Resummation of large IR logarithms for the Thrust distribution

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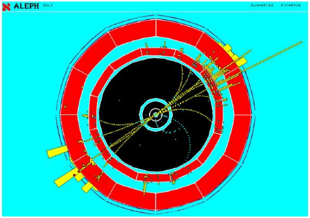
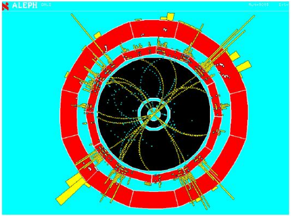
Event shapes are *IRC* safe observables which describe the topology of hadronic final states. They enjoy two peculiar properties:

- *Simple physical picture...*
  - *e.g.* Thrust $T = \max_{\{\vec{n}_T\}} \frac{\sum_i |\vec{p}_i \cdot \vec{n}_T|}{\sum_i |\vec{p}_i|}$

$T \simeq 1$: two jet event



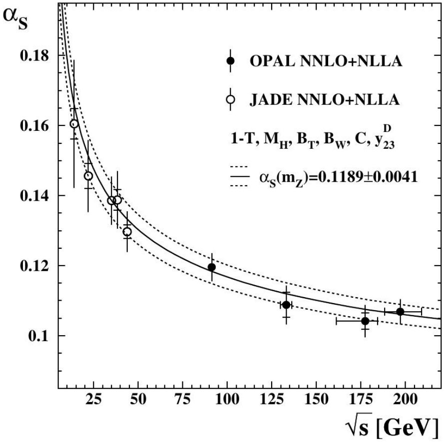
$T \simeq \frac{1}{2}$: spherical event



Event shapes are *IRC* safe observables which describe the geometry of hadronic final states. They enjoy two peculiar properties:

- *Simple physical picture...*
- *High sensitivity to QCD properties...*

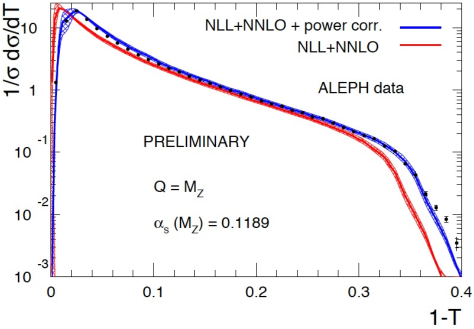
- Precise measurement of the strong coupling α_s [OPAL collaboration '11]



Event shapes are *IRC* safe observables which describe the geometry of hadronic final states. They enjoy two peculiar properties:

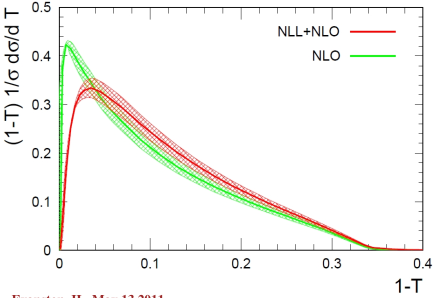
- *Simple physical picture...*
- *High sensitivity to QCD properties...*

- Precise measurement of the strong coupling α_s
- Analysis of hadronization corrections ($\sim \frac{1}{Q}$, shape function,...)



- Fixed order predictions for the event shape distribution **diverge** in the dijet limit ($T \to 1$)...
  - We compute the probability of real emission constraining the event such that $\Theta(1-T<\tau)$
  - Dijet limit $T \simeq 1 \to$ real gluon radiation forbidden
  - Imbalance between real (*constrained*) and virtual (*unaffected*) radiation leads to large logarithms in the distribution of the event shape
- The perturbative series is **poorly convergent** in the high T region due to the presence of terms $\alpha_s \log \frac{1}{1-T} \simeq 1$
- For a physical prediction to be reliable we need to resum logarithmically enhanced terms

- LL $\to \alpha^n L^{n+1}$
- NLL $\to \alpha^n L^n$
- N^{2}LL $\to \alpha^n L^{n-1}$
- ...



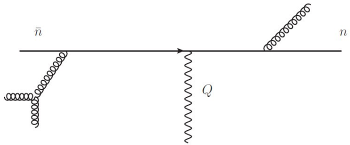
- State-of-the-art at LEP times (until 2007):
  - fixed NLO calculations,[Ellis, Ross, Terrano; Kunszt, Nason; Giele, Glover; Catani, Seymour]
  - DL resummation, [Binétruy; Schierholz '80]
  - NLL resummation, [Catani, Trentadue, Turnock, Webber '93; Banfi, Salam, Zanderighi '04].
- Very important progresses in the last three years:
  - for all event-shape observables (in particular LEP standard set):
    - fixed NNLO computation of jet rates and event-shape observables [Gehrmann, Gehrmann-De Ridder, Glover, Heinrich; Weinzierl]
    - matching of NLL+NNLO [Gehrmann, Luisoni, Stenzel]
    - non-perturbative corrections to moments at NNLO [Gehrmann, Jaquier, Luisoni]
  - for single event shapes:
    - T: non-perturbative corrections to NLL+NNLO distribution, [Davison, Webber]
    - T: N^3LL resummation (except $\Gamma^{(3)}_{\text{cusp}}$) in SCET and matching to NNLO, [Becher, Schwartz]
    - T: power corrections to N^3LL+NNLO, [Abbate, Fickinger, Hoang, Mateu, Stewart]
    - M_H: N^3LL resummation (except $\Gamma^{(3)}_{\text{cusp}}$) in SCET and matching to NNLO, [Chien, Schwartz]
    - B_W, B_T: general factorization beyond NLL, [Becher, Bell, Neubert]

Step I: factorization of the observable in the dijet region

- **Recoil effects**: the effect of the recoil due to large angle soft emissions is of relative order $\mathcal{O}(1-T) \longrightarrow$ power suppressed!
  $T \simeq$ *sum of soft* $(q,\bar{q})$ *and collinear* $(k,\bar{k})$ *contributions*

$$n = (1, \vec{n}_T), \qquad \bar{n} = (1, -\vec{n}_T)$$

$$1 - T \simeq \frac{\bar{k}^2}{Q^2} + \frac{k^2}{Q^2} + \frac{\bar{q}\cdot\bar{n}}{Q} + \frac{q\cdot n}{Q}$$

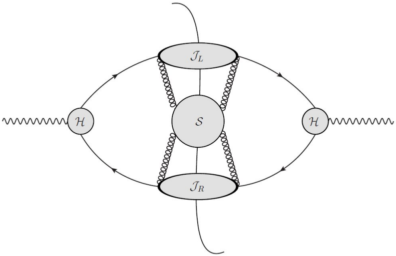


$$\Theta(1 - T < \tau) = \frac{1}{2\pi i}\int_C \frac{d\nu}{\nu} e^{\nu\tau} e^{-\frac{\bar{k}^2}{Q^2}\nu} e^{-\frac{k^2}{Q^2}\nu} e^{-\left(\frac{\bar{q}\cdot\bar{n}}{Q}+\frac{q\cdot n}{Q}\right)\nu}$$

Step II: factorization of the cross section ($\nu_0 = e^{-\gamma_E}$) [Collins, Soper; Berger, Kucs, Sterman] [Fleming, Hoang, Mantry, Stewart; Schwartz]

$$R_T(\tau) \simeq \mathscr{H}\left(\frac{Q^2}{\mu^2}, \alpha_s(\mu)\right) \int_C \frac{d\nu}{2\pi i\nu} e^{\nu\tau}\, \tilde{\mathscr{J}}_n\left(\frac{\nu_0 Q^2}{\nu\mu^2}, \alpha_s(\mu)\right) \tilde{\mathscr{J}}_{\bar{n}}\left(\frac{\nu_0 Q^2}{\nu\mu^2}, \alpha_s(\mu)\right) \tilde{\mathrm{S}}\left(\frac{\nu_0^2 Q^2}{\nu^2\mu^2}, \alpha_s(\mu)\right)$$

- single quark leg connecting $\mathscr{H}$ to $\mathscr{J}_{L,R}$
  - trivial in axial gauge
  - longitudinally polarized gluons are still present in Feynman gauge (decoupled by Ward Identities)
- soft gluons factorize
- graphs involving soft gluon exchange between S and $\mathscr{H}$ are suppressed or cancel



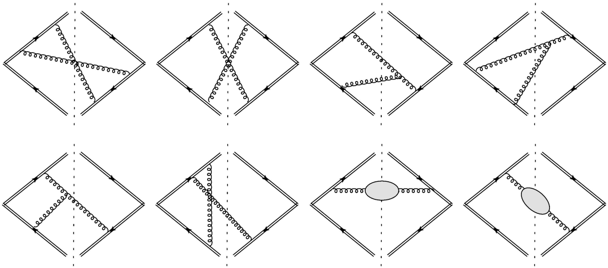
- $\tilde{\mathrm{S}}(\frac{\nu_0^2 Q^2}{\nu^2\mu^2}, \alpha_s(\mu))$
  - In emitting soft gluons, the hard collinear quarks behave as classical relativistic particles $\longrightarrow$ interactions with soft gluons are factorized into eikonal lines
  - Includes both soft and soft-collinear corrections
  - Known at $\mathcal{O}(\alpha_s^2)$ (this talk)
- $\tilde{\mathscr{J}}_n(\frac{\nu_0 Q^2}{\nu\mu^2}, \alpha_s(\mu))$
  - Describes the decay of a massless quark into a jet of collinear particles moving along the n direction
  - Defined as the cut quark propagator in the axial gauge
  - **Caution**: double counting with the soft subprocess and the $\bar{n}$-collinear jet has to be avoided!
  - Known at $\mathcal{O}(\alpha_s^2)$, [Becher, Neubert]
- $\mathscr{H}(\frac{Q^2}{\mu^2}, \alpha_s(\mu))$
  - Hard virtual corrections ($\sim \delta(1-T) \longrightarrow$ constant contribution at large ν)
  - Defined such that

$$\mathscr{H}\, \mathscr{J}_{\mathrm{n}} \otimes \mathscr{J}_{\bar{\mathrm{n}}} \otimes \mathrm{S} = \text{fixed order} + \mathcal{O}(\tau)$$

Computation of the *Soft subprocess*

$$\tilde{S}\left(\frac{\nu_0^2 Q^2}{\nu^2\mu^2},\alpha_s(\mu)\right)=\frac{1}{N_C}\int d\tau e^{-\nu\tau}\sum_{k_{eik}}\langle 0|\Phi_{\bar{n}}^{\dagger}(0)\Phi_{n}^{\dagger}(0)|k_{eik}\rangle\,\mathcal{J}_{cut}(\tau)\langle k_{eik}|\Phi_n(0)\Phi_{\bar{n}}(0)|0\rangle$$

- Light-like Wilson lines along n, $\bar{n}$ ($n^2=\bar{n}^2=0$) $\rightarrow$ *No recoil*!
- Purely Virtual corrections are *scaleless*
- Abelian contributions from exponentiation
- Only a few diagrams survive involving either *single* or *double* cuts
- Correlation between the two hemispheres does not contribute to the Thrust soft subprocess



+ mirror symmetric

- *Single* cut: single soft emission

$$\mathscr{J}_{cut}(\tau) \rightarrow \delta^{(+)}(q^2)(\delta(\tau Q - q\cdot n)\Theta(q\cdot\bar{n} - q\cdot n) + \delta(\tau Q - q\cdot\bar{n})\Theta(q\cdot n - q\cdot\bar{n}))$$

$\longrightarrow$ analytical solution

- *Double* cut: double soft emission

$$\begin{aligned}
&\mathscr{J}_{cut}(\tau) \rightarrow \delta^{(+)}(q^2)\delta^{(+)}(k^2) \\
&\times(\delta(\tau Q - q\cdot n - k\cdot n)\Theta(q\cdot\bar{n} - q\cdot n)\Theta(k\cdot\bar{n} - k\cdot n) \\
&+\delta(\tau Q - q\cdot\bar{n} - k\cdot\bar{n})\Theta(q\cdot n - q\cdot\bar{n})\Theta(k\cdot n - k\cdot\bar{n}) \\
&+\delta(\tau Q - q\cdot n - k\cdot\bar{n})\Theta(q\cdot\bar{n} - q\cdot n)\Theta(k\cdot n - k\cdot\bar{n}) \\
&+\delta(\tau Q - k\cdot n - q\cdot\bar{n})\Theta(k\cdot\bar{n} - k\cdot n)\Theta(q\cdot n - q\cdot\bar{n}))
\end{aligned}$$

$\longrightarrow$ semi-analytical solution **+** numerical sector decomposition:
SecDec+BASES/VEGAS [Carter, Heinrich '11]

Result (in Laplace space):

- two loop *cusp* and *soft* anomalous dimensions:

$$\Gamma_{\text{cusp}}(\alpha_s) = \frac{\alpha_s}{\pi} C_F + \frac{\alpha_s^2}{\pi^2} C_F \left(C_A \left(\frac{67}{36} - \frac{\pi^2}{12} \right) - \frac{5}{9} T_F n_F \right) + \mathcal{O}(\alpha_s^3)$$

$$\Gamma_{\text{soft}}(\alpha_s) = -\frac{\alpha_s^2}{\pi^2} C_F \left(T_F n_F \left(\frac{14}{27} - \frac{\pi^2}{36} + (0.0 \pm 2.46 \times 10^{-5}) \right) + C_A \left(-\frac{101}{54} + \frac{11}{144}\pi^2 + \frac{7}{4}\zeta_3 + (0.0 \pm 1.5 \times 10^{-5}) \right) \right) + \mathcal{O}(\alpha_s^3)$$

- *non-logarithmic* part of the two loop soft subprocess

$$\tilde{S}_0^{(2)}\left(\frac{v_0^2 Q^2}{v^2 \mu^2}, \alpha_s(\mu)\right) = \frac{\alpha_s^2(\mu)}{(4\pi)^2} \left(\frac{\pi^4}{2} C_F^2 + C_F T_F n_F 16 \times \left(\frac{7}{9} + \frac{131}{432}\pi^2 - \frac{23}{144}\zeta_3 - 0.867213 \pm 3.32 \times 10^{-4} \right) - C_A C_F 16 \times \left(\frac{7}{9} + \frac{497}{864}\pi^2 - \frac{101}{720}\pi^4 + \frac{173}{144}\zeta_3 + 9.29536 \pm 4.19 \times 10^{-4} \right) \right)$$

$$= \frac{\alpha_s^2(\mu)}{(4\pi)^2} (48.7045 C_F^2 - (56.48403 \pm 0.0067) C_A C_F + (43.38297 \pm 0.00531) C_F T_F n_F)$$

- Comparison to previous estimates of the two loop constant: EVENT2
  - linear binning: $\frac{\alpha_s^2(\mu)}{(4\pi)^2}\left((58 \pm 2) C_F^2 - (60 \pm 1) C_F C_A + (43 \pm 1) C_F T_F n_F\right)$, [Becher,Schwartz]
  - logarithmic binning: $\frac{\alpha_s^2(\mu)}{(4\pi)^2}\left(48.7045 C_F^2 - (58.8 \pm 2.25) C_F C_A + (43.8 \pm 3.06) C_F T_F n_F\right)$, [Hoang, Kluth]

- Resummed cross section in Laplace space:

$$R_T(\tau) = \mathscr{H}\left(\frac{Q^2}{\mu^2}, \alpha_s(\mu)\right) \int_C \frac{d\nu}{2\pi i \nu} e^{\nu\tau} \tilde{\mathscr{J}}^2\left(1, \alpha_s\left(\sqrt{\frac{\nu_0}{\nu}} Q\right)\right) \tilde{S}\left(1, \alpha_s\left(\frac{\nu_0 Q}{\nu}\right)\right)$$
$$\times \exp\left\{ -2 \int_{\frac{\nu_0}{\nu}}^{1} \frac{du}{u} \left(\int_{u^2 Q^2}^{uQ^2} \frac{dk^2}{k^2} \mathscr{A}(\alpha_s(k^2)) + \mathscr{B}(\alpha_s(uQ^2)) \right) \right\} + \mathscr{O}(\tau)$$

- $\mathscr{A}(\alpha_s) = \Gamma_{\text{cusp}}(\alpha_s) - \beta(\alpha_s) \frac{\partial \Gamma_{\text{soft}}(\alpha_s)}{\partial \alpha_s} = \sum_{k\geq 1} A^{(k)} \left(\frac{\alpha_s}{\pi}\right)^k$ $\quad \mathscr{B}(\alpha_s) = \Gamma_{\text{soft}}(\alpha_s) + \Gamma_{\text{coll}}(\alpha_s) = \sum_{k\geq 1} B^{(k)} \left(\frac{\alpha_s}{\pi}\right)^k$
- Altarelli-Parisi splitting function $P_{qq}(\alpha_s, z) = 2\frac{\Gamma_{\text{cusp}}(\alpha_s)}{(1-z)_+} + 2\mathscr{B}(\alpha_s)\delta(1-z) + \ldots$ (known at $\mathscr{O}(\alpha_s^3)$, [Vogt, Vermaseren, Moch])
  - Easy to prove in DIS, [Becher, Neubert, Pecjak]
- Beyond NLL:
  - Large angle soft emission
  - Interplay between Logarithms and Constants
- $A^{(k)} \longrightarrow \alpha_s^n L^{n+2-k}$ while $B^{(k)} \longrightarrow \alpha_s^n L^{n+1-k}$ with $n \geq k$
- $B^{(3)}$ is enough to determine the (N^{3}LL) coefficient $G_{3,1}$ (required in the R-matching)

- Resummed cross section in Thrust space (from NLL to N^{2}LL):

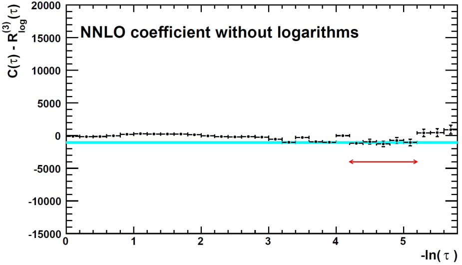
$$R_T(\tau) = \left(1 + \sum_{k=1}^{3} C_k \left(\frac{\alpha_s}{2\pi}\right)^k\right) e^{\mathscr{F}(\alpha_s(Q^2), \log\frac{1}{\tau})} \frac{1}{\Gamma(1-\gamma(\lambda))} \left[1 + \frac{\alpha_s}{\pi}\beta_0 f_2'(\lambda)\psi^{(0)}(1-\gamma(\lambda)) + \frac{1}{2}\frac{\alpha_s}{\pi}\beta_0\gamma'(\lambda)\Gamma(1-\gamma(\lambda))\frac{d^2}{d\gamma^2(\lambda)}\frac{1}{\Gamma(1-\gamma(\lambda))} + \frac{\alpha_s}{\pi} C_F\left(\gamma_E\left(\frac{3}{2}-\gamma_E\right) - \frac{\pi^2}{6}\right)\right] + \mathscr{O}(\tau)$$

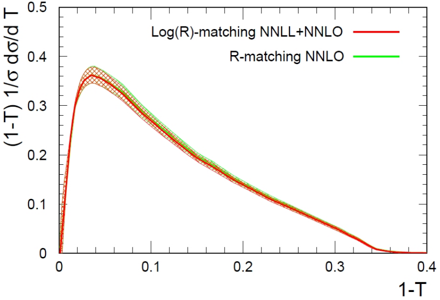
- $\mathscr{F}(\alpha_s(Q^2), L) = L f_1(\frac{\alpha_s}{\pi}\beta_0 L) + f_2(\frac{\alpha_s}{\pi}\beta_0 L) + \frac{\alpha_s}{\pi}\beta_0 f_3(\frac{\alpha_s}{\pi}\beta_0 L) + G_{3,1}\frac{\alpha_s^3}{(2\pi)^3} L + \mathscr{O}(\alpha_s^4 L^2)$
  $\gamma(\lambda) = f_1(\lambda) + \lambda f_1'(\lambda)$, $\lambda = \frac{\alpha_s}{\pi}\beta_0 \log\frac{1}{\tau}$

- C_1, C_2, f_3 and $G_{3,1}$ known!

- It reproduces the N^{2}LL Becher and Schwartz's results provided we expand $\alpha_s(Q\tau)$, $\alpha_s(Q\sqrt{\tau})$ in their formula around a fixed value of the coupling and we truncate the whole series neglecting terms of order $\mathscr{O}(\alpha_s^4 L^2)$
- The presence of the Landau pole does not affect the inversion of the Laplace transform and it is directly mapped onto the Thrust space

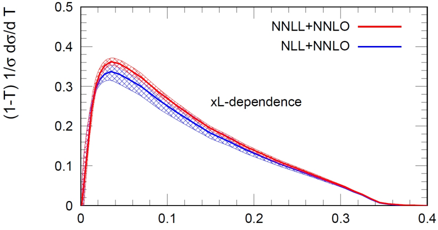
- We compare two different matching schemes (R vs $Log(R)$)
- The former requires the $\mathcal{O}(\alpha_s^3)$ constant term C_3
- C_3 unknown!
  - we fit it using EERAD3
  - good subtraction of the logarithms
  - high numerical instabilities below $L \sim -5.75$
    $\rightarrow$ high systematic error $\sim 40\%$!

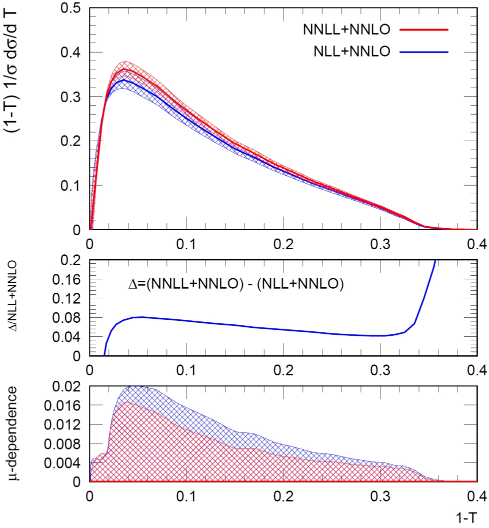




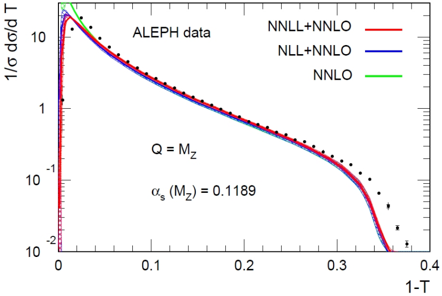
- R matching scheme improves the prediction in the 3 jet region with respect to the $Log(R)$ scheme

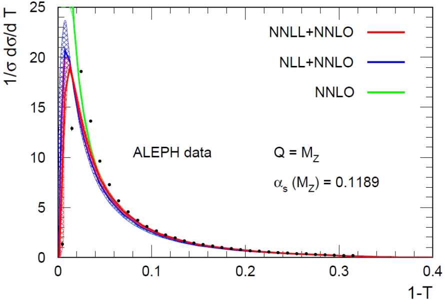
- comparison to NLL+N^{2}LO [Gehrmann, Luisoni, Stenzel]
- renormalization scale dependence reduces from $\sim 6\%$ to $\sim 4\%$
- resummation scale dependence reduces from $\sim 7.2\%$ to $\sim 2.7\%$ in the peak region
- better description of the 3 jets region
- multi-jet region dominated by fixed order prediction





- comparison to ALEPH data





- good agreement with data
- additional shift provided by hadronization

Conclusions

- Computation of the two loop soft subprocess for the Thrust distribution
  - Precise determination of the non-logarithmic part
- $\mathrm{N}^2\mathrm{LL}$ resummation using RG evolution
  - Full agreement with Becher-Schwartz up to subleading terms
- R and $Log(R)$ matching to $\mathrm{N}^2\mathrm{LO}$
  - Important reduction in both resummation and renormalization scale dependence
  - R scheme slightly better than $Log(R)$ in the 3 jet region