Rare B decays on the lattice

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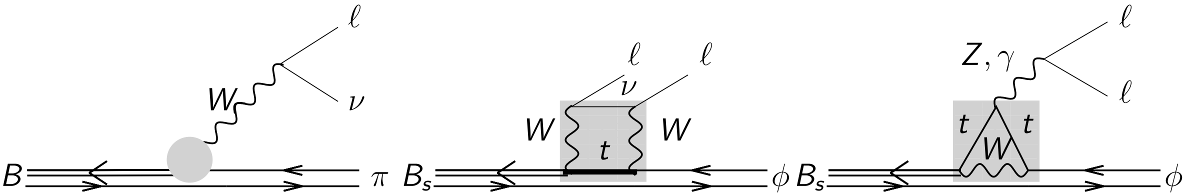
introduction

B decays

- Initial state is a pseudoscalar B or B_s meson
  $B^+ = (u\bar{b})$, $B^- = (\bar{u}b)$, $B^0 = (d\bar{b})$ and $\overline{B}^0 = (\bar{d}b)$ with mass ~5280 GeV
  $B_s^0 = (s\bar{b})$ and $\overline{B_s}^0 = (\bar{s}b)$ with mass ~5367 GeV
- Weak decays of the b-quark
  → Charged flavor changing currents mediated by $W^\pm$ (tree-level)
  → Flavor changing neutral currents (loop-level)



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  - → CKM suppressed
  - → GIM suppressed (no FCNC)

$$\begin{bmatrix} d' \\ s' \\ b' \end{bmatrix} = \begin{bmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{bmatrix} \begin{bmatrix} d \\ s \\ b \end{bmatrix} \quad \text{with} \quad \begin{bmatrix} |V_{ud}| & |V_{us}| & |V_{ub}| \\ |V_{cd}| & |V_{cs}| & |V_{cb}| \\ |V_{td}| & |V_{ts}| & |V_{tb}| \end{bmatrix} = \begin{bmatrix} 0.97434 & 0.22506 & 0.00357 \\ 0.22492 & 0.97351 & 0.0411 \\ 0.00875 & 0.0403 & 0.99915 \end{bmatrix}$$

[PDG 2016]

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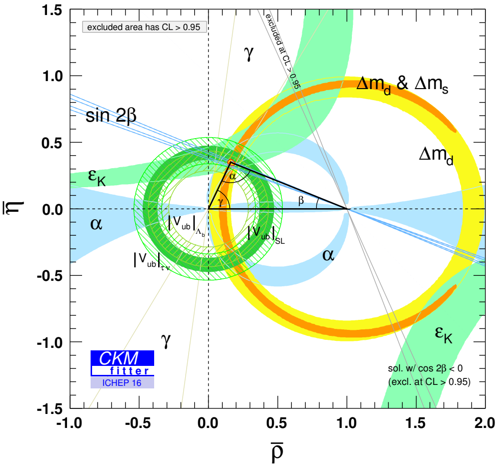
- Nonperturbative calculation of form factors
  → Exclusive semi-leptonic decays with one hadronic final state
  → Pseudoscalar or vector (narrow width approximation) final states
  → Only short distance contributions

Why are we interested in rare B decays?

- Charged current decays allow to determine CKM matrix elements $|V_{cb}|$ and $|V_{ub}|$
  - → Test unitarity of the CKM matrix
  - → Precision tests of the Standard Model
- Searches for / constraints on new physics

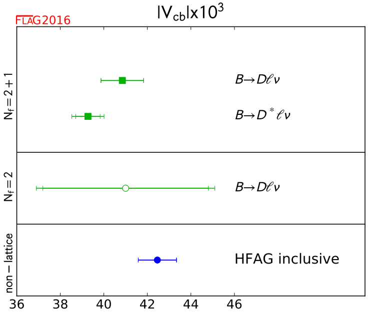


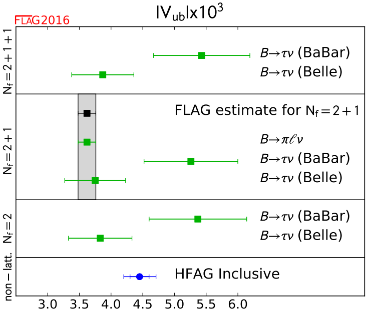
[http://ckmfitter.in2p3.fr]

Determination of $|V_{cb}|$ and $|V_{ub}|$

- Commonly $|V_{cb}|$ extracted from $B \to D^{(*)}\ell\nu$ and $|V_{ub}|$ extracted from $B \to \pi\ell\nu$
- Long standing tension between exclusive and inclusive determinations
  - $\to$ Revisit HQET constraints entering z-parametrizations

[Bigi, Gambino PRD94 (2016) 094008][Bigi, Gambino, Schacht PLB769 (2017) 441-445]





[FLAG2016]

[Fermilab/MILC PRD92 (2015) 034506] [HPQCD PRD92 (2015) 054510]
[Fermilab/MILC PRD89 (2014)114504]
[Atoui et al. EPJC74 (2014) 2861]

[HPQCD PRD75 (2006)119906]
[Fermilab/MILC PRD92 (2015) 014024]
[RBC-UKQCD PRD91 (2015) 074510]

Determination of $|V_{cb}|$ and $|V_{ub}|$: explore alternative channels

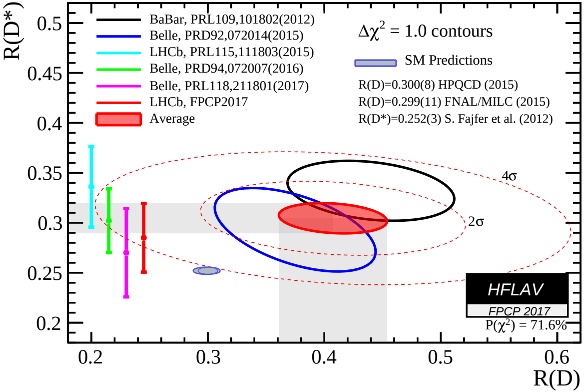
- $|V_{ub}|$ from $B \to \tau\nu$: errors too large
- $|V_{cb}|/|V_{ub}|$ from exclusive baryonic decays: $\Lambda_b \to \Lambda_c \ell\nu$ and $\Lambda_b \to p\ell\nu$
  [Detmold, Lehner, Meinel, PRD92 (2015) 034503]
- $|V_{cb}|$ from $B_s \to D_s \ell\nu$ and $|V_{ub}|$ from $B_s \to K\ell\nu$
  - $\to$ B-factories typically run at the $\Upsilon(4s)$ threshold i.e. B but no B_s mesons are produced
  - $\to$ Not (yet) experimentally measured with sufficient precision
  - $\to$ LHC energies are large enough to produce sufficient B_s mesons $\to$ LHCb
  - $\to$ Absolute normalization is challenging; ratios are preferred: determine $|V_{cb}|/|V_{ub}|$
- Use lattice techniques to compute inclusive decays
  [Hashimoto PTEP 2017 (2017) 053B03] [Hansen, Meyer, Robaina arXiv:1704.08993]

Searches for new physics (tree-level)

- Lepton flavor universality violations in $\mathcal{R}_D$ ratios

[HFLAV website]

$$\mathcal{R}^{\tau/\mu}_{D^{(*)}} \equiv \frac{d\Gamma(B \to D^{(*)}\tau\nu_\tau)/d_q^2}{d\Gamma(B \to D^{(*)}\mu\nu_\mu)/d_q^2}$$

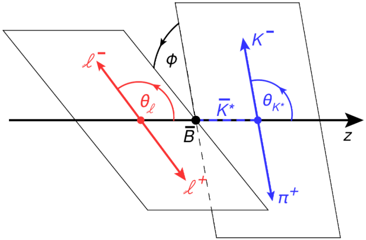


- Input: form factors over full q^2 range
  - → $B \to D\ell\nu$ [HPQCD PRD92 (2015) 054510] [Fermilab/MILC PRD92 (2015) 035606]
  - → $B \to D^*\ell\nu$ [Fajfer, Kamenik, Nisandzic PRD85 (2012) 094025]
  - → Theoretical uncertainty on $B \to D^*\ell\nu$ is suspiciously small

Searches for new physics (loop-level)

- $b \to s\ell^+\ell^-$ processes e.g. $B \to \pi\ell^+\ell^-$,$B \to K^{(*)}\ell^+\ell^-$, $B_s \to \phi\ell^+\ell^-$
- Decomposition into angular variables e.g. $B \to K^*\ell^+\ell^-$

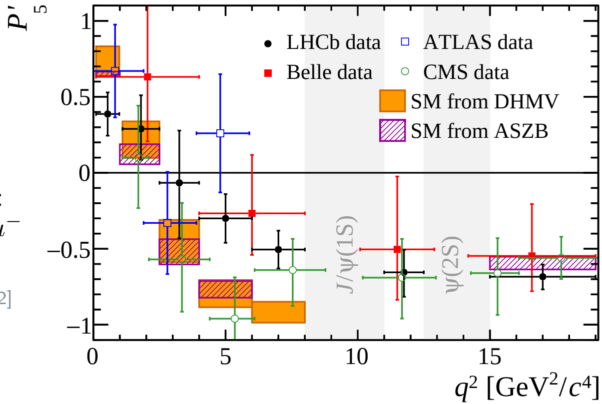
$$\frac{1}{d(\Gamma+\bar{\Gamma})/dq^2}\frac{d^4q(\Gamma+\bar{\Gamma})}{dq^2 d\vec{\Omega}} = \frac{9}{32\pi}\Big[\frac{3}{4}(1-F_L)\sin^2\theta_K + F_L\cos^2\theta_K$$
$$+\frac{1}{4}(1-F_L)\sin^2\theta_K\cos 2\theta_l - F_L\cos^2\theta_K\cos 2\theta_l + S_3\sin^2\theta_l\cos 2\phi$$
$$+S_4\sin 2\theta_K\sin 2\theta_l\cos\phi + S_5\sin 2\theta_K\sin\theta_l\cos\phi + \frac{4}{3}A_{FB}\sin^2\theta_K\cos\theta_l$$
$$+S_7\sin 2\theta_K\sin\theta_l\sin\phi + S_8\sin 2\theta_K\sin 2\theta_l\sin\phi + S_9\sin^2\theta_K\sin^2\theta_l\sin 2\phi\Big]$$



- F_L, A_{FB}, S_i are functions of Wilson coefficients ⇒ sensitive to new physics
- To reduce hadronic uncertainties, introduce $P'_{i=4,5,6,8} = -\frac{S_{j=4,5,7.8}}{\sqrt{F_L(1-F_L)}}$

Searches for new physics (loop-level)

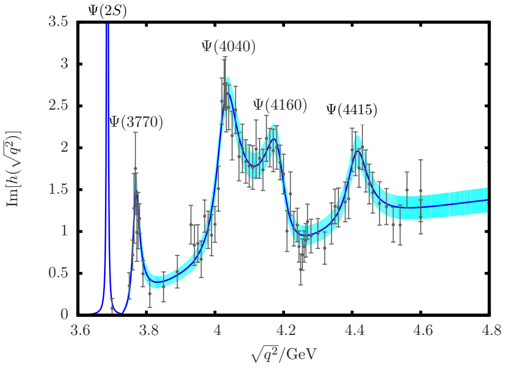
- Few sigma deviations from SM expectations seen for branching fractions and angular observables $3\,\text{GeV}^2 \lesssim q^2 \lesssim 8\,\text{GeV}^2$
- LHCb reported deviations for different processes: $B^0 \to K^0\mu^+\mu^-$, $B^+ \to K^+\mu^+\mu^-$, $B^0 \to K^*\mu^+\mu^-$ $B_s \to \phi\mu^+\mu^-$, $\Lambda_b \to \Lambda\mu^+\mu^-$
  [JHEP 06 (2014) 133][JHEP 11 (2016) 047][JHEP 04 (2017) 142] [JHEP 06 (2015) 115][JHEP 09 (2015) 179]
- Deviations seen by ATLAS, CMS, LHCb, Belle
- Hinting at new physics in Wilson coefficient C_9?
- Near J/Ψ resonance:
  $\to$ Are hadronic uncertainties under control?

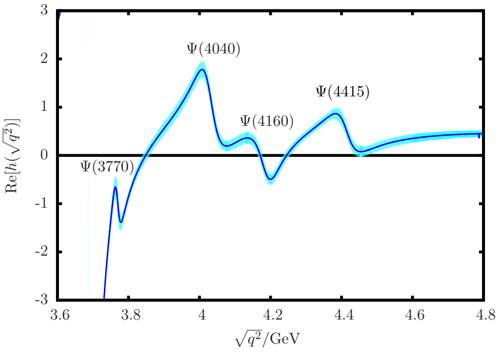


[LHCb JHEP 02 (2016) 104] [Belle PRL118 (2017) 111801]
[ATLAS-CONF-2017-023] [CMS-PAS-BPH-15-008]
[DHMV JHEP 12 (2014) 125, JHEP 10 (2016) 075]
[ASZB JHEP 08 (2016) 098, EPJC 75 (2015) 382]
plot: [Gershon arXiv:1707.05290]

Searches for new physics (loop-level)

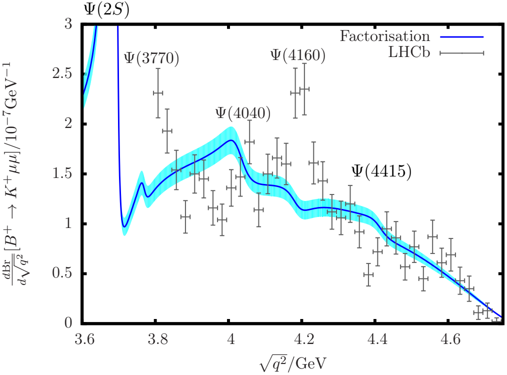
- Hadronic uncertainties: charm resonances [Lyon and Zwicky, arXiv:1406.0566]
  - → SM predictions rely on factorization approximation (FA)
  - → In the FA, charm-resonance contributions equal charm vacuum polarization
  - → Extract charm vacuum polarization via dispersion relation from BESII-data ($e^+e^- \to$ hadrons)





Searches for new physics (loop-level)

- Hadronic uncertainties: charm resonances [Lyon and Zwicky, arXiv:1406.0566]
  → Derived SM prediction for $B \to K\ell\ell$ based on FA and lattice QCD [HPQCD PRD88 (2013) 054509] disagrees with LHCb data

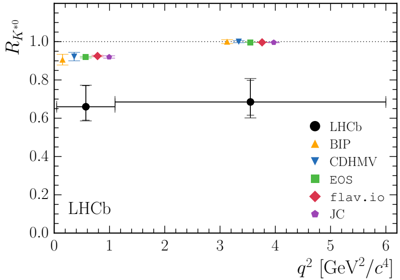


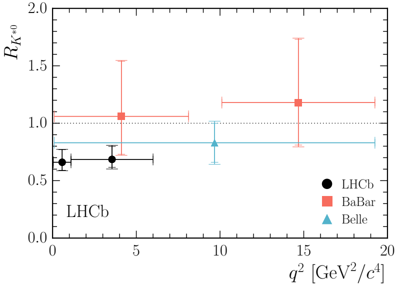
- Explore experimentally e.g. phase difference between short- and long-distance amplitude [LHCb EPJC77 (2017) 161]

Searches for new physics (loop-level)

- Lepton flavor universality violations in $\mathcal{R}_K$ ratios

$$\mathcal{R}_{K^*}^{\mu/e} \equiv \frac{d\Gamma(B^0 \to K^{*0}\mu^+\mu^-)/d_q^2}{d\Gamma(B^0 \to K^{*0}e^+e^-)/d_q^2}$$





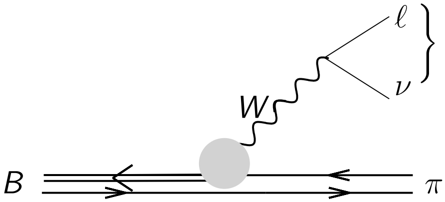
[BIP EPJC76 (2016) 440]
[CDHMV JHEP 10 (2016) 075]
[EOS PRD95 (2017) 035029]
[flav.io JHEP08 (2016) 098]
[JC PRD93 (2016) 014028]

[BaBar PRD86 (2012) 032012]
[Belle PRL103 (2009) 171801]
[LHCb JHEP 08 (2017) 055]

plots: [LHCb JHEP 08 (2017) 055]

calculation

$|V_{ub}|$ from exclusive semileptonic $B \to \pi\ell\nu$ decay



$$q^2 = M_B^2 + M_\pi^2 - 2M_B E_\pi$$

- Conventionally parametrized by (neglecting term $\propto m_\ell^2 f_0^2$)

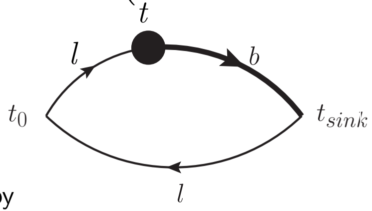
$$\frac{d\Gamma(B\to\pi\ell\nu)}{dq^2} = \frac{G_F^2}{192\pi^3 M_B^3}\left[\left(M_B^2 + M_\pi^2 - q^2\right)^2 - 4M_B^2 M_\pi^2\right]^{3/2} \times |f_+(q^2)|^2 \times |V_{ub}|^2$$

experiment known nonperturbative input CKM

$B \to \pi \ell \nu$ form factors

- Parametrize the hadronic matrix element for the flavor changing vector current in terms of the form factors $f_+(q^2)$ and $f_0(q^2)$

$$\langle \pi(k) | \bar{u}\gamma^\mu b | B(p) \rangle = f_+(q^2)\left(p^\mu + k^\mu - \frac{M_B^2 - M_\pi^2}{q^2} q^\mu\right) + f_0(q^2)\frac{M_B^2 - M_\pi^2}{q^2} q^\mu$$



- Calculate 3-point function by
  - → Inserting a quark source for a "light" propagator at t_0
  - → Allow it to propagate to t_{sink}, turn it into a sequential source for a b quark
  - → Use another "light" quark propagating from t_0 and contract both at t

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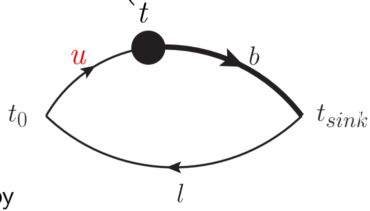

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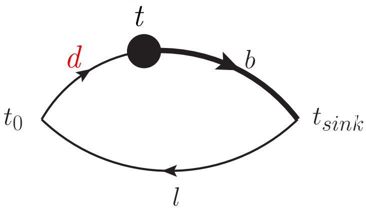
  - → Inserting a quark source for a "light" propagator at t_0
  - → Allow it to propagate to t_{sink}, turn it into a sequential source for a b quark
  - → Use another "light" quark propagating from t_0 and contract both at t
- On the lattice u and d quarks are degenerate (l); physically the daughter quark is a u-quark

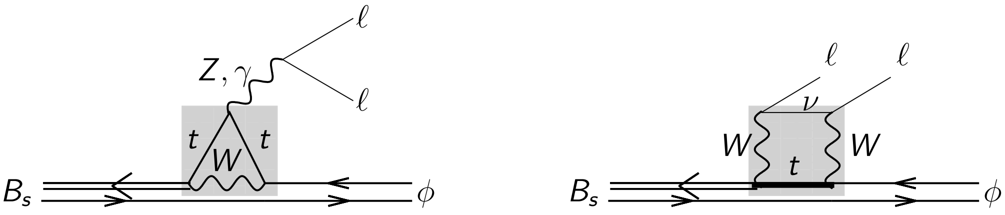
$B \to \pi \ell^+ \ell^-$ form factor

- If the daughter quark is a d-quark, we have a FCNC decay at loop-level
- Dominant contributions at short distance: f_0, f_+, and f_T

$$\langle \pi(k) | i\bar{d}\sigma^{\mu\nu} b(p) | B \rangle = 2\frac{p^\mu k^\nu - p^\nu k^\mu}{M_B + M_\pi} f_T(q^2)$$



$B_s \to \phi \ell^+ \ell^-$ form factors



- Vector final state treated in narrow width approximation
- Effective Hamiltonian

$$\mathcal{H}_{\text{eff}}^{b\to s} = -\frac{4G_F}{\sqrt{2}} V_{ts}^* V_{tb} \sum_i^{10} C_i O_i^{(\prime)}$$

- Leading contributions at short distance

$$O_7^{(\prime)} = \frac{m_b e}{16\pi^2} \bar{s}\sigma^{\mu\nu} P_{R(L)} b F_{\mu\nu} \qquad O_9^{(\prime)} = \frac{e^2}{16\pi^2} \bar{s}\gamma^\mu P_{L(R)} b \bar{\ell}\gamma_\mu \ell \qquad O_{10}^{(\prime)} = \frac{e^2}{16\pi^2} \bar{s}\gamma^\mu P_{L(R)} b \bar{\ell}\gamma_\mu \gamma^5 \ell$$

Seven form factors

$$\begin{aligned}
\langle\phi(k,\lambda)|\bar{s}\gamma^\mu b|B_s(p)\rangle =& f_V(q^2)\frac{2i\epsilon^{\mu\nu\rho\sigma}\varepsilon^*_\nu k_\rho p_\sigma}{M_{B_s}+M_\phi} \\
\langle\phi(k,\lambda)|\bar{s}\gamma^\mu\gamma_5 b|B_s(p)\rangle =& f_{A_0}(q^2)\frac{2M_\phi\varepsilon^*\cdot q}{q^2}q^\mu \\
&+ f_{A_1}(q^2)(M_{B_s}+M_\phi)\left[\varepsilon^{*\mu}-\frac{\varepsilon^*\cdot q}{q^2}q^\mu\right] \\
&- f_{A_2}(q^2)\frac{\varepsilon^*\cdot q}{M_{B_s}+M_\phi}\left[k^\mu+p^\mu-\frac{M_{B_s}^2-M_\phi^2}{q^2}q^\mu\right] \\
q_\nu\langle\phi(k,\lambda)|\bar{s}\sigma^{\nu\mu}b|B_s(p)\rangle =& 2f_{T_1}(q^2)\epsilon^{\mu\rho\tau\sigma}\varepsilon^*_\rho k_\tau p_\sigma \ , \\
q_\nu\langle\phi(k,\lambda)|\bar{s}\sigma^{\nu\mu}\gamma^5 b|B_s(p)\rangle =& if_{T_2}(q^2)\left[\varepsilon^{*\mu}(M_{B_s}^2-M_\phi^2)-(\varepsilon^*\cdot q)(p+k)^\mu\right] \\
&+ if_{T_3}(q^2)(\varepsilon^*\cdot q)\left[q^\mu-\frac{q^2}{M_{B_s}^2-M_\phi^2}(p+k)^\mu\right]
\end{aligned}$$

Other (recent) lattice calculations

- Further details on charged current semileptonic B decays
- HPQCD
  - → Form factors f_0, f_+, and f_T for $B \to K\ell^+\ell^-$ [PRL111 (2013) 162002, Erratum: PRL112 (2014) 149902] [PRD88 (2013) 054509, Erratum: PRD88 (2013) 079901]
- Fermilab/MILC
  - → Tensor form factor f_T for $B \to \pi\ell^+\ell^-$ [PRL115 (2015) 152002]
  - → Form factors f_0, f_+, and f_T for $B \to K\ell^+\ell^-$ [PRD93 (2016) 025026]
- Horgan, Liu, Meinel, and Wingate
  - → 7 form factors for $B \to K^*\ell^+\ell^-$ and $B_s \to \phi\ell^+\ell^-$ [PRD89 (2014) 090501][PoS Lattice2014 (2015) 372]
  - → Angular analysis [PRL112 (2014) 212003]

Challenges

- b-quark (4.18 GeV) is $\sim$ 1000 times heavy than d-quark (4.7 MeV) and larger than a^{-1}

↙	↘
simulate b-quark with effective action	extrapolate to physical b-quark
↓	↓
requires renormalization of mixed action	allows for full nonperturbative renormalization
↓	↓
Fermilab-action/RHQ, NRQCD, HQET	ETMC ratio method, heavy HISQ, heavy DWF

- Vector final states are unstable in QCD → narrow width approximation
  → Chiral perturbation theory cannot guide extrapolations of data at unphysically heavy pions
  → Resonances in range of kinematic extrapolation (hadronic uncertainties)
  → Pioneering work on $B \to K^* \to K\pi$ (cf. Leskovec, Meinel); new ideas [Hansen, Meyer, Robaina arXiv:1704.08993]
- Simulations with physical light and bottom quarks troubled by poor signal-to-noise ratio
  → So far form factors only at q^2_{max} calculated [HPQCD PRD93 (2016) 034502]
- Long distance contributions

RBC-UKQCD's project

Target quantities

- Decay constants f_B and f_{B_s}
- $B^0 - \overline{B}^0$ mixing matrix elements
- Semileptonic form factors with charged and neutral flavor changing currents

  $B \to \pi \ell \nu$, $B_s \to K \ell \nu$

  $B \to D^{(*)} \ell \nu$, $B_s \to D_s^{(*)} \ell \nu$, ...

  $B \to K^{(*)} \ell^+ \ell^-$, $B_s \to \phi \ell^+ \ell^-$, ...

$\rightarrow$ Ratios $R(D^{(*)})$, $R(K^{(*)})$, ...

Target quantities

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  $B \to \pi\ell\nu$, $B_s \to K\ell\nu$

  $B \to D^{(*)}\ell\nu$, $B_s \to D_s^{(*)}\ell\nu$, ...

  $B \to K^{(*)}\ell^+\ell^-$, $B_s \to \phi\ell^+\ell^-$, ...

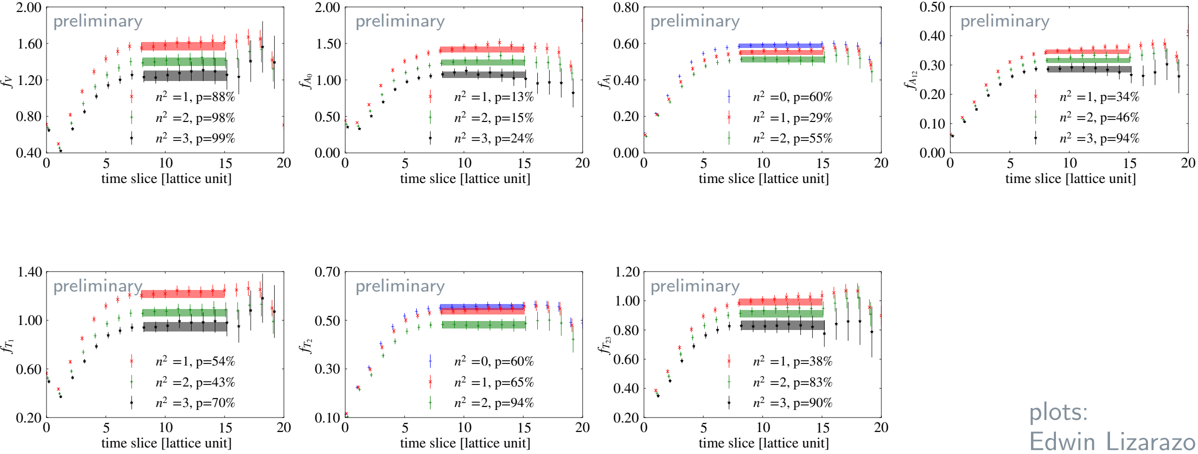
→ Ratios $R(D^{(*)})$, $R(K^{(*)})$, ...

Set-up

- RBC-UKQCD's 2+1 flavor domain-wall fermion and Iwasaki gauge field ensembles

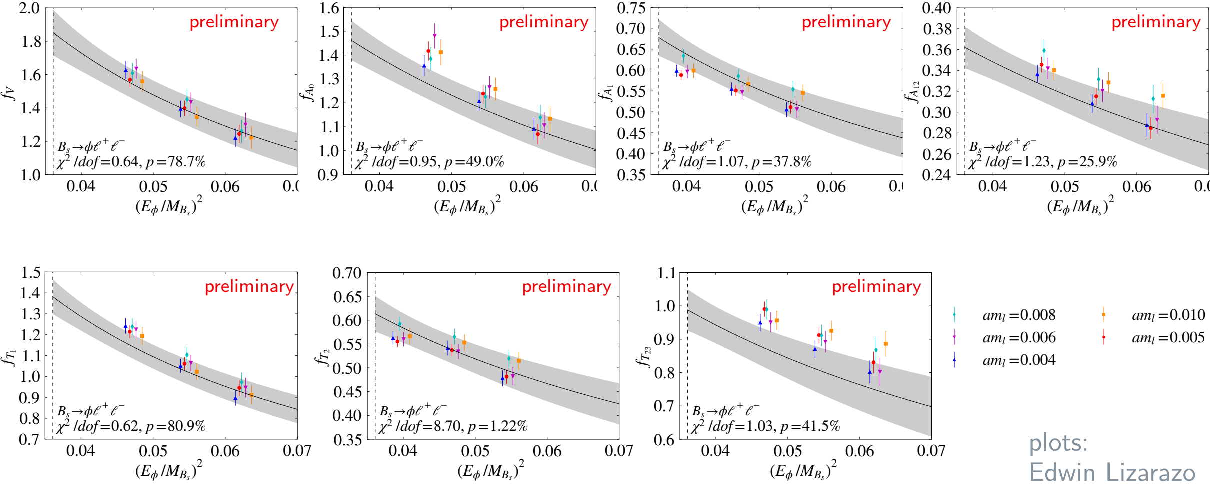
  → 3 lattice spacings incl. one with phys. pions
- Unitary DWF for light and strange quarks
- Heavy Möbius DWF for charm quarks
- RHQ for bottom quarks
- Operators $O(a)$-improved at 1-loop/treelevel
- Mostly nonperturbative renormalization

$B_s \to \phi\ell\ell$: Seven form factors ($a^{-1} = 1.784$ GeV, $am_l^{sea} = 0.005$, $am_s = 0.03224$)



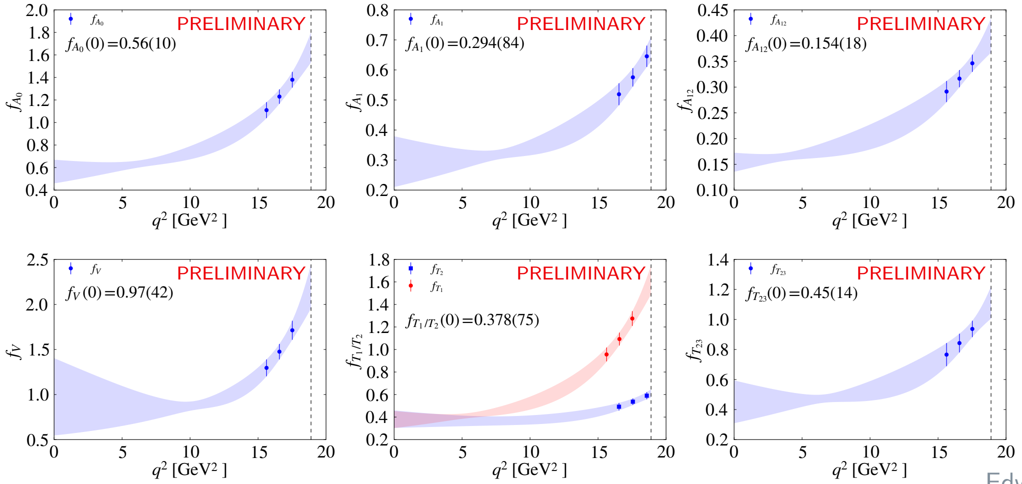
plots:
Edwin Lizarazo

$B_s \to \phi \ell\ell$: Seven form factors vs. q^2



plots: Edwin Lizarazo

$B_s \to \phi\ell\ell$: First attempt to use z-parametrization



plots: Edwin Lizarazo

→ Ignoring any implications from resonances!

conclusion

Conclusion

- Rare B decays allow many tests of the Standard Model and exhibit tantalizing signals
- Yet more data and improved theoretical predictions are needed
- Further lattice calculations are on the way but analysis takes time
- However, general code allows to share the computational costs for many processes of interest
  - → $B \to \pi\ell\nu$, $B \to \pi\ell\ell$
  - → $B \to K^*\ell\ell$
  - → $B \to D^{(*)}\ell\nu$
  - → $B_s \to K^{(*)}\ell\nu$, $B_s \to K^{(*)}\ell\ell$
  - → $B_s \to D_s^{(*)}\ell\nu$
  - → ...

Resources and Acknowledgments

USQCD: Ds, Bc, and pi0 cluster (Fermilab), qcd12s cluster (Jlab)
RBC qcdcl (RIKEN) and cuth (Columbia U)
UK: ARCHER, cirrus (EPCC) and DiRAC (UKQCD)

appendix

Set-up

- RBC-UKQCD's 2+1 flavor domain-wall fermion and Iwasaki gauge action ensembles
  - → Three lattice spacings $a \sim 0.11$ fm, 0.08 fm, 0.07 fm; one ensemble with physical pions
    [PRD 78 (2008) 114509][PRD 83 (2011) 074508][PRD 93 (2016) 074505][arXiv:1701.02644]

- Unitary and partially quenched domain-wall up/down quarks
  [Kaplan PLB 288 (1992) 342], [Shamir NPB 406 (1993) 90]
- Domain-wall strange quarks at/near the physical value
- Charm: Möbius domain-wall fermions optimized for heavy quarks [Boyle et al. JHEP 1604 (2016) 037]
  - → Simulate 3 or 2 charm-like masses then extrapolate/interpolate

- Effective relativistic heavy quark (RHQ) action for bottom quarks
  [Christ et al. PRD 76 (2007) 074505], [Lin and Christ PRD 76 (2007) 074506]
  - → Builds upon Fermilab approach [El-Khadra et al. PRD 55 (1997) 3933]
  - → Allows to tune the three parameters (m_0a, c_P, ζ) nonperturbatively [PRD 86 (2012) 116003]
  - → Smooth continuum limit; heavy quark treated to all orders in $(m_ba)^n$

2+1 Flavor Domain-Wall Iwasaki ensembles

L	a^{-1}(GeV)	am_l	am_s	M_π(MeV)	# configs.	#sources	
24	1.784	0.005	0.040	338	1636	1	[PRD 78 (2008) 114509]
24	1.784	0.010	0.040	434	1419	1	[PRD 78 (2008) 114509]
32	2.383	0.004	0.030	301	628	2	[PRD 83 (2011) 074508]
32	2.383	0.006	0.030	362	889	2	[PRD 83 (2011) 074508]
32	2.383	0.008	0.030	411	544	2	[PRD 83 (2011) 074508]
48	1.730	0.00078	0.0362	139	40	$81/1^\star$	[PRD 93 (2016) 074505]
64	2.359	0.000678	0.02661	139	—	—	[PRD 93 (2016) 074505]
48	2.774	0.002144	0.02144	234	70	24	[arXiv:1701.02644]

* All mode averaging: 81 “sloppy” and 1 “exact” solve [Blum et al. PRD 88 (2012) 094503]

- Lattice spacing determined from combined analysis [Blum et al. PRD 93 (2016) 074505]
- a: ~ 0.11 fm, ~ 0.08 fm, ~ 0.07 fm