Higher order threshold effects for top-pair production at hadron colliders

Christian Schwinn

— Univ. Freiburg —

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(Based on M.Beneke, P.Falgari, CS, arXiv:1007.5414 [hep-ph]

M.Beneke, P.Falgari, S. Klein, CS, in progress)

Top established at Tevatron:

plethora of measurements (Γ_t, spin correlations, charge asymmetry...)

Top quark precision physics:

$$\Delta\sigma_{t\bar{t}}/\sigma_{tt} \sim 7\%$$

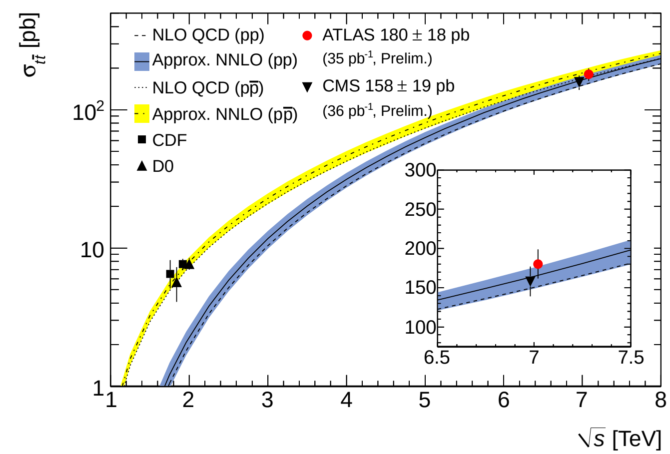
$$\Delta m_t/m_t < 1\%$$

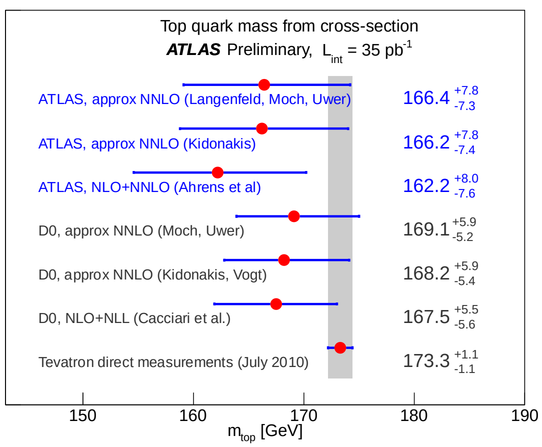
LHC catching up rapidly

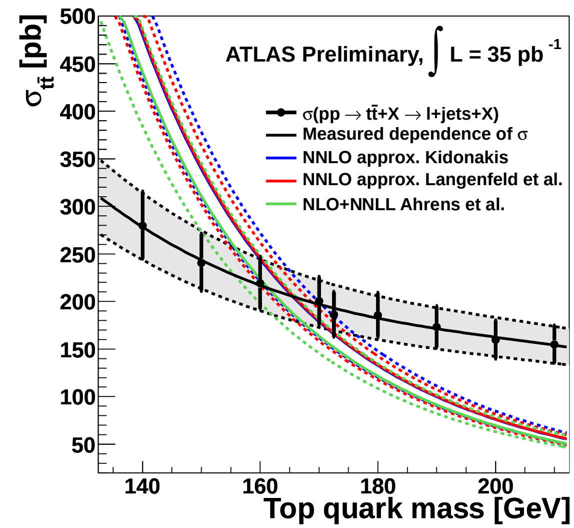
$\Rightarrow$ Challenge for theory!

Total cross section:

Sensitivity to m_t, gluon PDFs, constraints on new physics ...







NLO: long history

(QCD corrections (Nason, Dawson Ellis 88,Beenakker et.al. 89/91,. . .)

EW corrections (Bernreuther/Fuecker/Si; Kühn/Scharf/Uwer, 05/06)

Parton shower matching (Frixione/Nason/Webber 03, Frixione/Nason/Ridolfi 07))

NLO+NLL soft gluon resummation : $\Delta\sigma_{t\bar{t}} \approx 10 - 20\%$

(Cacciari et.al., Moch/Uwer; Kidonakis/Vogt,. . .)

NNLO: in progress (Czakon et.al. , Bonciani et.al.)

Progress for soft gluon resummation:

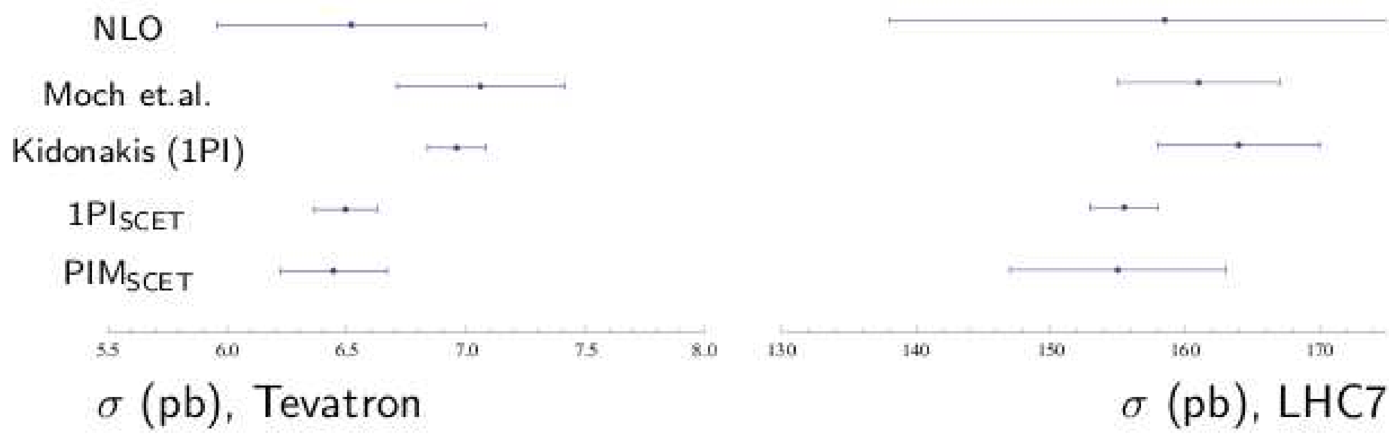
- massive 2-loop IR singularities (Becher/Neubert; Ferroglia et.al. 09)
- 2-loop soft anomalous dimension

  (Kidonakis; Mitov/Sterman/Sung; Beneke/Falgari/CS;Czakon Mitov/Sterman 09)

NNLO$_{\text{app.}}$ (Moch/Uwer(Langenfeld) 08/09, Beneke et.al.; Ahrens et.al. 09, Kidonakis 10)

NNLL resummation (Ahrens et.al. 10/11)

Status of NNLO$_{\text{approx}}$ results (B.Pecjak, Helmholtz Top-workshop Wuppertal 11)



Approximations for different observables

- total partonic cross section (Bonciani et.al. 98, Moch/Uwer/Langenfeld)

$$\hat{\sigma}(t\bar{t})(\hat{s}) \quad \Rightarrow \log^n \beta\,,\, \frac{1}{\beta^m}\,,\ \beta = \sqrt{1 - \frac{4m_t^2}{\hat{s}}}$$

- Pair invariant mass cross sections (Kidonakis,Sterman 97, Ahrens et.al. 10)

$$\frac{d\hat{\sigma}(t\bar{t})}{dM_{t\bar{t}}} \quad \Rightarrow \left[\frac{\log^n(1-z)}{1-z}\right]_+ \,,\ z = \frac{M_{t\bar{t}}^2}{\hat{s}}\,, \qquad \text{PIM}_{\text{SCET}} : \log\left(\frac{1-z}{\sqrt{z}}\right)$$

- One particle inclusive cross sections: (Laenen et.al. 98, Kidonakis 10, Ahrens et.al. 11)

$$\frac{d\hat{\sigma}(t+X)}{ds_4} \quad \Rightarrow \left[\frac{\log^n\left(s_4/m^2\right)}{s_4}\right]_+ \,,\ s_4 = p_X^2 - m_t^2\,, \qquad 1\text{PI}_{\text{SCET}} : \log\left(s_4/\sqrt{m^2+s_4}\right)$$

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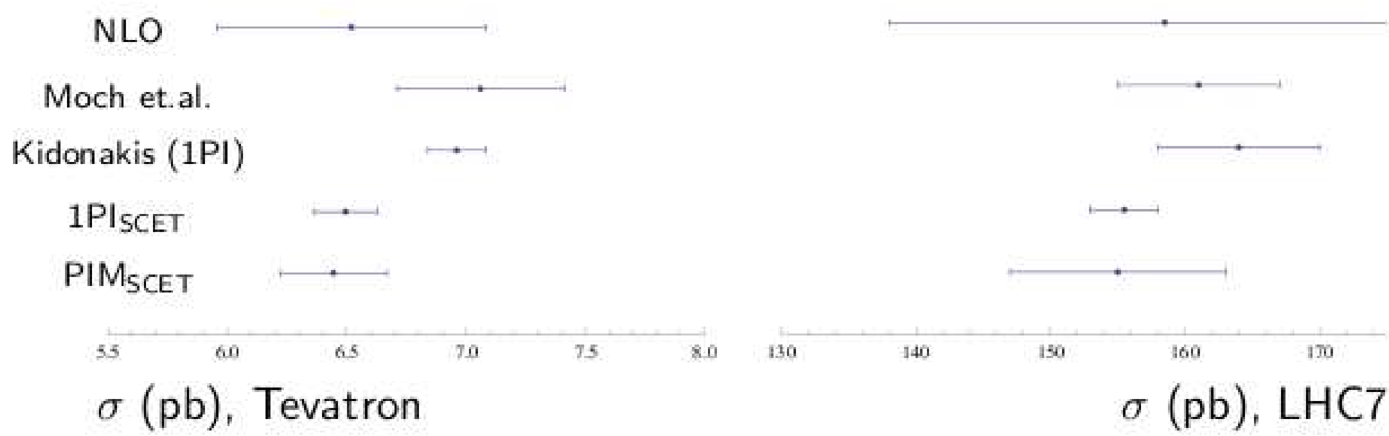
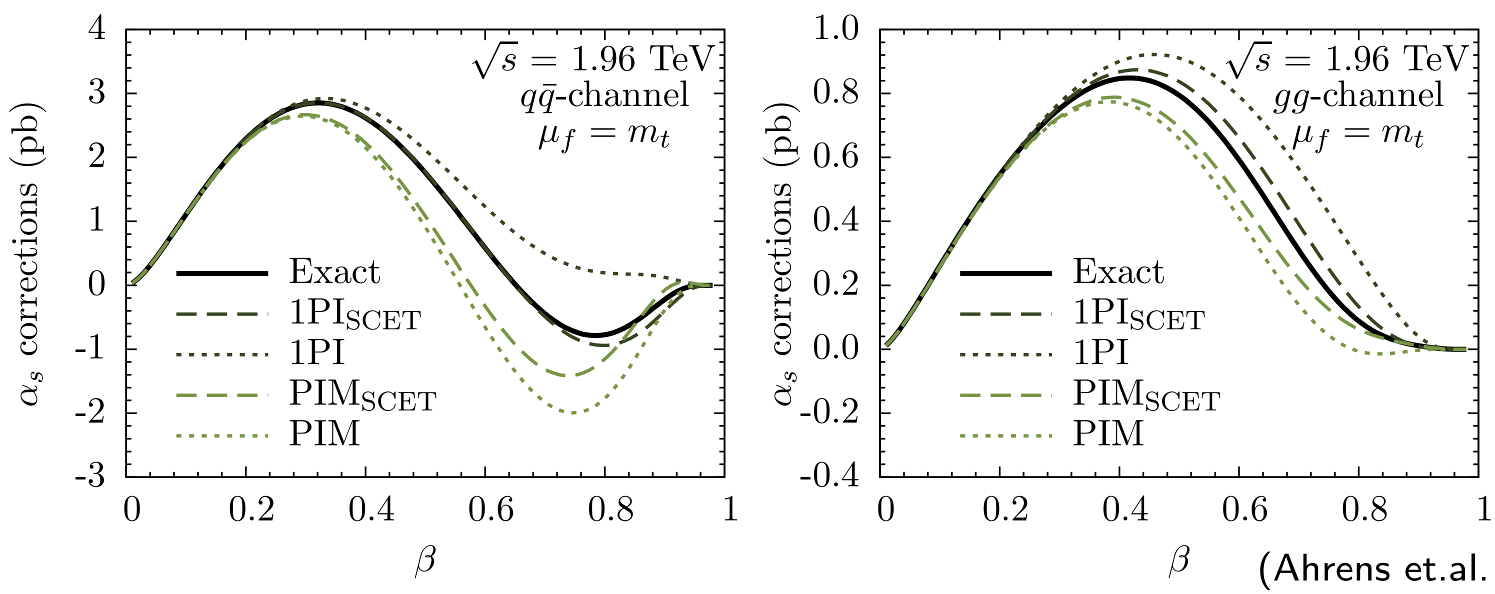
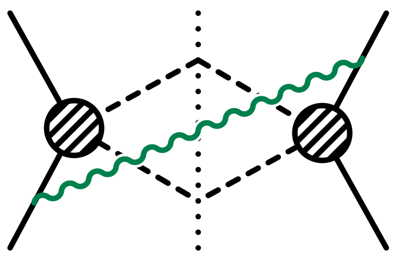


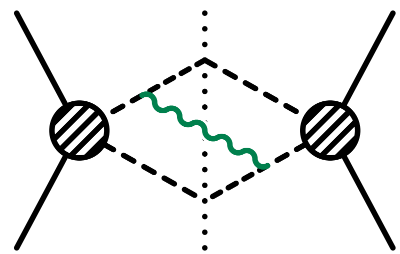


(Ahrens et.al. 11)

Soft corrections: (Resummation in Mellin space: Sterman 87; Catani, Trentadue 89, Kidonakis,Sterman 97, Bonciani et.al. 98, ...)

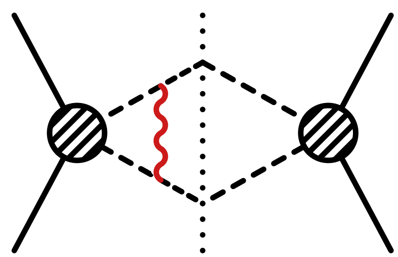


$\Rightarrow \alpha_s \; \log^2(8\beta^2)$



$\Rightarrow \alpha_s \; \log(8\beta^2)$

Coulomb gluon corrections (Fadin, Khoze 87; Peskin, Strassler 90, NRQCD,...)



$\Rightarrow \alpha_s \, \frac{1}{\beta}$

Counting of threshold corrections:

$$\hat\sigma_{pp'} \propto \sigma^{(0)} \exp\Big[\underbrace{\ln\beta\, g_0(\alpha_s \ln\beta)}_{\text{(LL)}} + \underbrace{g_1(\alpha_s \ln\beta)}_{\text{(NLL)}} + \underbrace{\alpha_s g_2(\alpha_s \ln\beta)}_{\text{(NNLL)}} + \dots\Big]$$

$$\times \sum_{k=0} \left(\frac{\alpha_s}{\beta}\right)^k \times \{1\,\text{(LL,NLL)}; \alpha_s, \beta\,\text{(NNLL)}; \dots\} :$$

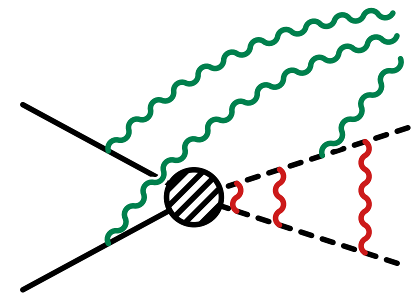
Combination of Coulomb- and soft effects?

Heavy particles nonrelativistic near threshold:

$$E \sim m\beta^2 \,, \qquad |\vec{p}| \sim m\beta$$

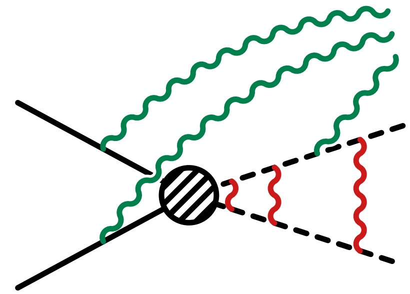
soft gluon momenta of same order: $q_s \sim m\beta^2 \sim E$

$\Rightarrow$ heavy particles "feel" soft radiation



Threshold resummation

Combination of Coulomb- and soft effects?



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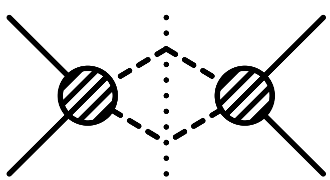
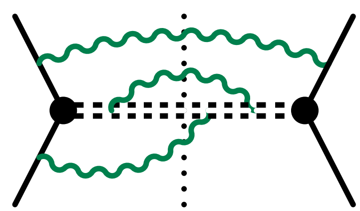
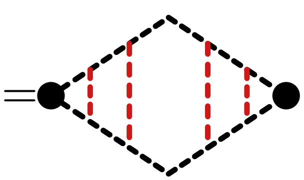
Factorization of cross section (Beneke, Falgari, CS 09/10)

$$\hat\sigma_{pp'\to HH'}|_{\hat s\to 4M^2} = \sum_{R,i} H_i \int d\omega\, W_i^R(\omega) J^R(E-\omega)$$

Hard, soft and Coulomb functions:

$H_i =$  , ${W_i}^R =$  , $J^R =$ 

Soft radiation "sees" only total colour charge R of heavy particles

(Singlet, octet,... Extends one-loop results by Sterman/Kidonakis 97, Bonciani et.al. 98, Kulesza/Moytka 08, Beenakker et.al. 09)

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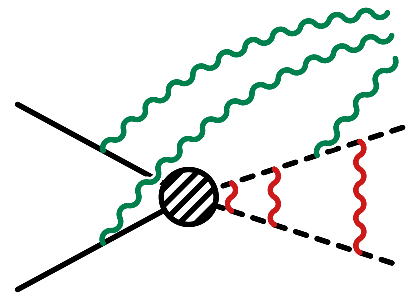

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- disentangles hard, soft and Coulomb contribution for total cross section for S-wave production and up to NNLL (more complicated colour structure for other observables: Ferroglia e.a., Ahrens e.a. 09)
- can perform simultaneous summation of threshold Logs and Coulomb corrections (also Hagiwara, Sumino, Yokoya; Kiyo et.al. 08)

Factorization scale dependence of H, W cancels against PDFs:

$$\frac{d\sigma}{d\mu} = \frac{d}{d\mu}\left(f_1 \otimes f_2 \otimes H \otimes W \otimes J\right) = 0$$

- $\frac{df_i}{d\mu}$ $\Rightarrow$ Altarelli-Parisi equation (3-loop: Moch/Vermaseren/Vogt 04/05)
- $\frac{dH_i}{d\mu}$ $\Rightarrow$ related to IR singlarities (2-loop: Becher, Neubert; Ferroglia et.al. 09)

$\Rightarrow$ RGE for soft function (NNLL: Beneke/Falgari/CS; Czakon/Mitov/Sterman 09)

$$\frac{d}{d\log\mu} W_i^{R_\alpha}(z^0,\mu) = \left(2\gamma_{\mathrm{cusp}}(C_r+C_{r'})\log\left(\frac{iz_0\bar{\mu}}{2}\right) - 2(\gamma_{H.s}^{R_\alpha} + \underbrace{\gamma_s^r + \gamma_s^{r'}}_{\text{as for Drell-Yan/Higgs}})\right) W_i^{R_\alpha}(z^0,\mu)$$

Solution in Mellin space (Korchemsky/Marchesini 92);

momentum space (Becher/Neubert 06)

Soft anomalous dimension (Beneke, Falgari, CS 09; Czakon, Mitov, Sterman 09)

$$\gamma_{H,s}^{R_\alpha} = \frac{\alpha_s}{4\pi}(-2C_{R_\alpha}) + \left(\frac{\alpha_s}{4\pi}\right)^2 C_{R_\alpha}\left[-C_A\left(\frac{98}{9} - \frac{2\pi^2}{3} + 4\zeta_3\right) + \frac{40}{18}n_f\right] + \mathcal{O}(\alpha_s^3).$$

(extracted from Becher/Neubert 09, Korchemsky/Radyushkin 92, Kidonakis 09)

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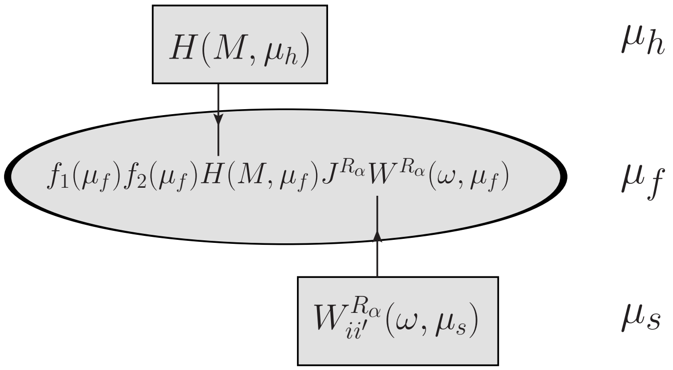
Resummation:

- evolve hard function from $\mu_h \sim 2m_t$ to μ_f
- evolve soft function from μ_s to μ_f
- (N)LO Coulomb-Green function (as in $e^-e^+ \to t\bar{t}$: Fadin/Khoze 87; Beneke/Signer/Smirnov 99,. . .)

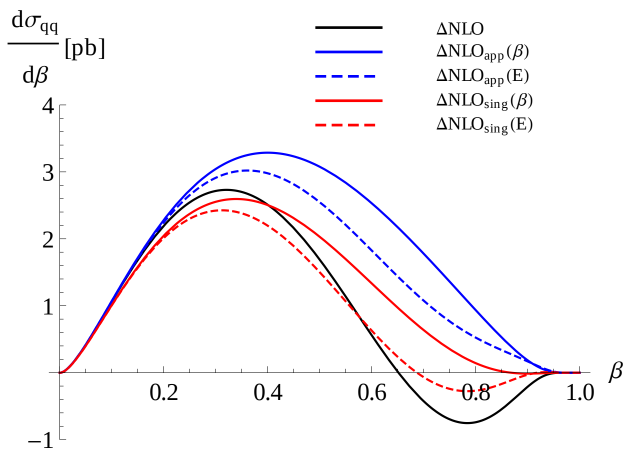


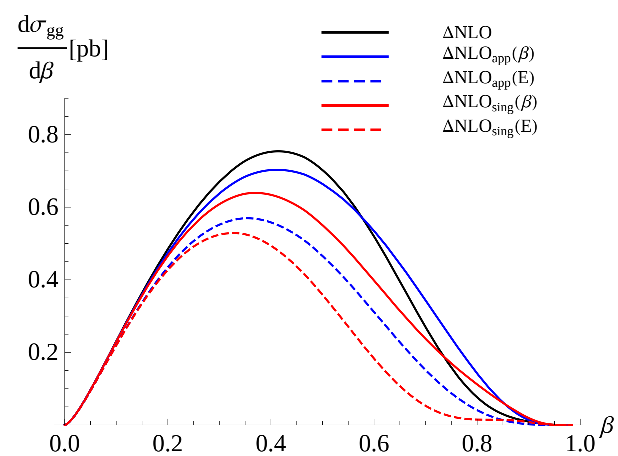
Accuracy of threshold approximation

NLO$_{\text{sing}}$: only $\log\beta$, $1/\beta$-terms

NLO$_{\text{app}}$: also constant terms (hard/soft function)

Ambiguity: $E=\sqrt{\hat{s}}-2m_t\approx m_t\beta^2$





$\frac{d\sigma}{d\beta}=\frac{8\beta m_{\tilde{q}}^2}{s(1-\beta^2)^2}L(\beta,\mu_f)\hat{\sigma}$ (Tevatron, L: parton luminosity (MSTW08))

$\Rightarrow$ large ambiguities, but exact result covered: ($\mu_f=\mu_r=m_t=173.1$ GeV)

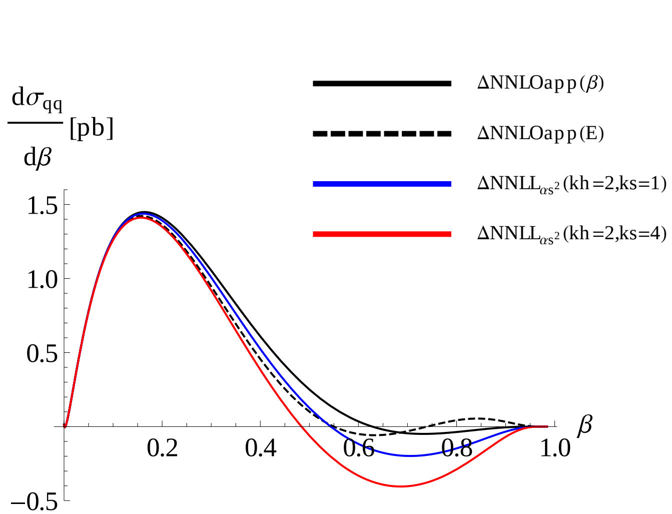
$\sigma_{\text{NLO,app}}(\text{Tev})=6.42-7.45$pb $\sigma_{\text{NLO}}(\text{Tev})=6.50$pb

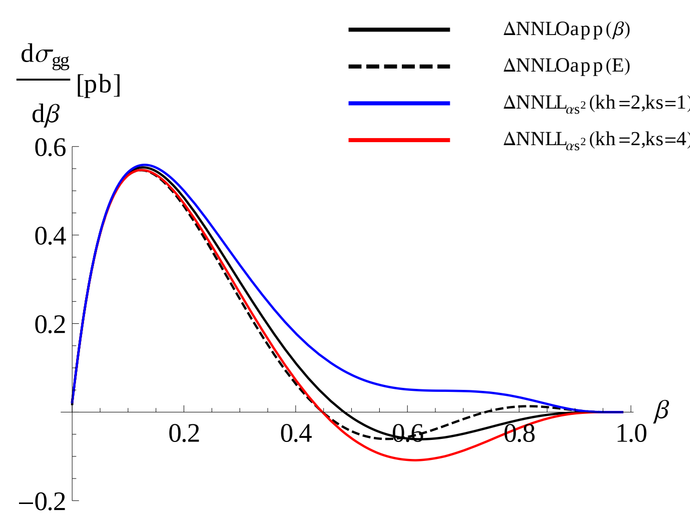
$\sigma_{\text{NLO,app}}(\text{LHC7})=130-158$ pb $\sigma_{\text{NLO}}(\text{LHC7})=150$ pb

NNLO$_{\text{approx}}$**:** all singular terms at $\mathcal{O}(\alpha_s^2)$ (constant not known yet)

NNLL(α_s^2)**:** all $\mathcal{O}(\alpha_s^2)$ terms in NNLL for $\mu_s = k_s m_t \beta^2$,

includes constant soft $\times$ hard, $\log \frac{\mu_h}{\mu_f} = \log k_h$, $\log k_s$





Estimate of $\mathcal{O}(\alpha_s^2)$ constant in threshold expansion

$$C^{(2)} \approx (C^{(1)})^2$$

$\Rightarrow$ similar effect on cross section as k_s, k_h variation

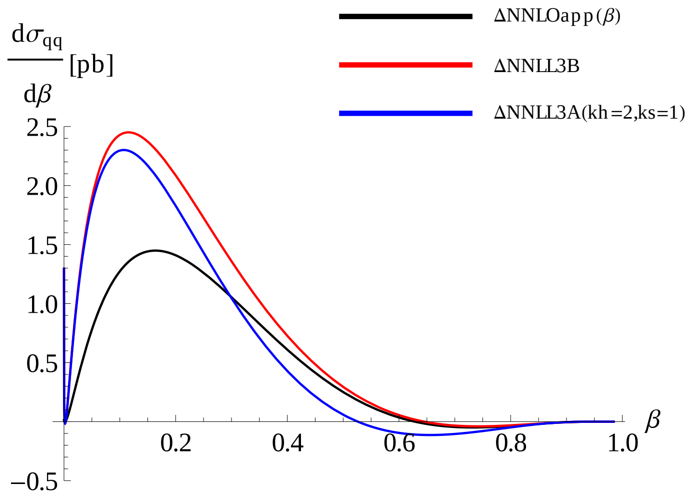
Estimates of residual uncertainties

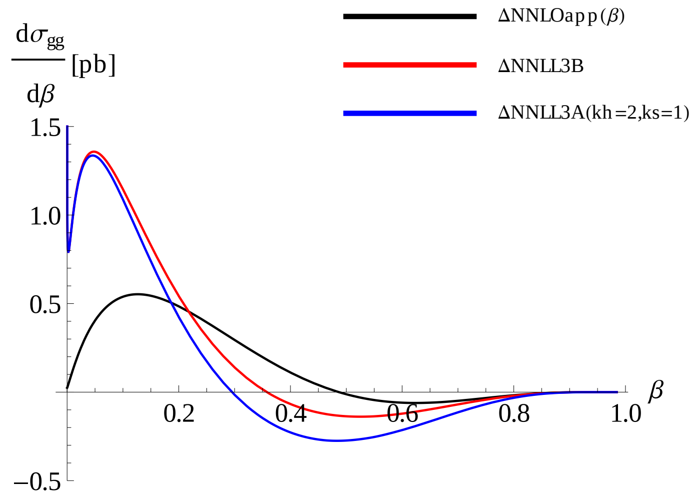
Expand NNLL to $\mathcal{O}(\alpha_s^3)$, e.g.

$$\Delta\sigma^{(3)}_{q\bar{q},\text{NNLL}} = \sigma^{(0)}\left(\frac{\alpha_s}{4\pi}\right)^3 \Big[12945.4\log^6\beta - 37369.1\log^5\beta + 27721.4\log^4\beta + 41839.4\log^3\beta$$
$$+\frac{1}{\beta}\left(-6278.5\log\beta + 3862.5\log^2\beta + 2804.7\log^3\beta - 2994.5\log^4\beta\right)$$
$$+\frac{153.9\log^2\beta + 122.9\log\beta - 145}{\beta^2} + \underbrace{\left\{\log\beta^{1,2}, 1/\beta, C^{(3)}\right\}}_{\text{not known exactly}} + \text{scale dep.}\Big]$$

NNLL$_3$A: keep all terms, including k-dependence and constants

NNLL$_3$B: only keep terms known exactly





Soft scale choice in momentum-space resummation

Structure of resummation formula (NLL, no Coulomb summation)

$$\hat{\sigma}^{\text{NLL}}(\hat{s}) = \sum_{i,R_\alpha} \hat{\sigma}^{i,(0)}(\hat{s})\, U_i^{R_\alpha}(\mu_s,\mu_f,\mu_h,m_t) \left[1-\frac{4m_t^2}{\hat{s}}\right]^{2\eta} \frac{\sqrt{\pi}e^{-2\eta\gamma_E}}{2\Gamma(2\eta+\frac{3}{2})}$$

$$U_i^{R_\alpha} = e^{-\frac{\alpha_s\Gamma_{\text{cusp}}^{(0)}}{2\pi}\log^2(\frac{\mu_s}{\mu_f})+\dots},$$

$$\eta = -2\int_{\alpha_s(\mu_f)}^{\alpha_s(\mu_s)} d\alpha_s \frac{\Gamma_{\text{cusp}}(\alpha_s)}{\beta(\alpha_s)} = \frac{\alpha_s\Gamma_{\text{cusp}}^{(0)}}{2\pi}\log(\tfrac{\mu_s}{\mu_f})+\dots$$

Expansion in α_s generates all logs in $\hat{\sigma}$ for $\mu_s \sim m_t\beta^2$

Resummation: Cannot convolute $\exp\left[+c\log(1-\frac{4m_t^2}{\hat{s}})\right]$ with PDFs

- multiply with PDFs in Mellin space (Catani et.al. 96)
- Introduce cutoffs (Berger/Contopagnaos 96; Bonvini/Forte/Ridolfi 10)

EFT approach: fixed μ_s that minimizes soft corrections to hadronic σ (Becher, Neubert, Xu 07)

Application to $t\bar{t}$:

ambiguities in soft scale from minimising $\sigma^{\text{NLO}}_{\text{soft}}(\mu)$ (e.g. LHC7):

$$\mu_s = 70 \text{ GeV } \left(\log m_t\beta^2/\mu\right) \; \mu_s = 147 \text{ GeV } \left(\log E/\mu = \log(\sqrt{\hat{s}} - 2m_t)/\mu\right)$$

Alternative approach: Introduce β_{cut}

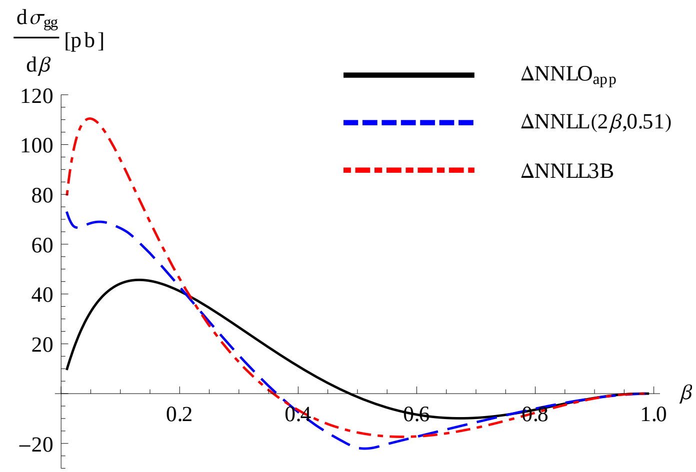
- allow for different implementations

  $\beta < \beta_{\textbf{cut}}$: NNLL $(\mu_s = k_s m_t \beta^2_{\text{cut}})$ with/without constant at $\mathcal{O}(\alpha_s^2)$

  $\beta > \beta_{\textbf{cut}}$: NNLL $(\mu_s = k_s m_t \beta^2)$; NNLO$_{\text{approx}}$; NNNL$_3$(A/B)

- Choose β_{cut} so that not too sensitive to
  - ambiguities for $\beta \to 1$
  - breakedown of perturbation theory for $\beta \to 0$

  (E.g. LHC7: $\mu_s = 2m_t\beta^2$, $\beta_{\text{cut}} = 0.51$)



Results for $t\bar{t}$ production

(Beneke, Falgari, Klein, CS preliminary)

$\sigma_{t\bar{t}}$(pb)	Tevatron	LHC7
NLO	$6.50^{+0.32+0.33}_{-0.70-0.24}$	150^{+18+8}_{-19-8}
NNLO$_{\rm app}(\beta)$	$7.10^{+0.0+0.36}_{-0.26,-0.26}$	162^{+2+9}_{-3-9}
NNLL	$7.24^{+0.48+0.37}_{-0.57-0.27}$	161^{+9+9}_{-9-9}

($m_t = 173.1$ GeV, $\mu_f = mt$, MSTW08NNLO)

Error breakedown for NNLL (added in quadrature above):

- Scale variation (μ_f, μ_h, μ_C):

$$\Delta_\mu \sigma_{\rm NNLL}(\text{TeV}) = \begin{matrix} +0.21 \\ -0.36 \end{matrix}\,, \quad \Delta_\mu \sigma_{\rm NNLL}(\text{LHC7}) = \begin{matrix} +1 \\ -1 \end{matrix}$$

- Uncertainty in resummation procedure:
  (vary β_{cut} by 20%, envelope of various approximations, ambiguity $E \leftrightarrow m_t\beta^2$)

$$\Delta_{\rm Res} \sigma_{\rm NNLL}(\text{TeV}) = \begin{matrix} +0.20 \\ -0.21 \end{matrix}\,, \quad \Delta_{\rm Res} \sigma_{\rm NNLL}(\text{LHC7}) = \begin{matrix} +4 \\ -5 \end{matrix}$$

- Estimate of missing constant at $\mathcal{O}(\alpha_s^2)$

$$\Delta_{\rm Const} \sigma_{\rm NNLL}(\text{TeV}) = \pm 0.38\,, \quad \Delta_{\rm Const} \sigma_{\rm NNLL}(\text{LHC7}) = \pm 7$$

Summary

Threshold corrections $\sim \log^n \beta$, $\frac{1}{\beta^n}$

- Factorization of soft and Coulomb corrections
- $\log \beta$ resummation from momentum space solution to RGEs
- combined Soft and Coulomb resummation possible
- theoretical progress: now NNLL resummation feasible

Threshold expansion to $\mathcal{O}(\alpha_s^2)$ of $t\bar{t}$ cross section

NNLL resummation for $t\bar{t}$

- Estimate of residual uncertainty

  approx. N3LO terms, kinematic ambiguities, uncertainties in resummation procedure
- discrepancy to NNLL from integrated $\frac{d\sigma}{dM_{tt}^2}$ (Ahrens et.al. 10) remains for central values, but results now marginally consistent

Hadron collider cross sections from QCD factorization

(Collins, Soper, Sterman)

$$\sigma_{NN'}(s) = \sum_{pp'} \int dx_1 dx_2 \; f_{N/p}(x_1, \mu_f) f_{N'/p'}(x_2, \mu_f) \, \hat{\sigma}_{pp'}(s x_1 x_2, \mu_f)$$

- $\hat{\sigma}_{pp'}$: partonic cross section: compute in perturbation theory
- $f_{p/N}(x)$: Parton distribution function for parton p in hadron N : fitted to experiment

PDF uncertainties for top: (e.g. Guffanti/Rojo arXiv:1008.4671 [hep-ph])

	CTEQ6.6	MSTW2008	NNPDF2.0	ABKM09	HERAPDF1.0
$\sigma_{t\bar{t}}^{\text{NLO}}(7\text{TeV})[pb]$	147.7 ± 6.4	159.0 ± 4.7	160.0 ± 5.9	131.9 ± 4.8	136.4 ± 4.7

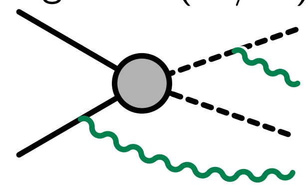
- Different α_s values
- Differences in gluon pdf at large x (impact of Tevatron jet-data)

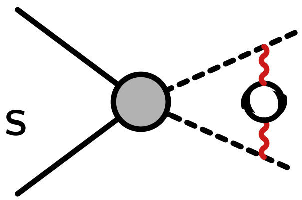
All threshold enhanced $\mathcal{O}(\alpha_s^2)$ terms (Beneke, Czakon, Falgari, Mitov, CS 09

Implemented in HATHOR, Aliev et.al. 10)

Pure soft corrections: (also Moch/Uwer+Langenfeld (08/09))

$$\Delta\sigma_{\mathsf{s}}^{(2)} \sim \alpha_s^2 (c_{\mathsf{LL}}^{(2)} \ln^4\beta + c_{\mathsf{NLL}}^{(2)} \ln^3\beta + c_{\mathsf{NNLL},2}^{(2)} \ln^2\beta + \underbrace{c_{\mathsf{NNLL},1}^{(2)} \ln\beta}_{\text{2-loop } \gamma_{H,s}})$$





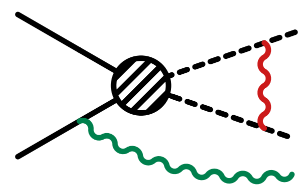
Potential corrections: 2nd Coulomb, NLO potentials

$$\Delta\sigma_{\mathsf{p}}^{(2)} \sim \alpha_s^2 \left(\frac{c_{C2}}{\beta^2} + \frac{1}{\beta}(c_{\mathsf{C},0}^{(2)} + c_{\mathsf{C},1}^{(2)} \log\beta) + \underbrace{c_{\mathsf{n\text{-}C}}^{(2)} \ln\beta}_{\text{spin-dependent}} \right)$$

(using Beneke, Signer, Smirnov 99, Czarnecki/Melnikov 97/01)

mixed Coulomb/soft, hard corrections:

$$\Delta\sigma_{\mathsf{p}\otimes\,\mathsf{sh}}^{(2)} \sim\sim \frac{\alpha_s}{\beta}\alpha_s(c_{\mathsf{LL}}^{(1)} \ln\beta^2 + c_{\mathsf{NLL}}^{(1)} \ln\beta + c + \underbrace{H^{(1)}}_{\text{process dependent}})$$



Potential corrections:

- 2nd Coulomb correction
- NLO Coulomb potentials:

$$\tilde{V}_{\mathrm{C}}^{(1)}(\boldsymbol{p},\boldsymbol{q}) = \frac{D_{R_\alpha}\alpha_s^2}{\boldsymbol{q}^2}\left(a_1 - \beta_0 \ln\frac{\boldsymbol{q}^2}{\mu^2}\right)$$

- Non-Coulomb potential:

$$\tilde{V}_{\mathrm{nC}}^{(1)}(\boldsymbol{p},\boldsymbol{q}) = \frac{4\pi D_{R_\alpha}\alpha_s}{\boldsymbol{q}^2}\left[\frac{\pi\alpha_s|\boldsymbol{q}|}{4m}\left(\frac{D_{R_\alpha}}{2}+C_A\right)+\frac{\boldsymbol{p}^2}{m^2}+\frac{\boldsymbol{q}^2}{m^2}\,v_{\mathrm{spin}}\right],$$

($v_{\mathrm{spin}} = 0$ (singlet); $-2/3$ (triplet))

Corrections to cross section:

$$\Delta\hat{\sigma}_{\mathrm{nC}} = \hat{\sigma}^{(0)}\,\alpha_s^2\,\ln\beta\,\left[-2D_{R_\alpha}^2\,(1+v_{\mathrm{spin}})+D_{R_\alpha}C_A\right]$$

(extracted from Beneke, Signer, Smirnov 99, Pineda, Signer 06)

Alternative threshold expansions

Pair invariant mass cross sections (Kidonakis,Sterman 97, Ahrens et.al. 10)

$$\frac{d\sigma(t\bar{t})}{dM_{t\bar{t}}} \Rightarrow \left[\frac{\log^n(1-z)}{1-z}\right]_+ , \; z = \frac{M_{t\bar{t}}^2}{\hat{s}}$$

One particle inclusive cross sections: (Laenen, Oderda, Sterman 98)

$$\frac{d\sigma(t+X)}{ds_4} \Rightarrow \left[\frac{\log^n(s_4/m^2)}{s_4}\right] , \; s_4 = p_X^2 - m_t^2$$

$\sigma_{t\bar{t}}$(pb)	Tevatron	LHC7	LHC10	LHC14
NLO	$6.50^{+0.32+0.33}_{-0.70-0.24}$	150^{+18+8}_{-19-8}	380^{+44+17}_{-46-17}	842^{+97+30}_{-97-32}
NNLO$_{\rm app}(\beta)$	$7.10^{+0.0+0.36}_{-0.26,-0.26}$	162^{+2+9}_{-3-9}	407^{+9+17}_{-5-18}	895^{+24+31}_{-6-33}
NLO + NNLL ($M_{t\bar{t}}$) (Ahrens et.al. 10)	$6.48^{+0.17+0.32}_{-0.21-0.25}$	146^{+7+8}_{-7-8}	368^{+20+19}_{-14-15}	813^{+50+30}_{-36-35}
NNLO$_{\rm app}(s_4)$ (mt=173; Kidonakis 10)	$7.08^{+0.00+0.36}_{-0.24-0.27}$	163^{+7+9}_{-5-9}	415^{+17+18}_{-21-19}	920^{+50+33}_{-39-35}

($m_t = 173.1$ GeV, $\mu_f = mt$, MSTW08NNLO)