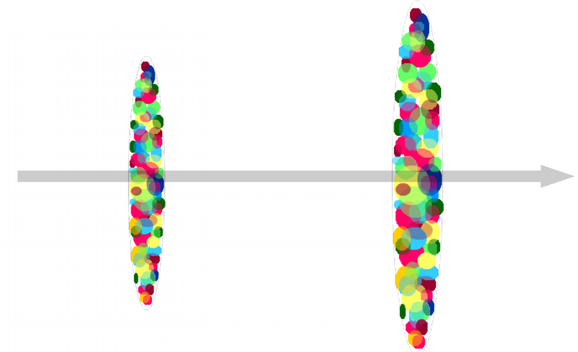


Proton – Proton
Proton – Nucleus
Nucleus – Nucleus

$p - p$



$p - A$

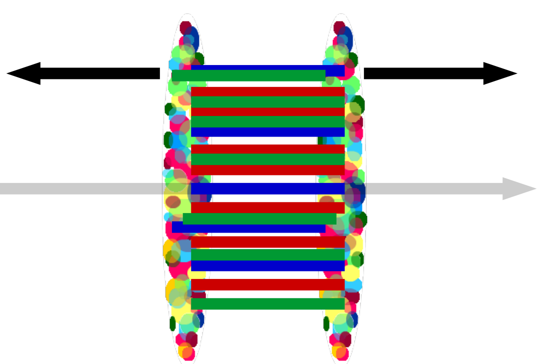


Classical Yang-Mills EOM:

$$[D_\mu, F^{\mu\nu}] = J^\nu$$

$$J^\mu = \delta^{\mu +}\rho_1 + \delta^{\mu +}\rho_2$$

$A - A$



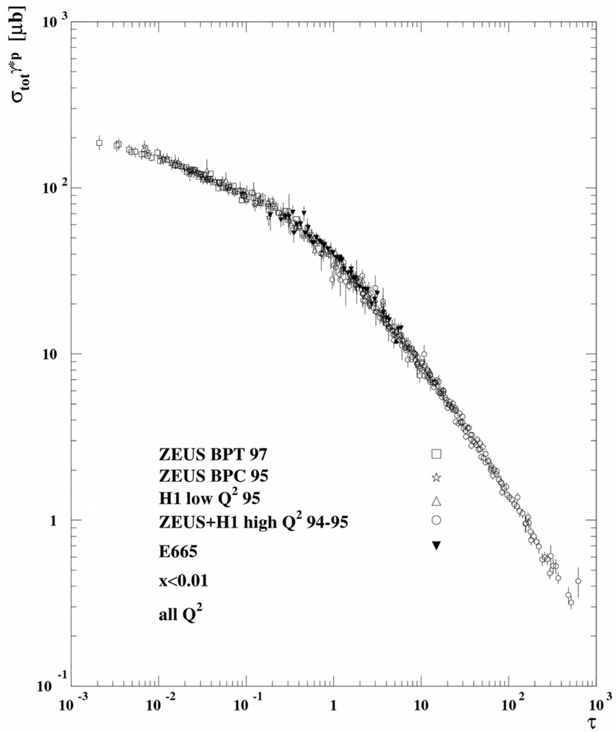
Observables:

$$\langle \mathcal{O} \rangle \equiv \int [\mathcal{D}\rho_1 \mathcal{D}\rho_2]\, W_1[\rho_1] W_2[\rho_2] \mathcal{O}[\rho]$$

Quantum corrections through small-*x* evolution:

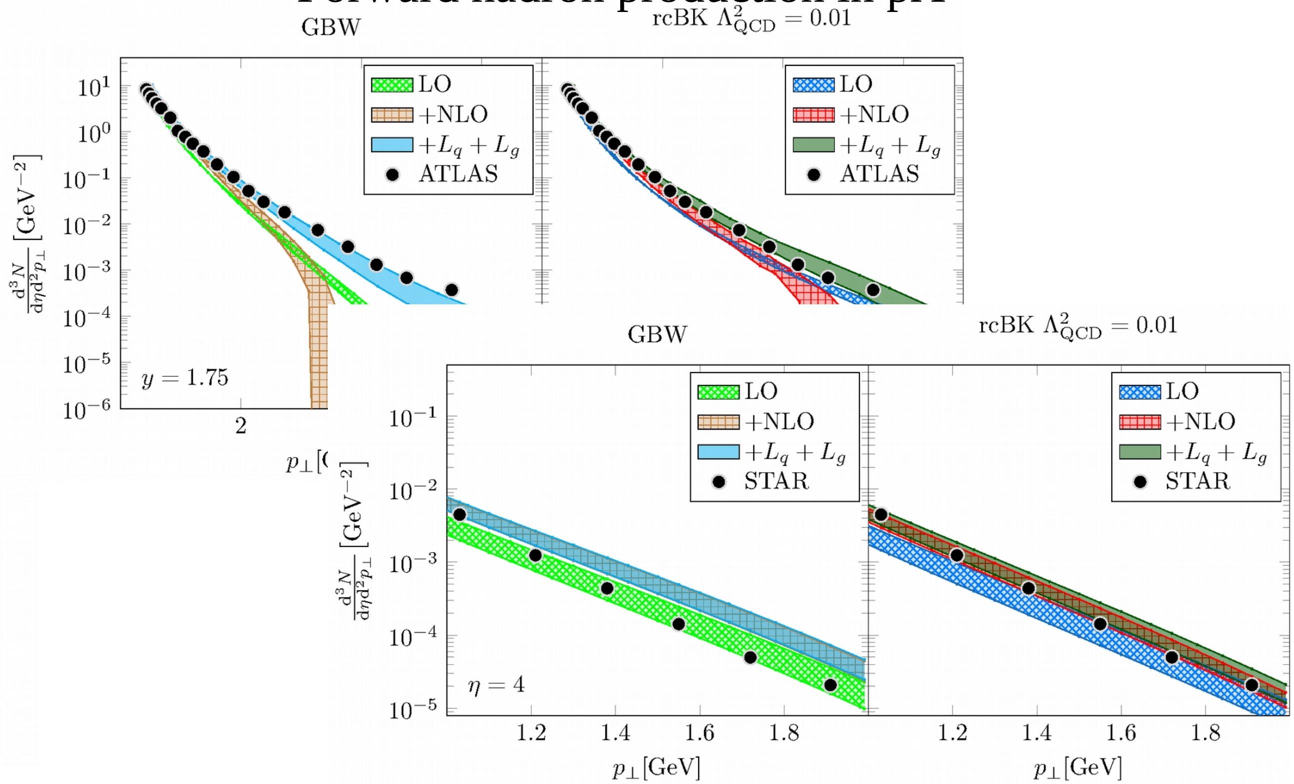
$$\frac{\partial W_{\Lambda^\pm}[\rho]}{\partial \ln(\Lambda^\pm)} = -\mathcal{H}_{1,2}\, W_{\Lambda^\pm}[\rho]$$

Geometric scaling in DIS



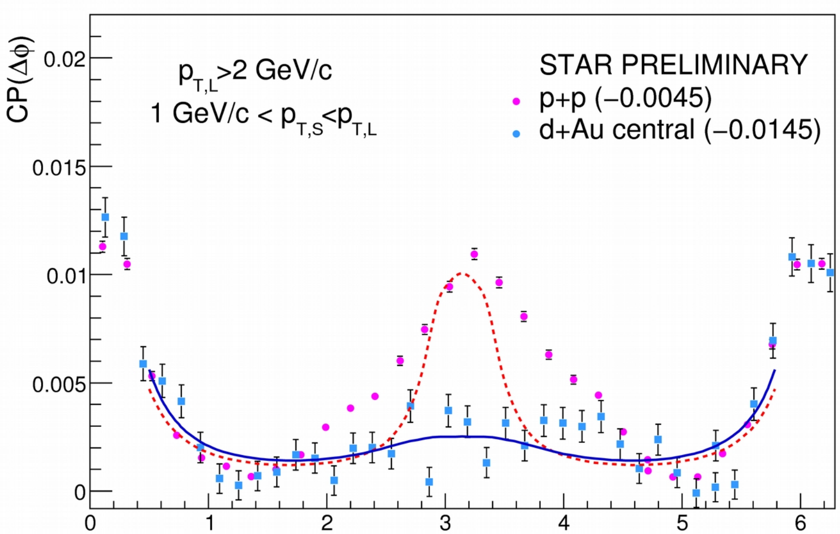
A. M. Stasto, K. Golec-Biernat, J. Kwiecinski, 2001

Forward hadron production in pA



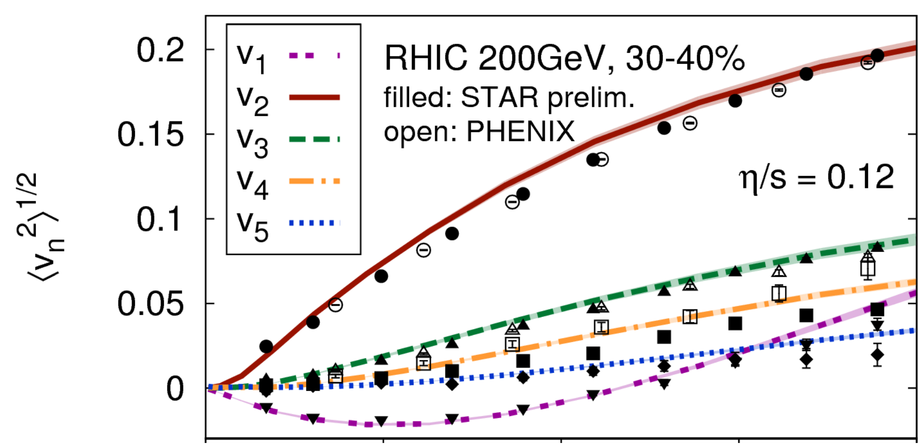
K. Watanabe, B. W. Xiao, F. Yuan, D. Zaslavsky, 2015

Di-hadron correlations in d+Au



J. L. Albacete, C. Marquet, 2010

Flow coefficients in AA



C. Gale, S. Jeon, B. Schenke, P. Tribedy, R. Venugopalan, 2013

The Color Glass Condensate provides a universal picture of the 3D momentum (and position) distribution of gluons inside protons and nuclei at small x.

Use CGC to study gluon TMD distributions at small x.

Transverse Momentum Dependent (TMD) factorization

- Ordering of momentum scales:

$$p_t \ll Q$$

- Any x;

- TMD gluon distributions:

$$\sim \text{F.T.} \left\langle P \middle| \mathrm{Tr}\left[F^{i-}\left(z^+, \boldsymbol{z}_t\right) W(z,0) F^{i-}\left(0^+, \mathbf{0}_t\right)\right] \middle| P \right\rangle$$

$$W = \mathcal{P} \exp\left[ig \int_a^b dx_\mu A_a^\mu\right]$$

Color Glass Condensate (CGC)

- No ordering of momentum scales;

- Small x;

- Averages of Wilson lines:

$$\sigma \sim \left\langle \mathrm{Tr}\left(U(\mathbf{b})U^\dagger(\mathbf{b}')t^d t^c\right) \left[V(\mathbf{x})V^\dagger(\mathbf{x}')\right]^{cd} \right\rangle_{W[\rho]}$$

$$U(\mathbf{x}) = \mathcal{P} \exp\left[ig \int_{-\infty}^{\infty} dx^- A_a^+(x^-, \mathbf{x}) t^a\right]$$

Transverse Momentum Dependent (TMD) factorization

- Ordering of momentum scales:

$$p_t \ll Q$$

- Any x → Small-x limit.

- TMD gluon distributions:

$$\sim \mathrm{F.T.} \langle P | \mathrm{Tr} \left[F^{i-}\left(z^+, \boldsymbol{z}_t\right) W(z,0) F^{i-}\left(0^+, \mathbf{0}_t\right)\right] |P\rangle$$

$$\Downarrow$$

$$\sim \mathrm{F.T.} \left\langle \mathrm{Tr} \left[(\partial_i U_{\mathbf{x}}) U^\dagger_{\mathbf{y}} \ldots (\partial_i U_{\mathbf{y}}) U^\dagger_{\mathbf{x}} \right] \right\rangle_{W[\rho]}$$

$$\Leftrightarrow$$

Color Glass Condensate (CGC)

- No ordering of momentum scales;

$$p_t \ll Q$$

- Small x;

- Averages of Wilson lines:

$$\sigma \sim \left\langle \mathrm{Tr}\left(U(\mathbf{b}) U^\dagger(\mathbf{b}') t^d t^c \right) \left[V(\mathbf{x}) V^\dagger(\mathbf{x}') \right]^{cd} \right\rangle_{W[\rho]}$$

$$\Downarrow$$

$$\sim \mathrm{F.T.} \left\langle \mathrm{Tr} \left[(\partial_i U_{\mathbf{x}}) U^\dagger_{\mathbf{y}} \ldots (\partial_i U_{\mathbf{y}}) U^\dagger_{\mathbf{x}} \right] \right\rangle_{W[\rho]}$$

TMD factorization $\equiv$ CGC

- Semi-inclusive DIS;
- Drell-Yan;
- Dijet production in DIS;
- Direct photon-jet production in pA;
- Dijet production in pp and pA.

F. Dominguez, C. Marquet, B. Xiao, F. Yuan, 2011;
F. Dominguez, B. Xiao, F. Yuan, 2011;
A. Metz, J. Zhou, 2011;
F. Dominguez J.W. Qiu, B. W. Xiao, F. Yuan, 2012 ;
P. Kotko, K. Kutak, C. Marquet, E. P., S. Sapeta, A. van Hameren, 2015;
C. Marquet, E.P., C. Roiesnel, 2016.

Non-universality

Dipole gluon distribution

$$\mathrm{TMD} \sim \mathrm{F.T.} \left\langle \mathrm{Tr}\left[F(\xi)\,\mathcal{U}^{[-]\dagger} F(0)\,\mathcal{U}^{[+]}\right]\right\rangle$$

$$\mathrm{CGC} \sim \mathrm{F.T.} \left\langle \mathrm{Tr}\left[(\partial_i U_{\mathbf{y}})\left(\partial_i U^\dagger_{\mathbf{x}}\right)\right]\right\rangle_W$$

Appears in:

- Semi-inclusive DIS;
- Drell-Yan;
- Hadron production in pA;
- Direct photon-jet production in pA;
- Dijet production in pp and pA.

- $\sim k_t^2$ at small transverse momentum.
- No probabilistic interpretation.

Weizsacker-Williams distribution

$$\mathrm{TMD} \sim \mathrm{F.T.} \left\langle \mathrm{Tr}\left[F(\xi)\,\mathcal{U}^{[+]\dagger} F(0)\,\mathcal{U}^{[+]}\right]\right\rangle$$

$$\mathrm{CGC} \sim \mathrm{F.T.} \left\langle \mathrm{Tr}\left[(\partial_i U_{\mathbf{x}})\, U^\dagger_{\mathbf{y}} \left(\partial_i U_{\mathbf{y}}\right) U^\dagger_{\mathbf{x}}\right]\right\rangle_W$$

Appears in:

- Dijet production in DIS;
- Higgs production from gluon fusion in pA;
- Dijet production in pp and pA.

- $\sim \ln(Q_s^2/k_t^2)$ at small transverse momentum.
- Probabilistic interpretation as number density.

- Express TMDs in terms of the same Wilson lines → Universality.

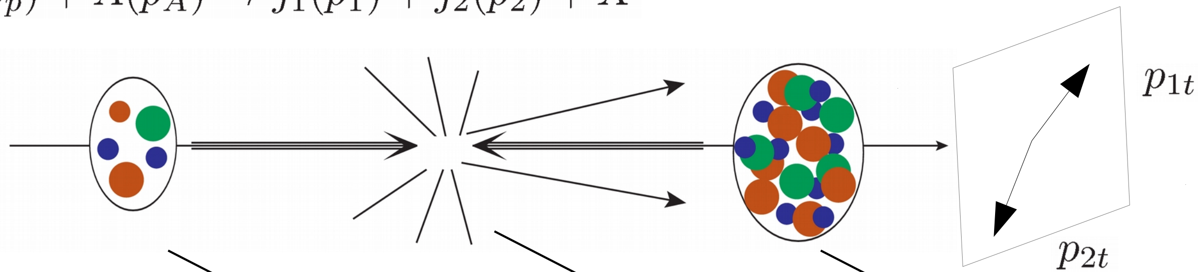
- Study TMDs using CGC methods:
    - Small-x evolution;
    - Perturbative behavior at large transverse momenta;
    - …

- Use CGC to derive relations between different TMDs;

- Derive effective TMD factorization from CGC at small x for processes where it cannot be derived in a general case.
    - Eg: Forward dijet production in pA.

Forward dijet production in pA

$$p(p_p) + A(p_A) \to j_1(p_1) + j_2(p_2) + X$$



Close to back-to-back:

$$|k_t| = |p_{1t} + p_{2t}| \ll |p_{1t}|, |p_{2t}|$$

Collinear PDF

$2 \to 2$
Hard factor

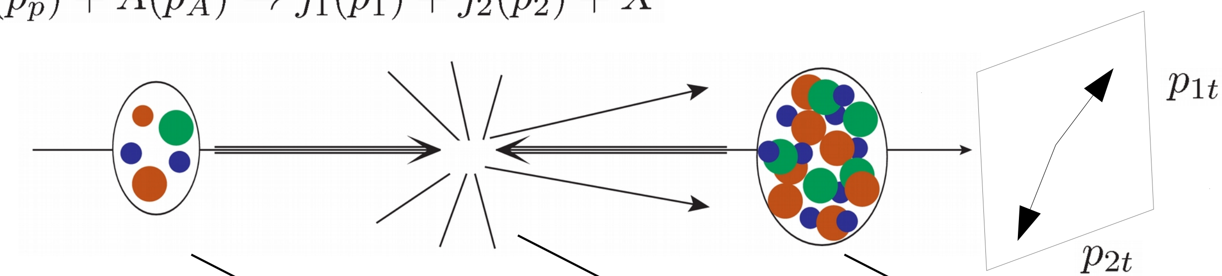
8 TMDs

$$\frac{d\sigma^{pA\to \text{dijets}+X}}{dy_1 dy_2 d^2p_{1t} d^2p_{2t}} = \frac{\alpha_s^2}{(x_1x_2s)^2} \sum_{a,c,d} x_1 f_{a/p}(x_1,\mu^2) \sum_i H^{(i)}_{ag\to cd} \mathcal{F}^{(i)}_{ag}(x_2,k_t) \frac{1}{1+\delta_{cd}}$$

Bomhof, Mulders and Pijlman (2006)
Dominguez, Marquet, Xiao and Yuan (2011)
Kotko, Kutak, Marquet, EP, Sapeta and van Hameren (2015)
Marquet, EP, Roiesnel (2016)

Forward dijet production in pA

$$p(p_p) + A(p_A) \to j_1(p_1) + j_2(p_2) + X$$



Close to back-to-back:

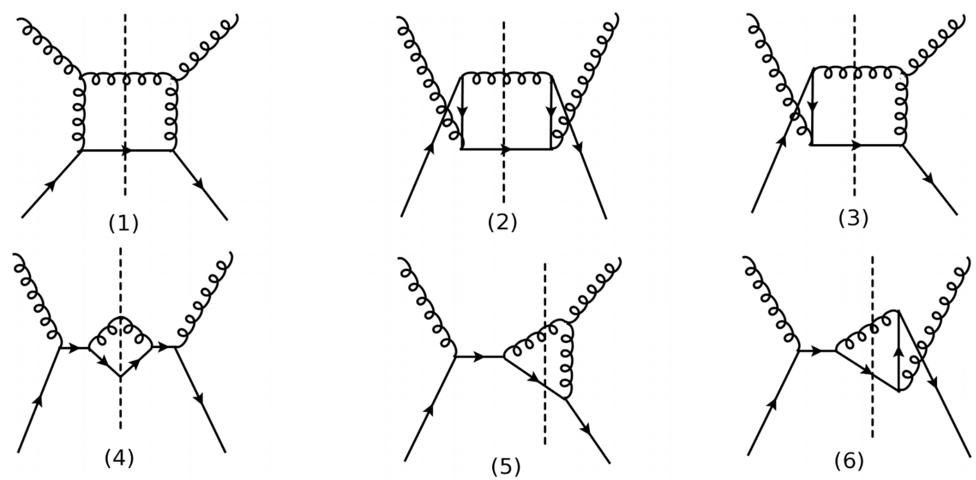
$$|k_t| = |p_{1t} + p_{2t}| \ll |p_{1t}|, |p_{2t}|$$

Collinear PDF

$2 \to 2$
Hard factor

8 TMDs

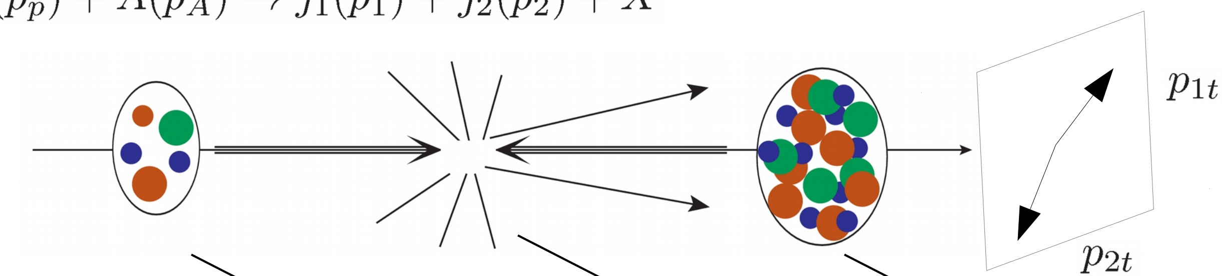
$$\frac{d\sigma^{pA\to \text{dijets}+X}}{dy_1 dy_2 d^2p_{1t} d^2p_{2t}} = \frac{\alpha_s^2}{(x_1 x_2 s)^2} \sum_{a,c,d} x_1 f_{a/p}(x_1, \mu^2) \sum_i H^{(i)}_{ag\to cd} \mathcal{F}^{(i)}_{ag}(x_2, k_t) \frac{1}{1+\delta_{cd}}$$



Bomhof, Mulders and Pijlman (2006)
Dominguez, Marquet, Xiao and Yuan (2011)
Kotko, Kutak, Marquet, EP, Sapeta and van Hameren (2015)
Marquet, EP, Roiesnel (2016)

Forward dijet production in pA

$$p(p_p) + A(p_A) \to j_1(p_1) + j_2(p_2) + X$$



Close to back-to-back:

$$|k_t| = |p_{1t} + p_{2t}| \ll |p_{1t}|, |p_{2t}|$$

Collinear PDF

$2 \to 2$
Hard factor

8 TMDs

$$\frac{d\sigma^{pA\to \text{dijets}+X}}{dy_1 dy_2 d^2p_{1t} d^2p_{2t}} = \frac{\alpha_s^2}{(x_1 x_2 s)^2} \sum_{a,c,d} x_1 f_{a/p}(x_1,\mu^2) \sum_i H^{(i)}_{ag\to cd} \mathcal{F}^{(i)}_{ag}(x_2,k_t) \frac{1}{1+\delta_{cd}}$$

$\left\langle \mathrm{Tr}\left[F(\xi)\,\mathcal{U}^{[-]\dagger} F(0)\,\mathcal{U}^{[+]}\right]\right\rangle$ Dipole distribution

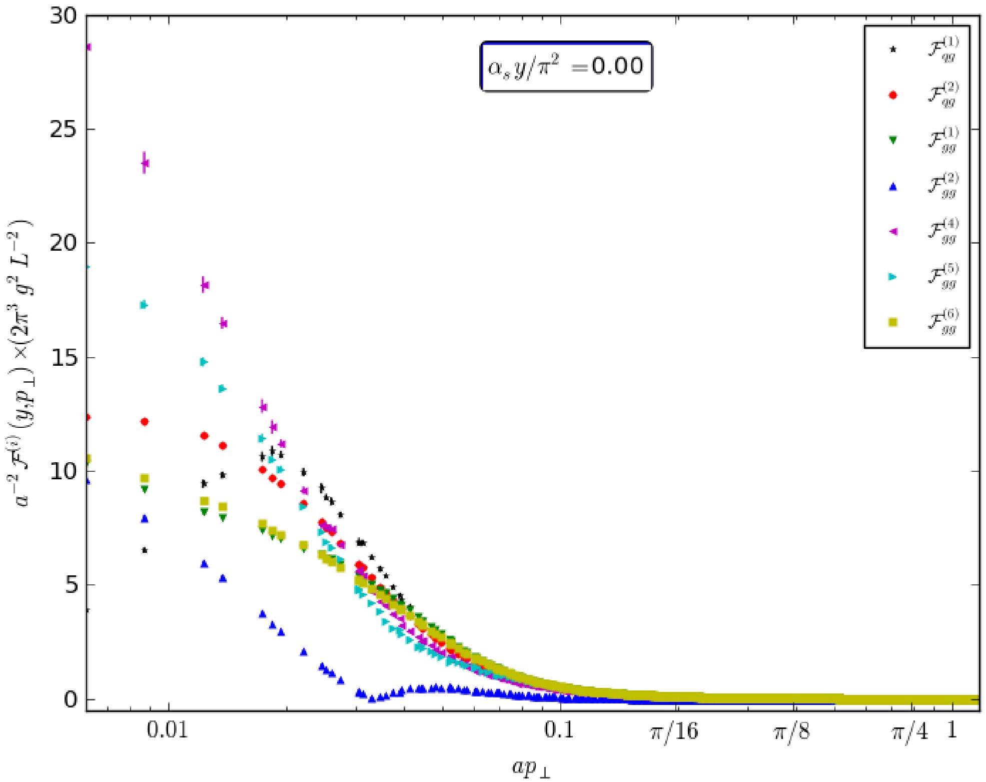
$\left\langle \mathrm{Tr}\left[F^{i-}(\xi)\;\mathcal{U}^{[+]\dagger} F^{i-}(0)\;\mathcal{U}^{[+]}\right]\right\rangle$ Weizsacker-Williams gluon distribution

$$\left\langle \mathrm{Tr}\left[F^{i-}(\xi)\;\mathcal{U}^{[+]\dagger} F^{i-}(0)\;\mathcal{U}^{[+]}\right] \mathrm{Tr}\left[\mathcal{U}^{[\square]}\right]\right\rangle \qquad \left\langle \mathrm{Tr}\left[F^{i-}(\xi)\;\mathcal{U}^{[\square]\dagger}_{\xi}\right] \mathrm{Tr}\left[F^{i-}(0)\;\mathcal{U}^{[\square]}_{0}\right]\right\rangle$$

$$\left\langle \mathrm{Tr}\left[F^{i-}(\xi)\;\mathcal{U}^{[-]\dagger} F^{i-}(0)\;\mathcal{U}^{[+]}\right] \mathrm{Tr}\left[\mathcal{U}^{[\square]\dagger}\right]\right\rangle \qquad \left\langle \mathrm{Tr}\left[F^{i-}(\xi)\;\mathcal{U}^{[\square]\dagger}_{\xi}\mathcal{U}^{[+]\dagger} F^{i-}(0)\;\mathcal{U}^{[\square]}_{0}\mathcal{U}^{[+]}\right]\right\rangle$$

$$\left\langle \mathrm{Tr}\left[F^{i-}(\xi)\;\mathcal{U}^{[-]\dagger} F^{i-}(0)\;\mathcal{U}^{[-]}\right]\right\rangle \qquad \left\langle \mathrm{Tr}\left[F^{i-}(\xi)\;\mathcal{U}^{[+]\dagger} F^{i-}(0)\;\mathcal{U}^{[+]}\right] \mathrm{Tr}\left[\mathcal{U}^{[\square]}\right] \mathrm{Tr}\left[\mathcal{U}^{[\square]\dagger}\right]\right\rangle$$

McLerran-Venugopalan model

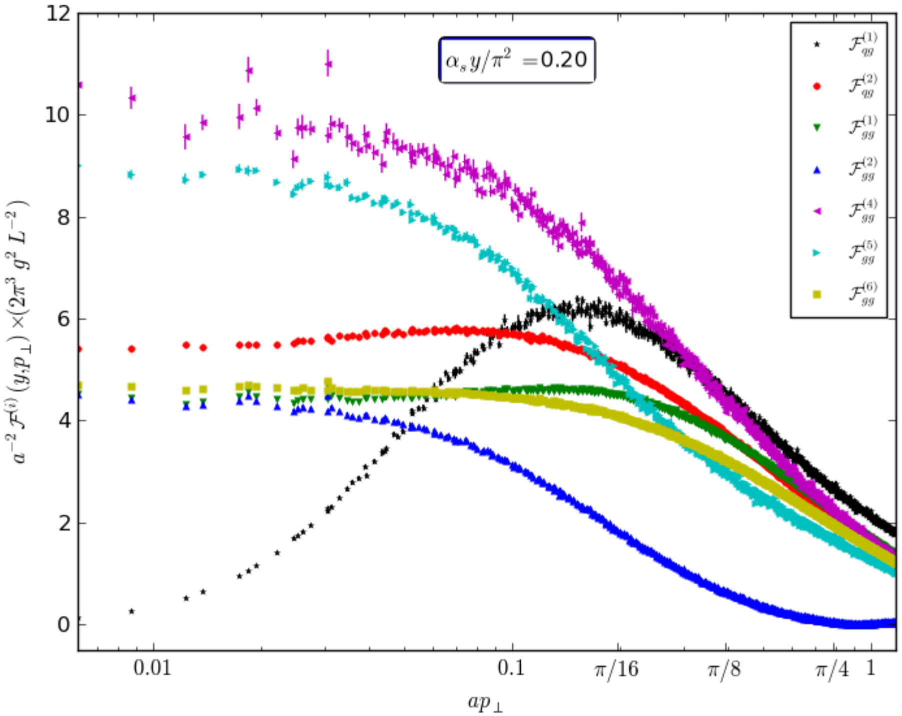


- High-kt behavior in the McLerran-Venugopalan model → All TMDs scale the same:

$$\mathcal{F}^{(i)} \simeq \gamma \frac{Q_s^2(x_2)}{k_t^2} + \mathcal{O}\left(\frac{Q_s^4(x_2)}{k_t^4} \log \frac{k_t^2}{\Lambda^2}\right)$$

Marquet, EP, Roiesnel (2016)

JIMWLK small-x evolution

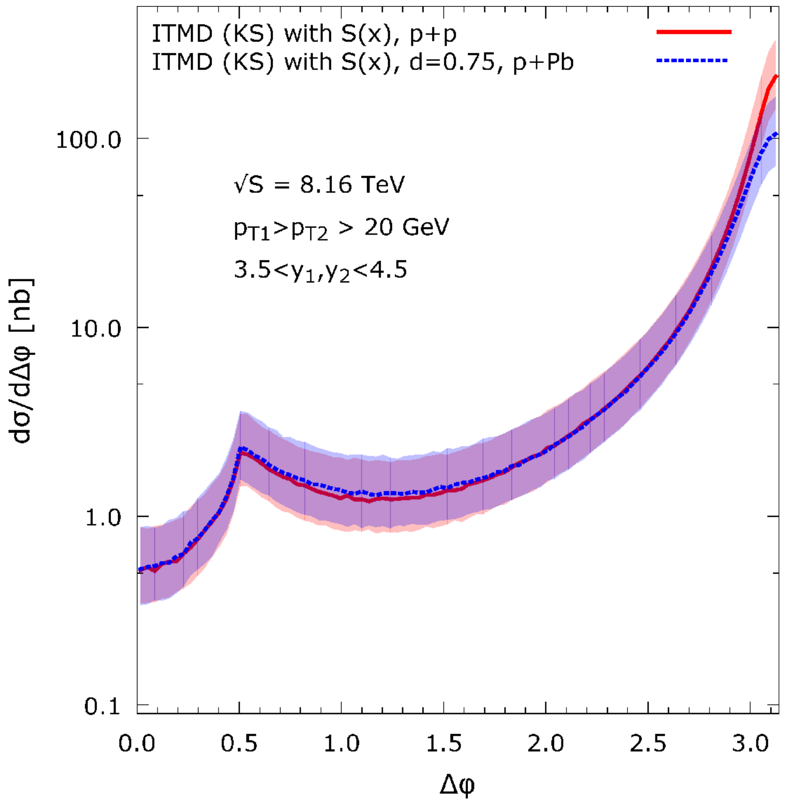


Different small-x behavior for different TMDs → Loss of universality at low transverse momentum

Marquet, EP, Roiesnel (2016)

code written by Claude Roiesnel

Forward dijet production in pA



van Hameren, Kotko, Kutak, Marquet, EP and Sapeta (2016)

Summary

Transverse-momentum-dependent (TMD) distributions

TMD factorization

Color Glass Condensate (CGC)

Probe the high-gluon-density matter;

EIC to probe the Weizsacker-Williams gluon distribution.

Non-universality $\rightarrow$ Test of QCD in the connection between experiments:

Quark distribution in polarized proton: SIDIS v.s. DY

Gluon distriubtion: Dipole from pA v.s. Weizsacker-Williams from EIC

Thank you!