Lattice QCD and Neutrino Physics

Aaron S. Meyer (ameyer@quark.phy.bnl.gov)

Brookhaven National Laboratory

October 12, 2017

In Search of New Paradigms - BNL Forum

Key People

Faculty:

- Richard Hill (U.Kentucky)
- Andreas Kronfeld (FNAL)
- James Simone (FNAL)
- Alexei Strelchenko (FNAL)

Postdocs:

- Ciaran Hughes (FNAL)
- Aaron Meyer (BNL)

Grad Students:

- Yin Lin (U.Chicago)

On behalf of the Fermilab Lattice & MILC Collaborations

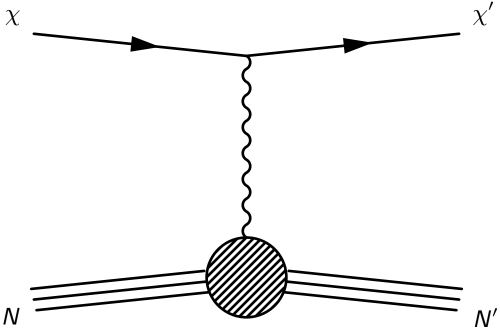
Outline

- Motivation
- Axial Form Factor in Neutrino Physics
- Lattice QCD - Spectrum + Axial Charge

Motivation

Lattice is well suited to compute matrix elements:

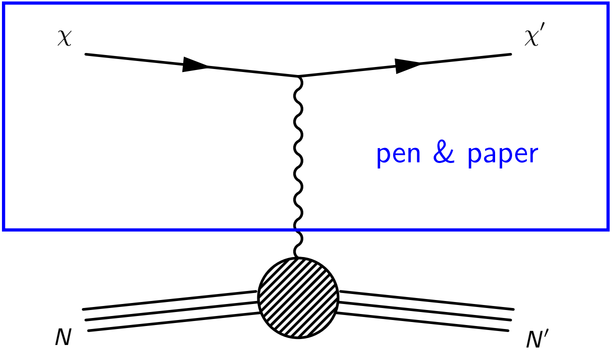
$$\mathcal{M}(q^2) \sim \langle \chi' | \Gamma_\chi | \chi \rangle \langle N' | \Gamma_N(q^2) | N \rangle$$



Motivation

Lattice is well suited to compute matrix elements:

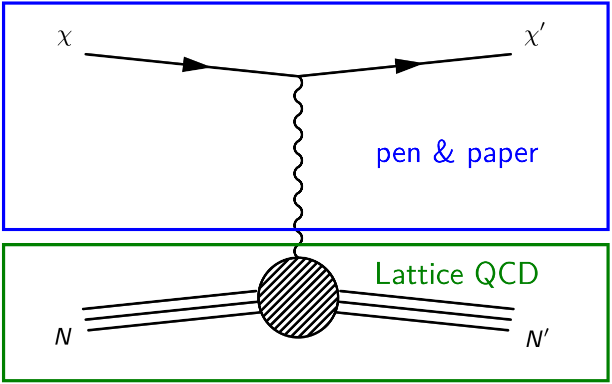
$$\mathcal{M}(q^2) \sim \langle \chi' | \Gamma_\chi | \chi \rangle \langle N' | \Gamma_N(q^2) | N \rangle$$



Motivation

Lattice is well suited to compute matrix elements:

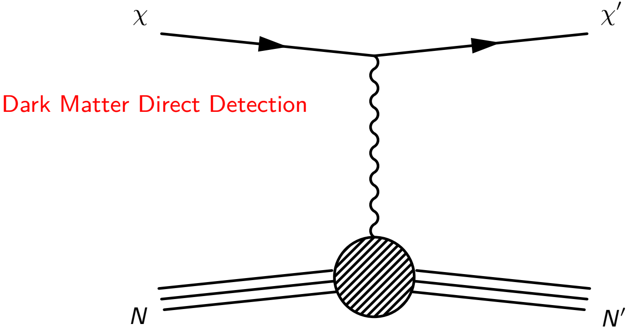
$$\mathcal{M}(q^2) \sim \langle \chi' | \Gamma_\chi | \chi \rangle \langle N' | \Gamma_N(q^2) | N \rangle$$



Motivation

Lattice is well suited to compute matrix elements:

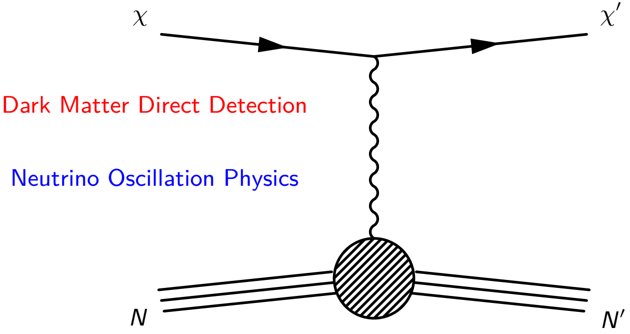
$$\mathcal{M}(q^2) \sim \langle \chi' | \Gamma_\chi | \chi \rangle \langle N' | \Gamma_N(q^2) | N \rangle$$



Motivation

Lattice is well suited to compute matrix elements:

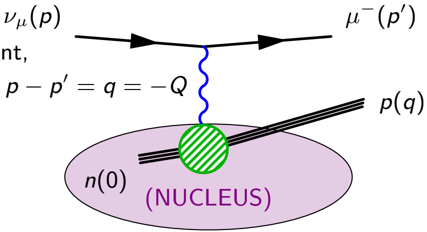
$$\mathcal{M}(q^2) \sim \langle \chi' | \Gamma_\chi | \chi \rangle \langle N' | \Gamma_N(q^2) | N \rangle$$



Quasielastic scattering

QE scattering is relatively easy measurement,
relatively theoretically clean:
ν interacts with nearly-free nucleon

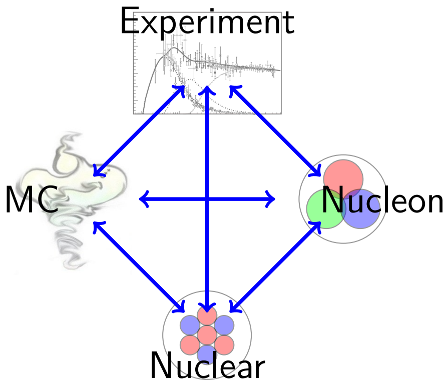
QE is primary signal measurement process
for neutrino oscillation experiments



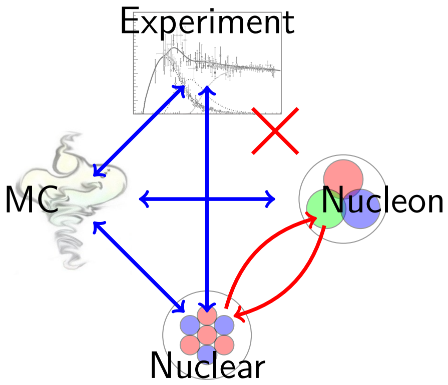
Current Monte Carlo nuclear models assume gas of weakly bound nucleons
$\Longrightarrow$ free nucleon amplitudes useful for determining nuclear matrix elements

QE matrix element involves many-body interactions
$\Longrightarrow$ parametrized by form factors

Axial form factor F_A is important, not well-known

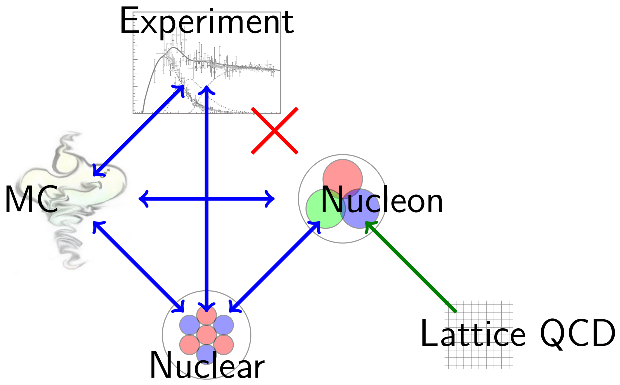


Ideally, lots of redundancy and checks between elements of analysis



Ideally, lots of redundancy and checks between elements of analysis

F_A not well determined by experiment,
$\implies$ nucleon amplitudes constrained by/used to constrain nuclear models



Ideally, lots of redundancy and checks between elements of analysis

F_A not well determined by experiment,
$\implies$ nucleon amplitudes constrained by/used to constrain nuclear models

Lattice QCD acts as a disruptive technology to break degeneracy

Focus

Want to inform nuclear models with theoretically clean/robust form factors

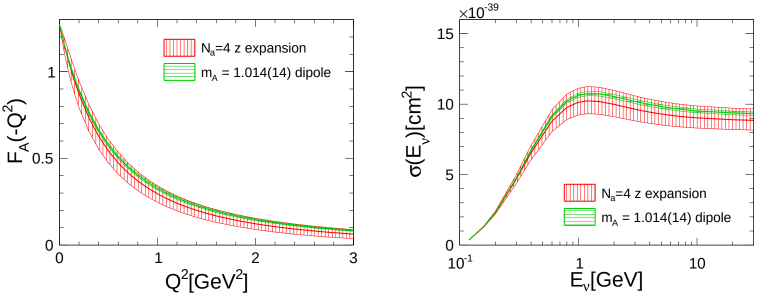
I take two approaches to constrain nucleon form factor:

- Reanalysis of deuterium bubble chamber data
  - use z expansion parametrization, motivated by analyticity, to study systematic uncertainties
- Lattice QCD calculation
  - compute the axial matrix element from first principles

First step is to compute axial charge: $g_A = F_A(Q^2)\big|_{Q^2=0}$

Future extensions of this work will compute Q^2 dependence, fit to z expansion parametrization

Deuterium Fitting - 1603.03048 [hep-ph]



$$\frac{1}{F_A(0)}\frac{dF_A}{dQ^2}\bigg|_{Q^2=0} \equiv -\frac{1}{6}r_A^2$$

$$r_A^2 = 0.46(22)\ \mathrm{fm}^2\,, \quad \sigma_{\nu n\to\mu p}(E_\nu = 1\ \mathrm{GeV}) = 10.1(0.9)\times 10^{-39}\mathrm{cm}^2$$

compared with Bodek *et al.* [Eur. Phys. J. C 53, 349]:

$$r_A^2 = 0.453(13)\ \mathrm{fm}^2\,, \quad \sigma_{\nu n\to\mu p}(E_\nu = 1\ \mathrm{GeV}) = 10.63(0.14)\times 10^{-39}\mathrm{cm}^2$$

Dipole model significantly underestimates error from nucleon form factor

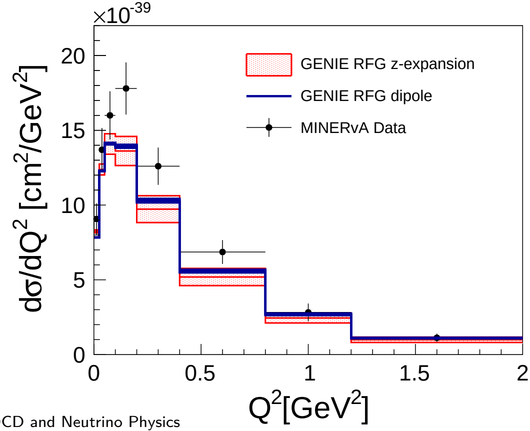
Most theoretically clean data do not constrain form factor precisely

z Expansion in GENIE

z expansion coded into Monte Carlo GENIE - turned on with user switch

Officially released in production version 2.12, will become default soon

Uncertainties on free-nucleon cross section as large as data-theory discrepancy
$\Longrightarrow$ need to improve F_A determination to make headway on nuclear effects



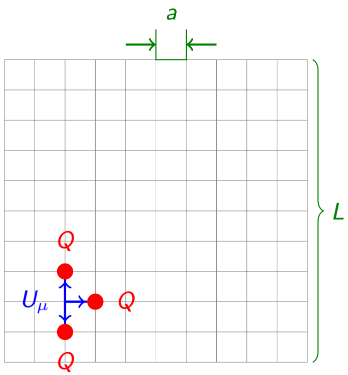
Lattice QCD

Lattice QCD: Formalism

- Numerical solution to path integral

$$\langle \mathcal{O} \rangle = \frac{1}{Z} \int \mathcal{D}\psi \, \mathcal{D}\overline{\psi} \, \mathcal{D}U \, \exp(-S) \, \mathcal{O}_\psi \, [U]$$

- Discretize spacetime
  $\Longrightarrow$ #DOF finite
- Quark fields defined on sites
  $\Longrightarrow$ $Q(x)$
- Gauge fields defined between sites
  $\Longrightarrow$ $U_\mu(x)$
- Euclidean time
  $\Longrightarrow$ correlators $\propto e^{-Et}$
- Lattice spacing a provides UV cutoff
- Lattice size L provides IR cutoff



Fermilab Lattice/MILC Effort

Fermilab Lattice & MILC computing the axial charge $g_A = F_A(Q^2)\big|_{Q^2=0}$ using staggered quarks on the MILC HISQ 2+1+1 gauge ensembles

- no explicit chiral symmetry breaking in $m \to 0$ limit
- physical pion mass for multiple lattice spacings
- large volumes
- absolutely normalized
- high-statistics (computationally fast)

Effort is needed to handle:

- Complicated group theory
- Lots of baryon "tastes" in correlation functions

Group Theory

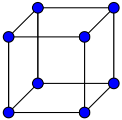
Extra baryon "tastes" show up in correlation functions,
but use them to diagonalize action in spin-taste space

Full lattice group is: $(((\mathcal{T}_M \times \mathbb{Q}_8) \rtimes W_3) \times D_4)/\mathbb{Z}_2$

Lowest lying states within irreducible representations of lattice group:

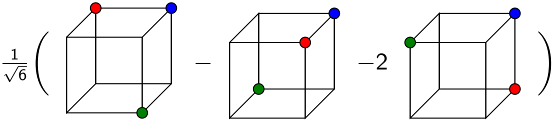
Irrep	$I = \frac{3}{2}$	$I = \frac{1}{2}$
8	$3N + 2\Delta$	$5N + 1\Delta$
$8'$	$0N + 2\Delta$	$0N + 1\Delta$
16	$1N + 3\Delta$	$3N + 4\Delta$

Focus on this irrep



$N_{\text{ops}} = N_{\text{states}}$ for lowest lying states in all irreps

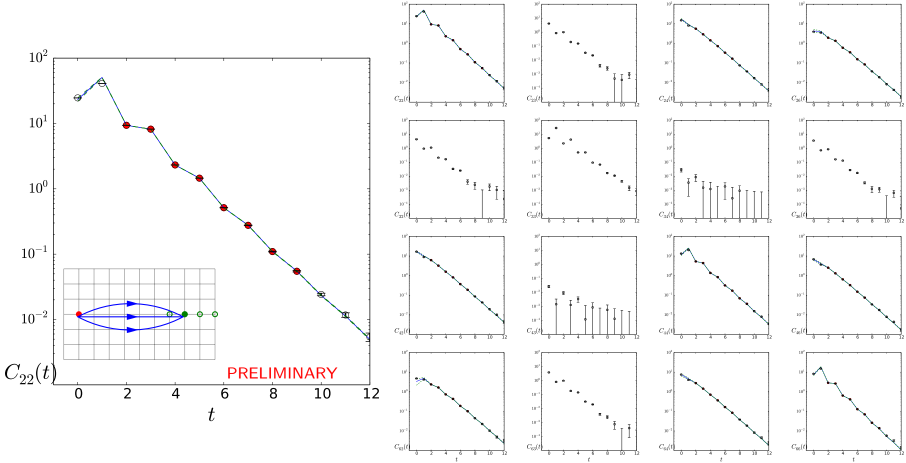
Operators constructed from displacing quarks within unit cube:



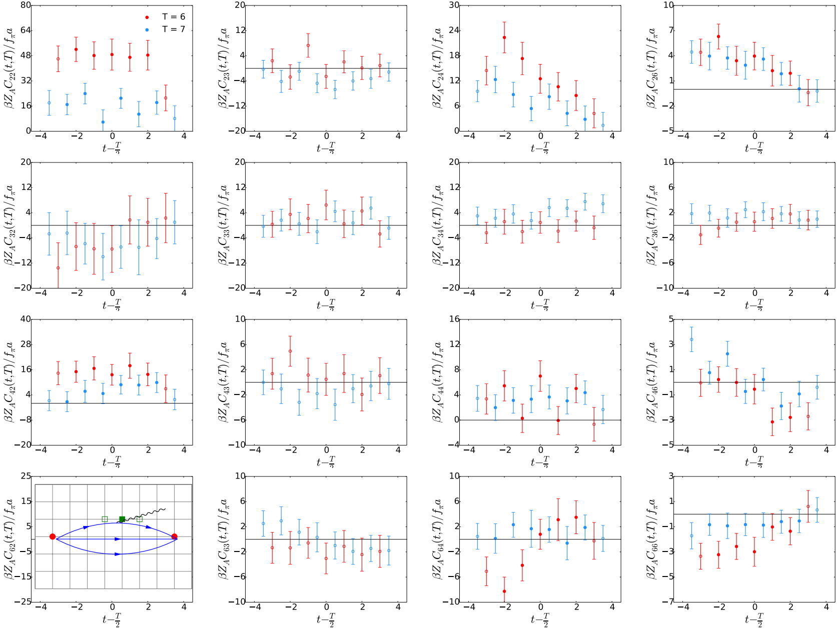
Spectrum - Matrix of Correlation Functions

Prerequisite for studying Interactions

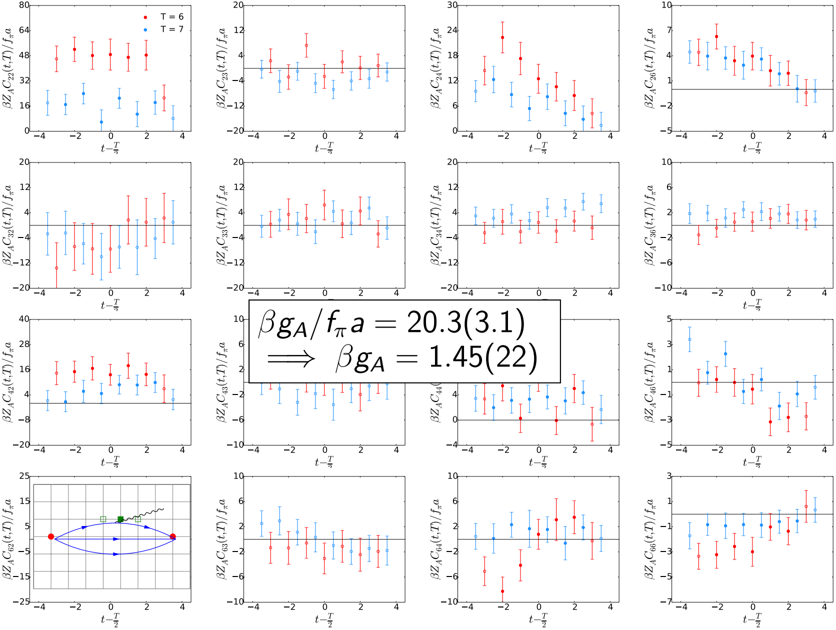
$$C_{ij}(t) = \sum_m a^E_{im} b^{E*}_{jm} e^{-E^E_m t} + \sum_m (-1)^t a^O_{im} b^{O*}_{jm} e^{-E^O_m t}$$



Interactions - Matrix of Correlation Functions

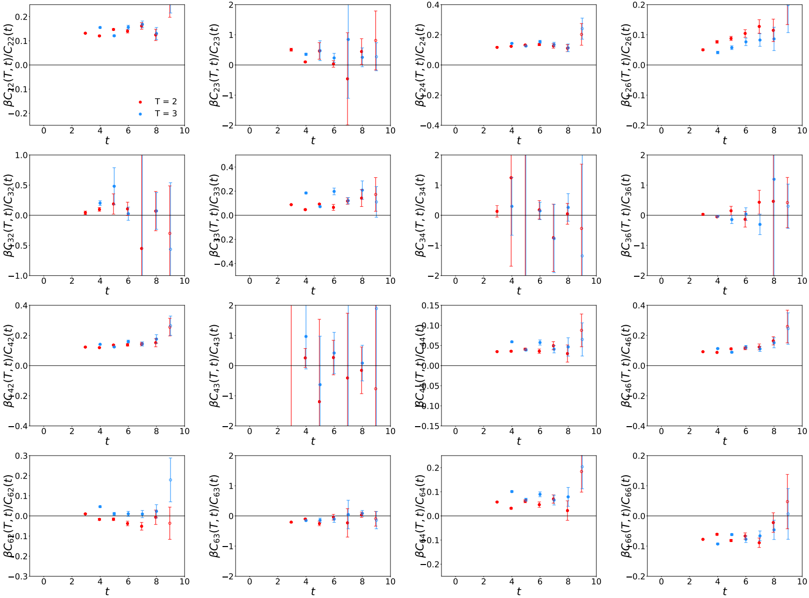


Interactions - Matrix of Correlation Functions



$$\beta g_A / f_\pi a = 20.3(3.1)$$
$$\implies \beta g_A = 1.45(22)$$

Interactions - New Strategy



Factor of 10 fewer measurements, factor of 10 better statistics

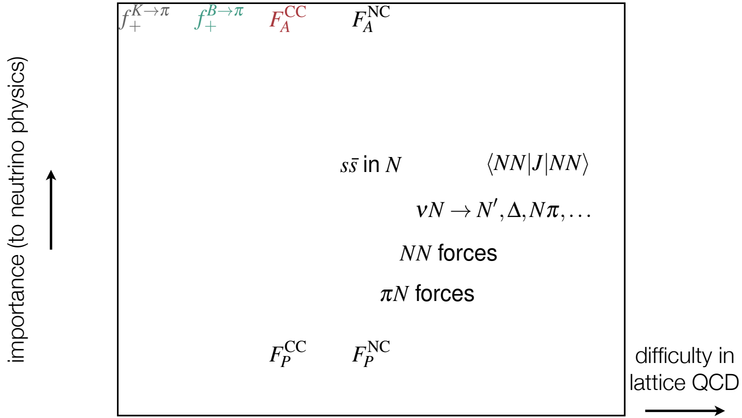
Conclusions

- Axial form factor is essential for the success of future neutrino oscillation experiments
- Need robust determination of nucleon amplitudes with realistic errors to support neutrino oscillation program

- z Expansion is a form factor parametrization consistent with QCD, motivated by analyticity
- Spectrum and axial charge have been calculated with Lattice QCD (world first for staggered baryons!)
- Signal to noise now under control, improvements to statistics and more ensembles in progress
- Results from Lattice QCD can be made available to experimental community as soon as they are ready

Backup

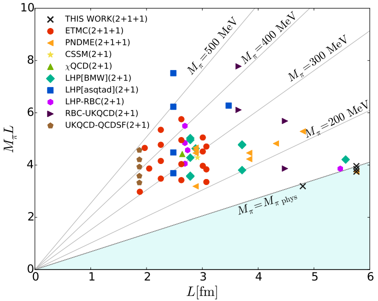
Further Calculations of Interest

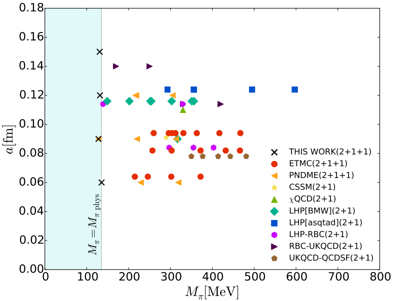


30

[A. Kronfeld]

Choices and Challenges

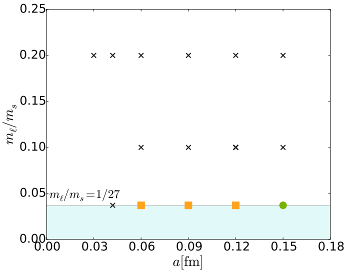




Current data:

- $m_{\text{valence}} = m_{\text{physical}}$
- $a = 0.15$ fm
- $32^3 \times 48$
- ~ 3000 Two-point measurements
- ~ 3000 Three-point measurements

(Measurements $\sim$ configurations)



Dipole Form Factor

Most analyses assume the Dipole axial form factor
(Llewellyn-Smith, 1972):

$$F_A^{\text{dipole}}(Q^2) = \frac{g_A}{\left(1 + \frac{Q^2}{m_A^2}\right)^2}$$

[Phys.Rept.3 (1972),261]

Dipole is an ansatz:
unmotivated in interesting energy region
$\implies$ uncontrolled systematics and therefore underestimated uncertainties

Large variation in m_A over many experiments
(dubbed the "axial mass problem"):

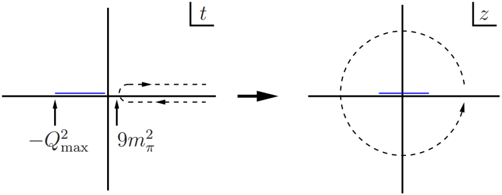
- $m_A = 1.026 \pm 0.021$ (Bernard *et al.*, [arXiv:00107088])
- $m_A^{\text{eff}} = 1.35 \pm 0.17$ (MiniBooNE, [arXiv:1002.2680])

Essential to use model-independent parameterization of F_A instead

z Expansion

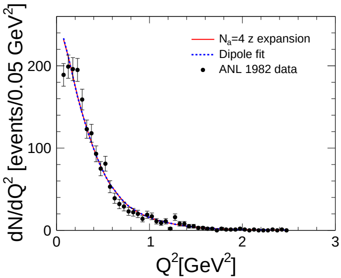
The z Expansion (1108.0423 [hep-ph]) is a conformal mapping which takes kinematically allowed region ($t = -Q^2 \leq 0$) to within $|z| < 1$

$$z(t; t_0, t_c) = \frac{\sqrt{t_c - t} - \sqrt{t_c - t_0}}{\sqrt{t_c - t} + \sqrt{t_c - t_0}} \qquad F_A(z) = \sum_{n=0}^{\infty} a_n z^n \qquad t_c = 9m_\pi^2$$



- Model independent: motivated by analyticity arguments from QCD
- Only few parameters needed: unitarity bounds
- Sum rules regulate large-Q^2 behavior

Deuterium Fitting - Differential Cross Section

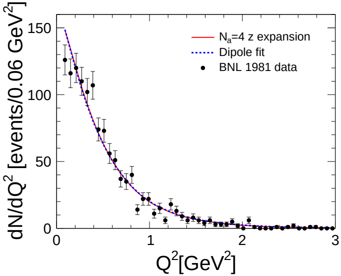


Dipole:

χ^2/N_{bins}	58.6/49
m_A	1.02(5)

z Expansion:

χ^2/N_{bins}	60.9/49
a_1	2.25(10)
a_2	0.2(0.9)
a_3	$-4.9(2.4)$
a_4	2.7(2.7)



Dipole:

χ^2/N_{bins}	70.9/49
m_A	1.05(4)

z Expansion:

χ^2/N_{bins}	73.4/49
a_1	2.24(10)
a_2	0.6(1.0)
a_3	$-5.4(2.4)$
a_4	2.2(2.7)

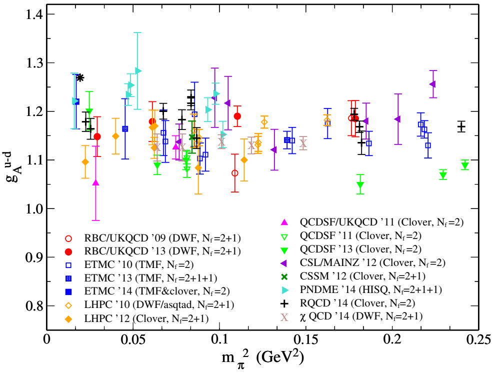
Analysis in arXiv:1603.03048[hep-ph] ASM, M. Betancourt, R. Gran, R. Hill

Axial Charge

Well-known from neutron β decay experiment: $n \to p + e^- + \bar{\nu}_e$

PDG value: $g_A = 1.2723(23)$

Lattice QCD has difficulty reproducing g_A:



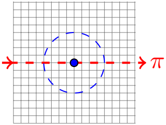
M. Constantinou, LAT2014 (arXiv:1411.0078)

Disclaimer: multi-dimensional space projected into two dimensions

g_A Problem

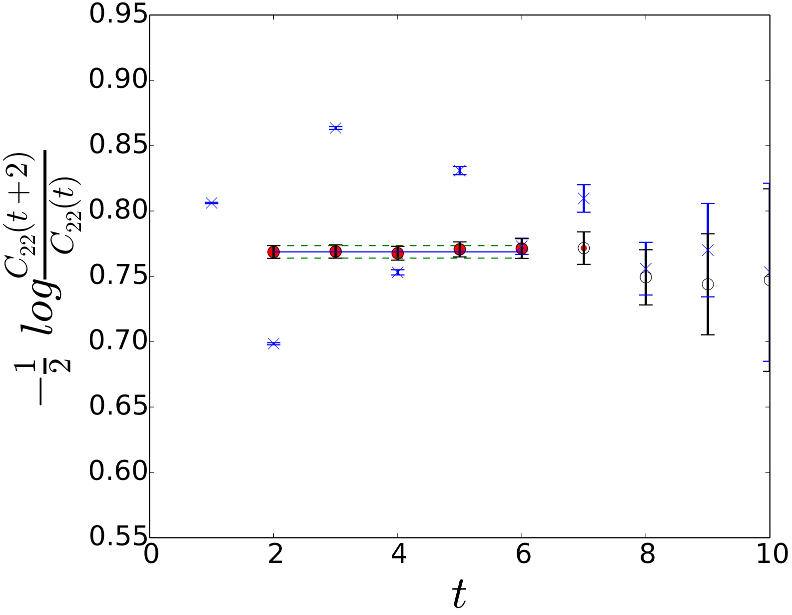
Why is g_A so difficult?

- Signal-to-Noise Grows Exponentially
  - Signal $\propto \langle \equiv \rangle \sim e^{-M_n t}$, noise$^2 \propto \langle | \equiv |^2 \rangle = \langle \equiv \rangle \sim e^{-3 m_\pi t}$
  - Noise gets contribution from 3-pion term
- Finite size effects
  - self-interaction via πs which wrap around periodic BC

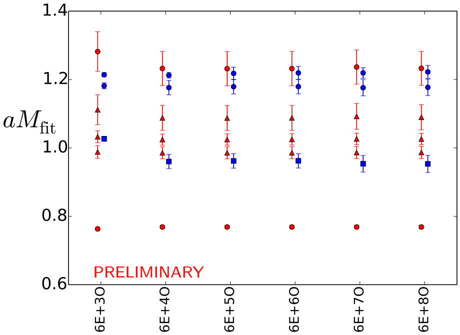


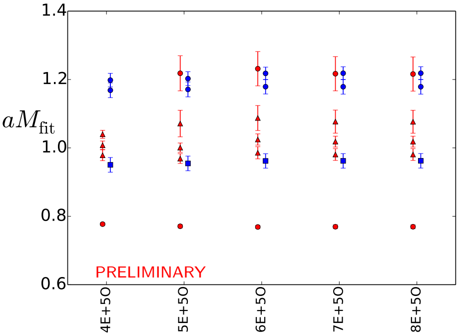
- Excited state contamination
  - Operators couple to ground state + excited states
  - Requires fitting $\sum_n e^{-E_n t}$ for many n
  - Rotation/tranlation symmetry broken by lattice

Spectrum - Effective Mass



Spectrum - Stability





Spectrum - Systematics

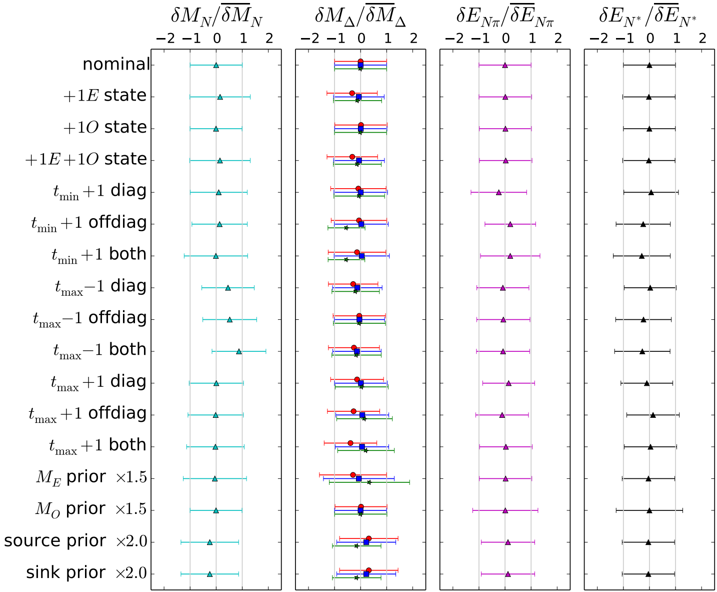
$\frac{\mathcal{O}_i - \overline{\mathcal{O}}}{\delta \overline{\mathcal{O}}} \pm \frac{\delta \mathcal{O}_i}{\delta \overline{\mathcal{O}}}$, $\overline{\mathcal{O}}$ =nominal

$\delta M_N / \overline{\delta M}_N$ −2 −1 0 1 2 | $\delta M_\Delta / \delta M_\Delta$ −2 −1 0 1 2 | $\delta E_{N\pi} / \delta E_{N\pi}$ −2 −1 0 1 2 | $\delta E_{N^*} / \delta E_{N^*}$ −2 −1 0 1 2

i:

i	
nominal	default fit
$+1E$ state	fit with 7 even + 5 odd states
$+1O$ state	fit with 6 even + 6 odd states
$+1E+1O$ state	fit with 7 even + 6 odd states
$t_{\min}+1$ diag	$t \in [3,8]$ if $i = j$; $t \in [2,7]$ if $i \neq j$
$t_{\min}+1$ offdiag	$t \in [2,8]$ if $i = j$; $t \in [3,7]$ if $i \neq j$
$t_{\min}+1$ both	$t \in [3,8]$ if $i = j$; $t \in [3,7]$ if $i \neq j$
$t_{\max}-1$ diag	$t \in [2,7]$ if $i = j$; $t \in [2,7]$ if $i \neq j$
$t_{\max}-1$ offdiag	$t \in [2,8]$ if $i = j$; $t \in [2,6]$ if $i \neq j$
$t_{\max}-1$ both	$t \in [2,7]$ if $i = j$; $t \in [2,6]$ if $i \neq j$
$t_{\max}+1$ diag	$t \in [2,9]$ if $i = j$; $t \in [2,7]$ if $i \neq j$
$t_{\max}+1$ offdiag	$t \in [2,8]$ if $i = j$; $t \in [2,8]$ if $i \neq j$
$t_{\max}+1$ both	$t \in [2,9]$ if $i = j$; $t \in [2,8]$ if $i \neq j$
M_E prior $\times 1.5$	priors on all even states widened by factor of 1.5
M_O prior $\times 1.5$	priors on all odd states widened by factor of 1.5
source prior $\times 2.0$	priors on all source overlap factors widened by factor of 2.0
sink prior $\times 2.0$	priors on all sink overlap factors widened by factor of 2.0

Spectrum - Systematics



Spectrum - Results

Taking nominal fits, statistics error only:

$8'$ Irrep

State		Prior [δM] [GeV]	Posterior [δM] [GeV]
Δ	[0]	1.22(13) [–]	1.289(18) [–]
Δ	[1]	1.29(13) [0.065(33)]	1.349(16) [0.060(19)]
N^*	[0]	1.51(20) [–]	1.536(12) [–]

16 Irrep

State		Prior [δM] [GeV]	Posterior [δM] [GeV]
N	[0]	0.994(65) [–]	1.003(6) [–]
Δ	[0]	1.289(74) [0.295(34)]	1.286(22) [0.283(23)]
Δ	[1]	1.349(80) [0.060(31)]	1.337(21) [0.051(21)]
Δ	[2]	1.409(89) [0.060(39)]	1.419(48) [0.082(48)]
$N\pi$	[0]	1.252(65) [–]	1.255(27) [–]
N^*	[0]	1.537(76) [0.284(39)]	1.538(13) [0.283(28)]

Correlation Function Normalization Strategy

Rather than computing g_A directly, instead compute ratio

$$\left.\frac{\langle N|\, Z_A A_\mu \,|N\rangle}{\langle 0|\, Z_A A_\mu \,|\pi^a\rangle}\right|_{Q^2=0} \propto \frac{g_A}{f_\pi}$$

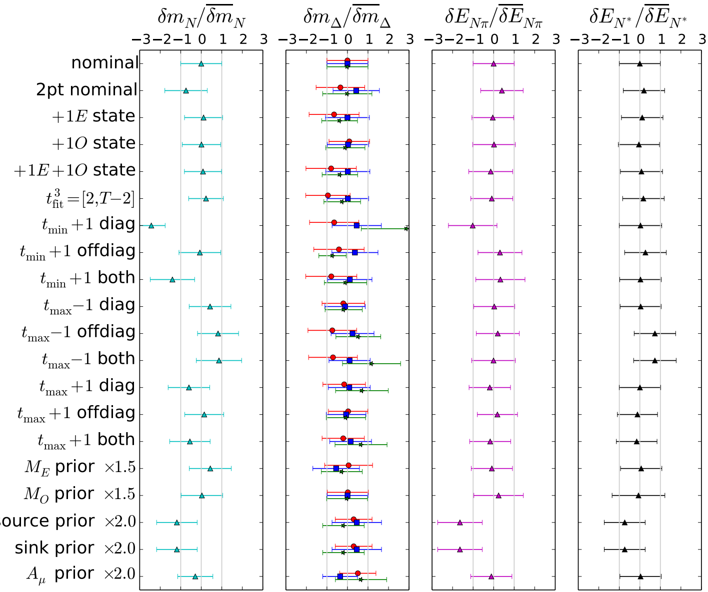
Benefits from statistical cancellation, absolutely normalized

Full computation to remove f_π dependence is

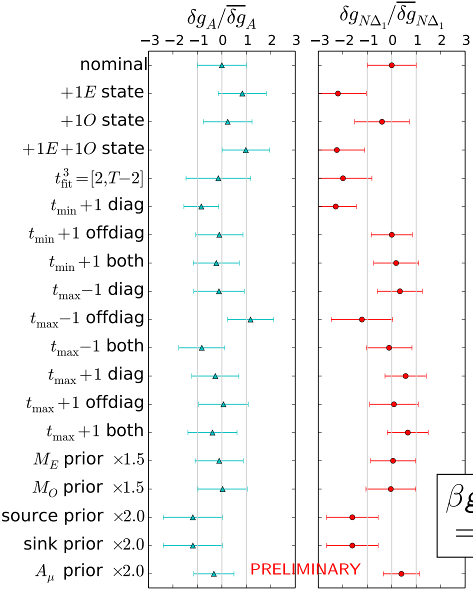
$$\beta g_A \bar{u}_N \gamma^\mu \gamma^5 u_N = \frac{\langle N|\beta Z_A A^\mu|N\rangle}{\langle 0|Z_A A^\mu|\pi\rangle} \frac{\langle 0|2\hat{m}P|\pi\rangle}{M_\pi}$$

Blinding factor β included in three point functions,
known only to few collaboration members

Interactions - Mass Systematics

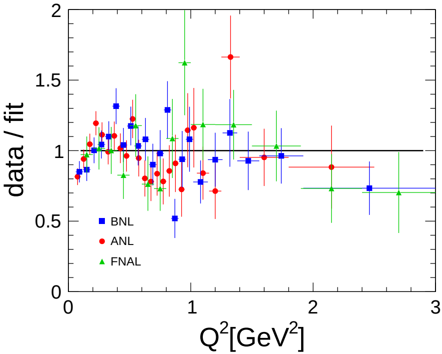


Interactions - Amplitude Systematics



$$\beta g_A/f_\pi a = 20.3(3.1)$$
$$\implies \beta g_A = 1.45(22)$$

Residuals



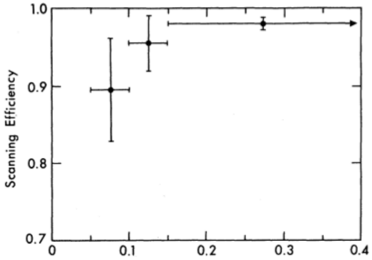
Neither z expansion, nor dipole can properly explain shape of data

Difficult to extract form factor from scattering data,
uncontrolled systematics introduced in process

Acceptance Corrections

Acceptance correction for fixing errors from hand scanning
Q^2 dependent correction, correlated between bins:

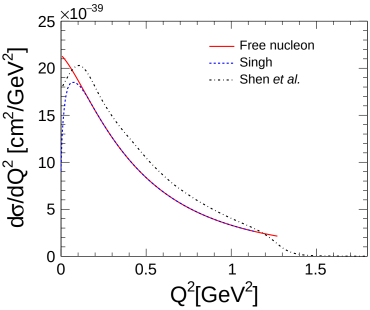
$$\frac{dN}{e(Q^2)} \to \frac{dN}{e(Q^2) + \eta\, de(Q^2)}\,, \quad \eta = 0 \pm 1$$



For ANL, BNL, FNAL respectively, $\eta = -1.9, -1.0, +0.01$; minimal improvement of goodness of fit

Deuterium Corrections

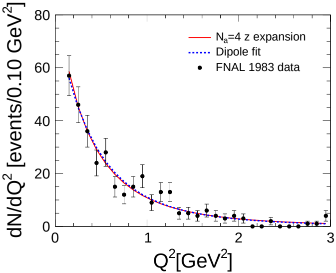
Corrections assumed to be E_ν independent
Two corrections tested:
Singh Nucl. Phys. B 36, 419,
Shen 1205.4337 [nucl-th]



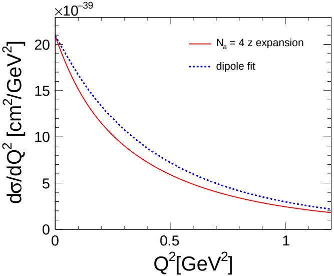
Central values of Shen, Singh are consistent with each other
Final fit done with Singh, inflated error bars

Normalization Degeneracy

Despite apparent similarity of dipole/z expansion cross sections, form factors quite different



$\rightarrow$



Consequence of self-consistency: cross section prediction

$$\frac{dN}{dE} \propto \frac{1}{\sigma}\frac{d\sigma}{dQ^2}$$

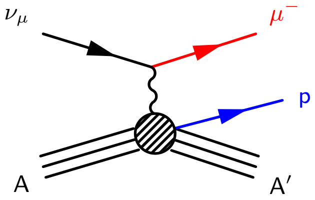
Cut of low-Q^2 data & floating normalization hide cross section differences

Nuclear Effects

Nuclear effects not well understood

$\rightarrow$ Models which are best for one measurement are worst for another

Need to break F_A/nuclear model entanglement



(assumed $m_A = 0.99$ GeV)

NuWro Model (χ^2/DOF)	RFG [GENIE]	RFG+ TEM	assorted others
leptonic(rate)	3.5	2.4	2.8-3.7
leptonic(shape)	4.1	1.7	2.1-3.8
hadronic(rate)	1.7[1.2]	3.9	1.9-3.7
hadronic(shape)	3.3[1.8]	5.8	3.6-4.8

(Minerva collaboration, 1305.2243,1409.4497[hep-ph])