Charm Mass Determination from QCD Sum Rules at $O(\alpha_s^3)$

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Outline

- **General remarks on heavy quark masses**
  - Different schemes. Renormalons.
  - Motivations for a precisse determination.
  - Recent results.
- **Treatment of experimental data**
  - How to combine data from different experiments?
  - How to treat errors and correlations?
  - Results.
- **Theoretical analysis**
  - Analytic properties. OPE expansion. Four loop results.
  - Estimate of (theoretical) perturbative errors.
- **Results for charm mass**

INTRODUCTION

Remarks on heavy quark masses

- Confinement ⟶ m_q not physical observable
- Parameter in QCD Lagrangian ⟶ formal definition (as strong coupling)

$$L_{\mathrm{QCD}} = \frac{1}{4} G^{A}_{\mu\nu} G^{\mu\nu\, A} + \sum_f \bar{q}_f \left(\not{D} - m_f \right) q_f$$

- Renormalization and scheme dependent object ⟶

$\delta m_q < \Lambda_{\mathrm{QCD}}$

possible

In general running mass $m(\mu)$ (RGE evolution)

$$m_q^{\text{scheme A}}(\mu) = m_q^{\text{scheme B}}(\mu)\left(1 + f_1\left[\log\left(\frac{m}{\mu}\right)\right]\alpha_s(\mu) + f_2\left[\log\left(\frac{m}{\mu}\right)\right]\alpha_s^2(\mu) + \cdots\right)$$

Only interested in short-distance schemes, which do not suffer from the $O(\Lambda_{\mathrm{QCD}})$ renormalon problem inherent to the pole mass scheme.

$\overline{\mathrm{MS}}$ scheme

- Short distance scheme.
- Standard mass for comparison: $\bar{m}_q(\bar{m}_q)$.
- And free of renormalon ambiguities.

Why high precision?

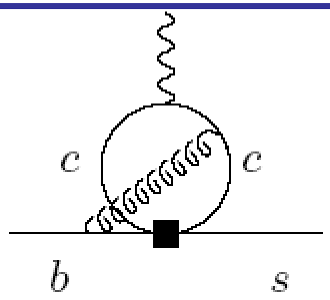
Strong dependence in flavor processes

Constrains new physics

$B \to X_s \gamma$

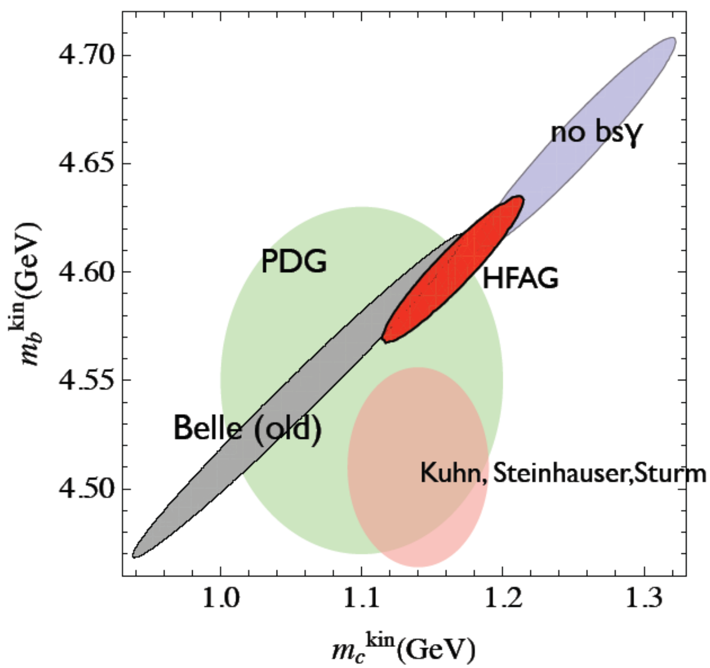
Strong charm mass (scheeme) dependence in NLO matrix elements

Misiak & Gambino



$K^+ \to \pi^+ \nu \bar{\nu}$

NNLO QCD computations for charm contributions



Taken from P. Gambino CKM'08

Determinations of m_c

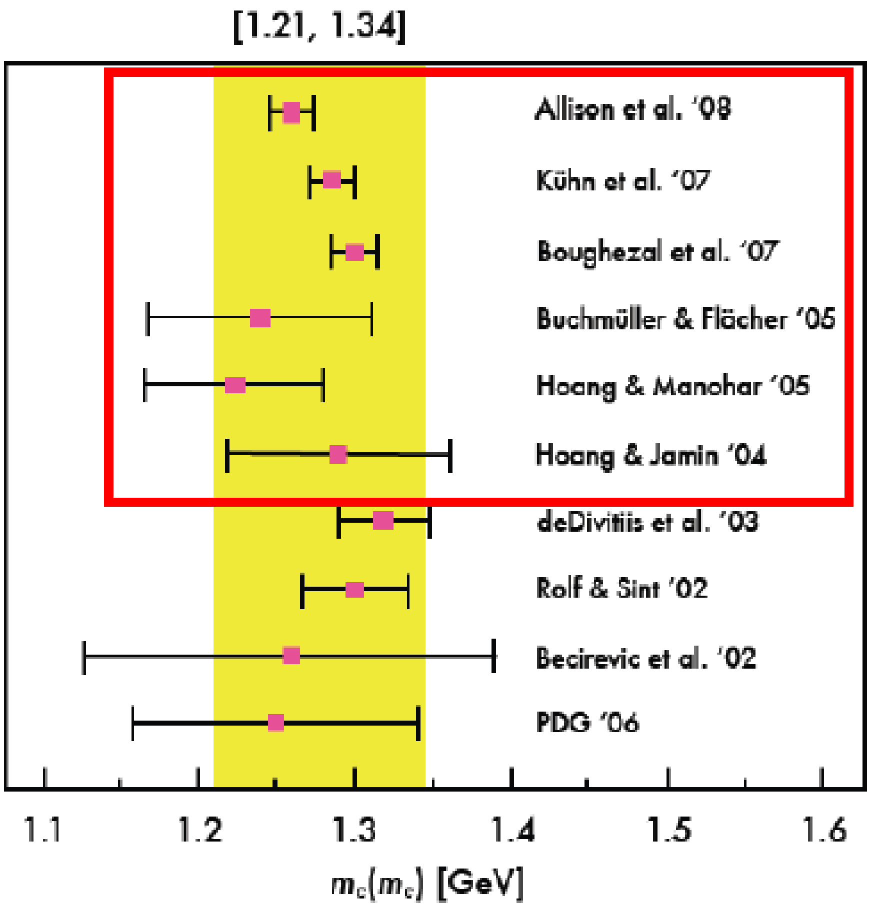
Spectral moments of inclusive B decays (nonrelativistic)

Charmominum sum rules (relativistic)

Lattice

Taken from A. Hoang

Flavor institute CERN 2008



$m_c(m_c)$ [GeV]	method
1.266 ± 0.014	lattice, unquenched, staggered
1.286 ± 0.013	low-momentum sum rules, N^3LO
1.295 ± 0.015	low-momentum sum rules, N^3LO
1.24 ± 0.07	fit to B-decay distribution, $\alpha_s^2\beta_0$
$1.224 \pm 0.017 \pm 0.054$	fit to B-decay data, $\alpha_s^2\beta_0$
1.29 ± 0.07	NNLO moments
1.319 ± 0.028	lattice, quenched
1.301 ± 0.034	lattice, quenched
$1.26 \pm 0.04 \pm 0.12$	lattice, quenched
1.25 ± 0.09	PDG 2006

Relativistic sum rules

Total hadronic cross section

$$R(s) = \frac{\sigma\left(e^+e^- \to \text{hadrons}\right)}{\sigma\left(e^+e^- \to \mu^+\mu^-\right)}$$

Moments of the cross section

$$M_n = \int_{4m^2}^{\infty} \frac{\mathrm{ds}}{s^{n+1}} R(s) \overset{z=\frac{s}{4m^2}}{=} \frac{1}{\left(4m^2\right)^n} \int_1^{\infty} \frac{\mathrm{dz}}{z^{n+1}} R(z)$$

Vacuum polarization function

$$\left(g_{\mu\nu} - q_\mu q_\nu\right)\Pi(q^2) = -i\int \mathrm{d}x\, e^{ix\cdot q} \left\langle 0 \middle| \mathrm{T}\left\{J_\mu(x) J_\nu(x)\right\} \middle| 0 \right\rangle$$

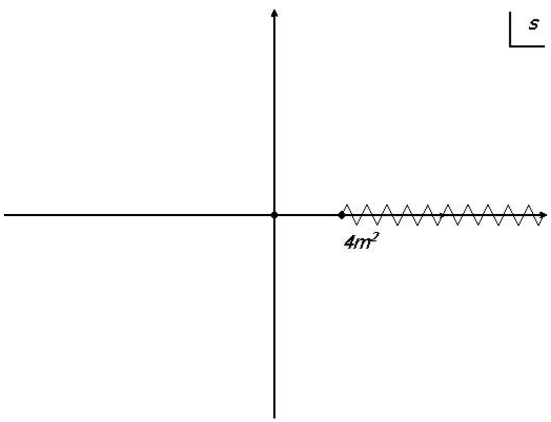
Vector current (electromagnetic)

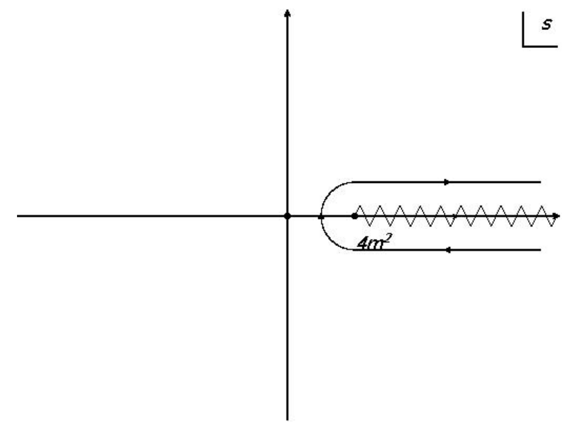
$$J_\mu(x) = \bar{q}(x)\gamma_\mu q(x)$$

electric charge

$$R(s) = 12\pi \boxed{Q^2}\, \mathrm{Im}\,\Pi(s + i0^+)$$

$$\Pi(q^2) - \Pi(0) = \frac{q^2}{12\pi^2 Q^2} \int_{4m^2}^{\infty} \mathrm{d}s \frac{R(s)}{s(q^2 - s)}$$





$$\Pi(q^2 \approx 0, m) = \frac{1}{12\pi^2 Q^2} \sum_{n=0}^{\infty} M_n\, q^{2n}$$

$$M_n = 6\pi i Q^2 \oint \mathrm{d}s \frac{\Pi(s)}{s^{n+1}}$$

Relativistic sum rules

Effective energy range: $E_{eff} = \frac{m_c}{n}$ (assimptotically correct for large n)

↓

$$\frac{m_c}{n} \gg \Lambda_{QCD}$$

- Since we want to apply perturbation theory for Wilson coefficients.
- Otherwise the OPE converges badly.

n=1 is the cleanest moment, and we will focuss on it for the analyses presented in this seminar.

(n = 2 is also fine)

Determination of m_c from sum rules

Fixed order analysis

(correlated variation)

$$\mu_\alpha = \mu_m$$

Kühn et al ('08)[3]	$\overline{m}_c(\overline{m}_c) = 1.286 \pm 0.009_{\exp} \pm 0.009_\alpha \pm 0.002_\mu$
Boughezal et al ('08) [4]	$1.295 \pm 0.012_{\exp} \pm 0.009_\alpha \pm 0.003_\mu$
Maier et al (08) [5]	$1.277 \pm 0.006_{\exp} \pm 0.014_\alpha \pm 0.005_\mu$

Only for n = 1 **[3,4]**, 2 **[5]** 3-loops in pert. theory. Updated experimental data.

Determination of m_c from sum rules

Fixed order analysis

(correlated variation)

$$\mu_\alpha = \mu_m$$

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Only for n = 1 **[3,4]**, 2 **[5]** 3-loops in pert. theory. Updated experimental data.

Tiny errors! (underestimated ?)

Need for more general analysis

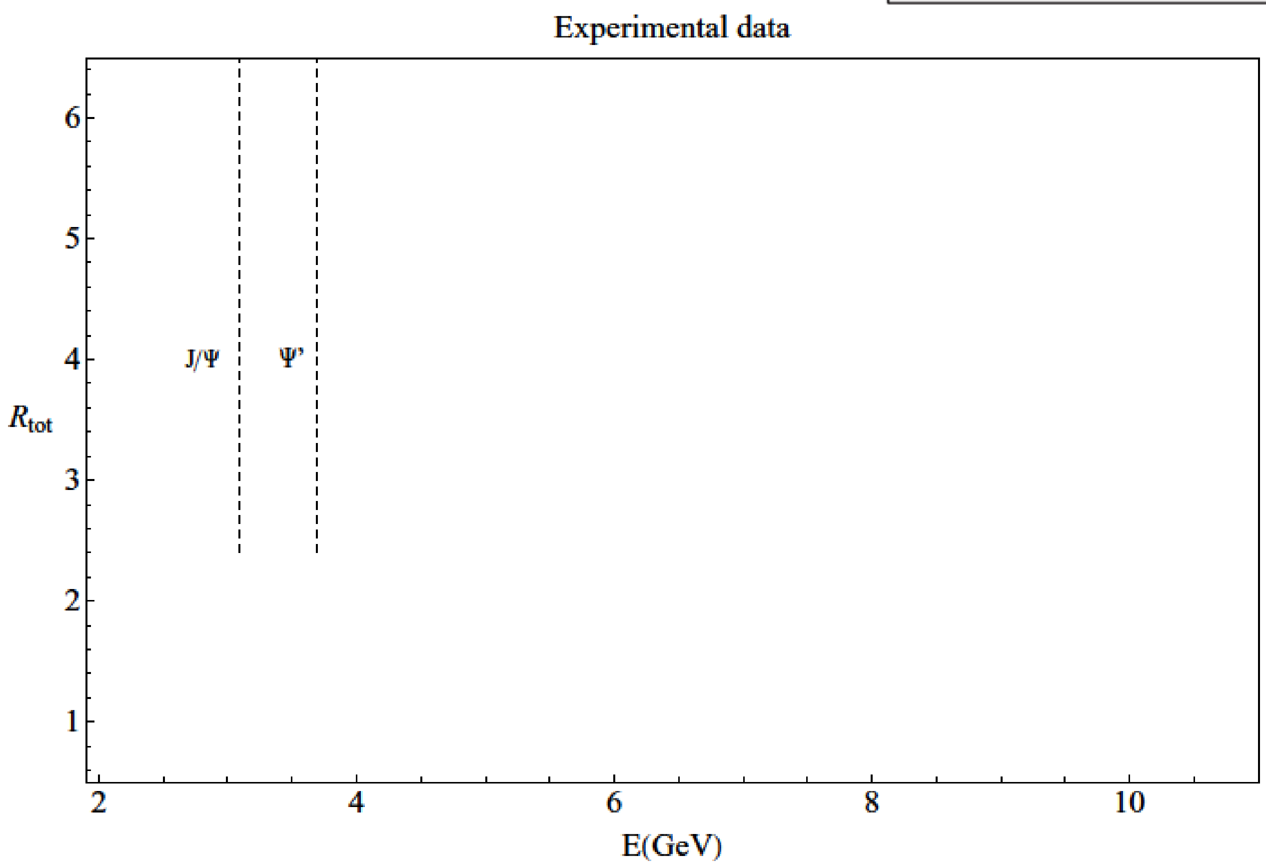
Similar for bottom mass determinations

Experimental data

Experimental data: charm

Narrow resonances

	J/Ψ	$\psi(2S)$
M (GeV)	3.096916(11)	3.686093(34)
Γ_{ee} (keV)	5.55(14)	2.48(6)
$(\alpha/\alpha(M))^2$	0.957785	0.95554

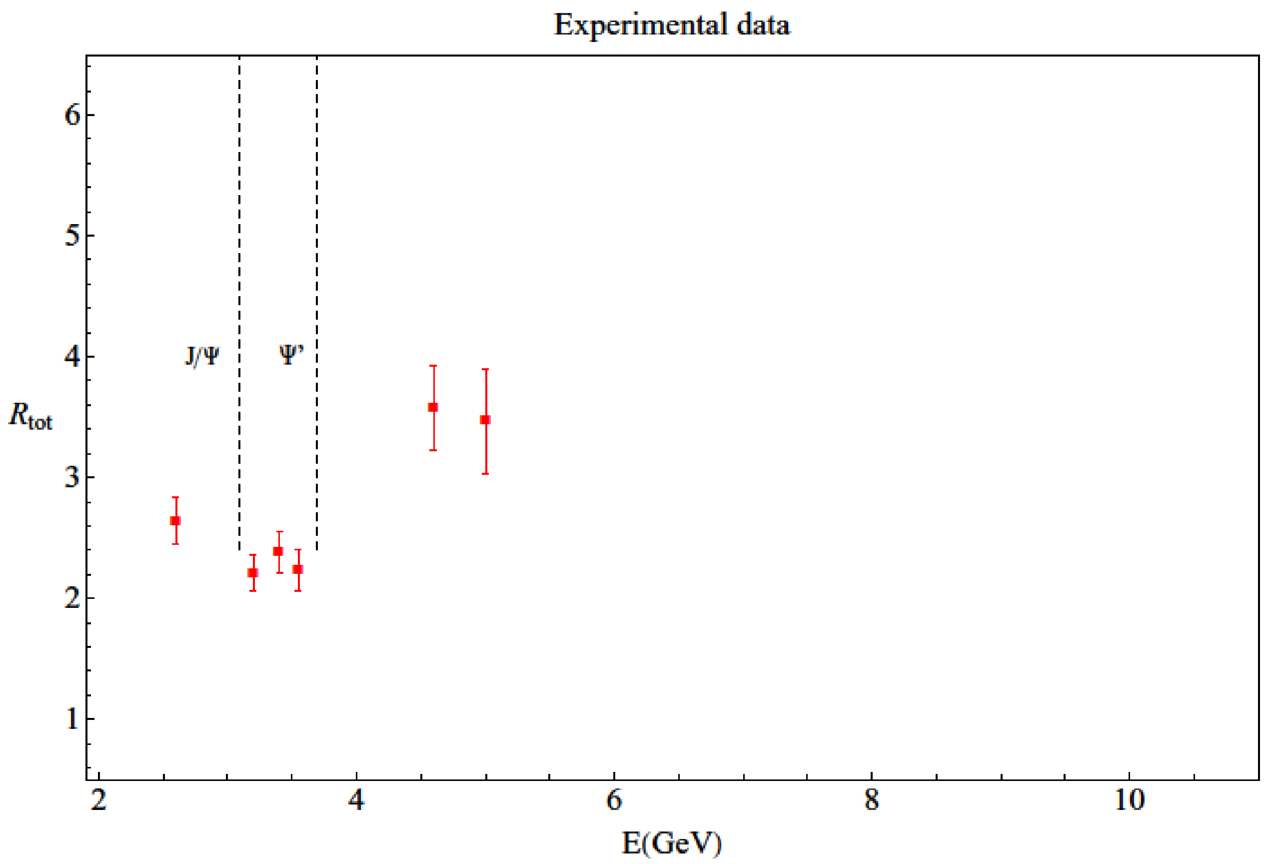


$$M_n^{\text{res}} = \frac{9\,\pi\,\Gamma_{ee}}{\alpha(M)^2 M^{2n+1}}$$

Narrow-width approximation

Experimental data: charm

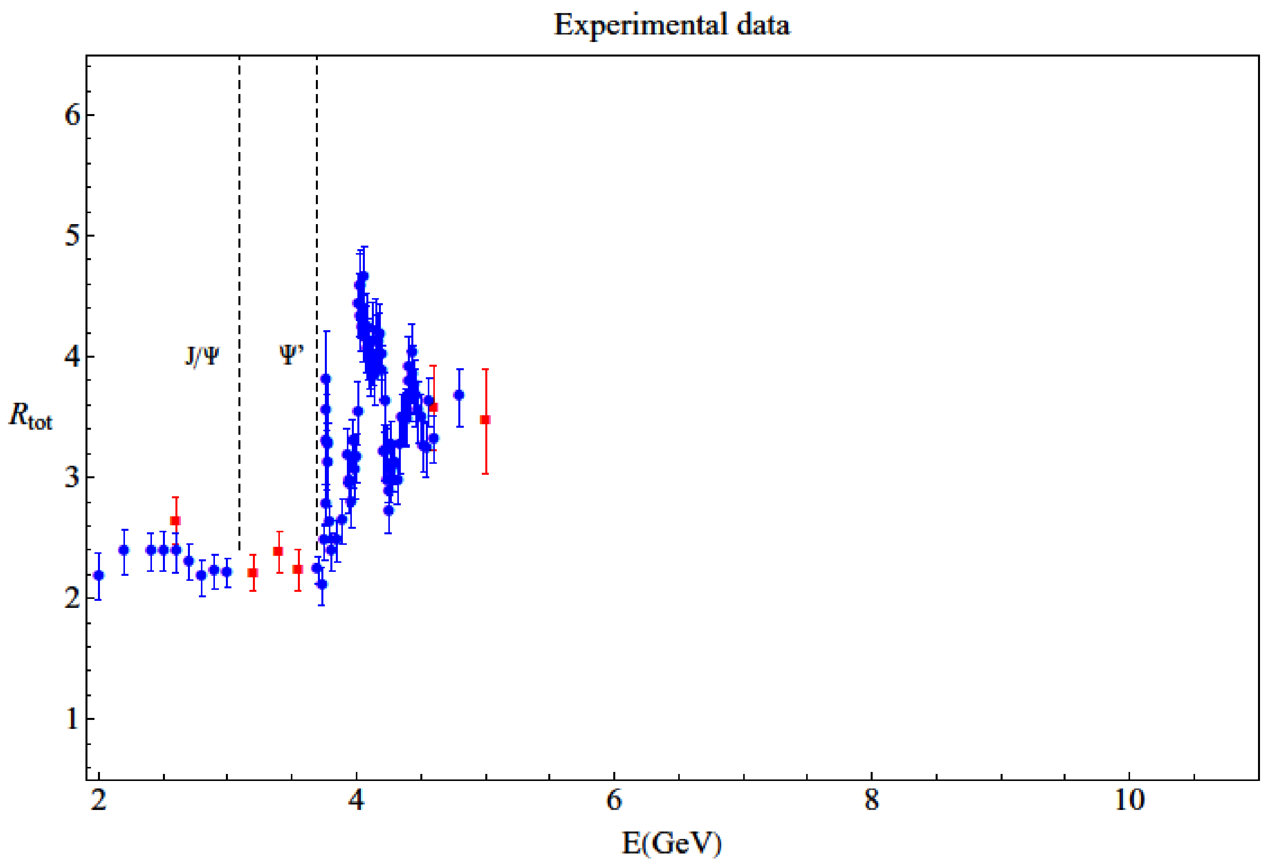
Sub-threshold and threshold   BES 1999 *



* Means that there is no information on the splitting of systematic errors in correlated and uncorrelated

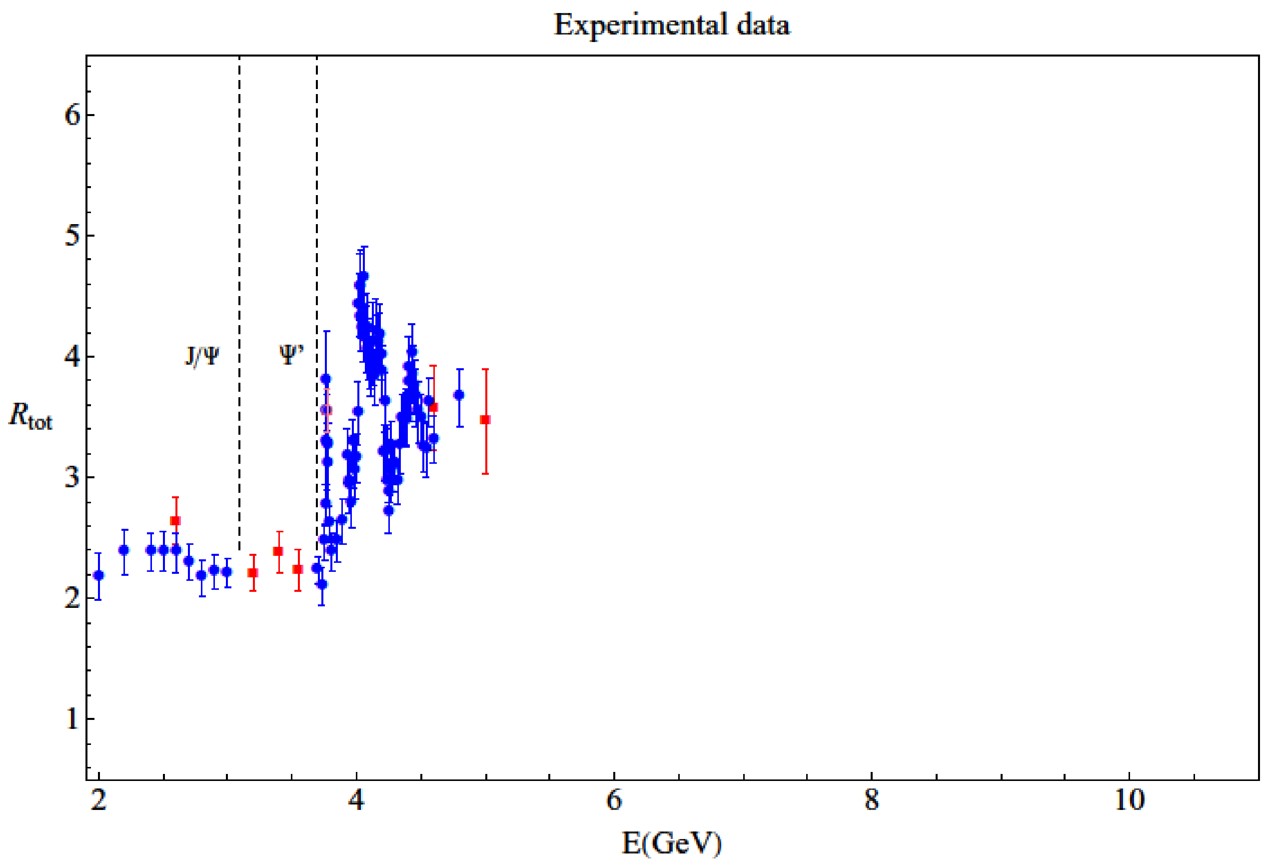
Experimental data: charm

Sub-threshold and threshold   BES 2001



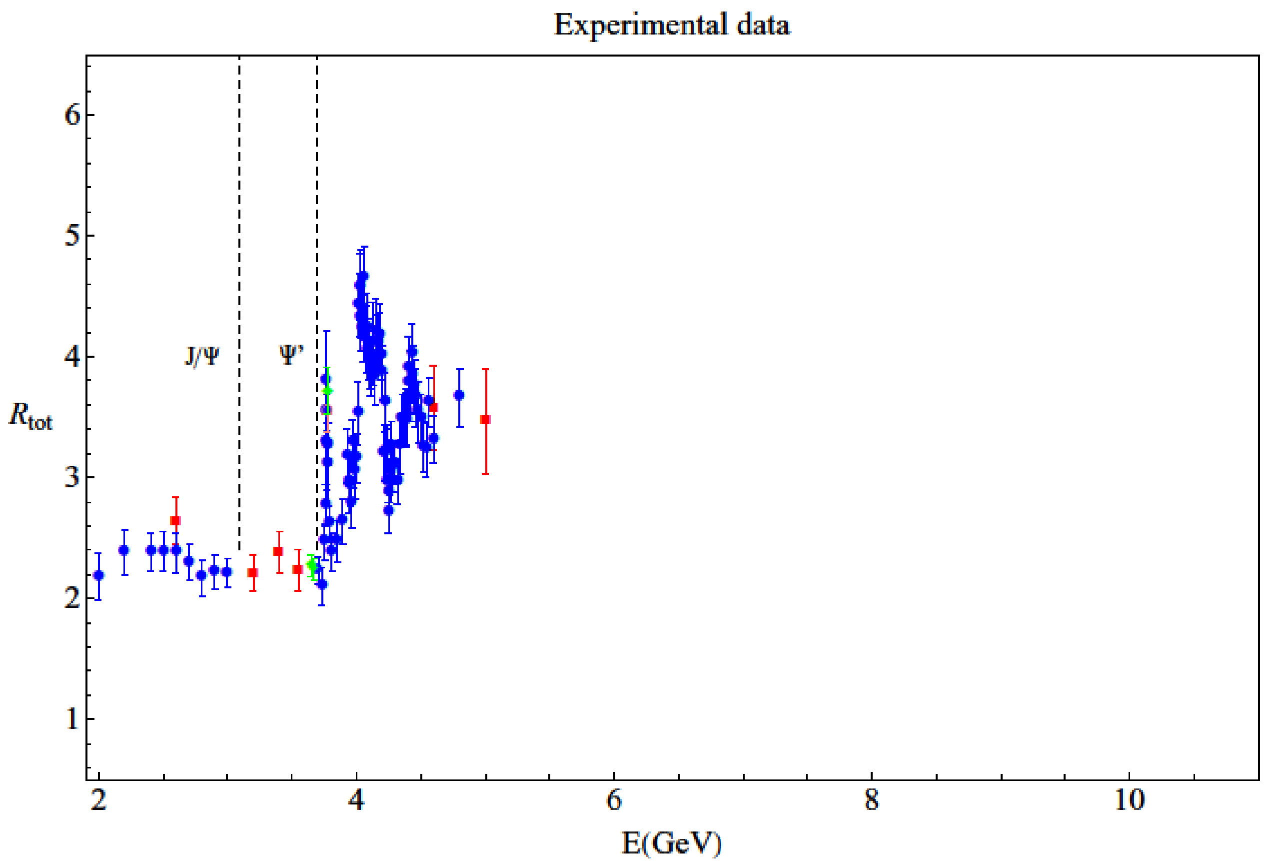
Experimental data: charm

Sub-threshold and threshold   BES 2004



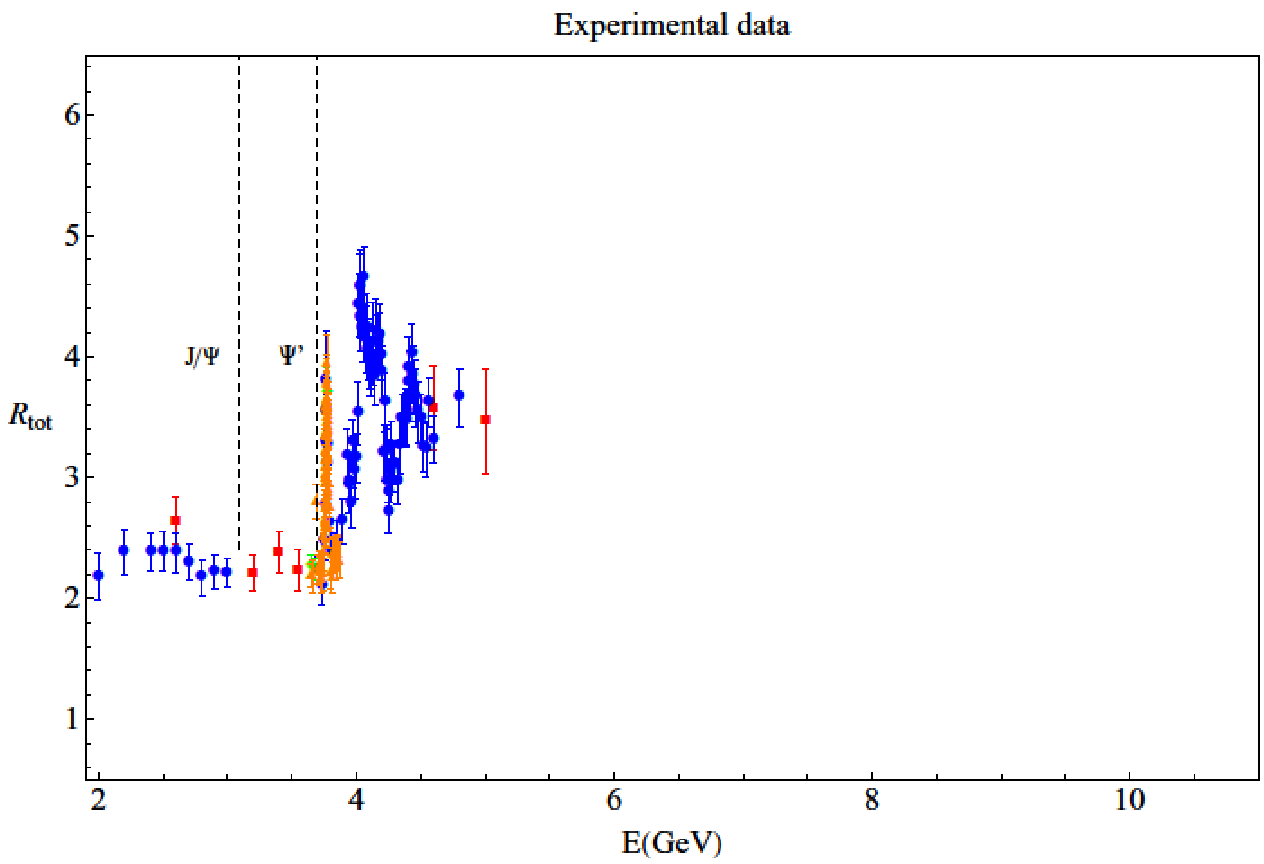
Experimental data: charm

Sub-threshold and threshold BES 2006 (I)



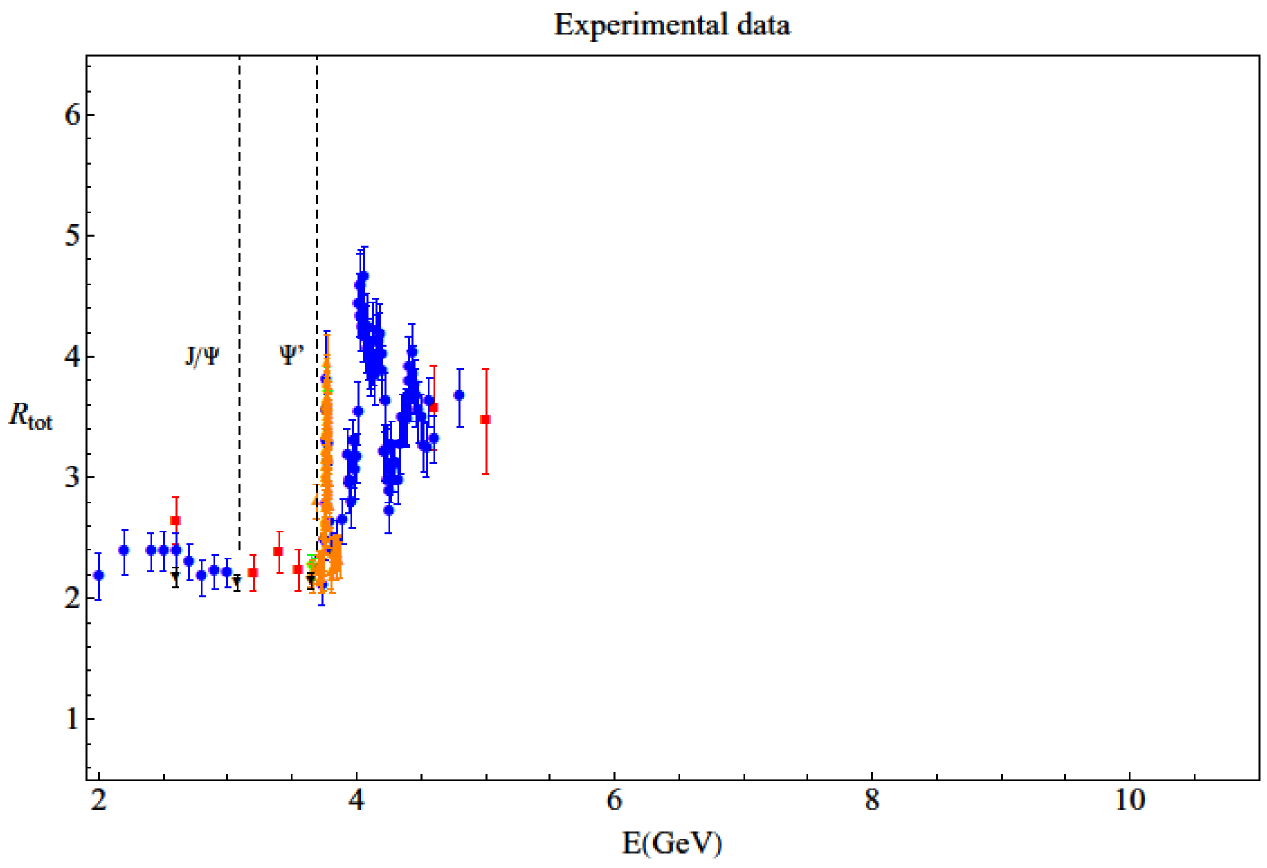
Experimental data: charm

Sub-threshold and threshold BES 2006 (II)



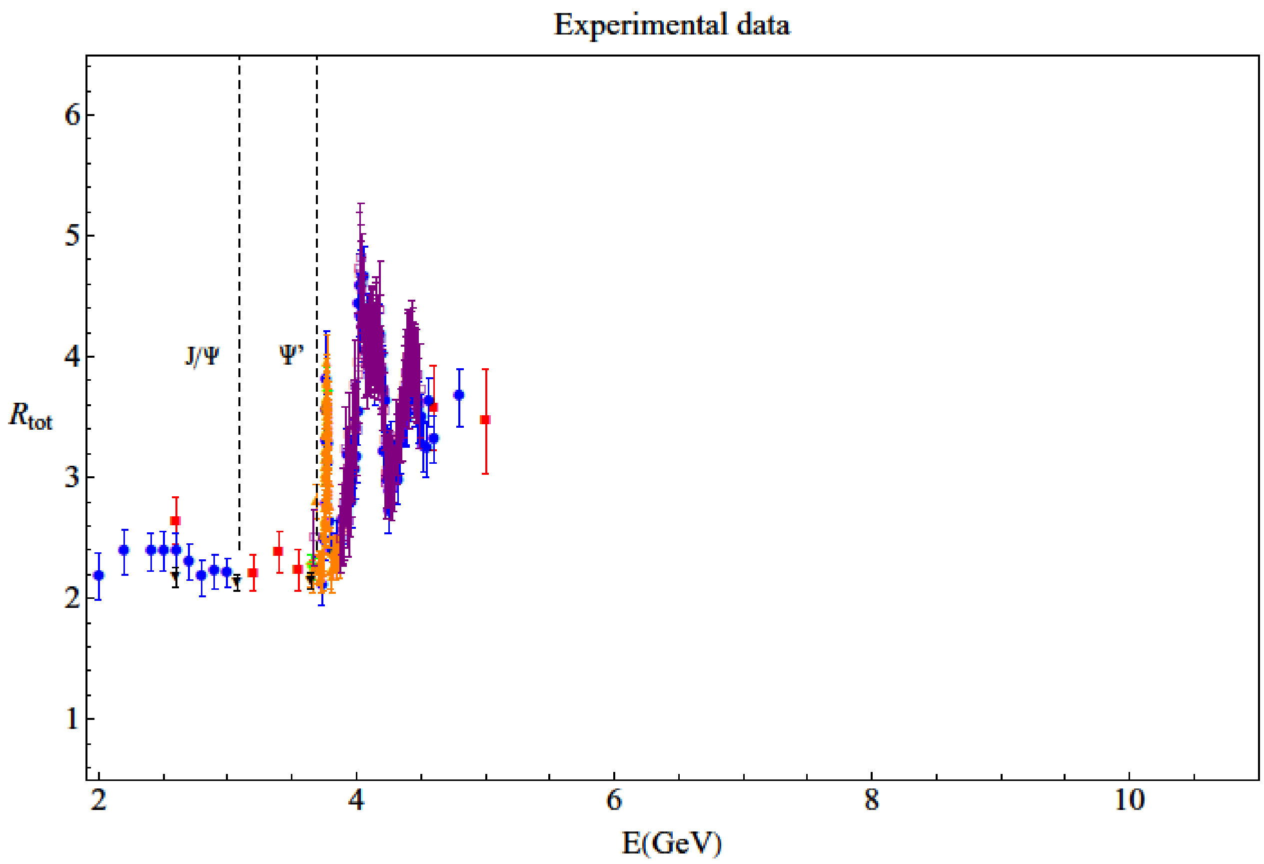
Experimental data: charm

Sub-threshold and threshold BES 2009 *



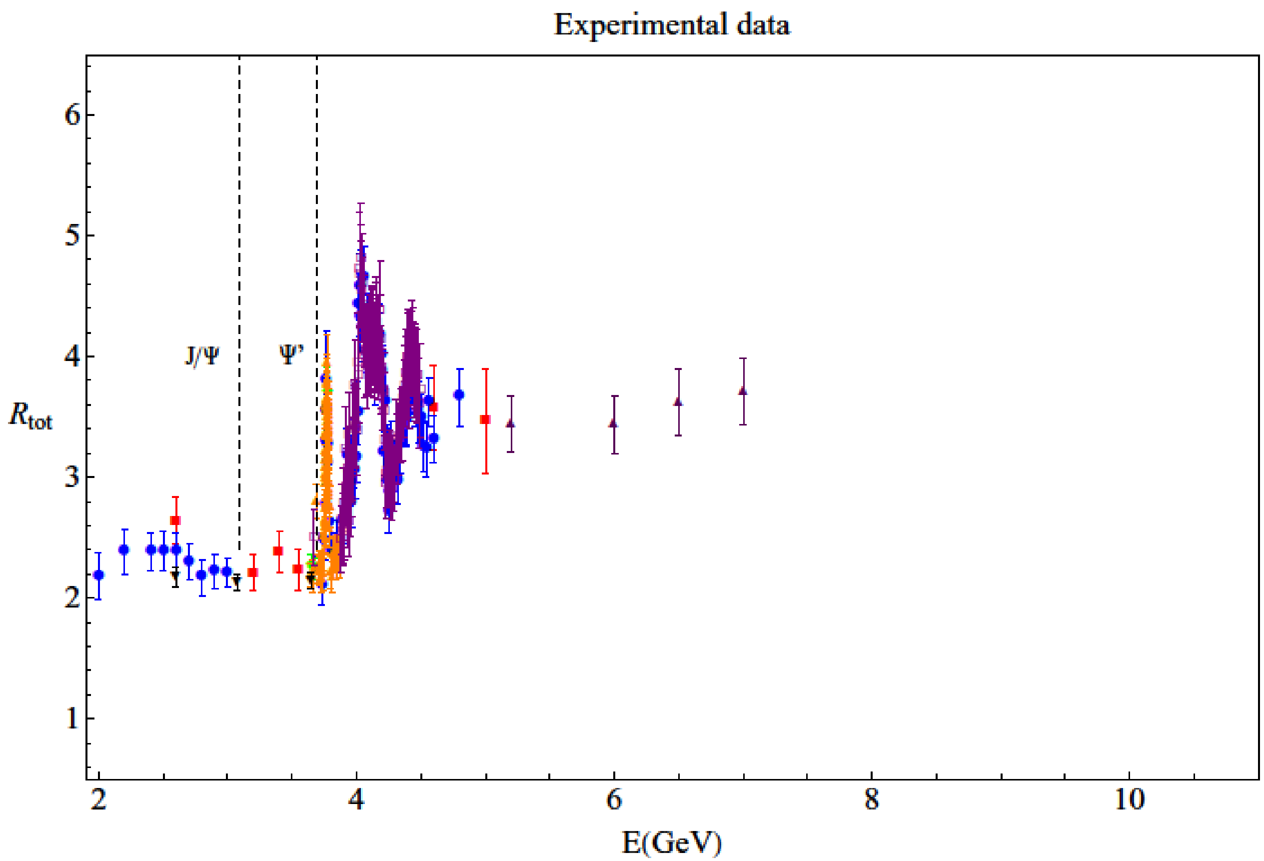
Experimental data: charm

Sub-threshold and threshold **Crystal Ball 1986**



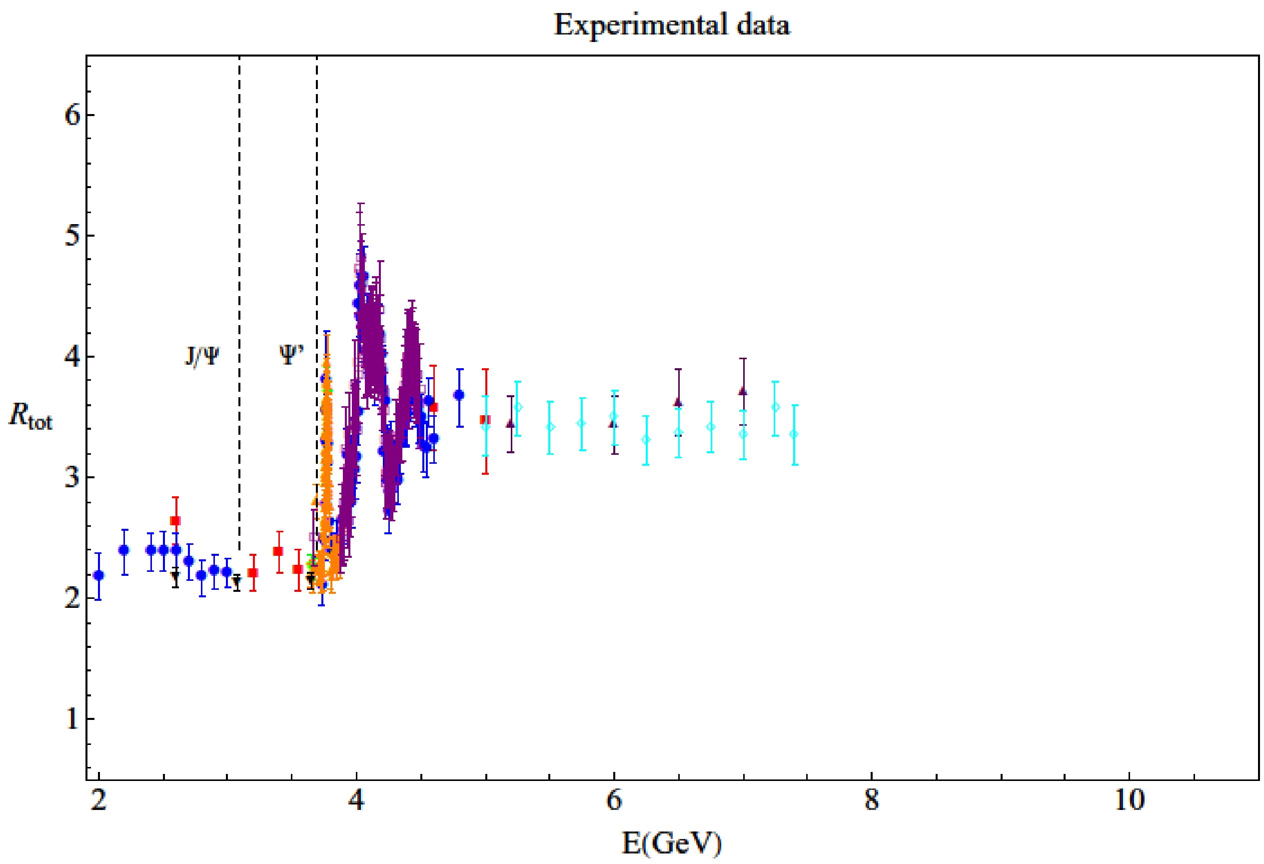
Experimental data: charm

Gap region **Crystal Ball 1990 (I)**



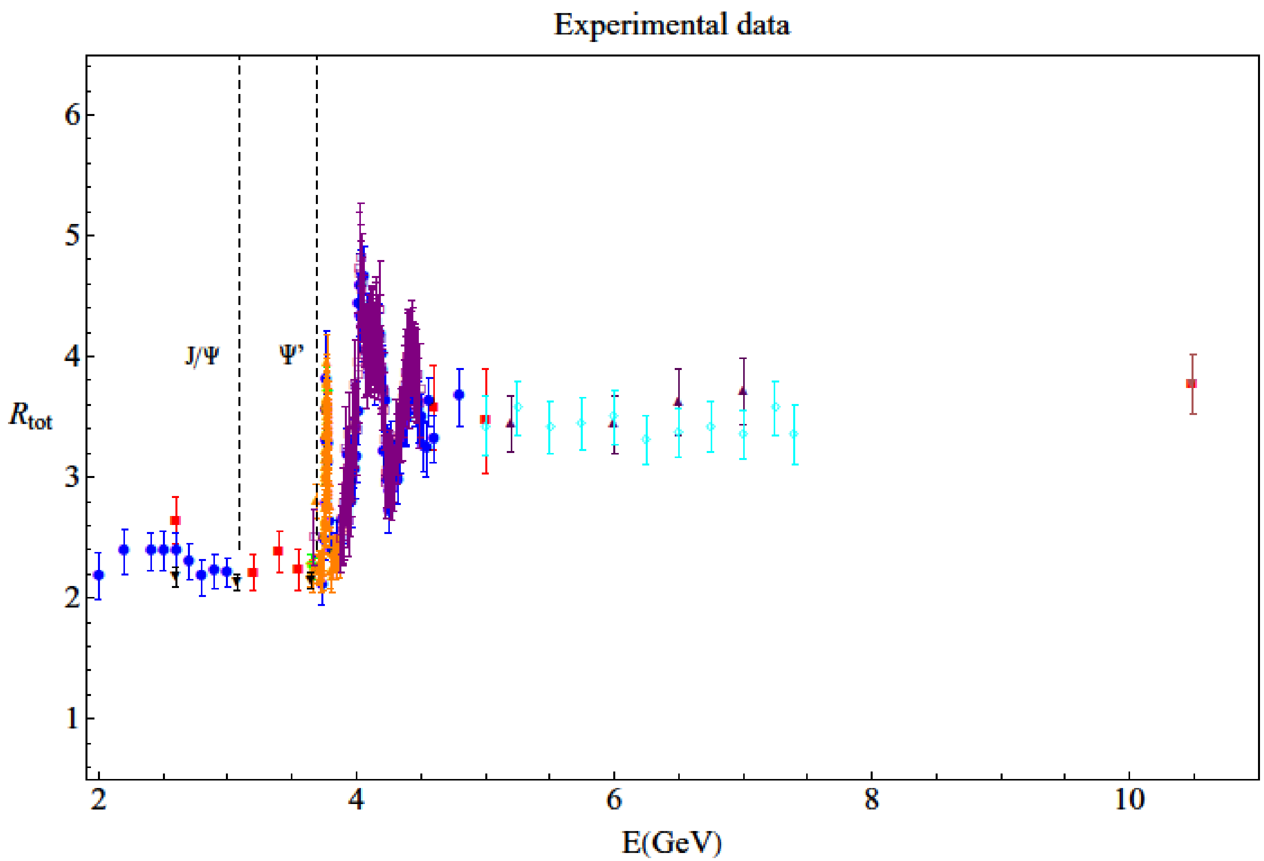
Experimental data: charm

Gap region **Crystal Ball 1990 (II)**



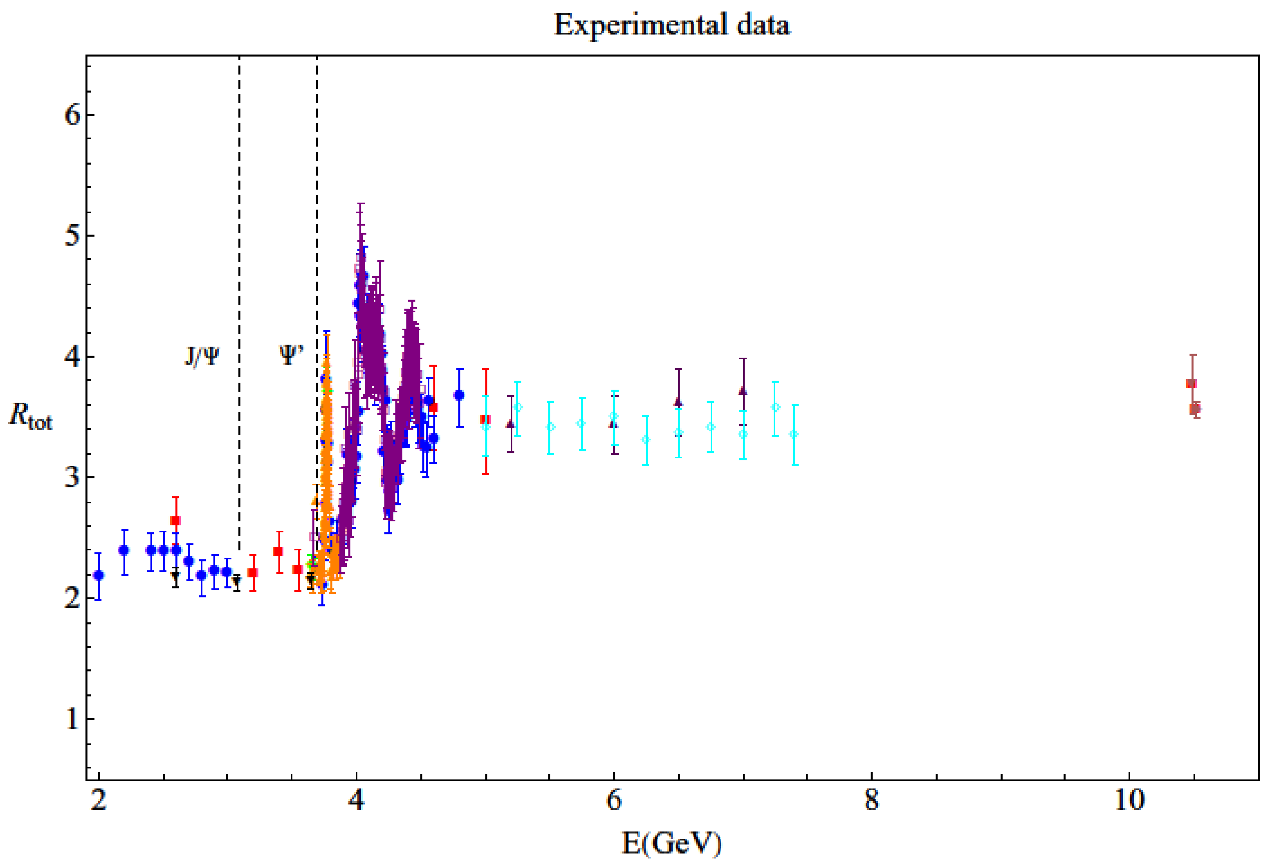
Experimental data: charm

High energy region **CLEO 1979**



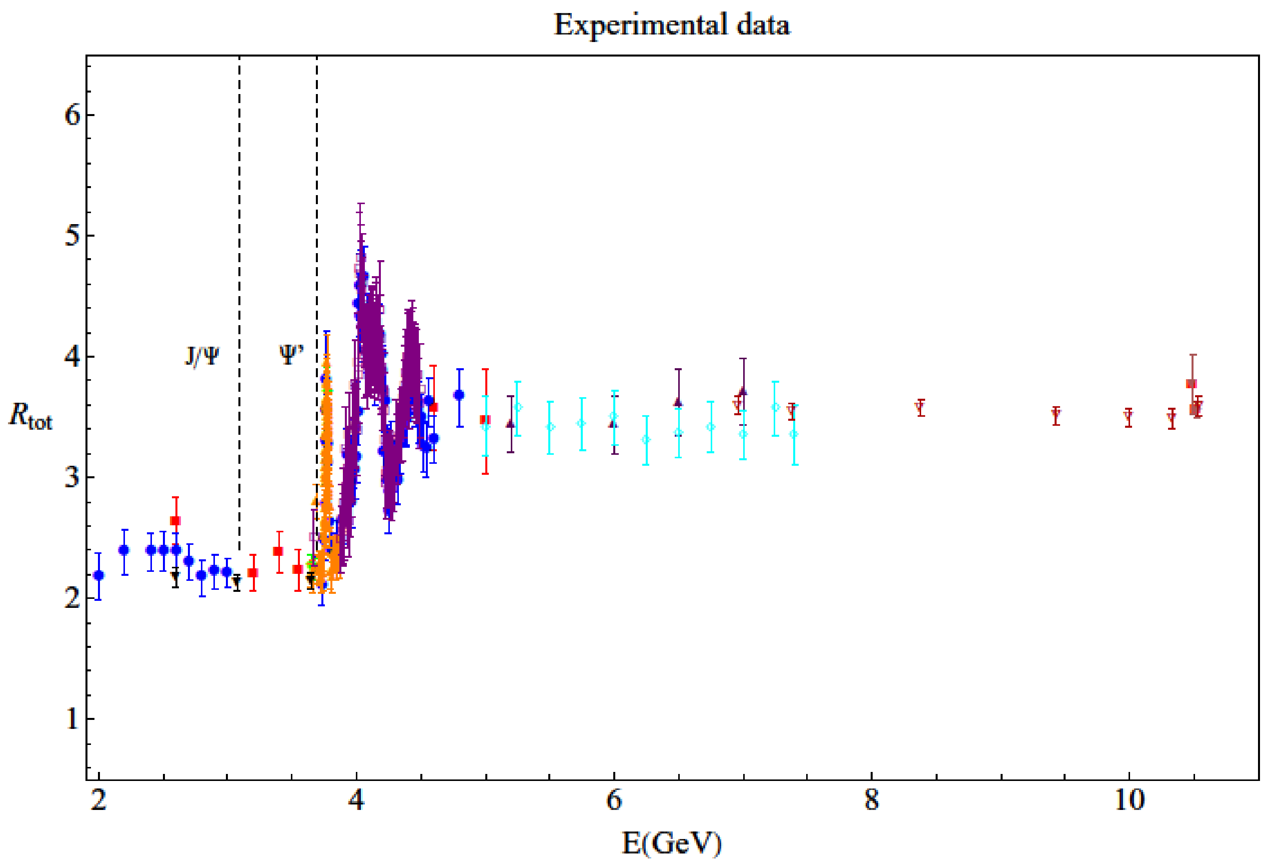
Experimental data: charm

High energy region **CLEO 1998**



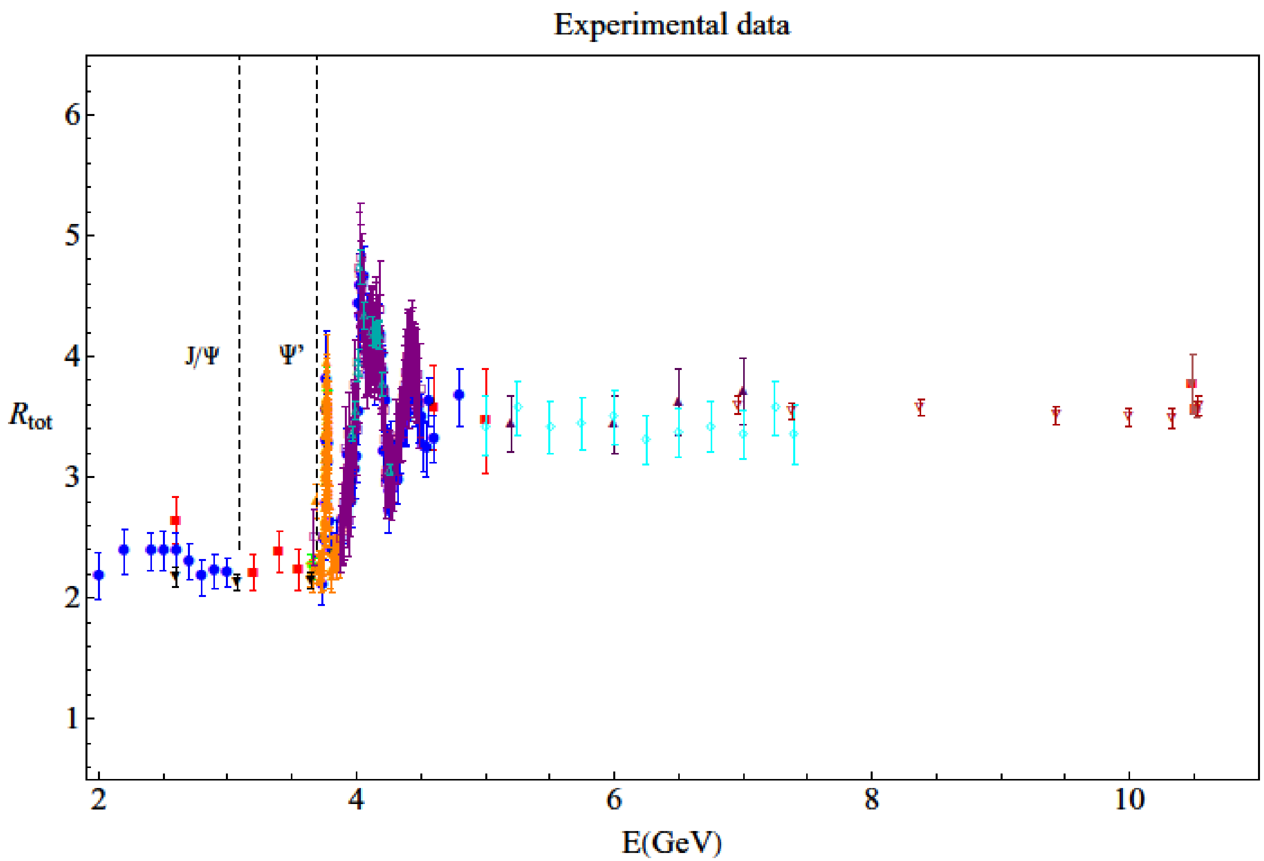
Experimental data: charm

High energy region **CLEO 2007**



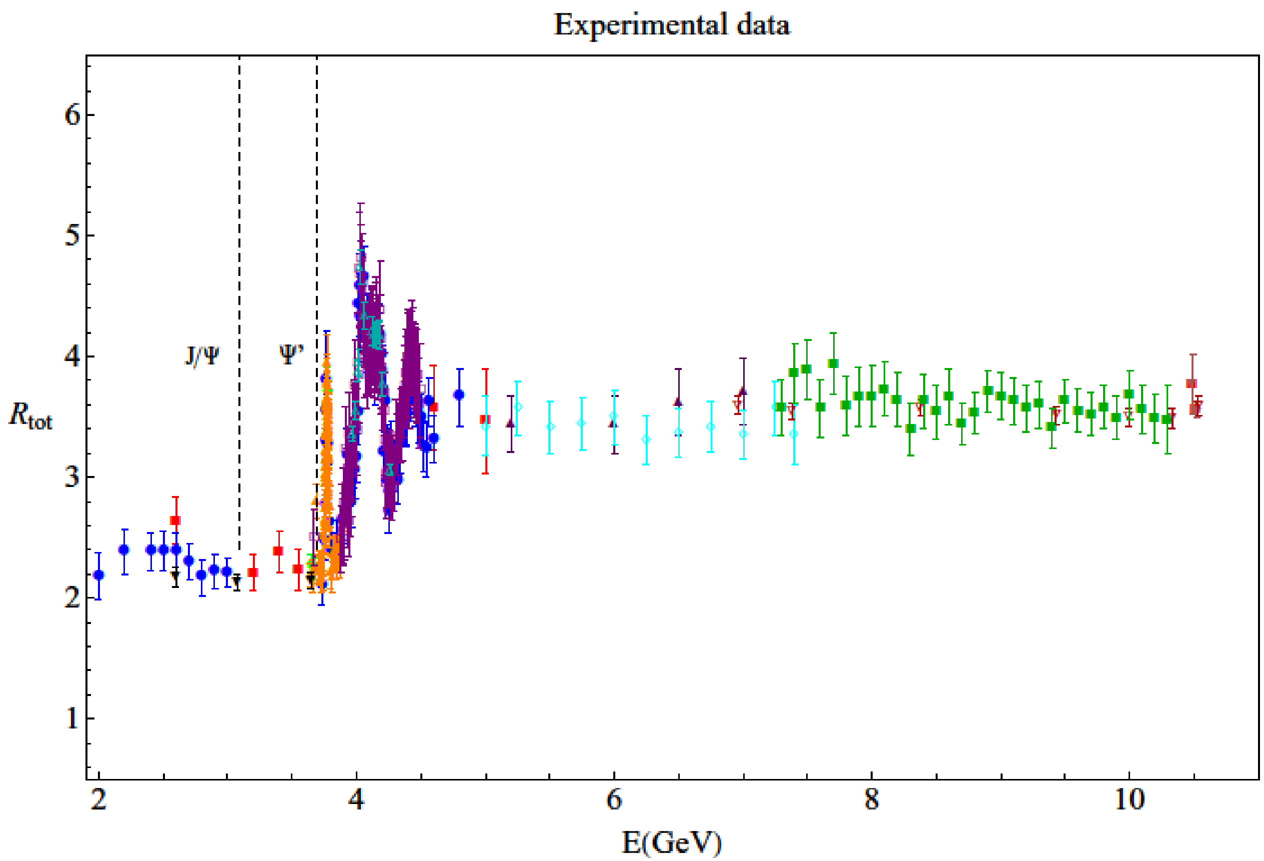
Experimental data: charm

Sub-threshold and threshold CLEO 2009 *



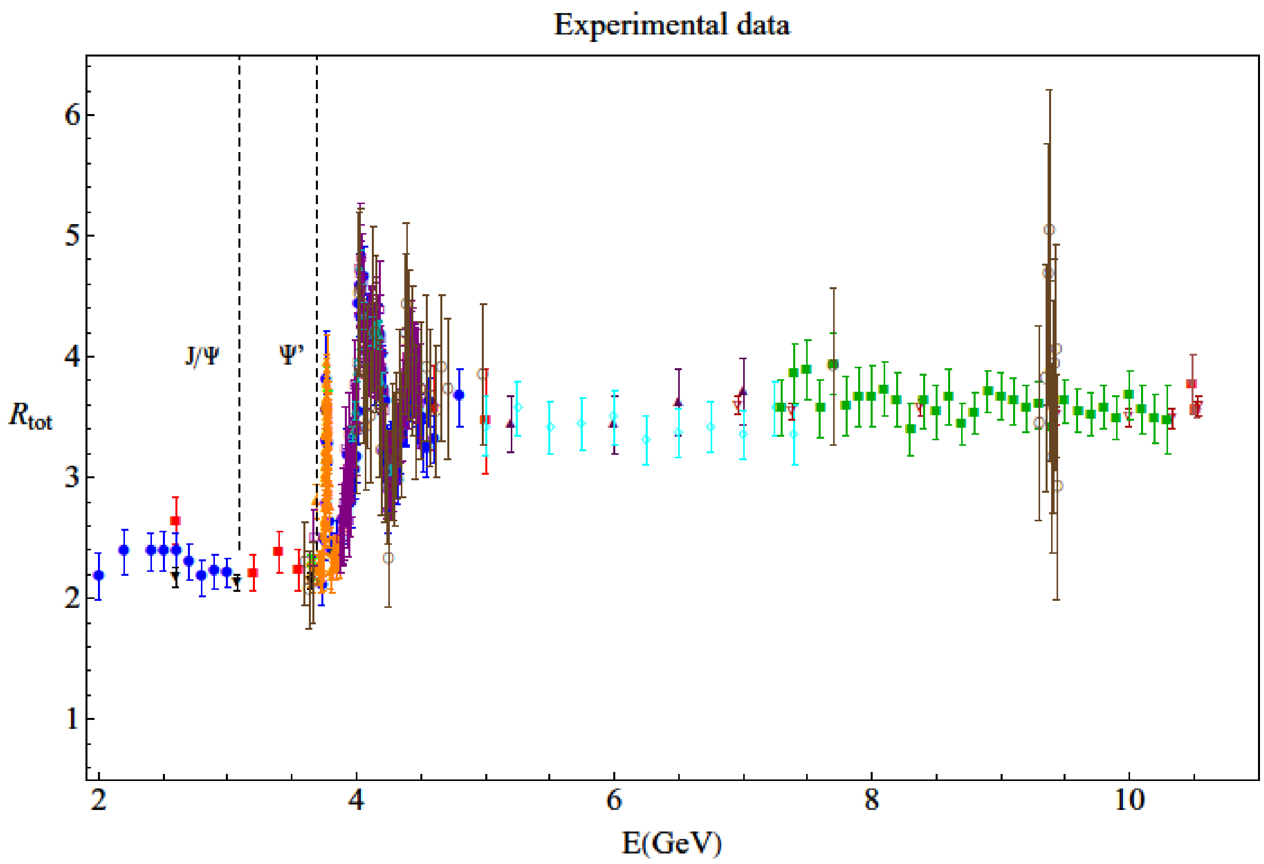
Experimental data: charm

High energy region **MD-1 1996**



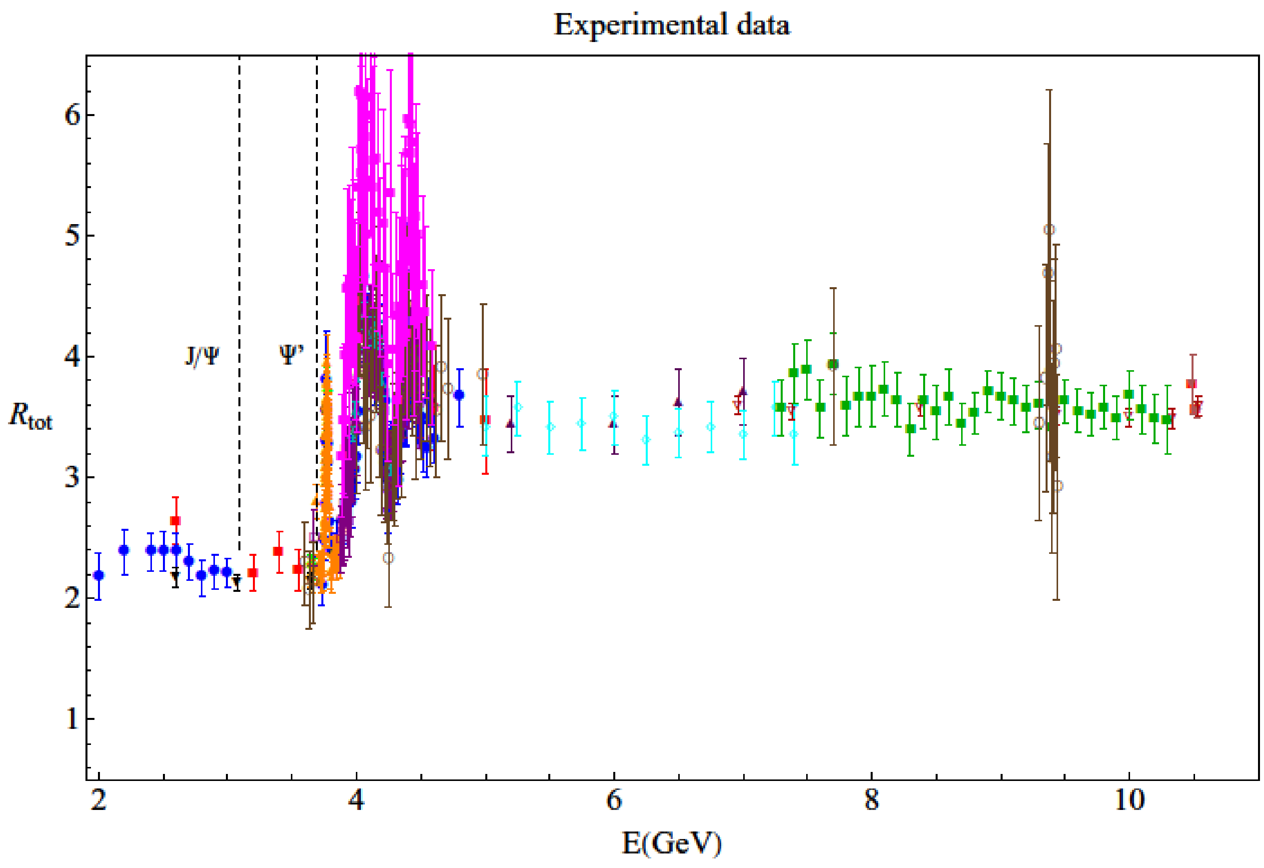
Experimental data: charm

Threshold and high energy **PLUTO 1982 ***



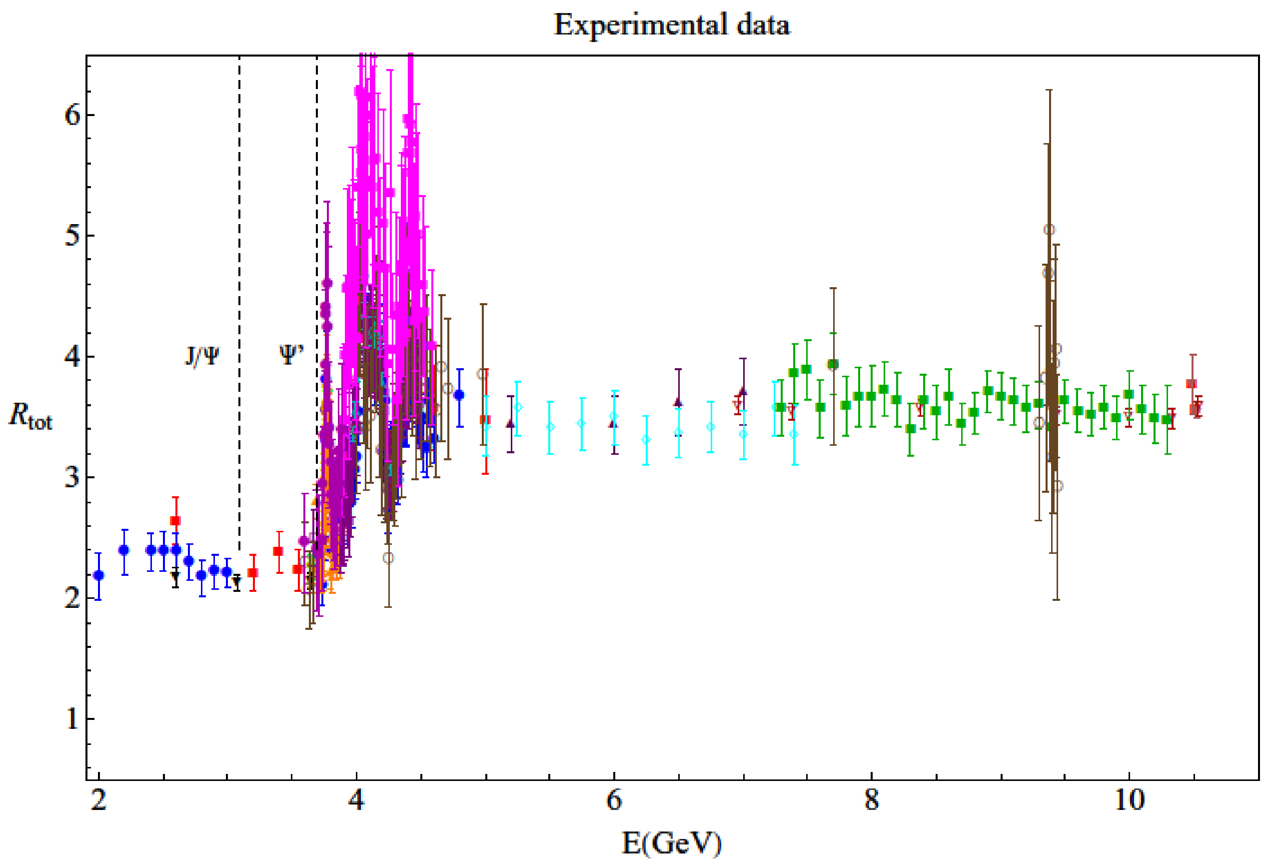
Experimental data: charm

Threshold region **MARKI 1976 ***



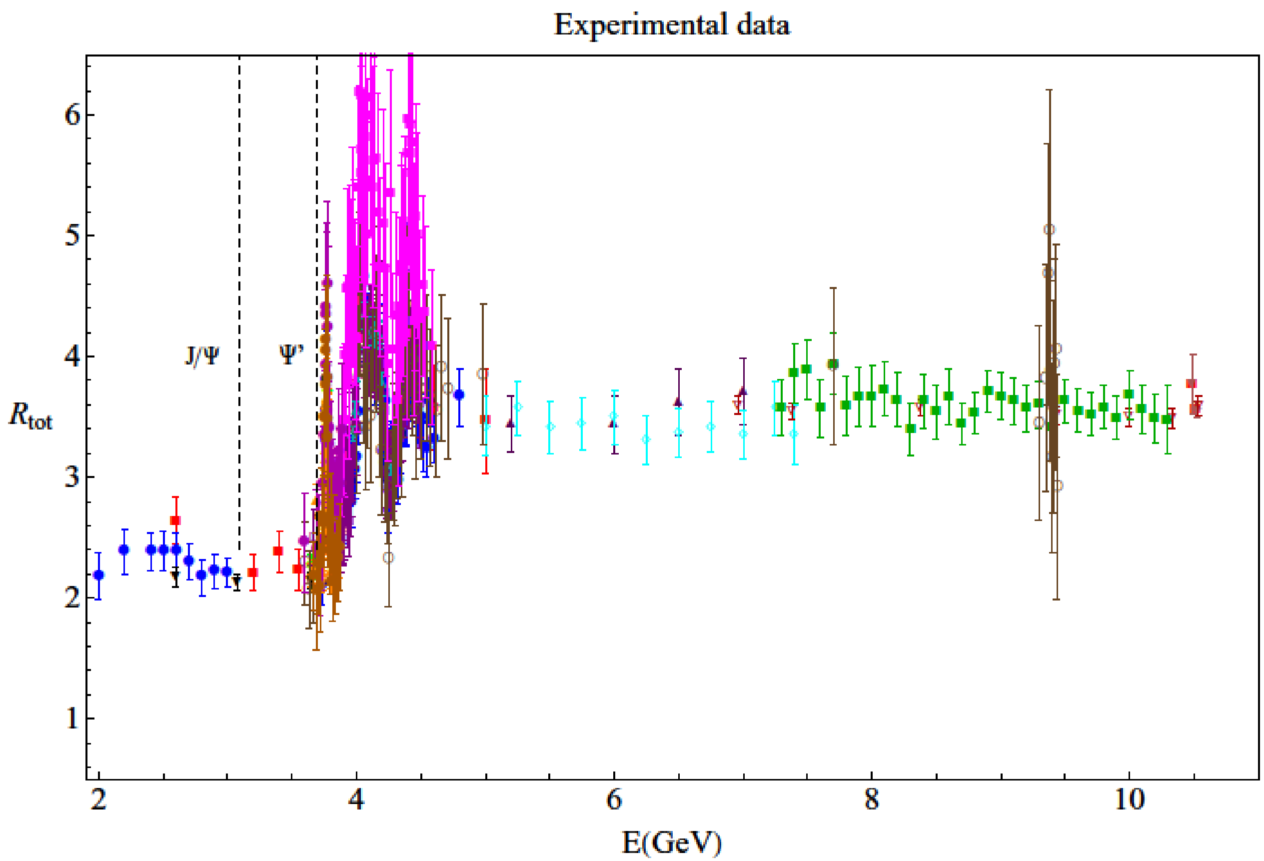
Experimental data: charm

Gap region **MARKI 1977 ***



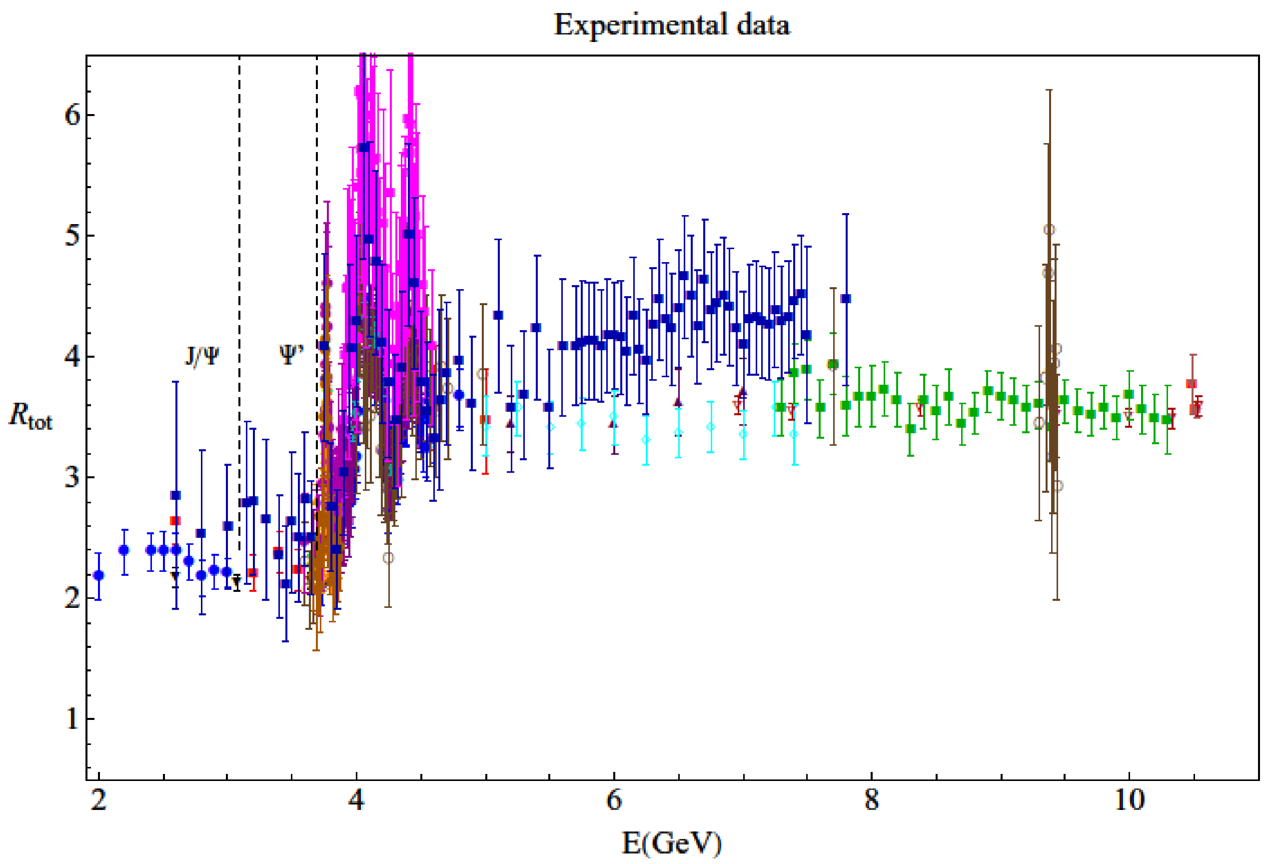
Experimental data: charm

Gap region **MARKII 1979**



Experimental data: charm

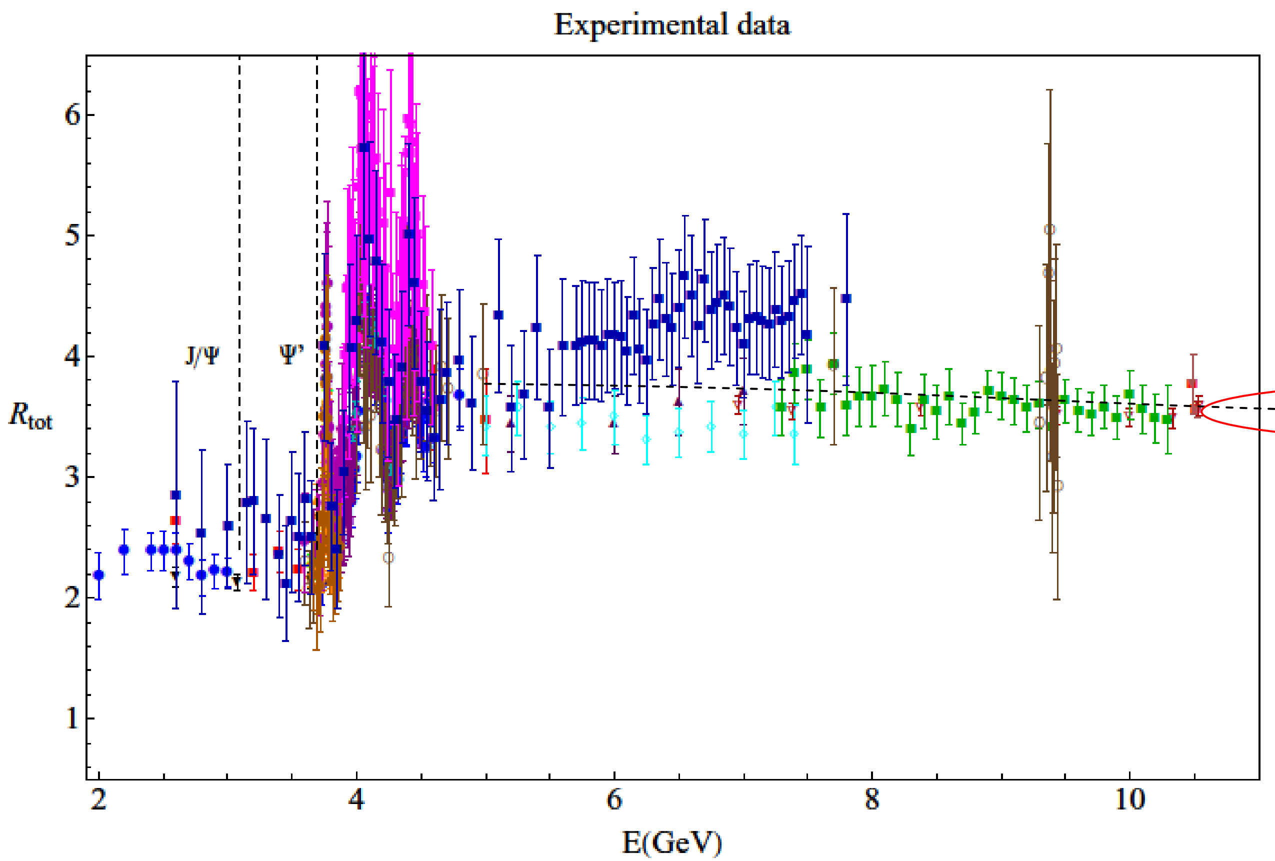
Threshold and gap regions   Mark-I 1981



Experimental data: charm

Perturbation theory

- Only where there is no data
- Assign a conservative 10% error to reduce model dependence

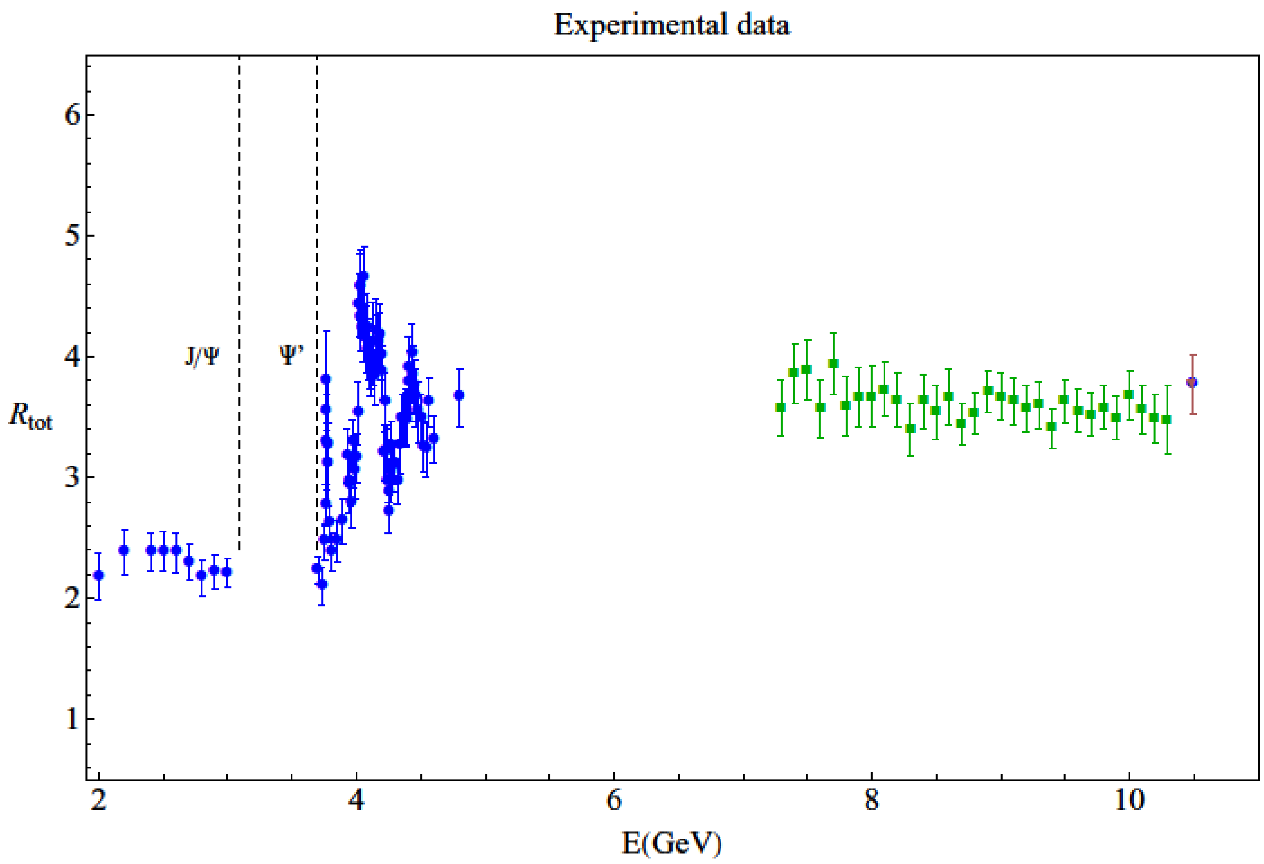


$M_1 \rightarrow 6\%$

$M_{n>1} \rightarrow < 1\%$

Experimental data: charm

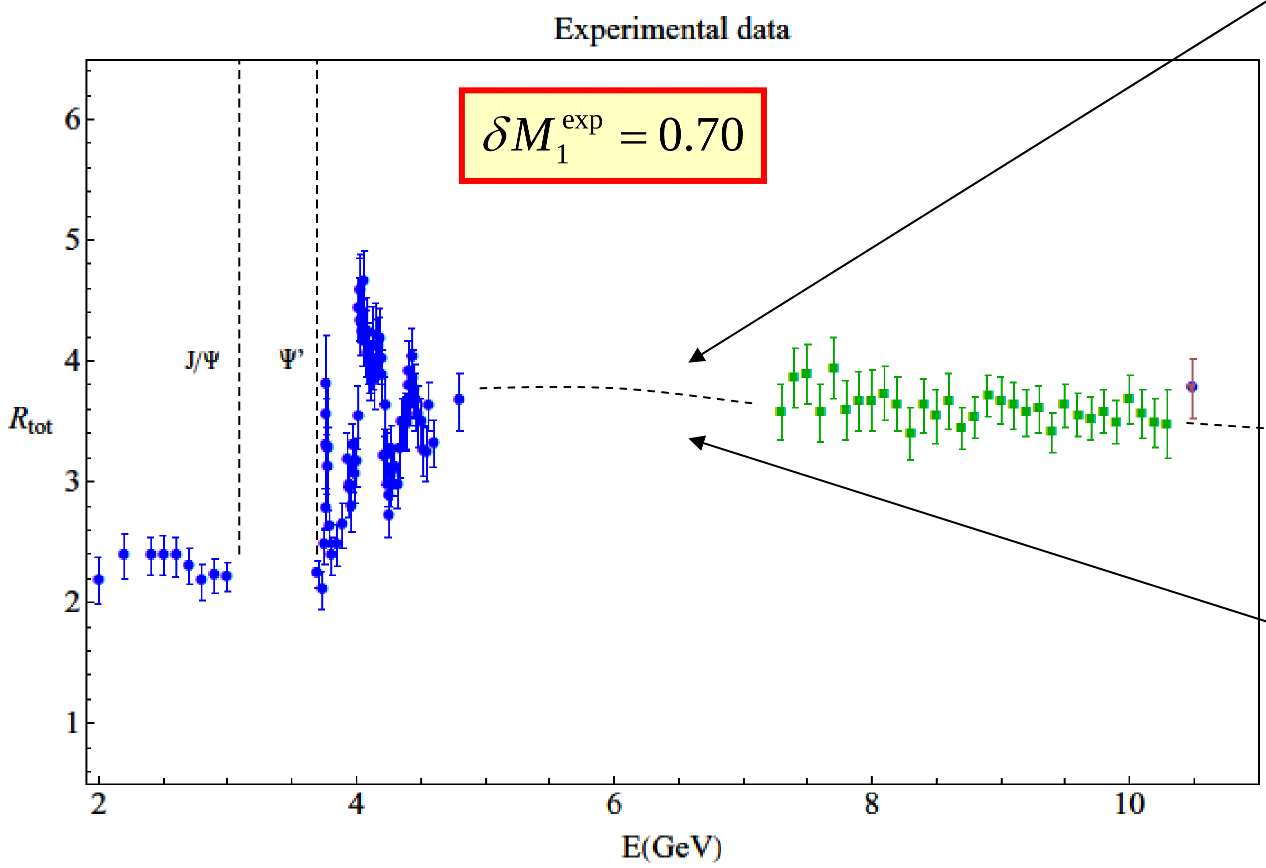
Data used in Hoang and Jamin (2004)



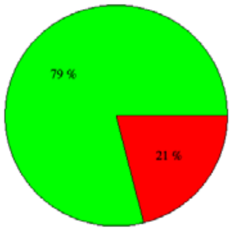
Experimental data: charm

Data used in Hoang and Jamin (2004)

- Perturbation theory only in gap and region with no data
- 10% error assigned as well

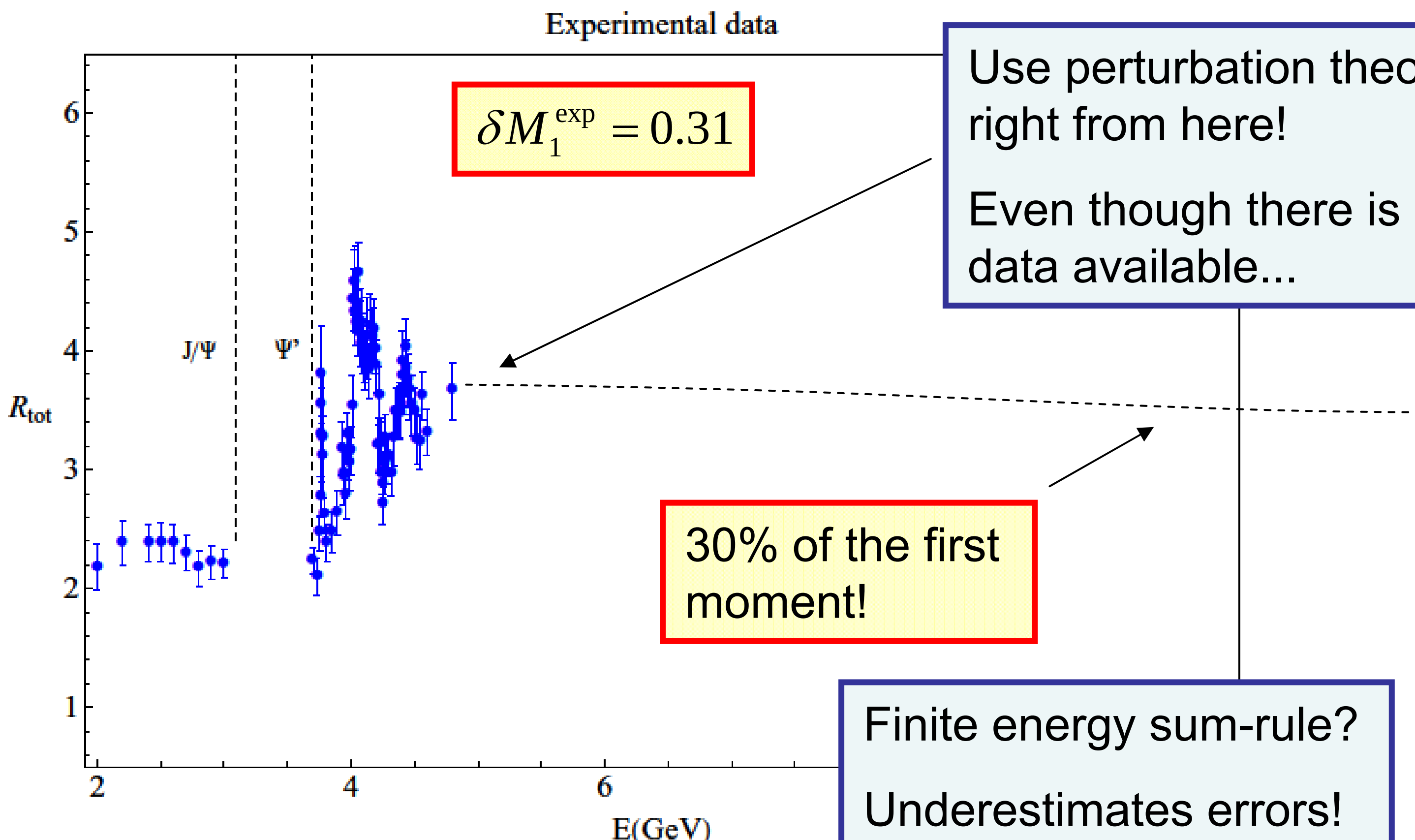


21% of the first moment



Experimental data: charm

Data used in Kühn et al (2001), Boughezal et al and Narison



$$\delta M_1^{\exp} = 0.31$$

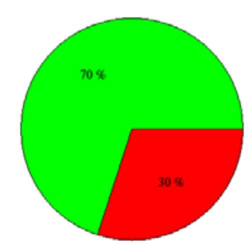
Use perturbation theory right from here!

Even though there is data available...

30% of the first moment!

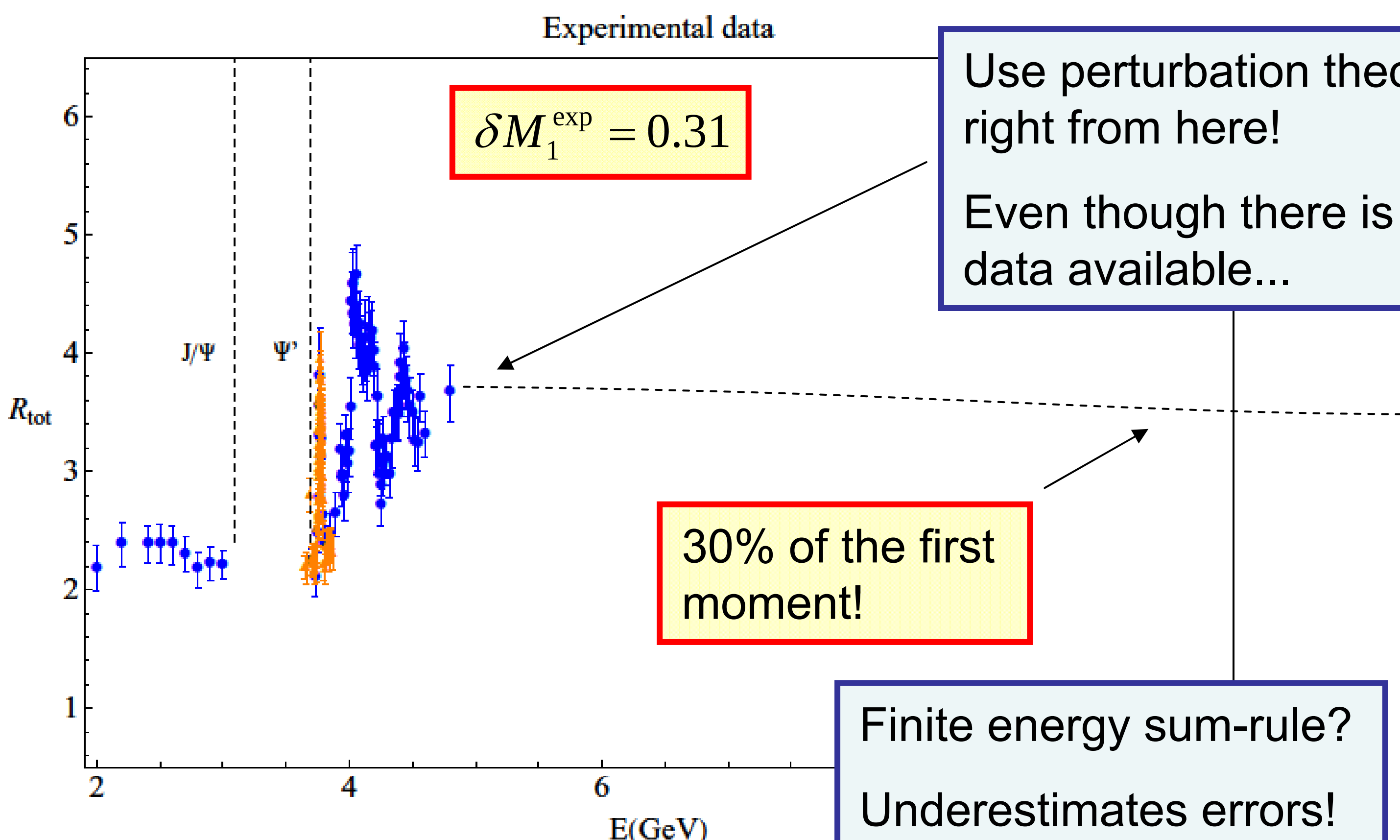
Finite energy sum-rule?

Underestimates errors! (they assing only naive theory error)



Experimental data: charm

Data used in Kühn et al (2004, 2005, ...)



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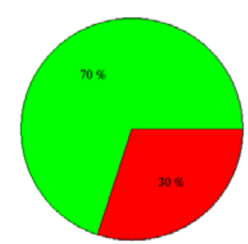
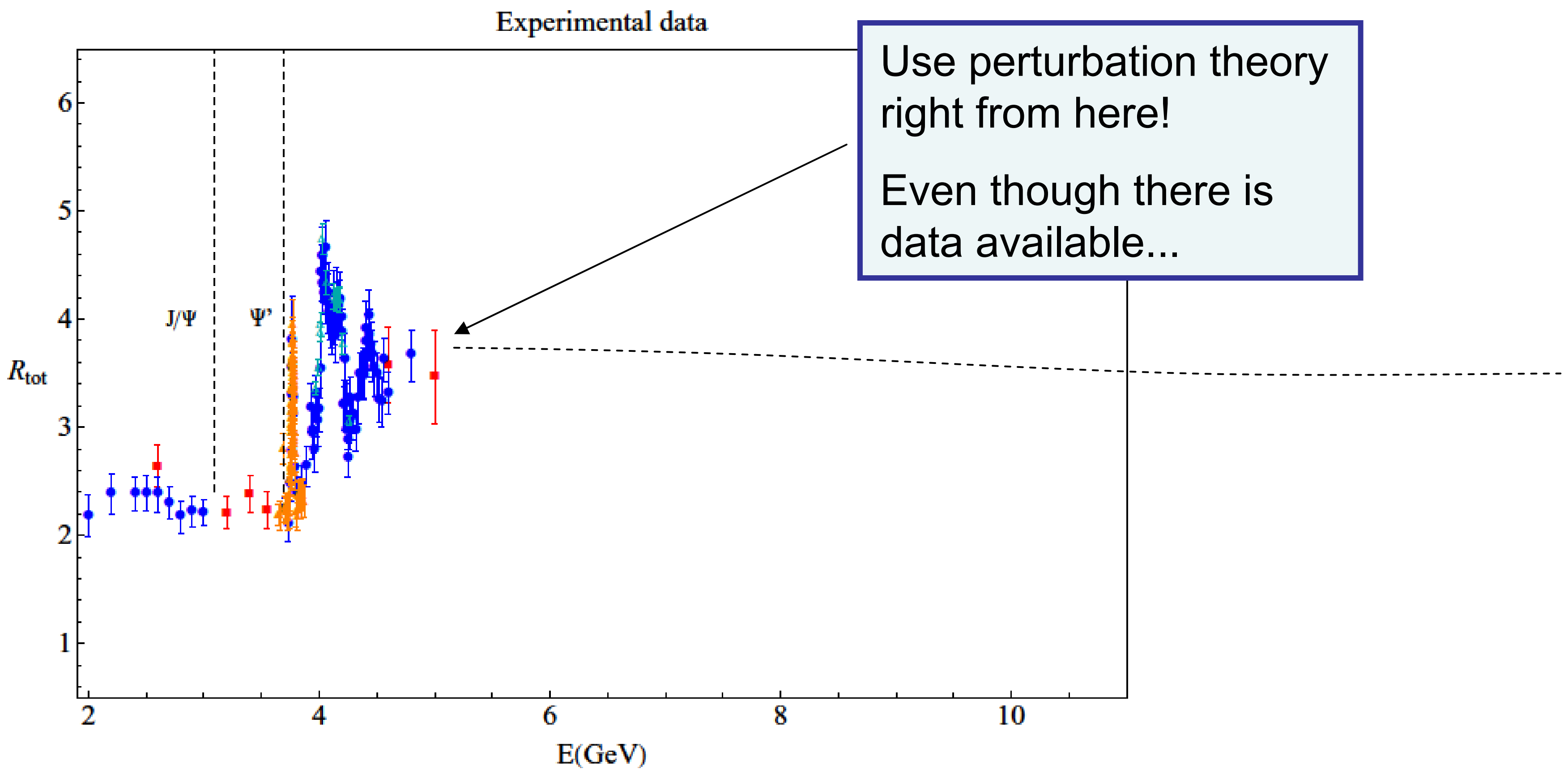
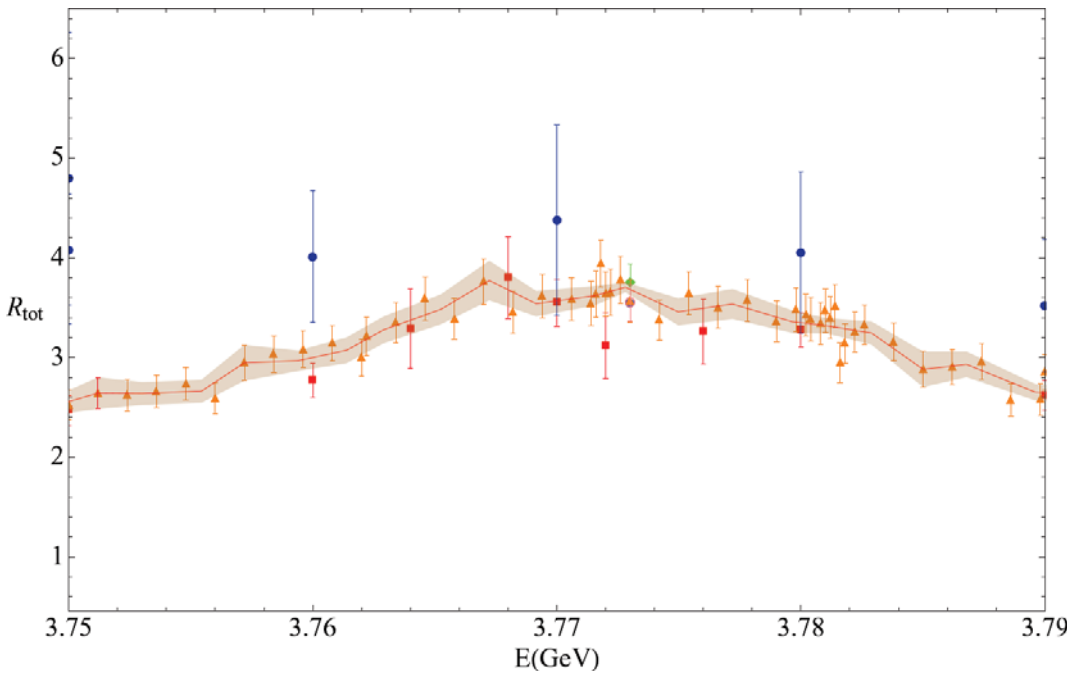


Experimental data: charm

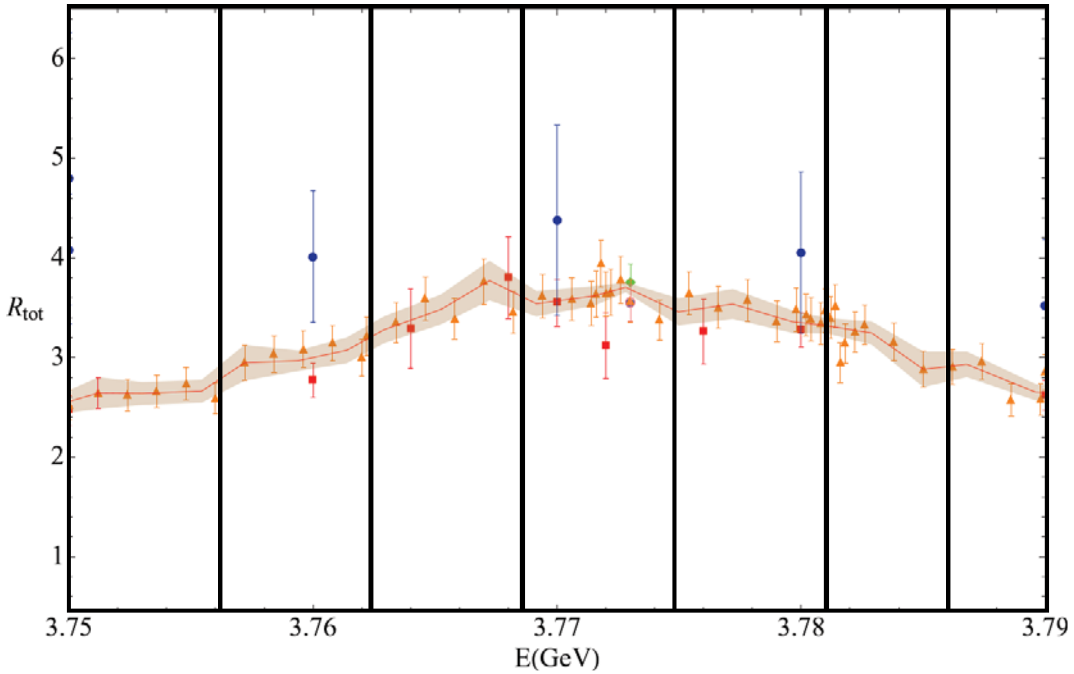
Data used in Bodenstein et al



Fit procedure



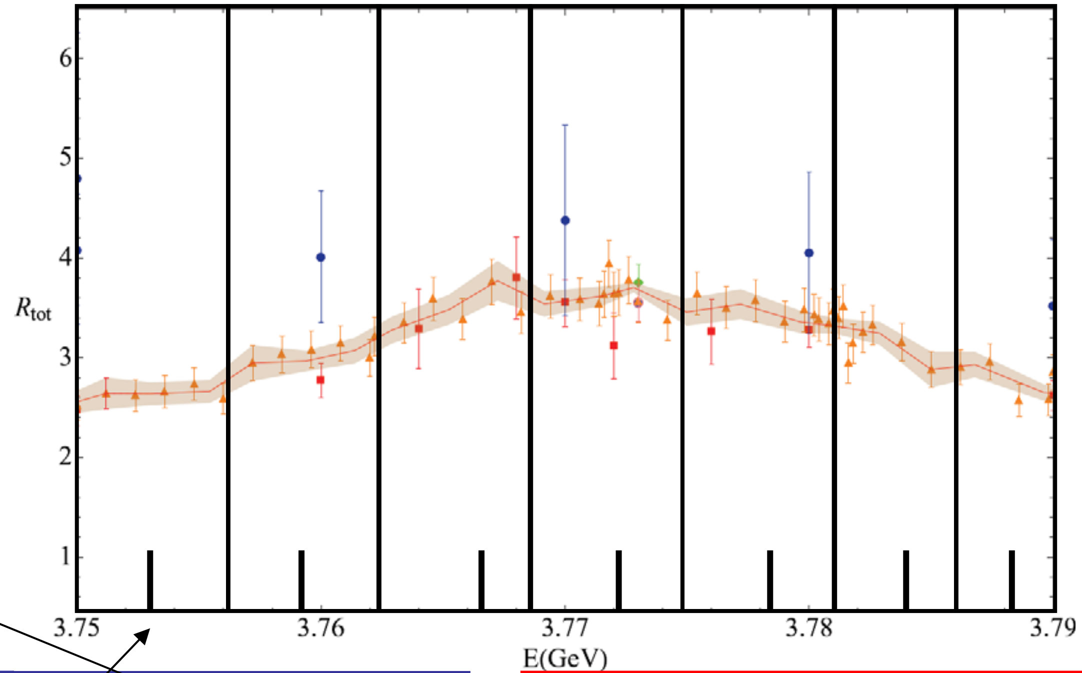
Fit procedure



1. Recluster data. Clusters not necessarily equally sized.

   Number of clusters and size of cluster according to the structure of the data

Fit procedure



Experimental energies

Cluster energy

$$E_m = \frac{\sum_{k=1}^{N_{exp}} \sum_{i=1}^{N^{k,m}} \frac{E_i^{k,m}}{\sigma_i^{k,m\,2} + \Delta_i^{k,m\,2}}}{\sum_{k=1}^{N_{exp}} \sum_{i=1}^{N^{k,m}} \frac{1}{\sigma_i^{k,m\,2} + \Delta_i^{k,m\,2}}}$$

k : Label for experiments

N_{exp} : Number of experiments

m : Label for clusters

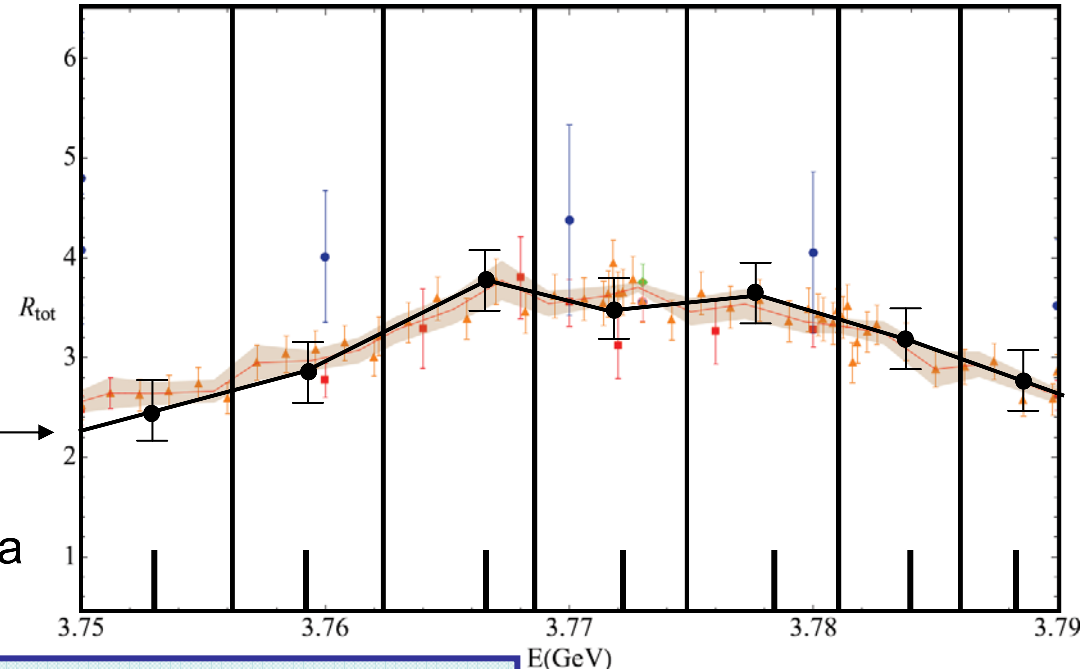
$N_{clusters}$: Number of clusters

i : Label for data points

$N^{k,m}$: Number of data points for experiment k in cluster m

2. Calculate the energy of the cluster. One weights the energy of the data points inside the clusters with their errors.

Fit procedure



R_n

Experimental data

$$\chi^2 = \sum_{k=1}^{N_{\exp}} \left(d_k^2 + \sum_{m=1}^{N_{\text{clusters}}} \sum_{i=1}^{N^{k,m}} \frac{\left[R_i^{k,m} - \left(1 + d_i \frac{\Delta_i^{k,m}}{R_i^{k,m}} \right) R_m \right]^2}{\sigma_i^{k,m\,2}} \right)$$

Fit parameters

k : Label for experiments

$N_{\exp}$: Number of experiments

m : Label for clusters

N_{clusters} : Number of clusters

i : Label for data points

$N^{k,m}$: Number of data points for experiment k in cluster m

3. Fit the value of R for each cluster. Data is allowed to “move” within its systematic error. The method renders errors and correlations among various clusters. One can then calculate errors and correlations for the moments.

Fit procedure

$$\chi^2_{\rm corr}(\{d_k\}) = \sum_{k=1}^{N_{\rm exp}} d_k^2$$

1-σ constraint to auxiliary parameters

$$R_{{\rm non}-c\bar{c}}(E) = n_{\rm ns}\, R_{uds}(E)$$

background (free normalization)

$$\chi^2_{\rm nc}(n_{\rm ns},\{d_k\}) = \sum_{k=1}^{N_{\rm exp}} \sum_{i=1}^{N^{k,1}} \left(\frac{R_i^{k,1} - (1+\Delta f_i^{k,1}\, d_k)\, n_{ns}\, R_{uds}(E_i^{k,1})}{\sigma_i^{k,1}} \right)^2$$

Data below threshold (only background), first cluster

$$\chi^2(\{R_m\},n_{\rm ns},\{d_k\}) =$$

Data above threshold (signal + background)

$$\sum_{k=1}^{N_{\rm exp}} \sum_{m=2}^{N_{\rm clusters}} \sum_{i=1}^{N^{k,m}} \left(\frac{R_i^{k,m} - (1+\Delta f_i^{k,m}\, d_k)\,(R_m + n_{ns}\, R_{uds}(E_i^{k,m}))}{\sigma_i^{k,m}} \right)^2$$

total χ^2

$$\chi^2(\{R_m\},n_{\rm ns},\{d_k\}) = \chi^2_{\rm corr}(\{d_k\}) + \chi^2_{\rm nc}(n_{\rm ns},\{d_k\}) + \chi^2(\{R_m\},n_{\rm ns},\{d_k\})$$

Fit procedure

- Method inspired by a similar one in Hagiwara, Martin & Teubner.
- Avoids the problems of a regular χ^2 in which the systematic errors are 100% correlated

$$d_i \rightarrow \text{Auxiliary parameters}$$

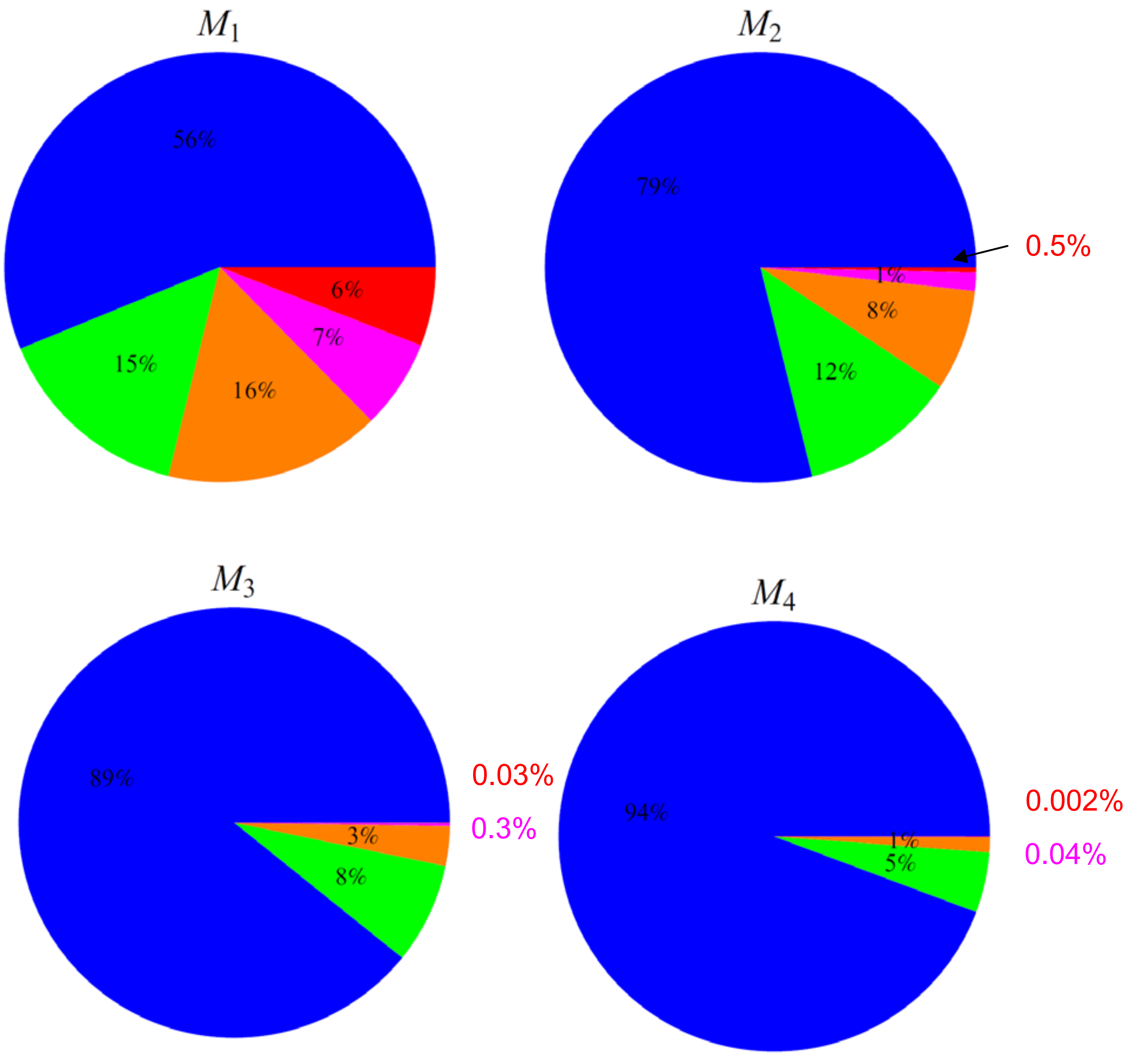
Prediction for moments $M_n = m_n 10^{n+1}$ GeV^{n+1}

$$M_1 = 21.38 \pm 0.20_{stat} \pm 0.46_{sys}$$
$$M_2 = 14.91 \pm 0.18_{stat} \pm 0.29_{sys}$$
$$M_3 = 13.10 \pm 0.19_{stat} \pm 0.25_{sys}$$
$$M_4 = 12.49 \pm 0.19_{stat} \pm 0.23_{sys}$$

We also predict correlations among the various moments, useful for simultaneous fits.

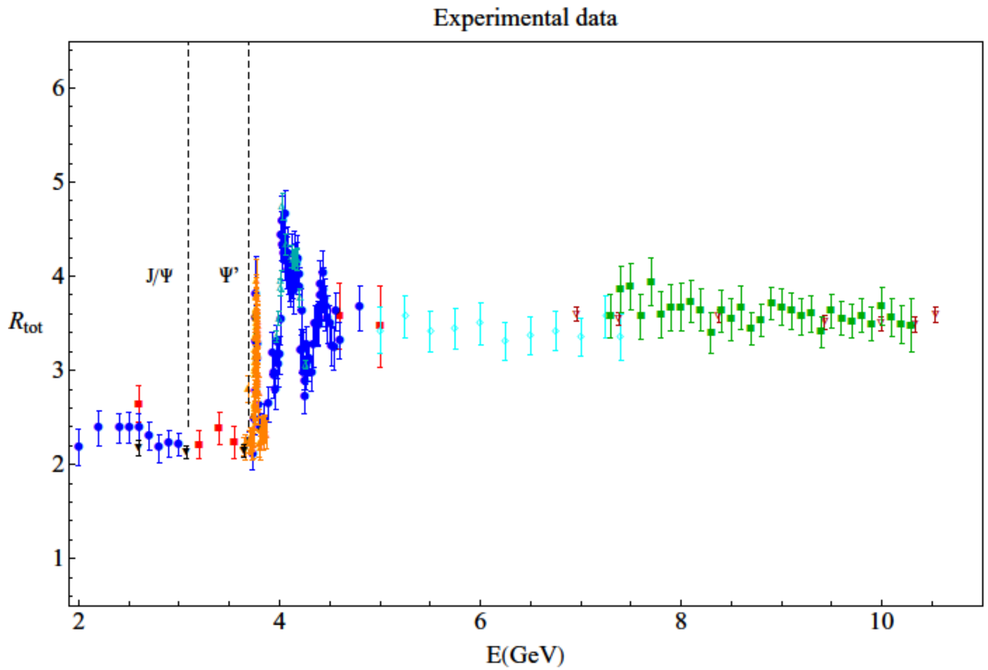
$$C^{\exp} = \begin{pmatrix} 0.250 & 0.167 & 0.147 & 0.142 \\ 0.167 & 0.120 & 0.107 & 0.103 \\ 0.147 & 0.107 & 0.095 & 0.092 \\ 0.142 & 0.103 & 0.092 & 0.090 \end{pmatrix} \qquad C^{\exp}_{\rm uc} = \begin{pmatrix} 0.041 & 0.035 & 0.034 & 0.034 \\ 0.035 & 0.034 & 0.034 & 0.035 \\ 0.034 & 0.034 & 0.035 & 0.036 \\ 0.034 & 0.035 & 0.036 & 0.037 \end{pmatrix}$$

Moments budget



Narrow resonances
3.73 – 4.8 GeV
4.8 – 7.25 GeV
7.25 – 10.54 GeV
10.54 GeV – Infinity

Minimal data selection



Data sets 1, 2, 5, 6, 9, 12, 13, and 14

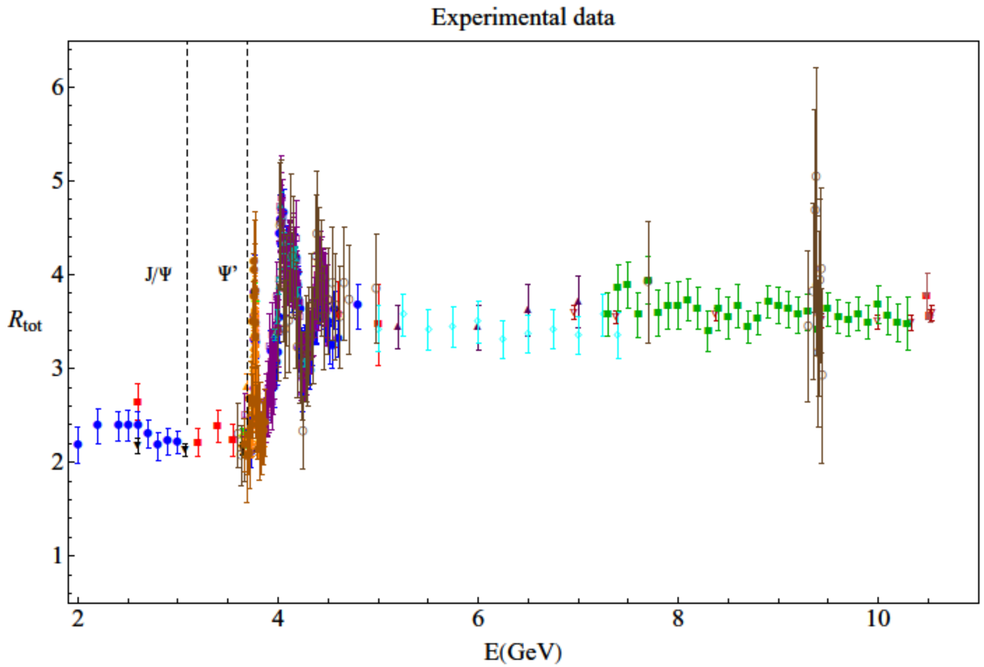
BES, CrystalBall, CLEO and MD1

$$\frac{\chi^2_{\rm minimal}}{\rm d.o.f} = 1.86$$

$$n_s^{\rm minimal} = 1.029 \pm 0.003_{\rm stat} \pm 0.015_{\rm syst}$$

n	Resonances	$3.73 - 4.8$	$4.8 - 7.25$	$7.25 - 10.538$	10.538-∞	Total
1	12.01(17|21)	3.11(6|8)	3.30(9|16)	1.40(2|6)	1.27(0|13)	21.09(22|51)
2	11.76(18|21)	1.73(3|4)	1.06(3|5)	0.199(4|9)	0.057(0|6)	14.80(19|31)
3	11.69(19|21)	0.98(2|3)	0.36(1|2)	0.030(1|1)	0.0034(0|3)	13.07(19|26)
4	11.77(19|21)	0.57(1|2)	0.131(5|5)	0.0046(1|2)	$2.3(0\|2) \times 10^{-4}$	12.47(19|23)

Standard data selection



All Data sets except 16,17 and 19

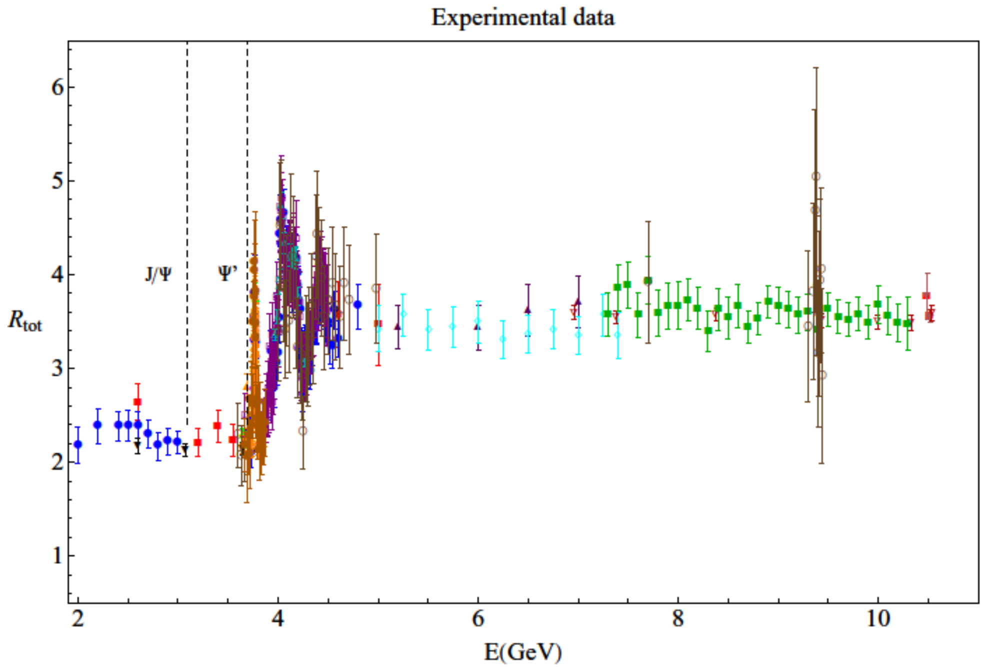
All periments except some MARKI and MARKII

$$\frac{\chi^2_{\text{standard}}}{\text{d.o.f}} = 1.89$$

$$n_s^{\text{standard}} = 1.039 \pm 0.003_{\text{stat}} \pm 0.012_{\text{syst}}$$

n	Resonances	$3.73 - 4.8$	$4.8 - 7.25$	$7.25 - 10.538$	10.538-∞	Total
1	12.01(17|21)	3.20(4|6)	3.47(8|13)	1.43(2|5)	1.27(0|13)	21.38(20|46)
2	11.76(18|21)	1.76(2|3)	1.13(3|4)	0.204(3|7)	0.057(0|6)	14.91(18|29)
3	11.69(19|21)	0.99(1|2)	0.390(9|12)	0.0305(5|10)	0.0034(0|3)	13.11(19|25)
4	11.77(19|21)	0.565(8|11)	0.143(3|4)	0.00474(9|15)	$2.3(0\|2) \times 10^{-4}$	12.49(19|23)

Standard data selection



All Data sets except 16,17 and 19

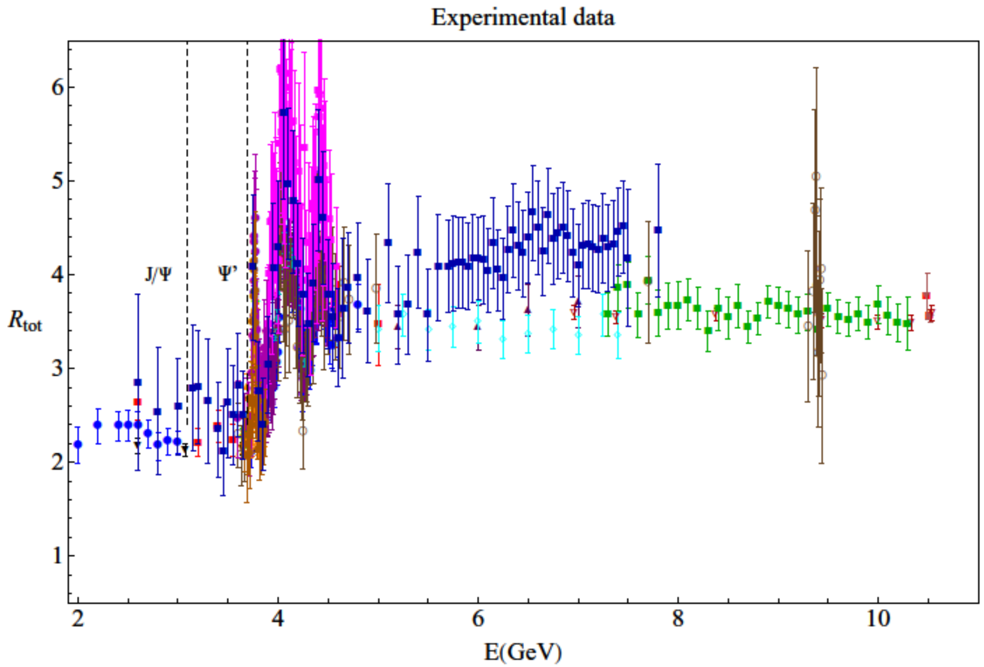
All periments except some MARKI and MARKII

Error in M_1^{exp} from unknown continuum where a 10% theory error has been assigned: 0.13

Acceptable model dependence !

n	Resonances	3.73 – 4.8	4.8 – 7.25	7.25 – 10.538	10.538–∞	Total
1	12.01(17|21)	3.20(4|6)	3.47(8|13)	1.43(2|5)	1.27(0|13)	21.38(20|46)
2	11.76(18|21)	1.76(2|3)	1.13(3|4)	0.204(3|7)	0.057(0|6)	14.91(18|29)
3	11.69(19|21)	0.99(1|2)	0.390(9|12)	0.0305(5|10)	0.0034(0|3)	13.11(19|25)
4	11.77(19|21)	0.565(8|11)	0.143(3|4)	0.00474(9|15)	$2.3(0\|2) \times 10^{-4}$	12.49(19|23)

Maximal data selection



All Data sets

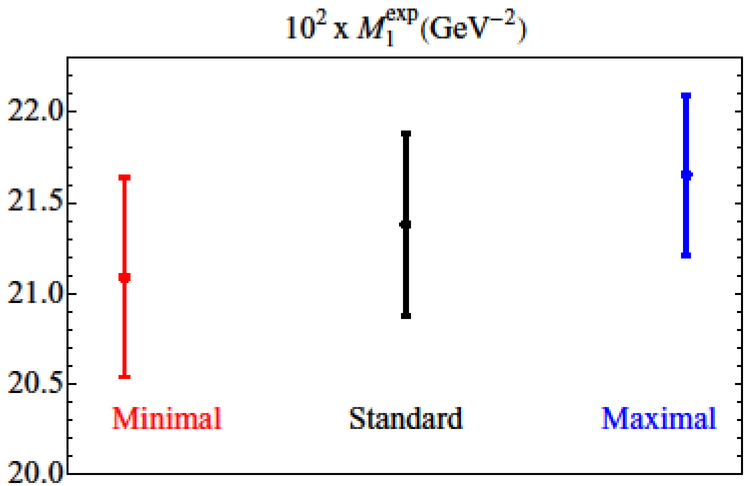
All experiments

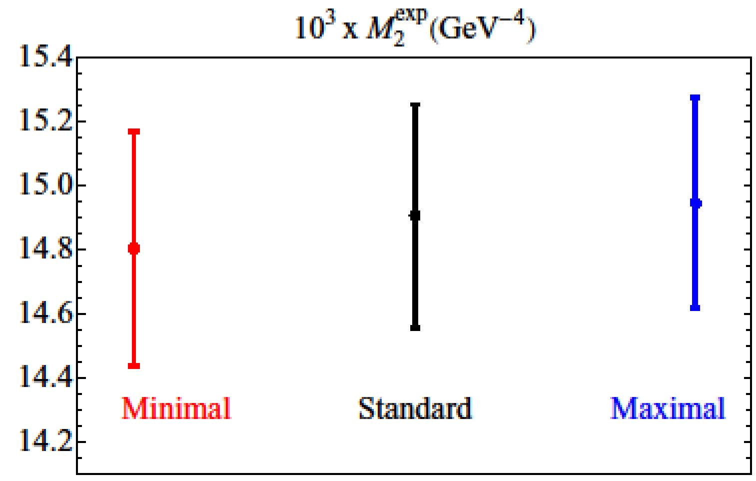
$$\frac{\chi^2_{\text{maximal}}}{\text{d.o.f}} = 1.81$$

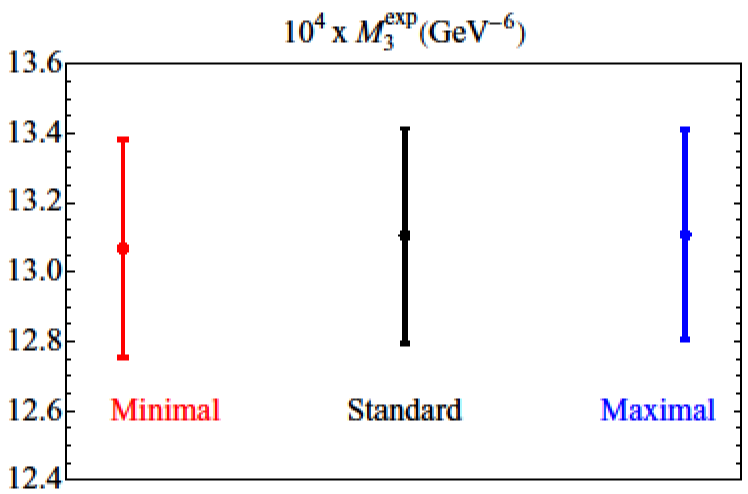
$$n_s^{\text{maximal}} = 1.023 \pm 0.003_{\text{stat}} \pm 0.011_{\text{syst}}$$

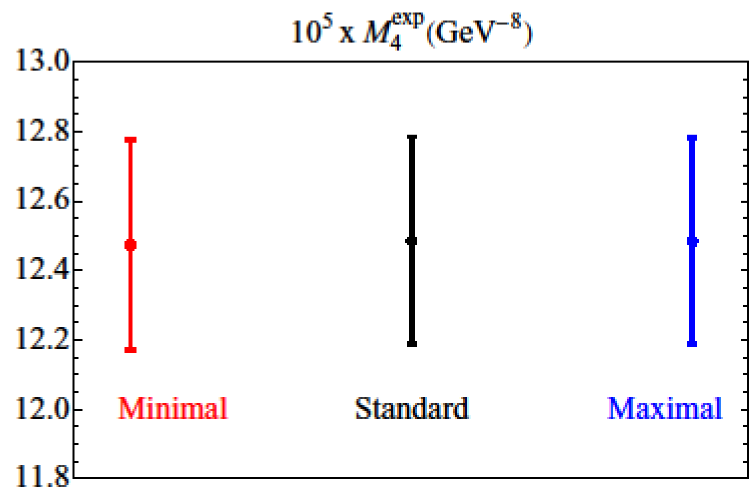
n	Resonances	$3.73 - 4.8$	$4.8 - 7.25$	$7.25 - 10.538$	10.538-∞	Total
1	12.01(17|21)	3.16(3|5)	3.66(6|7)	1.56(2|4)	1.27(0|13)	21.65(19|39)
2	11.76(18|21)	1.75(2|3)	1.16(2|2)	0.222(3|5)	0.057(0|6)	14.95(18|27)
3	11.69(19|21)	0.98(1|2)	0.40(1|1)	0.033(1|1)	0.0034(0|3)	13.11(19|24)
4	11.77(19|21)	0.56(1|1)	0.142(3|2)	0.0051(1|1)	$2.3(0\|2) \times 10^{-4}$	12.49(19|23)

Comparison selections

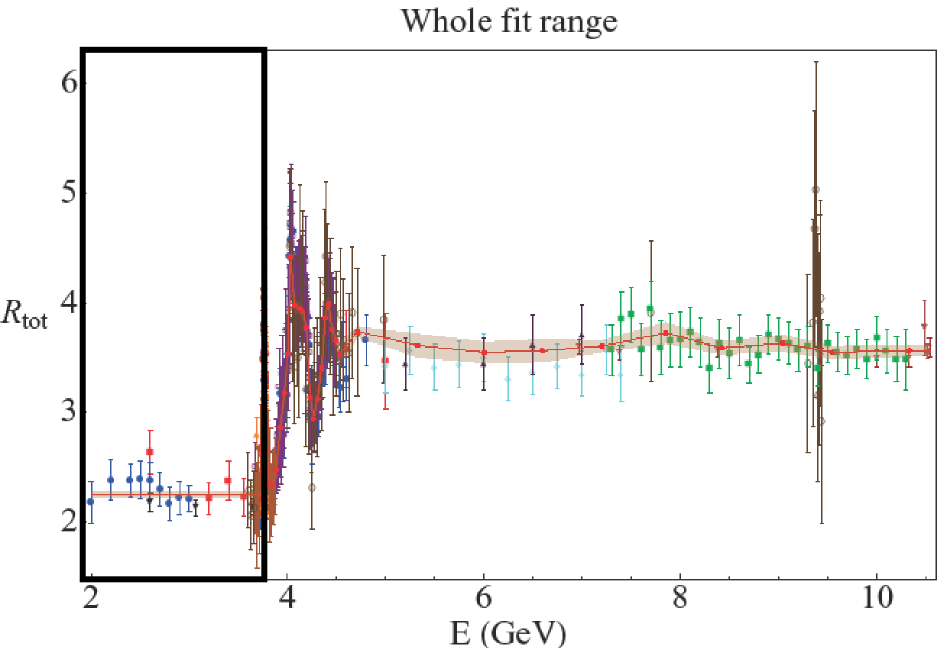


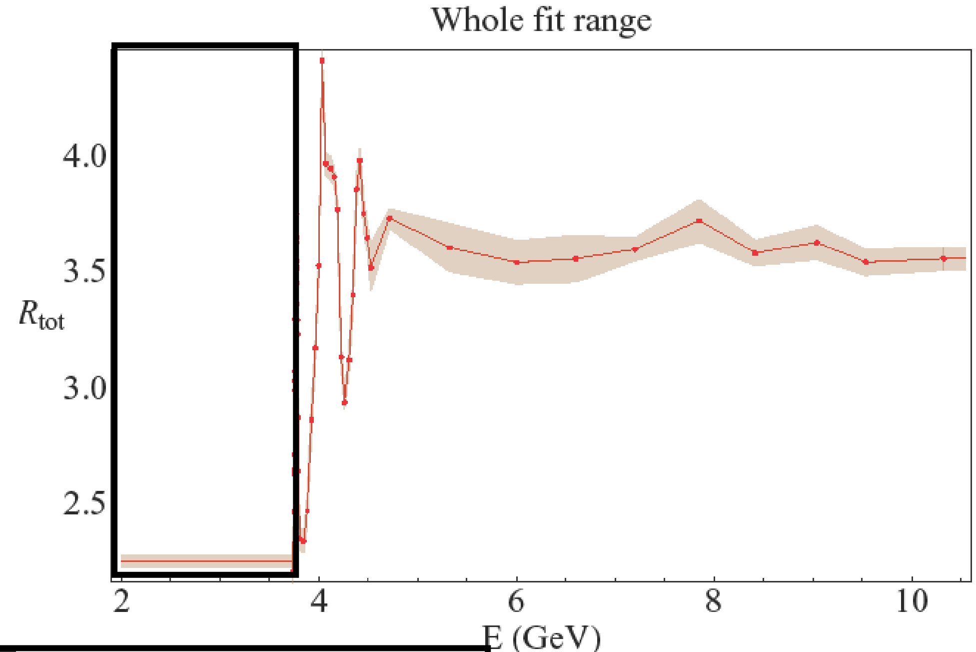






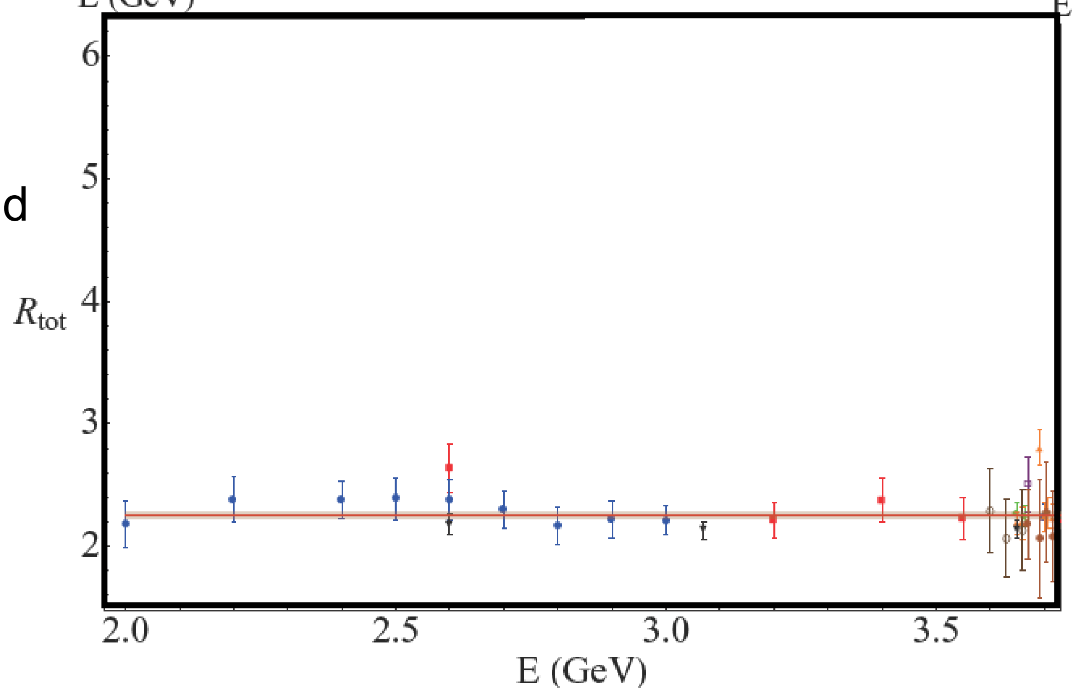
Fit results



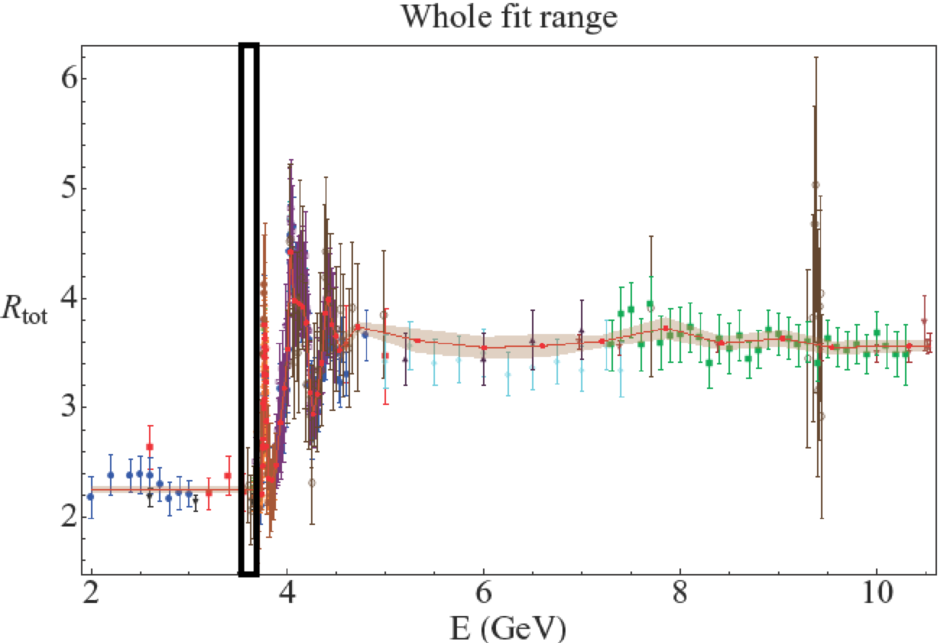


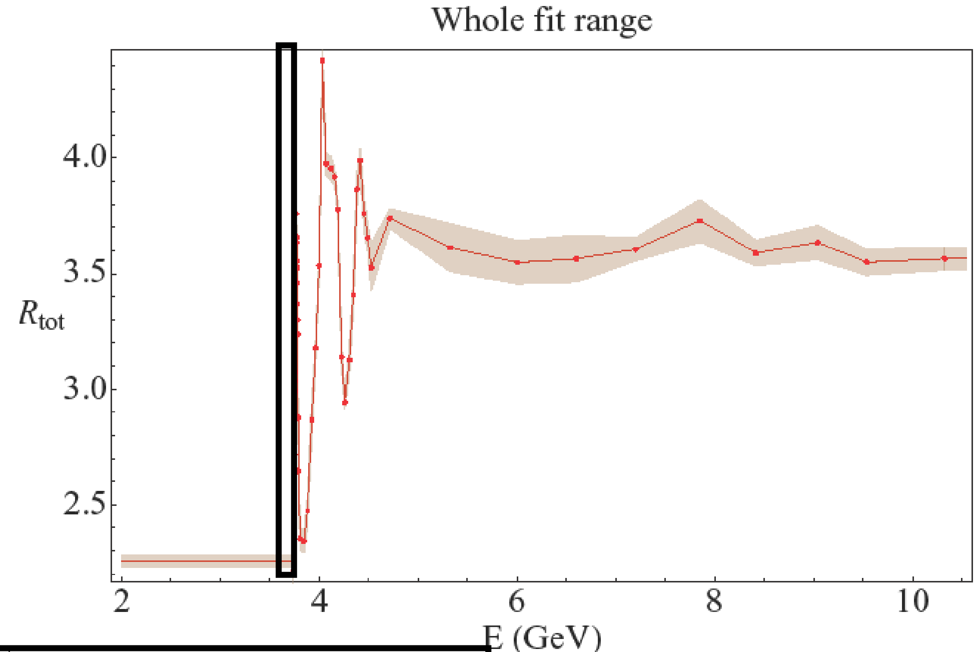
Below threshold

1 cluster



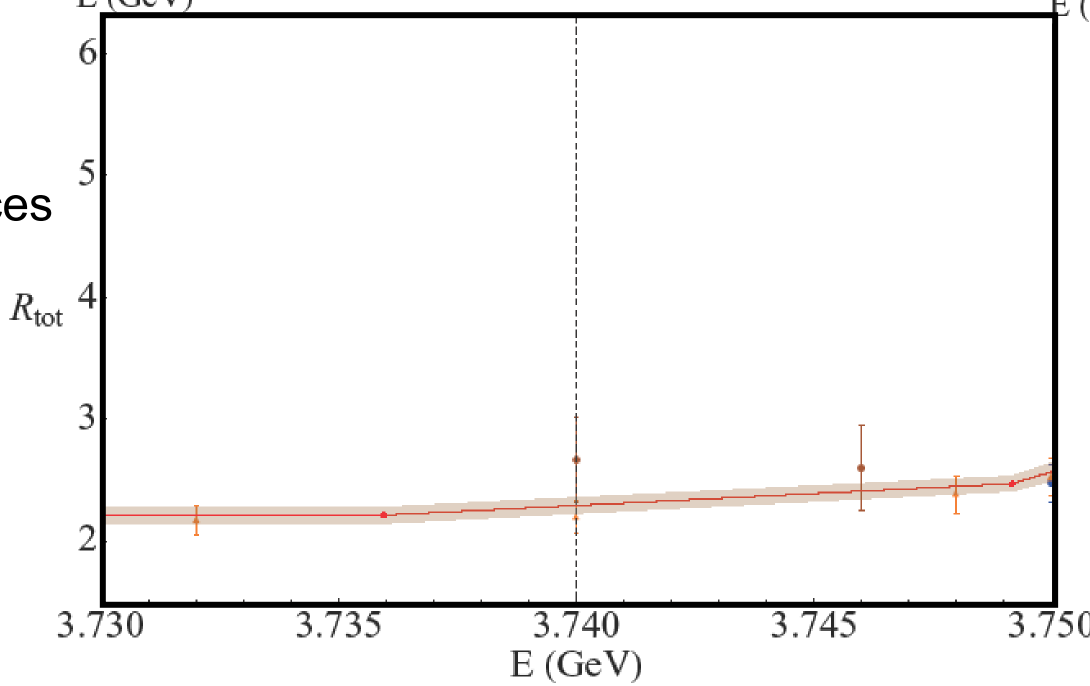
Fit results



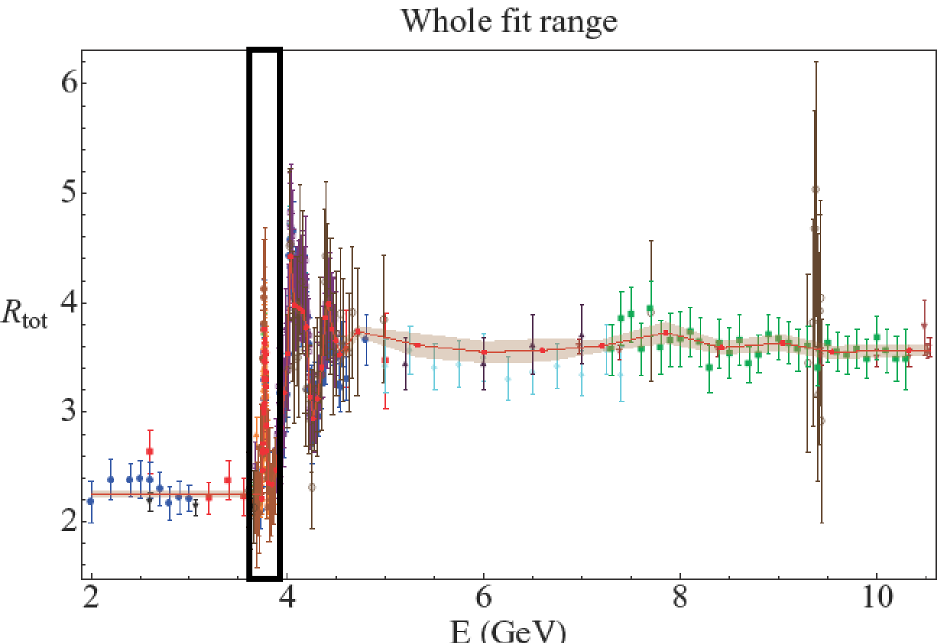


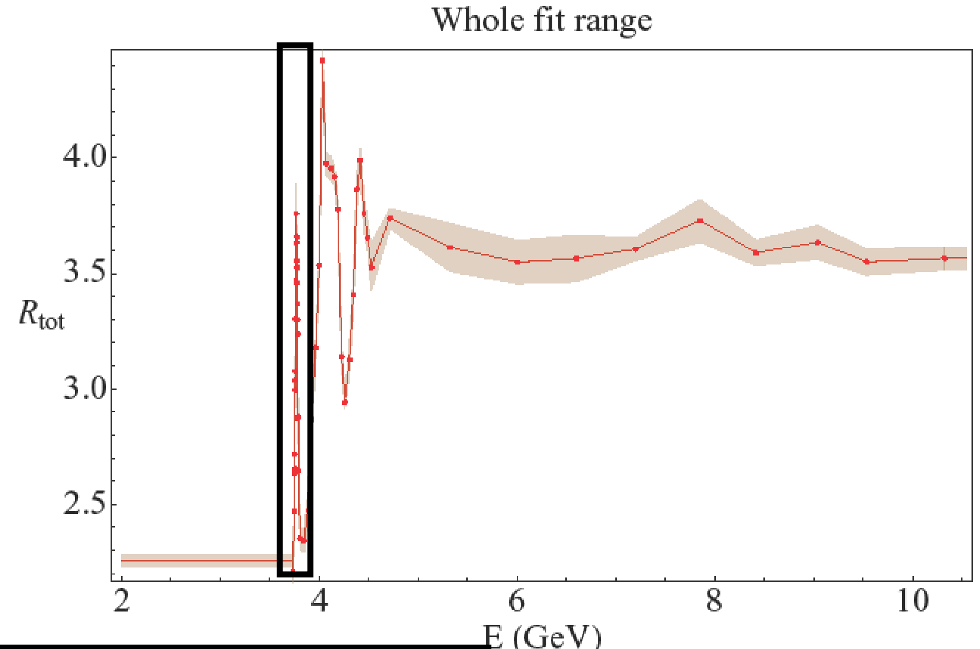
Below resonances

2 clusters



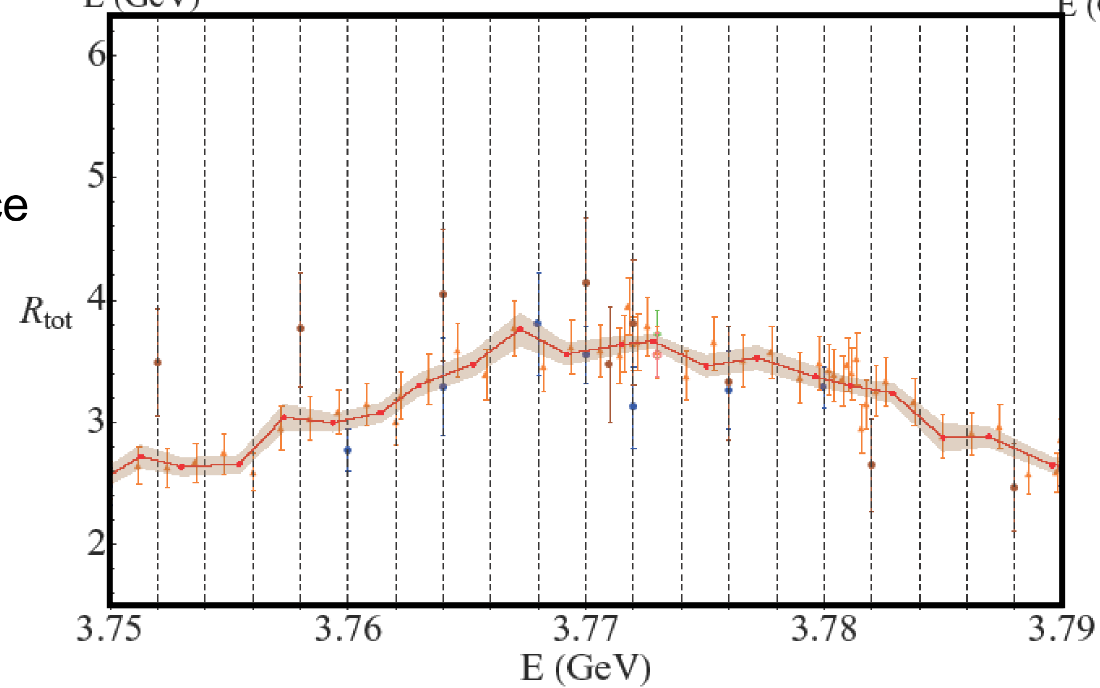
Fit results



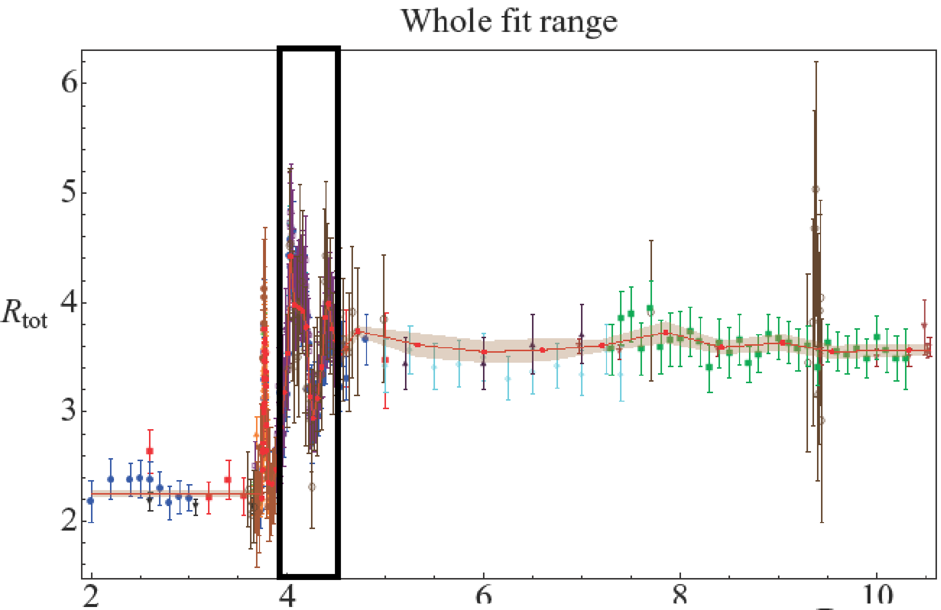


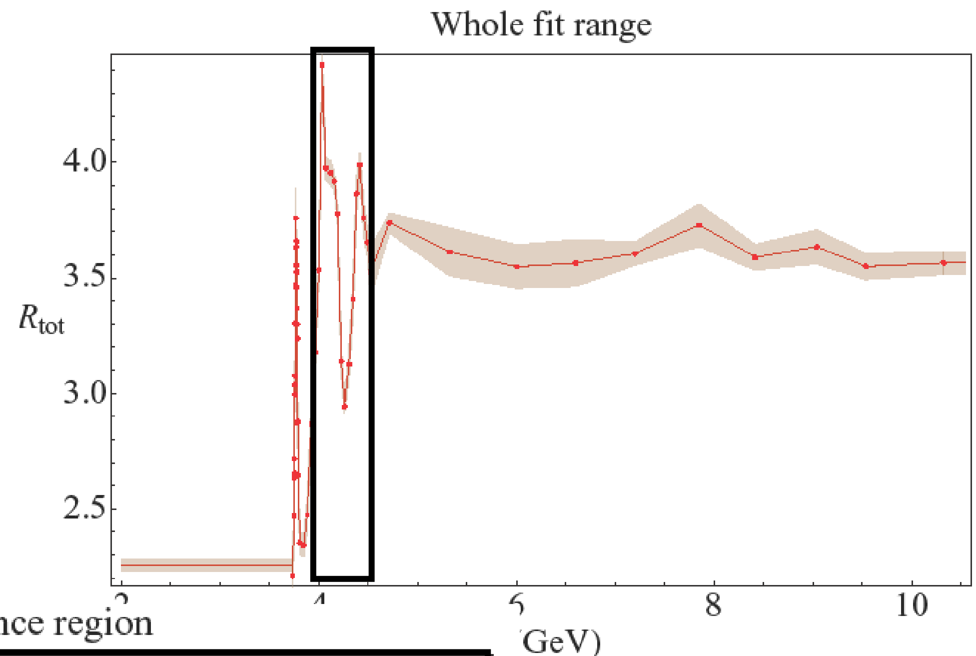
First resonance

20 clusters



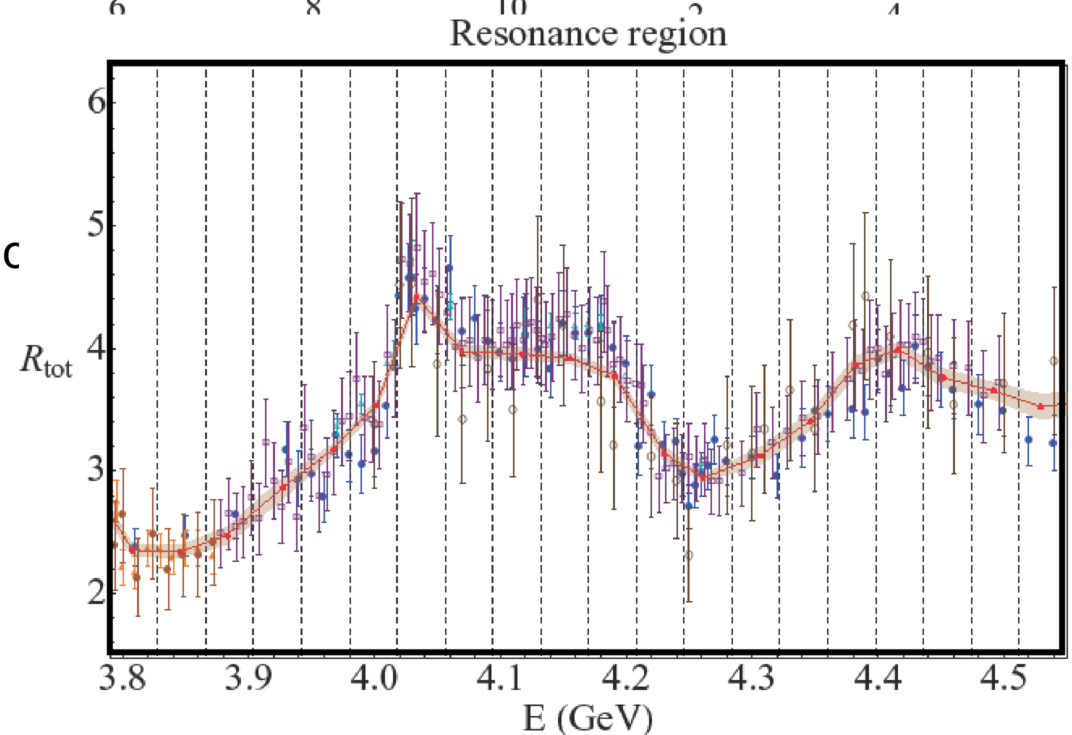
Fit results



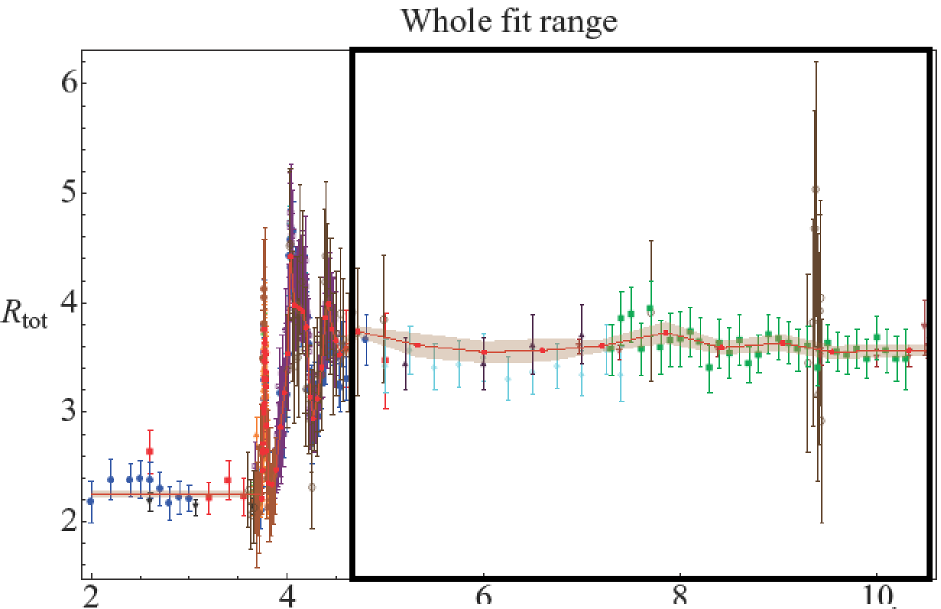


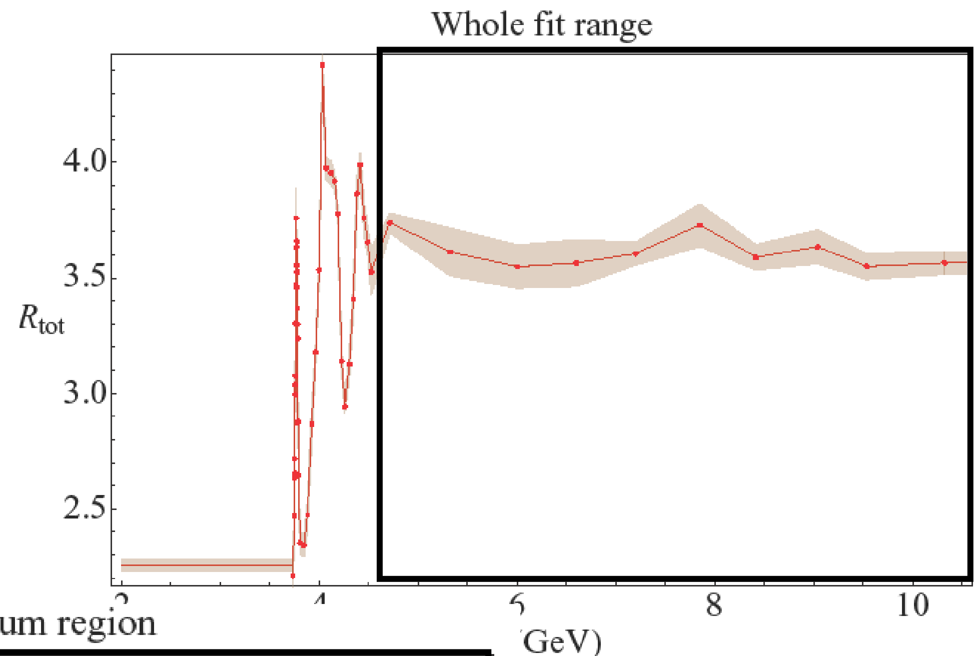
Second resonanc

20 clusters



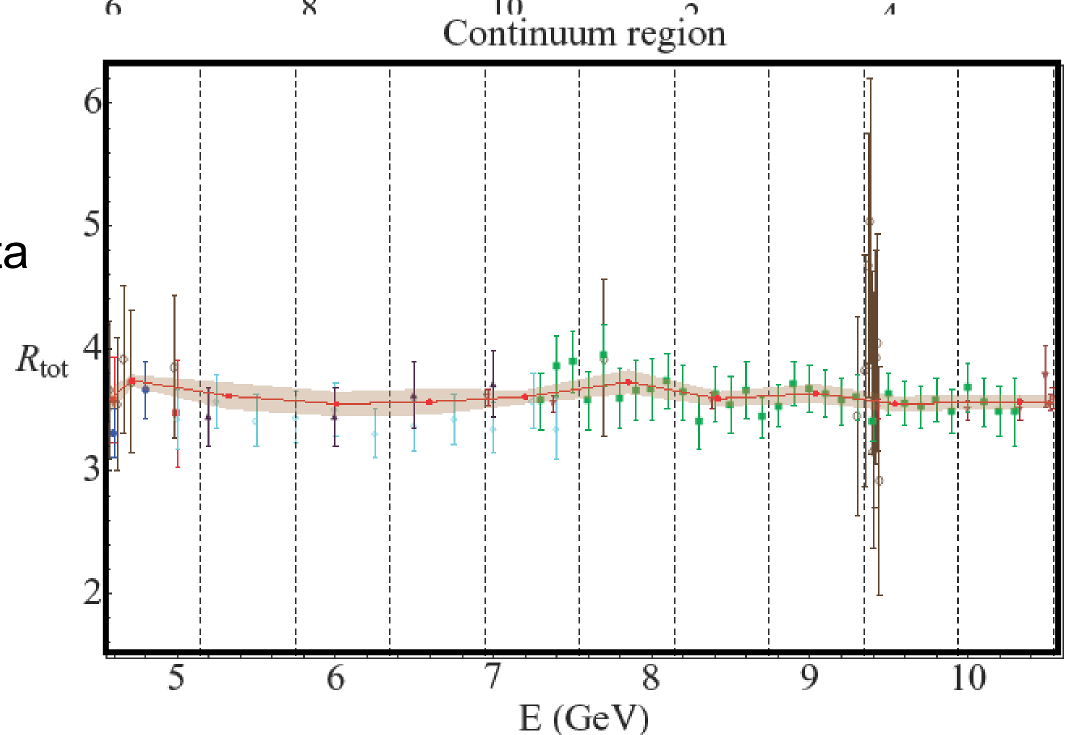
Fit results



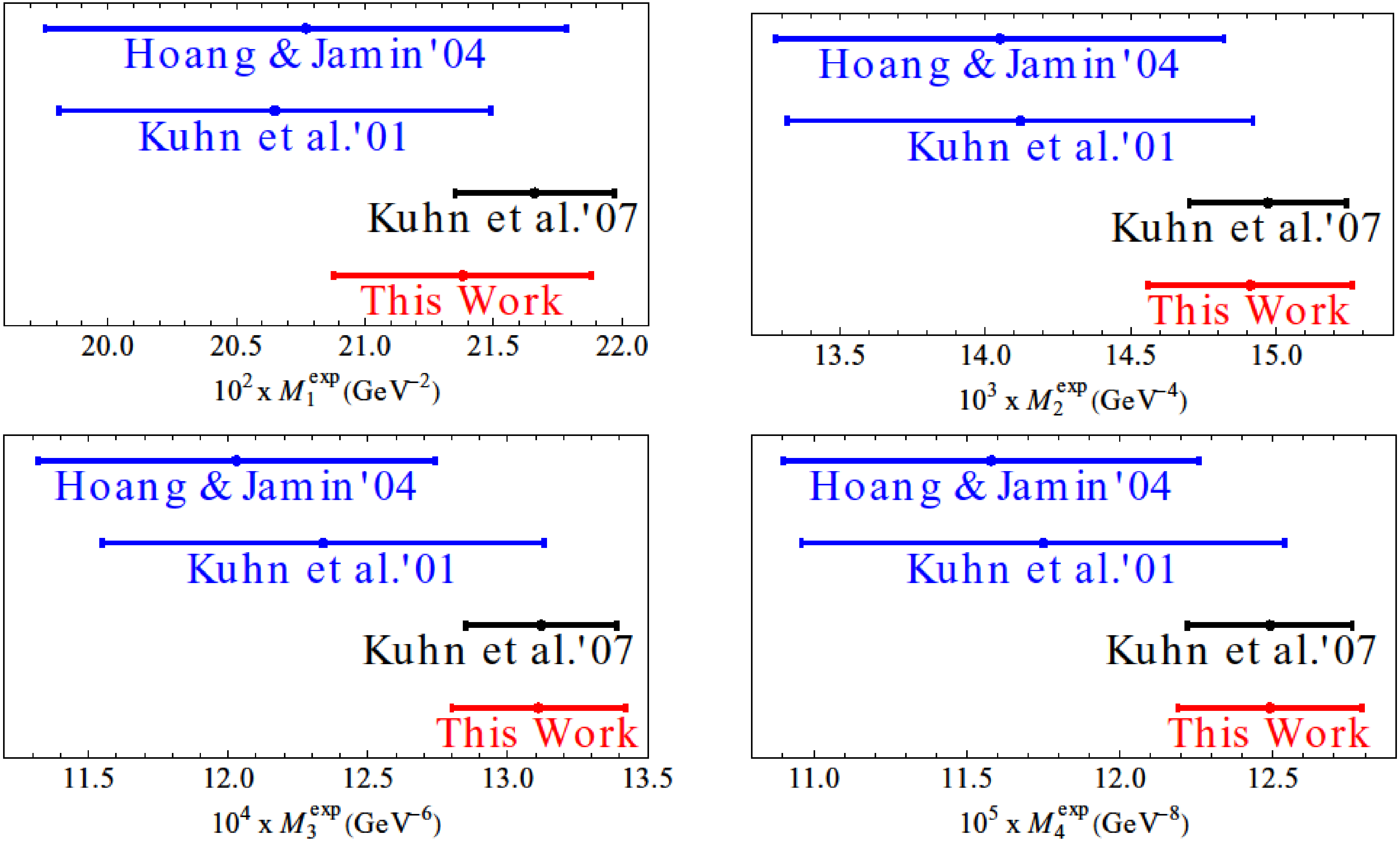


Continuum data

10 clusters



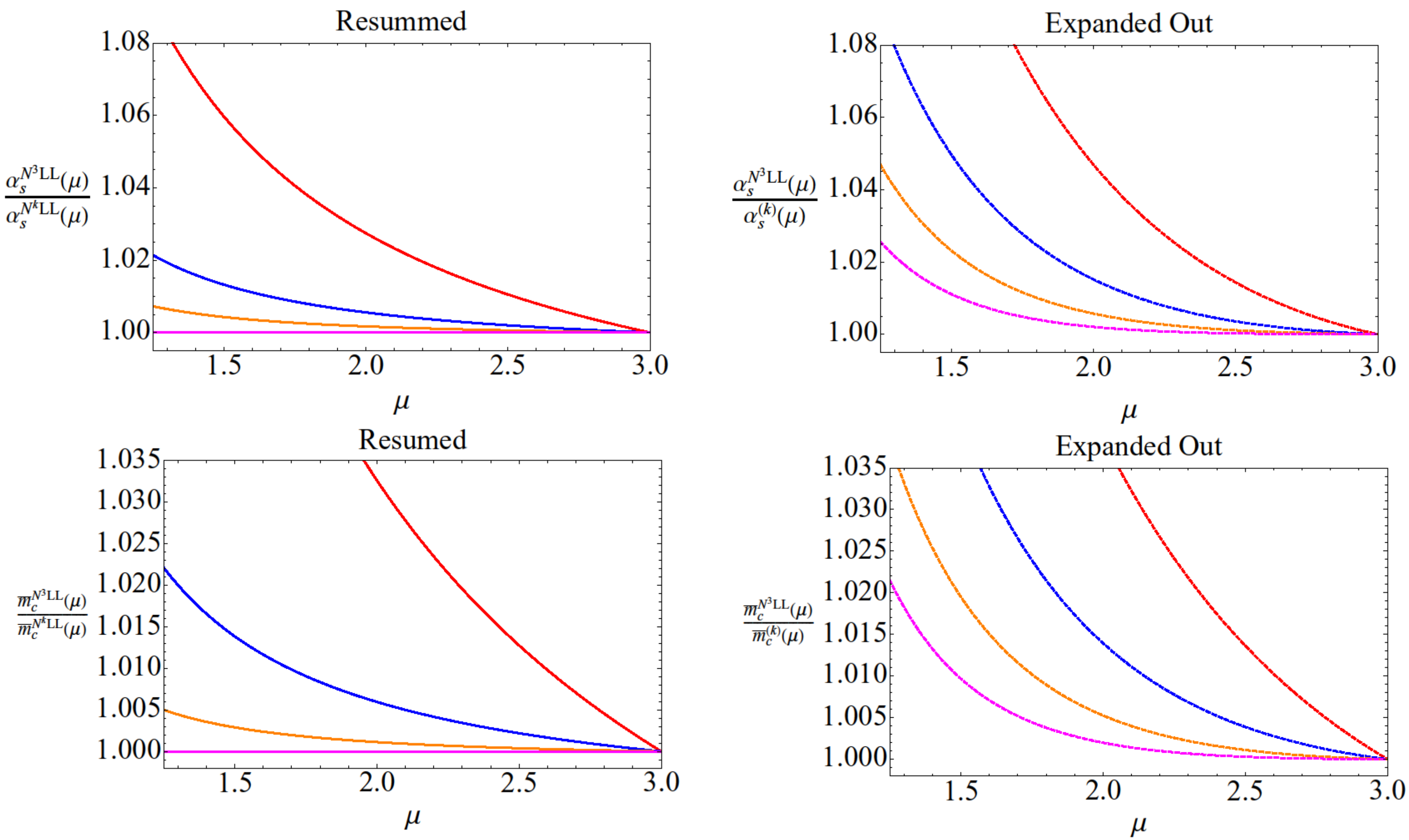
Comparison with other analyses



- Blue lines use outdated experimental data for narrow resonances.
- Different analyses tend to agree better for large n → Narrow resonances dominate

Theoretical developments

Mass and coupling running



- Excellent convergence of the running of quark masses and QCD coupling
- No failure of perturbative RG-evolution even down to 1 GeV

Use of $\overline{m}_c(\overline{m}_c)$ is fine!

Methods in perturbation theory

Fixed order

$$M_n = \frac{1}{\left[4\bar{m}_c^2(\mu_m)\right]^n} \sum_{i=0} \left(\frac{\alpha_s(\mu_\alpha)}{\pi}\right)^i \sum C_i^{a,b} \log^a\left[\frac{\bar{m}_c^2(\mu_m)}{\mu_m^2}\right] \log^b\left[\frac{\bar{m}_c^2(\mu_m)}{\mu_\alpha^2}\right] = M_n^{\exp}$$

Expanded out

$$\left(M_n\right)^{\frac{1}{2n}} = \frac{1}{2\bar{m}_c^2(\mu_m)} \sum_{i=0} \left(\frac{\alpha_s(\mu_\alpha)}{\pi}\right)^i \sum \tilde{C}_i^{a,b} \log^a\left[\frac{\bar{m}_c^2(\mu_m)}{\mu_m^2}\right] \log^b\left[\frac{\bar{m}_c^2(\mu_m)}{\mu_\alpha^2}\right] = \left(M_n^{\exp}\right)^{\frac{1}{2n}}$$

Iterative

$$\bar{m}_c^{(0)} = \left(\frac{M_n^{\exp}}{2C_{n,0}}\right)^{\frac{1}{2n}} = \frac{\left(M_n^{\exp}\right)^{\frac{1}{2n}}}{2\tilde{C}_{n,0}}$$

$$\bar{m}_c(\mu_m) = \bar{m}_c^{(0)} \left\{1 + \sum_{i=1} \left(\frac{\alpha_s(\mu_\alpha)}{\pi}\right)^i \sum \hat{C}_{n,i}^{a,b} \log^a\left[\frac{\bar{m}_c^{(0)\,2}}{\mu_m^2}\right] \log^b\left[\frac{\bar{m}_c^{(0)\,2}}{\mu_\alpha^2}\right]\right\}$$

Methods in perturbation theory

Fixed order $$M_n = \frac{1}{\left[4\bar{m}_c^2(\mu_m)\right]^n}\sum_{i=0}\left(\frac{\alpha_s(\mu_\alpha)}{\pi}\right)^i \sum C_i^{a,b}\log^a\left[\frac{\bar{m}_c^2(\mu_m)}{\mu_m^2}\right]\log^b\left[\frac{\bar{m}_c^2(\mu_m)}{\mu_\alpha^2}\right] = M_n^{\exp}$$

Numerical solution for mass:

sometimes there is no solution

Expanded out $$\left(M_n\right)^{\frac{1}{2n}} = \frac{1}{2\bar{m}_c^2(\mu_m)}\sum_{i=0}\left(\frac{\alpha_s(\mu_\alpha)}{\pi}\right)^i \sum \tilde{C}_i^{a,b}\log^a\left[\frac{\bar{m}_c^2(\mu_m)}{\mu_m^2}\right]\log^b\left[\frac{\bar{m}_c^2(\mu_m)}{\mu_\alpha^2}\right] = \left(M_n^{\exp}\right)^{\frac{1}{2n}}$$

Analytic solution for mass

always has a solution!

Iterative

$$\bar{m}_c^{(0)} = \left(\frac{M_n^{\exp}}{2C_{n,0}}\right)^{\frac{1}{2n}} = \frac{\left(M_n^{\exp}\right)^{\frac{1}{2n}}}{2\tilde{C}_{n,0}}$$

$$\bar{m}_c(\mu_m) = \bar{m}_c^{(0)}\left\{1+\sum_{i=1}\left(\frac{\alpha_s(\mu_\alpha)}{\pi}\right)^i \sum \hat{C}_{n,i}^{a,b}\log^a\left[\frac{\bar{m}_c^{(0)\,2}}{\mu_m^2}\right]\log^b\left[\frac{\bar{m}_c^{(0)\,2}}{\mu_\alpha^2}\right]\right\}$$

Methods in perturbation theory

Fixed order

$$M_n = \frac{1}{\left[4\bar{m}_c^2(\mu_m)\right]^n} \sum_{i=0} \left(\frac{\alpha_s(\mu_\alpha)}{\pi}\right)^i \sum C_i^{a,b} \log^a\left[\frac{\bar{m}_c^2(\mu_m)}{\mu_m^2}\right] \log^b\left[\frac{\bar{m}_c^2(\mu_m)}{\mu_\alpha^2}\right] = M_n^{\exp}$$

μ_α and μ_m independent

Expanded out

$$\left(M_n\right)^{\frac{1}{2n}} = \frac{1}{2\bar{m}_c^2(\mu_m)} \sum_{i=0} \left(\frac{\alpha_s(\mu_\alpha)}{\pi}\right)^i \sum \tilde{C}_i^{a,b} \log^a\left[\frac{\bar{m}_c^2(\mu_m)}{\mu_m^2}\right] \log^b\left[\frac{\bar{m}_c^2(\mu_m)}{\mu_\alpha^2}\right] = \left(M_n^{\exp}\right)^{\frac{1}{2n}}$$

Iterative

$$\bar{m}_c^{(0)} = \left(\frac{M_n^{\exp}}{2C_{n,0}}\right)^{\frac{1}{2n}} = \frac{\left(M_n^{\exp}\right)^{\frac{1}{2n}}}{2\tilde{C}_{n,0}}$$

$$\bar{m}_c(\mu_m) = \bar{m}_c^{(0)} \left\{1 + \sum_{i=1} \left(\frac{\alpha_s(\mu_\alpha)}{\pi}\right)^i \sum \hat{C}_{n,i}^{a,b} \log^a\left[\frac{\bar{m}_c^{(0)\,2}}{\mu_m^2}\right] \log^b\left[\frac{\bar{m}_c^{(0)\,2}}{\mu_\alpha^2}\right]\right\}$$

Methods in perturbation theory

Fixed order

$$M_n = \frac{1}{\left[4\bar{m}_c^2(\mu_m)\right]^n}\sum_{i=0}\left(\frac{\alpha_s(\mu_\alpha)}{\pi}\right)^i\sum C_i^{a,b}\log^a\left[\frac{\bar{m}_c^2(\mu_m)}{\mu_m^2}\right]\log^b\left[\frac{\bar{m}_c^2(\mu_m)}{\mu_\alpha^2}\right] = M_n^{\exp}$$

μ_α and μ_m independent

residual μ_α and μ_m dependence due to truncation of α series

Expanded out

$$\left(M_n\right)^{\frac{1}{2n}} = \frac{1}{2\bar{m}_c^2(\mu_m)}\sum_{i=0}\left(\frac{\alpha_s(\mu_\alpha)}{\pi}\right)^i\sum \tilde{C}_i^{a,b}\log^a\left[\frac{\bar{m}_c^2(\mu_m)}{\mu_m^2}\right]\log^b\left[\frac{\bar{m}_c^2(\mu_m)}{\mu_\alpha^2}\right] = \left(M_n^{\exp}\right)^{\frac{1}{2n}}$$

- residual μ_α dependence
- renders correct μ_m dependence to the order of truncation

Iterative

$$\bar{m}_c^{(0)} = \left(\frac{M_n^{\exp}}{2C_{n,0}}\right)^{\frac{1}{2n}} = \frac{\left(M_n^{\exp}\right)^{\frac{1}{2n}}}{2\tilde{C}_{n,0}}$$

$$\bar{m}_c(\mu_m) = \bar{m}_c^{(0)}\left\{1+\sum_{i=1}\left(\frac{\alpha_s(\mu_\alpha)}{\pi}\right)^i\sum \hat{C}_{n,i}^{a,b}\log^a\left[\frac{\bar{m}_c^{(0)\,2}}{\mu_m^2}\right]\log^b\left[\frac{\bar{m}_c^{(0)\,2}}{\mu_\alpha^2}\right]\right\}$$

Contour improved analysis

First applied to hadronic tau decays Liberder & Pich ('92)

Now μ depends on s $\rightarrow$ rearrangement of higher order contributions

Reweights threshold versus continuum effects

$$\mu_\alpha^2 \rightarrow \mu_\alpha^2(1-z) \qquad z = \frac{q^2}{4\bar{m}_c^2(\mu_m)}$$

Residual dependence on μ_α

2 - loops

Hoang, Jamin (2004)

Contour improved analysis

First applied to hadronic tau decays Liberder & Pich ('92)

Now μ depends on s $\rightarrow$ rearrangement of higher order contributions

Reweights threshold versus continuum effects

$$\mu_\alpha^2 \rightarrow \mu_\alpha^2 (1-z) \qquad z = \frac{q^2}{4\,\overline{m}_c^2(\mu_m)}$$

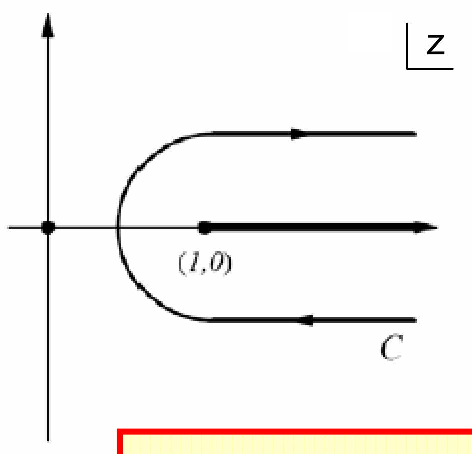
Residual dependence on μ_α

2 - loops

Hoang, Jamin

(2004)

Calculations $\longrightarrow$ easy to understand through vacuum polarization function



However one can derive analytic expressions (!) using properties of the running of the strong coupling constant.

Contour improved methods are (perturbatively) sensitive to the value of $\Pi(0)$

Nonperturbative contribution

$$M_n^{\rm th} = \boxed{M_n^{\rm pert}} + \boxed{\Delta M_n^{\langle G^2 \rangle}} + \ \ldots$$

already discussed

gluon condensate distribution

$$\Delta M_n^{\langle G^2\rangle} = \frac{1}{(4\overline{m}_c^2(\mu_m))^{n+2}} \boxed{\left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle_{\rm RGI}} \left\{ a_n^{0,0} + \frac{\alpha_s(\mu_\alpha)}{\pi} \left[a_n^{1,0} + a_n^{1,1} \log \frac{\overline{m}_c^2(\mu_m)}{\mu_m^2} \right] \right\}$$

Renormalization group invariant scheme for the gluon condensate

$$\left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle_{\rm RGI} = \underbrace{0.006 \pm 0.012 \ {\rm GeV}^4}$$

200% error

Compatible with 0

Contribution to the moments

n=1	n=2	n=3	n=4
0.02%	0.05%	0.08%	0.1%

State of the art of calculations

- For n=1,2,3 the $C_n^{0,0}$ coefficients are known at O(α_s^3)
- For n ≥ 4, $C_n^{0,0}$ are known in a semianalytic aproach (Padé approximants)

this method renders a central value and an error

- The rest of $C_n^{a,b}$ can be deduced by RGE evolution

Kühn et al, Maier et al,

Boughezal et al

Hoang, VM & Zebarjad

Kiyo et al

Greynat et al

State of the art of calculations

Kühn et al, Maier et al, Boughezal et al

- For n=1,2,3 the $C_n^{0,0}$ coefficients are known at $O(\alpha_s^3)$
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Hoang, VM & Zebarjad

Kiyo et al

Greynat et al

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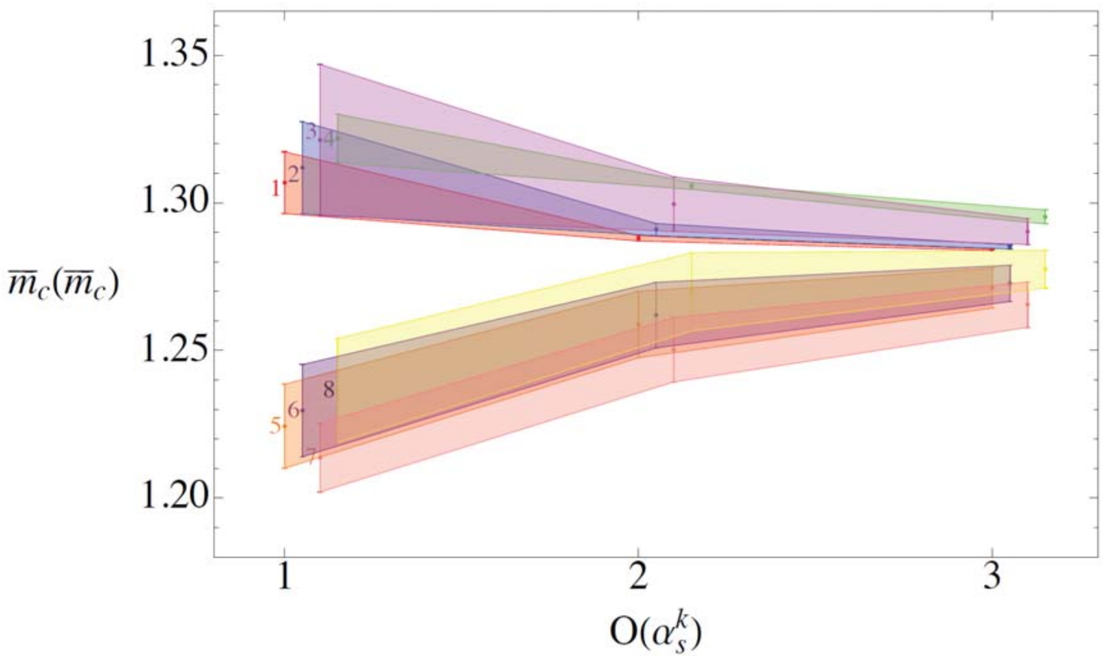
A first look into the various methods



State of the art of calculations

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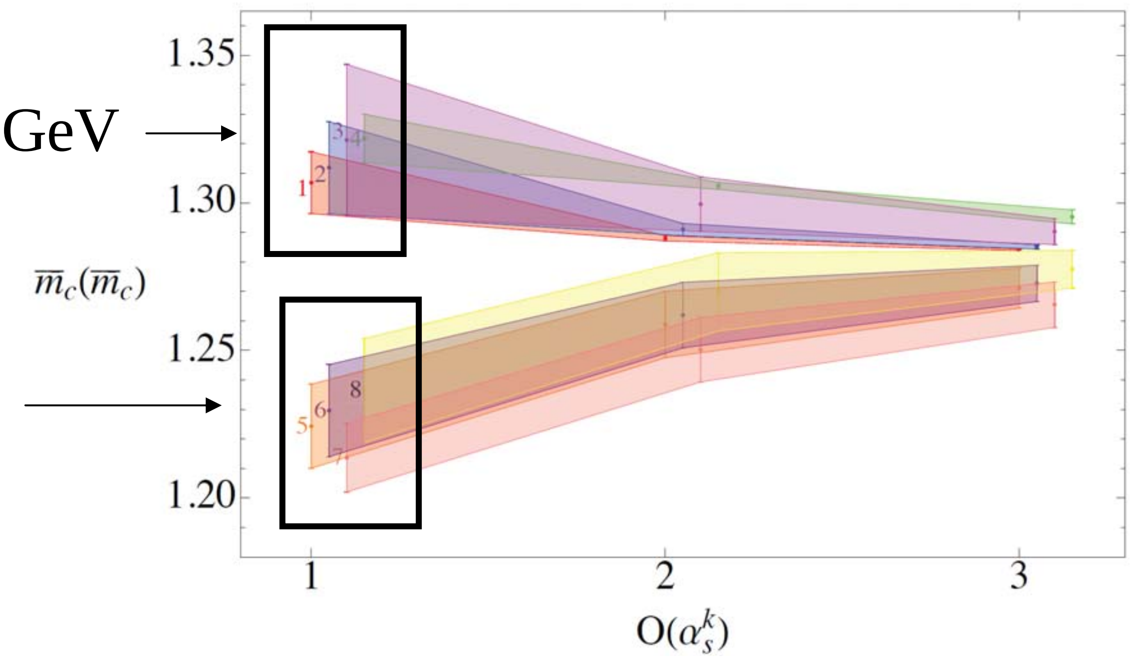
this method renders a central value and an error Hoang, VM & Zebarjad

- The rest of $C_n^{a,b}$ can be deduced by RGE evolution Kiyo et al Greynat et al

A first look into the various methods

$$2\,\mathrm{GeV} \le \mu_m = \mu_\alpha \le 4\,\mathrm{GeV}$$

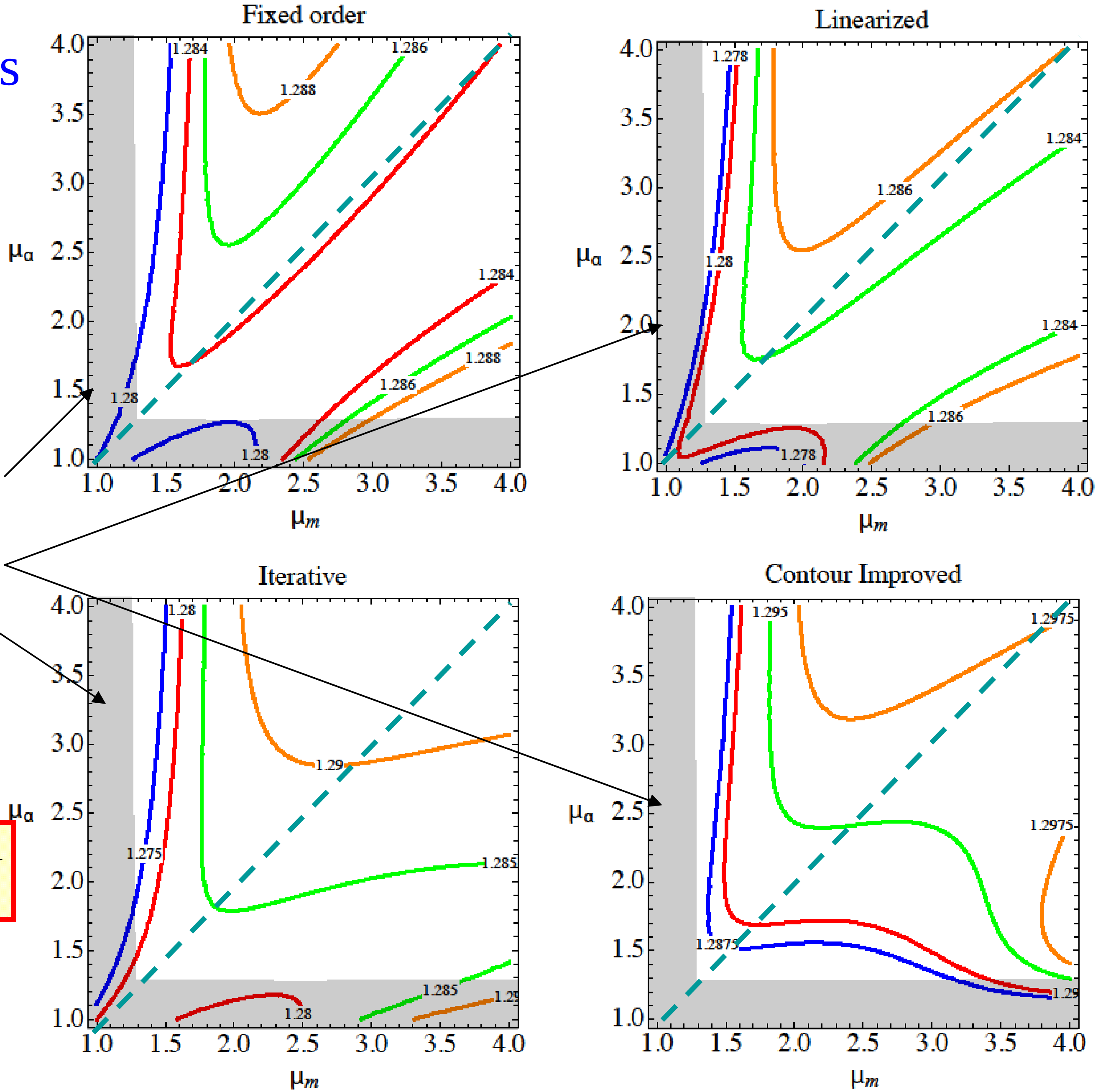
$$\mu_m = \overline{m}_c(\overline{m}_c)$$
$$2\,\mathrm{GeV} \le \mu_\alpha \le 4\,\mathrm{GeV}$$



Contours in the $\mu_\alpha - \mu_m$ plane

$O(\alpha_s^3)$ analyses

first moment



Exclude regions with $\mu_m, \mu_\alpha < \overline{m}_c(\overline{m}_c)$

$\overline{m}_c(\overline{m}_c) \leq \mu_m, \mu_\alpha \leq 4\,\text{GeV}$

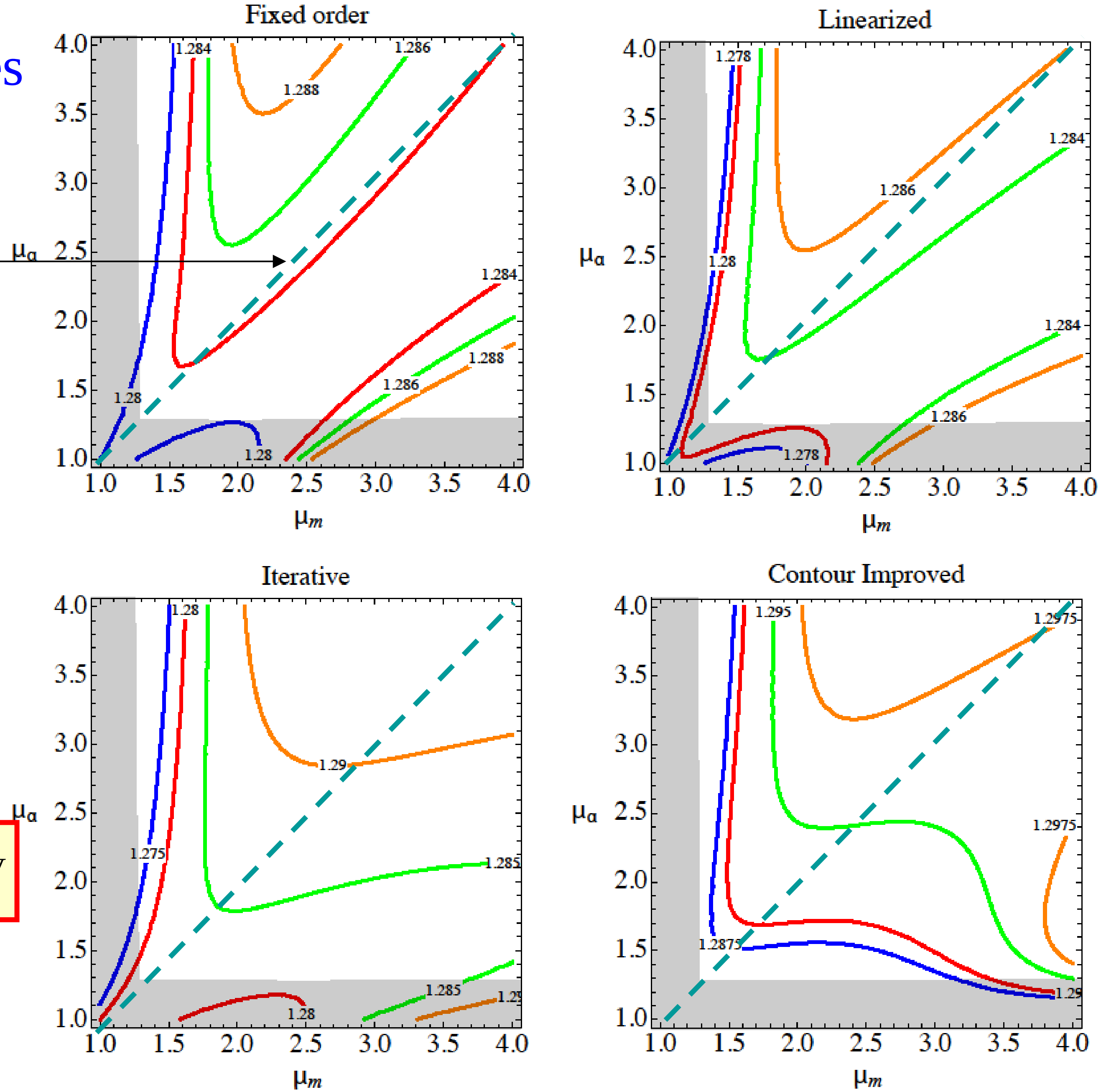
Contours in the $\mu_\alpha - \mu_m$ plane

$O(\alpha_s^3)$ analyses

first moment

Kühn et al path !

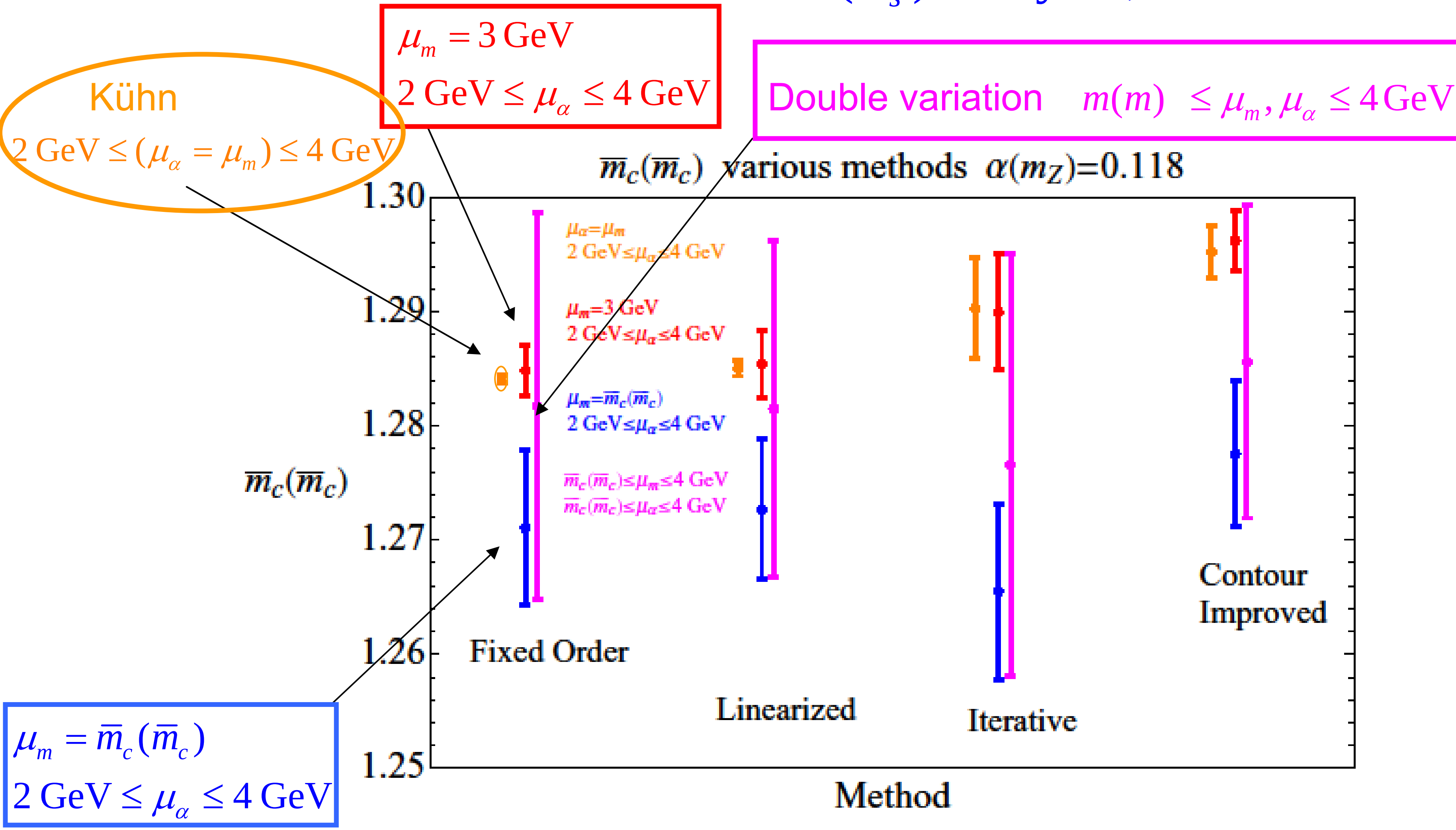
$\mu_\alpha = \mu_m$



$\overline{m}_c(\overline{m}_c) \le \mu_m, \mu_\alpha \le 4\,\mathrm{GeV}$

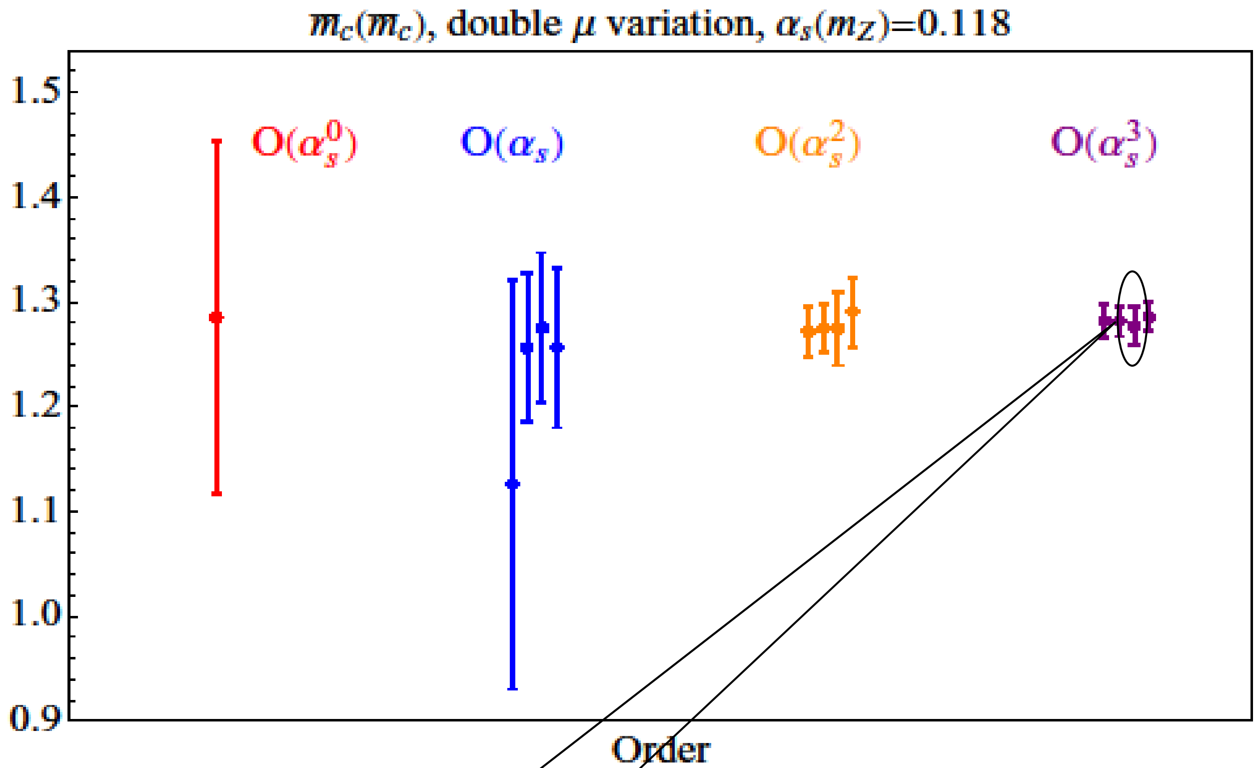
Various error estimates

$O(\alpha_s^3)$ analyses, first moment



Results

Convergence of errors



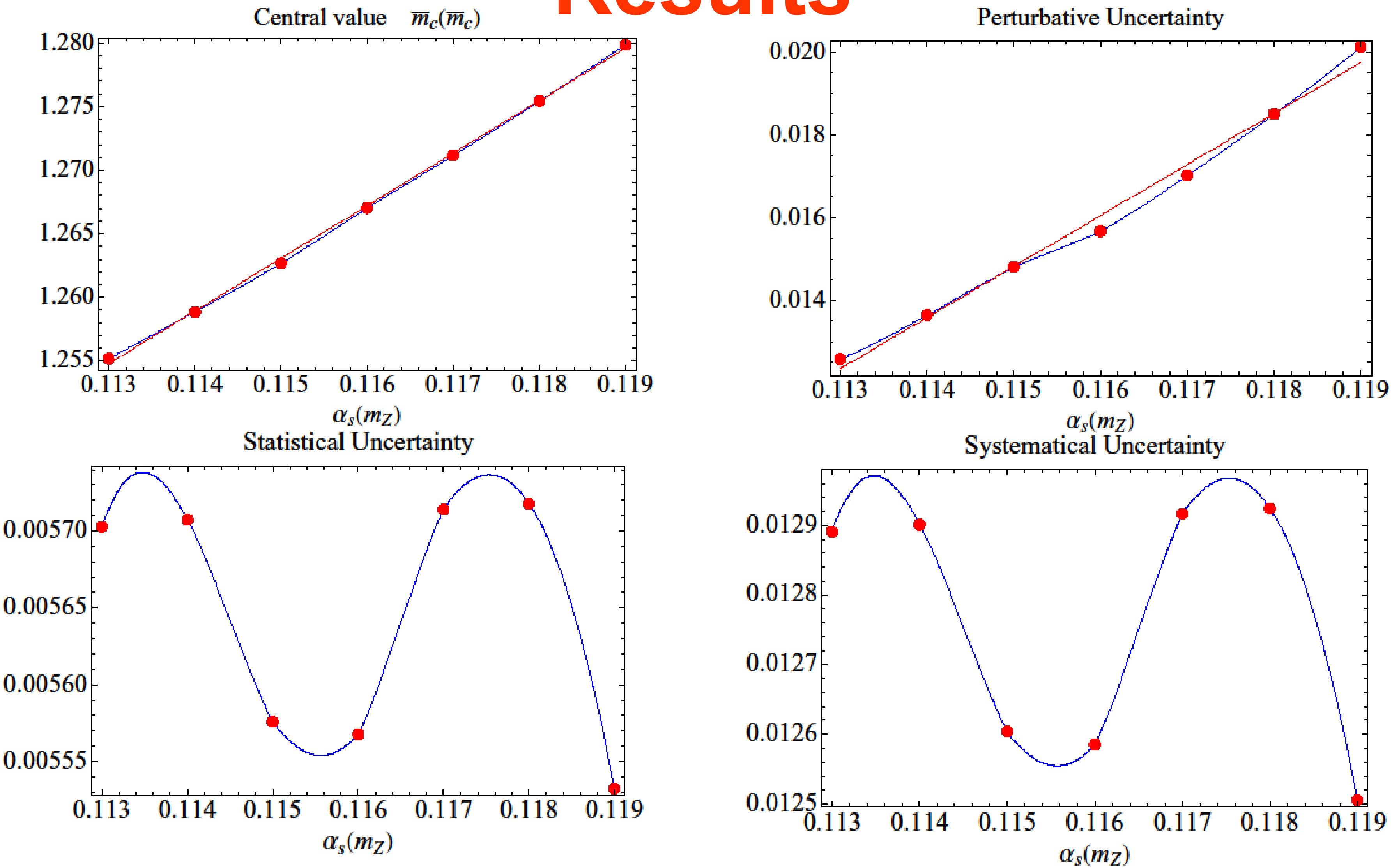
Using double variation all methods have similar values and errors

Result for $\alpha_s(m_Z) = 0.1184 \pm 0.0021$

$$\bar{m}_c(\bar{m}_c) = 1.277 \pm 0.006_{\text{stat}} \pm 0.013_{\text{sys}} \pm 0.019_{\text{th}} \pm 0.009_{\alpha} \pm 0.002_{\langle GG \rangle}$$

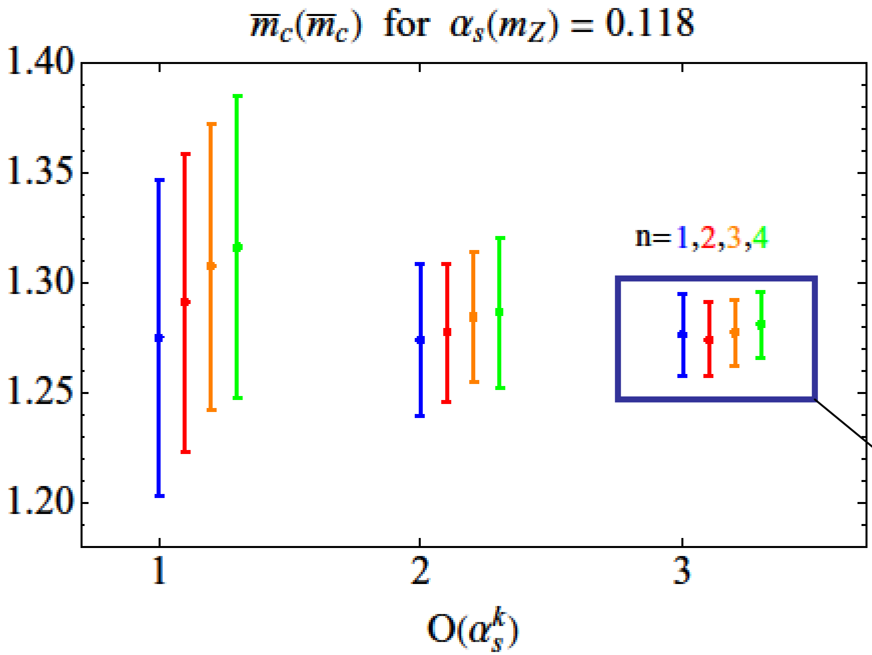
$$= 1.277 \pm 0.025$$

Results



$$\overline{m}_c(\overline{m}_c) = (0.788 + 4.13 \times \alpha_s(m_Z)) \pm (0.006)_{\text{stat}} \pm (0.013)_{\text{syst}}$$
$$\pm (-0.127 + 1.23 \times \alpha_s(m_Z))_{\text{pert}} \pm (0.009)_{\alpha_s} \pm (0.002)_{\langle GG \rangle}$$

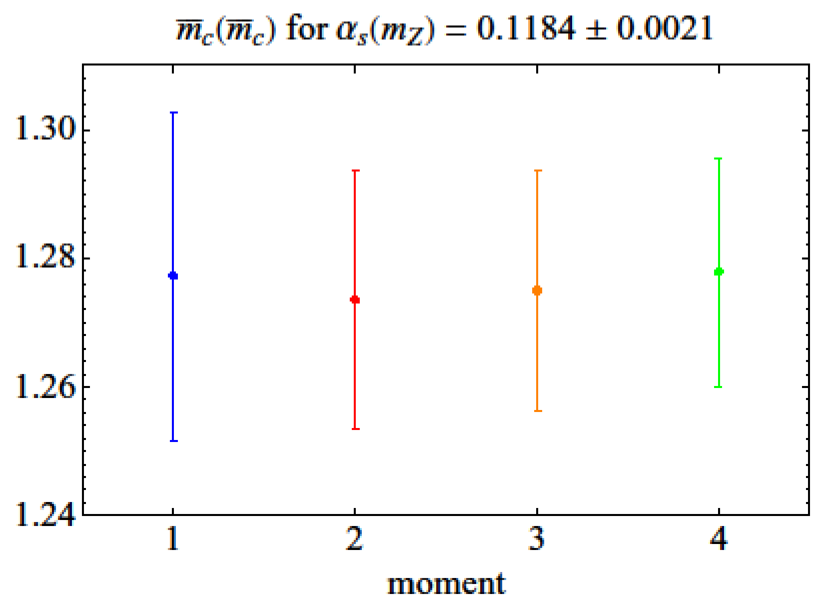
Glimpse on higher moments



Using only iterative method

Only perturbative errors

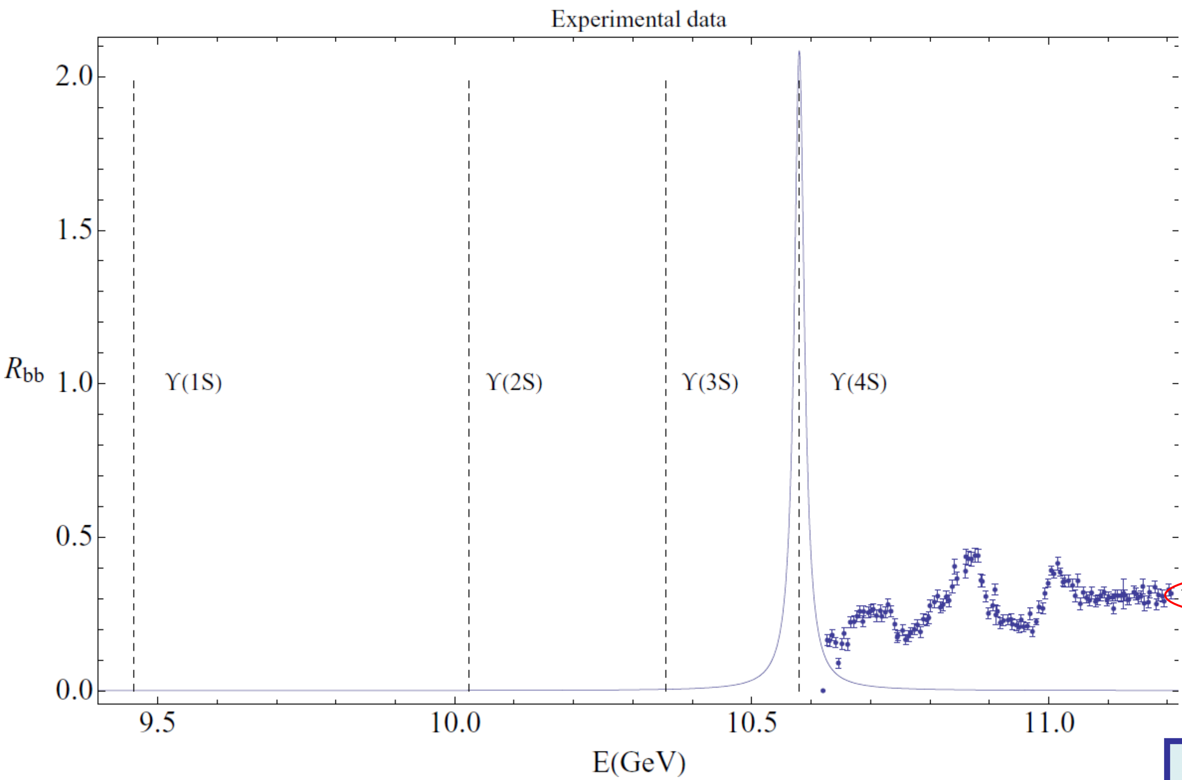
Results including gluon condensate

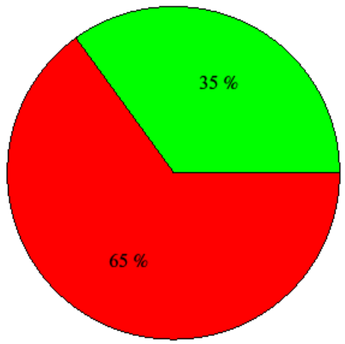


- Perfect agreement for central values
- Very similar error bars

Situation for bottom?

Perturbation theory





Perturbative QCD

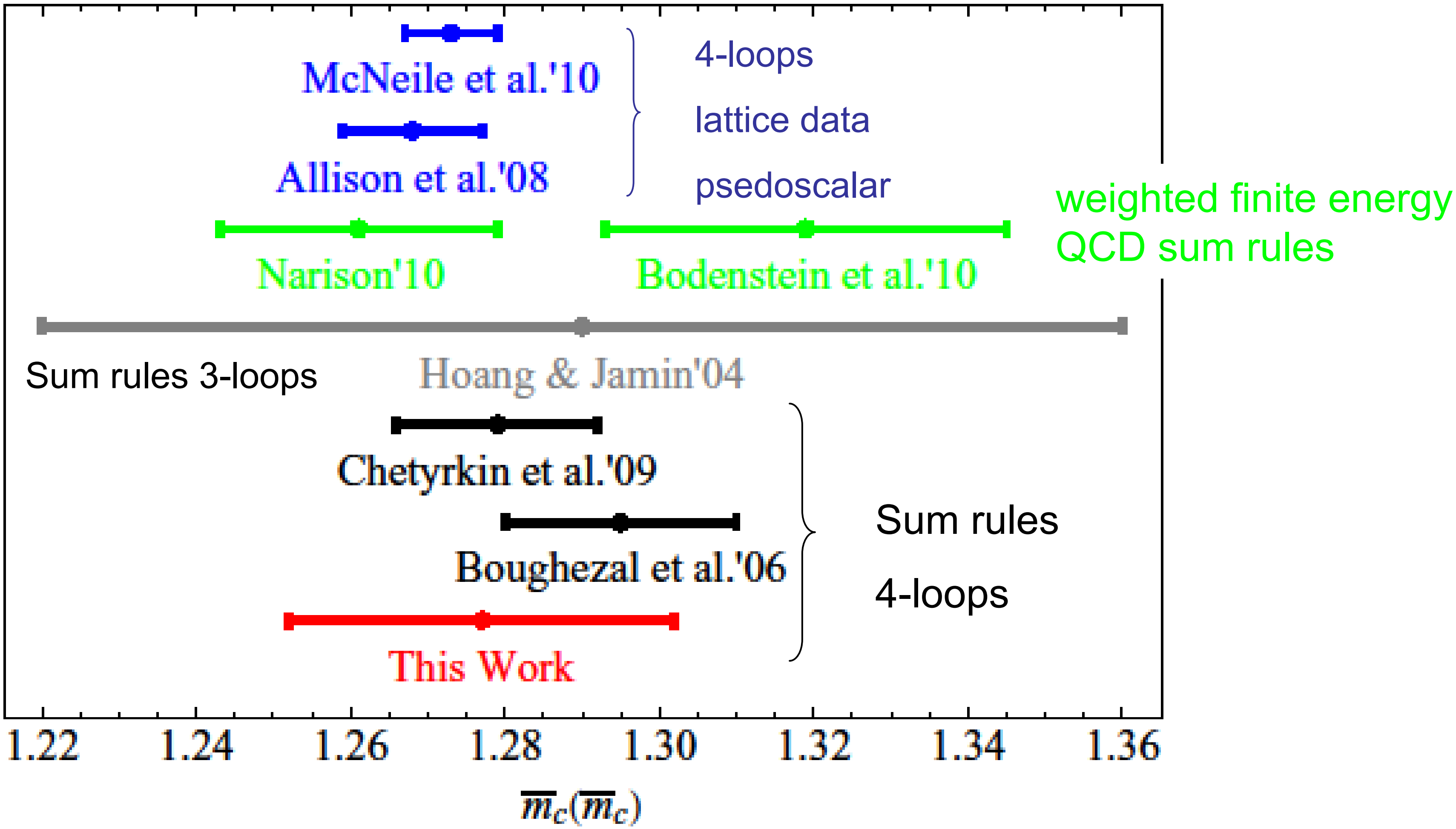
65% of the first moment for bottom sum rules !!

Aren't we comparing theory to theory?

10% error gives a huge error to the total moment

+ same issues with perturbative analysis

Comparison to similar analyses



Comparison to similar analyses

Sometimes it is hard to compare because different analyses use different values of $\alpha_s(m_Z)$

	$\overline{m}_c(\overline{m}_c)$	$\alpha_s(m_Z)$ used	$\overline{m}_c(\overline{m}_c)^{\alpha_s(m_Z)=0.1180}$
This work	1.277 ± 0.025	0.1184 ± 0.0021	1.275 ± 0.023
Chetyrkin et al. [35]	1.279 ± 0.013	0.1189 ± 0.0020	1.277 ± 0.012
Boughezal et al. [9]	1.295 ± 0.015	0.1182 ± 0.0027	–
Hoang & Jamin [34]	1.29 ± 0.07	0.1180 ± 0.0030	1.29 ± 0.07
Bodenstein et al. [61]	1.319 ± 0.026	0.1213 ± 0.0014	1.295 ± 0.026
Narison [63]	1.261 ± 0.018	0.1191 ± 0.0027	–
Allison et al. [38]	1.268 ± 0.009	0.1174 ± 0.0012	–
McNeile et al. [39]	1.273 ± 0.006	0.1183 ± 0.0007	–

These lattice analyses simultaneously fit for $\overline{m}_c(\overline{m}_c)$ and $\alpha_s(m_Z)$

Extrapolation to a common α_s value

Conclusions and outlook

- It is essential to have a reliable error estimate for the charm mass.
- Concerning relativistic sum rules, a revision of perturbative errors was mandatory.
- Experimental input must be treated with care (combining various sets of data, correlations, systematic errors …)
- Perturbative QCD should be used only where there is no data, and asigning a conservative error.

  - For charm PQCD is only a small fraction of the moment → small impact.
- The analysis can be easily extended to other correlators connection to lattice
- It can also be used to determine the bottom mass

Stay tunned for updated numbers on charm, and for results on bottom mass and pseudoscalar correlators.

Result for $\alpha_s(m_Z) = 0.1184 \pm 0.0021$

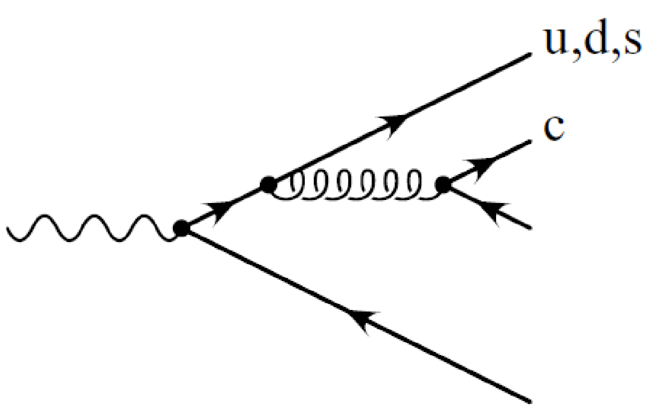
$$\overline{m}_c(\overline{m}_c) = 1.277 \pm 0.006_{\rm stat} \pm 0.013_{\rm sys} \pm 0.019_{\rm th} \pm 0.009_{\alpha} \pm 0.002_{\langle GG \rangle}$$

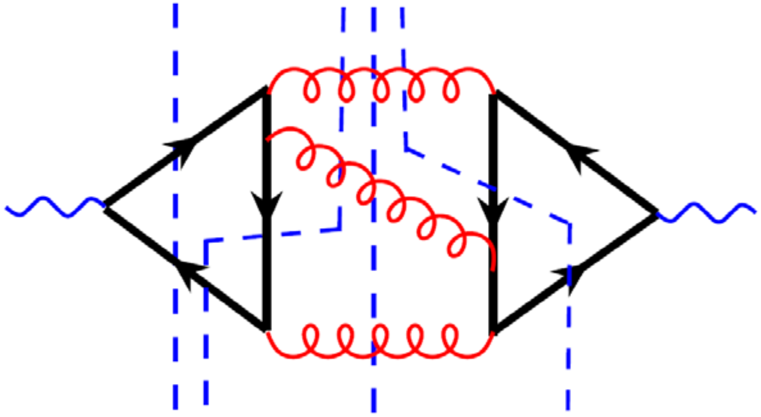
$$= 1.277 \pm 0.025$$

Back up slides

Size of neglected terms

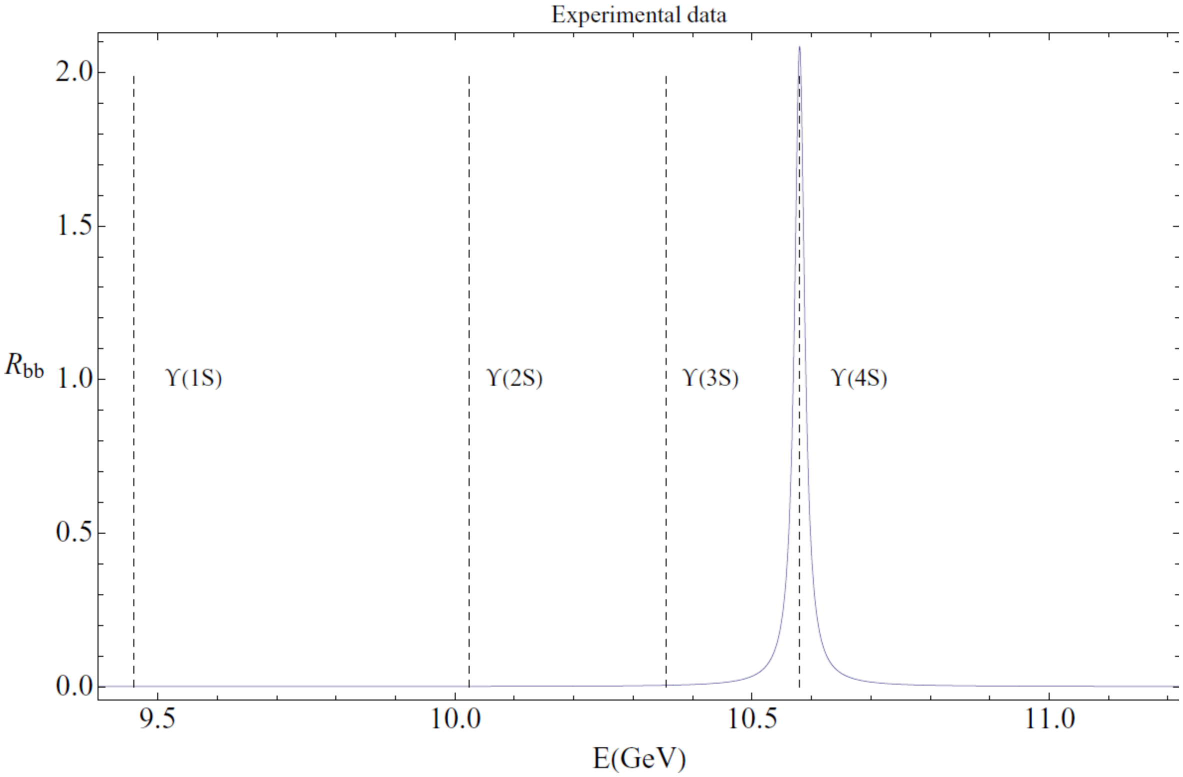
n	Mass corrections	Secondary Radiation	Singlet	Z-boson
1	0.02	0.038	3×10^{-4}	0.006
2	0.001	9×10^{-4}	2×10^{-5}	0.004
3	1×10^{-4}	4×10^{-5}	2×10^{-6}	0.003
4	8×10^{-6}	3×10^{-6}	1×10^{-7}	0.003





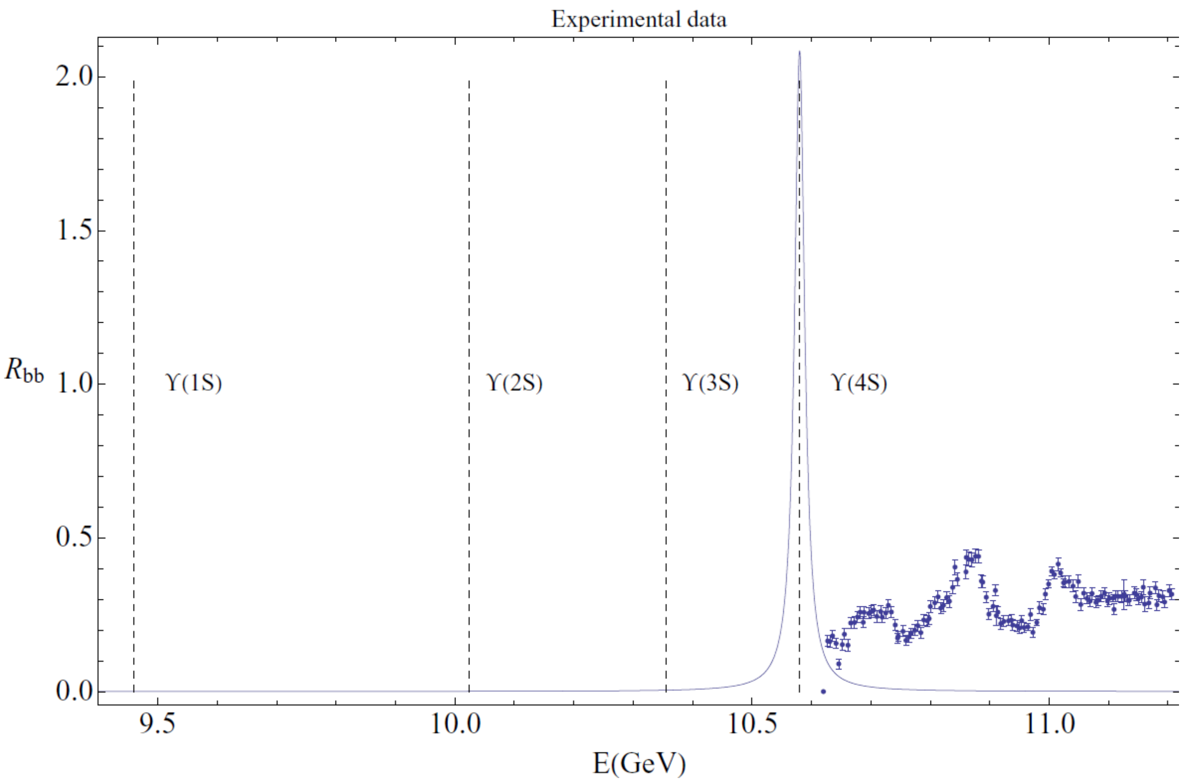
Experimental data: bottom

Narrow resonances



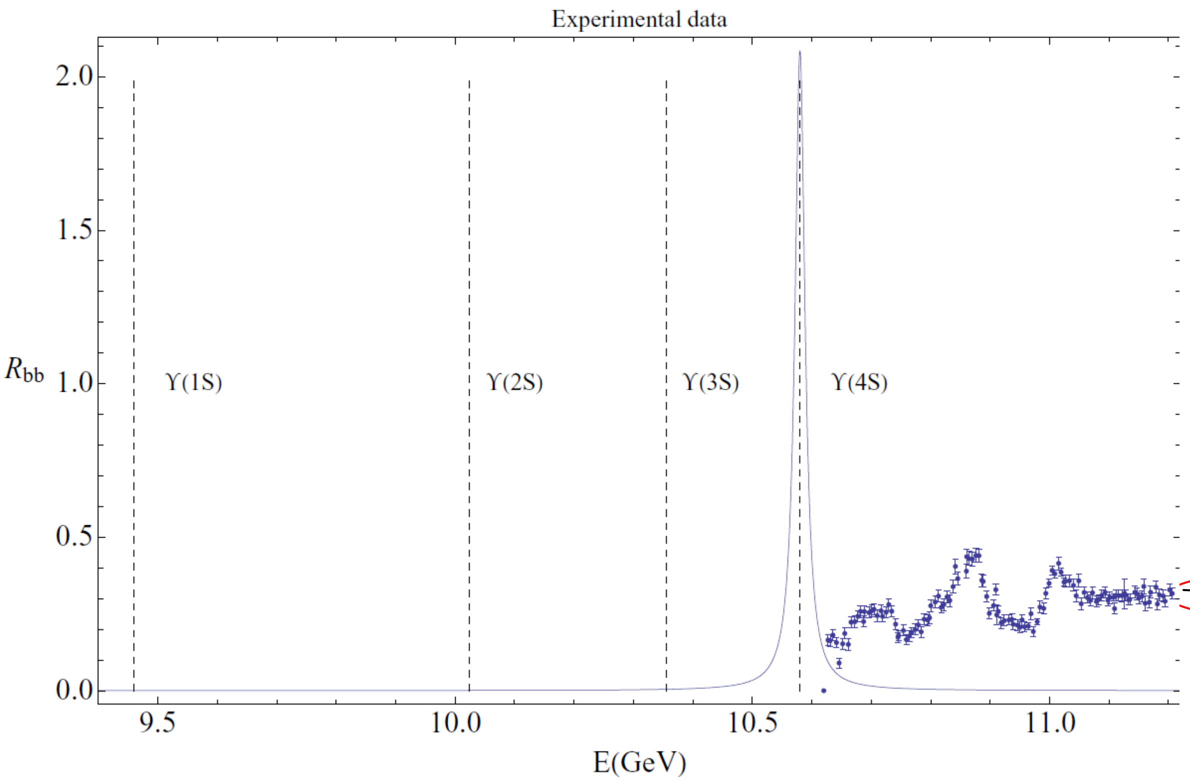
Experimental data: bottom

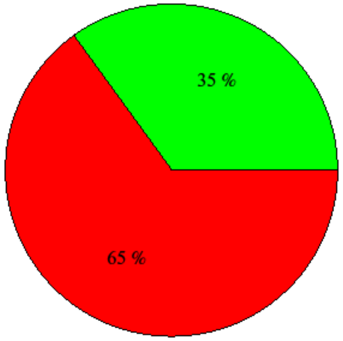
Babar data



Experimental data: bottom

Perturbation theory





Perturbative QCD

Aren't we comparing theory to theory?

10% error gives a huge error to the total moment

65% of the first moment for bottom sum rules !!

Stability of choices

Default: widths 50% correlated among themselves and with the continuous data sets.

For those sets with no information on correlations, assume a 100% correlation.

Different correlation between narrow resonances and data

Different correlation for some datasets

	Default	Minimal Overlap	No Correlation	50% Correlation	Uncorrelated
$n = 1$	21.38(20|46)	21.38(20|37)	21.38(20|30)	21.24(22|47)	21.09(28|25)
$n = 2$	14.91(18|29)	14.91(18|25)	14.91(18|22)	14.87(19|30)	14.84(20|21)
$n = 3$	13.10(19|25)	13.10(19|21)	13.10(19|22)	13.10(19|25)	13.10(19|21)
$n = 4$	12.49(19|23)	12.48(19|21)	12.49(19|21)	12.49(19|23)	12.49(19|21)

Different cluster energy definition

Different clustering

	Default	Regular average	Middle point	(2, 20, 40, 10)	(2, 10, 20, 10)	(2, 20, 20, 20)
$n = 1$	21.38(20|46)	21.37(20|46)	21.40(20|45)	21.38(20|46)	21.39(20|46)	21.34(20|46)
$n = 2$	14.91(18|29)	14.90(18|29)	14.91(18|29)	14.91(18|30)	14.91(18|29)	14.88(18|29)
$n = 3$	13.10(19|25)	13.10(19|25)	13.11(19|25)	13.11(19|25)	13.11(19|25)	13.08(19|25)
$n = 4$	12.49(19|23)	12.48(19|23)	12.49(19|23)	12.49(19|23)	12.49(19|23)	12.47(19|23)