Heavy quark production in association with a W boson

#### Keith Ellis, Fermilab

Loopfest, May 12, 2011 Work done with Simon Badger and John Campbell, arXiv:10116647 and work in progress with John Campbell and Fabrizio Caola.

# Importance of Wbb as a background



\* Final state produced from a number of important partonic processes.



# Single top Wbb background



# Higgs Wbb background

\* Many different analyses with different b-jet tagging requirements.

#### double SVT tag

#### single SVT tag

Niet	Diet	Niet	1 iet	2iet
Njet	Zjet	Ducto a Eventa	622204	100022
Pretag Events	145714	Pretag Events	033324	128033
Mistag	$28.6 \pm 8.5$	Mistag	$2551.7 \pm 468.3$	$1204.3 \pm 276.9$
$Wb\overline{b}$	$154.6\pm50.0$	Wbb	$1660.2 \pm 490.3$	$908.1 \pm 286.5$
$Wc\bar{c}/c$	$12.3\pm4.6$	$Wc\bar{c}/c$	$2826.2\pm1190.6$	$1117.7 \pm 490.0$
$t\bar{t}$	$92.5\pm13.0$	tt	$44.6 \pm 5.4$	$293.1\pm37.2$
Single top(s-ch)	$33.6 \pm 4.2$	Single top(s-ch)	$28.8 \pm 3.0$	$70.0 \pm 7.9$
Single top(t-ch)	$10.3 \pm 1.2$	Single top(t-ch)	$84.2 \pm 8.1$	$139.6\pm14.8$
WW	$0.8\pm0.1$	WW	$41.4 \pm 4.0$	$113.6\pm12.0$
WZ	$9.3 \pm 1.2$	WZ	$20.6 \pm 2.4$	$34.2 \pm 4.1$
ZZ	$0.4\pm0.1$	ZZ	$0.5\pm0.1$	$1.4 \pm 0.2$
Z + jets	$5.3\pm0.8$	Z + jets	$87.9 \pm 11.5$	$87.1 \pm 12.1$
nonW QCD	$23.1\pm10.9$	nonW QCD	$871.2 \pm 348.5$	$749.6 \pm 299.9$
Total background	$370.6 \pm 67.2$	Total background	$8217.3 \pm 2177.5$	$4719.0\pm1098.8$
WH/ZH signal (115GeV)	4.7	WH/ZH signal (115GeV)	Control region	9.6
Observed Events	334	Observed Events	7851	4431

**CDF note 10239** 

- # Just like single top, W+heavy flavor half the total background.
- \* Relaxing a tag increases signal but background grows enormously.

# Double tag sample

**★** Tagging both b-quarks  $\rightarrow$  above a given p<sub>T</sub>, not too close to beam, well-separated.

\* In this region can treat b-quarks as massless, expect corrections of order  $m_b^2/p_T^2$ .



\* Next-to-leading order (NLO) corrections originally computed in this approximation. Ellis, Veseli, hep-ph/9810489

# Single b-tag

\* If only one b-quark is explicitly tagged this is no longer sufficient.



\* Cross section for massless theory no longer finite in these regions.

$$(p_b + p_{\overline{b}})^2 \to 0 \qquad (p_b + p_{\overline{b}})^2 > 4m_b^2$$
  
massless massive

# First calculation with massive quarks

Massive calculation at NLO for an on-shell W boson. Febres Cordero, Reina, Wackeroth, hep-ph/0606102



#### Comments

The Febres-Cordero et al calculation uses a traditional diagrammatic approach, via Passarino-Veltman integral reduction

- \* no explicit analytic results, no public code.
- **\*** Unitarity methods now the tool of choice for NLO calculations.

# directly compute coefficients of a set of basis integrals.



\* well suited to numerical approaches  $\rightarrow$  new results for  $2\rightarrow 4$ ,  $2\rightarrow 5$ , ... processes.

- \* or, obtain compact analytic results analytically.
- Analytic results greatly simplified by treating massless particles.
  - \* few results available for calculations involving massive quarks.
- **\*** Test-case for analytic unitarity for massive particles.

Badger, Campbell, RKE, arXiv:1011.6647

# Spinor helicity formalism

**\*** Basic spinor notation for massless momenta  $k_i$  and  $k_j$ :

$$egin{array}{rl} |i
angle &= |i+
angle = u_+(k_i), \; |i] \; = |i-
angle = u_-(k_i) \; , \ \langle i| \; = \langle i-| = ar u_-(k_i), \; [i| \; = \langle i+| = ar u_+(k_i) \; . \end{array}$$

Spinor products:

\* Spinor products are square roots of dot products, up to a phase:

$$\langle i j \rangle \ [j i] = 2k_i \cdot k_j = s_{ij}$$
.

\* This language is the natural one for amplitudes in gauge theory.

\* e.g. *n*-gluon MHV amplitude:

$$\mathcal{A}\left(1^{+}\cdots i^{-}\cdots j^{-}\cdots n^{+}\right) = \frac{\langle ij\rangle^{4}}{\langle 12\rangle\langle 23\rangle\cdots\langle (n-1)n\rangle\langle n1\rangle}$$

#### Massive particles

- \* Decompose massive momentum  $(p_i)$  into two massless momenta  $(k_i)$ .
- \* Particularly convenient choice for two particles of equal mass:

$$p_2^{\mu} = rac{1+eta}{2}k_2^{\mu} + rac{1-eta}{2}k_3^{\mu} , \qquad eta = \sqrt{1-4m^2/s_{23}} , \qquad eta_{\pm} = rac{1}{2}(1\pmeta) \ p_3^{\mu} = rac{1+eta}{2}k_3^{\mu} + rac{1-eta}{2}k_2^{\mu} , \qquad p_2+p_3 = k_2+k_3 \qquad ext{Rodrigo, hep-ph/0508138}$$

\* Decompose massive spinors similarly, ensuring usual result for sum over polarizations:

$$\sum_{s=\pm} u_s(p,m)\bar{u}_s(p,m) = \not p + m$$

**\*** Solutions are:

$$\bar{u}_{\pm}(p_3,m) = \frac{\beta_{\pm}^{-1/2}}{\langle 2^{\mp}|3^{\pm} \rangle} \langle 2^{\mp}| (\not p_3 + m) , \qquad v_{\pm}(p_2,m) = \frac{\beta_{\pm}^{-1/2}}{\langle 2^{\mp}|3^{\pm} \rangle} (\not p_2 - m) |3^{\pm} \rangle$$
Not real helicities
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#### Fermionic currents

\* Easy to compute basic current from these spinors.

$$\begin{split} S^{\mu}(3_{Q}^{\pm}, 2_{\bar{Q}}^{\mp}) &= \bar{u}_{\pm}(p_{3}, m)\gamma^{\mu}v_{\mp}(p_{2}, m) & \text{helicity conserving} \\ &= \bar{u}_{\pm}(k_{3})\gamma^{\mu}v_{\mp}(k_{2}) & \text{current the same} \\ &= \langle 3^{\pm}|\gamma^{\mu}|2^{\pm}\rangle & \text{one, with } p_{i} \rightarrow k_{i} \end{split}$$

$$S^{\mu}(3^{-}_{Q}, 2^{-}_{\bar{Q}}) = \bar{u}_{-}(p_{3}, m)\gamma^{\mu}v_{-}(p_{2}, m) = 2\mathcal{N}_{--}(k_{2} - k_{3})^{\mu}$$
$$S^{\mu}(3^{+}_{Q}, 2^{+}_{\bar{Q}}) = \bar{u}_{+}(p_{3}, m)\gamma^{\mu}v_{+}(p_{2}, m) = 2\mathcal{N}_{++}(k_{2} - k_{3})^{\mu}$$

$$\mathcal{N}_{--} = \frac{m}{[2\,3]}, \quad \mathcal{N}_{++} = \frac{m}{\langle 2\,3 \rangle}$$
 these two currents differ only by overall phase

# Tree amplitudes



 $0 \to q(k_1) + \bar{Q}(p_2) + Q(p_3) + \bar{q}(k_4) + \bar{\ell}(k_5) + \ell(k_6)$ 

\* Simple current in the tree diagrams  $\rightarrow$  two amplitudes same as massless case.

$$\begin{split} -iA_{6}^{\text{tree}}(1_{q}^{-}, 2_{\bar{Q}}^{+}, 3_{Q}^{-}, 4_{\bar{q}}^{+}, 5_{\bar{\ell}}^{+}, 6_{\ell}^{-}) &= \left[\frac{\langle 1\,3\rangle\,[4\,5]\,\langle 6|(1+3)|2]}{s_{23}s_{56}s_{123}} - \frac{[4\,2]\,\langle 1\,6\rangle\,[5|(2+4)|3\rangle}{s_{23}s_{56}s_{234}}\right] \\ &\equiv \left[\frac{\langle 1\,3\rangle\,[4\,5]\,\langle 6|(1+3)|2]}{s_{23}s_{56}s_{123}} - \text{flip}\right]\,,\end{split}$$

symmetry operation: flip:  $(1 \leftrightarrow 4)$ ,  $(2 \leftrightarrow 3)$ ,  $(5 \leftrightarrow 6)$ ,  $[] \leftrightarrow \langle \rangle$ 

Bern, Dixon, Kosower, hep-ph/9708239

# Tree amplitudes

\* Amplitude not present in massless limit:  $\mathcal{N}_{--} = \frac{m}{[23]}, \quad \mathcal{N}_{++} = \frac{m}{\langle 23 \rangle}$ 

$$- iA_{6}^{\text{tree}}(1_{q}^{-}, 2_{\bar{Q}}^{-}, 3_{Q}^{-}, 4_{\bar{q}}^{+}, 5_{\bar{\ell}}^{+}, 6_{\ell}^{-}) = \mathcal{N}_{--} \\ \times \left[ [45] \left\{ \frac{\langle 6|(1+2)|3] \langle 31 \rangle - \langle 6|(1+3)|2] \langle 21 \rangle}{s_{23}s_{56}s_{123}} \right\} + \text{flip} \right]$$

**\*** Other amplitude obtained by replacing  $\mathcal{N}_{--}$  by  $\mathcal{N}_{++}$ .

\* equivalently, the two are related by the symmetry:

$$A_6^{\rm tree}(1^-_q,2^+_{\bar{Q}},3^+_Q,4^+_{\bar{q}},5^+_{\bar{\ell}},6^-_\ell) \ = \ -{\rm flip}\left[A_6^{\rm tree}(1^-_q,2^-_{\bar{Q}},3^-_Q,4^+_{\bar{q}},5^+_{\bar{\ell}},6^-_\ell)\right]$$

\* A similar relation holds for part of the 1-loop amplitudes;

\* only need to calculate 3 instead of 4 combinations of spin labels.

#### Color decomposition

- \* Amplitudes are color-stripped.
  - \* trivial at tree-level:

$$\mathcal{A}_{6}^{\text{tree}}(1_{q}, 2_{\bar{Q}}, 3_{Q}, 4_{\bar{q}}) = g_{W}^{2} g^{2} \mathcal{P}_{W}(s_{56}) A_{6}^{\text{tree}}(1_{q}, 2_{\bar{Q}}, 3_{Q}, 4_{\bar{q}}) \left(\delta_{j_{1}}^{\,\bar{j}_{2}} \,\delta_{j_{3}}^{\,\bar{j}_{4}} - \frac{1}{N_{c}} \delta_{j_{1}}^{\,\bar{j}_{4}} \,\delta_{j_{3}}^{\,\bar{j}_{2}}\right)$$

Breit-Wigner factor: 
$$\mathcal{P}_W(s) = \frac{s}{s - M_W^2 + i \Gamma_W M_W}$$

two structures at 1-loop:

$$\mathcal{A}_{6}^{1-\text{loop}}(1_{q}, 2_{\bar{Q}}, 3_{Q}, 4_{\bar{q}}) = g_{W}^{2} c_{\Gamma} g^{4} \mathcal{P}_{W}(s_{56})$$

$$\times \left[ N_{c} \, \delta_{j_{1}}^{\,\bar{j}_{2}} \, \delta_{j_{3}}^{\,\bar{j}_{4}} \, A_{6;1}(1_{q}, 2_{\bar{Q}}, 3_{Q}, 4_{\bar{q}}) + \delta_{j_{1}}^{\,\bar{j}_{4}} \, \delta_{j_{3}}^{\,\bar{j}_{2}} \, A_{6;2}(1_{q}, 2_{\bar{Q}}, 3_{Q}, 4_{\bar{q}}) \right]$$

\* Only one structure actually enters at NLO.

$$\sum_{\text{colours}} [\mathcal{A}_6^* \mathcal{A}_6]_{\text{NLO}} = 2g_W^4 c_{\Gamma} g^6 (N_c^2 - 1) N_c |\mathcal{P}_W(s_{56})|^2 \\ \times \text{Re} \Big\{ [A_6^{\text{tree}}(1_q, 2, 3, 4_{\bar{q}})]^* A_{6;1}(1_q, 2, 3, 4_{\bar{q}}) \Big\}$$

### Primitive amplitudes

\* Further decomposition into gauge invariant primitive amplitudes.

$$\begin{aligned} A_{6;1}(1_q, 2_{\bar{Q}}, 3_Q, 4_{\bar{q}}) &= A_6^{\rm lc}(1, 2, 3, 4) - \frac{2}{N^2} (A_6^{\rm cb}(1, 2, 3, 4) + A_6^{\rm lc}(1, 2, 3, 4)) \\ &- \frac{1}{N^2} A_6^{\rm sl}(1, 2, 3, 4) - \frac{n_{\rm lf}}{N} A_6^{\rm lf}(1, 2, 3, 4) - \frac{n_{\rm hf}}{N} A_6^{\rm hf}(1, 2, 3, 4) \end{aligned}$$

\* Other color structure not necessary here but made from same primitives:

$$\begin{aligned} A_{6;2}(1_q, 2_{\bar{Q}}, 3_Q, 4_{\bar{q}}) &= A_6^{\rm cb}(1, 2, 3, 4) + \frac{1}{N^2} (A_6^{\rm lc}(1, 2, 3, 4) + A_6^{\rm cb}(1, 2, 3, 4)) \\ &+ \frac{1}{N^2} A_6^{\rm sl}(1, 2, 3, 4) + \frac{n_{\rm lf}}{N} A_6^{\rm lf}(1, 2, 3, 4) + \frac{n_{\rm hf}}{N} A_6^{\rm hf}(1, 2, 3, 4) \end{aligned}$$

### Leading color and crossed box



\* New calculation required for A<sup>lc</sup>: use analytic unitarity methods.

\* Crossed box by symmetry:  $A_6^{cb}(1_q^{h_1}, 2_{\bar{Q}}^{h_2}, 3_Q^{h_3}, 4_{\bar{q}}^{h_4}) = -A_6^{lc}(1_q^{h_1}, 3_{\bar{Q}}^{h_3}, 2_Q^{h_2}, 4_{\bar{q}}^{h_4})$ 

# Subleading color



Subleading amplitude can be obtained from known massless result, Bern et al., hep-ph/9708239

trick: first two diagrams are just a heavy quark current, (equivalent to the massless current for two of the polarization choices). Other polarizations are also obtainable from this case.

 $\langle 3^{\pm}|\gamma^{\mu}|2^{\pm}\rangle \longrightarrow 2\mathcal{N}_{--}(k_2-k_3)^{\mu}$ 

\* last two diagrams are just massive vertex corrections.

Fermion loops



\* Simple to calculate.

\* Feynman diagrams as good as anything else.

# Examples (of the shortest!) coefficients

**\*** Box:

$$d_{1|2|3}(-,+) = \frac{s_{23}\langle 1|\mathbf{2}|1]}{2s_{123}} \left[ \frac{[2\,3]}{\langle 5\,6\rangle\,\langle 4|2+3|1]} \left( \frac{\langle 6|\mathbf{2}|1]}{\beta\,[1\,3]} + \frac{\langle 1\,6\rangle\,[1\,2]}{[2\,3]} \right)^2 - \frac{\langle 1\,3\rangle^2\,[4\,5]^2}{\langle 2\,3\rangle\,[5\,6]\,\langle 1|2+3|4]} \right]$$

**\*** Triangle:

$$\begin{split} c_{2|3}(-,+) &= \frac{1}{2s_{56}} \Biggl\{ \frac{[4\,5]}{s_{123}} \Bigl( -\frac{\beta_{-} \langle 1\,3 \rangle^{2} \langle 6|(1+2)|3]}{\langle 1\,2 \rangle} - \frac{\beta_{+}^{2} \langle 1\,2 \rangle [1\,2]^{2} \langle 6|(1+2)|3]}{\beta [1\,3]^{2}} \\ &+ \frac{\beta_{+}\beta_{-} \langle 1\,2 \rangle [1\,2] \langle 6|(1+3)|2]}{\beta [1\,3]} - \frac{\beta_{+}\beta_{-} \langle 1\,3 \rangle [1\,2] \langle 6|(1+2)|3]}{\beta [1\,3]} \\ &+ \frac{(\beta^{2} - \beta_{-}\beta_{+}\beta + 4\beta_{-}\beta_{+}^{2})}{\beta} \langle 1\,3 \rangle \langle 6|(1+3)|2] \Bigr) \\ &+ \frac{\beta_{-} \langle 1\,3 \rangle \langle 1\,6 \rangle [4\,5]}{\langle 1\,2 \rangle} - \frac{\beta_{+}^{2} \langle 2\,3 \rangle \langle 4|(2+3)|5] \langle 6|(1+2)|3]}{\beta \langle 2\,4 \rangle^{2} [1\,3]} + \frac{\beta_{+} \langle 3|(2+4)|5] \langle 6|(1+2)|3]}{\langle 2\,4 \rangle [1\,3]} \\ &- \text{flip} \Biggr\} \,. \end{split}$$

\* Relatively compact, but not short enough to publish the full amplitude.

# MCFM and checks

Amplitudes implemented in NLO code, MCFM v6.0 (May 2011)

 include also real corrections and ensure cancellation of singularities.



- Three levels of checks:
  - compare amplitudes with results of numerical implementation of D-dimensional unitarity, for a small set of phase space points.
  - \* check implementation of IR cancellation by changing extent of singular regions subtracted ("α-parameters").
  - comparison of final results for cross sections without the W decay, with the earlier calculation of Febres-Cordero et al.

# MCFM advert

**\*** MCFM represents a unified approach to NLO corrections.

http://mcfm.fnal.gov (v6.0, May 2011)

J. M. Campbell, R. K. Ellis (main authors) with celebrity appearances by R. Frederix, F. Maltoni, T. Melia, K.Melnikov, R. Rontsch, F. Tramontano, S. Willenbrock, C. Williams, G. Zanderighi

#### \* Next-to-leading order parton-level predictions.

- \* Cross sections and differential distributions.
- \* Standard Model processes involving vector boson+jets, top quarks, Higgs.
- \* Decays of unstable particles are included, maintaining spin correlations.
- **\*** Helicity amplitudes calculated from scratch or taken from the literature.
- \* Slightly-modified implementation of Catani-Seymour dipole subtraction.

# Differential distribution

Example: quantity that may now be computed at NLO, separation between lepton and nearest jet,



# Updated study for comparison with CDF

- Use 4-flavour calculation only.
  - \* no confusing separation of contributions.
  - \* no large gluon flux, so difference between 4- and 5-flavor schemes small.

**\*** with the CDF cuts, we find:

$$\frac{\sigma_{gq} (Wb+X)}{\sigma_{total} (Wb+X)} = 0.06$$

**\*** Other changes:

- newer PDF set (NNPDF2.1), previously CTEQ6.
- \* NLO calculation includes W decay products (previously estimated via LO).
- \* "well-isolated lepton" cut,  $\Delta R(lepton, jet) > 0.4$  (no such cut before).
- \* central scale choice  $\mu_R = \mu_F = M_W + 2m_b$  (previously M<sub>W</sub>).

#### Results

#### \* Cross-sections in picobarns

★ both W<sup>+</sup> and W<sup>-</sup> included, but decay into one flavor of lepton (same as CDF).

# of jets	1 jet		2 jets		
jet identities	b	(bb)	bj	(bb)j	bb
LO	0.430	0.105	I	-	0.162
NLO	0.582	0.130	0.090	0.030	0.150

**\*** NLO prediction:

$$\sigma_{\text{event}}$$
 (Wb) = 0.982 pb  
 $\sigma_{\text{b-jet}}$  (Wb) = 1.132 pb

(sum of NLO line) (include bb twice, per CDF)

#### Comments

- \* LO (b-jet) prediction: 0.86 pb (c.f. ALPGEN 0.78 pb).
- \* NLO slightly lower than before (compare with 1.22 pb quoted in note for CDF Campbell, Febres Cordero, Reina, unpublished)
  - \* this result is 4F only and different pdf set (few % changes in  $\alpha_s$  and valence u,d).
  - \* no significant difference when including W decays at NLO.
- Breakdown of contributions:





### Uncertainty estimates

\* Consider three sources of uncertainty on the W+b-jet cross section.

**\*** scale variation by a factor of two  $\rightarrow$  uncertainty ~ 14%.

 $\sigma_{b-jet}$  (Wb) = 1.132 + 0.156 - 0.145 pb

\* pdf variation (NNPDF prescription)  $\rightarrow$  uncertainty ~ 3%.

$$\sigma_{\text{b-jet}}$$
 (Wb) = 1.132 + 0.031 - 0.031 pb

\* variation of b-mass in the range 4.2 - 4.7 (central) - 5 GeV  $\rightarrow$  uncertainty ~ 5%.

 $\sigma_{b-jet}$  (Wb) = 1.132 + 0.070 - 0.043 pb

Combined prediction:

$$0.913 < \sigma_{b-jet}$$
 (Wb) < 1.389 pb

# LHC cross sections

$\sqrt{s}$		$7 { m TeV}$	8 TeV	14 TeV	
$W^+b\overline{b}$	LO	4.47	5.17	9.05	
	NLO	8.68	10.6	23.5	
$W^-b\overline{b}$	LO	2.59	3.11	6.27	
	NLO	5.06	6.36	15.6	

Cross section (pb) - no W decay

$$p_T^b > 25 \text{ GeV}$$

 $|\eta^b| < 2.5$ 

#### \* NLO corrections are very large.

- Iarge contribution from gluon pdf that is absent at LO.
- theoretical uncertainty still significant.



# LHC study

\* Use cuts from planned ATLAS measurement.

- **\*** jet definition:  $p_T > 25$  GeV,  $|\eta| < 2.5$ , anti- $k_T$  algorithm, D=0.4.
- no lepton cuts.
- ✤ work at 7 TeV.
- Use 4-flavor scheme again.

more susceptible to 4F/5F difference now:

$$\frac{\sigma_{gq}(W^+b+X)}{\sigma_{total}(W^+b+X)} = 0.41$$

Predicted NLO cross-section (W<sup>+</sup> only, one flavor of lepton):

$$\sigma_{event}$$
 (W+b) = 7.06 pb

# Composition: Tevatron vs. LHC

# 🔵 b 🌑 (bb) 🛑 bj 🛑 (bb)j 🌑 bb



**Tevatron** 



# LHC (7 TeV)

### Summary

\* First calculation of Wbb with massive b-quarks including correlations in W decay.

- \* computed using analytic unitarity, seldom used outside massless contexts.
- \* used special momentum decomposition, recycled some massless results.
- \* slight extension of standard methods to handle massive propagator.
- Results included in current version of MCFM
- At the Tevatron allows calculation of Wb,Wbb,W(bb),Wbj,Wbbj with complicated mix and match procedure.

**CDF, arXiv: 0909.1505** 

CDF	<b>2.74</b> ±0.27 (stat) ±0.42 (syst) pb
ALPGEN	<b>0.78</b> pb
PYTHIA	<b>1.10</b> pb
NLO	<b>0.913&lt; σ&lt;1.398</b> pb

Tevatron W+b-jet cross section