Towards NNLO corrections to jet cross sections in hadronic collisions

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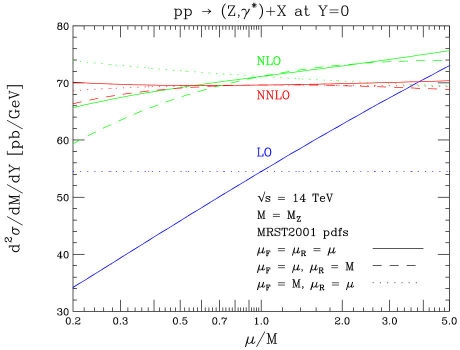
OUTLINE

- Motivation - why go beyond NLO
- NNLO ingredients of jet cross sections at hadron colliders
- Antenna subtraction method
- Towards pp$\rightarrow$ 2j at NNLO
- Conclusions and future work

WHY GO BEYOND NLO?

- improve the theoretical prediction truncated at NLO and reduce the sensitivity of the predictions on renormalisation and factorisation scales



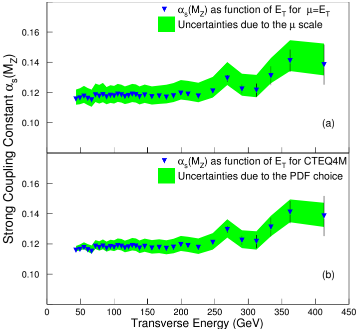
On-shell Z boson production at the LHC
[C.Anastasiou, L.Dixon, K.Melnikov, F.Petriello '03]

$pp \to 2j$ AT NNLO: MOTIVATIONS

- benchmark reaction: measurement to a few percent accuracy
- theoretical prediction with the same precision as the experimental data

  - α_s determination from hadronic jet observables limited by the unknown higher order corrections
  - CDF run I data gives

$$\alpha_s(M_Z) \quad = \quad 0.1178 \pm 0.0001(\text{stat})^{+0.0081}_{-0.0095}(\text{sys})$$
$$^{+0.0071}_{-0.0047}(\text{scale}) \pm 0.0059(\text{pdf})$$



- high E_T jet data constrains the gluon pdf
  - size of NNLO correction important for precise determination of parton distribution functions
- new physics signals in dijet mass distributions
- insight into infrared structure of QCD at NNLO

NNLO INGREDIENTS

QCD jet cross section perturbative expansion at hadron colliders

$$\mathrm{d}\sigma = \sum_{i,j} \int \left[\mathrm{d}\hat{\sigma}_{ij}^{LO} + \left(\frac{\alpha_s}{2\pi}\right) \mathrm{d}\hat{\sigma}_{ij}^{NLO} + \left(\frac{\alpha_s}{2\pi}\right)^2 \mathrm{d}\hat{\sigma}_{ij}^{NNLO} + \mathcal{O}(\alpha_s^3) \right] f_i(x_1) f_j(x_2) dx_1 dx_2$$

NNLO m-jet corrections contains three contributions:

$$\begin{aligned} \mathrm{d}\hat{\sigma}_{NNLO} \sim & \int \left[\langle \mathcal{M}^{(0)} | \mathcal{M}^{(0)} \rangle \right]_{m+4} \mathrm{d}\Phi_{m+2}\, J_m^{(m+2)} \\ & + \int \left[\langle \mathcal{M}^{(0)} | \mathcal{M}^{(1)} \rangle + \langle \mathcal{M}^{(1)} | \mathcal{M}^{(0)} \rangle \right]_{m+3} \mathrm{d}\Phi_{m+1}\, J_m^{(m+1)} \\ & + \int \left[\langle \mathcal{M}^{(1)} | \mathcal{M}^{(1)} \rangle + \langle \mathcal{M}^{(0)} | \mathcal{M}^{(2)} \rangle + \langle \mathcal{M}^{(2)} | \mathcal{M}^{(0)} \rangle \right]_{m+2} \mathrm{d}\Phi_{m}\, J_m^{(m)} \end{aligned}$$

$[\langle \mathcal{M}^{(i)} | \mathcal{M}^{(j)} \rangle]_M$ is the interference of M-particle i-loop and j-loop amplitudes

- three-loop splitting functions required for the evolution of parton distribution functions at NNLO

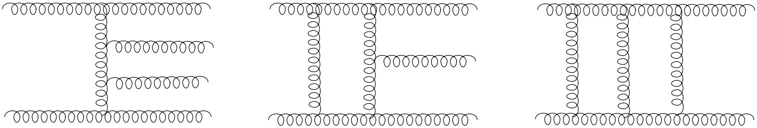
$$\frac{d}{dln\mu^2} f_i(x,\mu^2) = \sum_k \left[P_{ik}(\alpha_S(\mu^2)) \otimes f_k(\mu^2) \right](x)$$

$$P = \alpha_s P^{(0)} + \alpha_s^2 P^{(1)} + \alpha_s^3 P^{(2)} + \ldots$$

- DGLAP splitting kernels have been calculated to $\mathcal{O}(\alpha_s)^3$ and are needed for a consistent phenomenological treatment
  [S.Moch, J.A.M.Vermaseren, A.Vogt '04]

- NNLO parton distribution functions e.g.
  [A.D.Martin, R.Roberts, W.J.Stirling, R.S.Thorne, G.Watt]
  [S.Alekhin, J.Blümlein, S.Klein, S.Moch]

GLUONIC CONTRIBUTIONS



$A_6^{(0)}(gg \to gggg)$ $A_5^{(1)}(gg \to ggg)$ $A_4^{(2)}(gg \to gg)$

- tree level $2 \to 4$ matrix elements [F.A. Berends, W.T. Giele '87], [M.Mangano, S.J.Parke, Z.Xu '87], [R.Britto, F.Cachazo, B.Feng '06]
- 1-loop $2 \to 3$ matrix elements [Z.Bern, L.Dixon, D.A. Kosower '93]
- 2-loop $2 \to 2$ matrix elements [C. Anastasiou, E.W.N. Glover, C.Oleari, M.E. Tejeda-Yeomans '01], [Z.Bern, A.De Freitas, L.Dixon '02]

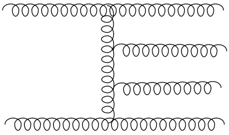
$$d\hat{\sigma}_{NNLO} = \int_{d\Phi_4} d\hat{\sigma}^{R}_{NNLO} + \int_{d\Phi_3} d\hat{\sigma}^{V,1}_{NNLO} + \int_{d\Phi_2} d\hat{\sigma}^{V,2}_{NNLO}$$

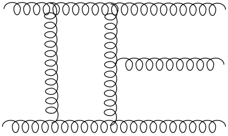
$$\begin{aligned}
d\hat{\sigma}^{R}_{NNLO} &= \mathcal{N}\, d\Phi_4(p_3,p_4,p_5,p_6;p_1,p_2)|\mathcal{M}^{(0)}_{gg\to gggg}|^2\, J_2^{(4)}(p_3,p_4,p_5,p_6) \\
d\hat{\sigma}^{V,1}_{NNLO} &= \mathcal{N}\, d\Phi_3(p_3,p_4,p_5;p_1,p_2) \\
&\quad \left(\mathcal{M}^{(0)\star}_{gg\to ggg}\mathcal{M}^{(1)}_{gg\to ggg} + \mathcal{M}^{(0)}_{gg\to ggg}\mathcal{M}^{(1)\star}_{gg\to ggg}\right)\, J_2^{(3)}(p_3,p_4,p_5) \\
d\hat{\sigma}^{V,2}_{NNLO} &= \mathcal{N}\, d\Phi_2(p_3,p_4;p_1,p_2) \\
&\quad \left(\mathcal{M}^{(2)\star}_{gg\to gg}\mathcal{M}^{(0)}_{gg\to gg} + \mathcal{M}^{(0)}_{gg\to gg}\mathcal{M}^{(2)\star}_{gg\to gg} + |\mathcal{M}^{(1)}_{gg\to gg}|^2\right)\, J_2^{(2)}(p_3,p_4)
\end{aligned}$$

- explicit infrared poles from loop integrations
  - pole structure agrees with prediction of [S. Catani '98]
- implicit poles in phase space regions for single and double unresolved gluon emission
- procedure to extract the infrared singularities and assemble all the parts

Goal

- construct parton-level generator for $(pp \to 2j)$ with NNLO accuracy
- extract singularities keeping the kinematics of the final state intact





- double unresolved configurations
  - double soft
  - triple collinear
  - double collinear
  - single soft and single collinear
- single unresolved configurations
  - single soft
  - single collinear
- remove overlapping of various single and double soft and/or collinear limits

NNLO CALCULATIONS

- sector decomposition: expansions in distributions, numerical integration [T.Binoth, G.Heinrich '02], [C.Anastasiou, K.Melnikov, F.Petriello '03]
  - applied to Higgs and vector boson production [C. Anastasiou, K. Melnikov, F. Petriello '04], [K. Melnikov, F. Petriello '06]

- subtraction: add and subtract counter-terms: process independent approximations in all unresolved limits, analytical integration
  - q_T subtraction for colorless high-mass systems [S.Catani, M.Grazzini '07]
    - applied to Higgs and vector boson production [D.de Florian, M.Grazzini '07], [S.Catani, L.Cieri, G.Ferrera, D.de Florian, M.Grazzini '09]
  - NNLO subtraction [G. Somogyi, Z.Trocsanyi '06]
  - sector decomposition and subtraction [M. Czakon '11]
    - numerical treatment for NNLO double real contribution of $pp \to t\bar{t}$
  - antenna subtraction: general subtraction scheme for jet cross sections in NNLO QCD [A.Gehrmann-De Ridder, T.Gehrmann, E.W.N.Glover]
    - jet observables in e^+e^- processes ✓
    - jet observables in ep processes ✓
    - jet observables in pp processes (under development)

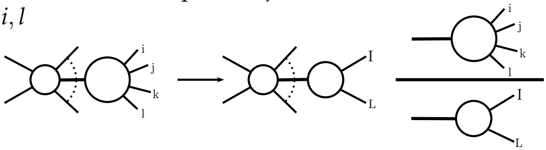
NNLO SUBTRACTION

$$
\begin{aligned}
d\hat{\sigma}_{NNLO} &= \int_{d\Phi_{m+2}} \left(d\hat{\sigma}^{R}_{NNLO} - d\hat{\sigma}^{S}_{NNLO} \right) \\
&+ \int_{d\Phi_{m+1}} \left(d\hat{\sigma}^{V,1}_{NNLO} - d\hat{\sigma}^{VS,1}_{NNLO} + d\hat{\sigma}^{MF,1}_{NNLO} \right) \\
&+ \int_{d\Phi_{m}} \left(d\hat{\sigma}^{V,2}_{NNLO} + d\hat{\sigma}^{MF,2}_{NNLO} \right) + \int_{d\Phi_{m+2}} d\hat{\sigma}^{S}_{NNLO} + \int_{d\Phi_{m+1}} d\hat{\sigma}^{VS,1}_{NNLO}
\end{aligned}
$$

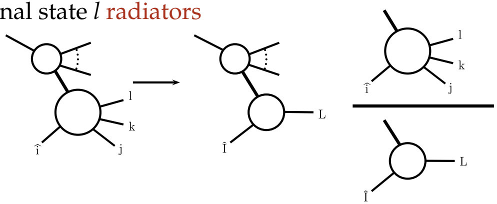
- $d\hat{\sigma}^{S}_{NNLO}$: real radiation subtraction term for $d\hat{\sigma}^{R}_{NNLO}$
- $d\hat{\sigma}^{VS,1}_{NNLO}$: one-loop virtual subtraction term for $d\hat{\sigma}^{V,1}_{NNLO}$
- $d\hat{\sigma}^{V,2}_{NNLO}$: two-loop virtual corrections
- $d\hat{\sigma}^{MF,1,2}_{NNLO}$: mass factorization counterterms
- each line above is free of infrared ϵ-poles and finite after numerical integration
- subtraction terms constructed using the antenna subtraction method at NNLO for hadron colliders → presence of initial state partons to take into account

Colour connected double unresolved case for hadron collider processes

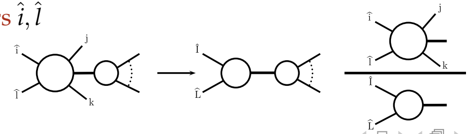
- final-final → 2 unresolved partons j, k emitted between hard final state radiators i, l



- initial-final → 2 unresolved partons j, k emitted between hard initial state $\hat{i}$ and final state l radiators



- initial-initial → 2 unresolved partons j, k emitted between hard initial state radiators $\hat{i}, \hat{l}$



ANTENNA FUNCTIONS AND TYPES

- colour-ordered pair of hard partons (radiators) with radiation in between
  - hard quark-antiquark pair
  - hard quark-gluon pair
  - hard gluon-gluon pair
- three-parton antenna $\rightarrow$ one unresolved parton
- four-parton antenna $\rightarrow$ two unresolved partons
- can be at tree level or at one loop
- can be massless or massive
- three configurations
  - final-final antenna
  - initial-final antenna
  - initial-initial antenna
- all three-parton and four-parton antenna functions can be derived fom physical matrix elements, normalised to two-parton matrix elements

INTEGRATED ANTENNAE

- massless tree-level antennae ($m = 0$)

	NLO	NNLO
final-final	✓[1]	✓[1]
initial-final	✓[2]	✓[3]
initial-initial	✓[2]	first results[4]

[1] A. Gehrmann-De Ridder, T. Gehrmann and E. W. N. Glover, *JHEP* **09** (2005) 056 [hep-ph/0505111];
[2] A. Daleo, T. Gehrmann and D. Maître, *JHEP* **04** (2007) 016 [hep-ph/0612257];
[3] A. Daleo, A. Gehrmann-De Ridder, T. Gehrmann and G. Luisoni, *JHEP* **01** (2010) 118 [0912.0374];
[4] R. Boughezal, A. Gehrmann-De Ridder and M. Ritzmann, *JHEP* **02** (2011) 098 [1011.6631];

- subtraction scheme fully worked out at NLO for e^+e^-, DIS ep and pp or $p\bar{p}$ processes
- subtraction scheme fully worked out at NNLO for e^+e^- and DIS ep processes

Integrated antennae

- massive tree-level antennae ($m \neq 0$)

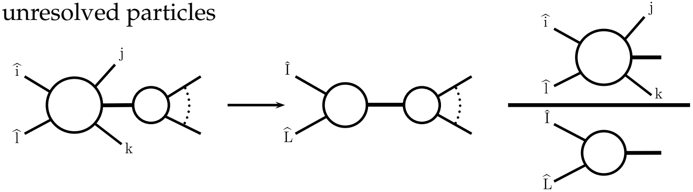
	NLO	NNLO
final-final	✓[5]	first results[7]
initial-final	✓[6]	✗
initial-initial	✓[6]	✗

[5] A. Gehrmann-De Ridder, M. Ritzmann, *JHEP* **07** (2009) 041 [0904.329711];
[6] A. Gehrmann-De Ridder, G. Abelof, *JHEP* **04** (2011) 063 [1102.2443];
[7] W. Bernreuther, C. Bogner, O Dekkers, [1105.0530];

- massive NLO subtraction scheme fully worked out for e^+e^-, DIS ep and pp or $p\bar{p}$ processes
- [6] contains $pp \to t\bar{t}j$ at NLO as byproduct
  - essential ingredient for $pp \to t\bar{t}$ at NNLO
- [7] integrated one four-parton massive antenna required for $t\bar{t}$ at NNLO

NNLO INITIAL-INITIAL ANTENNAE

- antenna factorisation for the initial-initial situation → colour connected unresolved particles



- momentum-mapping: [A. Daleo, T. Gehrmann, D. Maître '06]

$$p^\mu_{\hat{I}} = \hat{x}_i\, p^\mu_i \qquad \hat{x}_i = \left(\frac{s_{il}+s_{jl}+s_{kl}}{s_{il}+s_{ij}+s_{ik}}\,\frac{s_{ij}+s_{ik}+s_{il}+s_{jk}+s_{jl}+s_{kl}}{s_{il}}\right)^{1/2}$$

$$p^\mu_{\hat{L}} = \hat{x}_l\, p^\mu_l \qquad \hat{x}_l = \left(\frac{s_{il}+s_{ij}+s_{ik}}{s_{il}+s_{jl}+s_{kl}}\,\frac{s_{ij}+s_{ik}+s_{il}+s_{jk}+s_{jl}+s_{kl}}{s_{il}}\right)^{1/2}$$

- phase-space factorisation:

$$\begin{aligned} d\Phi_{m+2}(p_a,\ldots,p_j,p_k,\ldots,p_{m+2}) &= d\Phi_m(\tilde{p}_a,\ldots,\tilde{p}_{m+2};x_ip_i,x_lp_l) \\ &\quad \delta(x_i-\hat{x}_i)\,\delta(x_l-\hat{x}_l)\,[dk_j]\,[dk_k]\,dx_i\,dx_l \end{aligned}$$

NNLO INITIAL-INITIAL ANTENNAE

[R. Boughezal, A.Gehrmann-De Ridder, M.Ritzmann '10]

- crossings of final-final four-parton antennae

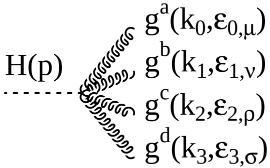
quark-antiquark antennae	
A_4^0	$A_4^0(\hat{q},\hat{g},g,\bar{q}),\, A_4^0(\hat{q},g,\hat{g},\bar{q}),\, A_4^0(\hat{q},g,g,\hat{\bar{q}}),\, A_4^0(q,\hat{g},\hat{g},\bar{q})$
$\widetilde{A}_4^0$	$\widetilde{A}_4^0(\hat{q},\hat{g},g,\bar{q}),\, \widetilde{A}_4^0(\hat{q},g,g,\hat{\bar{q}}),\, \widetilde{A}_4^0(q,\hat{g},\hat{g},\bar{q})$
B_4^0	$B_4^0(\hat{q},\hat{q}',\bar{q}',\bar{q}),\, B_4^0(\hat{q},q',\bar{q}',\hat{\bar{q}}),\, B_4^0(q,\hat{q}',\hat{\bar{q}}',\bar{q})^*$
C_4^0	$C_4^0(\hat{q},\hat{\bar{q}},q,\bar{q}),\, C_4^0(\hat{q},\bar{q},\hat{q},\bar{q}),\, C_4^0(q,\hat{\bar{q}},\hat{q},\bar{q})^*\; C_4^0(q,\bar{q},\hat{q},\hat{\bar{q}})^*$

quark-gluon antennae	
D_4^0	$D_4^0(\hat{q},\hat{g},g,g),\, D_4^0(\hat{q},g,\hat{g},g),\, D_4^0(q,\hat{g},\hat{g},g),\, D_4^0(q,\hat{g},g,\hat{g})$
E_4^0	$E_4^0(\hat{q},\hat{q}',\bar{q}',g),\, E_4^0(\hat{q},q',\bar{q}',\hat{g}),\, E_4^0(q,\hat{q}',\hat{\bar{q}}',g),\, E_4^0(q,\hat{q}',\bar{q}',\hat{g}),$
$\widetilde{E}_4^0$	$\widetilde{E}_4^0(\hat{q},\hat{q}',\bar{q}',g),\, \widetilde{E}_4^0(\hat{q},q',\bar{q}',\hat{g}),\, \widetilde{E}_4^0(q,\hat{q}',\hat{\bar{q}}',g),\, \widetilde{E}_4^0(q,\hat{q}',\bar{q}',\hat{g})$

NNLO INITIAL-INITIAL ANTENNAE

[R. Boughezal, A.Gehrmann-De Ridder, M.Ritzmann '10]

gluon-gluon antennae	
F_4^0	$F_4^0(\widehat{g},\widehat{g},g,g)$, $F_4^0(\widehat{g},g,\widehat{g},g)$
G_4^0	$G_4^0(\widehat{g},\widehat{q},\bar{q},g)$, $G_4^0(\widehat{g},q,\widehat{\bar{q}},g)$, $G_4^0(\widehat{g},q,\bar{q},\widehat{g})$, $G_4^0(g,\widehat{q},\widehat{\bar{q}},g)$
$\widetilde{G}_4^0$	$\widetilde{G}_4^0(\widehat{g},\widehat{q},\bar{q},g)$, $\widetilde{G}_4^0(\widehat{g},q,\bar{q},\widehat{g})$, $\widetilde{G}_4^0(g,\widehat{q},\widehat{\bar{q}},g)$
H_4^0	$H_4^0(\widehat{q},\widehat{\bar{q}},q',\bar{q}')$, $H_4^0(\widehat{q},\bar{q},\widehat{q}',\bar{q}')$



- gluonic contribution for $pp \to 2j$ at NNLO requires
  - kinematical crossing of final-final gluon-gluon antenna function

$$X_{1234} = \frac{|M_{gggg}|^2}{|M_{gg}|^2} \equiv F_4^0(1_g, 2_g, 3_g, 4_g)$$

  - $F_4^0(\hat{1}_g, \hat{2}_g, 3_g, 4_g)$, $F_4^0(\hat{1}_g, 3_g, \hat{2}_g, 4_g)$
  - $2 \to 3$ antenna in the initial-initial configuration

INTEGRATED ANTENNAE

- integrated antenna over the unresolved phase space with q^2 and x_1, x_2 fixed [R. Boughezal, A.Gehrmann-De Ridder, M.Ritzmann '10]

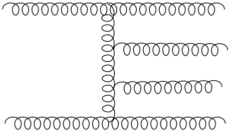
$$\mathcal{X}^0_{il}(x_i, x_l) = \frac{1}{C^2(\epsilon)} \int [\mathrm{d}k_j][\mathrm{d}k_k]\delta(x_i - \hat{x}_i)]\delta(x_l - \hat{x}_l)\, X^0_{il,jk}(p_i, p_j, p_k, p_l)$$

- linear combinations of 32 master integrals
- coefficients contain poles in ϵ and rational factors in x_1, x_2
- end point behaviour: $(1 - x_1)^{-1-2\epsilon}(1 - x_2)^{-1-2\epsilon}\, R(x_1, x_2)$
- expansion in distributions around endpoints $x_1, x_2 = 1$
- use differential equations in x_1, x_2 to compute the master integrals

- integrated antennae with two quark flavours
- crossings of:
  - quark-antiquark antenna: $B^0_4(q, q^{'}, \bar{q}^{'}, \bar{q})$
  - quark-gluon antenna: $\tilde{E}^0_4(q, q^{'}, \bar{q}^{'}, g)$
  - gluon-gluon antenna: $H^0_4(q, \bar{q}, q^{'}, \bar{q}^{'})$
- contain 12 (out of 32) master integrals
- full set in progress

NNLO DOUBLE REAL CORRECTIONS TO $pp \to 2j$

[N. Glover, JP '10]



- gluon scattering corrections to $gg \to gg$
- six parton tree level processes contributing to two jet final states (leading colour)

$$
\begin{aligned}
d\hat{\sigma}^{R}_{NNLO} = \; & N^2\, N_{born} \left(\frac{\alpha_s}{2\pi}\right)^2 d\Phi_4(p_3,\ldots,p_6;p_1,p_2) \Bigg(\\
& \frac{2}{4!} \sum_{P(i,j,k,l)\in(3,4,5,6)} A_6^0(\hat{1}_g,\hat{2}_g,i_g,j_g,k_g,l_g) J_2^{(4)}(p_i,\ldots,p_l) \\
& + \frac{2}{4!} \sum_{P(i,j,k,l)\in(3,4,5,6)} A_6^0(\hat{1}_g,i_g,\hat{2}_g,j_g,k_g,l_g) J_2^{(4)}(p_i,\ldots,p_l) \\
& + \frac{2}{4!} \sum_{P_C(i,j,k,l)\in(3,4,5,6)} A_6^0(\hat{1}_g,i_g,j_g,\hat{2}_g,k_g,l_g) J_2^{(4)}(p_i,\ldots,p_l) \Bigg)
\end{aligned}
$$

- three topologies according to position of the initial state gluons

SUBTRACTION TERMS - IIFFFF TOPOLOGY

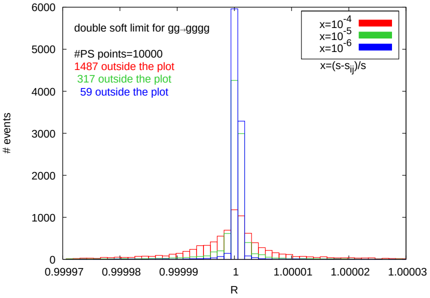
[N. Glover, JP '10]

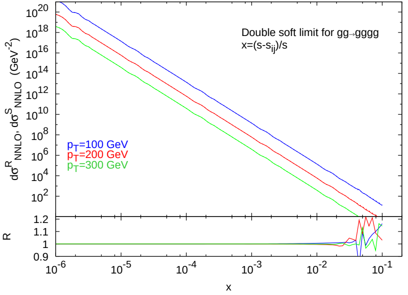
$$d\hat{\sigma}^{R}_{NNLO} = N^2\, N_{born} \left(\frac{\alpha_s}{2\pi}\right)^2 d\Phi_4(p_3,\dots,p_6;p_1,p_2)\, \frac{2}{4!} \sum_{P(3,4,5,6)} A_6^0(\hat{1}_g,\hat{2}_g,i_g,j_g,k_g,l_g) J_2^{(4)}(p_i,\dots,p_l)$$

$$\begin{aligned}
d\hat{\sigma}^{S,b}_{NNLO} &= N^2\, N_{born} \left(\frac{\alpha_s}{2\pi}\right)^2 d\Phi_4(p_3,\dots,p_6;p_1,p_2)\, \frac{2}{4!} \Bigg(\\
&\Big(F_4^0(\hat{2}_g,i_g,j_g,k_g) - f_3^0(\hat{2}_g,i_g,j_g) F_3^0(\hat{\bar{2}}_g,\widetilde{(ij)}_g,k_g) - f_3^0(i_g,j_g,k_g) F_3^0(\hat{2}_g,\widetilde{(ij)}_g,\widetilde{(jk)}_g) \\
&- f_3^0(j_g,k_g,\hat{2}_g) F_3^0(\hat{\bar{2}}_g,i_g,\widetilde{(kj)}_g)\Big) A_4^0(\hat{1}_g,\hat{\bar{2}}_g,\widetilde{(ijk)}_g,l_g) J_2^{(2)}(\widetilde{p_{ijk}},p_l)) \\
&+ \Big(F_{4,a}^0(i_g,j_g,k_g,l_g) - f_3^0(i_g,j_g,k_g) f_3^0(\widetilde{(ij)}_g,\widetilde{(jk)}_g,l_g) - f_3^0(j_g,k_g,l_g) f_3^0(i_g,\widetilde{(jk)}_g,\widetilde{(kl)}_g)\Big) \\
&A_4^0(\hat{1}_g,\hat{2}_g,\widetilde{(ijk)}_g,\widetilde{(lkj)}_g) J_2^{(2)}(\widetilde{p_{ijk}},\widetilde{p_{lkj}}) \\
&+ \Big(F_{4,b}^0(i_g,j_g,k_g,l_g) - f_3^0(i_g,j_g,k_g) f_3^0(\widetilde{(ij)}_g,l,\widetilde{(jk)}_g)\Big) A_4^0(\hat{1}_g,\hat{2}_g,\widetilde{(ijl)}_g,\widetilde{(klj)}_g) J_2^{(2)}(\widetilde{p_{ijl}},\widetilde{p_{klj}}) \\
&+ \Big(F_4^0(\hat{1}_g,l_g,k_g,j_g) - f_3^0(\hat{1}_g,l_g,k_g) F_3^0(\hat{\bar{1}}_g,\widetilde{(lk)}_g,j_g) - f_3^0(l_g,k_g,j_g) F_3^0(\hat{1}_g,\widetilde{(lk)}_g,\widetilde{(kj)}_g) \\
&- f_3^0(k_g,j_g,\hat{1}_g) F_3^0(\hat{\bar{1}}_g,l_g,\widetilde{(jk)}_g)\Big) A_4^0(\hat{\bar{1}}_g,\hat{2}_g,i_g,\widetilde{(lkj)}_g) J_2^{(2)}(p_i,\widetilde{p_{lkj}}) + \text{cyclic} + \text{l.reversal} + \dots \Bigg)
\end{aligned}$$

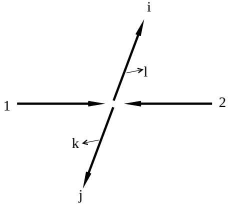
CHECK OF DOUBLE UNRESOLVED LIMITS

- double soft limit

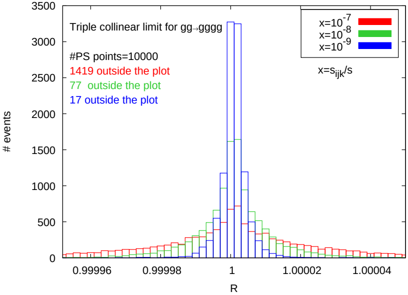


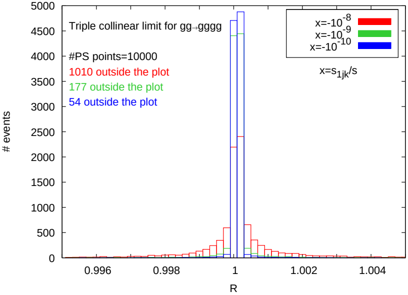


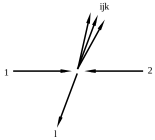
- double k, l soft limit when $s_{ij} \approx s$
- infrared behaviour of subtraction term coincides with the matrix element
- $R = \dfrac{d\sigma^R_{NNLO}}{d\sigma^S_{NNLO}} \xrightarrow{l_g, k_g \to 0} 1$

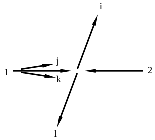


TRIPLE COLLINEAR LIMIT





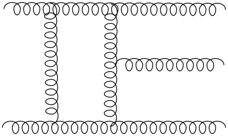




- generate phase space points with small triple invariant s_{ijk} or s_{1jk} mass
- soft and collinear limit ✓
- double collinear limit ✓
- similar agreement for all single unresolved limits ✓

NNLO REAL-VIRTUAL CORRECTIONS TO $pp \to 2j$

[A.Gehrmann-De Ridder, N.Glover, JP] (in preparation)



- gluon scattering corrections to $gg \to gg$
- five parton one-loop processes contributing to two jet final states (leading colour)

$$
\begin{aligned}
d\hat{\sigma}^{RV}_{NNLO} = \; & N^2 N_{born} \left(\frac{\alpha_s}{2\pi}\right)^2 d\Phi_3(p_3,\ldots,p_5;p_1,p_2) \Bigg(\\
& \frac{2}{3!} \sum_{P(i,j,k)\in(3,4,5)} A_5^1(\hat{1}_g,\hat{2}_g,i_g,j_g,k_g) J_2^{(3)}(p_i,p_j,p_k) \\
& + \frac{2}{3!} \sum_{P(i,j,k)\in(3,4,5)} A_5^1(\hat{1}_g,i_g,\hat{2}_g,j_g,k_g) J_2^{(3)}(p_i,p_j,p_k) \Bigg)
\end{aligned}
$$

- two topologies according to position of the initial state gluons
- simpler colour connection for one single unresolved emission

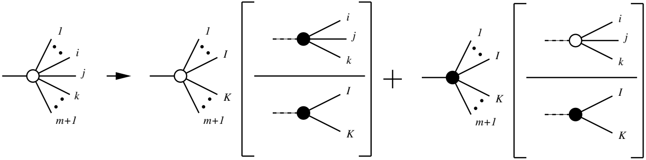
Real-Virtual subtraction terms

$$d\hat{\sigma}^{VS,1}_{NNLO} = \int d\hat{\sigma}^{S,a}_{NNLO} + d\hat{\sigma}^{VS,1,a}_{NNLO} + d\hat{\sigma}^{VS,1,b}_{NNLO} + \int d\hat{\sigma}^{A}_{NNLO}$$

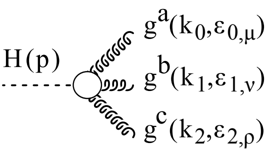
- $\int d\hat{\sigma}^{S,a}_{NNLO}$ is the integrated form of all single unresolved behaviour of the double real contributions
- removes the explicit poles in the virtual amplitude (KLN theorem)

$$\int d\hat{\sigma}^{S,a}_{NNLO} = \mathcal{N}\, d\Phi_{m+1}(p_3,\ldots,p_{m+3};p_1,p_2) \sum_{ik} \mathcal{X}^0_3 \begin{pmatrix} FF \\ IF \\ II \end{pmatrix} |\mathcal{M}_{m+3}|^2 J^{(m+1)}_m(p_3,\ldots,p_{m+3})$$

- $d\hat{\sigma}^{VS,1,a}_{NNLO}$ removes implicit poles from kinematical singularities of virtual amplitude → singular limits factorize as a combination of three parton one-loop antenna function and three parton tree level antenna function



$$d\hat{\sigma}^{VS,1,a}_{NNLO} = \mathcal{N}\, d\Phi_{m+1}(p_3,\ldots,p_{m+3};p_1,p_2)\sum_j X_3^0\begin{pmatrix}FF\\IF\\II\end{pmatrix}|\mathcal{M}^{(1)}_{m+2}|^2\, J_m^{(m)}(p_3,\ldots,p_{m+2})$$
$$+\ \mathcal{N}\, d\Phi_{m+1}(p_3,\ldots,p_{m+3};p_1,p_2)\sum_j X_3^1\begin{pmatrix}FF\\IF\\II\end{pmatrix}|\mathcal{M}^{(0)}_{m+2}|^2\, J_m^{(m)}(p_3,\ldots,p_{m+2})$$



- gluonic contribution for $pp \to 2j$ at NNLO requires 1-loop gluon gluon antenna function for initial-final and initial-initial configurations obtained by crossing from the $1 \to 3$ 1-loop antenna

$$X^1_{123} = \frac{2\Re(\mathcal{M}^0_{ggg}\mathcal{M}^{1,\star}_{ggg})}{|M_{gg}|^2} \equiv F^1_3(1_g,2_g,3_g)$$

- $d\hat{\sigma}_{NNLO}^{VS,1,b}$ removes oversubtraction of implicit and explicit poles of $\int d\hat{\sigma}_{NNLO}^{S,a}$ and $d\hat{\sigma}_{NNLO}^{VS,1,a}$

$$d\hat{\sigma}_{NNLO}^{VS,1,b} = \mathcal{N}\, d\Phi_{m+1}(p_3,\ldots,p_{m+3};p_1,p_2) \sum_{ik} \mathcal{X}_3^0(s_{ik}) \sum_{o} X_3^0 \begin{pmatrix} FF \\ IF \\ II \end{pmatrix} |\mathcal{M}_{m+2}|^2\, J_m^{(m)}$$

- large angle soft radiation contribution given by $\int d\hat{\sigma}_{NNLO}^{A}$

$$\int d\hat{\sigma}_{NNLO}^{A} = \mathcal{N}\, d\Phi_{m+1}(p_3,\ldots,p_{m+3};p_1,p_2) \sum_{j} \frac{1}{2} X_3^0 \begin{pmatrix} FF \\ IF \\ II \end{pmatrix}$$
$$\times \left[\mathcal{S}_{IK;ik} - \mathcal{S}_{ik;ik} + \mathcal{S}_{aI;ik} - \mathcal{S}_{ai;ik} + \mathcal{S}_{Kb;ik} - \mathcal{S}_{kb;ik}\right] |\mathcal{M}_{m+2}|^2\, J_m^{(m)}(p_3,\ldots,p_{m+2})$$

with $\mathcal{S}$ the integrated soft factors present in unintegrated form in the double real contributions

NNLO Real-Virtual corrections to $pp \to 2j$

[A.Gehrmann-De Ridder, N.Glover, JP] (in preparation)

- construction of

$$d\hat{\sigma}^{VS,1}_{NNLO} + d\hat{\sigma}^{MF,1}_{NNLO} = \int d\hat{\sigma}^{S,a}_{NNLO} + d\hat{\sigma}^{VS,1,a}_{NNLO} + d\hat{\sigma}^{VS,1,b}_{NNLO} + \int d\hat{\sigma}^{A}_{NNLO} + d\hat{\sigma}^{MF,1}_{NNLO}$$

  which removes all poles in the real virtual contribution $d\hat{\sigma}^{VS}_{NNLO}$ (under development)

- identified extra inclusive integrals needed for $pp \to 2j$ at NNLO

	$\mathcal{X}_3^1$	$\mathcal{S}_{ac;ik}$
final-final	✓	✓
initial-final	✓	✓
initial-initial	(in progress)	(in progress)

Conclusions

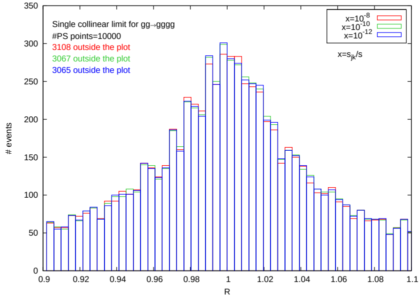
Towards $pp \to 2j$ at NNLO ($gg \to gg$ channel)

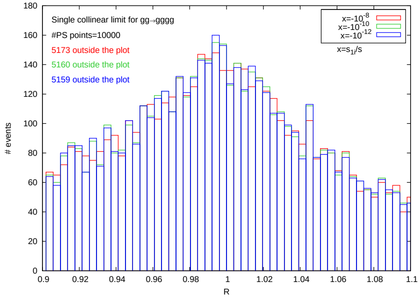
$$
\begin{aligned}
d\hat{\sigma}_{NNLO} \quad &= \int_{d\Phi_{m+2}} \left(d\hat{\sigma}^{R}_{NNLO} - d\hat{\sigma}^{S}_{NNLO} \right) \\
&+ \int_{d\Phi_{m+1}} \left(d\hat{\sigma}^{V,1}_{NNLO} - d\hat{\sigma}^{VS,1}_{NNLO} + d\hat{\sigma}^{MF,1}_{NNLO} \right) \\
&+ \int_{d\Phi_{m}} \left(d\hat{\sigma}^{V,2}_{NNLO} + d\hat{\sigma}^{MF,2}_{NNLO} \right) + \int_{d\Phi_{m+2}} d\hat{\sigma}^{S}_{NNLO} + \int_{d\Phi_{m+1}} d\hat{\sigma}^{VS,1}_{NNLO}
\end{aligned}
$$

- double real subtraction term $d\hat{\sigma}^{S}_{NNLO}$ constructed and tested
- infrared structure of the double real (NNLO) emission written in terms antenna functions
- $\int_{d\Phi_{m+2}} \left(d\hat{\sigma}^{R}_{NNLO} - d\hat{\sigma}^{S}_{NNLO} \right)$ finite and integrable in four dimensions
- real virtual subtraction terms $d\hat{\sigma}^{V,1}_{NNLO} - d\hat{\sigma}^{VS,1}_{NNLO} + d\hat{\sigma}^{MF,1}_{NNLO}$ in progress

Back up slides

Singly unresolved limits





Single collinear limits:

- generate phase space points with s_{jk} or s_{1i} small

Distributions with a broader shape due to:

- angular correlations in matrix elements and antenna functions when an initial/final state gluon splits into two gluons not accounted for by the subtraction term → non-locality of subtraction term

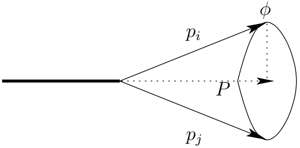
ANGULAR TERMS SUBTRACTION

- angular terms vanish after averaging over the azimuthal angle

$$\frac{1}{2\pi}\int_0^{2\pi} \mathrm{d}\phi\,(p_l\cdot k_\perp) = 0\,, \qquad \frac{1}{2\pi}\int_0^{2\pi} \mathrm{d}\phi\,(p_l\cdot k_\perp)^2 = -k_\perp^2\,\frac{p\cdot p_l\, n\cdot p_l}{p\cdot n}$$

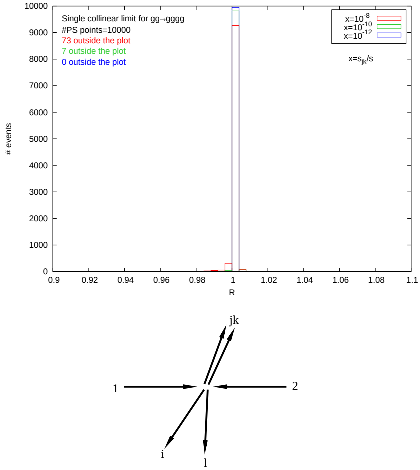
$$\Theta_{F_3^0}(i,j,z,k_\perp) \sim A\cos(2\phi+\alpha)$$

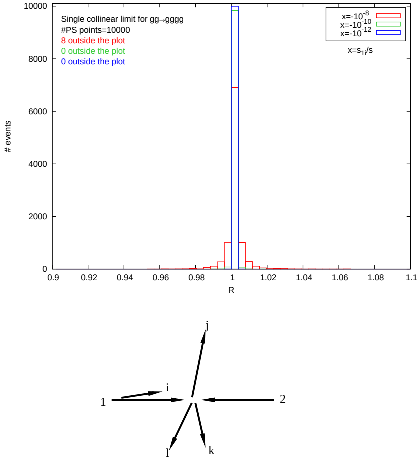
- combine phase space points related to each other by a rotation of the system of unresolved partons $\{p_i,p_j\} \to \{p_i',p_j'\}$



$$p_i^\mu = zp^\mu + k_\perp^\mu - \frac{k_\perp^2}{z}\frac{n^\mu}{2p\cdot n}\,, \qquad p_j^\mu = (1-z)p^\mu - k_\perp^\mu - \frac{k_\perp^2}{1-z}\frac{n^\mu}{2p\cdot n}\,,$$

$$\text{with } 2p_i\cdot p_j = -\frac{k_\perp^2}{z(1-z)}\,, \qquad p^2 = n^2 = k_\perp.p = k_\perp.n = 0$$





- in both collinear limits combining phase space points largely cancels angular dependent terms
- generalizable to multiple collinear emission
- subtract correlations systematically at the phase space generation level