# Status and outlook for precision calculations

relevant for SM measurements and New Physics searches

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### Why care about precision?

#### LHC direct searches



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### LHC direct searches



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### Typical conclusions of recent hep-ph/ hep-ex papers

EXP

We measure XXX and the observable is in agreement with the Standard Model predictions



ΤН

We compute XXX at **N<sup>i</sup>LO** and find a considerable reduction in scale dependence and a better description of the data

### LHC as a precision machine

The LHC conceived as a discovery machine:

e <sup>+</sup> e <sup>-</sup> collider	proton-proton collider
elementary	composite
weakly interacting	strongly interacting
light	heavy
clean environment	reach higher energies
precision machines	discovery machines

But: change of perspective with the Tevatron and revolution with the LHC: hadron collider as a precision machine

### LHC as a precision machine

#### NB:

precision measurements at the LHC are not a future opportunity, they are a reality now, e.g.

- W-boson mass measured with 20 MeV precision (0.02%)
- Higgs mass to 250 MeV (0.2%)
- Z-boson kinematic distributions to below a percent

- ...

### Precision: key to data

This is a game changer which doubles the potential of the LHC physics programme

- when new particles are found directly precision measurements of properties, which are needed to understand the new underlying theory (this is happening now with the Higgs sector of the SM)
- but also precision tests bring in new possibilities of precisiondriven discoveries, complementary to direct searches (like for the top quark at LEP)

Precise theory: key to exploit data

### Precision via perturbative expansions



Parton distribution functions (PDFs): extracted from data at one scale, evolution is perturbative Perturbative cross section: Expansion in the coupling constant (LO, NLO, NNLO ... )

$$\frac{d\sigma_{\rm pp\to hadrons}}{dO} = \sum_{i,j=q,g} \int dx_1 dx_2 \frac{f_i(x_1,\mu_F)f_j(x_2,\mu_F)}{f_i(x_1,\mu_F)f_j(x_2,\mu_F)} \times \frac{d\sigma_{\rm pp\to partons}}{dO} + \mathcal{O}\left(\frac{\Lambda_{\rm QCD}^n}{Q^n}\right)$$

### The NLO revolution

For a long time, the NLO calculation for each process required a separate non-trivial, manual calculation. Suddenly, thanks to theoretical conceptual breakthrough ideas

- input from supersymmetry/string theory
- connection between loop (NLO) amplitudes and tree (LO) ones
- sophisticated algebraic methods, OPP



generalised unitarity

the problem of computing NLO QCD corrections is now solved

### Automated NLO

#### An example: single Higgs production processes

#### Alwall et al '14

F	rocess	Syntax	Cross sec	ection (pb)	
Single	Higgs production		LO 13 TeV	NLO 13 $TeV$	
g.1 g.2	$pp \rightarrow H \text{ (HEFT)}$ $pp \rightarrow H i \text{ (HEFT)}$	pp>h pp>hi	$\begin{array}{rrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrr$	$\begin{array}{rrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrr$	
g.3	$pp \rightarrow Hjj (\text{HEFT})$	pp>hjj	$\begin{array}{rrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrr$	$5.124 \pm 0.020 \cdot 10^{0}  \begin{array}{r} -16.6\% & -1.4\% \\ +20.7\% & +1.3\% \\ -21.0\% & -1.5\% \end{array}$	
g.4 g.5	$pp \rightarrow Hjj$ (VBF) $pp \rightarrow Hjjj$ (VBF)	p p > h j j \$\$ w+ w- z p p > h j j j \$\$ w+ w- z	$\begin{array}{rrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrr$	$\begin{array}{cccc} 1.900 \pm 0.006 \cdot 10^{0} & {}^{+ 0.8 \% }_{- 0.9 \% }  {}^{+ 2.0 \% }_{- 0.9 \% } \\ 3.085 \pm 0.010 \cdot 10^{-1} & {}^{+ 2.0 \% }_{- 3.0 \% }  {}^{+ 1.5 \% }_{- 1.1 \% } \end{array}$	
g.6 g.7	$pp \rightarrow HW^{\pm}$ $pp \rightarrow HW^{\pm} j$	pp>hwpm pp>hwpmj	$\begin{array}{rrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrr$	$\begin{array}{rrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrr$	
g.8*	$pp \rightarrow HW^{\pm} jj$	pp>hwpmjj	$1.198 \pm 0.016 \cdot 10^{-1}  {}^{+26.1\%}_{-19.4\%}  {}^{+0.8\%}_{-0.6\%}$	$1.574 \pm 0.014 \cdot 10^{-1}  {}^{+ 5.0 \% }_{- 6.5 \% }  {}^{+ 0.9 \% }_{- 0.6 \% }$	
g.9 g.10 g.11*	$pp \rightarrow HZ$ $pp \rightarrow HZ j$ $pp \rightarrow HZ jj$	p p > h z p p > h z j p p > h z j j	$\begin{array}{rrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrr$	$\begin{array}{rrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrr$	
g.12* g.13* g.14* g.15*	$\begin{array}{l} pp \rightarrow HW^+W^- \ (\mathrm{4f}) \\ pp \rightarrow HW^\pm \gamma \\ pp \rightarrow HZW^\pm \\ pp \rightarrow HZZ \end{array}$	p p > h w+ w- p p > h wpm a p p > h z wpm p p > h z z	$\begin{array}{rrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrr$	$\begin{array}{rrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrr$	
g.16 g.17 g.18	$pp \rightarrow Ht\bar{t}$ $pp \rightarrow Htj$ $pp \rightarrow Hb\bar{b} (4f)$	$p p > h t t \sim$ $p p > h tt j$ $p p > h b b \sim$	$\begin{array}{rrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrr$	$\begin{array}{rrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrr$	
g.19 g.20*	$pp \rightarrow H t \bar{t} j$ $pp \rightarrow H b \bar{b} j$ (4f)	p p > h t t~ j p p > h b b~ j	$\begin{array}{cccc} 2.674 \pm 0.041 \cdot 10^{-1} & +45.6\% & +2.6\% \\ -29.2\% & -2.9\% \\ 7.367 \pm 0.002 \cdot 10^{-2} & +45.6\% & +1.8\% \\ & -29.1\% & -2.1\% \end{array}$	$\begin{array}{rrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrr$	

### Automated NLO

#### An example: single Higgs production processes

Alwall et al '14

Process Syntax		Cross section (pb)				
Single	Higgs production		LO 13  TeV	NLO 13 $TeV$		
g.1	$pp \rightarrow H (\text{HEFT})$	p p > h	$1.593 \pm 0.003 \cdot 10^{1}$ $^{+34.8\%}_{-26.0\%}$ $^{+1.2\%}_{-1.7\%}$	$3.261 \pm 0.010 \cdot 10^{1}$ $^{+20.2\%}_{-17.9\%}$ $^{+1.1\%}_{-1.6\%}$		
g.2	$pp \rightarrow Hj$ (HEFT)	pp>hj	$8.367 \pm 0.003 \cdot 10^{\circ}$ $^{+0.4\%}_{-26.4\%}$ $^{+1.2\%}_{-1.4\%}$	$1.422 \pm 0.006 \cdot 10^{1}$ $^{+10.07}_{-16.6\%}$ $^{+117.0}_{-1.4\%}$		
g.3	$pp \rightarrow Hjj$ (HEFT)	рр>пјј	$3.020 \pm 0.002 \cdot 10^{\circ}$ $_{-34.7\%}^{\circ}$ $_{-1.7\%}^{\circ}$	$5.124 \pm 0.020 \cdot 10^{\circ}$ $_{-21.0\%}$ $_{-1.5\%}$		
g.4	$pp \rightarrow Hjj$ (VBF)	pp>hjj\$\$ w+ w- z	$1.987 \pm 0.002 \cdot 10^{0}  {}^{+ 1.7 \% }_{- 2.0 \% }  {}^{+ 1.9 \% }_{- 1.4 \% }$	$1.900 \pm 0.006 \cdot 10^{0}  {}^{+ 0.8 \% }_{- 0.9 \% }  {}^{+ 2.0 \% }_{- 1.5 \% }$		
g.5	$pp \rightarrow Hjjj$ (VBF)	p p > h j j j \$\$ w+ w- z	$2.824 \pm 0.005 \cdot 10^{-1}  {}^{+ 15.7 \% }_{- 12.7 \% }  {}^{+ 1.5 \% }_{- 1.0 \% }$	$3.085 \pm 0.010 \cdot 10^{-1}  {}^{+ 2.0 \% }_{- 3.0 \% }  {}^{+ 1.5 \% }_{- 1.1 \% }$		
	<b>C</b>	• • •		CN4		
	Similar results available for all SM					
	Ŭ	•	• •1	•		
		processes of s	similar complex	ity		
		processes of s	similar complex	ity		
	$HW^+W^-(Af)$	processes of s	similar complex	1.065 + 0.002 10-2 +2.5% +2.0%		
g.12*	$pp \rightarrow HW^+W^-$ (4f) $pp \rightarrow HW^{\pm}\gamma$	p p > h w+ w-	$\frac{15.72}{8.325 \pm 0.139 \cdot 10^{-3}}$	$1.065 \pm 0.003 \cdot 10^{-2} +2.5\% +2.0\% -1.9\% -1.5\% +2.7\% +1.7\%$		
g.12* g.13* g.14*	$pp \rightarrow HW^+W^-$ (4f) $pp \rightarrow HW^{\pm}\gamma$ $pp \rightarrow HZW^{\pm}$	p p > h w+ w- p p > h wpm a	$\begin{array}{c} \text{similar complex} \\ \hline 8.325 \pm 0.139 \cdot 10^{-3} \\ 2.518 \pm 0.006 \cdot 10^{-3} \\ 3.763 \pm 0.007 \cdot 10^{-3} \\ \hline +0.0\% \\ +1.1\% \\ +2.0\% \end{array}$	$\begin{array}{c} 1.065 \pm 0.003 \cdot 10^{-2} & +2.5\% & +2.0\% \\ 1.065 \pm 0.003 \cdot 10^{-2} & -1.9\% & -1.5\% \\ 3.309 \pm 0.011 \cdot 10^{-3} & +2.7\% & +1.7\% \\ -2.0\% & -1.4\% \\ 5.202 \pm 0.015 \cdot 10^{-3} & +3.9\% & +1.8\% \end{array}$		
g.12* g.13* g.14* g.15*	$pp \rightarrow HW^+W^-$ (4f) $pp \rightarrow HW^{\pm}\gamma$ $pp \rightarrow HZW^{\pm}$ $pm \rightarrow HZZ$	p p > h w+ w- p p > h w+ w- p p > h wpm a p p > h z wpm	$\begin{array}{c} \text{similar complex} \\ \hline 8.325 \pm 0.139 \cdot 10^{-3} & +0.0\% & +2.0\% \\ \hline 8.325 \pm 0.006 \cdot 10^{-3} & -0.3\% & -1.6\% \\ \hline 2.518 \pm 0.006 \cdot 10^{-3} & +0.7\% & +1.9\% \\ \hline 3.763 \pm 0.007 \cdot 10^{-3} & +1.1\% & +2.0\% \\ \hline 2.093 \pm 0.003 \times 10^{-3} & +0.1\% & +1.9\% \end{array}$	$\begin{array}{c} 1.065 \pm 0.003 \cdot 10^{-2} & +2.5\% & +2.0\% \\ 1.065 \pm 0.003 \cdot 10^{-2} & -1.9\% & -1.5\% \\ 3.309 \pm 0.011 \cdot 10^{-3} & +2.7\% & +1.7\% \\ -2.0\% & -1.4\% \\ 5.292 \pm 0.015 \cdot 10^{-3} & +3.9\% & +1.8\% \\ -3.1\% & -1.4\% \\ 2.538 \pm 0.007 \cdot 10^{-3} & +1.9\% & +2.0\% \end{array}$		
g.12* g.13* g.14* g.15*	$pp \rightarrow HW^+W^-$ (4f) $pp \rightarrow HW^{\pm}\gamma$ $pp \rightarrow HZW^{\pm}$ $pp \rightarrow HZZ$	<pre>p p &gt; h w+ w- p p &gt; h wpm a p p &gt; h z wpm p p &gt; h z z</pre>	$\begin{array}{c} \underbrace{\text{similar complex}}_{-15.4\%} \\ \underline{\text{similar complex}}_{-15.4\%} \\ \underline{8.325 \pm 0.139 \cdot 10^{-3}}_{-0.3\%} & \underbrace{+0.0\% + 2.0\%}_{-0.3\% - 1.6\%} \\ \underline{2.518 \pm 0.006 \cdot 10^{-3}}_{-1.4\% - 1.5\%} \\ \underline{3.763 \pm 0.007 \cdot 10^{-3}}_{-1.5\% - 1.5\%} & \underbrace{+1.1\% + 2.0\%}_{-1.5\% - 1.5\%} \\ \underline{2.093 \pm 0.003 \cdot 10^{-3}} & \underbrace{+0.1\% + 1.9\%}_{-0.6\% - 1.5\%} \end{array}$	$\begin{array}{c} -\frac{1.065 \pm 0.003 \cdot 10^{-2}}{1.065 \pm 0.003 \cdot 10^{-2}} & +2.5\% +2.0\% \\ -1.9\% & -1.5\% \\ 3.309 \pm 0.011 \cdot 10^{-3} & +2.7\% +1.7\% \\ 5.292 \pm 0.015 \cdot 10^{-3} & +3.9\% +1.8\% \\ -3.1\% & -1.4\% \\ 2.538 \pm 0.007 \cdot 10^{-3} & +1.9\% +2.0\% \\ -1.4\% & -1.5\% \end{array}$		
g.12* g.13* g.14* g.15* g.16	$pp \rightarrow HW^+W^-$ (4f) $pp \rightarrow HW^{\pm}\gamma$ $pp \rightarrow HZW^{\pm}$ $pp \rightarrow HZZ$ $pp \rightarrow Ht\bar{t}$	<pre>p p &gt; h w+ w- p p &gt; h wpm a p p &gt; h z wpm p p &gt; h z z p p &gt; h t t~</pre>	$\begin{array}{c} \underbrace{\text{similar complex}}_{3.325 \pm 0.139 \cdot 10^{-3}} & \begin{smallmatrix} +0.0\% & +2.0\% \\ -0.3\% & -1.6\% \\ 2.518 \pm 0.006 \cdot 10^{-3} & \begin{smallmatrix} +0.0\% & +2.0\% \\ -0.3\% & -1.6\% \\ -0.3\% & -1.6\% \\ -1.4\% & -1.5\% \\ -1.4\% & -1.5\% \\ -1.5\% & -1.6\% \\ -1.5\% & -1.6\% \\ -1.5\% & -1.6\% \\ -1.5\% & -1.6\% \\ -0.6\% & -1.5\% \\ \hline \end{array}$	$\begin{array}{c} \hline & \hline $		
g.12* g.13* g.14* g.15* g.16 g.17	$\begin{array}{c} pp \rightarrow HW^+W^- \ (4f) \\ pp \rightarrow HW^\pm \gamma \\ pp \rightarrow HZW^\pm \\ pp \rightarrow HZZ \\ \\ pp \rightarrow Ht\bar{t} \\ pp \rightarrow Ht\bar{t} \\ pp \rightarrow Htj \\ \end{array}$	<pre>p p &gt; h w+ w- p p &gt; h wpm a p p &gt; h z wpm p p &gt; h z z p p &gt; h t t~ p p &gt; h tt j</pre>	$\begin{array}{c} \underbrace{\text{similar complex}}_{3.325 \pm 0.139 \times 10^{-3}} & \begin{smallmatrix} +0.0\% & +2.0\% \\ -0.3\% & -1.6\% \\ 2.518 \pm 0.006 \times 10^{-3} & \begin{smallmatrix} +0.0\% & +2.0\% \\ -0.3\% & -1.6\% \\ -0.3\% & -1.6\% \\ -1.4\% & -1.5\% \\ -1.4\% & -1.5\% \\ -1.4\% & -1.5\% \\ -1.5\% & -1.6\% \\ -1.5\% & -1.6\% \\ -1.5\% & -1.6\% \\ -1.5\% & -1.6\% \\ -1.5\% & -1.6\% \\ -1.5\% & -1.6\% \\ -1.5\% & -1.6\% \\ -1.5\% & -1.6\% \\ -1.5\% & -1.6\% \\ -0.6\% & -1.5\% \\ -1.5\% & -2.0\% \\ -21.5\% & -2.0\% \\ -21.5\% & -2.0\% \\ -21.5\% & -2.0\% \\ -21.5\% & -2.0\% \\ -2.0\% & -4.2\% & -1.3\% \end{array}$	$\begin{array}{c} \hline & \hline $		
g.12* g.13* g.14* g.15* g.16 g.17 g.18	$\begin{array}{c} pp \rightarrow HW^+W^- \ (4f) \\ pp \rightarrow HW^\pm \gamma \\ pp \rightarrow HZW^\pm \\ pp \rightarrow HZZ \\ \\ pp \rightarrow Ht\bar{t} \\ pp \rightarrow Ht\bar{t} \\ pp \rightarrow Htj \\ pp \rightarrow Hb\bar{b} \ (4f) \end{array}$	<pre>p &gt; h w+ w- p p &gt; h w+ w- p p &gt; h wpm a p p &gt; h z wpm p p &gt; h z z  p p &gt; h t t~ p p &gt; h t t p p &gt; h t b</pre>	$\begin{array}{c} \begin{array}{c} \text{similar complex} \\ \hline \text{similar complex} \\ \hline \text{s.325} \pm 0.139 \cdot 10^{-3} & +0.0\% & +2.0\% \\ & -0.3\% & -1.6\% \\ \hline \text{2.518} \pm 0.006 \cdot 10^{-3} & +0.7\% & +1.9\% \\ \hline \text{2.518} \pm 0.006 \cdot 10^{-3} & +0.7\% & +1.9\% \\ \hline \text{3.763} \pm 0.007 \cdot 10^{-3} & +1.1\% & +2.0\% \\ \hline \text{2.093} \pm 0.003 \cdot 10^{-3} & +0.1\% & +1.9\% \\ \hline \text{2.093} \pm 0.003 \cdot 10^{-1} & +30.0\% & +1.7\% \\ \hline \text{3.579} \pm 0.003 \cdot 10^{-1} & +30.0\% & +1.7\% \\ \hline \text{4.994} \pm 0.005 \cdot 10^{-2} & +2.4\% & +1.2\% \\ \hline \text{4.983} \pm 0.002 \cdot 10^{-1} & +28.1\% & +1.5\% \\ \hline \text{-21.0\% & -1.8\%} \end{array}$	$\begin{array}{c} \hline & \hline $		
g.12* g.13* g.14* g.15* g.16 g.17 g.18 g.19	$\begin{array}{c} pp \rightarrow HW^+W^- \ (4f) \\ pp \rightarrow HW^{\pm}\gamma \\ pp \rightarrow HZW^{\pm} \\ pp \rightarrow HZZ \\ \\ pp \rightarrow Ht\bar{t} \\ pp \rightarrow Ht\bar{t} \\ pp \rightarrow Ht\bar{j} \\ pp \rightarrow Hb\bar{b} \ (4f) \\ \\ pp \rightarrow Ht\bar{t}j \end{array}$	<pre>p &gt; h w+ w- p p &gt; h wpm a p p &gt; h z wpm p p &gt; h z z  p p &gt; h t t~ p p &gt; h t t </pre>	$\begin{array}{c} \begin{array}{c} \text{similar complex} \\ \hline \text{similar complex} \\ \hline \text{s.325 \pm 0.139 \cdot 10^{-3}} & \begin{array}{c} +0.0\% + 2.0\% \\ -0.3\% - 1.6\% \\ -0.3\% - 1.6\% \\ -0.3\% - 1.6\% \\ -0.3\% - 1.6\% \\ -0.3\% - 1.6\% \\ -0.3\% - 1.6\% \\ -1.4\% - 1.5\% \\ -1.4\% - 1.5\% \\ -1.4\% - 1.5\% \\ -1.5\% - 1.6\% \\ -1.5\% - 1.6\% \\ -0.6\% - 1.5\% \\ -1.5\% - 1.6\% \\ -0.6\% - 1.5\% \\ -1.5\% - 1.6\% \\ -0.6\% - 1.5\% \\ -0.6\% - 1.5\% \\ -21.5\% - 2.0\% \\ -21.5\% - 2.0\% \\ -2.0\% \\ -2.0\% - 1.3\% \\ -21.0\% - 1.3\% \\ -21.0\% - 1.3\% \\ -21.0\% - 1.3\% \\ -21.0\% - 1.3\% \\ -21.0\% - 1.3\% \\ -21.0\% - 1.3\% \\ -21.0\% - 1.3\% \\ -21.0\% - 1.3\% \\ -21.0\% - 1.3\% \\ -21.0\% - 1.8\% \\ -21.0\% - 1.8\% \\ -20.0\% \\ -29.2\% - 2.9\% \end{array}$	$\begin{array}{c} \hline & \hline & \hline \\ 1.065 \pm 0.003 \cdot 10^{-2} & +2.5\% & +2.0\% \\ & -1.9\% & -1.5\% \\ 3.309 \pm 0.011 \cdot 10^{-3} & +2.7\% & +1.7\% \\ & -2.0\% & -1.4\% \\ 5.292 \pm 0.015 \cdot 10^{-3} & +3.9\% & +1.8\% \\ & -3.1\% & -1.4\% \\ 2.538 \pm 0.007 \cdot 10^{-3} & +1.9\% & +2.0\% \\ & -1.4\% & -1.5\% \\ \hline & 4.608 \pm 0.016 \cdot 10^{-1} & +5.7\% & +2.0\% \\ & -9.0\% & -2.3\% \\ & 6.328 \pm 0.022 \cdot 10^{-2} & +2.9\% & +1.5\% \\ & -1.8\% & -1.6\% \\ & 6.085 \pm 0.026 \cdot 10^{-1} & +7.3\% & +1.6\% \\ & -9.6\% & -2.0\% \\ \hline & 3.244 \pm 0.025 \cdot 10^{-1} & +3.5\% & +2.5\% \\ & -8.7\% & -2.9\% \\ \hline \end{array}$		

### NLO calculations

Various (public) tools developed: Blackhat+Sherpa, GoSam+Sherpa, Helac-NLO, Madgraph5\_aMC@NLO, NJet+Sherpa, OpenLoops+Sherpa, Samurai, Recola ...

- Practical limitation: high-multiplicity processes still difficult because of numerical instabilities, need long run-time on clusters to obtain stable results (edge: 5-6 particles in the final state, depending on the process)
- Today focus on
  - automation of NLO for BSM signals
  - Ioop-induced processes: formally higher-order, but enhanced by gluon PDF
  - automation of NLO electroweak corrections (necessary to match accuracy of NNLO)

# NNLO

#### NNLO is one of the most active areas in QCD now

After pioneering calculations for Higgs and Drell-Yan more than 15 years ago, recently many  $2 \rightarrow 2$  processes computed at NNLO

#### NNLO most important in three different situations

Benchmark processes measured with highest accuracy

- $Z \rightarrow \parallel$
- $W \rightarrow Iv$
- Z + jet

— ....

Input to PDF fits + background to Higgs studies

- diboson
- boson + jet
- top-pairs
- **–** dijets, ...

Very large NLO corrections (moderate precision requires NNLO)

– Higgs

. . .

– Higgs + jet

# Two main difficulties at NNLO

calculation of two-loop amplitudes/ master integrals methods to cancel (overlapping) divergences before integration



Cancelation manifest after phase space integration, but to have fully differential results must achieve cancelation before integration

# 1. Cancelation of divergences

#### Two strategies

#### Slicing methods:

partition the phase space with a (small) slicing parameter so that divergences are all below the slicing cut. In the divergent region use an approximate expression, neglecting finite terms, above use the exact (finite) integrand

(need to test independent of slicing parameter)

#### Subtraction methods:

since IR singularities of amplitudes are knows, add and subtract counterterms so as to make integrals finite. "Easy" at NLO, but complicated at NNLO due to the more intricate structure of (overlapping) singularities

(possible to use local subtraction terms)

## Practical realisations

#### Slicing methods:

- qT subtraction Catani, Grazzini

- N-jettiness subtraction Boughezal, Focke, Liu, Petriello; Gaunt, Stahlhofen, Tackmann, Walsh

#### Subtraction methods:

- Sector decomposition Anastasiou, Melnikov, Petriello; Binoth, Heinrich
- Antenna subtraction Kosower; Gehrmann, Gehrmann De Ridder, Glover
- Sector Improved residue subtraction Czakon; Boughezal, Melnikov, Petriello; Czakon Heymes; Caola, Melnikov, Rontsch
- Colourful subtraction Del Duca, Duhr, Kardos, Somogyi, Trocsanyi
- Projection to Born Cacciari, Dreyer, Karlberg, Salam, GZ

### Practical realisations

In principle, the problem of cancelation of singularities solved in theory in a generic way In practise, methods applied to  $2 \rightarrow 2$  processes. Require long runs on large computer farms (plus, possibly, a way to deal with outliers/ spikes)

NB: the attitude "Today we have big farms, so why care?" is not acceptable. The phenomenology that one gets out of a calculation scales as inverse power of the computation time

#### Lots of progress still to come ...

# 2. Two-loop amplitudes

- A number of massless amplitudes computed long time ago (e.g.  $gg \rightarrow \gamma\gamma$ ) Bern, De Freitas, Dixon 2002
- 2 to 2 amplitudes with internal or external masses are the stateof-the-art today, computed either analytically or numerically (e.g. pp → VV, pp → tt, pp → HH ...)

Caola,Henn, Melnikov, Smirvov,Smirnov (2014-2015); Gehrmann, Mantueffel,Tancredi (2014-2015) Czakon, Birowks, Greiner, Heinrich, Jones, Kerner, Schlenk, Schubert,Zirke (2016)

The calculation of the amplitude requires

- a reduction from tensor to scalar (master) integrals
- the calculation of the master integrals

Unlike at one loop, both steps are still not well-understood/ automated (many master integrals, choice of basis not clear, integrals not all known, no connection to tree level ...)

# 2. Two-loop integrals

 Rather than brute-force calculation, master integrals in many cases computed solving differential equations

Kotikov 1991; Remiddi 1997; Henn 2013; Papadopoulos 2014

 Method well-understood when only generalised polylogarithmic functions (GPL) are involved

e.g. method pushed to 3-loop 4-point functions in N=4 SYM Henn & Mistlberger 1608.00850 or to 2-loop planar 5-point functions Gehrmann, Henn, Lo Presti 1511.05409 ...

- Internal masses complicate the problem considerably: elliptic functions appear
- Steady progress with internal masses, but no complete understanding yet

Tancredi, Remiddi (2016); Adams, Bogner, Weinzierl (2015-2016); Abreu, Britto, Duhr, Gardi (2017)

 One case-study involving elliptic functions: two-loop planar results for Higgs pt with full mass dependence

# Two upcoming challenges

 Fully mastering conceptual challenges related to internal masses (internal masses necessary for Higgs physics at high pt)
 Extension of NNLO to 2 to 3 processes

A few encouraging results for going to higher multiplicity from generalised unitarity methods

- 5/6 all-plus gluon amplitudes computed analytically at two loops
   Badger, Frellesvig, Zhang (2013); Badger, Modull,Ochiruv, O'Connel
   (2015); Badger, Scabinger (2015); Dunbar, Perkins (2016)
- numerical unitarity at two-loops: full numerical results of pp  $\rightarrow$  2j

Abreu, Febres-Cordero, Ita, Jaquier, Page, Zeng (2017)

## Is NNLO really needed?

### NNLO vs data



LHC data clearly prefers NNLO

Same conclusion in all measurements examined so far With more data NLO likely to be insufficient

# Comparison to NNLO for Z + jet



Gehrmann-De Ridder, Gehrmann, Glover, Huss, Morgan '16 Boughezal, Liu, Petriello '16 Boughezal, Ellis, Focke, Giele, Liu, Petriello '15

- NNLO and EW alleviate tension between data and theory
- better agreement in normalised distribution
- remember 2-3%
   luminosity error on data



# Impact of Z + jet on luminosities



10<sup>2</sup>

M<sub>v</sub> [GeV]

 $10^{3}$ 

1.1

... 20. Iatio

0.95

0.9

0.85

0.8

Boughezal, Guffanti, Petriello, Ubiali 1705.00343

Significant reduction of uncertainty in all luminosities (e.g. 30% impact on PDF uncertainty of Higgs crosssection)



# Recent progress in PDFs

#### HXSWG Yellow Report 3 (2013) HXSWG Yellow Report 4 (2016)



Still, in many cases PDF uncertainty dominant or not negligible. Further improvements soon.

# Recent progress in PDFs

#### Nadolsky QCD@LHC2017

#### 2017: explicit and implicit advancements

All major groups rush toward implementing LHC data on **jet**, **W/Z**,  $Z p_T t\bar{t}$  production in the PDF analysis

- ABMP'16 (arXiv:1701.05838) includes a large LHC W/Z data set, got closer to the other PDF sets
- The NNPDF3.1 set has been released (arXiv:1706.00428), including a compatible subset of the new LHC data
- CT1X and MMHT'XX to be released within a few months, once compatibility of the new experiments is understood

Recurring issues:

- properly assessing the associated error (bias due to choice of data, methodology, etc.)
- making sure that new physics is not fitted in PDFs

# H + one jet at NNLO

3 calculations with 3 methods  $\Rightarrow$  cross-checks and validation Decays of Higgs to bosons also included. Fiducial cross-sections compared to ATLAS and CMS data



Caola, Melnikov, Schulze 1508.02684 Boughezal, Melnikov, Petriello, Schulze 1504.07922 Chen, Gehrmann, Glover, Jaquier 1607.08817



Good agreement on normalised distributions, less good agreement on unnormalised ones (but current data have large errors)

# H + one jet at NNLO



 Better agreement with theory with 13 TeV data, compared to 8 TeV

 Fixed-order predictions for fiducial cross-sections and merging to parton showers crucial

### Fully differential VBFH at NNLO

#### Cacciari, Dreyer, Karlberg, Salam, GZ 1506.02660





- Allows to study realistic observables, with realistic cuts
- NNLO corrections much larger (10%) than expected (1%)
- Important for coupling measurements

# Тор

#### Top unique in the SM:

- decays before hadronizing ⇒ direct access to bare quark (spin, couplings)
- heavy ⇒ window to New Physics



## Top: scale choice

#### Differential NNLO results now available in the stable top approximation

Czakon, Fiedler, Heynes, Mitov 1601.05375 Czakon, Heynes, Mitov 1704.08551

- Non-trivial (and often overlooked) problem: difference between various dynamic scales can be substantial.
- This is now well-understood for inclusive top-pair production

arXiv:1606.03350

Criteria for fixing the scales:

$$u_0 = \begin{cases} \frac{m_T}{2} & \text{for} : p_{T,t}, p_{T,\bar{t}} \text{ and } p_{T,t/\bar{t}}, \\\\ \frac{H_T}{4} & \text{for} : \text{ all other distributions.} \end{cases}$$

Require minimal K-factors, both at NLO and NNLO and for the full kinematics

Mitov QCD@LHC2017

## Top: scale choice

Differential NNLO results now available in the stable top approximation



# Тор

Differential NNLO top cross section reduces uncertainty on gluon-gluon luminosity:



### Top mass

Scheme choice no longer an issue: conversion from pole- to MSbarscheme known to better than 200 MeV

Beneke, Marquard, Nason, Steinhauser1605.03609 But most extractions rely on generators: intrinsic uncertainty hard to quantify



ATLAS: combine several <u>leptonic observables</u> which are less sensitive to MC modelling (theory=NLO MCFM). Soon: first application of NNLO production with NNLO decay?

# NNLO inclusive jet spectrum

#### Scale ( $\mu_R$ , $\mu_F$ ): pt of leading jet

#### Scale ( $\mu_R$ , $\mu_F$ ): pt of jet



Currie, Glover, Pires 1611.01460

Low transverse momentum region

# NNLO inclusive jet spectrum

#### Scale ( $\mu_R$ , $\mu_F$ ): pt of leading jet

#### Scale ( $\mu_R$ , $\mu_F$ ): pt of jet



Currie, Glover, Pires 1611.01460

High transverse momentum region

# Di-jet invariant mass

Larger y1-y2

#### Small y1-y2



 $2p_t \lesssim m_{jj} \lesssim 3p_t$  (LO)

 $\Rightarrow$  pt and mjj scales similar

 $\Rightarrow$  pt and mii very different

 $9p_t \lesssim m_{jj} \lesssim 15p_t$  (LO)

"We choose the dijet invariant mass as the theoretical scale on the grounds of perturbative convergence and residual scale variation ..." Currie, A. Gehrmann-De Ridder, Gehrmann, Glover, Huss, Pires 1705.10271

# Scale setting: the usual questions

- How should one set the renormalization and factorization scale in a given process? It is fair to set the scale a posteriori ...?
- Can one trust the scale uncertainty band, i.e.the factor two variation around central scale [in particular if set a posteriori]?
- How should the scale uncertainty be interpreted? as a flat 100% interval\* or as a 1σ (3σ?,5σ?) gaussian ...?

(\*) Scale uncertainty interpreted as a 100% flat interval e.g. in the N3LO Higgs cross-section in the HXSWG and in the first extraction of  $\alpha_s$  from ttbar at the LHC

# Scale setting: the usual questions

Mostly, there are no good answers. A few approaches:

- dynamical, a priori procedure to set the scales based on clustering scales (CKKW, MiNLO) [typically yields larger uncertainty bands]
- uncertainty extracted from convergence of the PT series [but needs a few orders...]
- Cacciari-Houdeau Bayesian approach [suggests scale band is less then 1σ]

# Scale setting

Some (obvious) considerations are:

- The question is not what is the right scale (BLM? PCM?) but rather what is the theoretical uncertainty, and its interpretation
- The more orders one computes, the less relevant the question is, however the more precise data are, the more important the question is. Altogether, the question likely to remain relevant
- In all cases examined, when the scale uncertainty band fails badly to estimate size of the next order there is a reason (e.g. new channels, Born zeros, large logarithms ...). Still scale variation has serious limitations and should never to overrated
- In the only two cases for which the N<sup>3</sup>LO result is available, it does lie in the NNLO scale band (but inclusive results only)

More experience with NNLO and N3LO will guide us further

# NNLO+PS

#### NNLO:

good perturbative accuracy, accurate inclusive cross-sections, but limited to low multiplicity and parton level only



#### Parton shower:

less accurate, but realistic description, including multi-parton interactions, resummation, hadronization effects



# NNLO+PS

Merging NNLO and parton shower (NNLOPS) is a must to have the best perturbative accuracy with a realistic description of final state



- First NNLOPS codes: Higgs, Drell-Yan & associated Higgs production
- currently, three different methods: MiNLO, UNNLOPS, Geneva

Hoeche, Li,Prestel '14-'15 [UNNLOPS Astill, Bizon, Hamilton, Karlberg, Nason, Re, GZ '13-'16 [MiNLO Alioli, Bauer, Berggren, Guns, Tackmann, Walsh '15-'16 [Geneva

# NNLO+PS for HW

<u>One sample NNLOPS result</u>: associated HW production with cuts suggested by HXSWG

 Parton shower and hadronization cause migration between jetbins

 Difficult to reach high accuracy in jet-binned observables

#### HW-NNLOPS(Pythia8-part) HW-NNLOPS(Pythia8-hadr) NNLO Fiducial XS [fb] 75.95 7.51 2.02 58.65 10.65 3.97 77.93 8.20 2.34 56.52 10.05 3.65 80.13 8.46 2.43 54.33 9.79 3.57 $0 \text{ GeV} < p_{t,H} < 150 \text{ GeV}$ BIN1: 1.20 BIN2: 150 GeV < pt.H < 250 GeV Ratio to NNLO BIN3: 250 GeV < pt. H 1.10 1.00 0.90 BIN 1 BIN 2 BIN 3 BIN 1 BIN 2 BIN 3 (no jets) (no jets) (no jets) (with jets) (with jets) (with jets)

LOW D

#### Astill, Bizon, Re, GZ 1603.01620

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# N3LO

Two LHC processes known at N<sup>3</sup>LO

Gluon fusion Higgs production (in the limit of infinite top-quark mass) Vector boson fusion Higgs production (in the structure function approximation, i.e. double DIS process)

### N<sup>3</sup>LO Higgs production



- dashed lines include resummation of even higher orders (essentially no impact on central value at preferred renormalisation scale m<sub>H</sub>/2)
- N<sup>3</sup>LO stabilises the perturbative expansion (N<sup>3</sup>LO band contained in NNLO band, while NNLO was not in the NLO band)

### Data vs theory



Next challenge: extend N<sup>3</sup>LO accuracy to differential distributions (hard but within reach?)

### ... and inclusive VBFH at N<sup>3</sup>LO

#### Dreyer & Karlberg 1606.00840





Again, NNLO was outside the NLO uncertainty band, while N<sup>3</sup>LO band (with sensible scale) is fully contained in the NNLO band

### The strong coupling

The strong coupling is the fundamental parameter of QCD. It is not an observable, but observables depend on it, hence  $\alpha_s(Q)$  extracted by comparing calculations and data. Four key considerations: sensitivity, accuracy, control of non-perturbative, scale Q probing  $\alpha_s$ 



Summary of extractions from e<sup>+</sup>e<sup>-</sup>, DIS and hadron collider experiments

running probed to TeV scales

 good agreement
 between various fits (but the devil is in the details)

### The strong coupling

<u>Tevatron + LHC average</u> (NLO, not contributing to world average)

 $\alpha_s(M_Z) = 0.1172 \pm 0.0059$ 

agrees well with the world average\*

 $\alpha_s(M_Z) = 0.1181 \pm 0.0011$ 

PDG 2016

(\*) world average obtained as average of sub-averages. Removing each sub-average changes the average by less than its quoted error

Further improvements likely to come from lattice (but one never knows)

Two examples where precision brings in new opportunities

### 1. Pinning down the Higgs potential



### 1. Pinning down the Higgs potential

<u>Current and future bounds on  $\lambda$  based on double Higgs production:</u>

 $\frac{\lambda}{\lambda_{\rm SM}} \in [-14.5, \, 19.1]$ LHC Run I, 20.3 fb<sup>-1</sup> 2y2b, 1406.5053; 4b, 1506.00285; 2b2T, 2y2W, 1509.04670 LHC Run II, 13.3 fb<sup>-1</sup>  $\longrightarrow \frac{\lambda}{\lambda_{SM}} \in [-8.4, 13.4]$ 4b, ATLAS-CONF-2016-049  $\rightarrow \frac{\lambda}{\lambda_{\rm SM}} \in [-0.8, 7.7]$ HL-LHC, 3 ab-1 2y2b, ATL-PHYS-PUB-2017-001

### 1. Pinning down the Higgs potential

Alternative: exploit indirect sensitivity to  $\lambda$  of single Higgs production



Bizon, Gorbahn, Haisch, GZ '16 Degrassi, Maltoni, Giardino, Pagani '16 Degrassi, Fedele, Giardino '17 Di Vita, Grojean, Panico, Riembau, Vantalon '17

### 2. Higgs coupling to 2<sup>nd</sup> generation

- we know quite well that the Higgs couples to vector bosons and to 3<sup>rd</sup> generation (heavy) quarks as predicted in the SM
- couplings to 2<sup>nd</sup> (and 1<sup>st</sup>) generation notoriously more difficult
- a number of ways to constraint the coupling of Higgs to charm:
  - rare exclusive Higgs decays
  - Higgs + charm production
  - ► constraint from VH (H → bb) including charm mis-tagging
  - constraint from Higgs width

still largely unconstraint



### 2. Higgs coupling to 2<sup>nd</sup> generation

- Higgs produced dominantly via topquark loop (largest coupling)
- but interference effects with light quarks are not negligible
- provided theoretical predictions are accurate enough, constraint on charm (and possible strange) Yukawa can be significantly improved





Bishara, Haisch, Monni, Re '16 [similar ideas in Soreq, Zhu, Zupan '16]

similar sensitivity in leading jet  $p_t$ 

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Bishara, Haisch, Monni, Re '16 [similar ideas in Soreq, Zhu, Zupan '16]





### Conclusions

- The Higgs discovery leaves many open questions for the LHC Run II to explore
- Precision calculations, crucial to address those questions, are making giant steps: NLO automated, NNLO the frontier, first N<sup>3</sup>LO and NNLO+parton shower results
- Uncertainties reach the level of the few percent for cross-sections (larger for distributions)
- Perturbative QCD uncertainty often already not the dominant theory uncertainty, other corrections must be included (EW corrections, PDF and α<sub>s</sub> uncertainties, non-perturbative effects, corrections to large-mt effective theory in gluon-fusion production ... )
- Given the outcome of Run I and Run II measurements, role of precision predictions has never been as important at the LHC