

# $W^\pm/Z/h$ +jets with POWHEG in SHERPA



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LoopFest X

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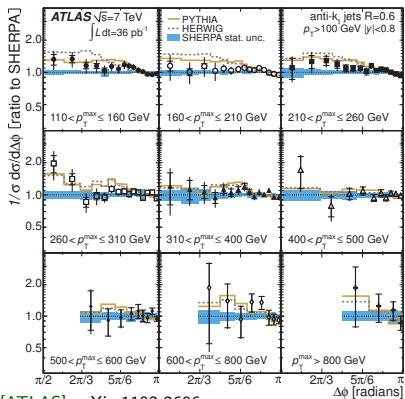
<sup>1</sup> In collaboration with: F. Krauss, M. Schönherr, F. Siegert JHEP04(2011)024

# Multi-jet event simulation: Where do we stand?

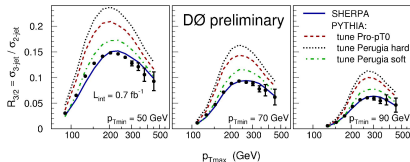
ME $\otimes$ PS as today's standard approach

- automatically include arbitrary higher-order tree-level ME
- naturally extend PS phase space
- miss out on virtual corrections

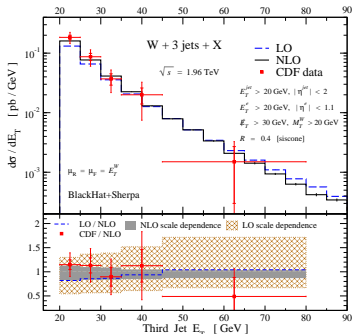
Want full NLO as *new standard*!



[ATLAS] arXiv:1102.2696



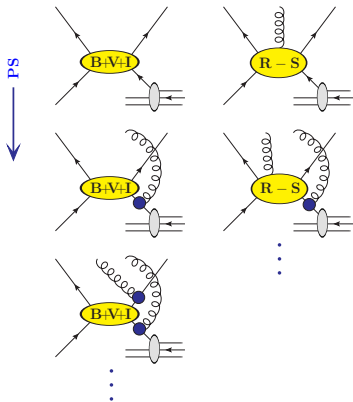
[DØ] DØ Note 6032-CONF



[Berger et al.] PRD80(2009)074036

# Combining NLO calculations and parton showers

$$\sigma^{NLO} = \int d\Phi_B (B + \tilde{V}) + \int d\Phi_R R = \int d\Phi_B \left[ (B + \tilde{V} + I) + \int d\Phi_{R|B} (R - S) \right]$$



Requirements for NLO $\otimes$ PS:

- Preserve resummation as in PS
- Implement  $\mathcal{O}(\alpha_s)$  accuracy from ME

Problems:

- Two kinematically different configurations B-/R-like
- Real-emission term and PS populate same phase-space region
- Naively adding PS on top of ME leads to double-counting

General solutions by MC@NLO [Frixione,Webber] JHEP06(2002)029

and POWHEG (positive weights only) [Frixione,Nason,Oleari] JHEP11(2004)040

# NLO calculations and parton-shower Monte Carlo

→ **Real-emission contribution to NLO cross section**  $\{\vec{a}\} \rightarrow$  set of partons

$$d\sigma_R(\{\vec{p}\}) = \sum_{\{\vec{f}\}} d\sigma_R(\{\vec{a}\}) \quad d\sigma_R(\{\vec{a}\}) = d\Phi_R(\{\vec{p}\}) R(\{\vec{a}\})$$

where  $R(\{\vec{a}\}) = \mathcal{L}(\{\vec{a}\}) \mathcal{R}(\{\vec{a}\})$  and  $\mathcal{L}(\{\vec{a}\}; \mu^2) = x_1 f_1(x_1, \mu^2) x_2 f_2(x_2, \mu^2)$

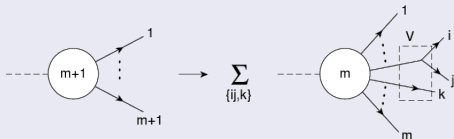
$d\Phi_R$  contains initial-state phase space  $d \log x_1 d \log x_2$

$\mathcal{R}(\{\vec{a}\}) = |\mathcal{M}_R|^2(\{\vec{a}\}) / [F(\{\vec{a}\}) S(\{\vec{f}\})]$  with symmetry factor  $S$ , flux  $F$

Similar formulas for Born-level term  $B(\{\vec{a}\})$  one parton less, of course

**Assume generalized “dipole terms”, such that** think of Catani-Seymour dipoles

$$\mathcal{R}(\{\vec{a}\}) \xrightarrow{\text{soft/collinear}} \sum_{\{i,j\}} \sum_{k \neq i,j} \mathcal{D}_{ij,k}(\{\vec{a}\})$$



**Define partition of real-emission term**  $\mathcal{R}(\{\vec{a}\}) = \sum_{\{i,j\}} \sum_{k \neq i,j} \mathcal{R}_{ij,k}(\{\vec{a}\})$

$$\mathcal{R}_{ij,k}(\{\vec{a}\}) := \rho_{ij,k}(\{\vec{a}\}) \mathcal{R}(\{\vec{a}\}), \quad \text{where} \quad \rho_{ij,k}(\{\vec{a}\}) = \frac{\mathcal{D}_{ij,k}(\{\vec{a}\})}{\sum_{\{m,n\}} \sum_{l \neq m,n} \mathcal{D}_{mn,l}(\{\vec{a}\})}$$

Note: **Holds throughout the phase space !**

# NLO calculations and parton-shower Monte Carlo

$\mathcal{D}_{ij,k}(\{\vec{a}\})$  defines parton maps think of Catani-Seymour dipoles

$$b_{ij,k}(\{\vec{a}\}) = \begin{cases} \{\vec{f}\} \setminus \{f_i, f_j\} \cup \{f_{ij}\} \\ \{\vec{p}\} \rightarrow \{\vec{p}\} \end{cases} \leftrightarrow r_{ij,\tilde{k}}(f_i, \Phi_{R|B}; \{\vec{a}\}) = \begin{cases} \{\vec{f}\} \setminus \{f_{ij}\} \cup \{f_i, f_j\} \\ \{\vec{p}\} \rightarrow \{\vec{p}\} \end{cases}$$

- $b_{ij,k}$  converts real-emission configuration to Born-level
- $r_{ij,\tilde{k}}$  converts Born-level to real-emission needs extra flavor & phase space

Trivially factorize real-emission term into **Born** and **radiative** contribution

$$d\sigma_R(\{\vec{a}\}) = \sum_{\{i,j\}} \sum_{k \neq i,j} d\sigma_B(b_{ij,k}(\{\vec{a}\})) dP_{ij,k}(\{\vec{a}\})$$

differential emission probability is  $dP_{ij,k}(\{\vec{a}\}) = d\Phi_{R|B}^{ij,k}(\{\vec{p}\}) \frac{R_{ij,k}(\{\vec{a}\})}{B(b_{ij,k}(\{\vec{a}\}))}$

Subtraction algorithms predict  $dP_{ij,k}$  in the soft/collinear limits via

$$\mathcal{D}_{ij,k}(\{\vec{a}\}) \xrightarrow{\text{soft/collinear}} \frac{S(b_{ij,k}(\{\vec{f}\}))}{S(\{\vec{f}\})} \frac{1}{2 p_i p_j} 8\pi \alpha_s \mathcal{B}(b_{ij,k}(\{\vec{a}\})) \otimes V_{ij,k}(p_i, p_j, p_k),$$

Note the symmetry factors  $\leftrightarrow$  factorization of invariant ME, not of specific process

$\otimes \rightarrow$  spin & color-correlations between  $\mathcal{B}$  and  $V$

# NLO calculations and parton-shower Monte Carlo

**Now make an approximation** replace correlated with uncorrelated dipole kernel

$$\mathcal{B}(b_{ij,k}(\{\vec{a}\})) \otimes V_{ij,k}(p_i, p_j, p_k) \rightarrow \mathcal{B}(b_{ij,k}(\{\vec{a}\})) \mathcal{K}_{ij,k}(p_i, p_j, p_k)$$

Parametrize radiative phase space:  $d\Phi_{R|B}^{ij,k}(\{\vec{p}\}) = \frac{1}{16\pi^2} dt dz \frac{d\phi}{2\pi} J_{ij,k}(t, z, \phi)$

Assume phase space gets filled successively in  $t \leftrightarrow$  partons can be distinguished

Must adapt symmetry factors:  $\frac{S(b_{ij,k}(\{\vec{f}\}))}{S(\{\vec{f}\})} \rightarrow \frac{1}{S_{ij}} = \begin{cases} 1/2 & \text{if } i, j > 2, b_i = b_j \\ 1 & \text{else} \end{cases}$

**Combining everything gives PS expression for radiation probability**

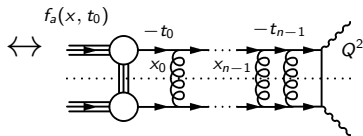
$$dP_{ij,k}^{(\text{PS})}(\{\vec{a}\}) = \frac{dt}{t} dz \frac{d\phi}{2\pi} \frac{\alpha_s}{2\pi} \frac{1}{S_{ij}} J_{ij,k}(t, z, \phi) \mathcal{K}_{ij,k}(t, z, \phi) \frac{\mathcal{L}(\{\vec{a}\}; t)}{\mathcal{L}(b_{ij,k}(\{\vec{a}\}); t)}$$

Iterate this equation for higher-multi ME

→ ladder-like structure of amplitude squared  
with strong ordering in scales  $t_0 < \dots < t_n$

**Factorization at any stage above  $\Lambda_{QCD}$**

can split emissions off ME one by one



**Corrections induced by  $dP_{ij,k}^{(\text{PS})}$  can be large and must be resummed**

In inclusive case  $t \in [0, \infty)$  divergences in  $\mathcal{K}_{ij,k}$  cancel  $\varepsilon$ -poles in  $V \rightarrow$  unitarity!

# NLO calculations and parton-shower Monte Carlo

→ **No-emission probability** from Poisson statistics implementing unitarity constraint

$$\mathcal{P}_{\tilde{ij},\tilde{k}}^{(\text{PS})}(t', t''; \{\vec{a}\}) = \exp \left\{ - \sum_{f_i=q,g} \int_{t'}^{t''} \int_{z_{\min}}^{z_{\max}} \int_0^{2\pi} dP_{ij,k}^{(\text{PS})}(r_{\tilde{ij},\tilde{k}}(\{\vec{a}\})) \right\}.$$

Note:  $r_{\tilde{ij},\tilde{k}}$  implicitly and uniquely defined by subtraction scheme, i.e.  $\mathcal{K}_{ij,k}$

Assume IF-splitting → Lumi ratio  $\frac{x}{z} f_{f_i}(\frac{x}{z}, t) / x f_{f_i}(x, t)$ , symmetry factor 1

$$\frac{\partial \log \mathcal{P}_{\tilde{ij},\tilde{k}}^{(\text{PS})}(t, t'; \{\vec{a}\})}{\partial \log(t/\mu^2)} = \int_x^{z_{\max}} \frac{dz}{z} \int_0^{2\pi} \frac{d\phi}{2\pi} \sum_{f_i=q,g} \frac{\alpha_s}{2\pi} J_{ij,k}(t, z, \phi) \mathcal{K}_{ij,k}(t, z, \phi) \frac{f_{f_i}(\frac{x}{z}, t)}{f_{f_i}(x, t)}$$

**Voilà, the DGLAP equation !** imagine  $J_{ij,k}(t, z, \phi) \mathcal{K}_{ij,k}(t, z, \phi) \rightarrow P_{i;\tilde{ij}}(z)$

$$\frac{d}{d \log(t/\mu^2)} \begin{array}{c} f_q(x,t) \\ \rightarrow \text{circle} \\ \text{outgoing } q \end{array} = \int_x^1 \frac{dz}{z} \frac{\alpha_s}{2\pi} \begin{array}{c} P_{qq}(z) \\ \rightarrow \text{circle} \\ \text{outgoing } q \end{array} + \int_x^1 \frac{dz}{z} \frac{\alpha_s}{2\pi} \begin{array}{c} P_{gq}(z) \\ \rightarrow \text{circle} \\ \text{outgoing } q \end{array}$$

$$\frac{d}{d \log(t/\mu^2)} \begin{array}{c} f_g(x,t) \\ \rightarrow \text{circle} \\ \text{outgoing } g \end{array} = \sum_{i=1}^{2n_f} \int_x^1 \frac{dz}{z} \frac{\alpha_s}{2\pi} \begin{array}{c} P_{qg}(z) \\ \rightarrow \text{circle} \\ \text{outgoing } g \end{array} + \int_x^1 \frac{dz}{z} \frac{\alpha_s}{2\pi} \begin{array}{c} P_{gg}(z) \\ \rightarrow \text{circle} \\ \text{outgoing } g \end{array}$$

## Recover NLO-accurate radiation pattern in PS through correction weight

$$w_{ij,k}(\{\vec{a}\}) = dP_{ij,k}(\{\vec{a}\}) / dP_{ij,k}^{(\text{PS})}(\{\vec{a}\})$$

Easy to compute in general-purpose Monte-Carlo, all input is tree-level only

## Approximate “seed cross section” using local $K$ -factor $\bar{B}/B$

$$\frac{\bar{B}(\{\vec{a}\})}{B(\{\vec{a}\})} = 1 + \frac{\tilde{V}(\{\vec{a}\}) + I(\{\vec{a}\})}{B(\{\vec{a}\})} + \sum_{\{\tilde{ij}, \tilde{k}\}} \sum_{f_i=q,g} \int d\Phi_{R|B}^{ij,k} \frac{R_{ij,k}(r_{\tilde{ij}, \tilde{k}}(\{\vec{a}\})) - S_{ij,k}(r_{\tilde{ij}, \tilde{k}}(\{\vec{a}\}))}{B(\{\vec{a}\})}$$

Note: Implies wrong dependence of observables on final-state momenta  $\{\vec{p}\} \rightarrow$  resolved by PS

## Combine ME-correction and local $K$ -factor $\rightarrow$ observable $O$ to $\mathcal{O}(\alpha_s)$ from

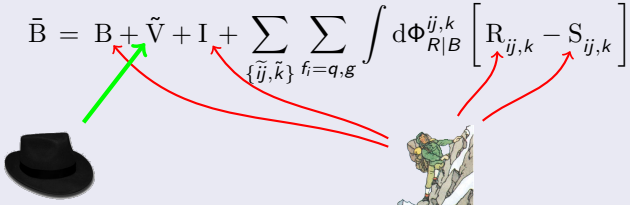
$$\begin{aligned} \langle O \rangle^{(\text{POWHEG})} &= \sum_{\{\vec{f}\}} \int d\Phi_B(\{\vec{p}\}) \bar{B}(\{\vec{a}\}) \left[ \mathcal{P}^{(\text{ME})}(t_0, \mu^2; \{\vec{a}\}) O(\{\vec{p}\}) \right. \\ &+ \sum_{\{\tilde{ij}, \tilde{k}\}} \sum_{f_i=q,g} \frac{1}{16\pi^2} \int_{t_0}^{\mu^2} dt \int_{z_{\min}}^{z_{\max}} dz \int_0^{2\pi} \frac{d\phi}{2\pi} J_{ij,k}(t, z, \phi) \\ &\left. \times \frac{1}{S_{ij}} \frac{S(r_{\tilde{ij}, \tilde{k}}(\{\vec{f}\}))}{S(\{\vec{f}\})} \frac{R_{ij,k}(r_{\tilde{ij}, \tilde{k}}(\{\vec{a}\}))}{B(\{\vec{a}\})} \mathcal{P}^{(\text{ME})}(t, \mu^2; \{\vec{a}\}) O(r_{\tilde{ij}, \tilde{k}}(\{\vec{p}\})) \right] \end{aligned}$$

**POWHEG master formula** [Nason] JHEP11(2004)040 [Frixione, Nason, Oleari] JHEP11(2007)070



# Assembling the local $K$ -factor

Virtual corrections not automated in SHERPA  $\Rightarrow$  **share the workload**

$$\bar{B} = B + \tilde{V} + I + \sum_{\{\tilde{j}, \tilde{k}\}} \sum_{f_i=q,g} \int d\Phi_{R|B}^{ij,k} \left[ R_{ij,k} - S_{ij,k} \right]$$


Standardized interface exists as Binoth Les Houches accord CPC181(2010)1612

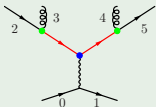
- “One-Loop Engines” like BlackHat PRD78(2008)036003, PRL102(2009)222001 or GOLEM CPC180(2009)2317, PLB683(2010)154 provide virtual piece or more
- ME generator takes care of Born, real emission and subtraction
- Phase-space generator employs modified tree-level integrators  
Specialized two-step procedure for underlying Born plus real emission

# Assembling the local $K$ -factor

Integration of  $\bar{B}(\Phi_B)$  proceeds in two steps ...

**Step I: Born phase space** via recycling

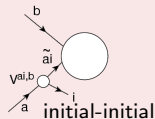
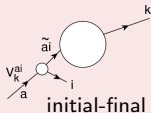
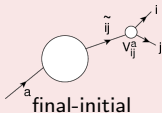
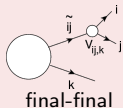
Standard phase-space generator, e.g. single channels from NPB9(1969)568  
VEGAS-refined JCP27(1978)192 and combined in multi-channel CPC83(1994)141



$$D_{iso}(23, 45) \otimes P_0(23) \otimes P_0(45) \\ \otimes D_{iso}(2, 3) \otimes D_{iso}(4, 5)$$

**Step II: Constrained real-emission phase space** new

Extra emission generator (EEG) produces additional parton starting from  $\Phi_B$   
Kinematics according to CS dipole terms NPB485(1997)291, NPB627(2002)189




# Assembling the local $K$ -factor

Extra emission generator (EEG)  
with multi-channeling over all  
dipole configurations

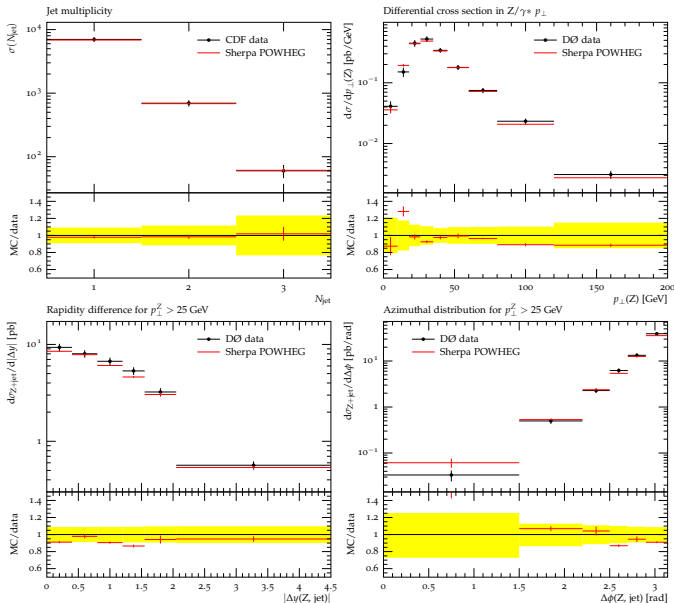


Separate integration (sep)  
of Born and real-emission kinematics  
with (modified) standard integrator

Process	$\sigma$ [pb] (EEG)	$\sigma$ [pb] (sep)	$\sigma$ [pb] (LO)
$e^+e^- \rightarrow 2jets$ $E_{\text{cms}}=91.2$ GeV	29449(19)	29454(18)	28381(18)
$e^+e^- \rightarrow 3jets$ as above, $y_{\text{cut}} = 10^{-1.92}$	9399(38)	9418(60)	7724(21)
$e^+e^- \rightarrow 4jets$ as above, $y_{\text{cut}} = 10^{-1.92}$	1377(14)	1357(21)	907(10)
$p\bar{p} \rightarrow e^- \bar{\nu}_e$ $E_{\text{cms}} = 1.96$ TeV, CTEQ 6.6	1331.7(5)	1332.2(4)	1098.6(3)
$p\bar{p} \rightarrow e^- \bar{\nu}_e + jet$ as above, $k_T = 10$ GeV, $D = 0.7$	389.0(16)	390.6(17)	282.9(5)
$p\bar{p} \rightarrow e^- \bar{\nu}_e + 2jets$ as above, $k_T = 10$ GeV, $D = 0.7$	104.2(7)	105.5(9)	73.9(2)

500k MC-points before cuts, no time limit, no error target  
virtual part delivered by BlackHat PRD78(2008)036003 

# Results for $Z+\text{jet}$



Preliminary

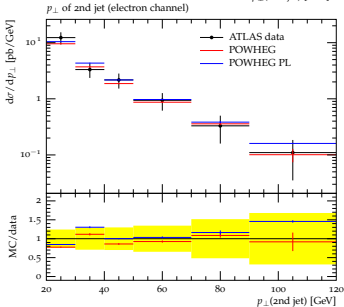
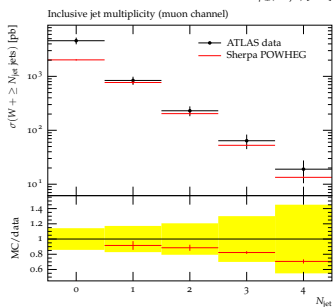
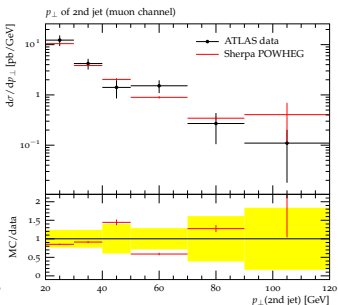
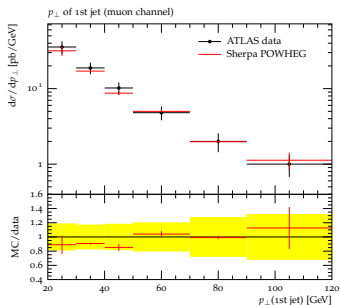
SHERPA POWHEG  
vs. Tevatron data

[CDF] PRL100(2008)102001

[DØ] PLB669(2008)278

[DØ] PLB682(2010)370

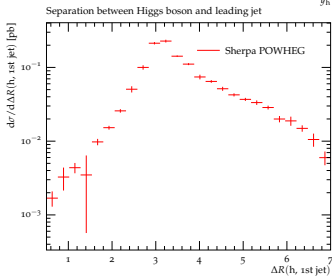
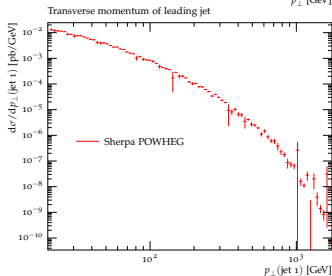
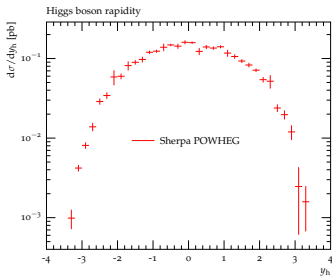
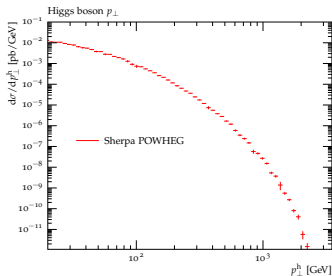
# Results for $W+\text{jet}$



Preliminary

SHERPA POWHEG  
vs. LHC data

[Atlas] PLB698(2011)325



**Preliminary**

SHERPA POWHEG  
 Loop-ME: MCFM  
 [Campbell, Ellis, Williams]

## Status quo

- First “non-trivial” POWHEG processes available in SHERPA
- General, automated algorithm, only virtual ME to be provided
- Precise predictions for Tevatron and LHC

## What's next?

- Apply to more processes
- Merge with higher multiplicity through MENLOPS
- Merge with lower multiplicity POWHEG

## More and more higher-order pQCD built into MC

Computing jet- $p_T$  spectra at NLO through MC feasible