Little hierarchy in the minimally specified MSSM

Radovan Dermisek

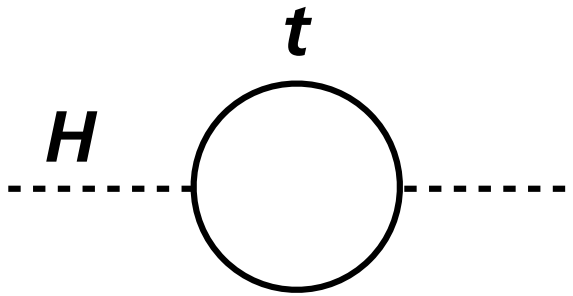
Indiana University, Bloomington

Brookhaven Forum 2017, BNL, October 12, 2017

Fine tuning problem in EWSB

in a bottom up approach is seen as the cancellation of two numbers:

the top loop contribution to the Higgs mass squared



$$\Delta m_H^2 = -\frac{\lambda_t^2}{8\pi^2}\Lambda^2 + \dots$$

and the boundary condition at the scale of new physics.

Quantifying fine tuning

In a model with two parameters A, B ~ 1 contributing to X,

$$X = A - B$$

in order to get e.g. X = 0.001, parameters A and B have to be specified with 3 digits and carefully chosen/tuned

e.g.: A = 0.963
B = 0.962

Tuning often quantified by:

$$\max_{p=\{A,B\}} \left| \frac{\partial \ln X}{\partial \ln p} \right| \simeq \frac{A}{X} \simeq 1000$$

0.1% tuning

Quantifying fine tuning

In a model with two parameters A, B ~ 1 contributing to X,

$$X = A - B$$

in order to get e.g. X = 0.001, parameters A and B have to be specified with 3 digits and carefully chosen/tuned

e.g.: A = 0.963
B = 0.962

Tuning often quantified by:

$$\max_{p=\{A,B\}} \left| \frac{\partial \ln X}{\partial \ln p} \right| \simeq \frac{A}{X} \simeq 1000$$

0.1% tuning

Note that ~10% tuning corresponds to specifying A and B with one digit: no matter what the remaining digits are, we need to adjust just the first digit of model parameters to get X ~ 0.1

Fine tuning in the MSSM

In the MSSM there are several parameters contributing to the electroweak scale, e.g:

$$M_Z^2 \simeq -1.9\mu^2 + 5.9M_3^2 - 1.2m_{H_u}^2 + 1.5m_{\tilde{t}}^2 - 0.8A_t M_3 + 0.2A_t^2 + \dots$$

boundary conditions at the GUT scale

$\tan\beta = 10$

The usual naturalness measure,

$$\max_p \left| \frac{\partial \ln M_Z^2}{\partial \ln p} \right|$$

gives us the size of the largest individual contribution. This however does not directly translate into how model parameters need to be tuned/specified.

Little hierarchy from complexity

e.g. in a model with more parameters contributing to X:

$$X = A - B - c - d$$

in order to get X = 0.001 for randomly chosen A ~ 1, no parameter needs to be carefully chosen, e.g.:

A = 0.963

B = 0.9
c = 0.06
d = 0.002

only the first digit of all parameters need to be adjusted no matter what the following digits are

what is a tuned outcome in a model with 2 parameters may be a completely ordinary outcome in a more complex model

RD, arXiv:1611.03188

Outline

Naturalness criteria based on the largest contribution are too strong for models with more parameters (they do not necessarily indicate how parameters have to be tuned).

Naturalness based on parameter specification:

Natural outcome for an observable in a given model is any outcome that does not require specifying model parameters with more than 10% precision (no more than one digit needs to be specified).

Let's see what it means for EWSB in the CMSSM.

Minimally specified CMSSM

Assume model parameters of the same order

$$\mathbf{M_{1/2},\ m_0,\ \mu,\ -A_0\ \simeq\ M_{SUSY}}$$

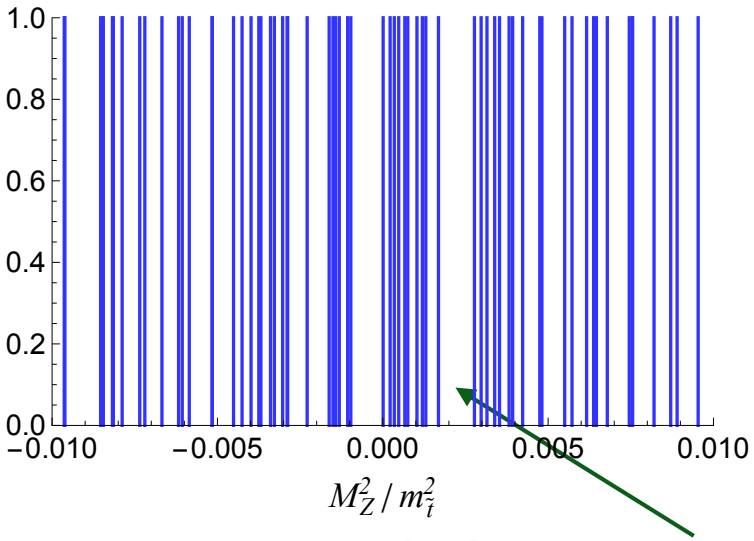
and vary them in ± 50% range keeping only one digit (10% precision) specifying the departure from $\mathbf{M_{SUSY}}$, e.g.:

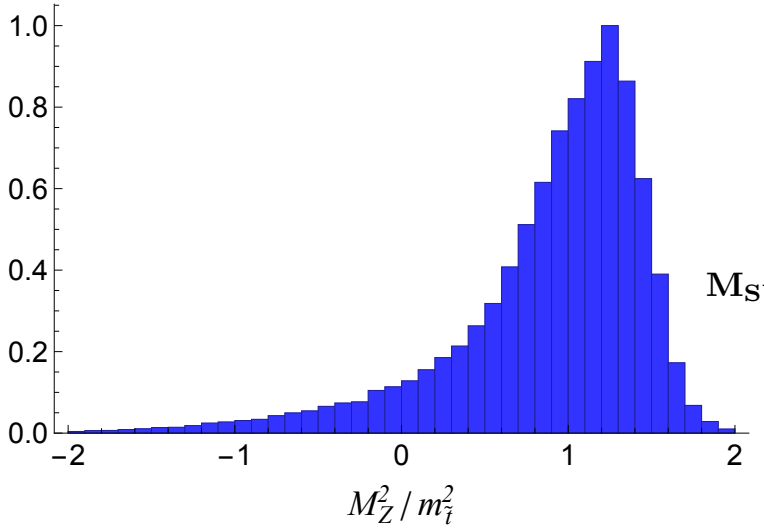
$$\begin{aligned} \mathbf{M_{1/2}} &= \mathbf{0.6\,M_{SUSY}} \\ \mathbf{m_0} &= \mathbf{1.1\,M_{SUSY}} \\ \mu &= \mathbf{1.4\,M_{SUSY}} \\ \mathbf{-A_0} &= \mathbf{0.9\,M_{SUSY}} \\ &\vdots \end{aligned} \qquad \begin{aligned} \mathbf{M_{1/2}} &= \mathbf{0.9\,M_{SUSY}} \\ \mathbf{m_0} &= \mathbf{1.2\,M_{SUSY}} \\ \mu &= \mathbf{0.6\,M_{SUSY}} \\ \mathbf{-A_0} &= \mathbf{0.8\,M_{SUSY}} \\ &\vdots \end{aligned} \qquad \begin{aligned} \mathbf{M_{1/2}} &= \mathbf{0.5\,M_{SUSY}} \\ \mathbf{m_0} &= \mathbf{1.1\,M_{SUSY}} \\ \mu &= \mathbf{1.5\,M_{SUSY}} \\ \mathbf{-A_0} &= \mathbf{0.8\,M_{SUSY}} \\ &\vdots \end{aligned}$$

Minimally specified CMSSM

Maximal hierarchy from minimally specified inputs:

RD and N. McGinnis, arXiv:1705.01910





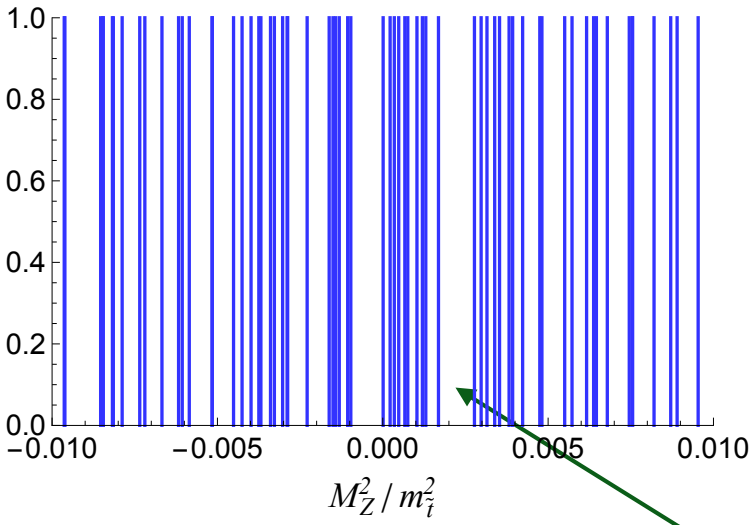
the largest gaps ~0.001 (fairly uniform away from edges of the distribution)

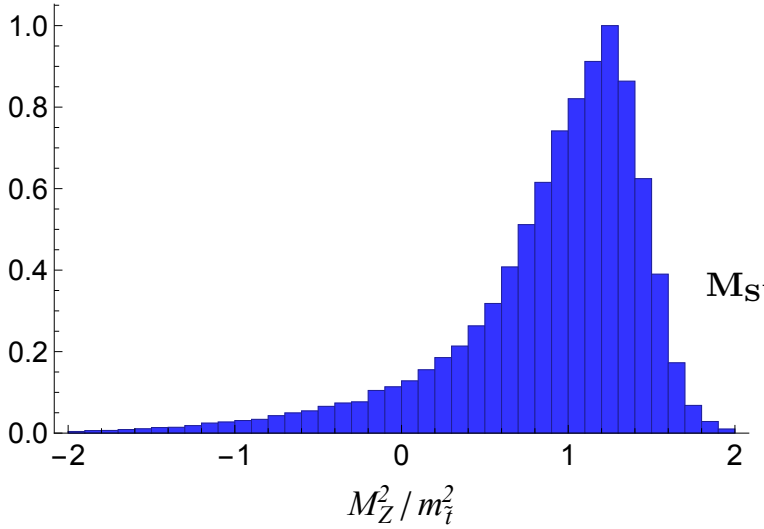
The smallest outcome that does not depend on specifying parameters with more than 1 digit is indicated by the largest gap found in the distribution. Outcomes smaller than the largest gap are accidental.

Minimally specified CMSSM

Maximal hierarchy from minimally specified inputs:

RD and N. McGinnis, arXiv:1705.01910





the largest gaps ~0.001 (fairly uniform away from edges of the distribution)

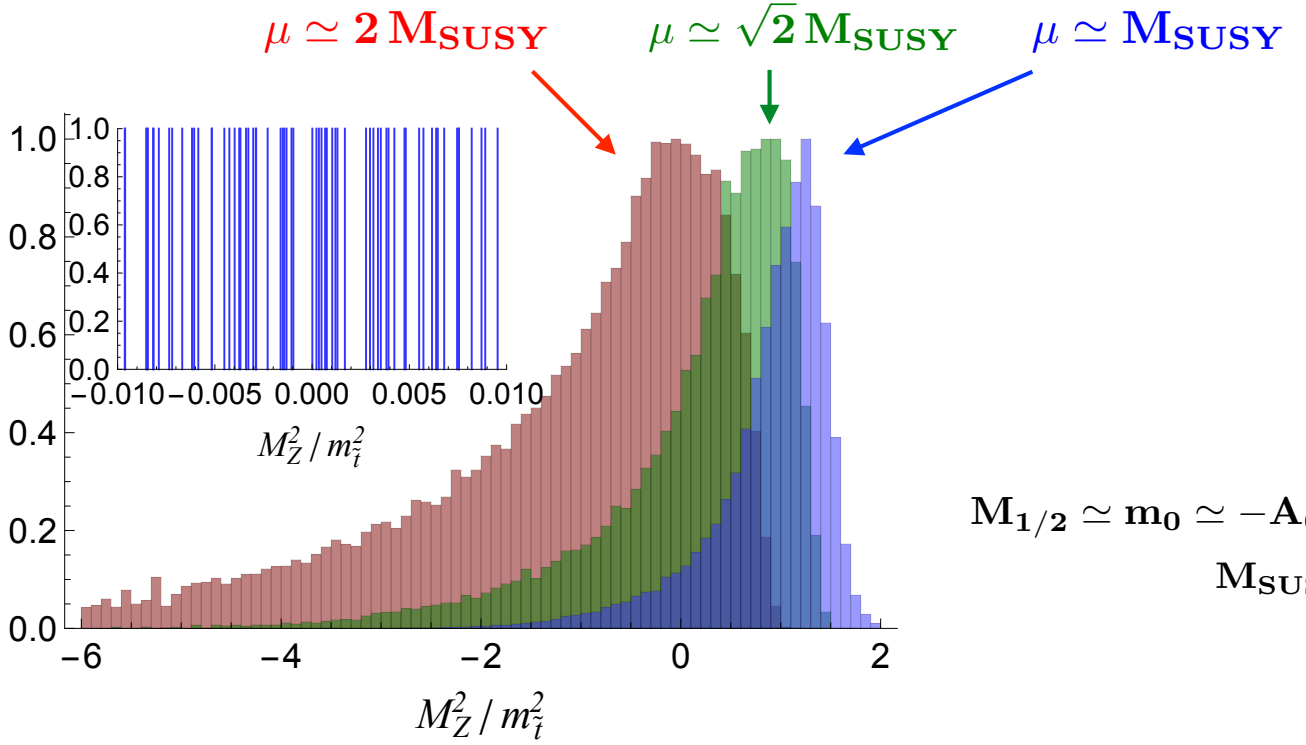
The smallest outcome that does not depend on specifying parameters with more than 1 digit is indicated by the largest gap found in the distribution. Outcomes smaller than the largest gap are accidental.

$$\mathbf{m_{\tilde{t}} \simeq 30\, M_Z}$$

is an ordinary outcome from minimally specified parameters

Minimally specified CMSSM

For different central values of parameters, the peak of the distribution changes



$\mathbf{M_{1/2} \simeq m_0 \simeq -A_0 \simeq M_{SUSY}}$

$\mathbf{M_{SUSY} = 3\ TeV}$

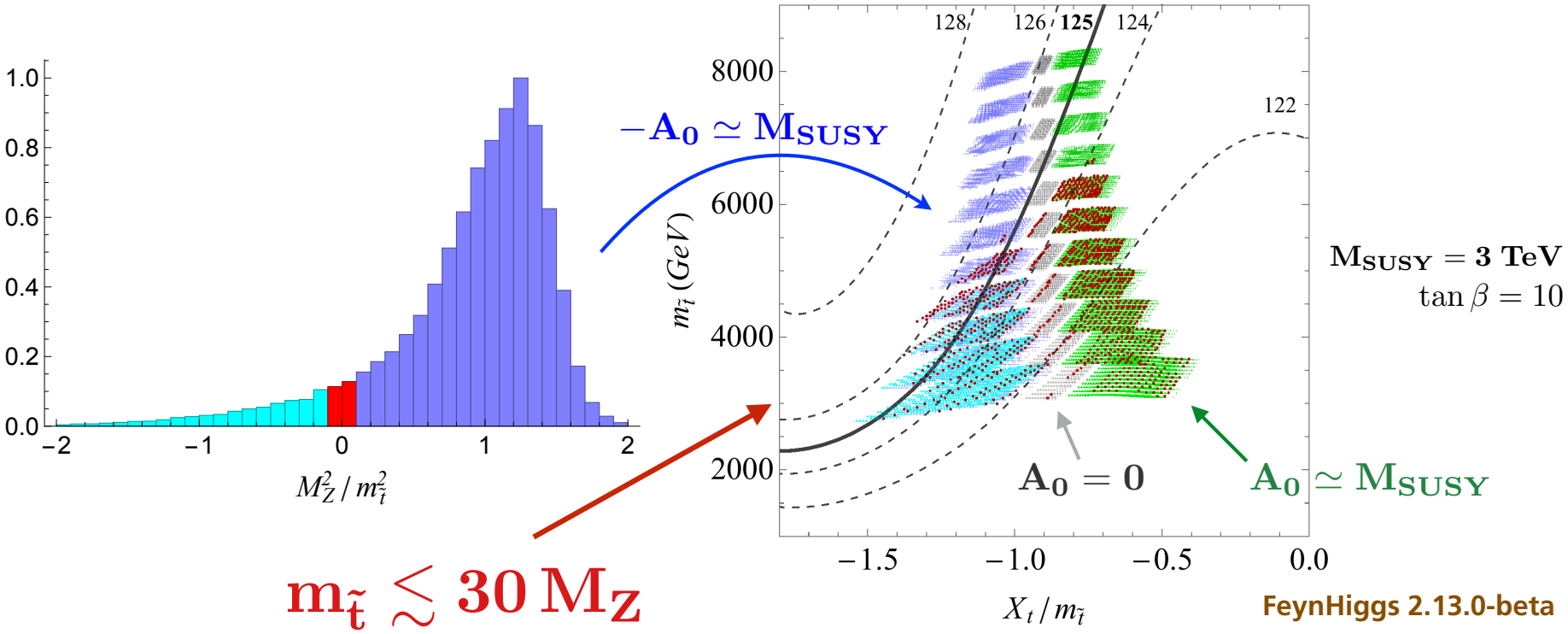
$\tan\beta = 10$

but the maximal gap size remains almost the same.

Prediction for maximal hierarchy is very robust.

Minimally specified CMSSM

Higgs boson mass from minimally specified inputs:



Only scenarios with negative A-terms can have sufficiently heavy stops to explain the Higgs boson mass

Small probability not a sign of fine tuning

It is often argued that small outcomes have a small probability compared to large outcomes.

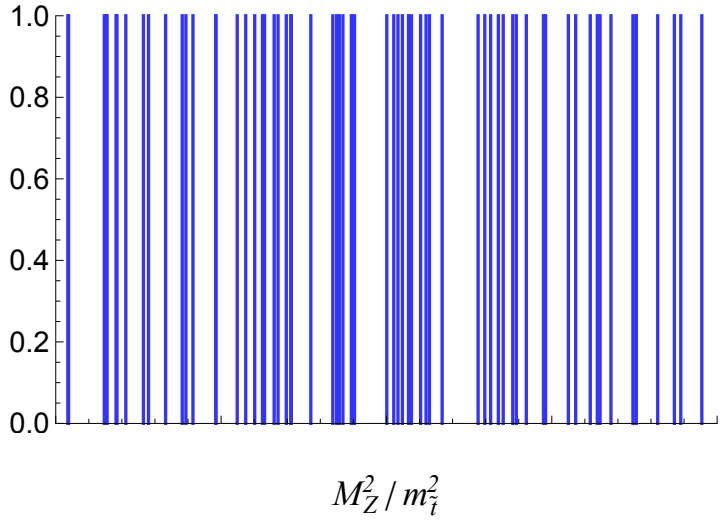
true, but it contains no useful information and it has nothing to do with fine tuning (e.g. lottery)

If there are several parameters contributing then just few choices for each parameter lead to a small probability of any specific outcome (small or large).

Small probability of any outcome is normal in complex models.

Large sensitivity not a sign of fine tuning

It is often argued that small outcomes are highly sensitive to variations of model parameters.



$$\max_p \left| \frac{\partial \ln M_Z^2}{\partial \ln p} \right|$$

Variations of model parameters shift the whole distribution little to the left or right; some outcome is guaranteed within the largest gap size from any point.

The 0 is not special, the same result applies to any other point.

Conclusions

The usual naturalness criteria do not necessarily indicate how model parameters have to be tuned.

I advocated considering any outcome that does not require specifying model parameters with more than 10% precision (one digit) as natural.

In the CMSSM, up to ~3 TeV superpartners are natural in this sense. Only one digit of model parameters needs to be adjusted to get ~100 GeV electroweak scale, no matter what the remaining digits are. Parameter choices with negative A terms can give the correct Higgs mass.