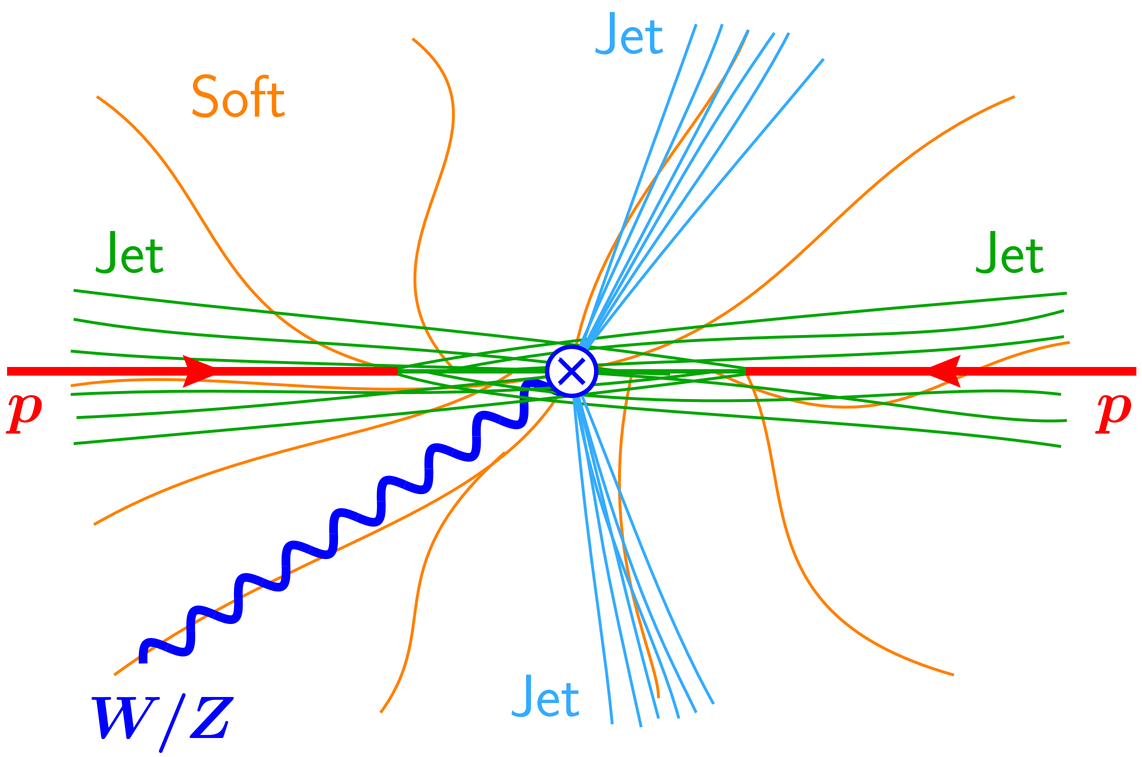
NNLL Resummation for N-jet Production at Hadron Colliders



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LoopFest X, Northwestern University
May 12 - 14, 2011

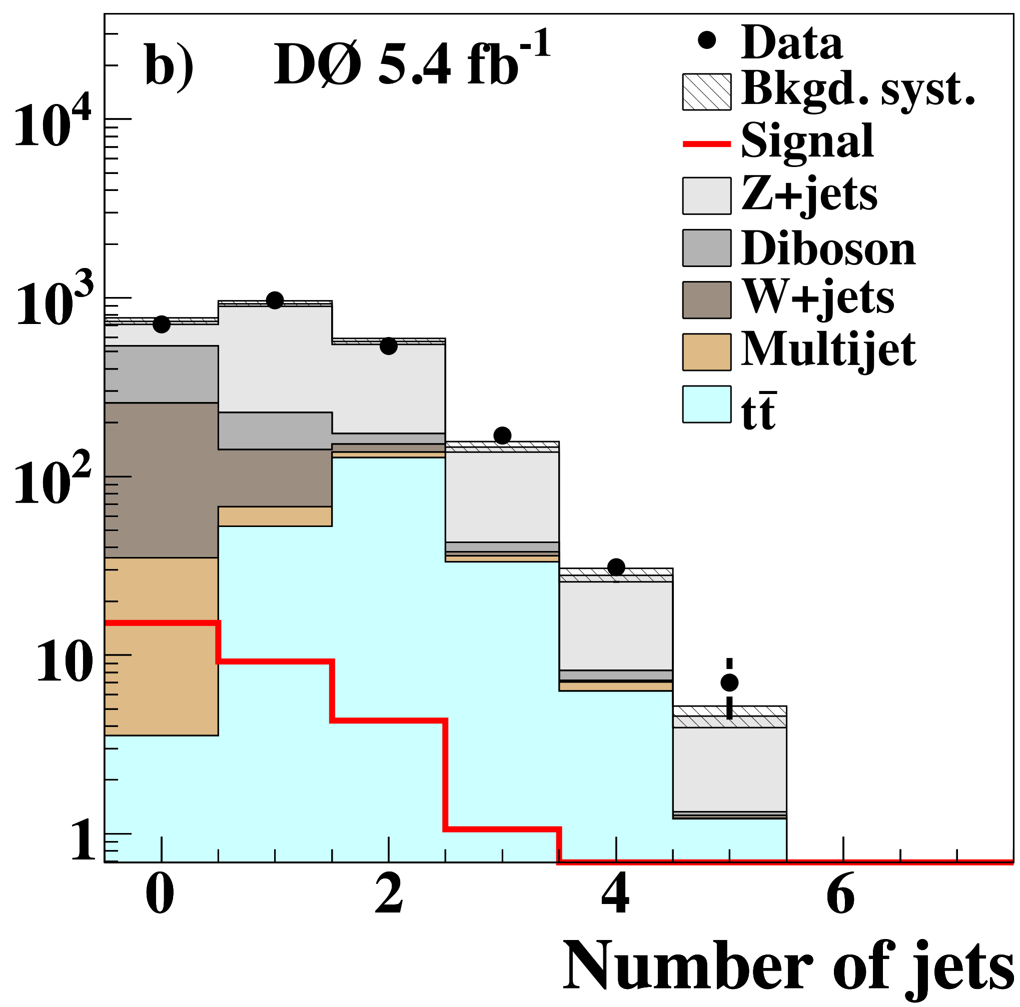
Work with Iain Stewart,
Frank Tackmann, Wouter Waalewijn

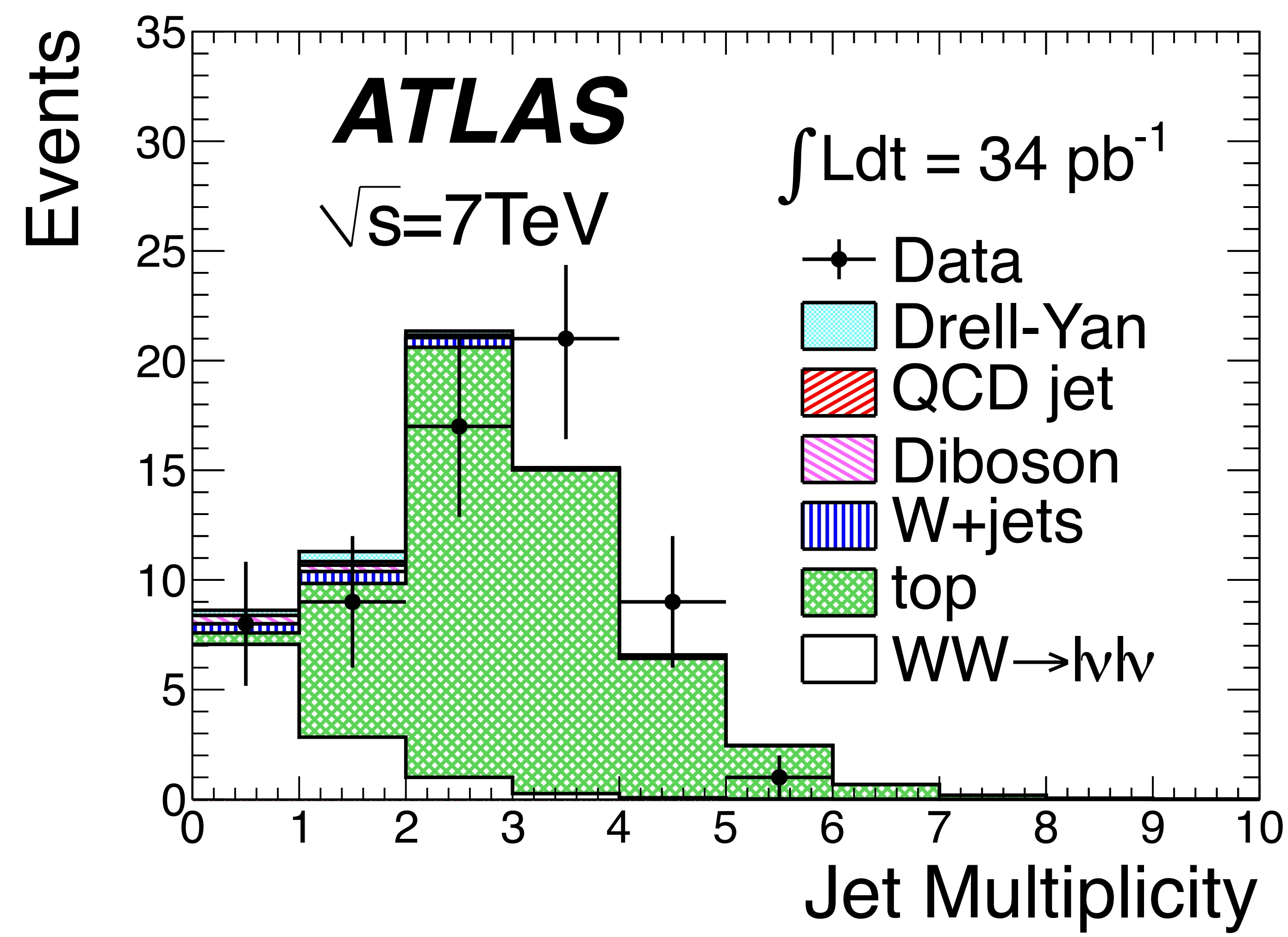
arXiv:1102.4344

LHC and Tevatron searches for Higgs require exclusive (differential) jet cross sections

Signal and backgrounds for $H \to WW$ vary differently as a function of jet multiplicity

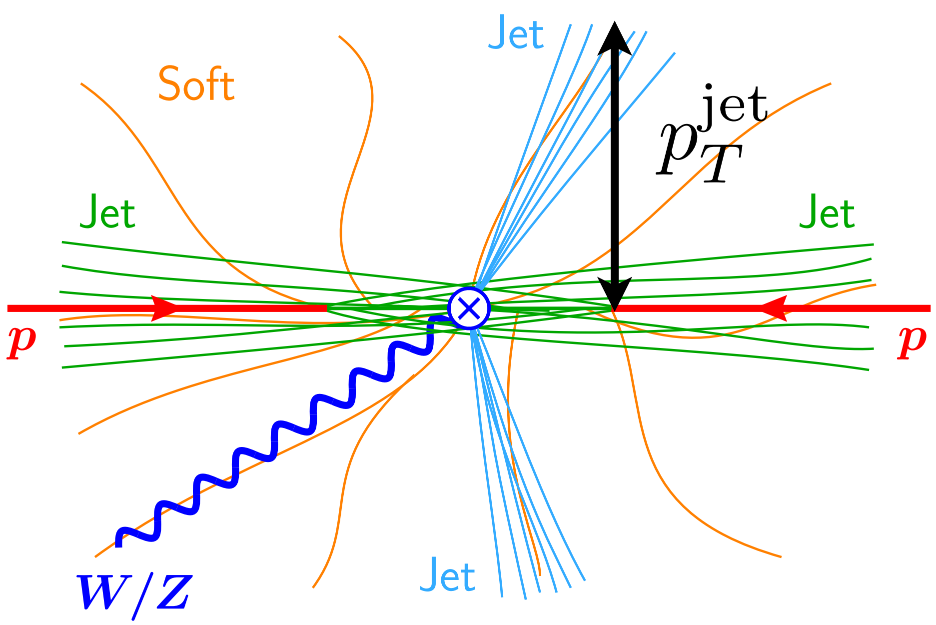
Same effect is illustrated below for $pp \to WW \to l\nu l\nu$





signal is for $m_H = 165 \text{ GeV}$

Exclusive N-jet cross sections are defined with a jet veto



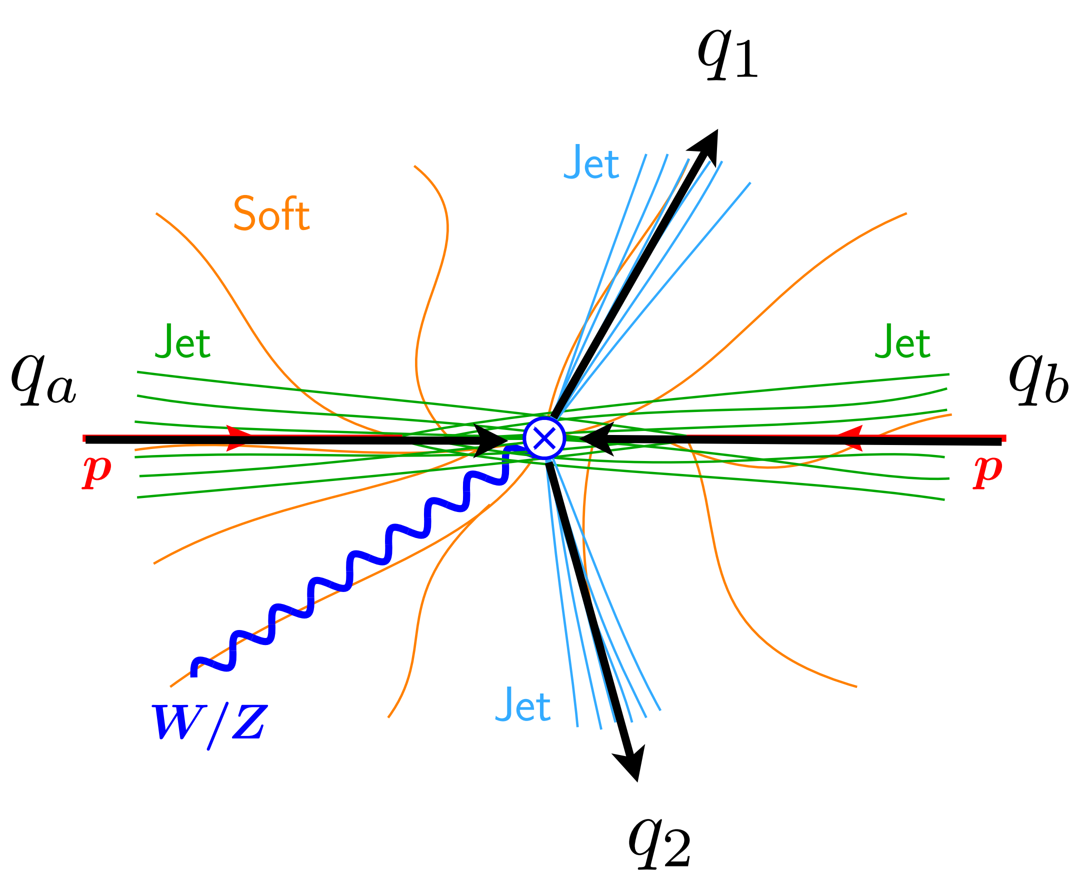
We require that all but N jets have $p_T^{\rm jet} < p_T^{\rm cut}$

$\Rightarrow \ln^2 \dfrac{p_T^{\rm cut}}{Q}$ large logarithms

Exclusive N-jet cross sections are defined with a jet veto

N-jettiness quantifies the distance of particles from beam $d_{a,b}(p)$ and jet $d_i(p)$ directions   Stewart, Tackmann, Waalewijn

$$\mathcal{T}_N = \sum_k |\vec{p}_{kT}| \min\{d_a(p_k), d_b(p_k), d_1(p_k), d_2(p_k), \ldots, d_N(p_k)\}$$



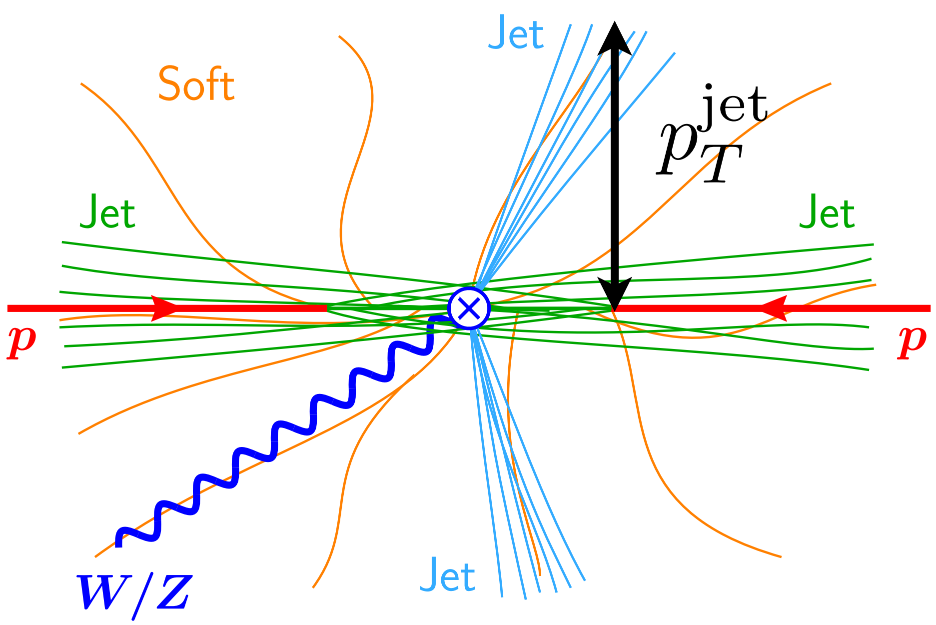
$\mathcal{T}_N$ has dimensions of momentum

It is similar to thrust

- N pencil-like jets: $\mathcal{T}_N = 0$
- More than N jets: $\mathcal{T}_N \sim Q$

$$\mathcal{T}_N = \mathcal{T}_N[q_a, q_b, q_1, \ldots, q_N]$$

Exclusive N-jet cross sections are defined with a jet veto



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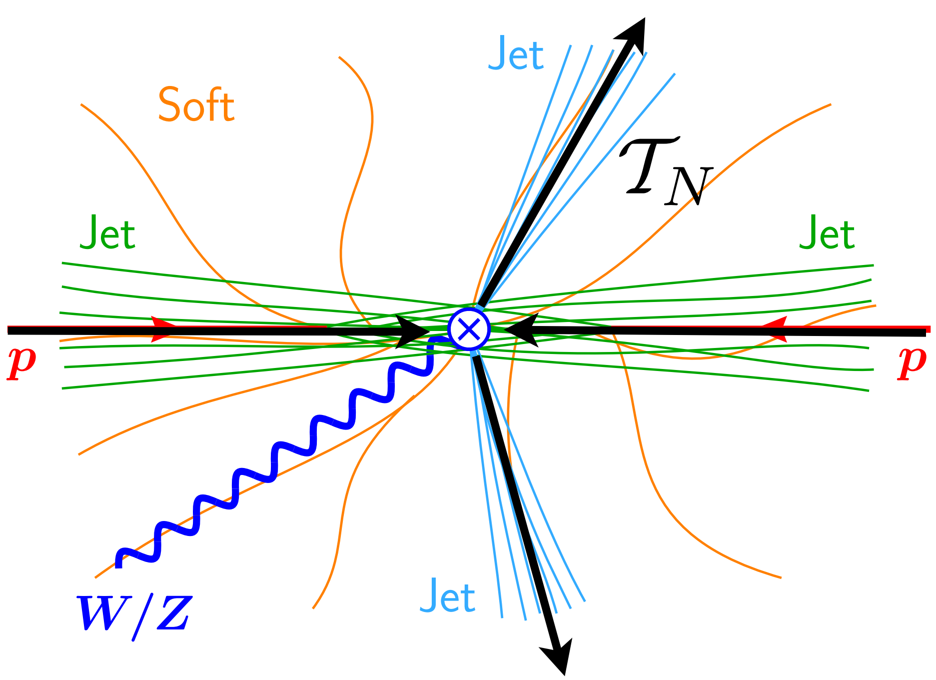
$$\Rightarrow \ln^2 \frac{p_T^{\rm cut}}{Q}$$ large logarithms



$\mathcal{T}_N < \mathcal{T}_N^{\rm cut}$ directly constrains all radiation to be collinear to N+2 directions or soft

$$\Rightarrow \ln^2 \frac{\mathcal{T}_N^{\rm cut}}{Q}$$ large logarithms

N-jettiness factorizes naturally

$\longrightarrow$ resummation is easier

Large logarithmic terms dominate exclusive jet cross sections

$$L = \ln \frac{\mathcal{T}_N^{\mathrm{cut}}}{Q}$$

$$\begin{array}{lcccccccl}
\sigma_{\mathrm{N-jet}} = 1 & & & & & & & & \\
+ \, \alpha_s L^2 & + \, \alpha_s L & + \, \alpha_s & + \, \alpha_s n_1(\mathcal{T}_N^{\mathrm{cut}}) & & & & & \mathrm{NLO} \\
+ \, \alpha_s^2 L^4 & + \, \alpha_s^2 L^3 & + \, \alpha_s^2 L^2 & + \, \alpha_s^2 L & + \, \alpha_s^2 & + \, \alpha_s^2 n_2(\mathcal{T}_N^{\mathrm{cut}}) & & & \mathrm{NNLO} \\
+ \, \alpha_s^3 L^6 & + \, \alpha_s^3 L^5 & + \, \alpha_s^3 L^4 & + \, \alpha_s^3 L^3 & + \, \alpha_s^3 L^2 & + \, \alpha_s^3 L & + & & \cdots \\
+ \, \vdots & + \, \vdots & + \, \vdots & + \, \vdots & + \, \vdots & + \, \vdots & + & & \ddots \\
\mathrm{LL} & & & & & & & &
\end{array}$$

Inclusive cross section has $\mathcal{T}_N^{\mathrm{cut}} \sim Q$ so $L \lesssim 1$

- Fixed order calculation gives accurate results

Exclusive cross section: $\mathcal{T}_N^{\mathrm{cut}} \ll Q$, tight cut can give $\alpha_s L^2 \sim 1$ or $\alpha_s L \sim 1$

- Resummation is necessary

When $1 < L < 1/\alpha_s$, resummation improves accuracy

Large logarithmic terms dominate exclusive jet cross sections

$$L = \ln \frac{\mathcal{T}_N^{\mathrm{cut}}}{Q}$$

$$
\begin{aligned}
\sigma_{\mathrm{N-jet}} = 1 & \\
& + \alpha_s L^2 + \alpha_s L + \alpha_s + \alpha_s n_1(\mathcal{T}_N^{\mathrm{cut}}) && \text{NLO} \\
& + \alpha_s^2 L^4 + \alpha_s^2 L^3 + \alpha_s^2 L^2 + \alpha_s^2 L + \alpha_s^2 + \alpha_s^2 n_2(\mathcal{T}_N^{\mathrm{cut}}) && \text{NNLO} \\
& + \alpha_s^3 L^6 + \alpha_s^3 L^5 + \alpha_s^3 L^4 + \alpha_s^3 L^3 + \alpha_s^3 L^2 + \alpha_s^3 L + && \cdots \\
& + \vdots + \vdots + \vdots + \vdots + \vdots + \vdots + && \ddots
\end{aligned}
$$

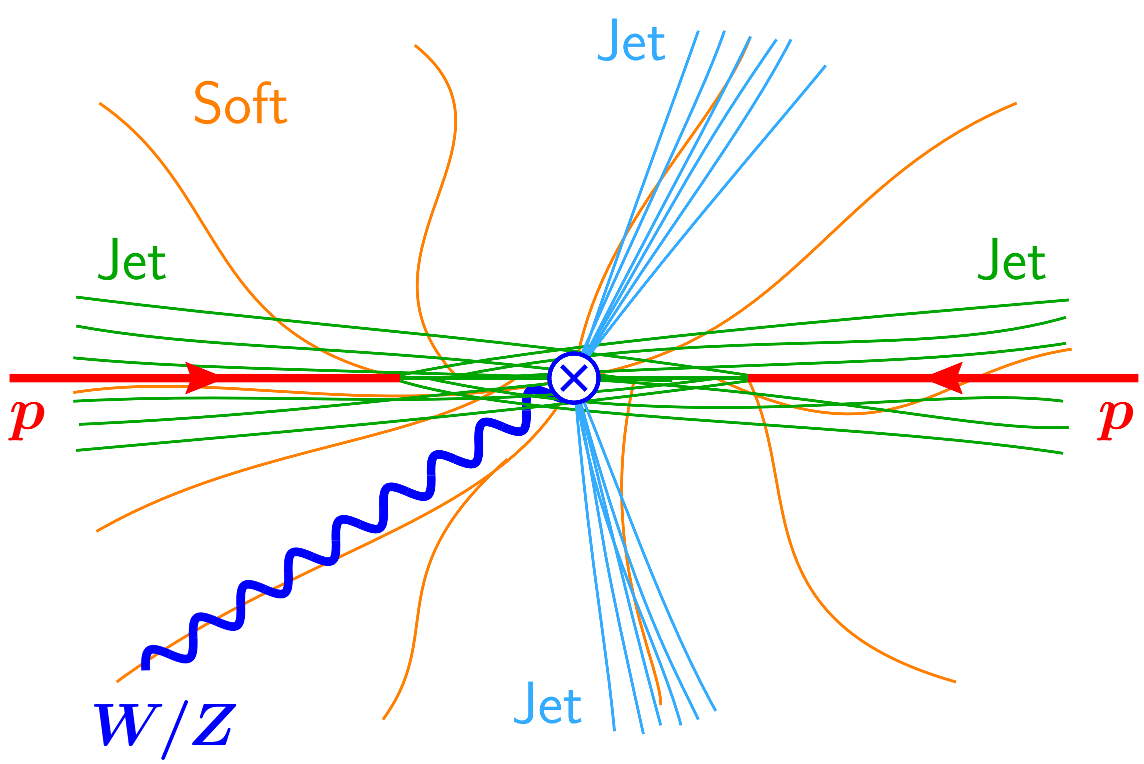
NNLL

	Cusp anomalous dimension	Non-cusp anomalous dimension	Fixed order for matching QCD to SCET
LL + LO	1-loop	-	tree
⋮	⋮	⋮	⋮
NNLL + NLO	3-loop	2-loop	1-loop

Parton shower codes

Enabled by our work

NNLL Resummation for N-jet Production at Hadron Colliders



Resummation ingredients

Two facets of N-jettiness

- Jet definition
- Observable

Soft calculation

Soft-Collinear Effective Theory (SCET) factorizes hard, collinear and soft physics

Bauer, Fleming, Pirjol, Stewart

Factorization enables resummation of large logarithms between different scales

$$\sigma_N = H_N(\mu) \times \left[B_a(\mu) B_b(\mu) \prod_{i=1}^{N} J_i(\mu) \right] \otimes S_N(\mu)$$

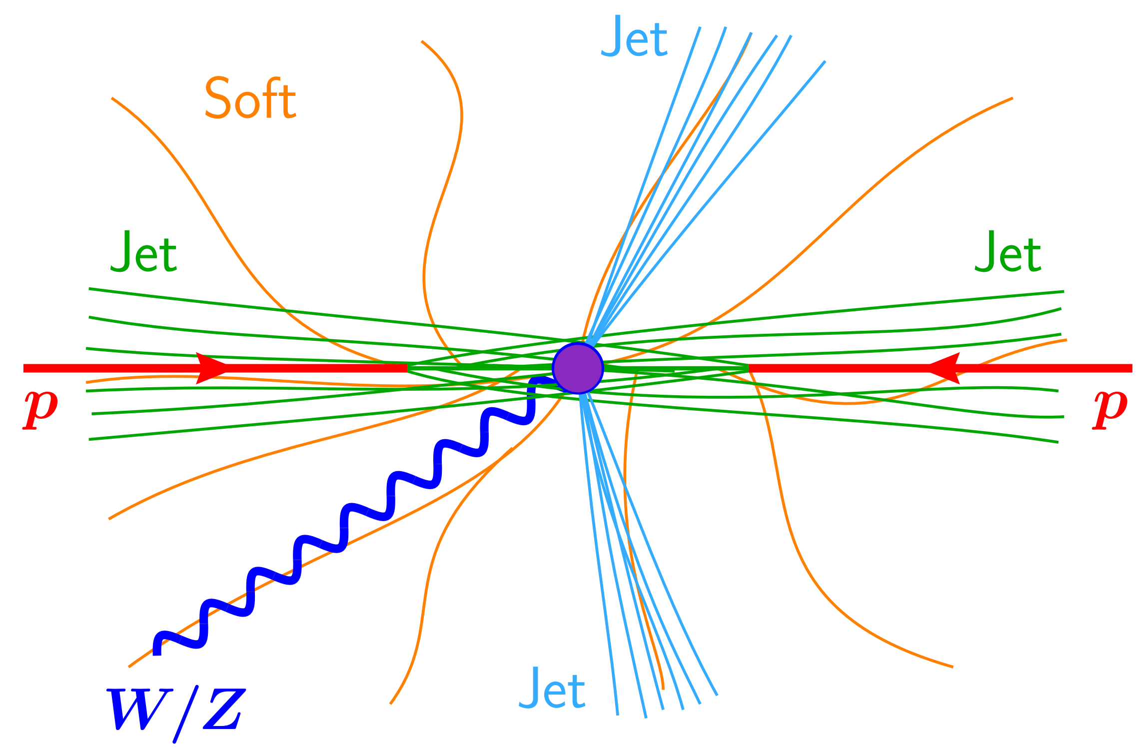
$$\ln^2 \frac{\mathcal{T}_N}{Q} = 2 \ln^2 \frac{Q}{\mu} - \ln^2 \frac{\mathcal{T}_N Q}{\mu^2} + 2 \ln^2 \frac{\mathcal{T}_N}{\mu}$$

Hard function can be obtained using 1-loop QCD calculation

$$\mu_H \simeq Q, \quad \mu_B, \mu_J \simeq \sqrt{\mathcal{T}_N Q}, \quad \mu_S \simeq \mathcal{T}_N$$

Collinear radiation is described by beam functions and jet functions

Our 1-loop soft function calculation is the last missing ingredient for NNLL resummation



Hard function is obtained by matching QCD to SCET

$$\mathcal{L}_{\rm QCD} \sim \vec{C}_N \cdot \vec{\mathcal{O}}_N$$

$$\widehat{H}_N = \vec{C}_N \vec{C}_N^\dagger$$

The matching coefficient is given by the IR-finite part of the QCD-amplitude (in dim.reg.)

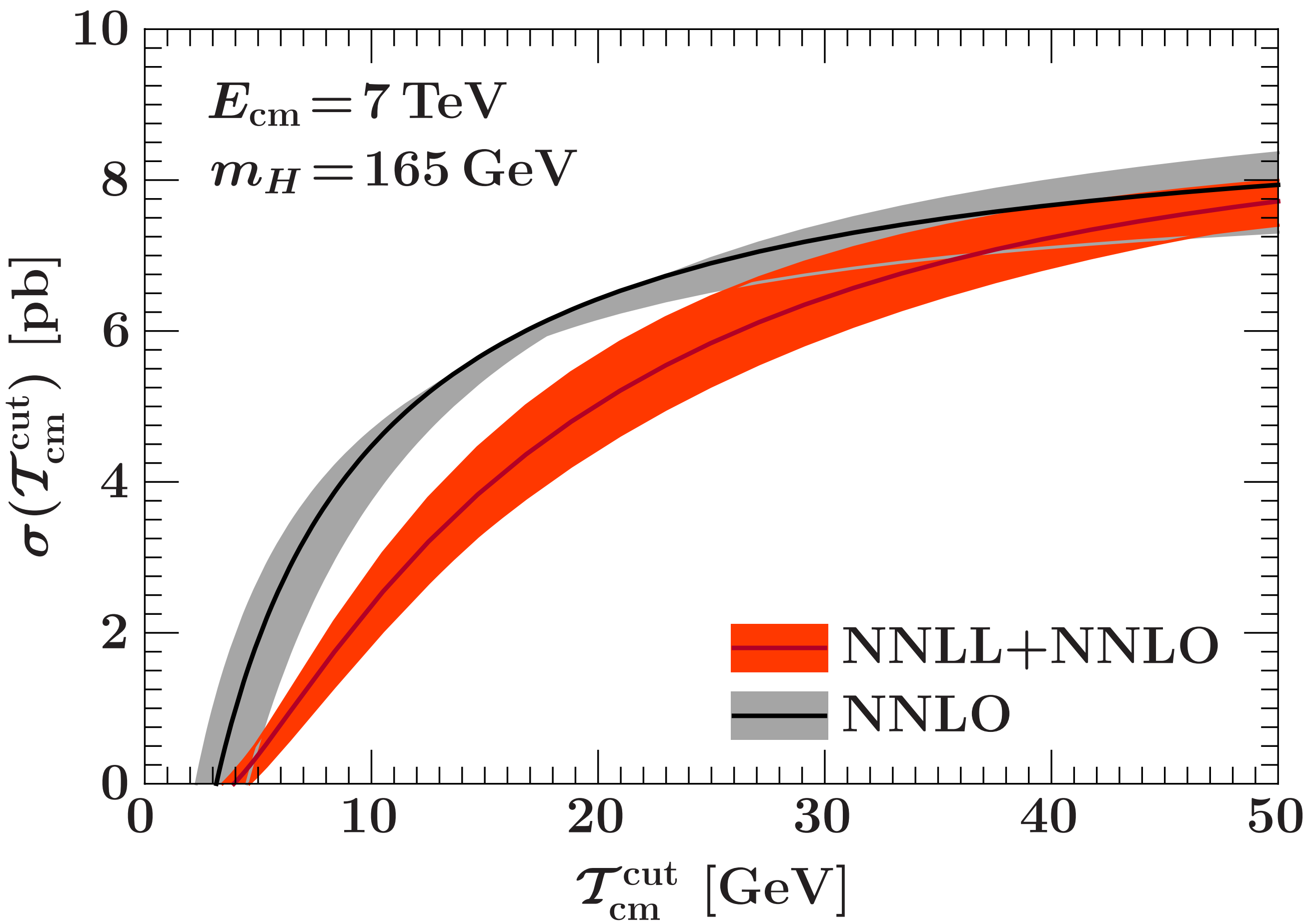
$$C_N^{a_1 a_2 \cdots \alpha_{n-1} \alpha_n}(p_1, p_2, \ldots, p_{n-1}, p_n)$$
$$= \mathcal{A}_{\rm finite}(q^{a_1}(p_1), q^{a_2}(p_2), \ldots, g^{\alpha_{n-1}}(p_{n-1}), g^{\alpha_n}(p_n))$$

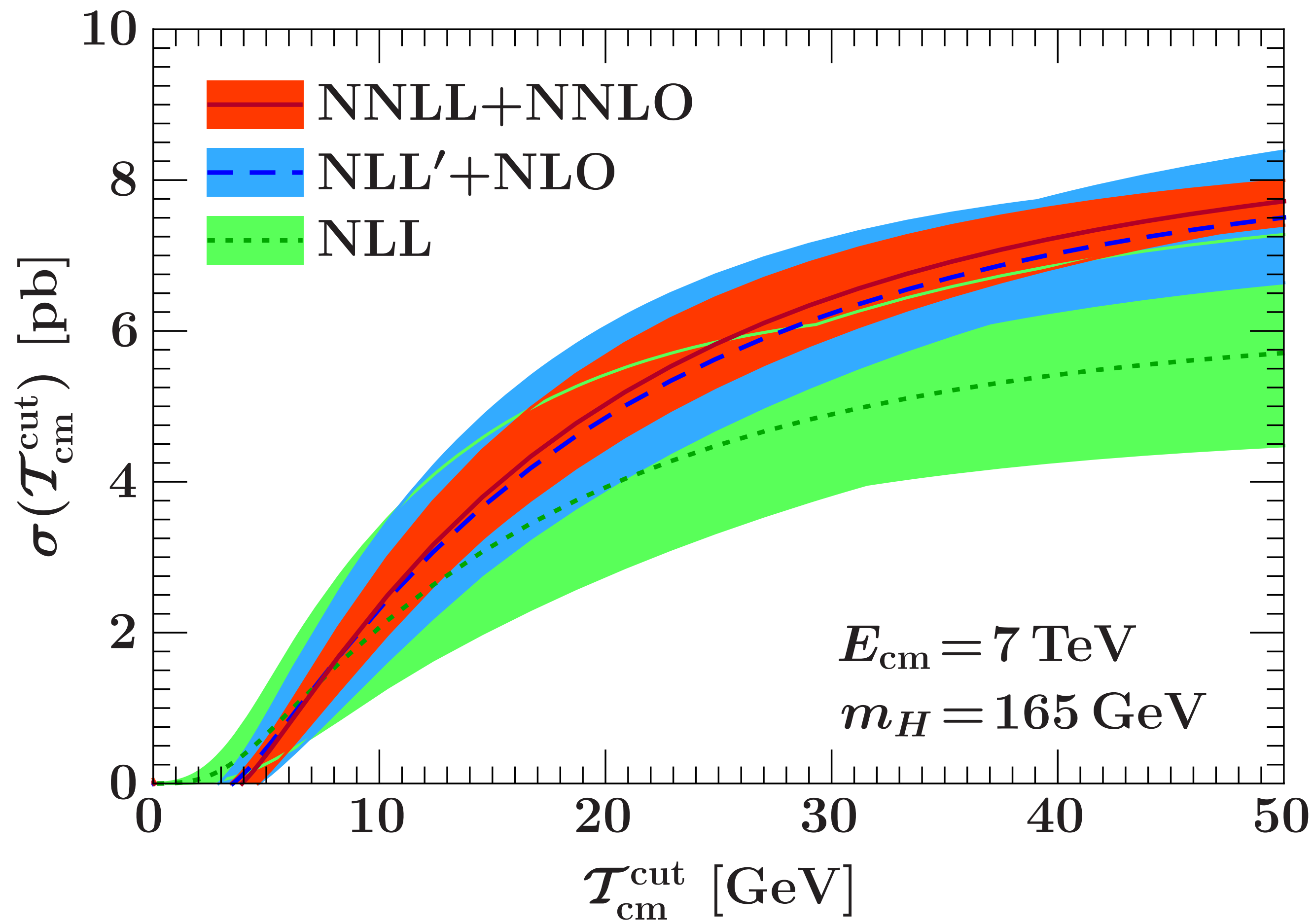
Some QCD calculations that have been used in matching

- $gg \to H$ at NNLO
  Dawson
  Djouadi, Spira, Graudenz, Zerwas
  Harlander, Kilgore
  Anastasiou, Melnikov
  Ravindran, Smith, van Neerven
  Pak, Rogal, Steinhauser
  Harlander, Mantler, Marzani, Ozeren
- Drell-Yan at NNLO
  Altarelli, K. Ellis, Martinelli
  Hamberg, van Neerven, Matsuura
  Harlander, Kilgore
  Anastasiou, Dixon, Melnikov, Petriello
- Direct photon production at NLO
  Aurenche, Douiri, Baier, Fontannaz, Schiff
  Gordon, Vogelsang
- Two jet production at hadron colliders at NLO
  K. Ellis, Furman, Haber, Hinchliffe
  K. Ellis, Sexton
  Kunszt, Signer, Trocsanyi
- $p\bar{p}, pp \to t\bar{t}$ at NLO
  Nason, Dawson, K. Ellis
  Beenakker, Kuijf, van Neerven, Meng, Schuler, Smith
  Czakon, Mitov
  Bonciani, Ferroglia, Gehrmann, Maitre, Studerus
  Frixione, Mangano, Nason, Ridolfi

Cross section for $pp \to H \to WW$ shows the significant impact of resummation

Berger, Marcantonini, Stewart, Tackmann, Waalewijn

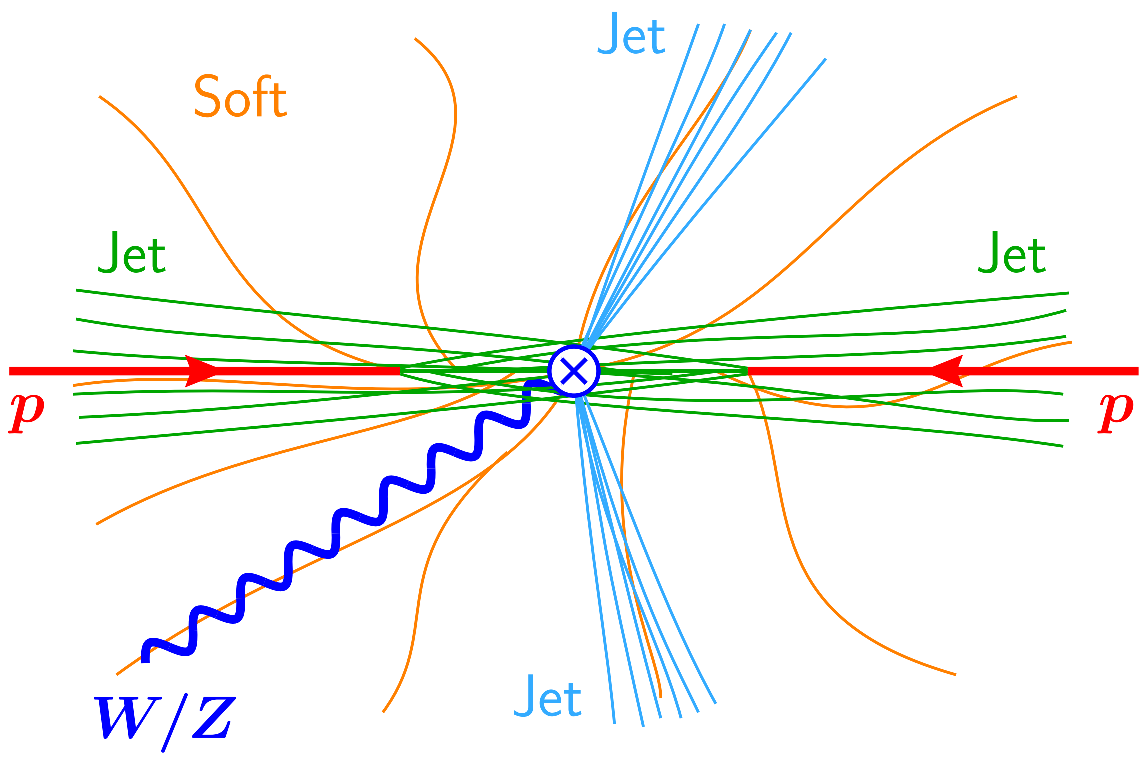




Resummation matters most for small values of $\mathcal{T}_{\rm cm}^{\rm cut} = \mathcal{T}_0^{\rm cut}$

Resummation improves convergence of perturbation theory

NNLL Resummation for N-jet Production at Hadron Colliders



Resummation ingredients

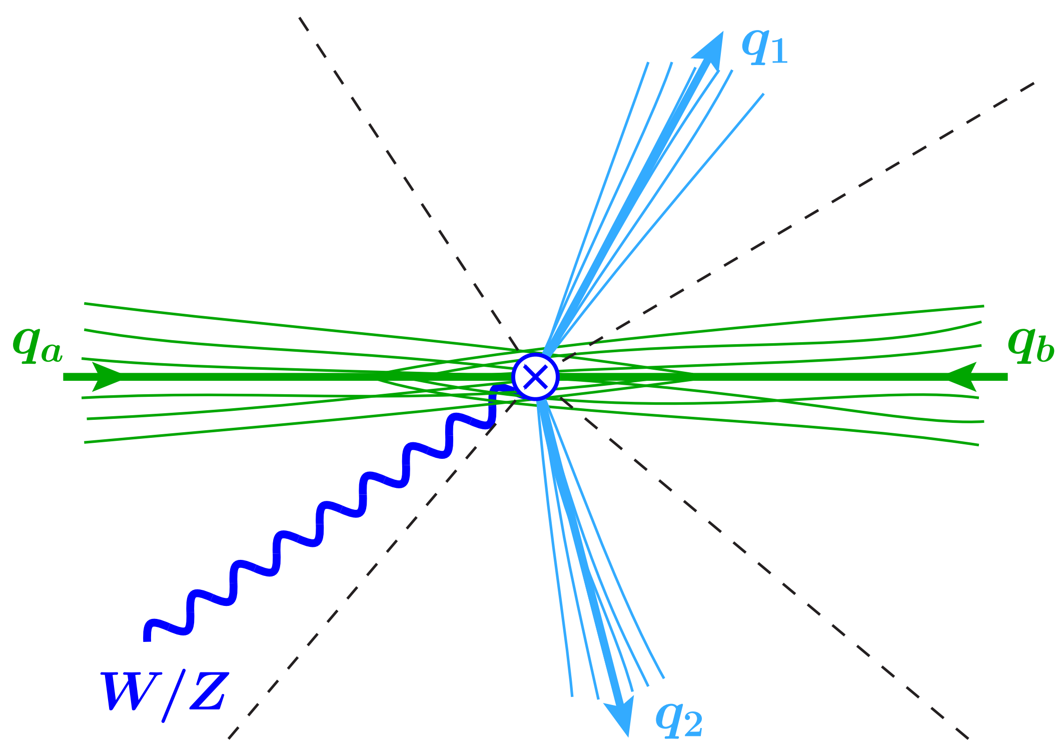
Two facets of N-jettiness
- Jet definition
- Observable

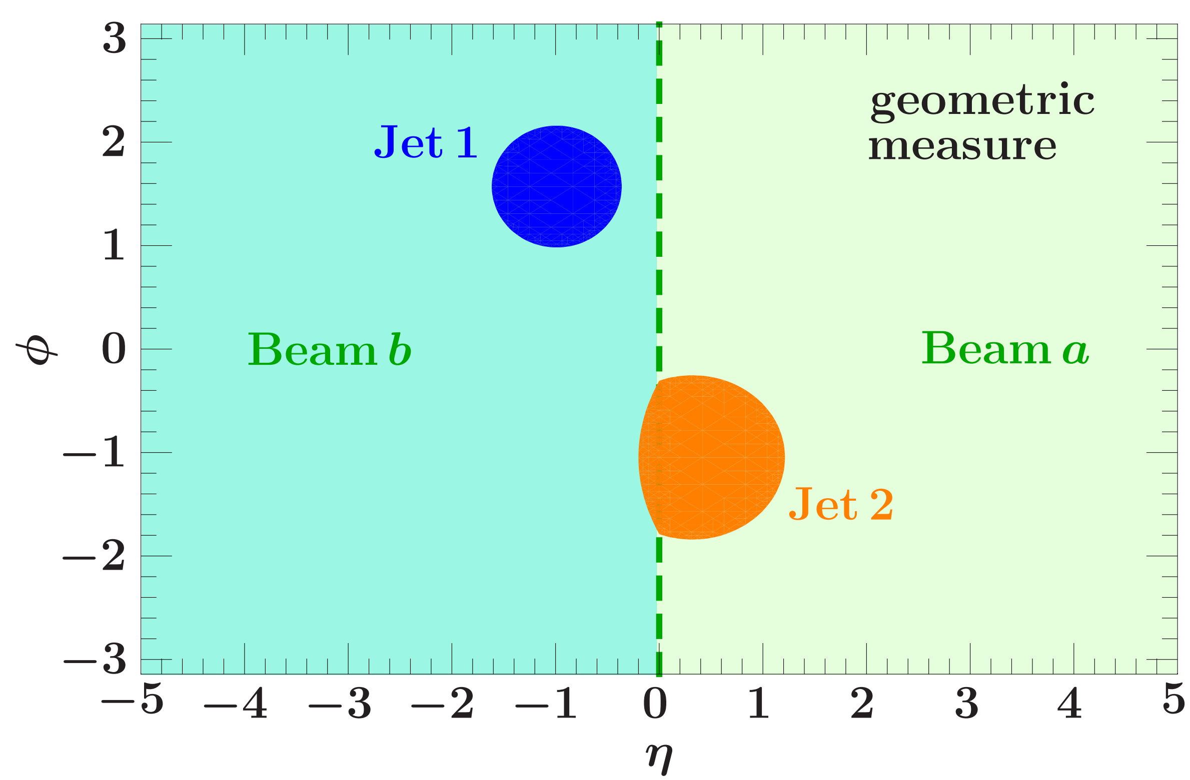
Soft calculation

Jet definition:

N-jettiness divides particles into jet and beam regions

$$\mathcal{T}_N = \sum_k |\vec{p}_{kT}| \min\{d_a(p_k), d_b(p_k), d_1(p_k), d_2(p_k), \ldots, d_N(p_k)\}$$





Jets are in general non-planar

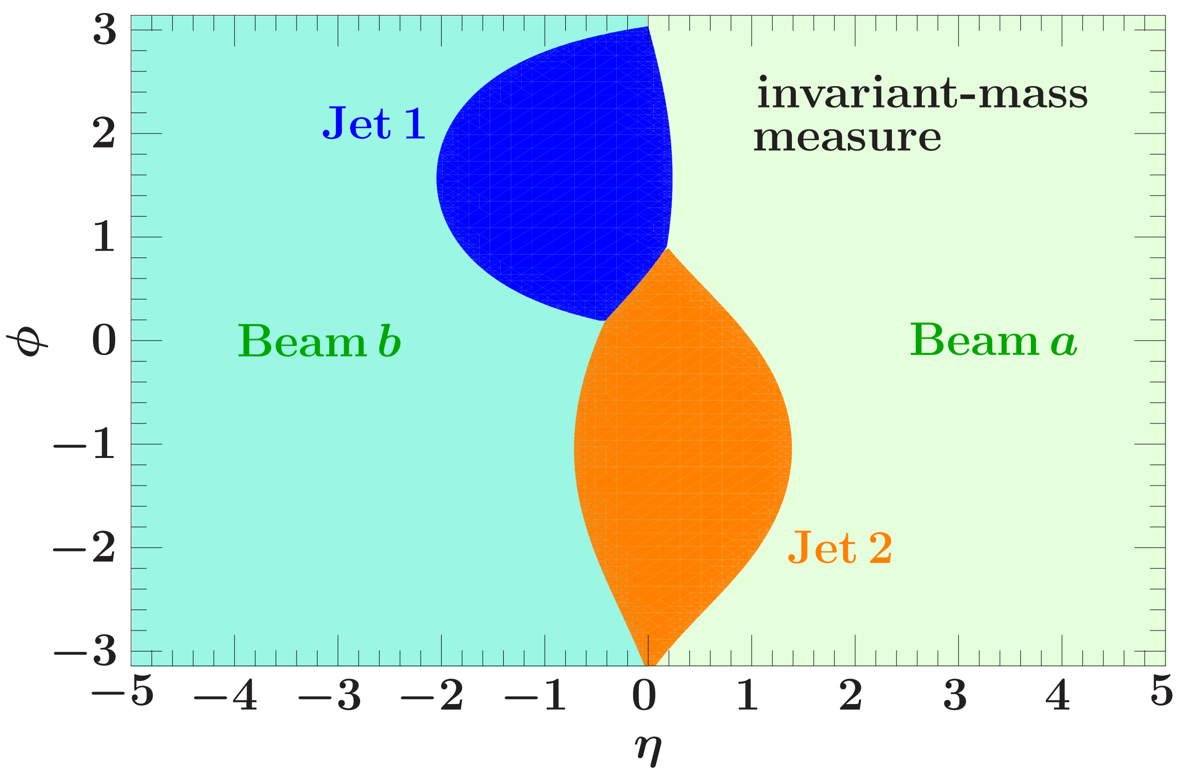
$$d_{a,b}(p_k) = e^{\mp\eta_k}$$

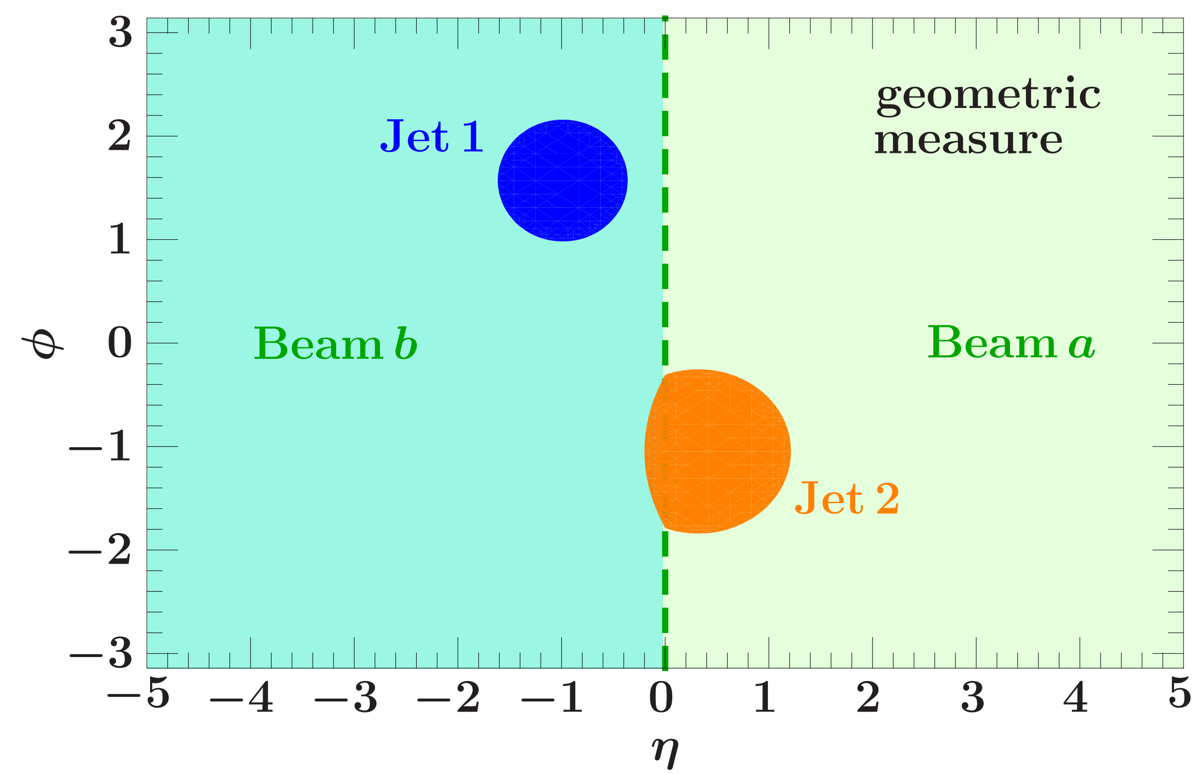
$$d_j(p_k) = 2\cosh\Delta\eta_{jk} - 2\cos\Delta\phi_{jk}$$
$$\approx (\Delta\eta_{jk})^2 + (\Delta\phi_{jk})^2$$

Jet definition:

N-jettiness divides particles into jet and beam regions

$$\mathcal{T}_N = \sum_k |\vec{p}_{kT}| \min\{d_a(p_k), d_b(p_k), d_1(p_k), d_2(p_k), \ldots, d_N(p_k)\}$$





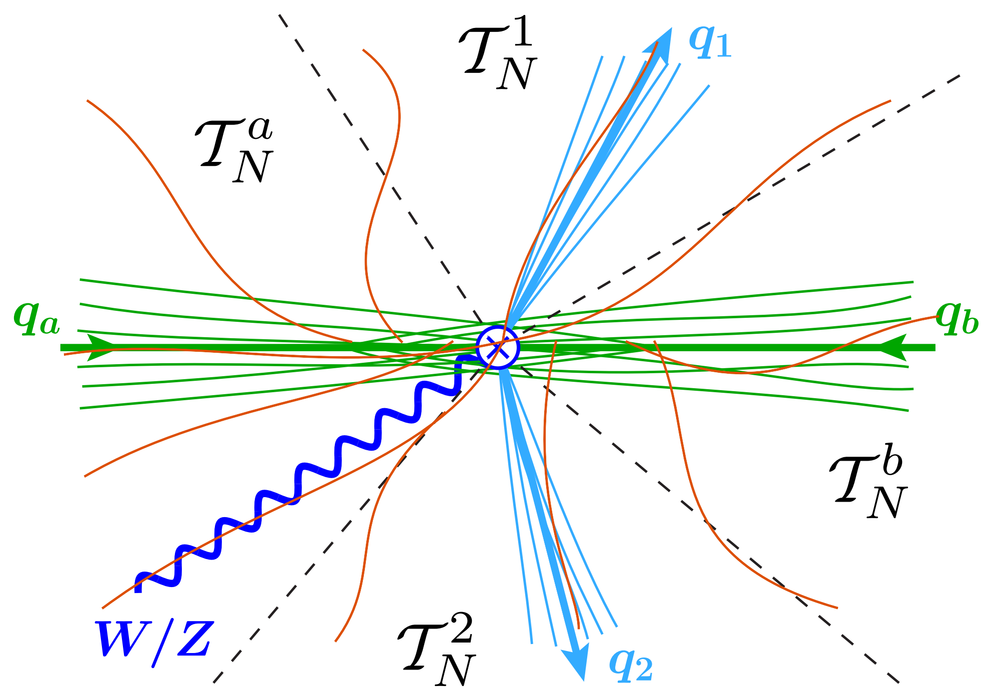
$$d_i(p_k) = \frac{2q_i \cdot p_k}{Q|\vec{p}_{kT}|}$$

$$d_{a,b}(p_k) = e^{\mp\eta_k}$$

$$d_j(p_k) = 2\cosh\Delta\eta_{jk} - 2\cos\Delta\phi_{jk} \approx (\Delta\eta_{jk})^2 + (\Delta\phi_{jk})^2$$

Observable:

Measuring N-jettiness defines exclusive N-jet cross sections



Cross section fully differential in

$$\mathcal{T}_N^j = \sum_{k\in j} |\vec{p}_{kT}|\, d_j(p_k)$$

N-jettiness measures jet mass

$$M_j^2 = P_j^2 = |\vec{p}_T^{\,j}|\,\mathcal{T}_N^j,$$

$$\text{where } P_j = \textstyle\sum_{k\in j} p_k$$

Factorization requires small jet masses

$$\mathcal{T}_N = \textstyle\sum_i \mathcal{T}_N^i \ll Q$$

$$\frac{d\sigma}{d\mathcal{T}_N^a\, d\mathcal{T}_N^b \cdots d\mathcal{T}_N^N} = \int dx_a dx_b \int d\begin{pmatrix}\text{phase}\\ \text{space}\end{pmatrix}$$

$$\times \operatorname*{Tr}_{\text{color}}\Big[\widehat{H}_N \int \mathbf{d}\boldsymbol{t_a}\, \boldsymbol{B_a(t_a, x_a)} \int \mathbf{d}\boldsymbol{t_b}\, \boldsymbol{B_b(t_b, x_b)} \prod_{J=1}^{N} \int \mathbf{d}\boldsymbol{s_J}\, \boldsymbol{J_J(s_J)}$$

$$\times\, \widehat{\boldsymbol{S}}_{\boldsymbol{N}}\left(\boldsymbol{\mathcal{T}_N^a - \frac{t_a}{Q_a}, \mathcal{T}_N^b - \frac{t_b}{Q_b}, \mathcal{T}_N^1 - \frac{s_1}{Q_1}, \ldots, \mathcal{T}_N^N - \frac{s_N}{Q_N}}\right)\Big]$$

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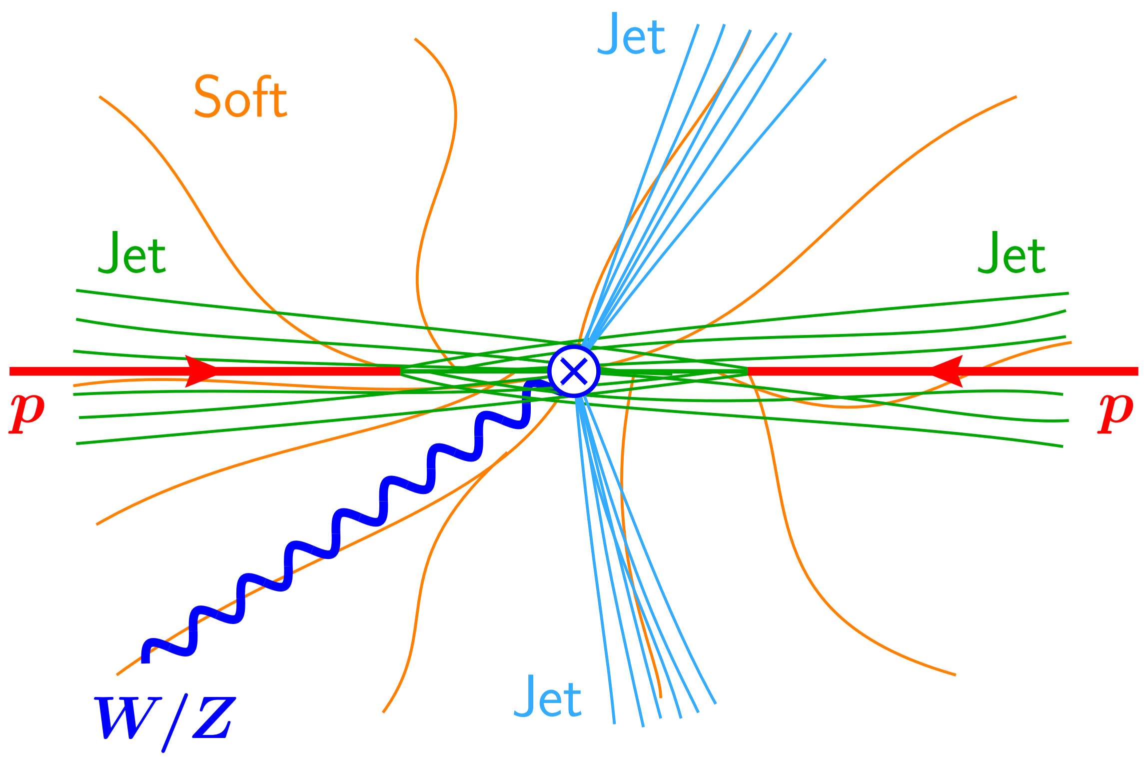


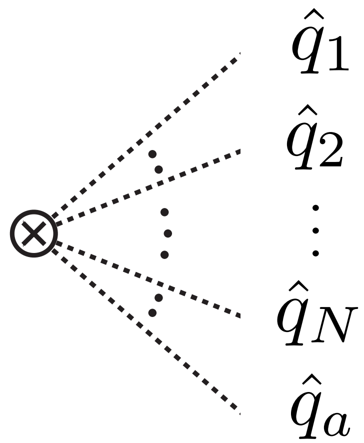
Resummation ingredients

Two facets of N-jettiness

- Jet definition
- Observable

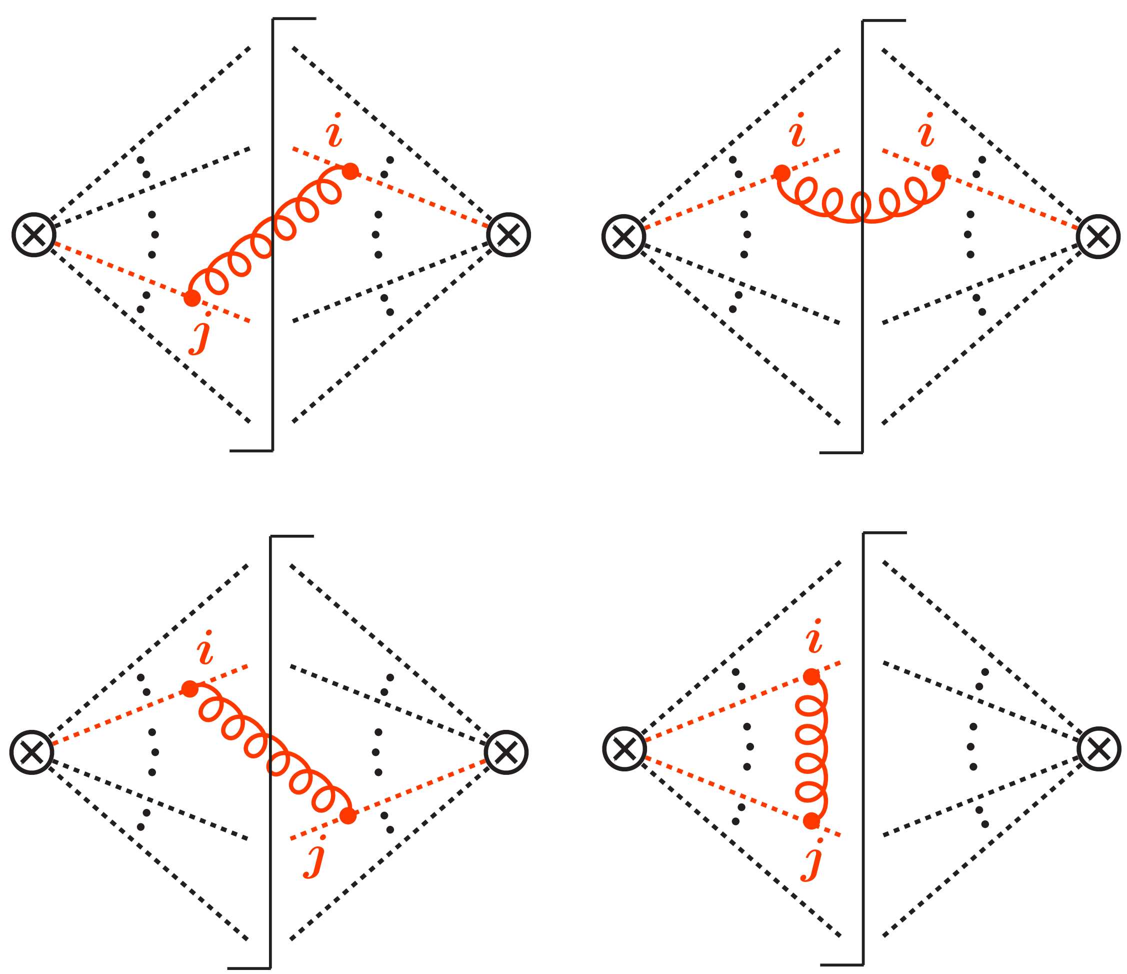
Soft calculation

The soft function describes radiation connecting the collinear sectors

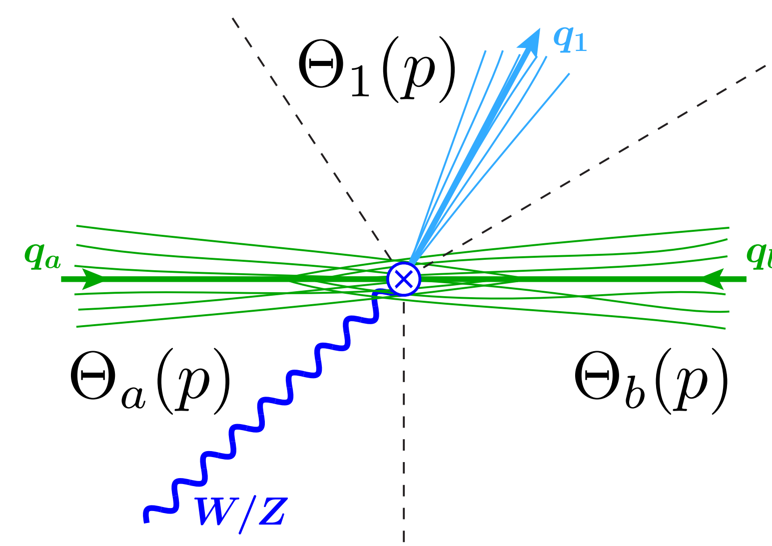


$$\widehat{S}_N \sim \langle 0 | \Big(\prod_i \widehat{Y}_i \Big) \Big(\prod_l \delta(k_l - \mathcal{O}_l) \Big) \Big(\prod_j \widehat{Y}_j \Big) | 0 \rangle$$

$$\hat{q}_i^\mu \equiv q_i^\mu / Q_i, \quad i \in \{a, b, 1, \ldots, N\}$$



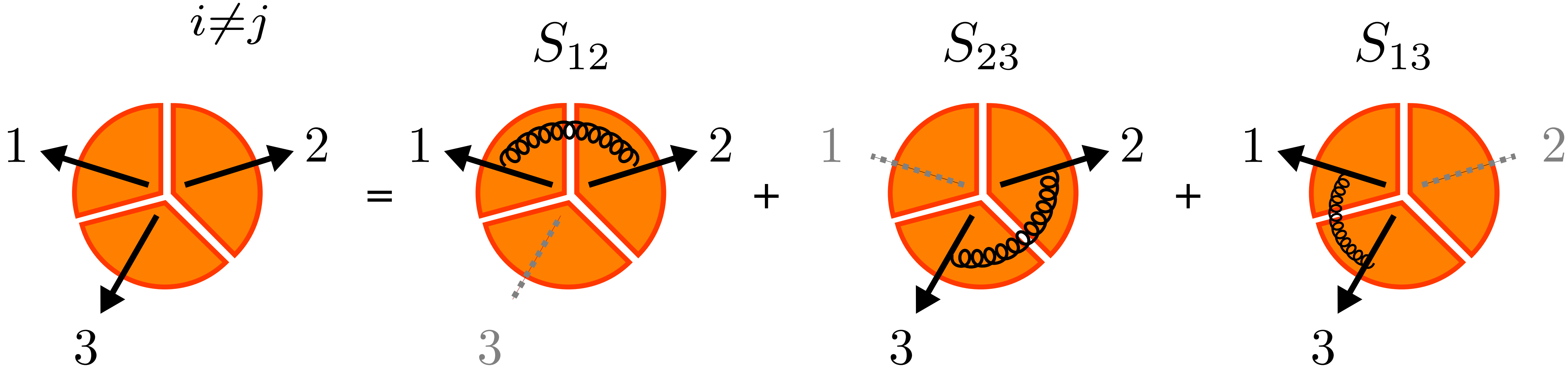
1-jettiness angular phase space is divided into three jet regions



$$\Theta_1(p) \equiv \theta(\hat{q}_a \cdot p - \hat{q}_1 \cdot p)\theta(\hat{q}_b \cdot p - \hat{q}_1 \cdot p)$$

Calculation simplifies by first choosing the eikonal lines producing the gluon

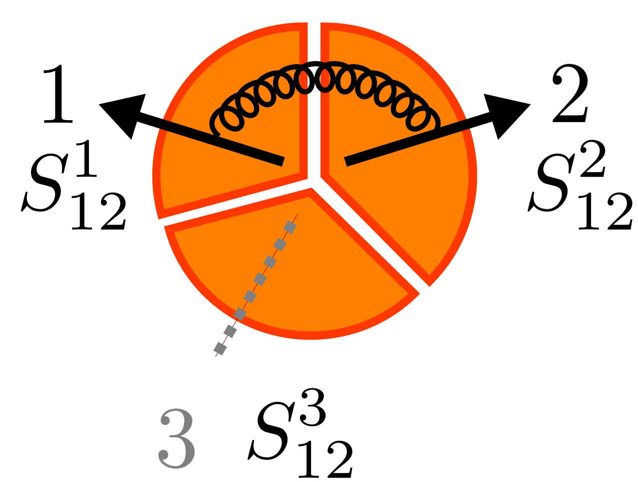
$$\widehat{S}_N = \sum_{i \neq j} \mathbf{T}_i \cdot \mathbf{T}_j \, S_{\boxed{ij}}$$ eikonal lines producing the gluon



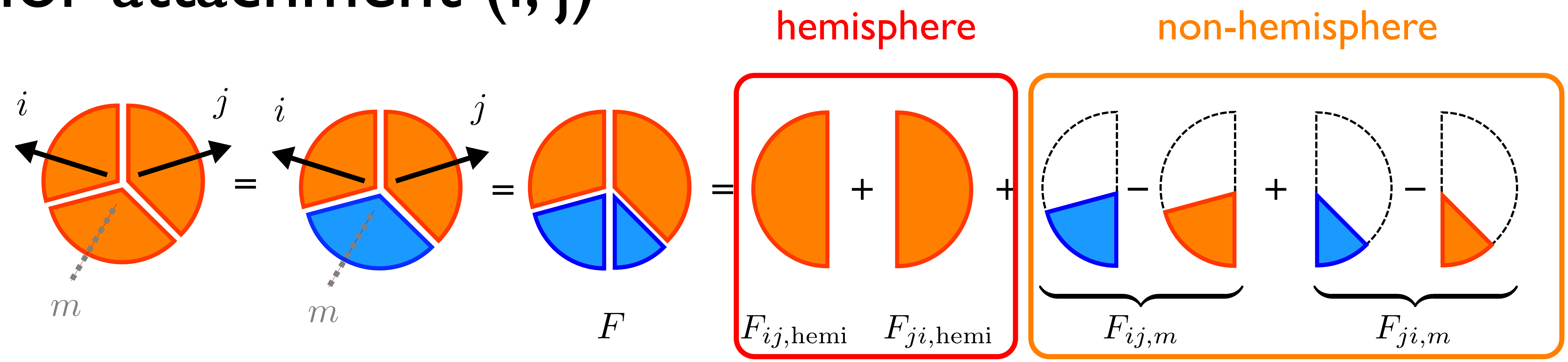
and then seeing which region it ends up in

$$S_{ij} = \sum_m S_{ij}^{\boxed{m}}$$ direction of the outgoing gluon

$$S_{12} = S_{12}^1 + S_{12}^2 + S_{12}^3 =$$



Hemisphere decomposition for attachment (i, j)



$$\widehat{S}_N \sim \sum_{i \neq j} \int \mathrm{d}^d p \, \frac{1}{(\hat{q}_i \cdot p)(\hat{q}_j \cdot p)} F$$

Produce a gluon from eikonal lines (i, j)

Double divergences are along these directions

Split remaining region(s) into two hemispheres

1-jettiness has only three regions

Extend regions (i, j) to full hemispheres

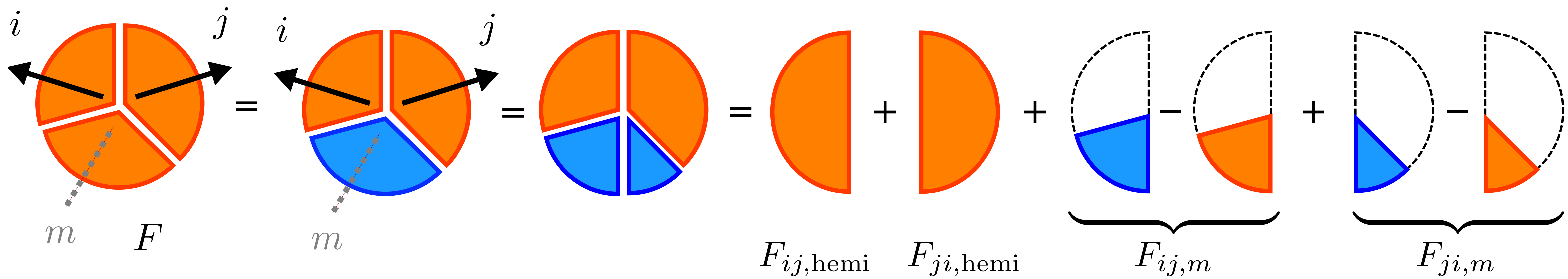
Hemispheres contain all the divergences

and give the expected anomalous dimension

Non-hemisphere contributions are UV and IR finite

as will be shown later

Hemisphere decomposition for attachment (i, j)



$$F = F_{ij,\mathrm{hemi}} + F_{ji,\mathrm{hemi}} + \sum_{m \neq i} F_{ij,m} + \sum_{m \neq j} F_{ji,m}$$

$$F_{ij,\mathrm{hemi}} = \Theta_{ij}^{\mathrm{hemi}}(p)\,\delta[k_i - \hat{q}_i \cdot p] \prod_{l \neq i} \delta(k_l)$$

$$F_{ij,m} = \Theta_{ij}^{\mathrm{hemi}}(p)\,\Theta_m(p) \prod\nolimits_{l \neq i,m} \delta(k_l) \times \Big\{ \delta(k_i)\,\delta[k_m - \hat{q}_m \cdot p] - \delta[k_i - \hat{q}_i \cdot p]\,\delta(k_m) \Big\}$$

$$\Theta_{ij}^{\mathrm{hemi}}(p) \equiv \theta(\hat{q}_j \cdot p - \hat{q}_i \cdot p)$$

$$\Theta_m(p) \equiv \prod_{l \neq m} \theta(\hat{q}_l \cdot p - \hat{q}_m \cdot p)$$

$$k_i = \mathcal{T}^i_{1,\mathrm{soft}}$$

Hemisphere contributions contain all UV divergences

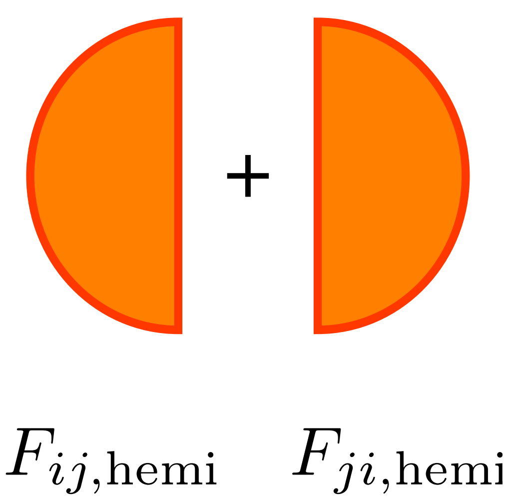
(removed by adding counterterms)

$$\widehat{S}_N(\{k_i\}) = \mathbf{1}\prod_i \delta(k_i) + \sum_{i\neq j} \mathbf{T}_i \cdot \mathbf{T}_j \left[S^{(1)}_{ij,\mathrm{hemi}}(\{k_i\}) + \sum_{m\neq i,j} S^{(1)}_{ij,m}(\{k_i\})\right] + \mathcal{O}(\alpha_s^2)$$

$$\mathcal{L}_n(x) = \left[\frac{\theta(x)\ln^n x}{x}\right]_+$$

$$S^{(1)}_{ij,\mathrm{hemi}}(\{k_i\},\mu) = \frac{\alpha_s(\mu)}{4\pi}\left[\frac{8}{\sqrt{2\hat{q}_i\cdot\hat{q}_j}\,\mu}\mathcal{L}_1\!\left(\frac{k_i}{\sqrt{2\hat{q}_i\cdot\hat{q}_j}\,\mu}\right) - \frac{\pi^2}{6}\,\delta(k_i)\right]\delta(k_j)\,\delta(k_m)$$

The (i, j) hemisphere contribution only depends on $\hat{q}_i\cdot\hat{q}_j$, not on other $\hat{q}_m$



The anomalous dimension is determined by the hemisphere contribution

Non-hemisphere contributions depend on angles between all jets

$$S^{\mathrm{bare}(1)}_{ij,m}(\{k_i\}) \sim \int \mathrm{d}\Omega_{d-2}\, \mathrm{d}p^i\, \mathrm{d}p^j\, \frac{\theta(p^i)\,\theta(p^j)}{(p^i p^j)^{1+\epsilon}} \qquad p^l \equiv 2\hat{q}_l \cdot p$$

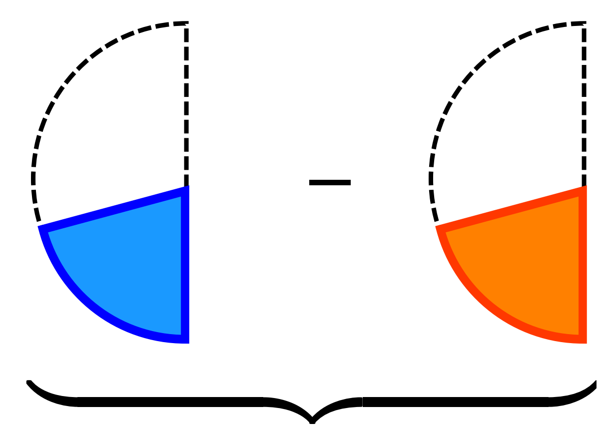
$$\times \Big[\delta(k_i)\,\delta(k_m - p^m) - \delta(k_i - p^i)\,\delta(k_m)\Big]$$

$$\times\, \delta(k_j)\,\theta(p^j - p^i)\,\Theta_m(p)$$

There is a non-hemisphere term for each jet region $m \neq i, j$

Divergences are regulated as follows

- Soft — taking the difference
- UV — measurement
- Collinear — restriction to region m



$$F_{ij,m}$$

Explicit expression for 1-jettiness non-hemisphere contribution

$$
\begin{aligned}
&S^{(1)}_{ij,m}(\{k_i\},\mu) \\
&\quad = \frac{\alpha_s(\mu)}{\pi}\Big\{ I_0 \Big[\frac{1}{\mu}\mathcal{L}_0\Big(\frac{k_i}{\mu}\Big)\,\delta(k_m) - \delta(k_i)\,\frac{1}{\mu}\mathcal{L}_0\Big(\frac{k_m}{\mu}\Big) + \ln\frac{\hat{q}_j\cdot\hat{q}_m}{\hat{q}_i\cdot\hat{q}_j}\,\delta(k_i)\,\delta(k_m)\Big]\delta(k_j) \\
&\qquad\qquad + I_1\,\delta(k_i)\,\delta(k_j)\,\delta(k_m)\Big\}
\end{aligned}
$$

$$
I_0(\alpha,\beta) = 2\int_0^{\phi_{\rm cut}(\alpha,\beta)} \frac{\mathrm{d}\phi}{\pi}\,\ln\frac{y_+(\phi,\alpha)}{\sqrt{\beta/\alpha}} + 2\theta(\alpha-\beta-1)\,\int_{\phi_{\rm cut}(\alpha,\beta)}^{\phi_{\max}(\alpha)}\frac{\mathrm{d}\phi}{\pi}\,\ln\frac{y_+(\phi,\alpha)}{y_-(\phi,\alpha)}
$$

$$
\begin{aligned}
I_1(\alpha,\beta) &= 2\int_0^{\phi_{\rm cut}(\alpha,\beta)}\frac{\mathrm{d}\phi}{\pi}\,\Big[G\big(y_+(\phi,\alpha),\phi\big) - G\big(\sqrt{\beta/\alpha},\phi\big)\Big] \\
&\quad + 2\theta(\alpha-\beta-1)\,\int_{\phi_{\rm cut}(\alpha,\beta)}^{\phi_{\max}(\alpha)}\frac{\mathrm{d}\phi}{\pi}\,\Big[G\big(y_+(\phi,\alpha),\phi\big) - G\big(y_-(\phi,\alpha),\phi\big)\Big]
\end{aligned}
$$

Explicit expression for 1-jettiness non-hemisphere contribution

$$I_0(\alpha,\beta) = 2\int_0^{\phi_{\rm cut}(\alpha,\beta)} \frac{\mathrm{d}\phi}{\pi}\,\ln\frac{y_+(\phi,\alpha)}{\sqrt{\beta/\alpha}} + 2\theta(\alpha-\beta-1)\int_{\phi_{\rm cut}(\alpha,\beta)}^{\phi_{\rm max}(\alpha)} \frac{\mathrm{d}\phi}{\pi}\,\ln\frac{y_+(\phi,\alpha)}{y_-(\phi,\alpha)}$$

$$I_1(\alpha,\beta) = 2\int_0^{\phi_{\rm cut}(\alpha,\beta)} \frac{\mathrm{d}\phi}{\pi}\,\Big[G\big(y_+(\phi,\alpha),\phi\big) - G\big(\sqrt{\beta/\alpha},\phi\big)\Big]$$
$$+ 2\theta(\alpha-\beta-1)\int_{\phi_{\rm cut}(\alpha,\beta)}^{\phi_{\rm max}(\alpha)} \frac{\mathrm{d}\phi}{\pi}\,\Big[G\big(y_+(\phi,\alpha),\phi\big) - G\big(y_-(\phi,\alpha),\phi\big)\Big]$$

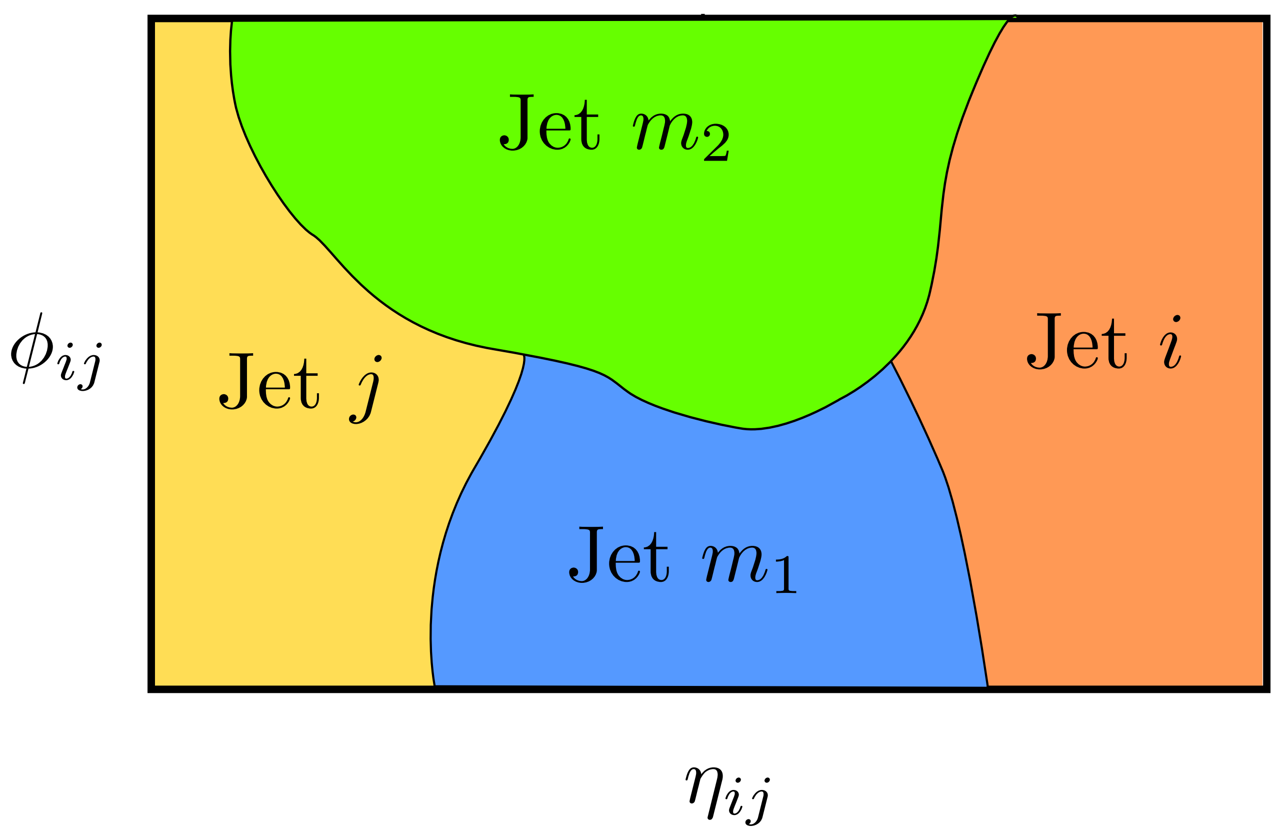
$$\phi_{\rm max}(\alpha) = \arcsin\frac{1}{\sqrt{\alpha}}, \quad \phi_{\rm cut}(\alpha,\beta) = \begin{cases} 0 & |\sqrt{\alpha}-\sqrt{\beta}| \geq 1\,, \\ \pi & \sqrt{\alpha}+\sqrt{\beta} \leq 1\,, \\ \arccos\dfrac{\alpha+\beta-1}{2\sqrt{\alpha\beta}} & \text{otherwise}\,. \end{cases}$$

$$\alpha = \frac{\hat{q}_j\cdot\hat{q}_m}{\hat{q}_i\cdot\hat{q}_j} \qquad \beta = \frac{\hat{q}_i\cdot\hat{q}_m}{\hat{q}_i\cdot\hat{q}_j}$$

$$G(y,\phi) = -2\mathrm{Re}\big[\mathrm{Li}_2(y e^{\mathrm{i}\phi})\big]$$

$$y_-(\phi,\alpha) = \cos\phi - \sqrt{1/\alpha - \sin^2\phi}\,,$$
$$y_+(\phi,\alpha) = \cos\phi + \sqrt{1/\alpha - \sin^2\phi}$$

Hemisphere decomposition works for N-jettiness and other observables



Definition of F does not depend on

- number of jets

$$F_{ij,\mathrm{hemi}} = \theta(\hat{q}_j \cdot p - \hat{q}_i \cdot p)\,\delta[k_i - \hat{q}_i \cdot p\,] \boxed{\prod_{l \neq i}} \delta(k_l)$$

$$F_{ij,m} = \theta(\hat{q}_j \cdot p - \hat{q}_i \cdot p)\ \Theta_m(p)\ \boxed{\prod_{l \neq i,m}} \delta(k_l)$$
$$\times \Big\{ \delta(k_i)\,\delta[k_m - \hat{q}_m \cdot p\,] - \delta[k_i - \hat{q}_i \cdot p\,]\,\delta(k_m) \Big\}$$

N-jettiness result is very similar to 1-jettiness

More delta functions

$$S^{(1)}_{ij,\mathrm{hemi}}(\{k_i\},\mu)$$
$$= \frac{\alpha_s(\mu)}{4\pi}\left[\frac{8}{\sqrt{2\hat{q}_i\cdot\hat{q}_j}\,\mu}\mathcal{L}_1\left(\frac{k_i}{\sqrt{2\hat{q}_i\cdot\hat{q}_j}\,\mu}\right) - \frac{\pi^2}{6}\,\delta(k_i)\right]\boxed{\prod_{m\neq i}}\,\delta(k_m)$$

Same form but more theta functions in $I_0,\ I_1$

$$S^{(1)}_{ij,m}(\{k_i\},\mu)$$
$$= \frac{\alpha_s(\mu)}{\pi}\left\{I_0\left[\frac{1}{\mu}\mathcal{L}_0\left(\frac{k_i}{\mu}\right)\delta(k_m) - \delta(k_i)\,\frac{1}{\mu}\mathcal{L}_0\left(\frac{k_m}{\mu}\right) + \ln\frac{\hat{q}_j\cdot\hat{q}_m}{\hat{q}_i\cdot\hat{q}_j}\,\delta(k_i)\,\delta(k_m)\right]\right.$$
$$\left. + I_1\,\delta(k_i)\,\delta(k_m)\right\}\boxed{\prod_{l\neq i,m}}\,\delta(k_m)$$

N-jettiness result is very similar to 1-jettiness

Same form but more theta functions in $I_0, \ I_1$

$$I_0(\alpha, \beta, \{\alpha_l, \beta_l, \phi_l\})$$

$$= \frac{1}{\pi}\int_{-\pi}^{\pi} \mathrm{d}\phi \int \frac{\mathrm{d}y}{y}\, \theta\big(y - \sqrt{\beta/\alpha}\big)\, \theta\Big(\frac{1}{\alpha} - 1 - y^2 + 2y\cos\phi\Big)$$

$$\boxed{\times \prod_l \theta\Big[\alpha_l - 1 + (\beta_l - 1)y^2 - 2y\big[\sqrt{\alpha_l\beta_l}\,\cos(\phi+\phi_l) - \cos\phi\big]\Big]}$$

$$I_1(\alpha, \beta, \{\alpha_l, \beta_l, \phi_l\})$$

$$= \frac{1}{\pi}\int_{-\pi}^{\pi} \mathrm{d}\phi \int \frac{\mathrm{d}y}{y}\, \ln(1 + y^2 - 2y\cos\phi)\, \theta\big(y - \sqrt{\beta/\alpha}\big)\, \theta\Big(\frac{1}{\alpha} - 1 - y^2 + 2y\,\cos\phi\Big)$$

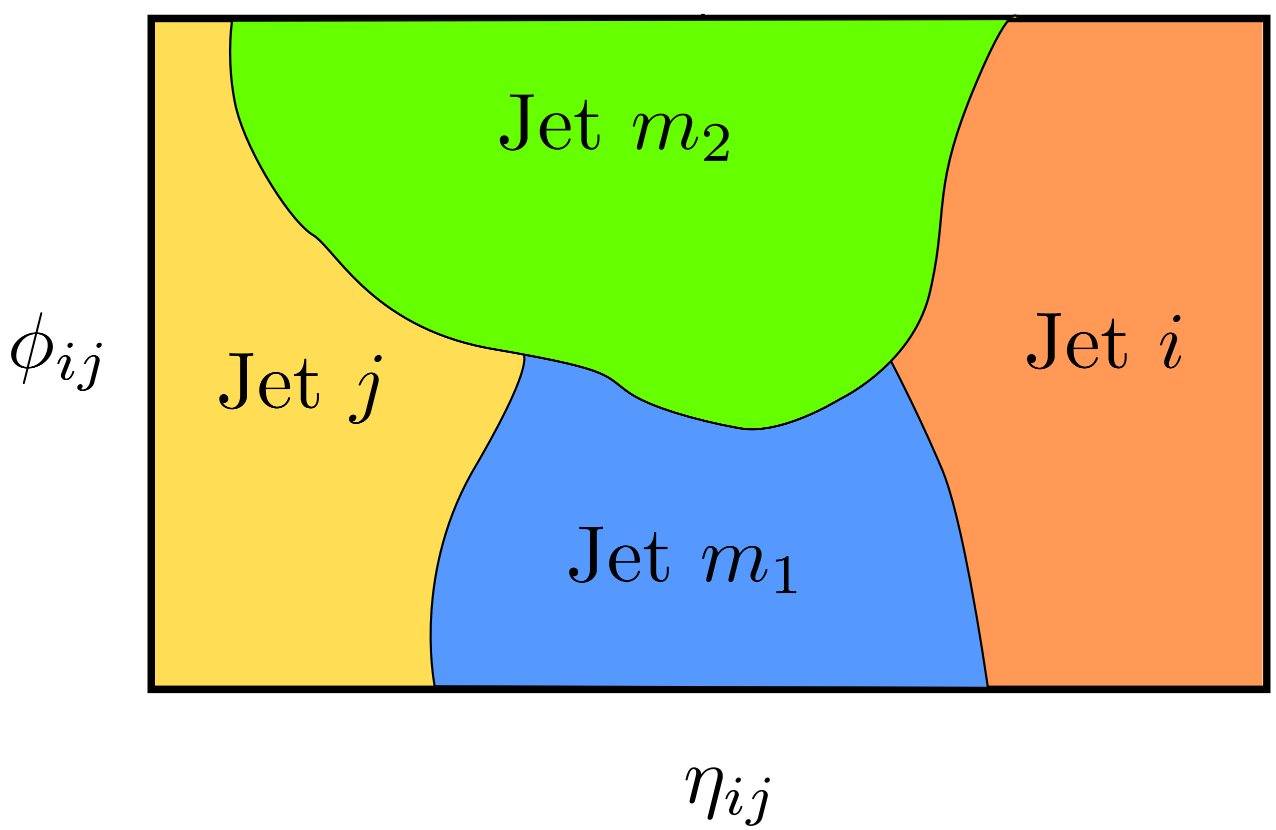
$$\boxed{\times \prod_l \theta\Big[\alpha_l - 1 + (\beta_l - 1)y^2 - 2y\big[\sqrt{\alpha_l\beta_l}\,\cos(\phi+\phi_l) - \cos\phi\big]\Big]}$$

This is the minimal extra complication from additional jets

$$\alpha = \frac{\hat{q}_j\cdot\hat{q}_m}{\hat{q}_i\cdot\hat{q}_j}, \qquad \beta = \frac{\hat{q}_i\cdot\hat{q}_m}{\hat{q}_i\cdot\hat{q}_j}$$

$$\alpha_l = \frac{\hat{q}_j\cdot\hat{q}_l}{\hat{q}_j\cdot\hat{q}_m}, \qquad \beta_l = \frac{\hat{q}_i\cdot\hat{q}_l}{\hat{q}_i\cdot\hat{q}_m}$$

Hemisphere decomposition works for N-jettiness and other observables



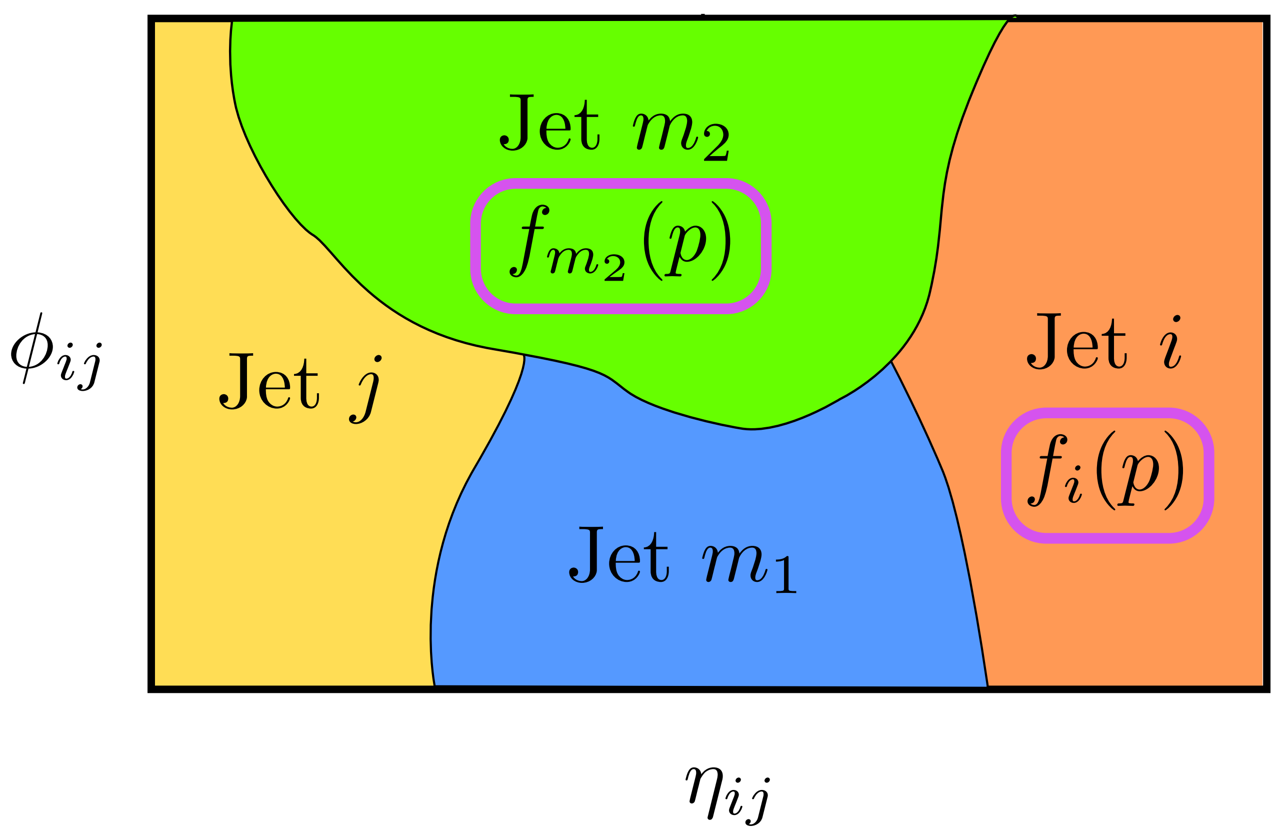
Definition of F does not depend on

- number of jets
- specification of regions

$$F_{ij,\mathrm{hemi}} = \theta(\hat{q}_j \cdot p - \hat{q}_i \cdot p)\, \delta[k_i - \hat{q}_i \cdot p\,] \prod_{l \neq i} \delta(k_l)$$

$$F_{ij,m} = \theta(\hat{q}_j \cdot p - \hat{q}_i \cdot p) \boxed{\Theta_m(p)} \prod\nolimits_{l \neq i,m} \delta(k_l) \\ \times \Big\{ \delta(k_i)\, \delta[k_m - \hat{q}_m \cdot p\,] - \delta[k_i - \hat{q}_i \cdot p\,]\, \delta(k_m) \Big\}$$

Hemisphere decomposition works for N-jettiness and other observables



Definition of F does not depend on

- number of jets
- specification of regions
- observables

$$F_{ij,\mathrm{hemi}} = \theta(\hat{q}_j \cdot p - \hat{q}_i \cdot p)\, \delta[k_i - \boxed{f_i(p)}] \prod_{l \neq i} \delta(k_l)$$

$$F_{ij,m} = \theta(\hat{q}_j \cdot p - \hat{q}_i \cdot p)\ \Theta_m(p)\ \ \textstyle\prod_{l \neq i,m} \delta(k_l)$$
$$\times \Big\{ \delta(k_i)\, \delta[k_m - \boxed{f_m(p)}] - \delta[k_i - \boxed{f_i(p)}]\, \delta(k_m) \Big\}$$

Conclusions

- SCET is powerful for analyzing jet processes with experimental cuts
  - Exclusive jet cross sections include large logarithms
  - Can systematically improve fixed order calculations by resummation
- Our calculation enables NNLL resummation for exclusive jet production
  - 1-loop soft function was the last missing ingredient
- Future work
  - Direct photon / W / Higgs + 1 jet phenomenology
  - Use the freedom in the choice of jet regions (jet algorithms, etc.)
  - Calculate N-jet soft functions for other observables
  - Hemisphere structure of the anomalous dimension suggests that extension to two loops is feasible