Towards a non-perturbative calculation of Weak Hamiltonian Wilson Coefficients

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Introduction

Weak decays of hadrons

rich phenomenology
(e.g. CP violation in $K \to \pi\pi$)

QCD $\to$ confinement, light objects

Weak interactions $\to$ short range, heavy mediators

There is a natural scale separation in these decays

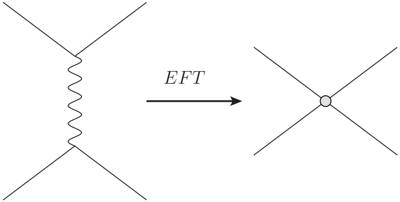
$\downarrow$

build an **effective low-energy theory**

Integrate out heavy degrees of freedom: heavy quarks, weak bosons

Effective theory

Integrating out weak bosons generates **four-quark vertices**



Current-current diagrams:
$c \to s u \bar{d}$

new divergences in the EFT
$\downarrow$
operator mixing

$$\mathcal{H}_{\text{eff}} \propto G_{\text{F}} \sum_i C_i Q_i \quad \text{with} \quad i = 1, 2 \text{ in our example}$$

New Physics (heavy d.o.f.) captured by Wilson Coefficients

Long distance matrix elements $\langle Q_i \rangle \to$ Lattice

Wilson Coefficients $C_i \to$ PT

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OPE

We use W boson propagator in unitary gauge (Euclidean)

$$W_{\mu\nu}(q) = \frac{1}{q^2 + m_{\mathrm{W}}^2}\left(\delta_{\mu\nu} - \frac{q_\mu q_\nu}{m_{\mathrm{W}}^2}\right) \overset{m_{\mathrm{W}}\to\infty}{\approx} \frac{1}{m_{\mathrm{W}}^2}\left[\delta_{\mu\nu} + O\left(\frac{q^2}{m_{\mathrm{W}}^2}\right)\right]$$

Four-quark operators Q_i are first terms in the expansion

$$\mathcal{H}_{\mathrm{eff}} \propto G_{\mathrm{F}}\left[\sum_i C_i Q_i + \sum_i \frac{c_i^{(d)}}{m_{\mathrm{W}}^{d-6}} O_i^{(d)}\right] \quad , \quad d \geq 8$$

$O_i^{(d)}$ can be gauge-invariant operators

if we fix the (QCD) gauge $O_i^{(d)}$ can be gauge-noninvariant operators

$O_i^{(d)}$ depend on momenta p_i of external states

In the limit $p_i/m_{\mathrm{W}} \to 0$, $\forall i$, only Q_1 and Q_2 survive

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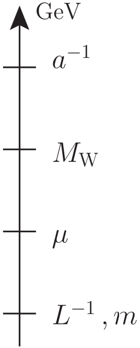
Window problem

μ is the matching scale:



$am_W \ll 1$ for discretization effects

$\mu \ll m_W$ for higher order operators

$\mu \gg m\,, \mu L \gg 1$ for infrared effects

Present study is focused on unphysically small $m_W \approx 2$ GeV

Non-perturbative effects $O(\Lambda_{QCD}/m_W)$

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Systematic uncertainties

Wilson Coefficients are ultraviolet quantities

- related to $p \gtrsim m_{\mathrm{W}} \to$ potentially large discretization errors ✓
- independent from infrared regulators, up to
  - finite volume effects ✓
  - finite quark mass effects ✓
  - non-perturbative effects ✓

We study all these effects

- current available lattices $m_{\mathrm{W}} \approx 2$ GeV
- neglect disconnected ($\to$penguin) diagrams (for larger operator basis)

[Dawson, Martinelli, Rossi, Sachrajda, Sharpe, Talevi, Testa '98]

Seminal ideas for a non-perturbatively defined weak hamiltonian

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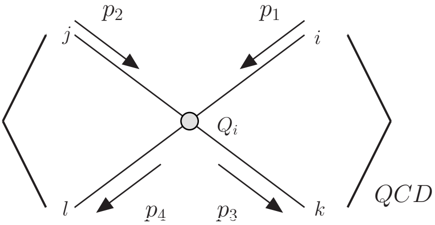
Lattice Observables - EFT

$G(Q_i) =$



Green's function $G(Q_i)$

$Q_1 = (\bar{s}_i c_j)_{\mathrm{V-A}} \otimes (\bar{u}_j d_i)_{\mathrm{V-A}}$

$Q_2 = (\bar{s}_i c_i)_{\mathrm{V-A}} \otimes (\bar{u}_j d_j)_{\mathrm{V-A}}$

RI schemes

$p_1 = p_3 = p$, $p_2 = p_4 = -p$

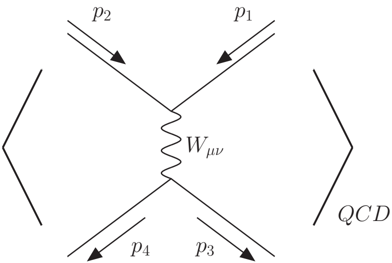
$p_1 \neq p_2 \neq p_3 \neq p_4$, $p_i^2 = p^2$

$\Lambda(Q_i)$: amputated $G(Q_i)$ with quark propagators $S(p_i, m_i)$

Projectors: $P_1 = \delta_{il}\delta_{kj}\ (\Gamma_1 \otimes \Gamma_2)$, $P_2 = \delta_{ij}\delta_{kl}\ (\Gamma_1 \otimes \Gamma_2)$ [RBC/UKQCD '10]

We define $M_{ij} = P_j\left[\,\Lambda(Q_i)\,\right]$

Lattice Observables - Full theory

$G_{\rm SM} =$ 

W boson in unitary gauge

RI schemes:
$p_1 = p_3 = p$, $p_2 = p_4 = -p$
$p_1 \neq p_2 \neq p_3 \neq p_4$, $p_i^2 = p^2$

Weak vertex factor $\propto g_2$

$\Lambda_{\rm SM}$: amputated $G_{\rm SM}$ with quark propagators $S(p_i, m_i)$

3. Define $W_i = P_i(\Lambda_{\rm SM})$
4. Note that $W_i^{\rm RI}(\mu) = Z_{\rm q}^{-2}(\mu)\ Z_V^2\ W_i^{\rm lat}|_{p^2=\mu^2}$
   Z_V: vector bilinear operator renormalization factor

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Matching procedure

Matching equation for RI conditions

$$\frac{G_F}{\sqrt{2}} C_i^{\mathrm{RI}}(\mu) M_{ij}^{\mathrm{RI}}(\mu) = \frac{g_2^2}{8} W_j^{\mathrm{RI}}(\mu)$$

CKM matrix elements simplify

$G_F/\sqrt{2}$ and $g_2^2/8$ simplification $\to 1/m_W^2$

$$C_i^{\mathrm{RI}}(\mu) = m_W^2 \left(W_j^{\mathrm{lat}} [M^{\mathrm{lat}}]_{jk}^{-1} \right) \left([Z^{\mathrm{RI}}(\mu)]_{ki}^{-1} Z_V^2 \right)$$

Bare lattice Wilson Coefficients: $C_k^{\mathrm{lat}} = m_W^2 W_j^{\mathrm{lat}} [M^{\mathrm{lat}}]_{jk}^{-1}$

1. The matching procedure on the lattice
   - study effects of higher order operators $O(p^2/m_W^2)$
   - study infrared/non-perturbative effects in limit $p^2 \to 0$

2. Renormalization of the lattice theory to RI (or $\overline{\mathrm{MS}}$)

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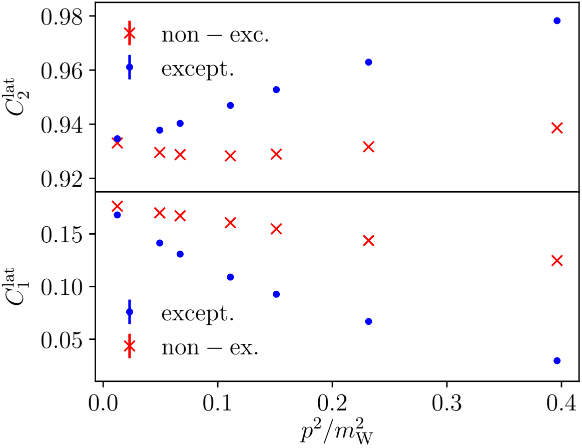
Higher order operators



Excellent statistical precision

Different external states
↓
different p^2 behaviors

Polynomial fits in $\frac{p^2}{m_W^2}$

Projectors: $VA + AV$
$m_W \approx 1.7$ GeV

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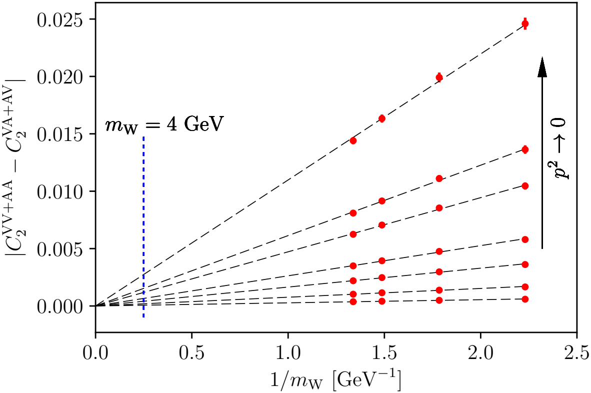
From $m_\mathrm{W} \to \infty$ expansion of propagator only powers of $1/m_\mathrm{W}^2$

observed $1/m_\mathrm{W}$ from non-perturbative effects (e.g. condensates)



Example of such operator

$$\frac{\langle \bar{q} q \rangle}{p^2 m_\mathrm{W}} O^{6\text{-dim}}$$

$p^2 \gg \Lambda_\mathrm{QCD}$ vanishes

Systematic error

$$|C_i^{\mathrm{VV+AA}} - C_i^{\mathrm{VA+AV}}|$$

On-going calculation $m_\mathrm{W} \approx 4$ GeV

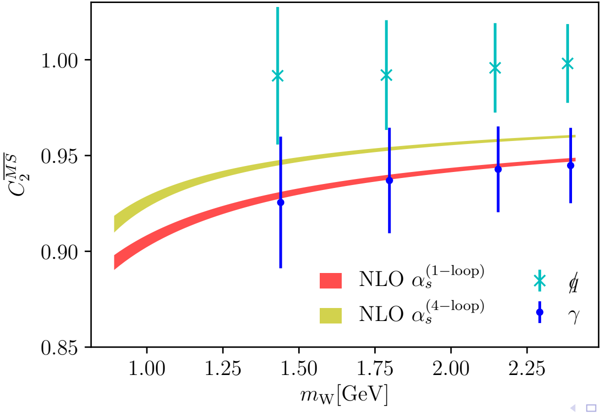
RESULTS - C_2

Large error from matching RI $\rightarrow \overline{\text{MS}}$

Error dominated by systematics (90%) over statistical (10%)

Systematics correlated $\rightarrow$ fit $\rightarrow$ predict 1-loop coefficient

analytic results from [Buchalla, Buras, Lautenbacher '95]



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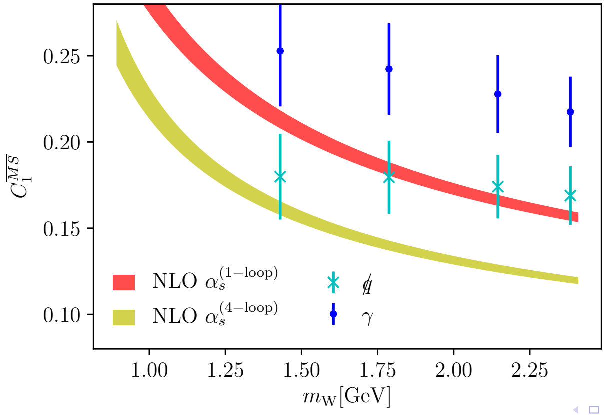
Results - C_1

Large error from matching $\mathrm{RI} \to \overline{\mathrm{MS}}$

Error dominated by systematics (90%) over statistical (10%)

Systematics correlated → fit → predict 1-loop coefficient (1 σ)

analytic results from [Buchalla, Buras, Lautenbacher '95]



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Towards the Standard Model

With our strategy can we reach the Standard Model?

1. W boson mass of 80 GeV

   fit our data in RI scheme → predict higher loop coefficients
   → run α_s at 80 GeV → estimate or bound Wils.Coeff.

2. EFT with 5-flavors in the sea

   need simulations with more dynamical quarks in the sea

3. integrating out top quark

   future studies, difficult problem on lattice

Repeat calculation with $N_\mathrm{f} = 2 + 1 + 1$ quarks at $a^{-1} \approx 4$ GeV

systematic errors below 1% → precise fits → **1.** ✓

$N_\mathrm{f} = 2 + 1$ vs. $N_\mathrm{f} = 2 + 1 + 1$ → flavor dependence → **2.** ✓

→ On-going calculation Fall 2017

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Conclusions

We have developed a method to compute (weak) wilson coefficients to all-orders in α_s

- controlled quark mass and finite volume errors
  - discretization effects removed with 2 lattice spacings
- excellent statistical precision
- account for non-perturbative contributions
- possibility to bound perturbative error

Outlook:

- push towards higher values of $m_{\mathrm{W}} \to$ reduce systematics
- study flavor dependence
- extend the basis of operators

Thanks for the attention!

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Backup slides

RI/(S)MOM scheme

[Martinelli, Pittori, Sachrajda, Testa, Vladikas 95]

Given renormalized amputated Green's function Λ^R
Regularization Independent conditions (RI-MOM)

$$\Lambda^R|_{p^2=\mu^2} = Z_{\rm q}^{-n/2}\ Z\ \Lambda^{\rm bare}|_{p^2=\mu^2} = \Lambda^{\rm tree}$$

The renormalization scheme is defined by the choice of the external states:

- we use off-shell external quark states
  with momentum p_i, $i = 1, 2, 3, 4$
  with masses $m_i = m$, $\forall i$
  with Projectors P_i to project onto definite spin-color states
- we use Landau gauge

Lattice Setup

Ensembles $N_{\mathrm{f}} = 2 + 1$ Shamir Domain-Wall fermions

$a^{-1} \approx 1.78\ \mathrm{GeV} \approx 0.11\ \mathrm{fm}$

$L \approx 1.8\ \mathrm{fm}$ and $2.6\ \mathrm{fm}$

$a^{-1} \approx 2.38\ \mathrm{GeV} \approx 0.08\ \mathrm{fm}$

$L \approx 2.6\ \mathrm{fm}$

Bare operators with external p between 0.2 and $1.0\ \mathrm{GeV}$

RI schemes with external p between 1.4 and $2.4\ \mathrm{GeV}$

Artificially small $m_{\mathrm{W}} \in [1.4\,, 2.4]\ \mathrm{GeV} \to 0.6 < am_{\mathrm{W}} < 1.2$

Momentum sources and Twisted BC

Can we define a method to safely extract Wilson Coefficients?

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